AC loss in superconductors

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Some useful formulas:

**magnetic moment** of a current loop

\[ \vec{m} = I\vec{S} \quad [\text{Am}^2] \]

**magnetization** of a sample

\[ \vec{M} = \frac{\sum \vec{m}}{V} \quad [\text{A/m}] \]

alternative (preferred in SC community)

\[ \vec{M} = \mu_0 \frac{\sum \vec{m}}{V} \quad [\text{T}] \]

Measurable quantities:

**magnetic field** \( B \) [T] – *Hall probe, NMR*

**voltage** from a pick-up coil [V]

\[ u_i = -\frac{d\Psi}{dt} \approx -N \frac{d\Phi}{dt} = -NS \frac{d\bar{B}}{dt} \]

linked magnetic flux  \quad number of turns  \quad area of single turn
Outline:

1. Hard superconductor in varying magnetic field

2. Magnetization currents: Flux pinning  Coupling currents

3. Possibilities for reduction of magnetization currents

4. Methods to measure magnetization and AC loss
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Superconductors used in magnets - what is essential?

- type II. superconductor (critical field)
- high transport current density
Superconductors used in magnets - what is essential?

Type II. superconductor (critical field)

Mechanism(s) hindering the change of magnetic field distribution

=> pinning of magnetic flux = hard superconductor

\[ \vec{F}_L = \vec{j} \times \vec{B} \]

Gradient in the flux density

\[ \frac{\partial B_z}{\partial x} = -\mu_0 j_y \]

Pinning of flux quanta

Distribution persists in static regime (DC field), but would require a work to be changed

=> dissipation in dynamic regime
(repulsive) interaction of flux quanta
=> flux line lattice

summation of microscopic pinning forces
+ elasticity of the flux line lattice
= macroscopic pinning force density \( F_p \) [N/m\(^3\)]

macroscopic behavior described by the critical state model [Bean 1964]:

\[
\Phi_0 = 2 \times 10^{-15} \text{ Vs}
\]
\[
B = \frac{\Phi_0}{a^2}
\]
\[
a = \sqrt{\frac{\Phi_0}{B}}
\]
\[
B = 1 \text{ T; } a = 45 \text{ nm}
\]

**local density of electrical current in hard superconductor is either 0 in the places that have not experienced any electric field or it is the critical current density, \( j_c \), elsewhere**

in the simplest version (first approximation) \( j_c = \text{const.} \)
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Transport of electrical current

e.g. the critical current measurement

\begin{array}{ccc}
0 \text{ A} & 20 \text{ A} & 100 \text{ A} \\
80 \text{ A} & 20 \text{ A} & 0 \text{ A} \\
\end{array}
Transport of electrical current

e.g. the critical current measurement

\[ j = 0 \]

\[ j = + j_c \]
Transport of electrical current

e.g. the critical current measurement

0 A

20 A

80 A

100 A
Transport of electrical current

e.g. the critical current measurement

0 A

20 A

100 A

80 A

20 A

0 A
Transport of electrical current

e.g. the critical current measurement

\[ j = +j_c \]

\[ j = 0 \]

\[ j = -j_c \]
Transport of electrical current

e.g. the critical current measurement

\[ j = + j_c \]

\[ j = 0 \]

\[ j = - j_c \]

\[ I_p \]

persistent magnetization current
Transport of electrical current

AC cycle with $I_a$ less than $I_c$: neutral zone

$80 \rightarrow 60 \rightarrow -80 \rightarrow -60 \rightarrow 0$ A

persistent magnetization current
AC transport in hard superconductor is not dissipation-less (AC loss)

\[ Q = \int_{T} I Ud t = -\int I d\Phi \]

neutral zone:
\[ j = 0,\ E = 0 \]

check for hysteresis in \( I \) vs. \( \Phi \) plot
AC transport loss in hard superconductor

hysteresis → dissipation → AC loss
Hard superconductor in changing magnetic field

0 → 30 → 50 → 40 mT

→ 0 → -50 → -40 → 0 → 50 mT
Hard superconductor in changing magnetic field

$0 \rightarrow 30 \rightarrow 50 \rightarrow 40 \text{ mT}$

$\rightarrow 0 \rightarrow -50 \rightarrow -40 \rightarrow 0 \rightarrow 50 \text{ mT}$
Hard superconductor in changing magnetic field

\[
0 \rightarrow 30 \rightarrow 50 \rightarrow 40 \text{ mT}
\]

\[
\rightarrow 0 \rightarrow -50 \rightarrow -40 \rightarrow 0 \rightarrow 50 \text{ mT}
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Hard superconductor in changing magnetic field

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$\rightarrow 0 \rightarrow -50 \rightarrow -40 \rightarrow 0 \rightarrow 50 \text{ mT}$
Hard superconductor in changing magnetic field
dissipation because of flux pinning

volume loss density \( Q \) [J/m\(^3\)]

\[
\frac{Q}{V} = \oint B_a \, dM
\]
magnetization:

\[
M = \frac{1}{S} \oint_S -x \cdot j(x, y) \, dx \, dy
\]

(2D geometry)
Round wire from hard superconductor in changing magnetic field

\[ M_s \] \textit{saturation magnetization}, \quad B_p \textit{penetration field}
Round wire from hard superconductor in changing magnetic field

estimation of AC loss at $B_a >> B_p$

$$\frac{Q}{V} \approx 4B_a M_s$$
(infinite) slab in parallel magnetic field – analytical solution

\[ B_p = \mu_0 j_c \frac{w}{2} \]

\[ M_s = j_c \frac{w}{4} = \frac{B_p}{2\mu_0} \]

\[ \frac{Q}{V} = \frac{1}{\mu_0} \left\{ \begin{array}{ll}
\frac{2}{3} \frac{B_a^3}{B_p} & \text{for } B_a < B_p \\
2B_p B_a - \frac{4}{3} \frac{B_p^2}{B_a} & \text{for } B_a > B_p
\end{array} \right\} \]

\[ Q \approx 4B_a M_s \]
Slab in parallel magnetic field – analytical solution

\[ Q \approx 4B_a M_s \]
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Two parallel superconducting wires in metallic matrix

in the case of a perfect coupling:

\[ B_a \]

0 \rightarrow 20 \rightarrow 80 \rightarrow 60 \text{ mT}
Magnetization of two parallel wires

![Diagram showing magnetization curves for coupled and uncoupled states.](image)
Magnetization of two parallel wires

how to reduce the coupling currents?
Composite wires – twisted filaments

\[ \bigotimes \dot{B} \]

- good interfaces: \( \rho_t = \rho_m \frac{1 - \lambda}{1 + \lambda} \)
- bad interfaces: \( \rho_t = \rho_m \frac{1 + \lambda}{1 - \lambda} \)

\[ \lambda = \frac{S_{SC}}{S_m} \]

\[ j_\perp = \frac{l_p \dot{B}}{2\pi \rho_t} \]
Composite wires – twisted filaments
coupling currents (partially) screen the applied field

\[ B_i = B_{\text{ext}} - \tau \dot{B} \]

\( \tau \) - time constant of the magnetic flux diffusion

\[ \tau = \frac{\mu_0}{2 \rho_t} \left( \frac{l_p}{2\pi} \right)^2 \]

in AC excitation

\[ \frac{Q}{V} = \frac{B_{\text{max}}^2}{\mu_0} \frac{2\pi \omega \tau}{1 + \omega^2 \tau^2} \]

shape factor 
(\( \sim \) aspect ratio)

\[ \frac{Q}{V} = \frac{B_{\text{max}}^2}{\mu_0} \frac{\chi_0 \pi \omega \tau}{1 + \omega^2 \tau^2} \]
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Persistent currents:

at large fields proportional to $B_p \sim j_c w$

= magnetization reduction by either lower $j_c$ or reduced $w$

lowering of $j_c$ would mean more superconducting material required to transport the same current

thus only plausible way is the reduction of $w$
effect of the field orientation

parallel field

perpendicular field

\[ \frac{Q}{V} \approx 4B_a M_s \]
Magnetization loss in strip with aspect ratio 1:1000

\[ \frac{Q}{V} \approx 4B_a M_s \]
in the case of flat wire or cable the orientation is not a free parameter

= reduction of the width

e.g. striation of CC tapes

\[ B \approx 6 \text{ times lower magnetization} \]
striation of CC tapes

but in operation the filaments are connected at magnet terminations

coupling currents will appear

=> transposition necessary
Coupling currents:

at low frequencies proportional to the time constant of magnetic flux diffusion

\[ \tau = \frac{\mu_0}{2\rho_i} \left( \frac{l_p}{2\pi} \right)^2 \]

= filaments (in single tape) or strands (in a cable) should be transposed

= low loss requires high inter-filament or inter-strand resistivity

*but good stability needs the opposite*
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Different methods necessary to investigate

- Wire (strand, tape)
- Cable
- Magnet

shape of the excitation field (current) pulse transition unipolar harmonic

relevant information can be achieved in harmonic regime

final testing necessary in actual regime
ideal magnetization loss measurement:
pick-up coil wrapped around the sample

induced voltage \( u_m(t) \) in one turn:

\[
  u_m(t) = -\frac{d\phi_m(t)}{dt} = -S \frac{d\bar{B}(t)}{dt}
\]

\[
  \bar{B}(t) = \frac{1}{S} \int_B B_{int}(t)dS = B_{ext}(t) + \mu_0 M(t)
\]

\[
  u_m(t) = -S \left[ \frac{dB_{ext}(t)}{dt} + \mu_0 \frac{dM(t)}{dt} \right]
\]

pick-up coil voltage processed by integration
either numerical or by an electronic integrator:

\[
  \mu_0 M(t) = -\frac{1}{S} \int u_m(t) dt - B_{ext}(t)
\]
Method 1: double pick-up coil system with an electronic integrator:
measuring coil, compensating coil

\[ \mu_0 M(t) = -\frac{1}{S} \int u_M(t) dt \]

AC loss in one magnetization cycle \([J/m^3]\):

\[ Q = \int B dM = \int_0^T B(t) \frac{dM}{dt} dt \]
Harmonic AC excitation – use of complex susceptibilities

\[ B_{\text{ext}}(t) = B_a \cos \omega t \]

\[ \mu_0 M(t) = B_a \sum_{n=1}^{\infty} \left( \chi_n' \cos n\omega t + \chi_n'' \sin n\omega t \right) \]

fundamental component \( n = 1 \)

AC loss per cycle
\[ W_q = -\pi \chi'' \frac{B_a^2}{\mu_0} \]

energy of magnetic shielding
\[ W_m = \chi' \frac{B_a^2}{2\mu_0} \]

Temperature dependence:
Method 2: Lock-in amplifier
– phase sensitive analysis of voltage signal spectrum in-phase and out-of-phase signals

\[ B_{\text{ext}} = B_a \cos \omega t \]

\[ U_{nS} = \frac{1}{\pi} \int_{0}^{2\pi} u_M(t) \sin n\omega t \, d\omega t \]

\[ U_{nC} = \frac{1}{\pi} \int_{0}^{2\pi} u_M(t) \cos n\omega t \, d\omega t \]

reference signal necessary to set the frequency phase taken from the current energizing the AC field coil

**Diagram:**
- \( B_{\text{ext}} \)
- \( u_m \)
- \( u_M \)
- \( u_c \)
- Lock-in amplifier
- Reference signal
- \( U_{nS} \)
- \( U_{nC} \)
Method 2: Lock-in amplifier – only at harmonic AC excitation

\[ B_{ext} = B_a \cos \omega t \]

\[ u_M(t) = S \omega B_a \left[ \sin \omega t + \sum_{n=1}^{\infty} n \left( \chi_n' \sin n\omega t - \chi_n'' \cos n\omega t \right) \right] \]

- empty coil
- sample magnetization

fundamental susceptibility

\[ \chi' = \frac{U_{1S}}{S \omega B_a} - 1 = \frac{U_{1S}}{U_N} - 1 \]

higher harmonic susceptibilities

\[ \chi_n' = \frac{U_{nS}}{n U_N} \]

\[ \chi_n'' = \frac{U_{nC}}{n U_N} \]
Real magnetization loss measurement:

Pick-up coil

Calibration necessary

\[ M = C \int u \, dt \]

by means of:

measurement on a sample with known properties

calibration coil

numerical calculation

…
AC loss can be determined from the balance of energy flows
Solution 1 - detection of power flow to the sample
Solution 2- elimination of parasitic power flows

AC power supply -> AC power flow -> AC loss in SC object
Loss measurement from the side of AC power supply:

\[ P_{sample} = I_m U_B \]
Loss measurement from the side of AC power supply:

\( \Psi(I) \) hysteresis loop registration for superconducting magnet (Wilson 1969)
Conclusions:

1) Hard superconductors in dynamic regime produce heat because of magnetic flux pinning -> transient loss, AC loss

2) Extent of dissipation is proportional to macroscopic magnetic moments of currents induced because of the magnetic field change

3) Hysteresis loss (current loop entirely within the superconductor) can be reduced by the reduction of superconductor dimension

4) Coupling loss (currents connecting parallel superconductors) reduced by the transposition (twisting) and the control of transverse resistance

5) Minimization of loss often in conflict with other requirements

6) Basic principles are known, particular cases require clever approach and innovative solutions