AC loss in superconductors

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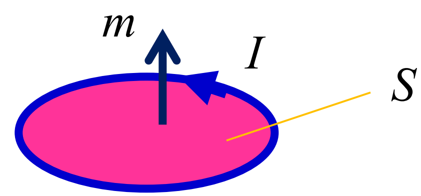
Some useful formulas:

magnetic moment of a current loop

$$\vec{m} = I\vec{S} \quad [\mathrm{Am}^2]$$

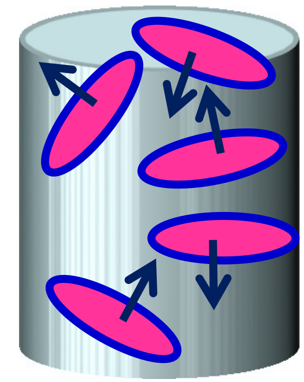


magnetization of a sample

$$\vec{M} = \frac{\sum \vec{m}}{V} \quad [\mathrm{A/m}]$$

alternative (preferred in SC community)

$$\boxed{\vec{M} = \mu_0 \frac{\sum \vec{m}}{V}} \quad \textbf{[T]}$$



Measurable quantities:

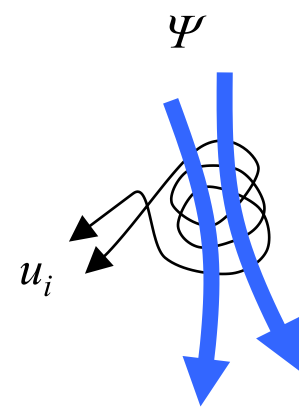
magnetic field B [T] – *Hall probe, NMR*

voltage from a pick-up coil [V]

average of magnetic field

$$u_i = -\frac{\mathrm{d}\Psi}{\mathrm{d}t} \approx -N\frac{\mathrm{d}\Phi}{\mathrm{d}t} = -NS\frac{\mathrm{d}\bar{B}}{\mathrm{d}t}$$

linked magnetic flux *number of turns* *area of single turn*



Outline:

1. Hard superconductor in varying magnetic field

2. Magnetization currents: Flux pinning
Coupling currents

3. Possibilities for reduction of magnetization currents

4. Methods to measure magnetization and AC loss

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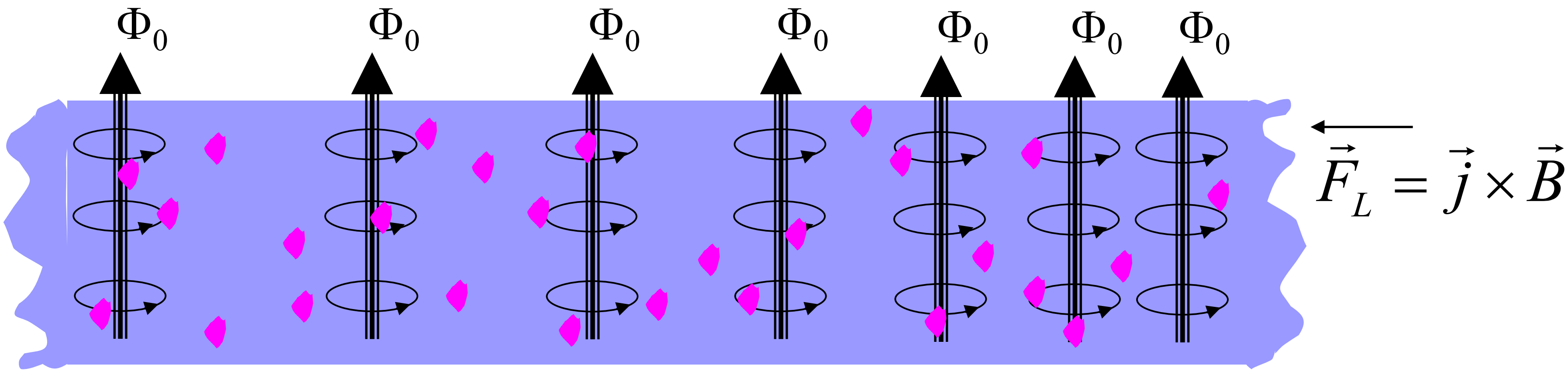
Superconductors used in magnets - what is essential?
type II. superconductor (critical field)
high transport current density

Superconductors used in magnets - what is essential?
type II. superconductor (critical field)
mechanism(s) hindering the change of magnetic field distribution
=> **pinning of magnetic flux** = hard superconductor



gradient in the flux density $$\frac{\partial B_z}{\partial x} = -\mu_0 j_y$$

pinning of flux quanta

distribution persists in static regime (DC field), but would require a work to be changed
=> dissipation in dynamic regime

(repulsive) interaction of flux quanta
=> flux line lattice

$$\Phi_0 = 2\times10^{-15}\ \mathrm{Vs}$$

$$\mathrm{B} = \frac{\Phi_0}{a^2}$$

$$a = \sqrt{\frac{\Phi_0}{B}}$$

summation of microscopic pinning forces
+ elasticity of the flux line lattice
= macroscopic pinning force density F_p [N/m^3]

$$B = 1\ \mathrm{T};$$

$$a = 45\ \mathrm{nm}$$

macroscopic behavior described by the critical state model [Bean 1964]:

local density of electrical current in hard superconductor is either 0 in the places that have not experienced any electric field or it is the critical current density, j_c, elsewhere

in the simplest version (first approximation) j_c=const.

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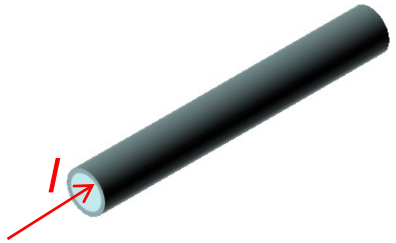
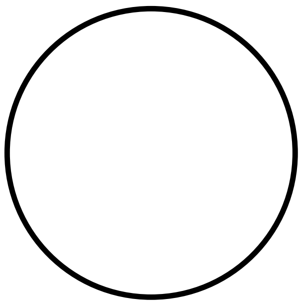
Transport of electrical current



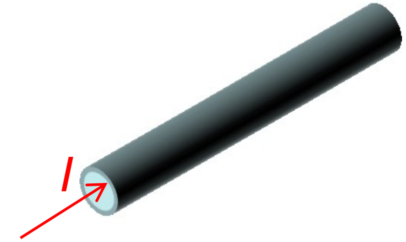
e.g. the critical current measurement

0 A 20 A 100 A



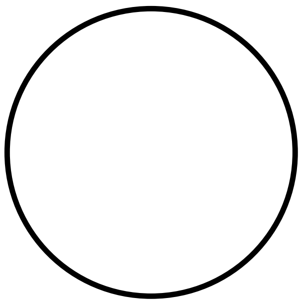
80 A 20 A 0 A

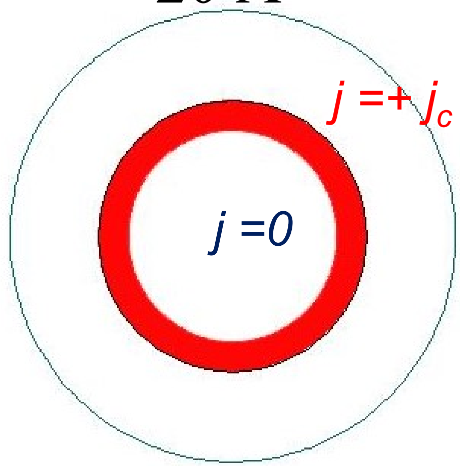
Transport of electrical current



e.g. the critical current measurement

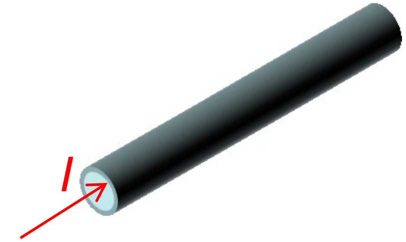
0 A | 20 A | 100 A





80 A | 20 A | 0 A

Transport of electrical current



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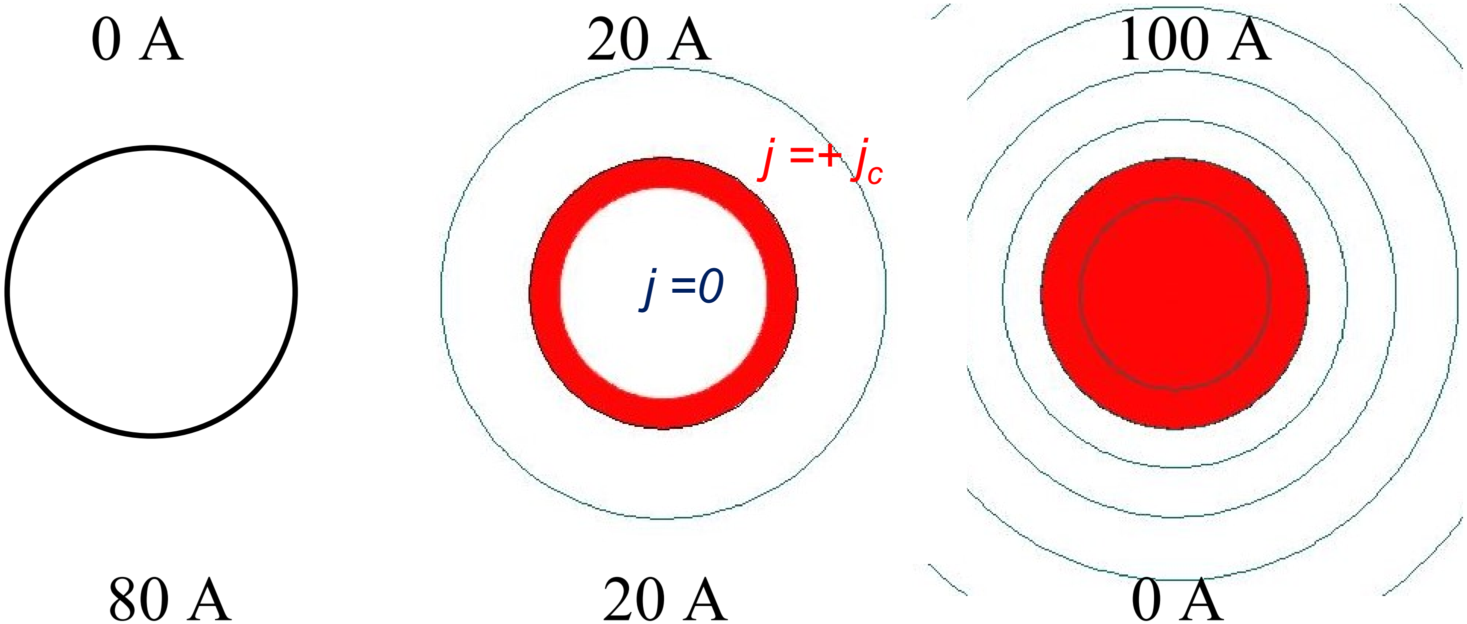
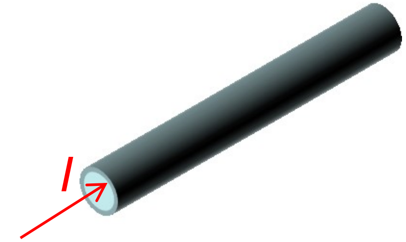


Transport of electrical current



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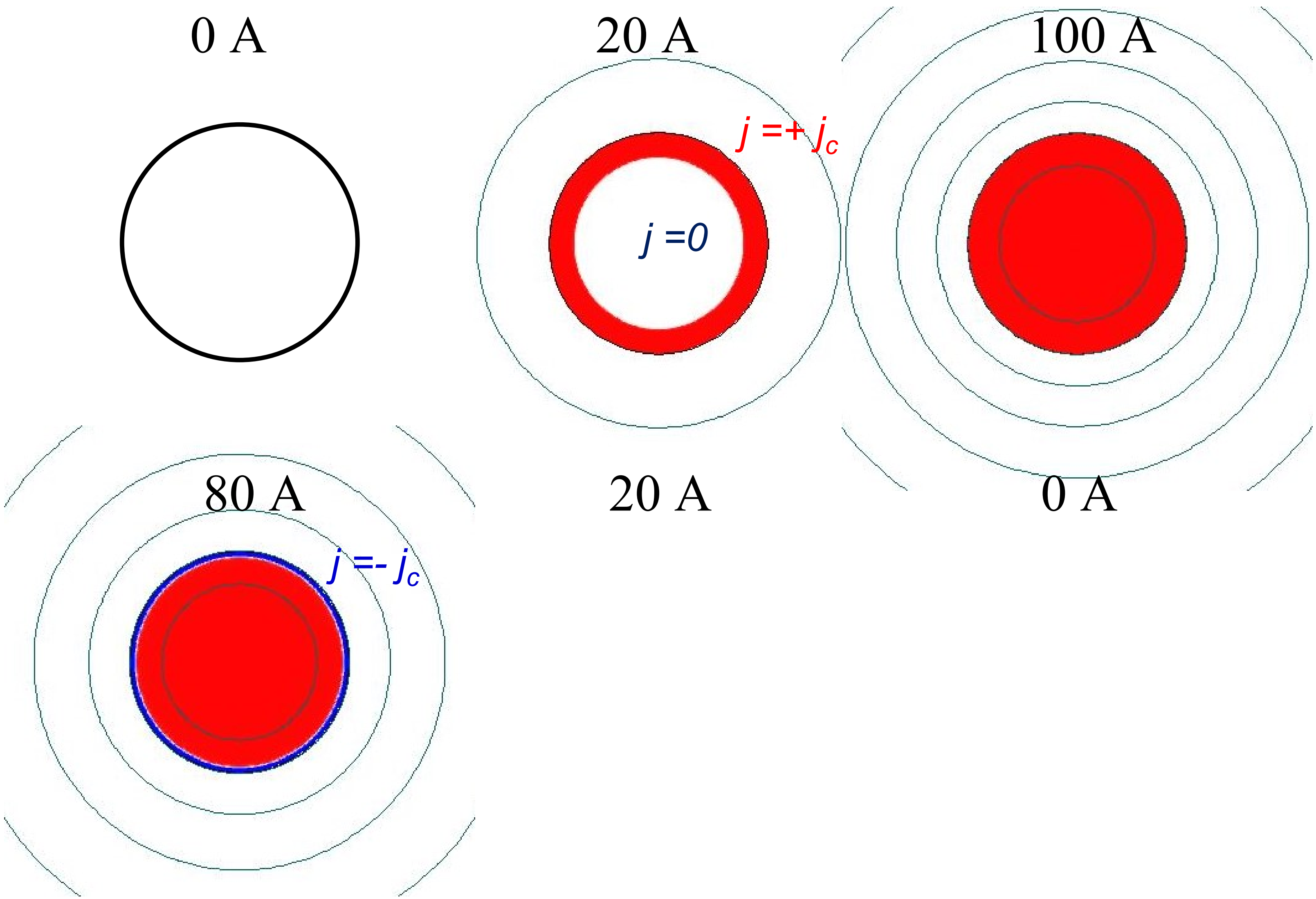
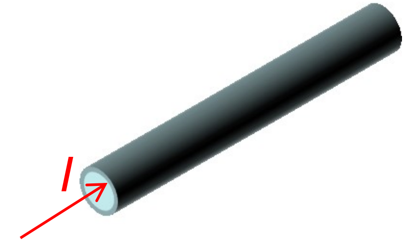
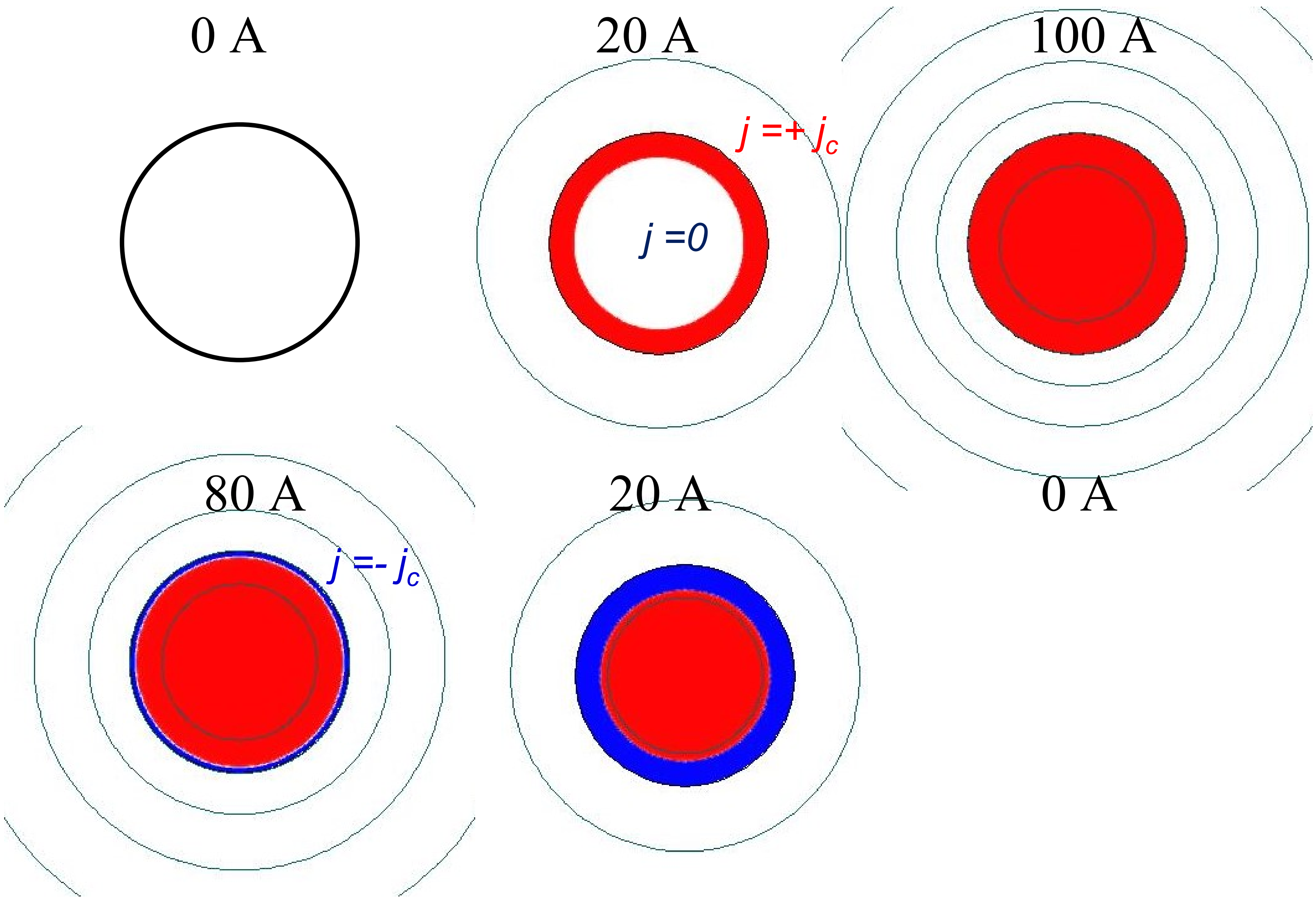


Transport of electrical current

e.g. the critical current measurement





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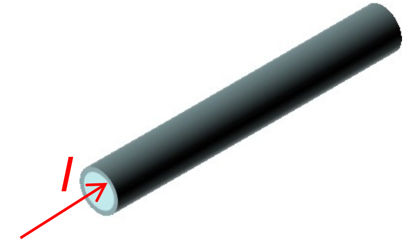
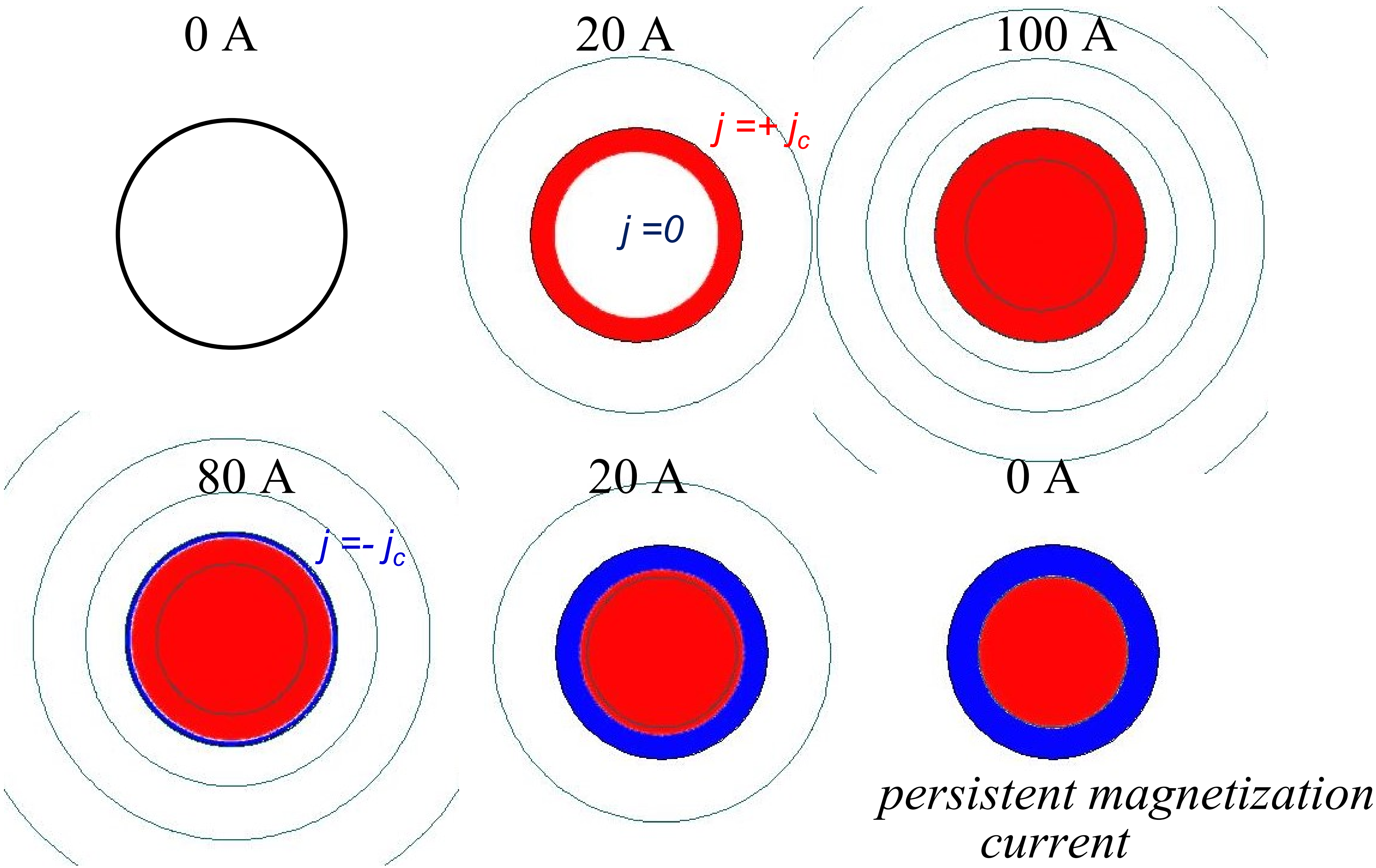
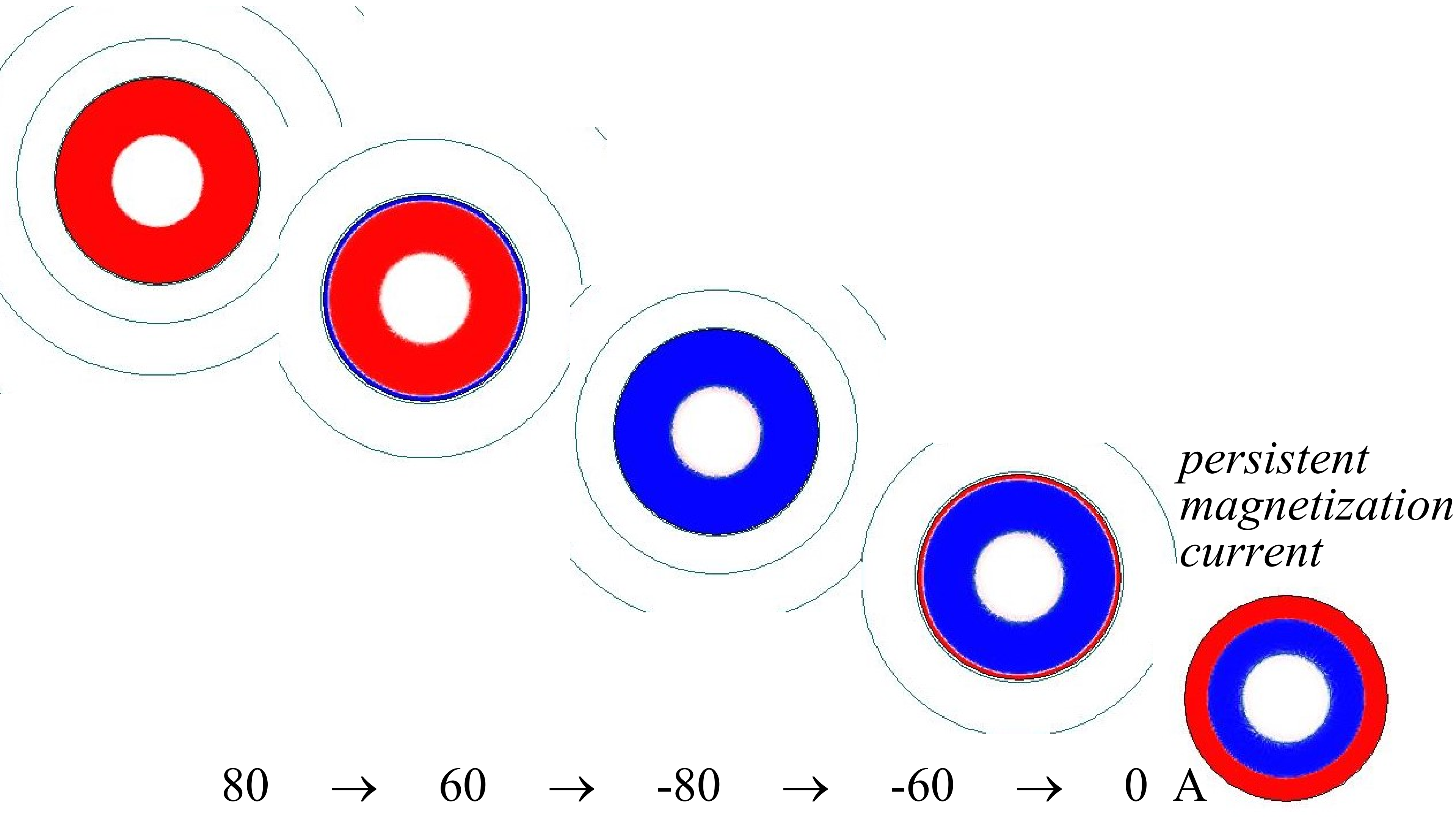


e.g. the critical current measurement



Transport of electrical current

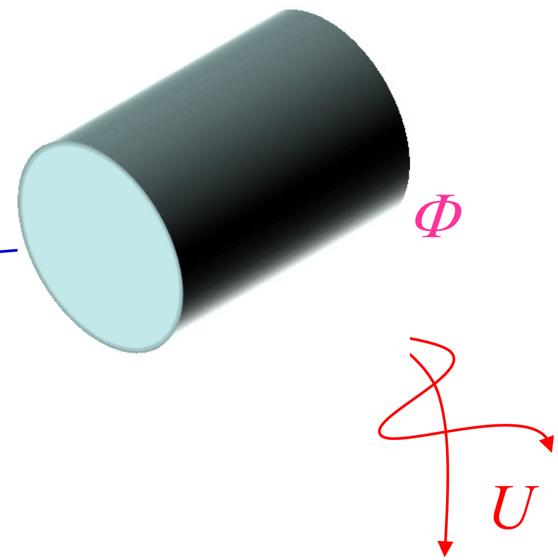
AC cycle with I_a less than I_c : *neutral zone*



AC transport in hard superconductor is not dissipation-less (AC loss)

$$Q = \int_T IU\,dt = -\int I\,d\Phi$$

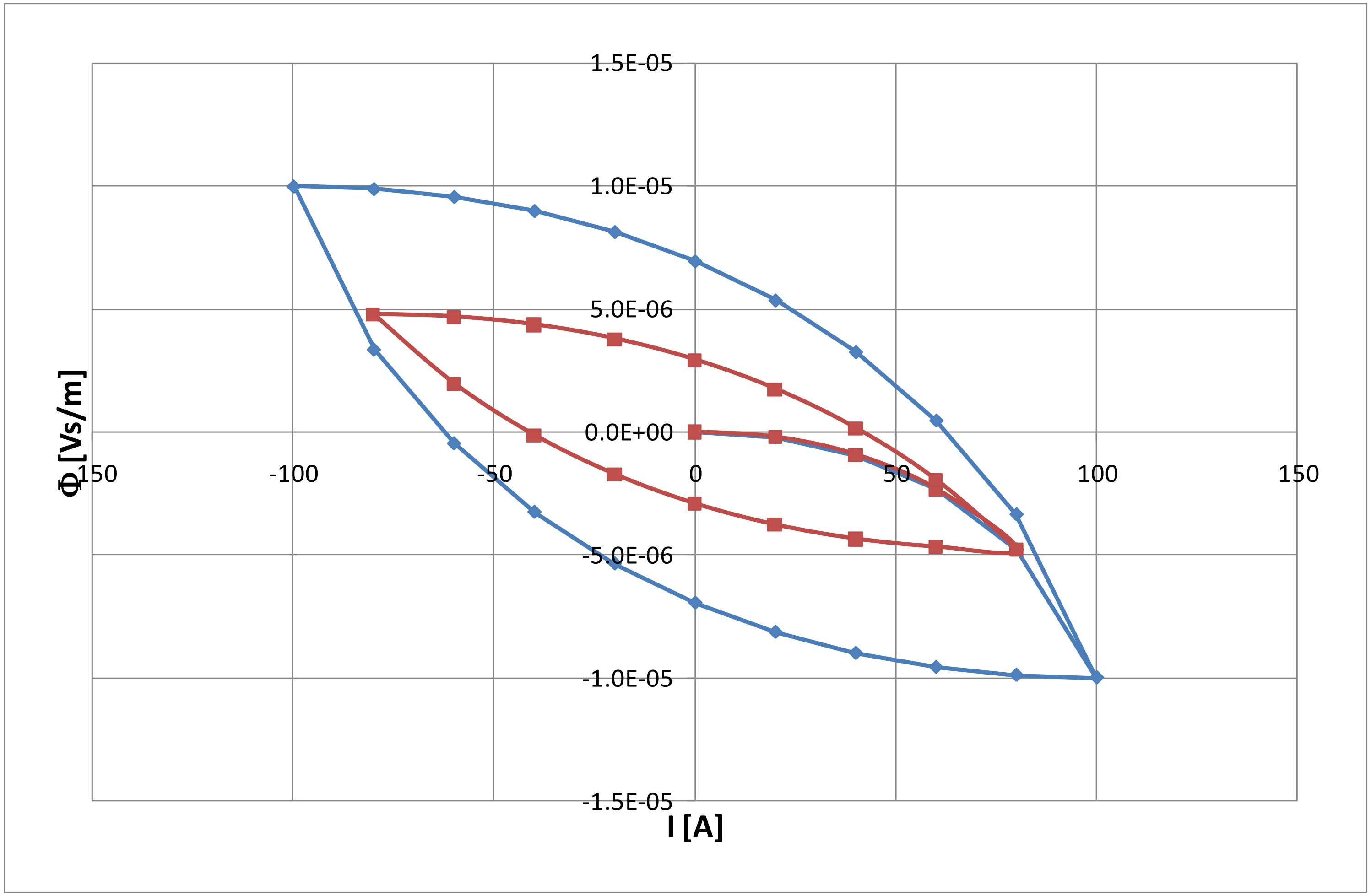
neutral zone:
$j = 0$, $E = 0$



$$U = -\frac{\partial \Phi}{\partial t}$$

check for hysteresis in I *vs.* Φ plot

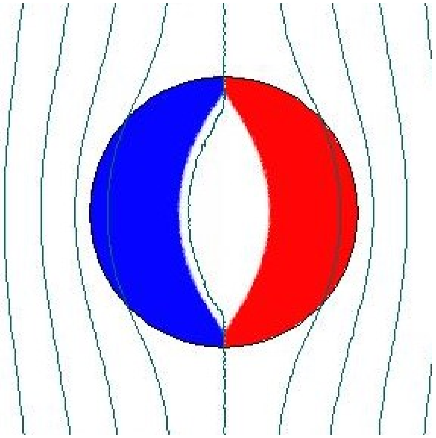
AC transport loss in hard superconductor



hysteresis → dissipation → **AC loss**

Hard superconductor in changing magnetic field

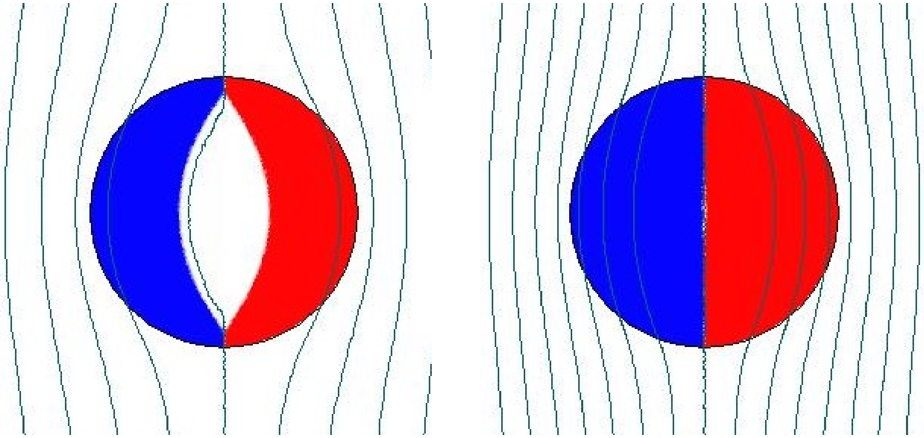
0 → 30 → 50 → 40 mT



→ 0 → -50 → -40 → 0 → 50 mT

Hard superconductor in changing magnetic field

0 → 30 → 50 → 40 mT



→ 0 → -50 → -40 → 0 → 50 mT

Hard superconductor in changing magnetic field

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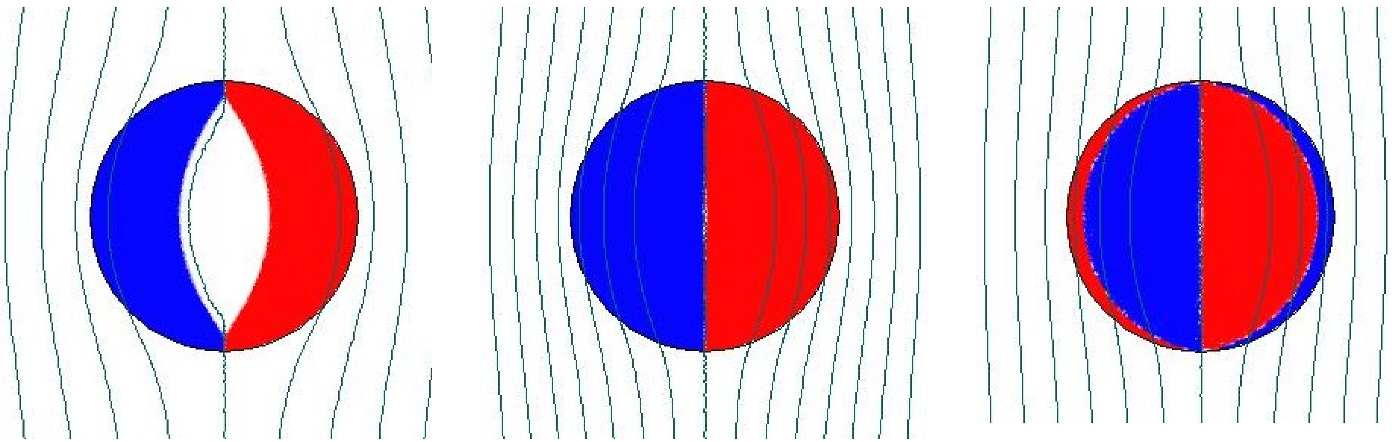

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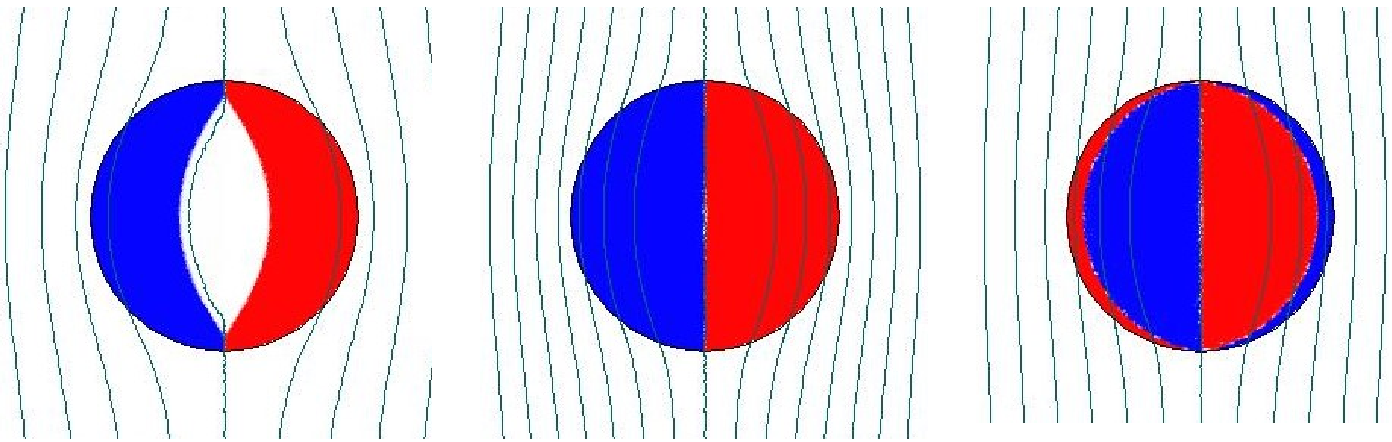

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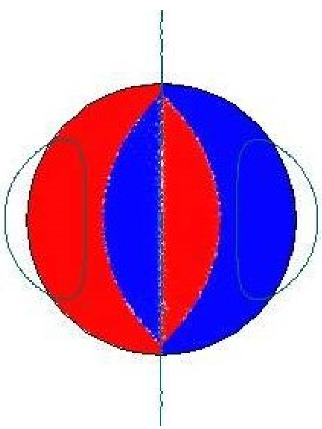

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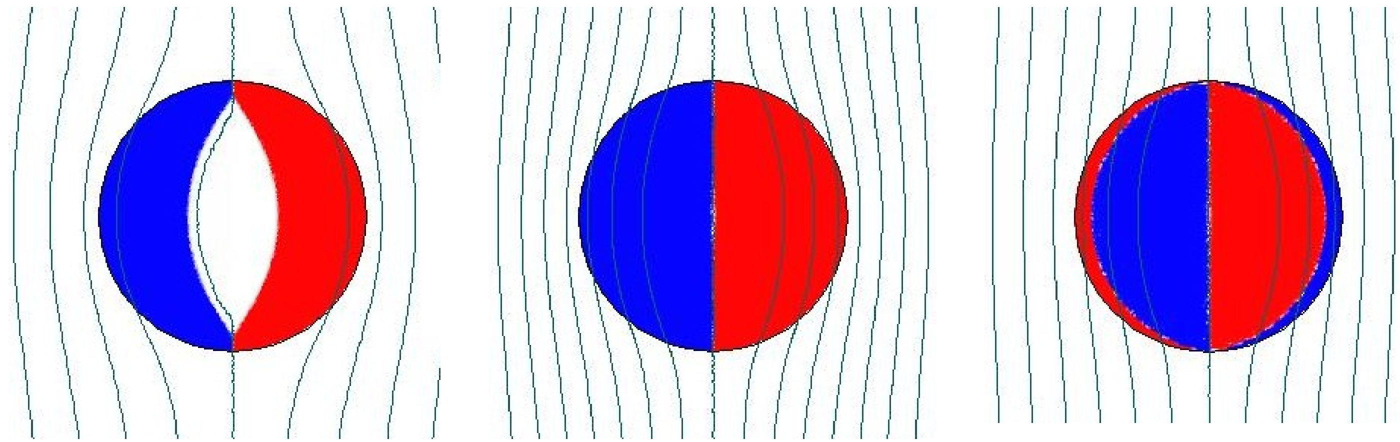

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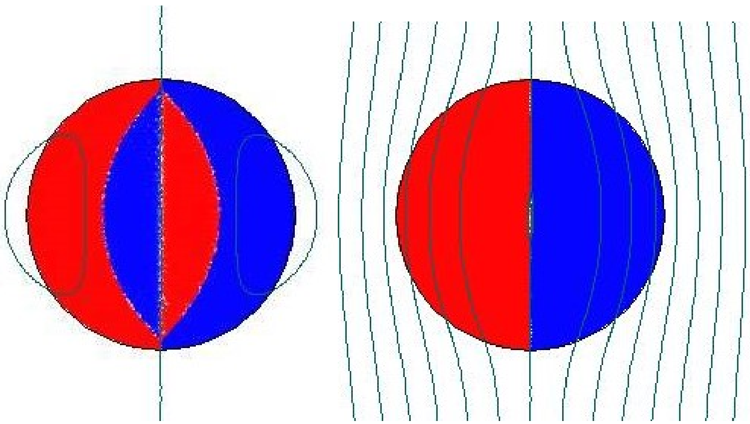

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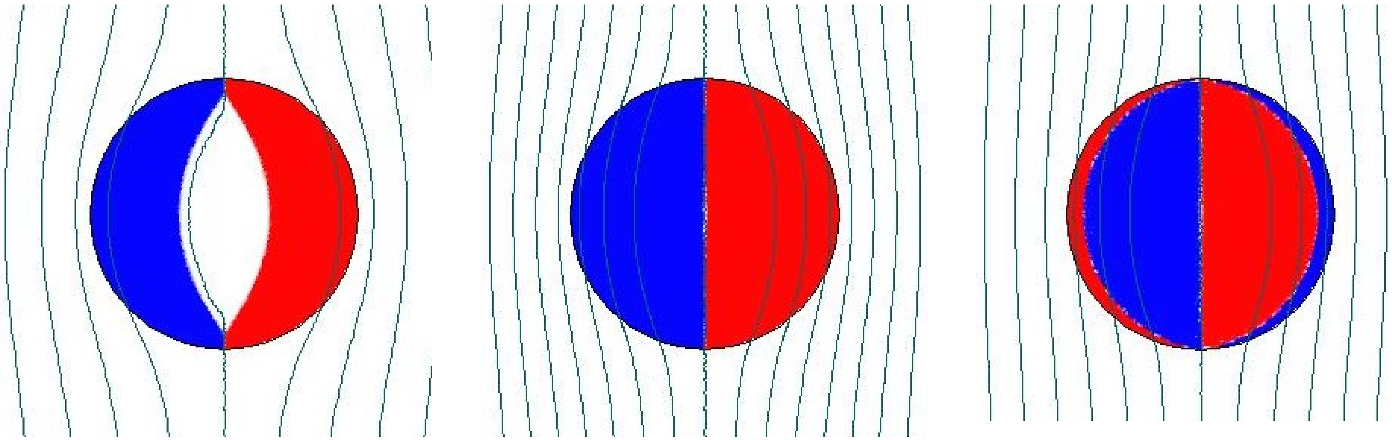

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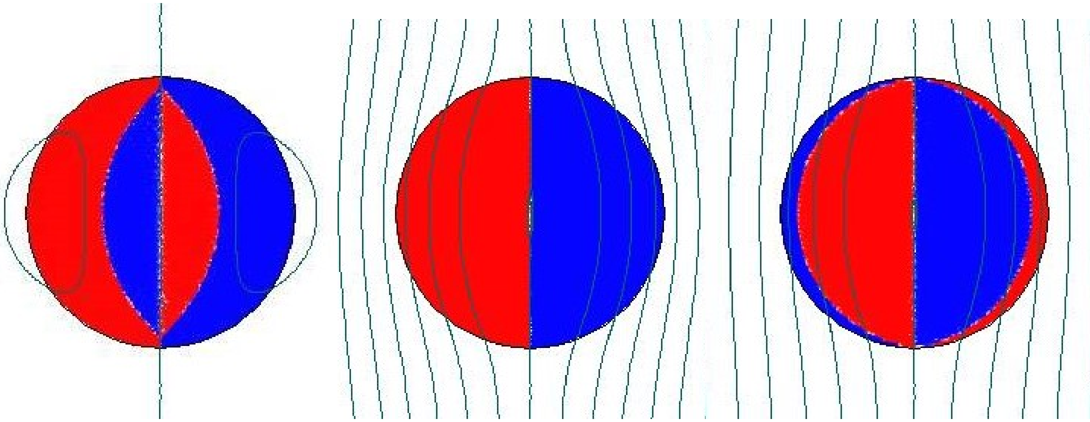
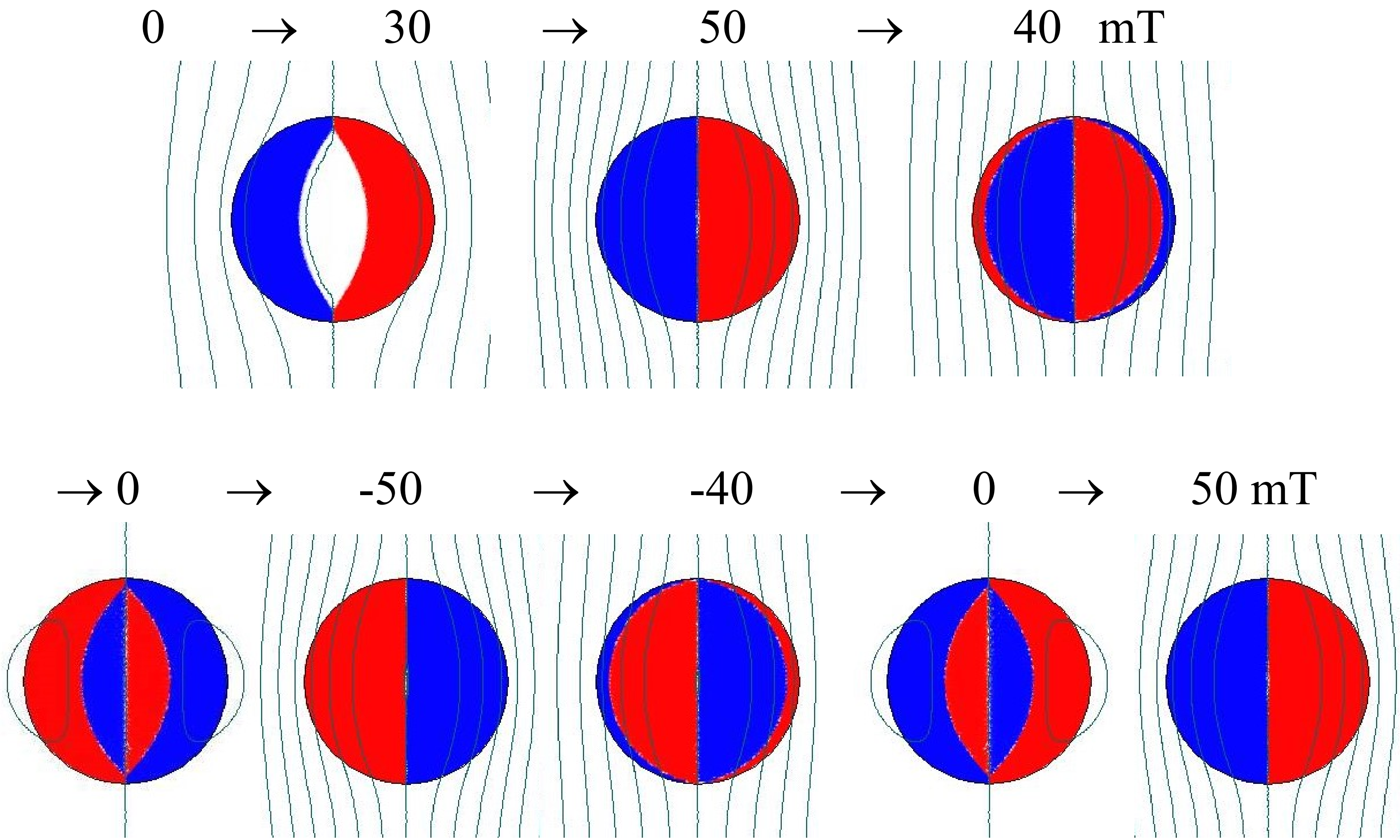
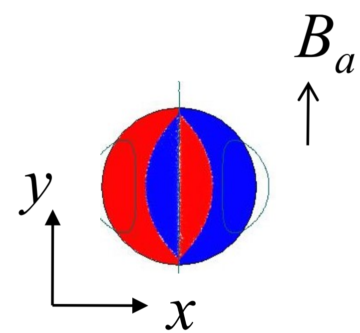


Hard superconductor in changing magnetic field



Hard superconductor in changing magnetic field

dissipation because of flux pinning



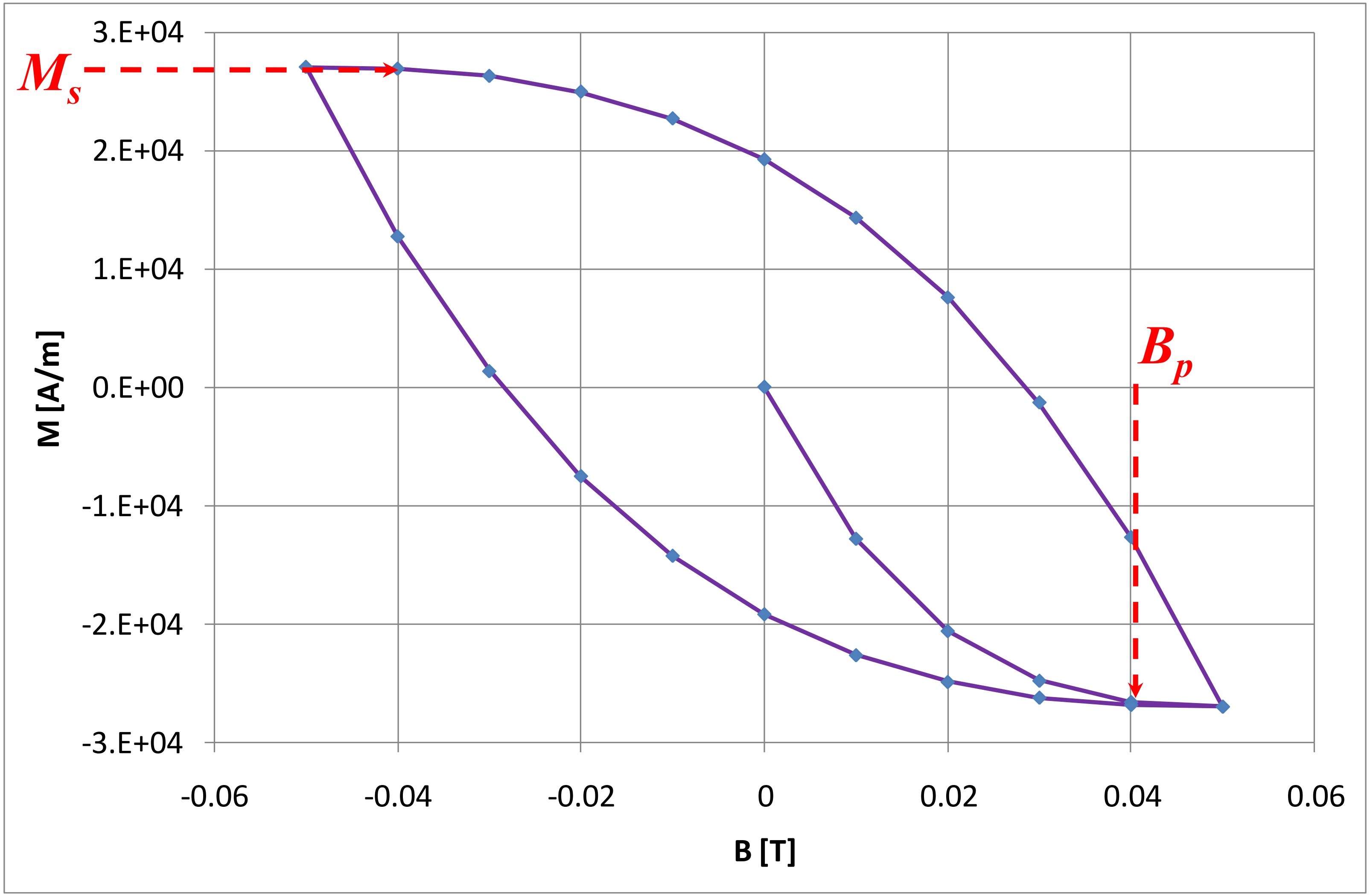
volume loss density Q [J/m^3]

$$\frac{Q}{V} = \oint B_a \mathrm{d}M$$

magnetization:
(*2D geometry)*

$$M = \frac{1}{S}\int_S -x.j(x,y)\mathrm{d}x\mathrm{d}y$$

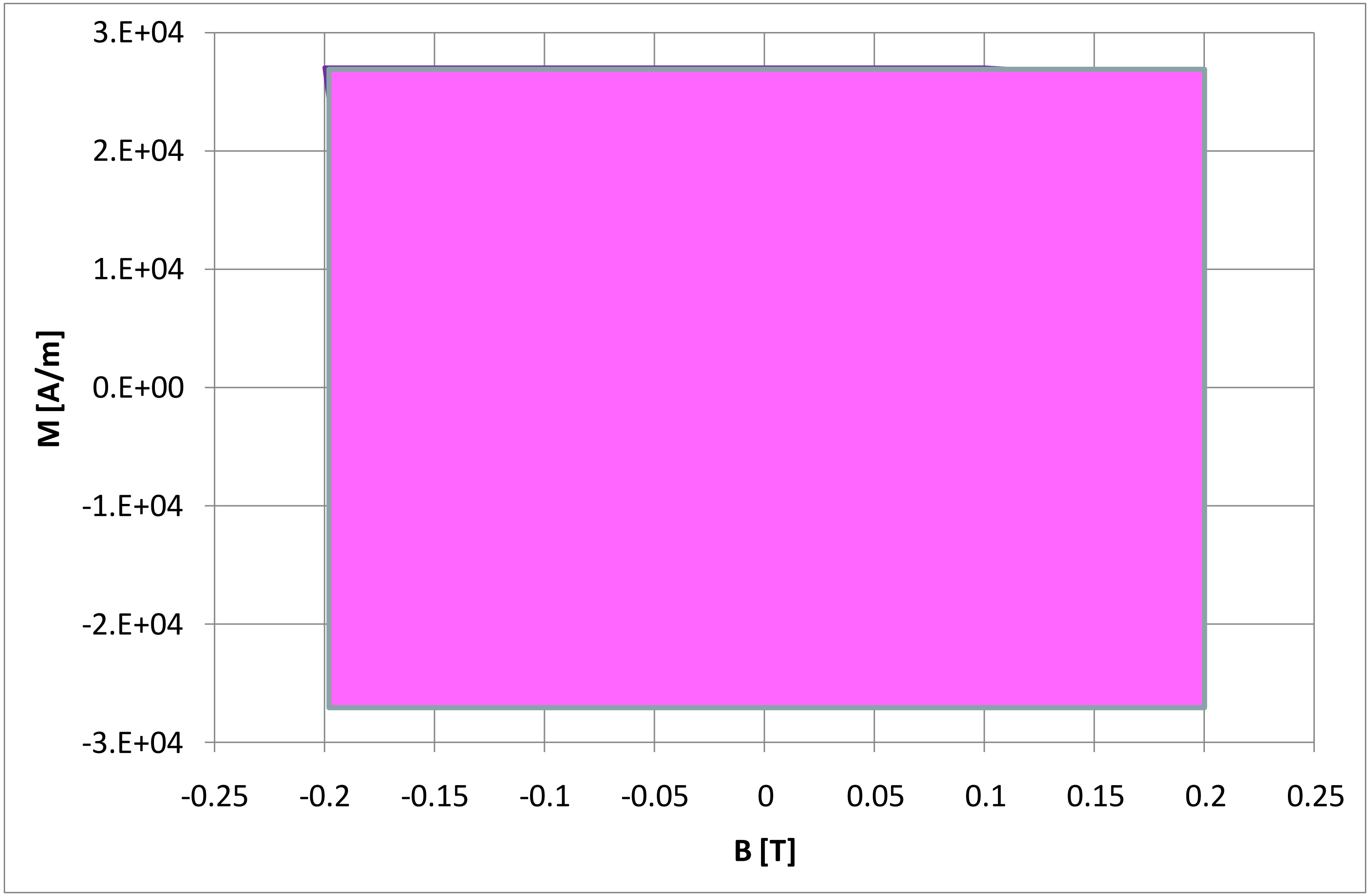
Round wire from hard superconductor in changing magnetic field



M_s saturation magnetization, B_p penetration field

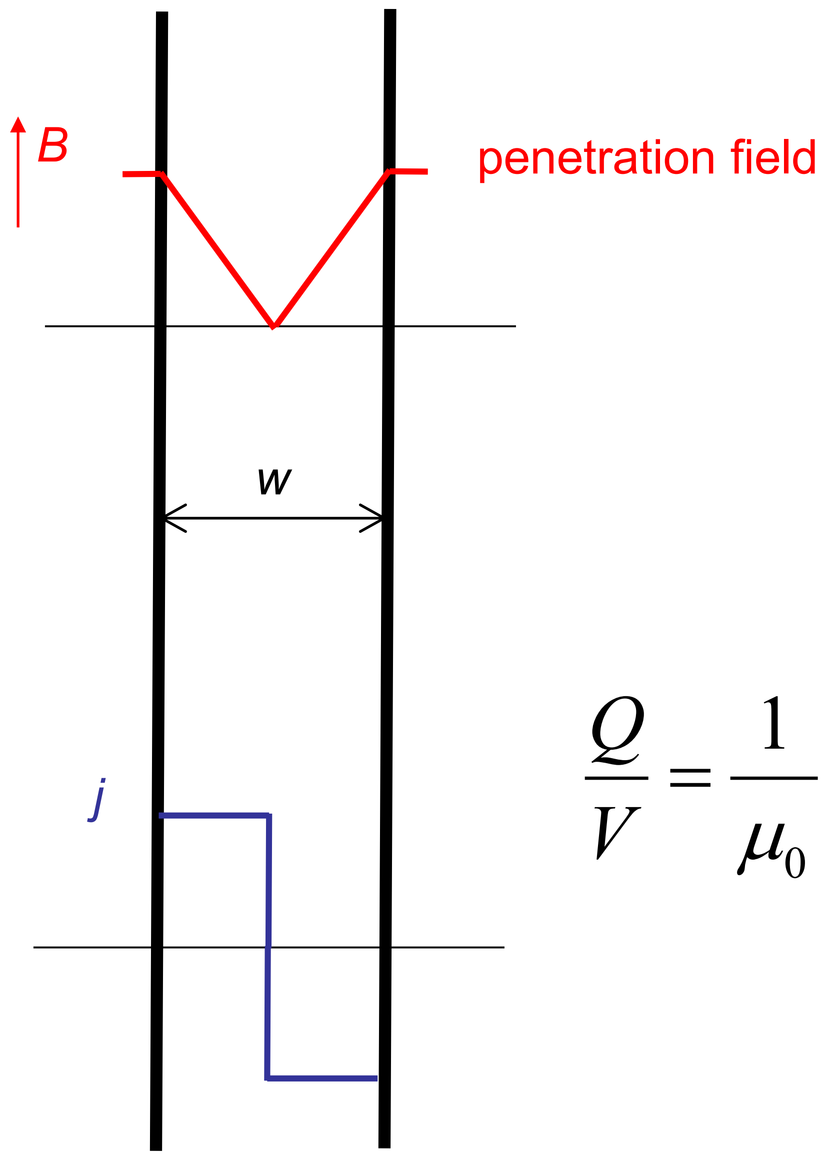
Round wire from hard superconductor in changing magnetic field

estimation of AC loss at $B_a >> B_p$



$$\frac{Q}{V} \approx 4 B_a M_s$$

(infinite) slab in parallel magnetic field – analytical solution



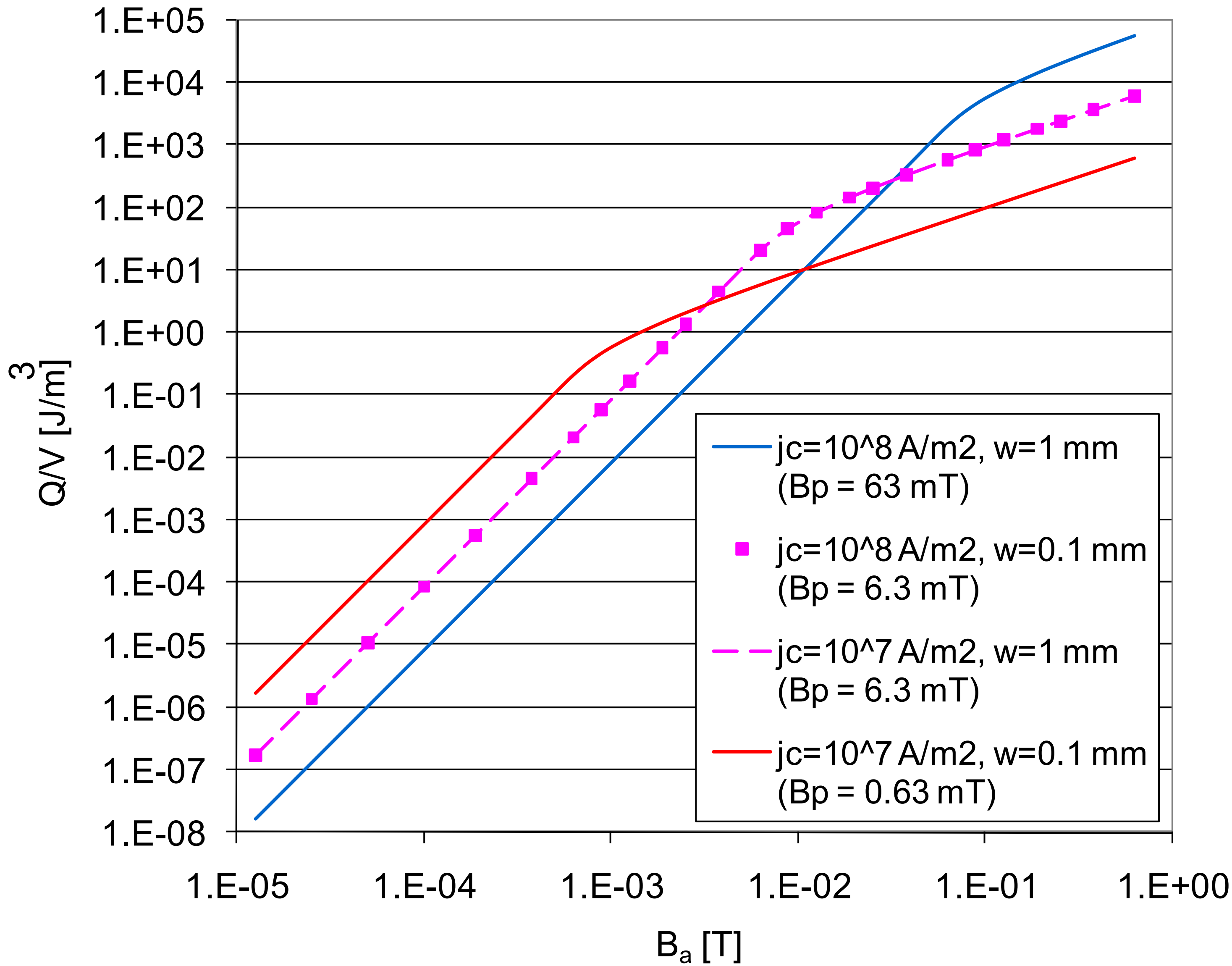
$$B_p = \mu_0 j_c \frac{w}{2}$$

$$M_s = j_c \frac{w}{4} = \frac{B_p}{2\mu_0}$$

$$\frac{Q}{V} = \frac{1}{\mu_0}\begin{cases} \dfrac{2}{3}\dfrac{B_a^3}{B_p} & \text{for } B_a < B_p \\ 2B_pB_a - \dfrac{4}{3}B_p^2 & \text{for } B_a > B_p \end{cases}$$

$$\boxed{Q \approx 4B_aM_s}$$

Slab in parallel magnetic field – analytical solution



$$Q \approx 4 B_a M_s$$

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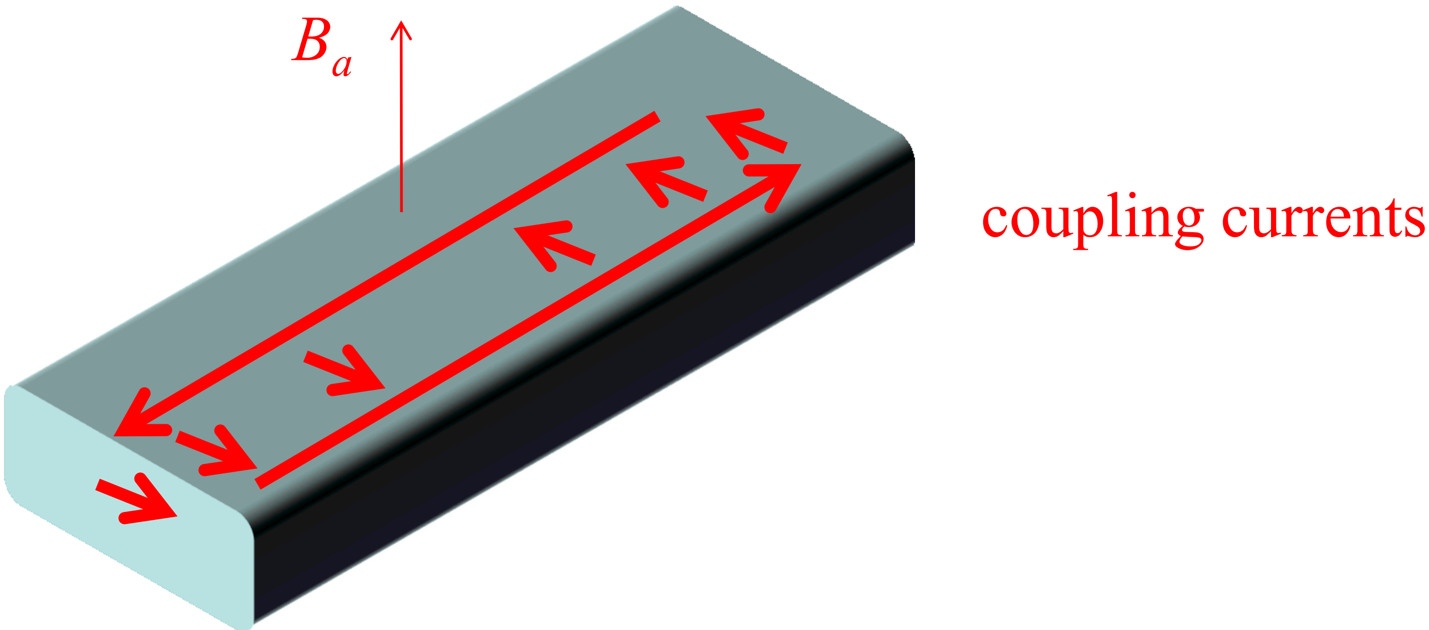
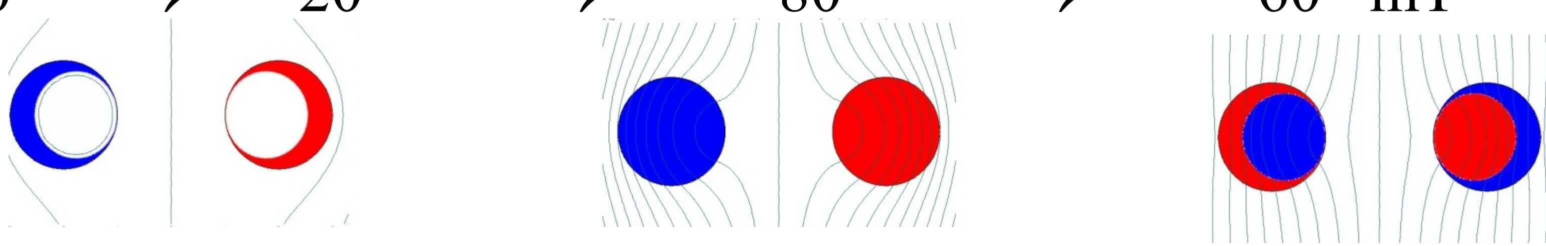
Two parallel superconducting wires in metallic matrix

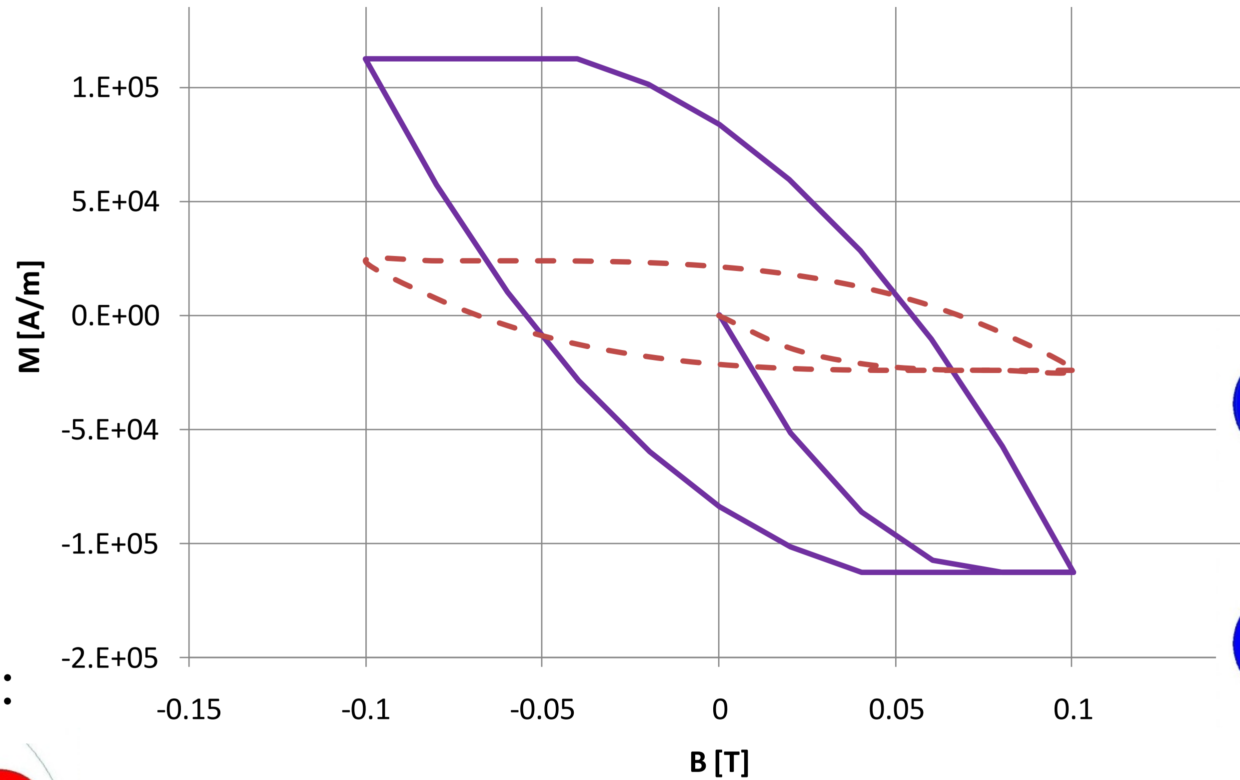


in the case of a perfect coupling:

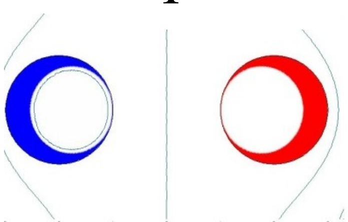
0 → 20 → 80 → 60 mT



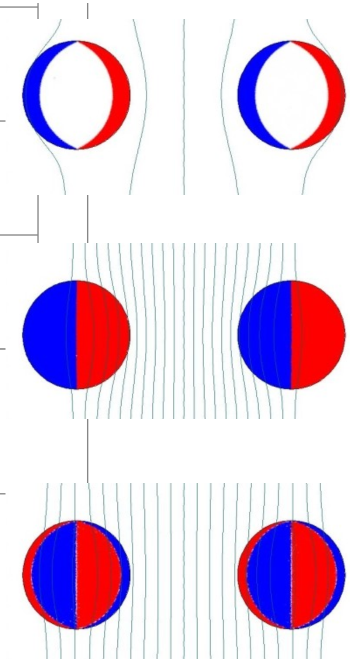
Magnetization of two parallel wires



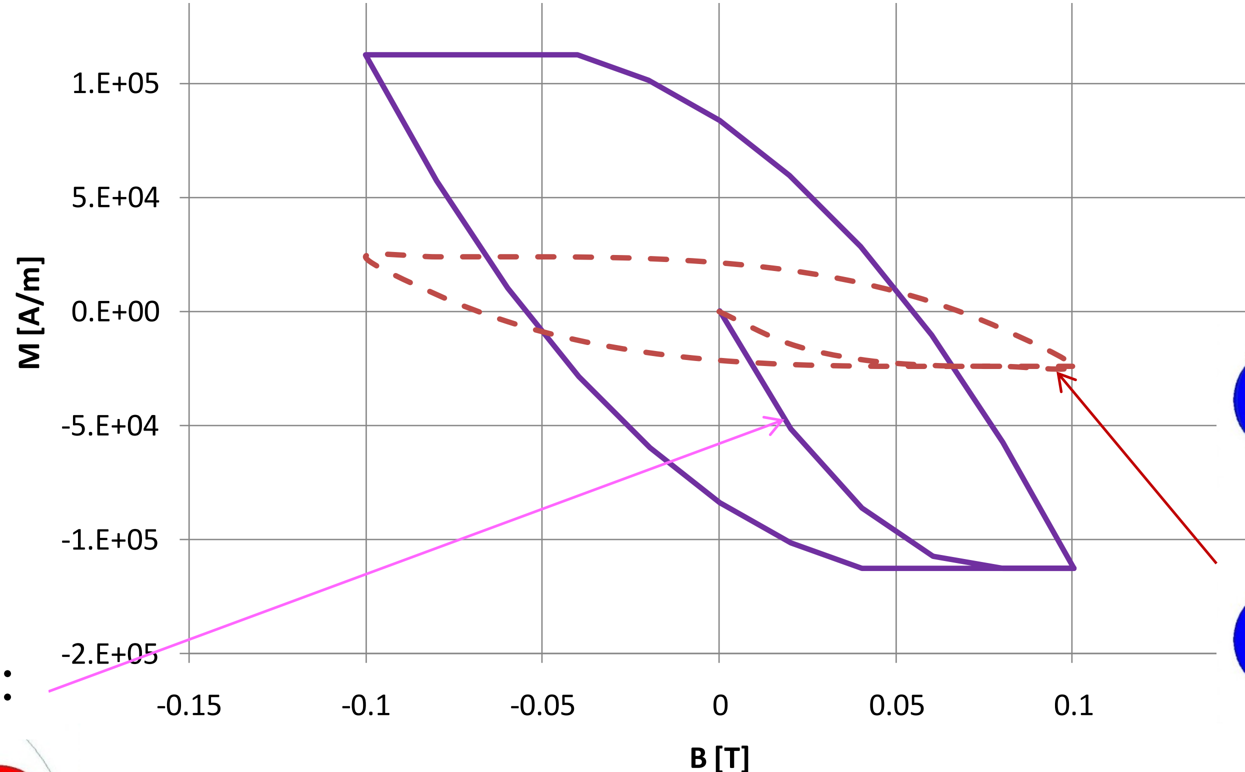
coupled:



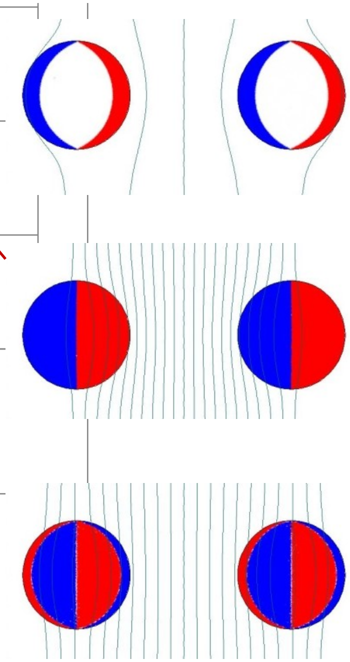
uncoupled:



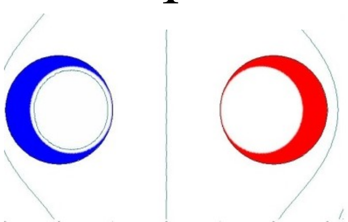
Magnetization of two parallel wires



uncoupled:



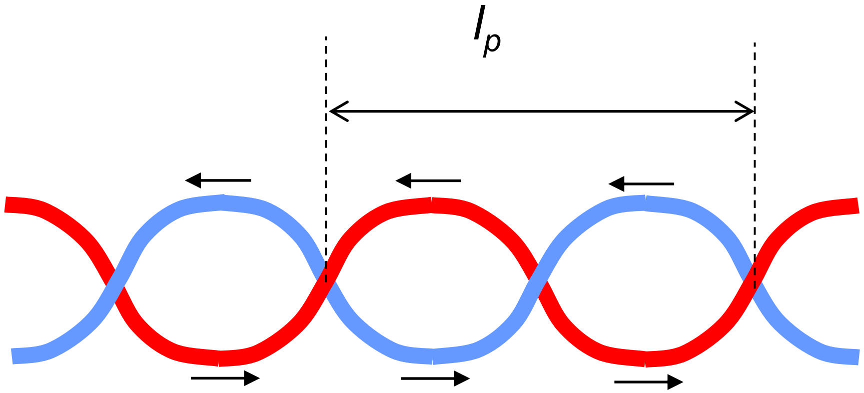
coupled:

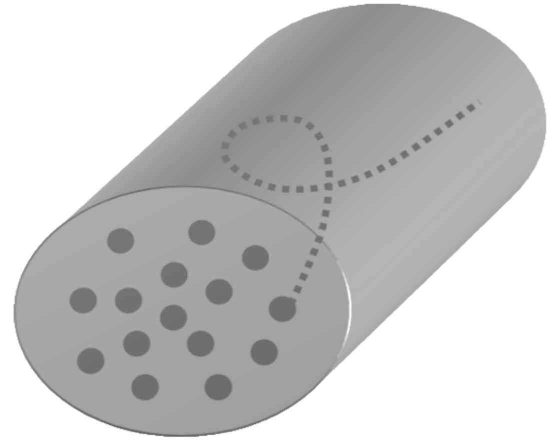


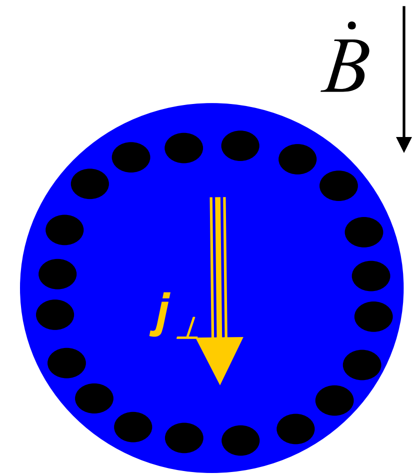
how to reduce the coupling currents ?

Composite wires – twisted filaments

$\otimes \dot{B}$







good interfaces $$\rho_t = \rho_m \frac{1-\lambda}{1+\lambda}$$

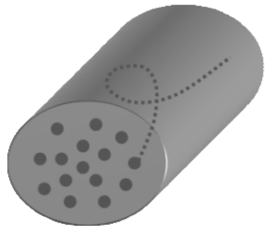
bad interfaces $$\rho_t = \rho_m \frac{1+\lambda}{1-\lambda}$$

$$\lambda = \frac{S_{SC}}{S_m}$$

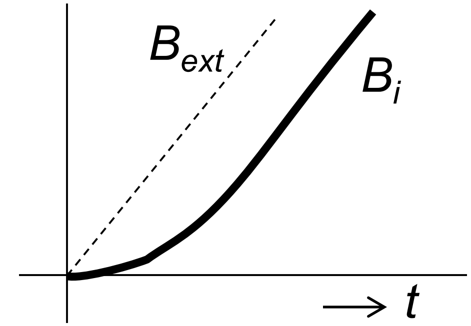
$$j_\perp = \frac{l_p \dot{B}}{2\pi\rho_t}$$

Composite wires – twisted filaments

coupling currents (partially) screen the applied field



$$B_i = B_{ext} - \tau \dot{B}$$

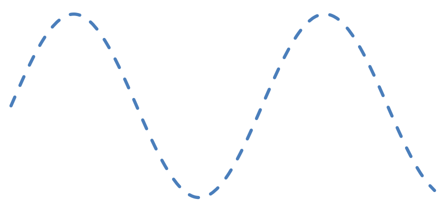


τ - time constant of the magnetic flux diffusion

$$\tau = \frac{\mu_0}{2\rho_t}\left(\frac{l_p}{2\pi}\right)^2$$

A.Campbell (1982) Cryogenics 22 3
K. Kwasnitza, S. Clerc (1994) Physica C 233 423
K. Kwasnitza, S. Clerc, R. Flukiger, Y. Huang (1999) Cryogenics 39 829

in AC excitation



$$\frac{Q}{V} = \frac{B_{\max}^2}{\mu_0}\frac{2\pi\omega\tau}{1+\omega^2\tau^2}$$

round wire

shape factor
(~ aspect ratio)

$$\frac{Q}{V} = \frac{B_{\max}^2}{\mu_0}\frac{\chi_0\pi\omega\tau}{1+\omega^2\tau^2}$$

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Persistent currents:

at large fields proportional to $B_p \sim j_c\, w$

width of superconductor (perpendicular to the applied magnetic field)

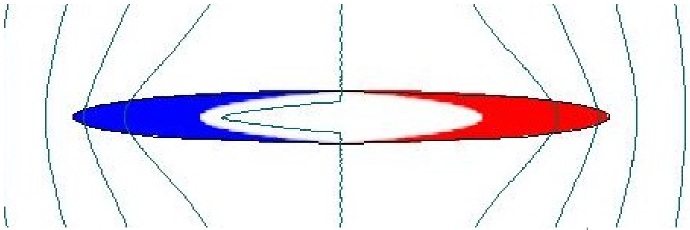
= magnetization reduction by either lower j_c or reduced w

lowering of j_c would mean more superconducting material required to transport the same current

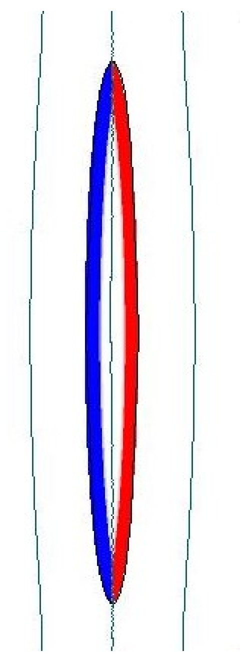
thus only plausible way is the reduction of w

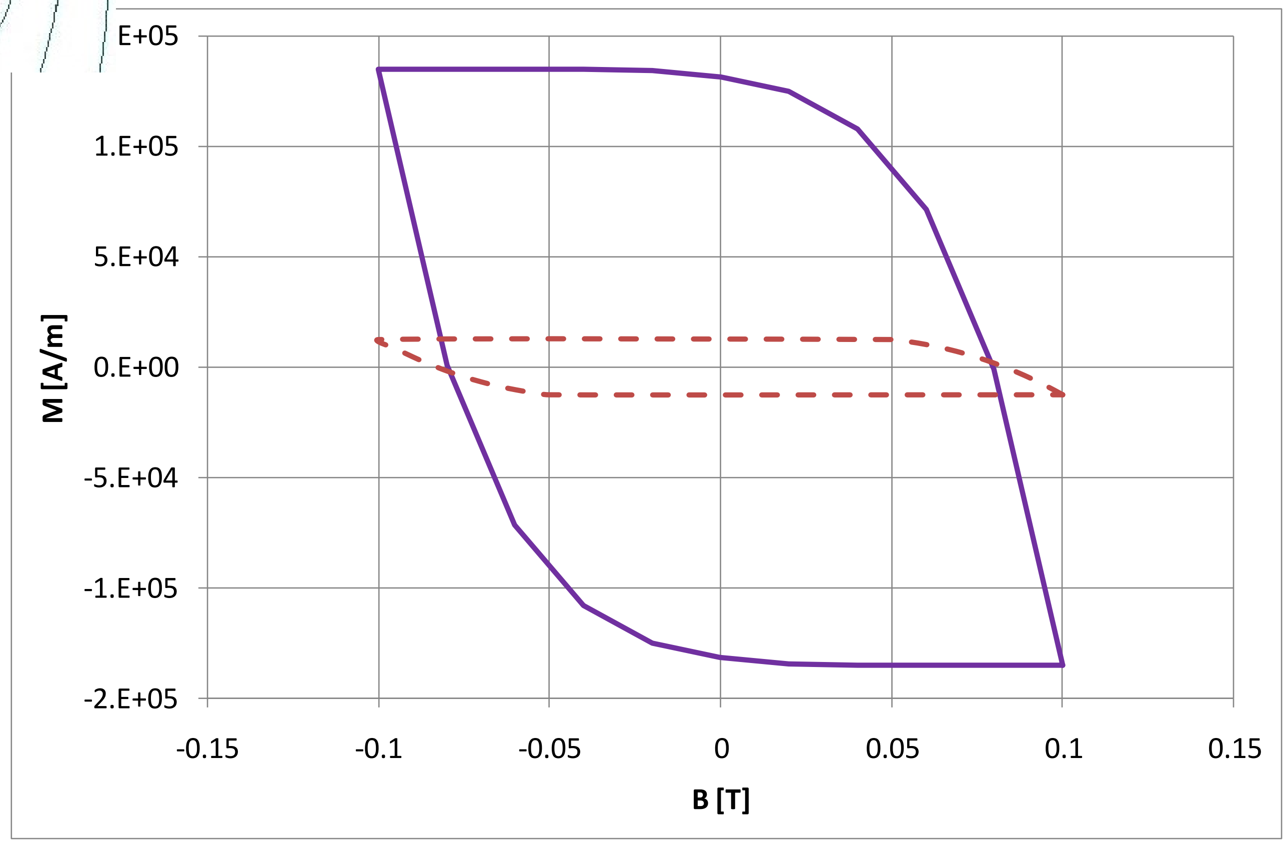
effect of the field orientation

perpendicular field



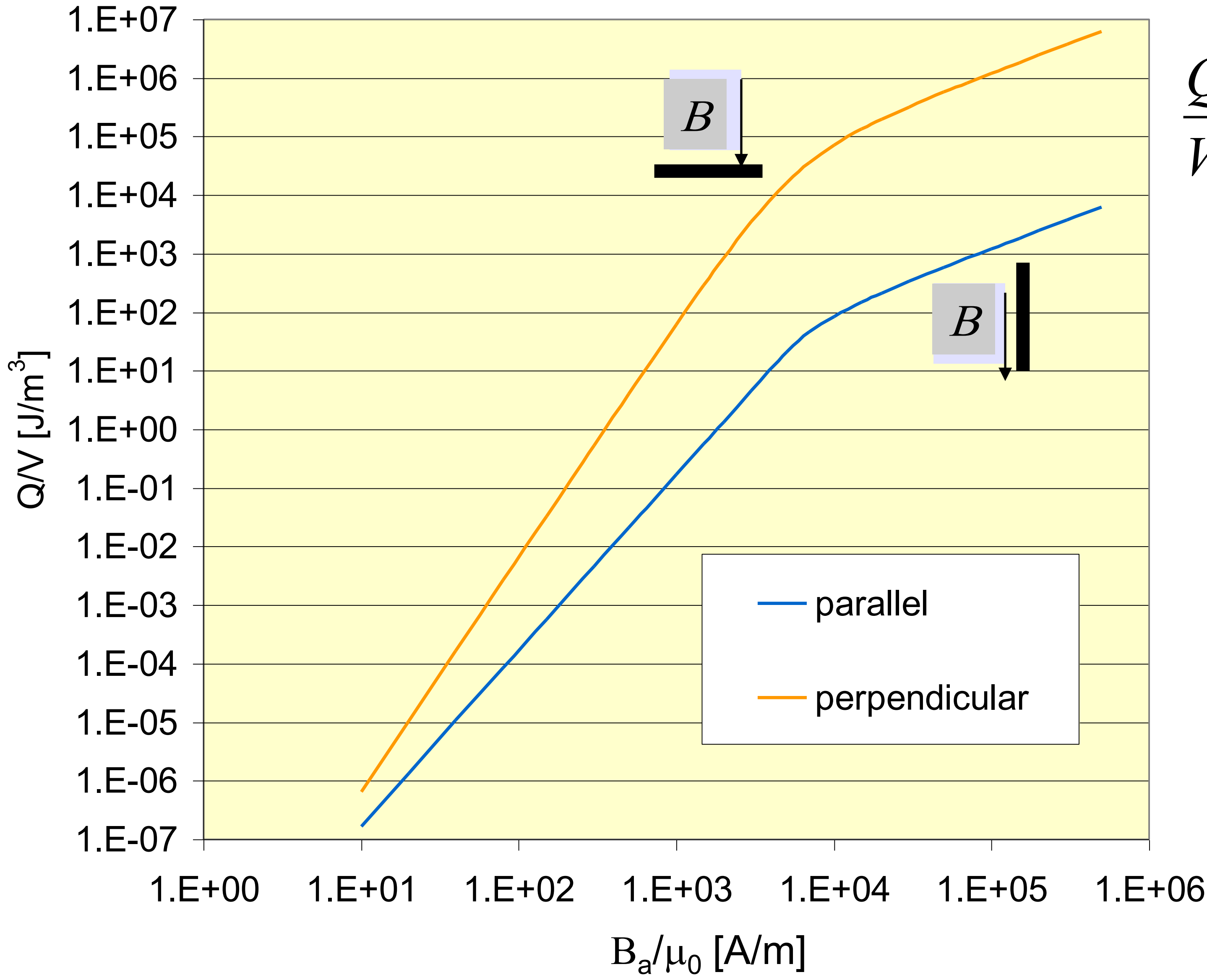
parallel field





$$\frac{Q}{V} \approx 4 B_a M_s$$

Magnetization loss in strip with aspect ratio 1:1000

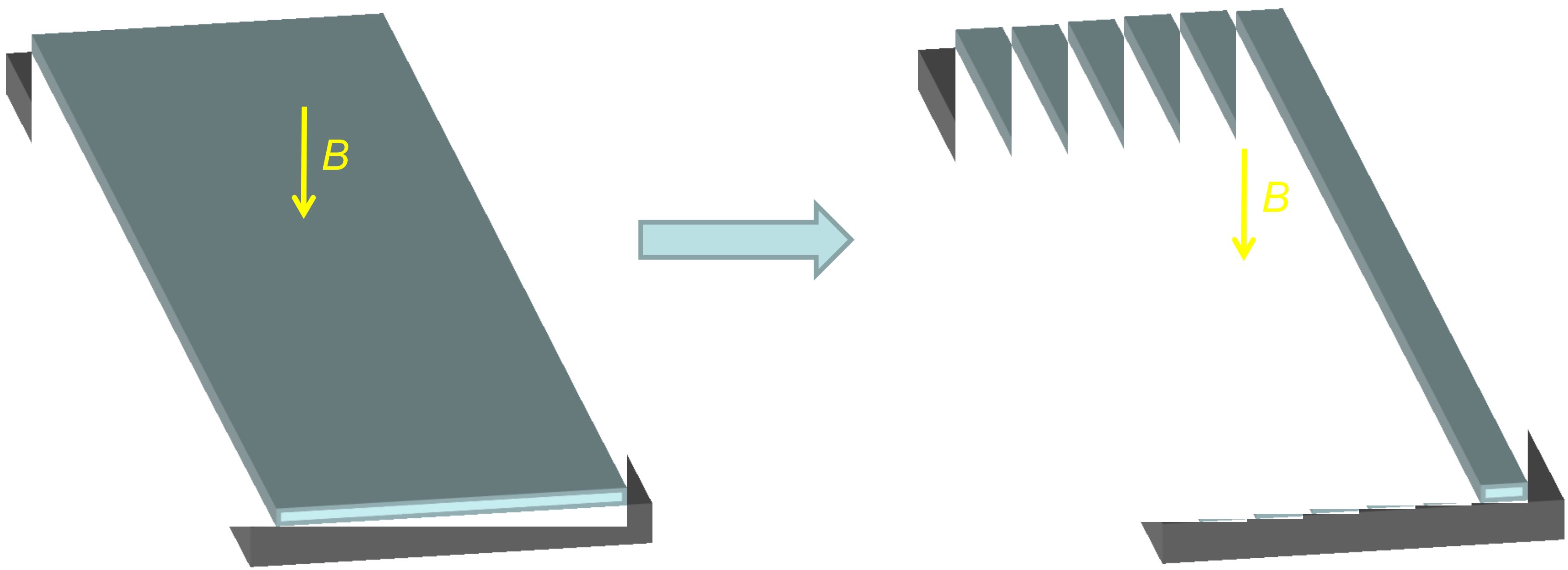


$$\frac{Q}{V} \approx 4 B_a M_s$$

in the case of flat wire or cable the orientation is not a free parameter

= reduction of the width

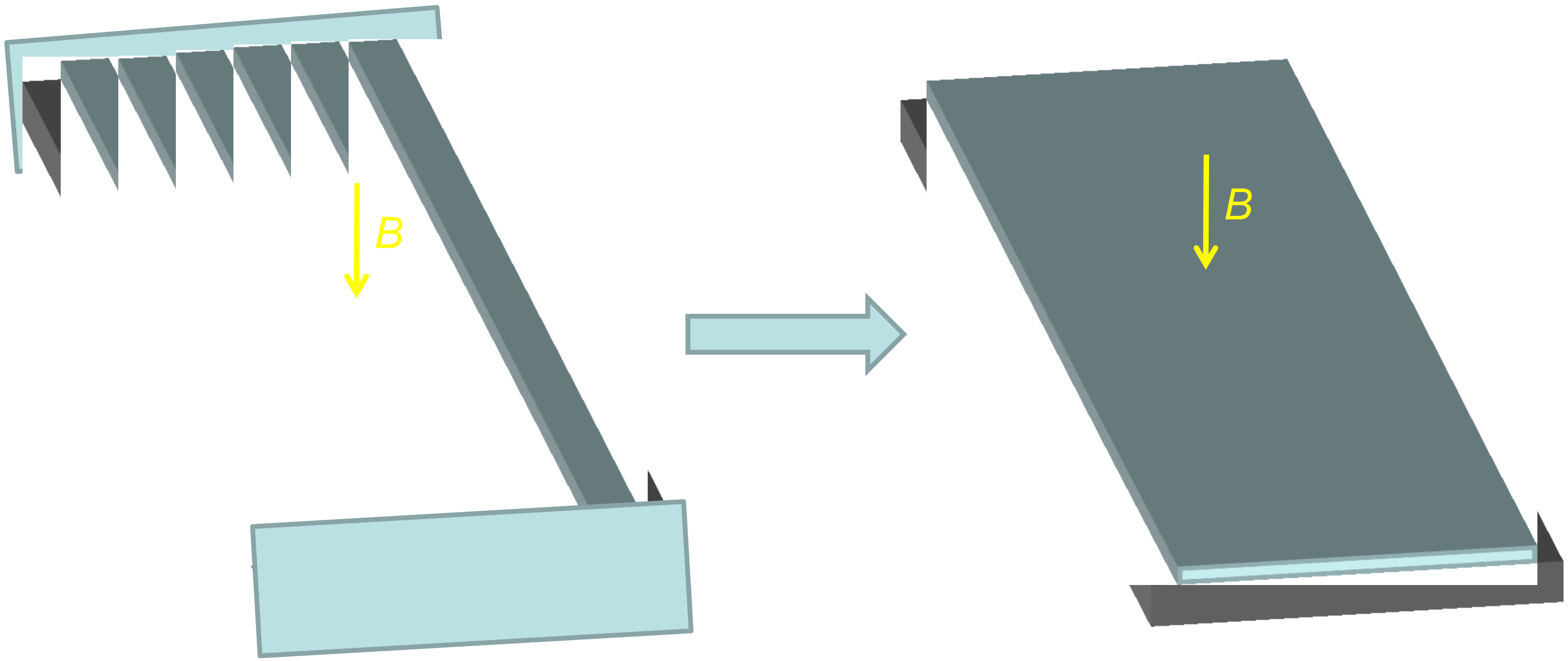
e.g. striation of CC tapes



~ 6 times lower magnetization

striation of CC tapes

but in operation the filaments are connected at magnet terminations



coupling currents will appear
=> transposition necessary

Coupling currents:

at low frequencies proportional to the time constant of magnetic flux diffusion

$$\tau = \frac{\mu_0}{2\rho_t}\left(\frac{l_p}{2\pi}\right)^2$$

transposition length

effective transverse resistivity

= filaments (in single tape) or strands (in a cable) should be transposed

= low loss requires high inter-filament or inter-strand resistivity

but good stability needs the opposite

Outline:

1. Hard superconductor in varying magnetic field

2. Magnetization currents: Flux pinning
   Coupling currents

3. Possibilities for reduction of magnetization currents

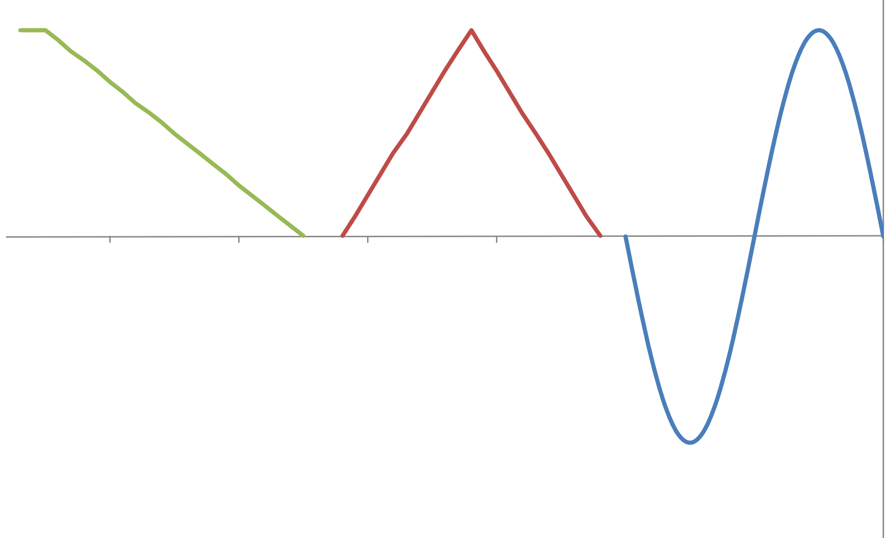
4. Methods to measure magnetization and AC loss

Different methods necessary to investigate

- Wire (strand, tape)
- Cable
- Magnet

shape of the excitation field (current) pulse

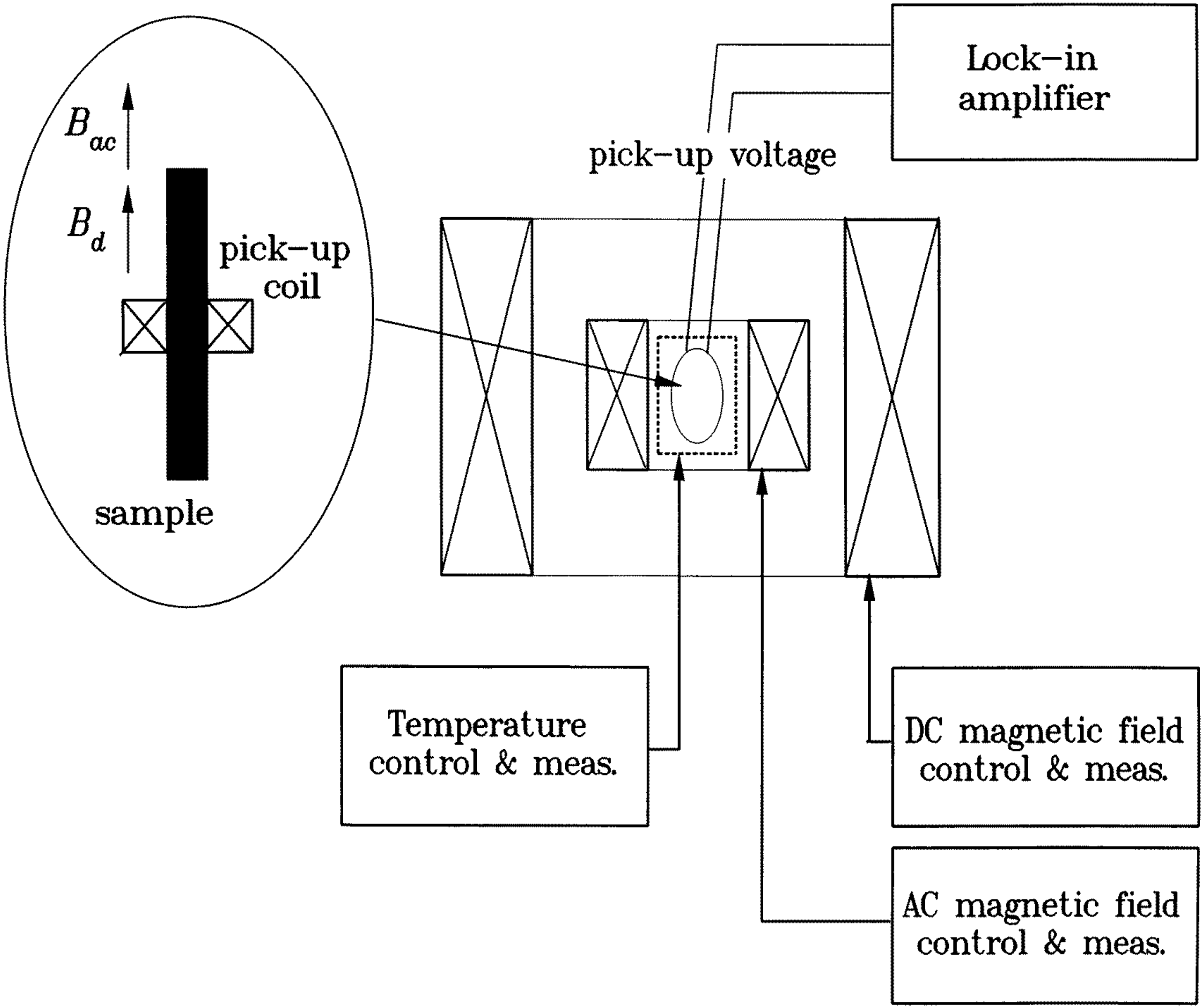
transition unipolar harmonic



relevant information can be achieved in harmonic regime

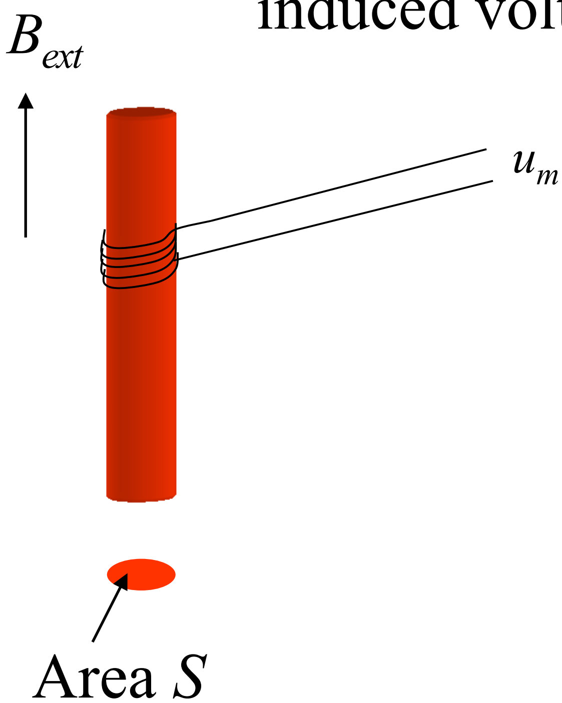
final testing necessary in actual regime

ideal magnetization loss measurement:



pick-up coil wrapped around the sample

induced voltage $u_m(t)$ in one turn:



$$u_m(t) = -\frac{\mathrm{d}\phi_m(t)}{\mathrm{d}t} = -S\frac{\mathrm{d}\overline{B}(t)}{\mathrm{d}t}$$

$$\overline{B}(t) = \frac{1}{S}\int_S B_{\mathrm{int}}(t)\mathrm{d}S = B_{ext}(t) + \mu_0 M(t)$$

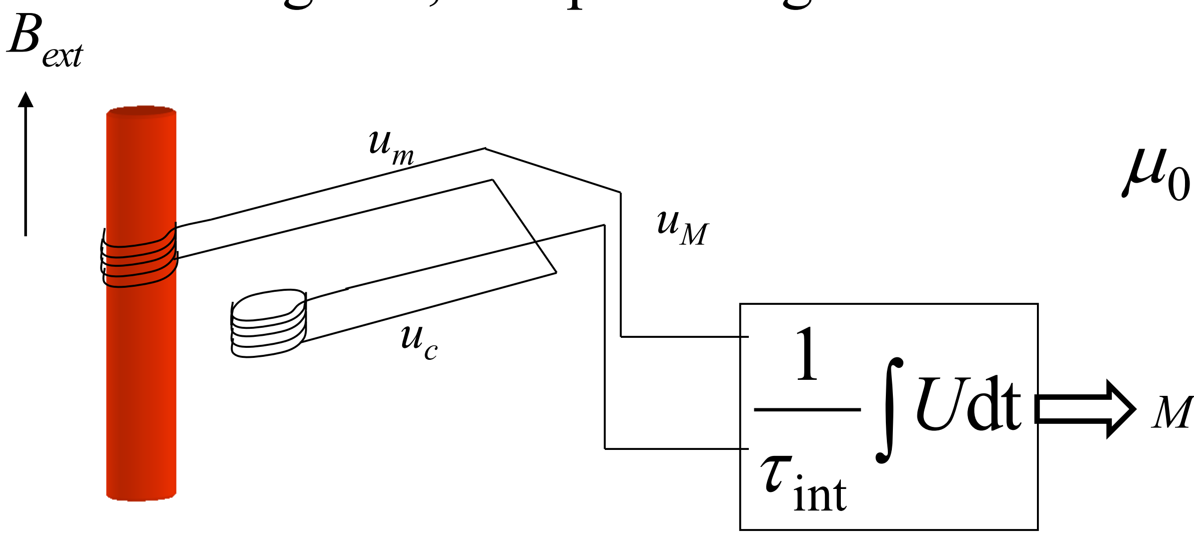
$$u_m(t) = -S\left[\frac{dB_{ext}(t)}{dt} + \mu_0\frac{dM(t)}{dt}\right]$$

pick-up coil voltage processed by integration
either numerical or by an electronic integrator:

$$\mu_0 M(t) = -\frac{1}{S}\int u_m(t)\mathrm{d}t - B_{ext}(t)$$

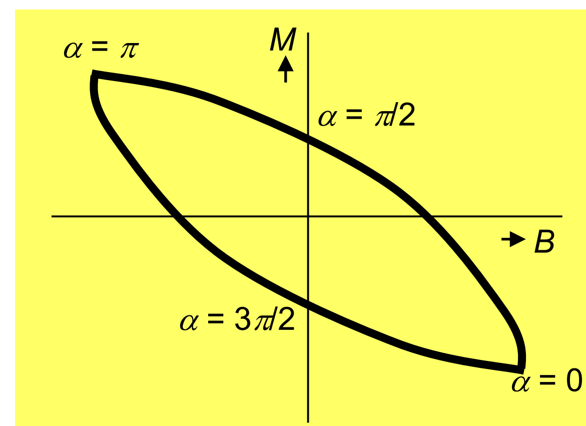
Method 1: double pick-up coil system with an electronic integrator : measuring coil, compensating coil

$$\mu_0 M(t) = -\frac{1}{S}\int u_M(t)\mathrm{d}t$$



AC loss in one magnetization cycle [J/m^3]:

$$Q = \oint B\mathrm{d}M = \int_0^T B(t)\frac{\mathrm{d}M}{\mathrm{d}t}\mathrm{d}t$$



Harmonic AC excitation – use of complex susceptibilities

$$B_{ext}(t) = B_a \cos \omega t$$

$$\mu_0 M(t) = B_a \sum_{n=1}^{\infty} \left(\chi_n' \cos n\omega t + \chi_n'' \sin n\omega t \right)$$

fundamental component $n = 1$

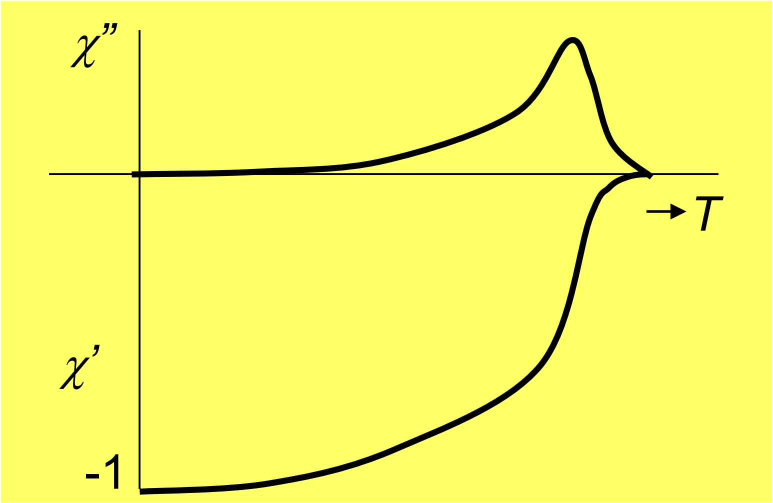
AC loss per cycle

$$W_q = -\pi \chi'' \frac{B_a^2}{\mu_0}$$

energy of magnetic shielding

$$W_m = \chi' \frac{B_a^2}{2\mu_0}$$

Temperature dependence:



Method 2: Lock-in amplifier
– phase sensitive analysis of voltage signal spectrum

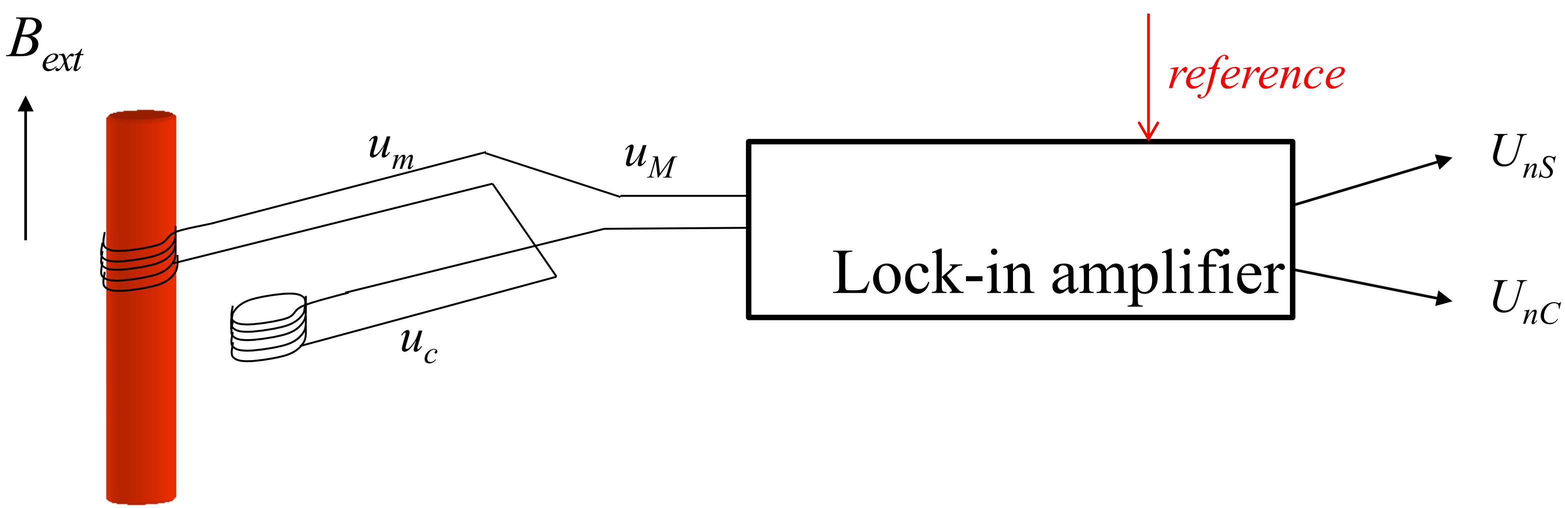
in-phase and out-of-phase signals

$$B_{ext} = B_a \cos \omega t$$

$$U_{nS} = \frac{1}{\pi} \int_0^{2\pi} u_M(t) \sin n\omega t \, \mathrm{d}\omega t$$

$$U_{nC} = \frac{1}{\pi} \int_0^{2\pi} u_M(t) \cos n\omega t \, \mathrm{d}\omega t$$

reference signal necessary to set the
frequency
phase
taken from the current energizing the AC field coil



Method 2: Lock-in amplifier – only at harmonic AC excitation

$$B_{ext} = B_a \cos \omega t$$

$$u_M(t) = S\omega B_a \left[\sin \omega t + \sum_{n=1}^{\infty} n\left(\chi_n' \sin n\omega t - \chi_n'' \cos n\omega t \right) \right]$$

empty coil

sample magnetization

fundamental susceptibility

$$\chi' = \frac{U_{1S}}{S\omega B_a} - 1 = \frac{U_{1S}}{U_N} - 1$$

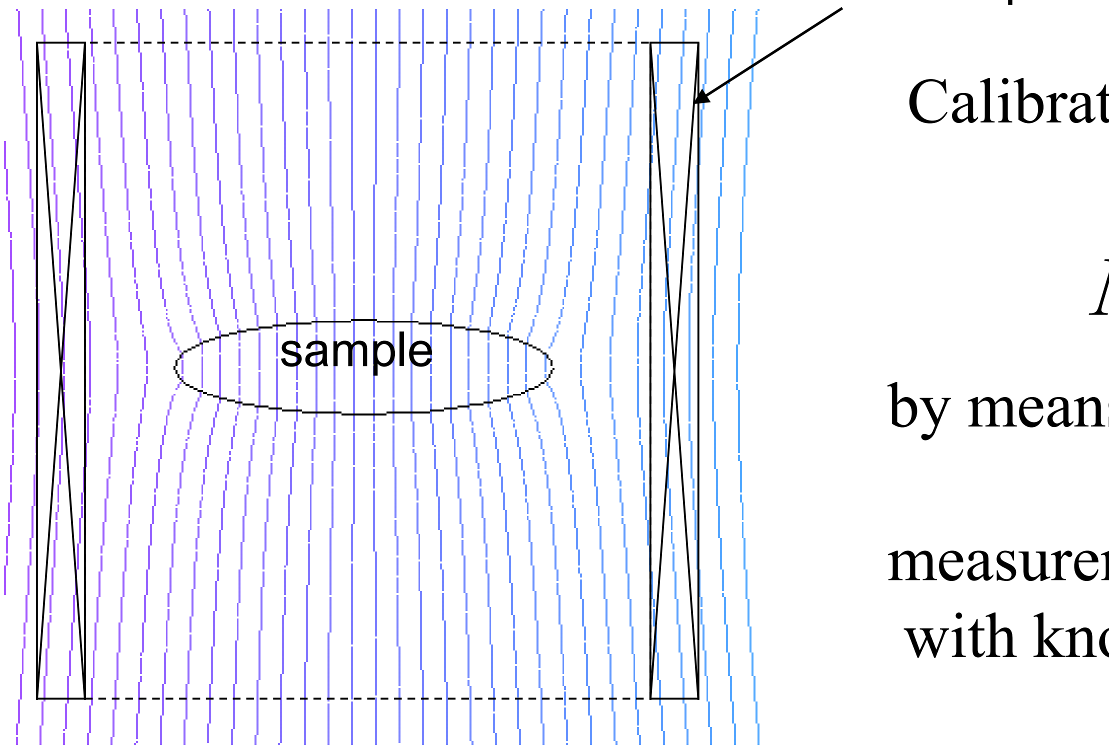
$$\chi'' = \frac{-U_{1C}}{S\omega B_a} = \frac{-U_{1C}}{U_N}$$

higher harmonic susceptibilities

$$\chi_n' = \frac{U_{nS}}{nU_N}$$

$$\chi_n'' = \frac{U_{nC}}{nU_N}$$

Real magnetization loss measurement:



Calibration necessary

$$M = C\int u\mathrm{d}t$$

by means of:

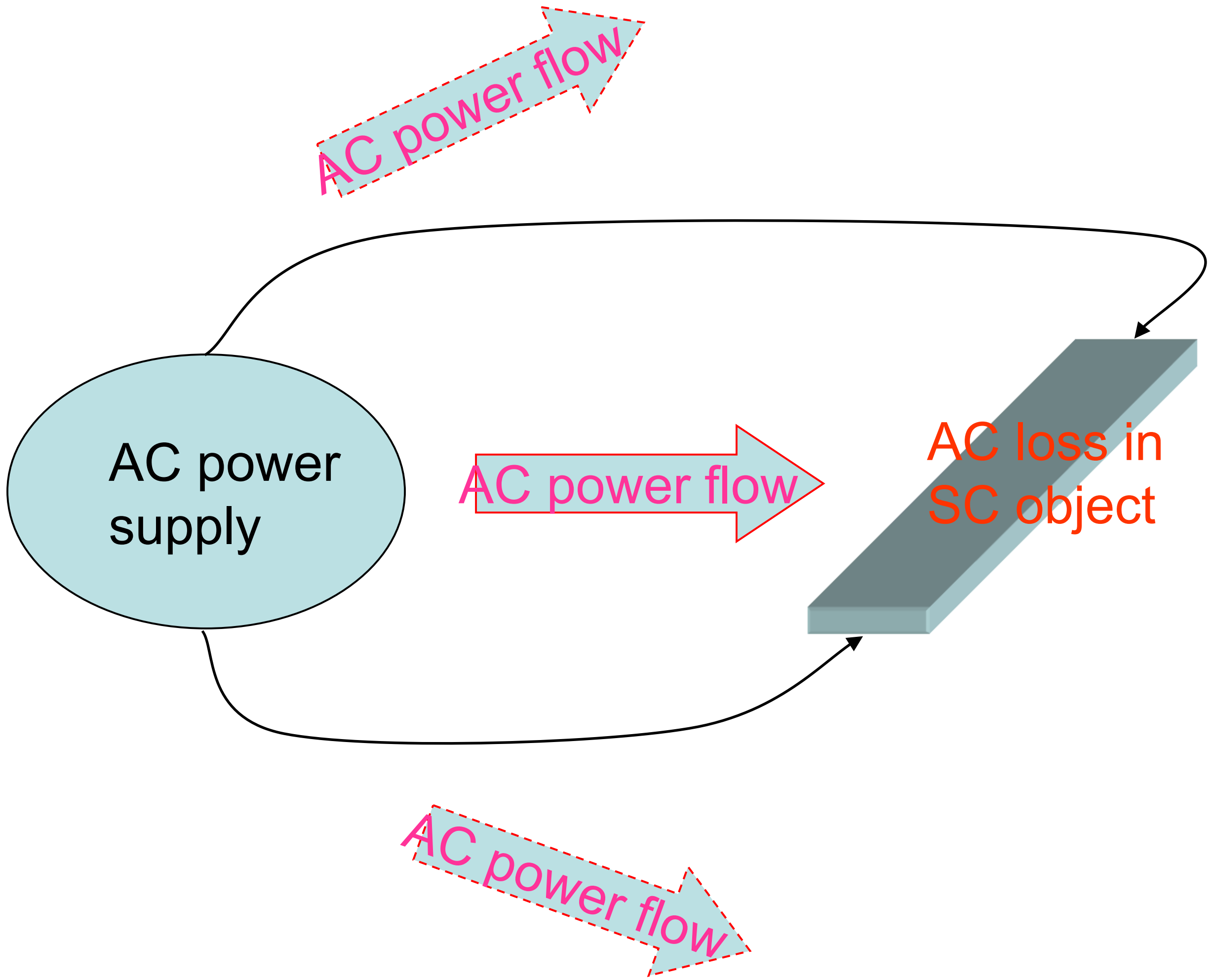
measurement on a sample
with known properertties

calibration coil

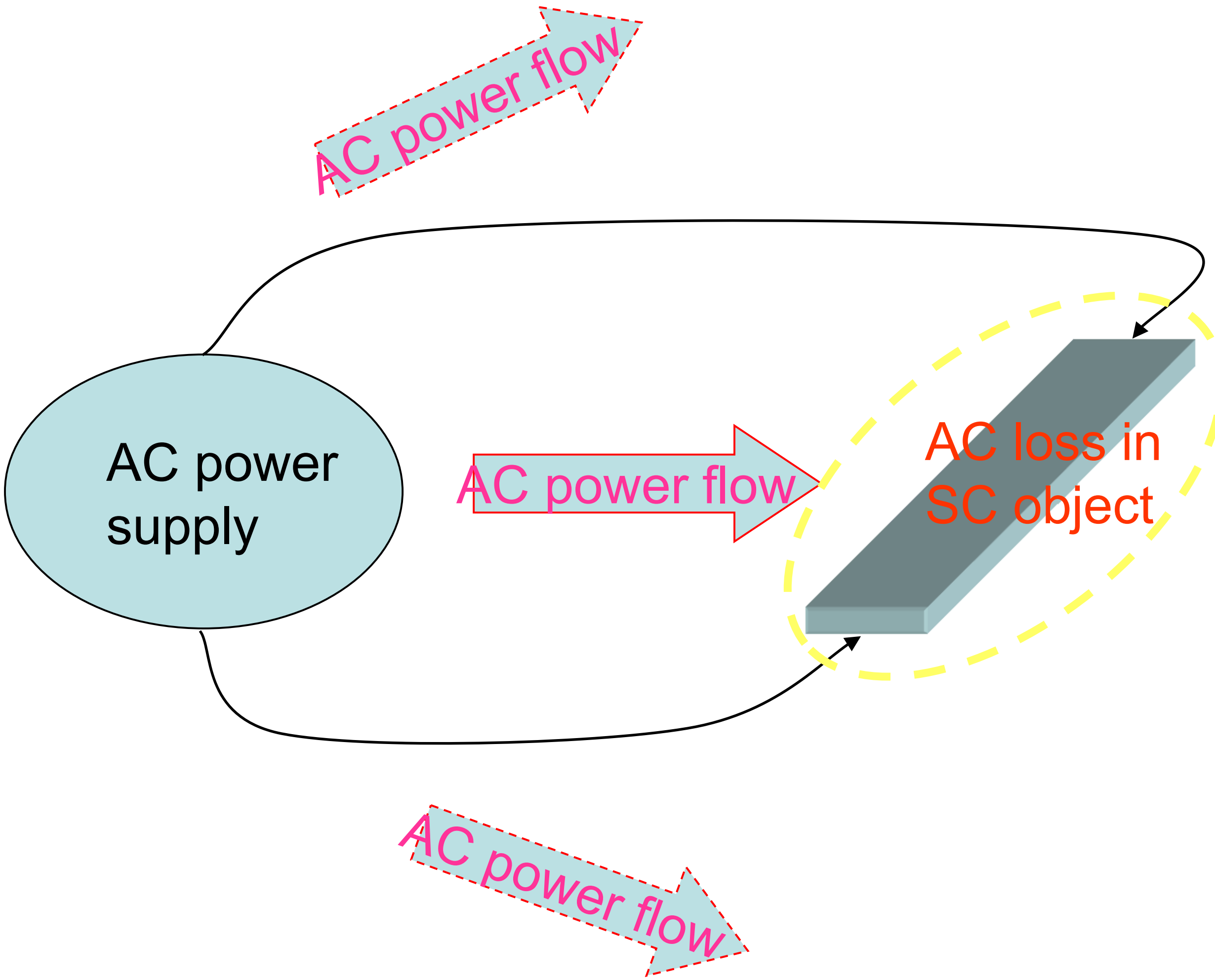
numerical calculation

…

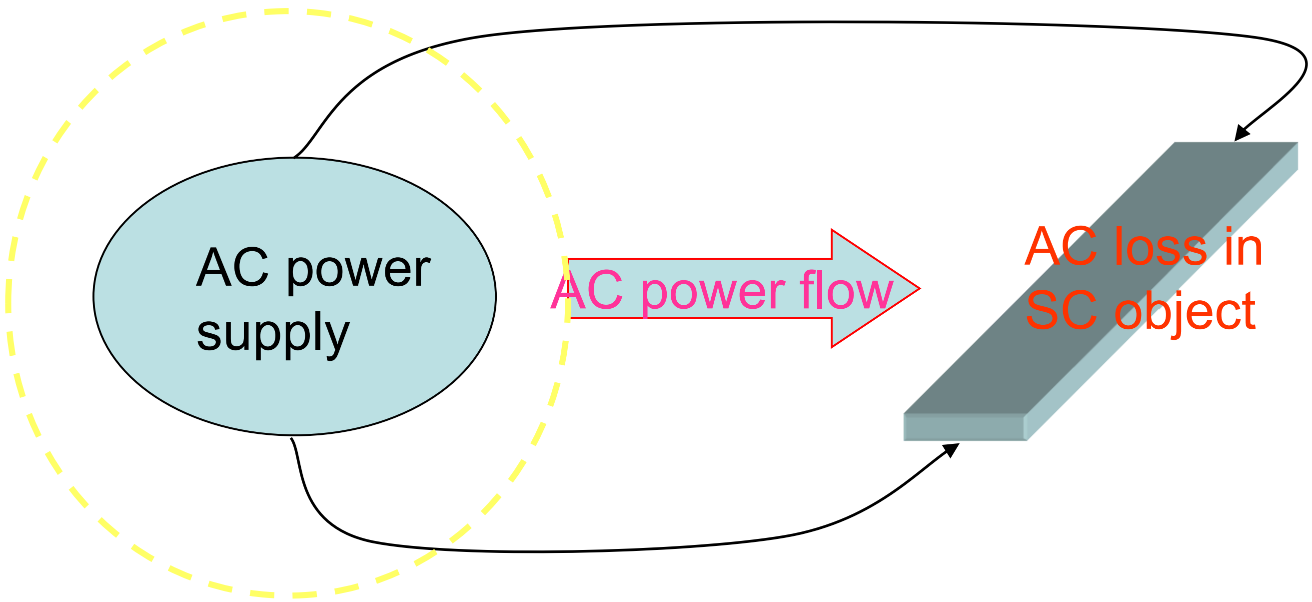
AC loss can be determined from the balance of energy flows



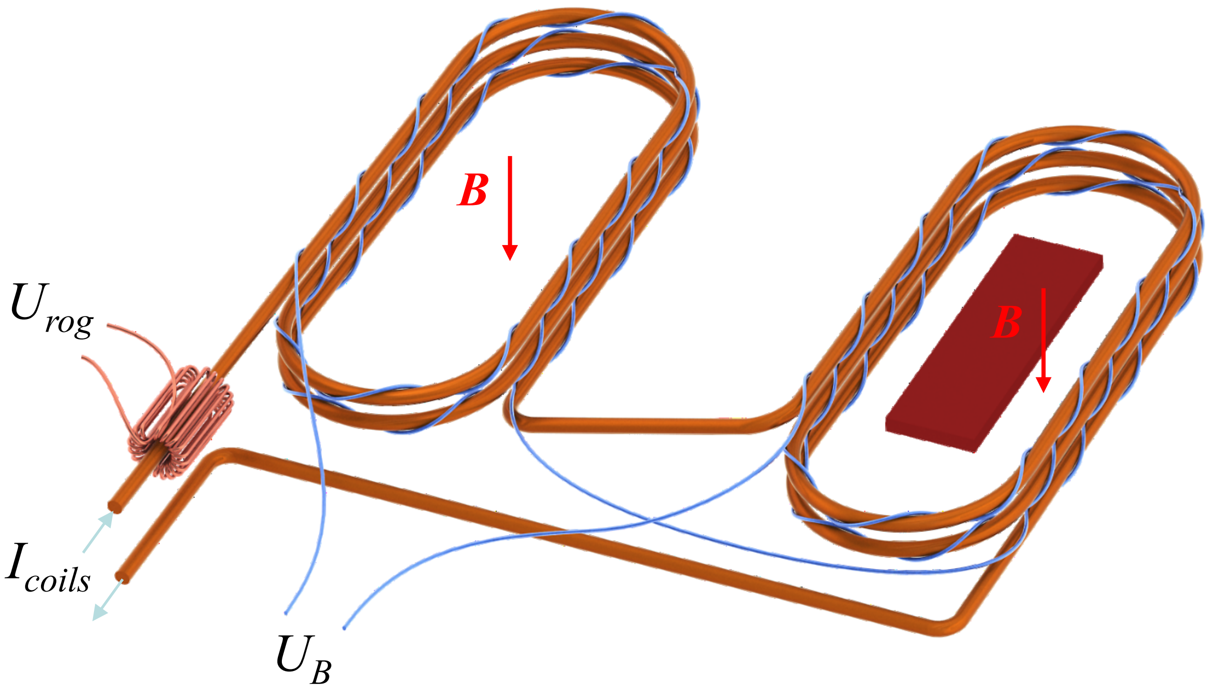
Solution 1- detection of power flow to the sample



Solution 2- elimination of parasitic power flows



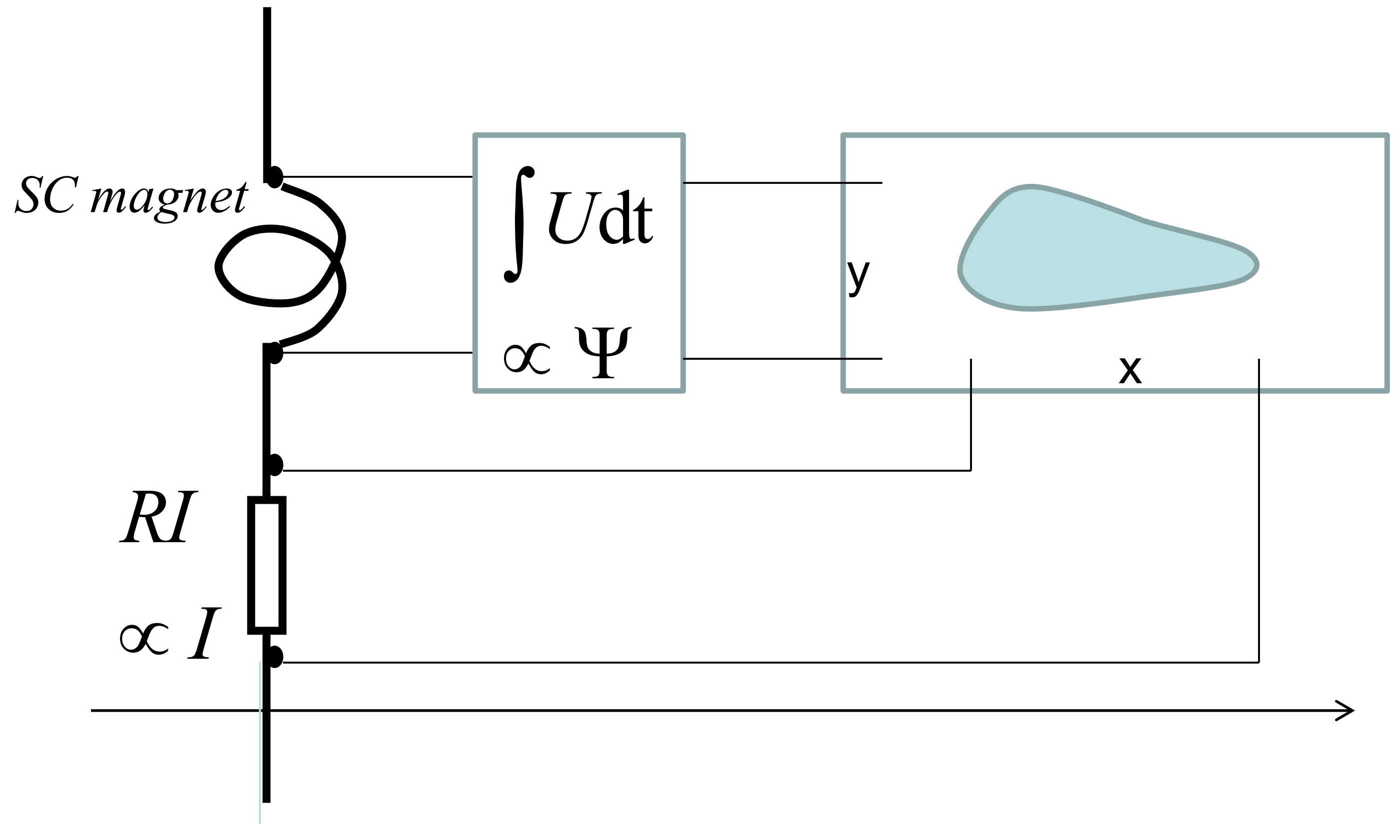
Loss measurement from the side of AC power supply:



$$P_{sample} = I_m U_B$$

Loss measurement from the side of AC power supply:

Ψ(I) hysteresis loop registration for superconducting magnet (Wilson 1969)



Conclusions:

1) Hard superconductors in dynamic regime produce heat because of magnetic flux pinning -> transient loss, AC loss

2) Extent of dissipation is proportional to macroscopic magnetic moments of currents induced because of the magnetic field change

3) Hysteresis loss (current loop entirely within the superconductor) can be reduced by the reduction of superconductor dimension

4) Coupling loss (currents connecting parallel superconductors) reduced by the transposition (twisting) and the control of transverse resistance

5) Minimization of loss often in conflict with other requirements

6) Basic principles are known, particular cases require clever approach and innovative solutions