Characterization Techniques

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Outline

- Quantities of interest
  - Intrinsic material parameters: $T_c$, $\lambda$, $\xi$;
  - Extrinsic property: $J_c$ (flux pinning)

- Measurement Techniques
  - Resistivity
  - Magnetization
  - Some others
Intrinsic Properties

- Transition Temperature, $T_c$
  - BCS: phonon frequency, DOS, coupling
  - Unconventional SC: $T_c \sim 1/\lambda^2$ (empirical)
  - Tuning normally difficult, e.g. pressure, doping
  - Highest in clean materials: scattering is pair breaking for anisotropic energy gap (or multi-gap), only second order effects in s-wave SC
Intrinsic Properties

- Magnetic penetration depth, $\lambda$
  - Length scale for field changes in a superconductor
  - Superfluid density: $\sim 1/\lambda^2$
  - Increases with impurity scattering

- Coherence length, $\xi$
  - Length scale for changes of the superconducting order parameter
  - “Size” of a Cooper pair (clean limit)
  - Decreases with impurity scattering

- Other intrinsic sc parameters can be calculated
  - Condensation energy ($E_c$), critical fields ($H_c, H_{c1}, H_{c2}$), depairing current density….
Importance of coherence length

• Upper critical field: \( B_{c2} = \frac{\phi_0}{2\pi \xi^2} \)

• Efficient pinning centers should have a radius of about \( \xi \) (orthogonal to field).

• A small \( \xi \) potentially harms the inter-grain coupling

• A small \( \xi \) increases the condensation energy \( \mu_0 \frac{H_c^2}{2} \):
  \[
  H_c = \frac{\phi_0}{2\pi\sqrt{2}\mu_0\lambda\xi}
  \]

• A small \( \xi \) favors high critical currents.

Depairing current density \( J_d = \frac{\phi_0}{3\sqrt{3}\pi\mu_0\xi\lambda^2} \)

Maximum achievable \( J_c \) by optimized pinning landscape: \( \sim 0.2 J_d \)
Determination of coherence length

• From upper critical field: $B_{c2} = \frac{\phi_0}{2\pi\xi^2}$
  - Resistivity measurements
  - Magnetization measurements
  - Specific heat measurements
• Scanning Tunneling Microscopy (STM): Size of the vortex core
• Small angle neutron scattering
Importance of magnetic penetration depth

- Lower critical field: \( B_{c1} = \frac{\phi_0}{2\pi \lambda^2} \ln(\kappa) \)

- A small \( \lambda \) is a manifestation of a large superfluid density: \( n_s = \frac{m_e}{\mu_0 \lambda^2 e^2} \)

- Condensation energy \( \mu_0 \frac{H_c^2}{2} \): \( H_c = \frac{\phi_0}{2\pi \sqrt{2} \mu_0 \lambda \xi} \)

- Depairing current density \( J_d = \frac{\phi_0}{3\sqrt{3} \mu_0 \xi \lambda^2} \) (impacts on \( J_c \))

- Surface impedance: \( Z_S = R_S + iX_S, X_S = \mu_0 \omega \lambda \)

- Unconventional SC (cuprates, IBS): \( T_c \sim 1/\lambda^2 \)
Determination of magnetic penetration depth

- From Meissner susceptibility
- From lower critical field: \( B_{c1} = \frac{\phi_0}{2\pi\lambda^2} \ln(\kappa) \)
- Reversible magnetization
  - \( B_c \) from area below rev. magnetization curve: \( \frac{B_c^2}{2\mu_0} \)
  - Fit of (available parts of) M(H) to theory.
- Magnetic Force Microscopy (MFM): Field of flux-lines, surface currents
- Small-angle neutron scattering
- Muon spin rotation (\( \mu \)SR)
- Surface impedance, e.g., cavity perturbation method: change of Q of superconducting cavity
Ginzburg-Landau Theory

Two important parameters:

1) Coherence length $\xi$: variation length of superconducting order parameter.
2) Magnetic penetration depth $\lambda$: variation length of magnetic field.

Ginzburg-Landau parameter $\kappa := \frac{\lambda}{\xi}$

- $\kappa < \frac{1}{\sqrt{2}}$: type-I superconductors
- $\kappa > \frac{1}{\sqrt{2}}$: type-II superconductors

GL-theory is in principle restricted to temperatures close to $T_c$. Magnetic behavior does not change qualitatively at low temperatures.
Relation to BCS and London Theory

- BCS coherence length $\xi_0 = \frac{\hbar v_F}{\pi \Delta(0)}$ ("size of a cooper pair")

$$\xi(T) = 0.74 \frac{\xi_0}{\sqrt{1 - t}} \quad \text{(clean limit: } \xi \ll l)$$

$$\xi(T) = 0.885 \frac{\sqrt{\xi_0} l}{\sqrt{1 - t}} \quad \text{(dirty limit: } \xi \gg l \ldots \text{ mean free path)}$$

- London penetration depth $\lambda_L(0) = \sqrt{\frac{m_e}{\mu_0 n_s e^2}} = \sqrt{\frac{3}{2\mu_0 e^2 n_F^2 N_0}}$

$$\lambda(T) = \frac{1}{\sqrt{2}} \frac{\lambda_L(0)}{\sqrt{1 - t}} \quad \text{(clean limit)}$$

$$\lambda(T) = \lambda_L(T) \frac{\xi_0}{1.33 l} \quad \text{(dirty limit)}$$
Extrinsic property: Critical current density $J_c$

- Intrinsic: depairing current density $J_d = \frac{\phi_0}{3\sqrt{3}\pi\mu_0\xi\lambda^2}$

$k_c$ is given by flux pinning: $J_c = \eta J_d$

- Extrinsic: Pinning efficiency $0 \leq \eta \lesssim 0.2$

- Pinning centers: crystal defects, grain boundaries

- Pinning engineering: (artificial pinning) nano-inclusions
Measurements of Transition Temperature

The superconductor should not be disturbed too much by the measurement: applied fields and currents etc. should be small. Most frequently used techniques:

- Resistive ($I_c$)
- Magnetic ($H_{c1}$, $H_c$, $H_{c2}$)
- Specific heat (bulk probe!)

Possible issues: surface effects, material inhomogeneities, thermal gradients
Jump in the specific heat at $T_c$

C. Senatore et al., 2007
Resistive transitions

Four point measurements: separate current and voltage contacts:

Current reversal to get rid of non-dissipative components (thermal voltage, chemical potentials)

Arbitrary evaluation criteria: 90 %, 50%, 10% (irreversibility line), tangent criteria, “zero” resistivity…
The real onset may be masked by a highly conductive sheath. Representative for the bulk?
MAGNETIZATION
MEISSNER STATE
Perfect Diamagnetism

Type I superconductor
Type II superconductor

Infinite sample: $B = \mu_0(H + M) = 0 \Rightarrow H = -M$
AC Susceptibility (SQUID)

\[ \chi = \frac{M}{H_{eff}} \]

Applied ac-field: \( H_0 \sin(\omega t) \)
Magnetization in Meissner state:
\[ -\frac{H_0}{1 - D} \sin(\omega t) \]

In-phase component of ac susceptibility (fourrier component of \( \sin(\omega t) \)):
\[ \chi' = -1 \]

Out-of-phase component (fourrier component of \( \cos(\omega t) \)):
\[ \chi'' = 0 \]

Close to \( T_c \): \( H_{c1} < H_0 \rightarrow \) Mixed state \rightarrow dissipation: \( \chi'' > 0 \) (loss peak)
DC susceptibility:
\[ \chi = \chi' \] zero field cooled (zfc), \( H_0 << H_{c1} \)
\[ \chi < \chi' \] field cooled (fc) (flux pinning)

Advantage of ac technique: Better resolution at the same \( H_0 \).
Perfect Diamagnetism

Field enhancement caused by return-flux

\textit{normaleitend} \hspace{3cm} \textit{supraleitend}
Perfect Diamagnetism

Effectively applied magnetic field below \((1-D)H_c\):

\[ H_{\text{eff}} = H_a + DM \]

\[ M = -H_{\text{eff}} \]

\[ H_{\text{eff}} = \frac{H_a}{(1 - D)} \]

(along the equator of spheroids)

\[ m = -DH_a V, \] Meissner slope \( \frac{dm}{dH_a} \) is universally \(-DV\).
Demagnetization factors

![Graph](image)

Numerical calculation for cuboids \(a \times b \times h, a > b\)

\[ D = 1 - \frac{\pi h}{2d} \]

Asymptotic behavior for thin disks (\(d \times h\)):

\[ D = 1 - \frac{\pi h}{2d} \]

Empirical:

\[ \frac{a}{h} \rightarrow \frac{d}{h} \left( \frac{4d}{P} \right) \]

\(d\): larger lateral dimension, \(P\): perimeter

Field at the edges diverges with \(h \rightarrow 0\).

\[ H_{\text{eff}} = \frac{H_a}{(1 - D)} \]
**Thin plate of lead (vector magnetometer)**

\[
M_{||sf} = -\frac{H_0 \cos(\theta)}{1 - D_{||}}
\]

\[
M_{||H_0} = M_{||sf} \cos(\theta) + M_{\perp sf} \sin(\theta)
\]

\[
M_{||H_0} = -\frac{H_0 \cos^2(\theta)}{1 - D_{||}} - \frac{H_0 \sin^2(\theta)}{1 - D_{\perp}}
\]

\[
M_{\perp sf} = -\frac{H_0 \sin(\theta)}{1 - D_{\perp}}
\]

\[
M_{\perp H_0} = M_{||sf} \sin(\theta) + M_{\perp sf} \cos(\theta)
\]

\[
M_{\perp H_0} = -H_0 \cos(\theta) \sin(\theta) \left( \frac{1}{1 - D_{||}} + \frac{1}{1 - D_{\perp}} \right)
\]
Shielding Fraction

- Experimental assessment of Meissner slope
- Calculation of shielded volume: \( V_{\text{shielding}} = - \frac{1}{D} \frac{dm}{dH_a} \)
- Shielding fraction: \( \eta = \frac{V_{\text{shielding}}}{V_{\text{sample}}} \)
  - \( \eta < 1 \): sample is not entirely superconducting: cracks, secondary phases, etc.
  - \( \eta = 1 \): sample may be OK.
- Shielding fraction may differ from superconducting volume fraction! E.g. hollow sphere, powder
- Problematic:
  - Thin samples: \( D \) is very sensitive to sample thickness
  - Sample dimensions comparable to or smaller than magnetic penetration depth \( \lambda \).
Perfect Diamagnetism?

- $B=0$ only in the bulk of the superconductor.
- $B$ penetrates the superconductor in a surface layer with thickness of the order of $\lambda$.
- If $\lambda$ is not much smaller than the smallest sample dimension, $\eta$ is reduced.

Meissner susceptibility can be used to determine the magnetic penetration depth.
Meissner susceptibility

MgB$_2$ powder

- Low shielding fraction
- Temperature dependence of susceptibility down to low temperatures
- Inter-grain coupling?

MgB$_2$ bulk (cube)

- Shielding fraction close to one below $T_c$
- Susceptibility hardly depends on temperature below 30 K.
- Material inhomogeneities or inter-grain (de-)coupling manifest near $T_c$. 
Meissner susceptibility

Sintered Sm-1111 bulks

- Grains decouple with field in sample LIC
Thin plate parallel to field

\[ \chi = \frac{M}{H_0} \]

\[ M = M_0 \left( 1 - \frac{2\lambda}{h} \tanh \left( \frac{h}{2\lambda} \right) \right), \quad M_0 = -DH_0 \]

Empirical two-fluid model: \( \lambda(T) = \frac{\lambda(0)}{\sqrt{1-t^4}}, \quad t = \frac{T}{T_c} \)
Determination of $\lambda$ from Meissner susceptibility: experimental issues

- $\chi \approx -1$ at low temperature (unless sample is very thin): high experimental resolution is needed.
- $D$ and $V$ (superconducting sample volume) have to be known very accurately.
- Fit of $D$ and $\lambda(0)$ to $\chi(T)$ for known temperature dependence of $\lambda$:
  - $\lambda(T)$ is influenced by strong coupling, symmetry of pairing, multi-bands, impurity scattering….
  - Inhomogeneities influence $\chi(T)$ close to $T_c$.
- Powder samples: geometry of grains, inter-grain coupling.
- Very thin samples: Misalignment between sample and applied field.

$\Delta \lambda(T) := \lambda(T) - \lambda(0)$ can be determined more easily/reliably.
Two-coil mutual inductance technique

\[ M \approx \pi \mu_0 \frac{r_{dr} r_{pu}}{h} \int_0^\infty dx \]

\[ \times \frac{xe^{-x}}{x \cosh(d \sqrt{i \mu_0 \omega \sigma} + \frac{i \mu_0 \omega \sigma h}{2}} \frac{\sinh(d \sqrt{i \mu_0 \omega \sigma})}{\sqrt{i \mu_0 \omega \sigma}} \]

\[ \times J_1 \left( x \frac{r_{dr}}{h} \right) J_1 \left( x \frac{r_{pu}}{h} \right) . \]
Self-Oscillating Tunnel-Diode Resonator (TDR)

- Resonance frequency \( f = \frac{1}{2\pi\sqrt{LC}} \) (e.g. 14 MHz)
- Superconductor in coil changes \( L \)
- Change of resonance frequency

Infinite slab (volume \( V_s \), width \( 2w \)): \[
\frac{\Delta f}{f} = \frac{V_s}{2V_c} \left( 1 - \frac{\lambda}{w} \tanh\left( \frac{w}{\lambda} \right) \right)
\]

(\( V_c \): Volume of inductor)

e.g. R. Prozorov and V. G. Kogan, RPP 74 (2011) 124505
Summary: Meissner susceptibility

- $M = -H_{\text{eff}}$ only if $\lambda \ll R$. (Hence, never near $T_c$!)
  - $\lambda = \lambda(T) \implies \chi \rightarrow \chi(T)$
    - Possibility for the determination of $\lambda$.
- The demagnetization effect increases the effectively applied field (quite significantly in thin samples): $H_{\text{eff}} > H_0$
- $|\chi|$ is reduced by granularity
- $\chi(T)$ is changed by inhomogeneities (in particular near $T_c$).
MAGNETIZATION
MIXED STATE
Vortices in type-II superconductors

Vortices: normal conducting core (radius $\sim \xi$), circular currents produce magnetic flux quantum $\phi_0$. 
**Ginzburg-Landau Theory**

\[
H_c = \frac{\phi_0}{2\pi\sqrt{2\mu_0\lambda\xi}}
\]

\[
H_{c1} = \frac{\phi_0}{4\pi\mu_0\lambda^2} \ln \kappa
\]

\[
H_{c2} = \frac{\phi_0}{2\pi\mu_0\xi^2} = \sqrt{2\kappa}H_c
\]

Many important parameters can be obtained from the reversible magnetization curve. Unfortunately, flux pinning normally inhibits its measurement.
Magnetization measurements of $H_{c2}$

1) $M(T)$ at fixed $H_0$

$T_c(H) \Rightarrow H_{c2}(T)$

2) $M(H)$ at fixed $T_0$

$H_{c2}(15 \text{ K})$

“$T_c(1 \text{ T})$”
Magnetization measurements of $H_{c2}$

Possible issues:

1) Magnetic background (normal state properties, sample holder, vibrations in VSM)
2) Surface contamination ($\rho_n \uparrow \rightarrow H_{c2} \uparrow$)
3) Surface superconductivity $H_{c3} = 1.69(?) H_{c2}$ (resistive measurements)
4) Fluctuations, Ginzburg number

$$G_i := 1 - \frac{T_r}{T_c} = \frac{1}{2} \left( \frac{\mu_0 k_B}{4 \pi} \frac{\gamma T_c}{B_c^2(0) \xi^3(0)} \right)^2 = \frac{1}{2} \left( \frac{2 \pi \mu_0 k_B}{\phi_0^2} \frac{\gamma \lambda^2(0) T_c}{\xi(0)} \right)^2$$
Estimation of large $H_{c2}$

Large $H_{c2}$: superconductors with large $\kappa$. $\rightarrow$ London theory is a good approximation for $H<0.5H_{c2}$: 
\[ \frac{\partial M}{\partial \ln H} = \frac{\phi_0}{8\pi\mu_0\lambda^2} \]

- Plot $m$ vs. $\ln(H)$.
- Extrapolate to $m=0$ to obtain $H_{c2}$ (underestimation)
- Alternative: Fit to alternative models for $M(H)$
Resistive transitions

Arbitrary evaluation criteria: 90 %, 50%, 10% (irreversibility line, $B_{\text{irr}}$)…

$T_c(B) \Rightarrow B_{c2}(T)$ (also specific heat)

Cuprates: $B_{c2} >> B_{\text{irr}}$
Magnetic determination of thermodynamic critical field

Usually impeded by pinning!

If pinning is weak, the reversible magnetic moment can be obtained by

\[ m_{\text{rev}} = \frac{1}{2} (m_+ + m_-) \]

\( m_+ \) (\( m_- \)): magnetic moment on increasing (decreasing) field branch
Summary: Mixed State

- Two important parameters: $\lambda$, $\zeta$.
- Determination of $\zeta$ from $H_{c2}$.
- $\lambda$ is a very important parameter but hard to assess.
- Flux pinning normally impedes the determination of the thermodynamic (or reversible) magnetization curve.
FLUX PINNING
CRITICAL CURRENTS
Critical State Model

Flux pinning balances the Lorentz force:

\[ F_L = J_c0 \times B = -F_p^{\text{max}} \]

Current (or pinning) causes field gradients:

\[ \nabla \times B = \mu_0 J \]

Current results in a magnetic moment:

\[ m = \frac{1}{2} \int r \times J d^3r \]

Bean Model: Current density in a superconductor is either 0 or \( J_c \). The direction of the current depends on geometry and history. If \( I < I_c \), \( J_c \) does not flow in the entire sample volume.
Thermally activated depinning

\[ \nu = \nu_0 e^{-\frac{U_0}{k_B T}} \]  

depinning rate

\[ U(J) = U_0 \left( 1 \pm \frac{J}{J_{c0}} \right) \]  

activation barrier

Net forward jump rate:

\[ e^{-\frac{U_0 (1-J/J_{c0})}{k_B T}} - e^{-\frac{U_0 (1+J/J_{c0})}{k_B T}} = 2e^{-\frac{U_0}{k_B T}} \sinh \frac{JU_0}{J_{c0} k_B T} \]

\[ \rightarrow E(J) = 2\rho_c J_{c0} e^{-\frac{U_0}{k_B T}} \sinh \frac{JU_0}{J_{c0} k_B T} \]

1) \( E(J) = \rho_c J_{c0} e^{-\frac{U_0}{k_B T}} \left( e^{\frac{U_0}{k_B T} J_{c0}} - 1 \right), \quad J \ll J_{c0} \)  

flux creep

2) \( E(J) = 2\rho_c \frac{U_0}{k_B T} e^{-\frac{U_0}{k_B T} J} =: \rho_{\text{T AFF}} J, \quad J \ll J_{c0} \)  

thermally assisted flux flow

\[ \rightarrow J_{c} := J(E_{\text{crit}}) < J_{c0} \]
Thermally activated depinning

\[ J_c := J(E_{\text{crit}}) < J_{c0} \]

The electric field criterion, \( E_{\text{crit}} \), is chosen arbitrarily or imposed by the experiment.
Thermally activated depinning

\[ E_{\text{crit}}(E_c) \]

\[ J_{c0}(B,T) \rightarrow J(E,B,T) \]

\[ J_c^{\text{exp}} = J(E_c) < J_{c0} \]
## Typical electric field criteria

<table>
<thead>
<tr>
<th>Experimental technique</th>
<th>$E_c$ (µV/cm)</th>
<th>Depending on</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistive transport</td>
<td>0.1-10</td>
<td>Your choice, sample size</td>
</tr>
<tr>
<td>SQUID</td>
<td>$10^{-8}-10^{-3}$</td>
<td>Sample size, $J_c$, $S$ (material, field, temperature)</td>
</tr>
<tr>
<td>VSM (sweep mode)</td>
<td>$10^{-4}-0.1$</td>
<td>Sample size, field sweep rate</td>
</tr>
<tr>
<td>VSM (step mode)</td>
<td>$10^{-7}-10^{-2}$</td>
<td>Sample size, material, field, temperature</td>
</tr>
<tr>
<td>AC-susceptibility</td>
<td>$&lt;10^{-5} - 1$</td>
<td>Sample size, amplitude and frequency, $J_c$</td>
</tr>
<tr>
<td>Scanning Hall probe</td>
<td>$10^{-9}-10^{-3}$</td>
<td>Sample size, material, field, temperature</td>
</tr>
</tbody>
</table>

Cuboid in a VSM:

\[
E = \frac{U}{2(a + b)} = -\frac{1}{2(a + b)} \frac{d\phi}{dt} = -\frac{ab}{2(a + b)} \frac{dB}{dt}
\]

Field ramp rate

SQUID and SHP measurements: Electric field given by the relaxation of the magnetization.
SQUID vs. Resistive

Huge difference at high magnetic fields!

Transport (1µV/cm) vs. SQUID

$J_c (\text{A/m}^2)$

$\mu_0H(T)$

$77 \text{ K}$
Assessment of $J(E)$ in a wide range

$E = E_c \left( \frac{J}{J_c} \right)^n$

Combination of different techniques
CRITICAL CURRENT TRANSPORT MEASUREMENTS
I-V curves

“Textbook”
**I-V curves**

Empirical: \( E = E_c \left( \frac{J}{J_c} \right)^n \)  

\( n \)-value (large \( n \) is desired)
$J_c$-evaluation

$J_c = J_c^{\exp} = J(E_c)$

- Common electric field criteria: $E_c=0.1$ or 1 µV/cm
  - Simple intersection with I-V curve
- Fit of I-V curve by $E = E_c \left( \frac{J}{J_c} \right)^n \rightarrow J_c, n$
- Slope of I-V curve in double logarithmic representation: $n$
Experimental Issues

- Heating/thermal voltages
- Strain: thermal expansion, Lorentz force
- Thermal instabilities: quench
- Transfer length (linear component) (minimum distance between current and voltage taps)

![Graph showing V/V_c vs. I/I_c^exp](image)
CRITICAL CURRENT
MAGNETIC MEASUREMENTS
Bean Model

\[ m = \frac{1}{2} \int r \times J \, d^3r = J_c \frac{1}{2} \int r \times e \, d^3r \]
# Bean Model: Formulae

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Relation between $J_c$ and the irreversible component of $m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cylinder of radius $R$ and height $h$, axis parallel to $H$.</td>
<td>$J_c = \frac{3m_{irr}}{\pi R^3 h}$</td>
</tr>
<tr>
<td>Cylinder of radius $R$ and height $h$, axis normal to $H$, $h &gt; 2R$</td>
<td>$J_c = \frac{3m_{irr}}{4R^3 h(1 - 3\pi R/16h)}$</td>
</tr>
<tr>
<td>Cuboid with dimensions $a \times b \times c$, $H$ parallel to $c$, $a \geq b$.</td>
<td>$J_c = \frac{4m_{irr}}{ab^2c(1 - b/3a)}$</td>
</tr>
<tr>
<td>Cuboid with dimensions $a \times b \times c$, $H$ parallel to $c$, $b \geq a$.</td>
<td>$J_c = \frac{4m_{irr}}{a^2bc(1 - a/3b)}$</td>
</tr>
<tr>
<td>Sphere of radius $R$.</td>
<td>$J_c = \frac{8m_{irr}}{\pi^2 R^4}$</td>
</tr>
</tbody>
</table>

For the fully penetrated state (all current loops have the same orientation)
Different Symmetry of Reversible and Irreversible Magnetization

\[ m = m_{\text{irr}} + m_{\text{rev}} \]

\[ m_{\text{irr}}(H) = -m_{\text{irr}}(-H) \]

\[ m_{\text{irr}}(H) = m_{\text{irr}}(-H) \]

\[ m_{\text{irr}}(H) = \frac{|m_-(H) - m_+(H)|}{2} \]

\[ m_{\text{irr}}(H) = \frac{|m_-(H) + m_+(-H)|}{2} \]

\[ m_{\text{rev}}(H) = -m_{\text{rev}}(-H) \]

\[ m_{\text{rev}}(H) = \frac{m_-(H) + m_+(H)}{2} \]
**Self field correction**

\[ J_c \neq J_c(H) \]
\[ J_c = J_c(|B|) \]

\[ B = \mu_0 (H_0 + M_{rev}) + B_{self} \]

\[ \nabla \times B = \mu_0 J \]

Infinite cylinder: \( \frac{\partial B_z}{\partial r} = -\mu_0 J_\phi \) \( \frac{\partial B_z}{\partial r} \) linear field profile, slope proportional to \( J_c \).

Calculation of self-field is normally based on the Biot-Savart law:

\[ B_{self}(r) = \int \frac{J \times r'}{|r - r'|^3} d^3 r' = J_c \int \frac{e_J \times r'}{|r - r'|^3} d^3 r' \]

Influence of aspect ratio on self-field

\[ H^* = H(r = 0) = \frac{J_c h}{2} \ln \frac{\sqrt{h^2 + d^2 + d}}{h} \]

Rough estimate:

\[ H^* = J_c \frac{a}{2} \]

\( a \): smallest sample dimension
Self-field correction

Shift of measured points along x-axis:

\[ m(H) \rightarrow m(B) \]

\[
B_{av} = \frac{1}{N} \int_V |\mu_0(H_0 + M_{rev}) + B_{self}(r)| r^2 d^3r
\]

\[ J_c = J_c(|B|)! \]

Iterative calculation:

1) Calculate \( J_c(H) \) and \( M_{rev}(H) \).
2) Calculate \( B_{av} \) from \( B_{self}(H,J_c(H)) \) and \( M_{rev}(H) \).
3) Calculate \( J_c(B_{av}), M_{rev}(B_{av}) \) from \( m_+(B_{av}) \) and \( m_-(B_{av}) \).
4) Re-iterate \( B_{av} \) from \( B_{self}(B_{av},J_c(B_{av})) \), and \( M_{rev}(B_{av}) \).
5) Continue with 3) until \( B_{av} \) remains constant for all \( m \).
6) \( J_c(B) = J_c(B_{av}) \).

e.g. M. Zehetmayer, Phys. Rev. B 80 (2009) 104512
Self-field correction

\[ J_c \text{ not assessable for small fields } (B<\sim B') \text{ with transport nor magnetization measurements.} \]
Sloppy Treatment in Literature

- \( J_c(H) \)!
- Flux jumps
- \( H^* \) not taken into account properly

Diagram:
- Y-axis: \( J_c \) (A cm\(^{-2}\))
- X-axis: \( B \) (T)
- Legends for samples with different concentrations and magnetic fields.
Surface barriers/pinning

Strongly asymmetric magnetization loop

Decomposition in reversible and irreversible moment

- Weak bulk pinning or decoupled grains (powder)
- Distinction of surface effects from significant contribution of $M_{\text{rev}}$ often difficult:
  - Self field correction (including demagnetization).
  - Estimation of $M_{\text{rev}} (H_{c1})$ from $\lambda$ and $\zeta$ (if known).
Granularity

So far: perfectly homogeneous sample. Limit of sample consisting of decoupled (cubic) grains: Sum of contribution of all grains.

$$J^\text{intra}_c = N \frac{6m^\text{intra}_\text{irr}}{a_g^4} = \frac{6m^\text{irr}}{a^3 a_g}$$

- $a_g$: grain size
- $N = a^3 / a_g^3$: number of grains

Weakly coupled grains: $m^\text{irr} = m^\text{intra}_\text{irr} + m^\text{inter}_\text{irr}$

$$\frac{m^\text{inter}_\text{irr}}{m^\text{intra}_\text{irr}} = a \frac{J^\text{inter}_c}{a_g J^\text{intra}_c}$$ (cubic sample and grains)

If this ratio is either small or large, $J^\text{intra}_c$ or $J^\text{inter}_c$ can be derived approximately.
Granularity (Scanning Hall Probe Microscopiy, SHPM)

Calculation of local currents by numerical inversion of the Biot-Savart law.
Granularity (SHPM)
Granularity (SHPM)

Average $j_c^{\text{intra}}$ and $j_c^{\text{inter}}$ can be derived.

J. Hecher et al.,
SUST 29 (2016) 025004.
Field where the magnetization curve becomes reversible $\leftrightarrow J_c \to 0$.

Reality: $J(B, E_{\text{crit}}) < J_{\text{crit}}$
MAGNETOMETERS
Superconducting Quantum Interference Device (SQUID)

- The sample is moved through a pick-up coil system.
- The net flux is coupled to a SQUID sensor.
- Fit of the signal ($U(z)$) to the theoretical curve.
- Typical resolution limit:
  - $5 \times 10^{-11}$ Am$^2$ in zero field
  - $5 \times 10^{-10}$ Am$^2$ at a few Tesla
  - Even better in the VSM mode

Second order gradiometer
Vibrating Sample Magnetometer (VSM)

- First order gradiometer or Mallison coil set.
- Sample vibrates and induces a voltage:

\[
U \propto \frac{d\phi}{dt} = \frac{d}{dt} \int B_z(x, y, z - z_0) dx dy = \frac{d\phi(z - z_0)}{dt}
\]

\[
= - \frac{d\phi(z - z_0)}{dz_0} \frac{dz_0}{dt} = A\omega \frac{d\phi(z)}{dz}
\]

- Typical resolution: \(10^{-9}\) to \(10^{-8}\) Am\(^2\)
- Faster than a SQUID
- Field sweep mode