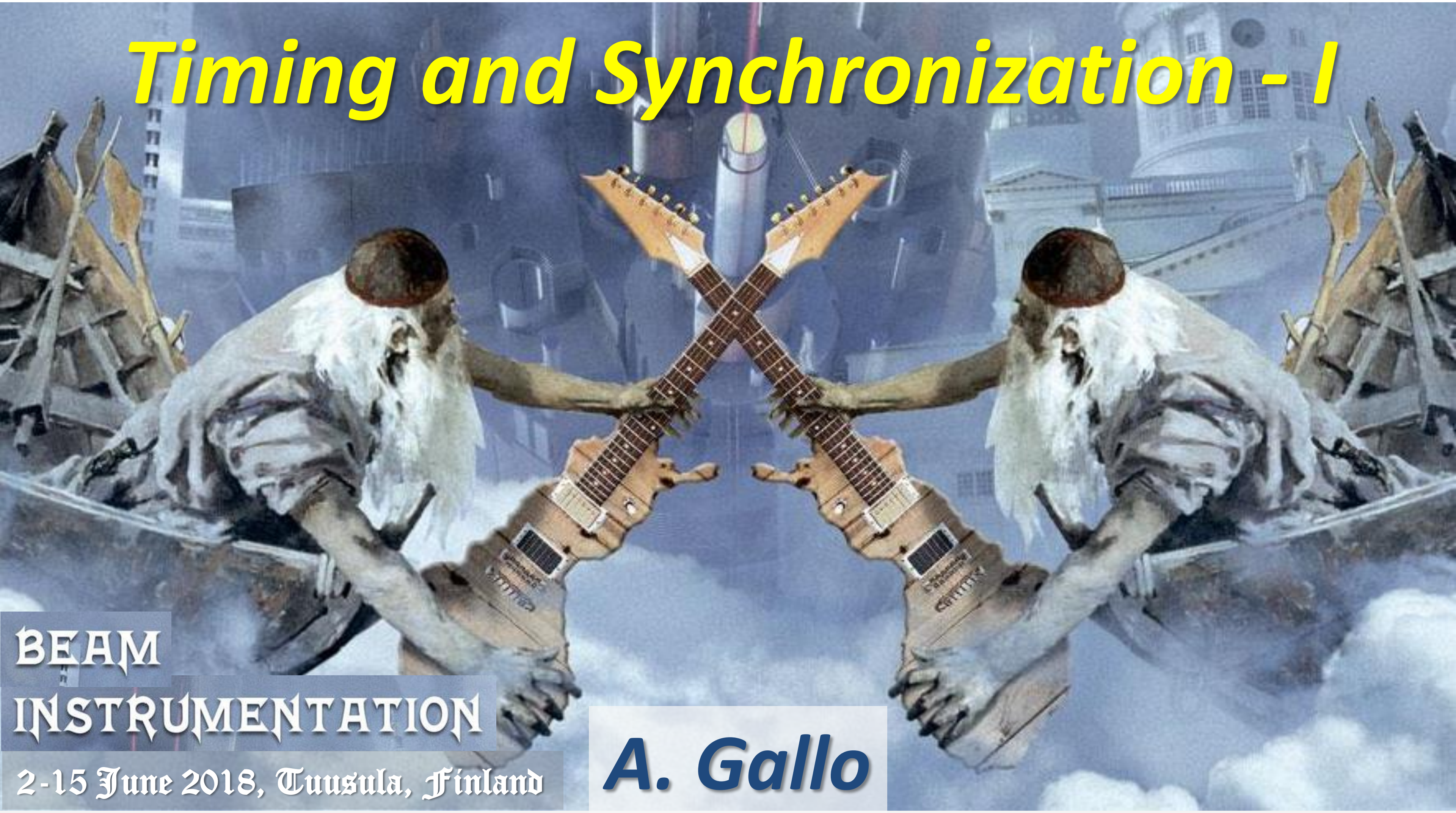
Timing and Synchronization - I



BEAM INSTRUMENTATION

2-15 June 2018, Tuusula, Finland

A. Gallo

Istituto Nazionale di Fisica Nucleare Laboratori Nazionali di Frascati

via Enrico Fermi 40 - 00044 Frascati(RM) - Italy

Lecture I&II Outline

Lecture I

- **MOTIVATIONS**
  - ✓ Why accelerators need synchronization, and at what precision level
- **DEFINITIONS AND BASICS**
  - ✓ Glossary: Synchronization, Master Oscillator, Drift vs. Jitter
  - ✓ Fourier and Laplace Transforms, Random processes, Phase noise in Oscillators
  - ✓ Phase detectors, Phase Locked Loops, Precision phase noise measurements

Lecture II

  - ✓ Electro-optical and fully optical phase detection
- **SYNCRONIZATION ARCHITECTURE AND PERFORMANCES**
  - ✓ Phase lock of synchronization clients (RF systems, Lasers, Diagnostics, ...)
  - ✓ Residual absolute and relative phase jitter
  - ✓ Reference distribution – actively stabilized links
- **BEAM ARRIVAL TIME FLUCTUATIONS**
  - ✓ Bunch arrival time measurement techniques
  - ✓ Expected bunch arrival time downstream magnetic compressors (an example)
  - ✓ Beam synchronization – general case

CONCLUSIONS AND REFERENCES

GENERAL DEFINITION

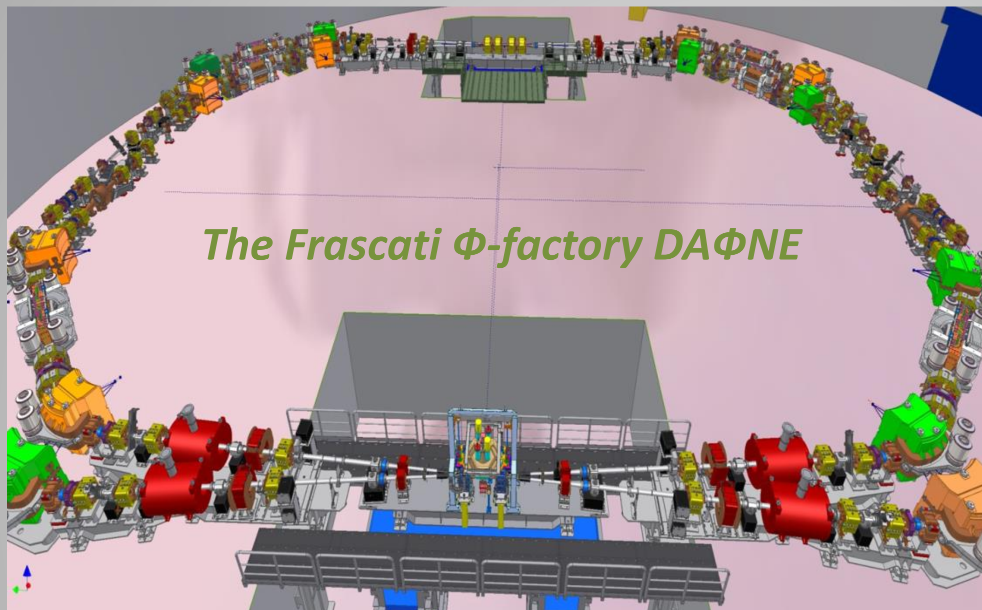
Every accelerator is built to produce some ***specific physical process***.

One ***necessary condition*** for an efficient and stable machine operation is that ***some events have to happen at the same time*** (simultaneously for an observer in the laboratory frame) or in a ***rigidly defined temporal sequence***, within a maximum allowed time error budget.

If the ***simultaneity*** or the time separation ***of the events fluctuates*** beyond the specifications, ***the performances of the machine are spoiled***, and the quantity and quality of the accelerator products are compromised.

Clearly, the tolerances on the time fluctuations are different for different kind of accelerators. The ***smaller the tolerances***, the ***tighter the level of synchronization required***. In the last 2 decades a new generation of accelerator projects such as FEL radiation sources or plasma wave based boosters ***has pushed the level of the synchronization specifications down to the fs scale***.

synchronization of
FLAT BEAM COLLIDERS

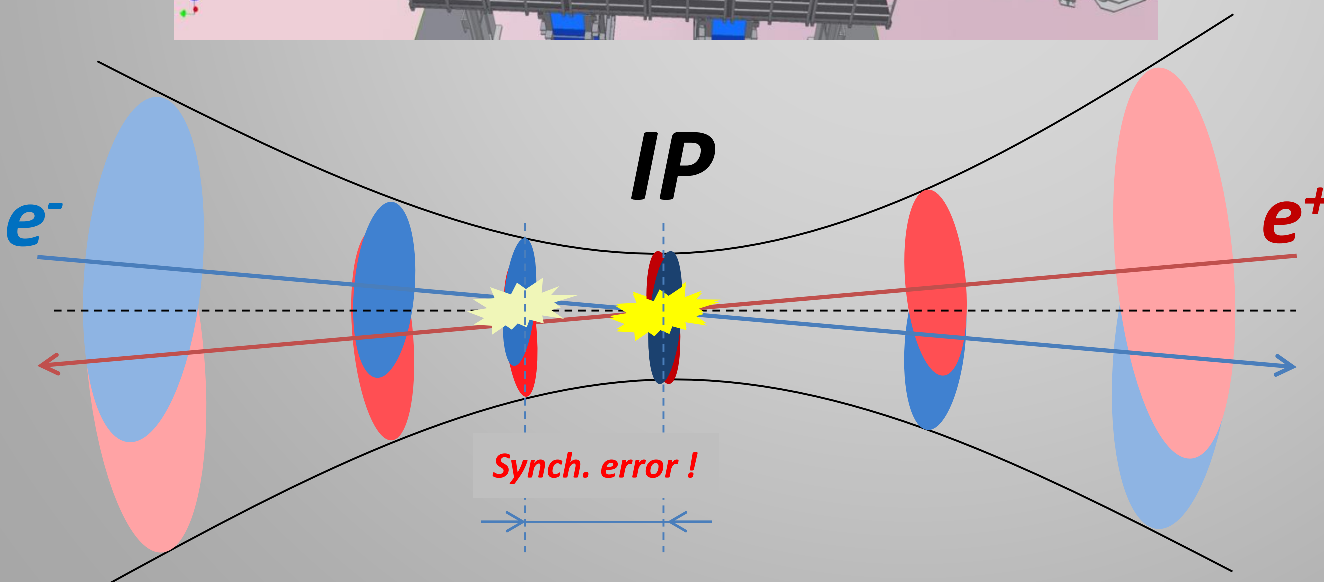


Bunches of the 2 colliding beams need to ***arrive*** at the ***Interaction Point*** (max vertical focalization) at the same time.

Waist length $\approx \beta_y \approx \sigma_z$ *(hourglass effect)*

Synchronization requirement:

$$\Delta t \ll \sigma_{t_{bunch}} = \frac{1}{c} \cdot \sigma_{z_{bunch}}$$



CIRCULAR COLLIDERS:

$\sigma_z \approx 1\,cm \rightarrow \Delta t < 10\,ps$

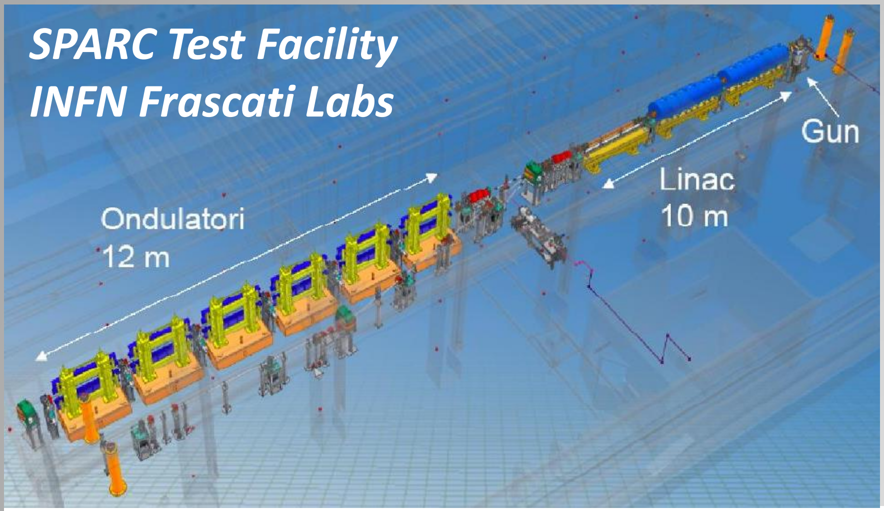
LINEAR COLLIDER (ILC):

$\sigma_z < 1\,mm \rightarrow \Delta t < 1\,ps$

RF Stability spec

synchronization of
SASE FELs

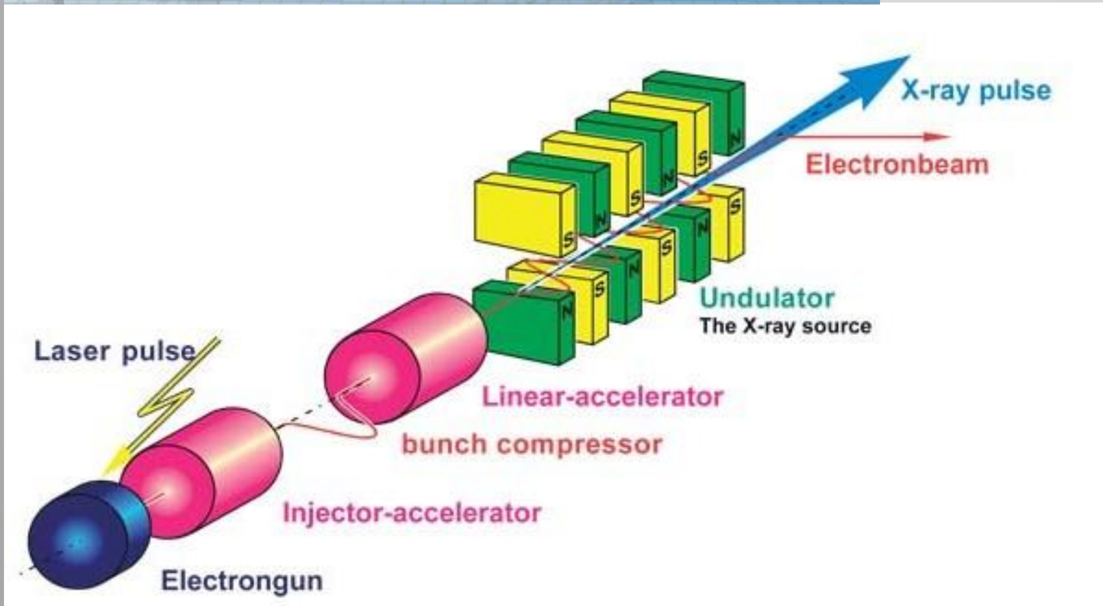
SPARC Test Facility
INFN Frascati Labs



Free Electro Laser machines had a crucial role in pushing the accelerator synchronization requirements and techniques to a new frontier in the last ≈15 years.

The simplest FEL regime, the ***SASE*** (***Self-Amplified Spontaneous Emission)***, requires high-brightness bunches, being:

$$B \div \frac{I_{bunch}}{\epsilon_{\perp}^{2}}$$



Large peak currents I_{bunch} are typically obtained by ***short laser pulses*** illuminating a ***photo-cathode*** embedded in an RF Gun accelerating structure, and furtherly increased with ***bunch compression*** techniques.

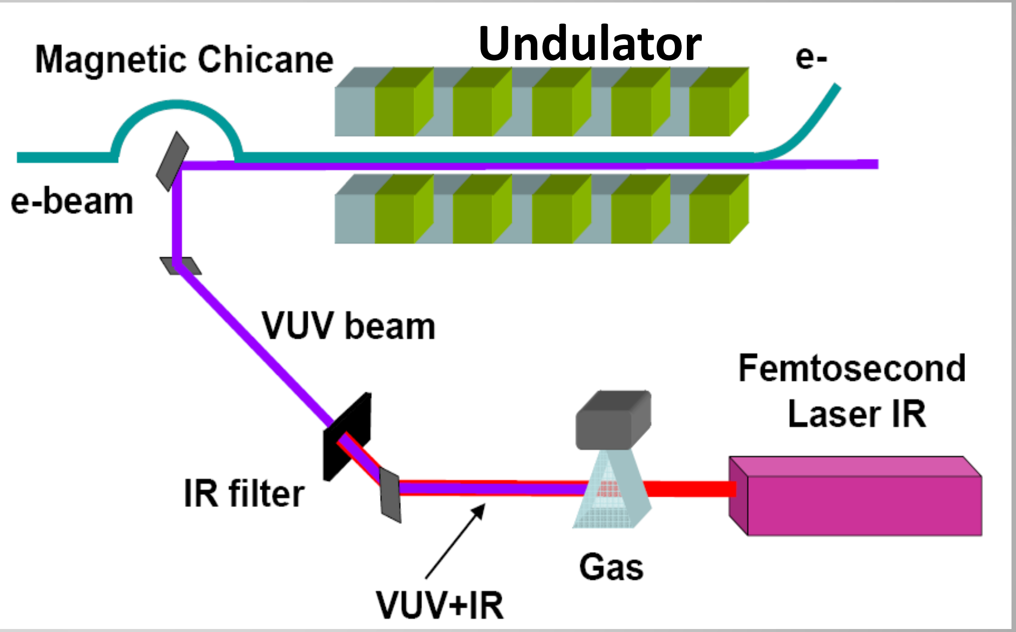
Small transverse emittances $\epsilon_{\perp}$ can be obtained with ***tight control*** of the global machine WP, including amplitude and phase of the RF fields, magnetic focusing, laser arrival time, …

Global Synchronization requirements: < 500 fs rms

synchronization of Seeded FELs

In a simple SASE configuration the ***micro-bunching process***, which is the base of the FEL radiation production, starts from ***noise***. Characteristics such as radiation intensity and envelope profile can vary considerably from shot to shot.

A better control of the radiation properties resulting in more ***uniform*** and ***reproducible*** shot to shot pulse characteristics can be achieved in the "***seeded***" FEL configuration.



To "trigger" and guide the avalanche process generating the exponentially-growing radiation intensity, the ***high brightness bunch*** is made to interact with a ***VUV*** short and intense ***pulse*** obtained by HHG (High Harmonic Generation) in gas driven by an IR pulse generated by a dedicated high power laser system (typically TiSa). The presence of the external radiation since the beginning of the micro-bunching process inside the magnetic undulators seeds and drives the FEL radiation growth in a steady, repeatable configuration. The ***electron bunch*** and the ***VUV pulse***, both ***very short***, must constantly ***overlap*** in ***space*** and ***time*** shot to shot.

Synchronization requirements (e- bunch vs TiSa IR pulse): < 100 fs rms

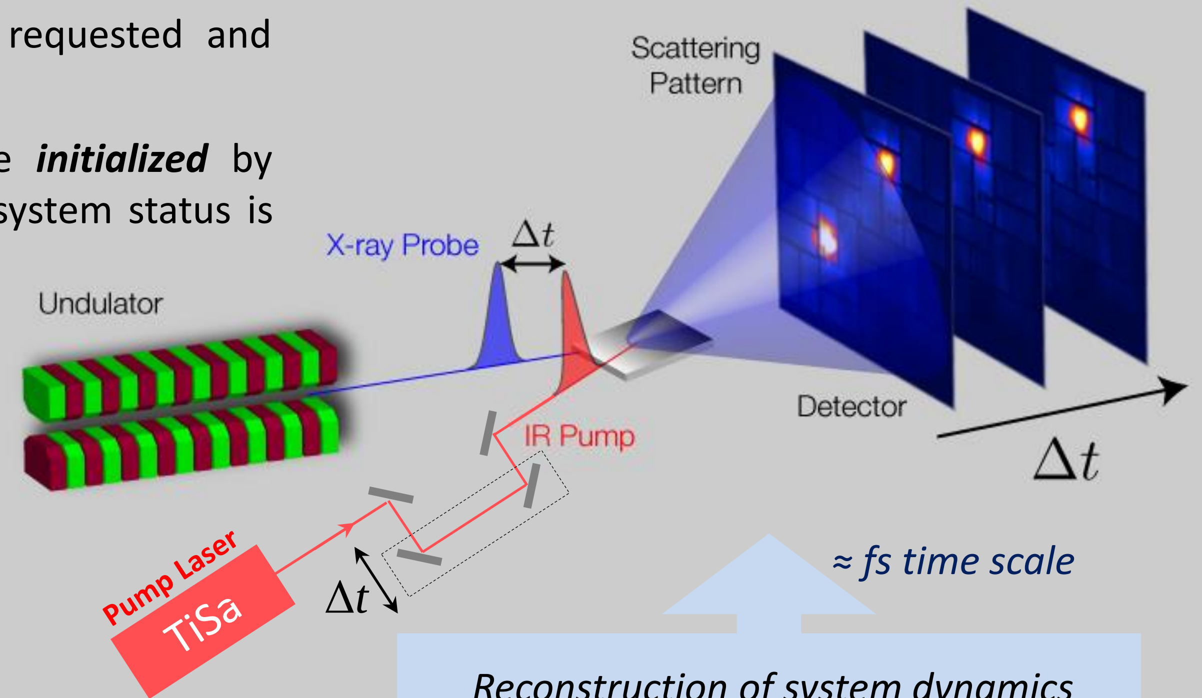
Pump-probe with FELs

Pump-probe technique is widely requested and applied by user experimentalists.

Physical / chemical processes are ***initialized*** by ultra-short ***laser pulses***, then the system status is ***probed*** by ***FEL radiation***.

The dynamics of the process under study is captured and stored in a "snapshots" record.

Pump laser and FEL pulses need to be ***synchronized*** at level of the ***time-resolution*** required by the experiments (down to ≈ ***10 fs***).



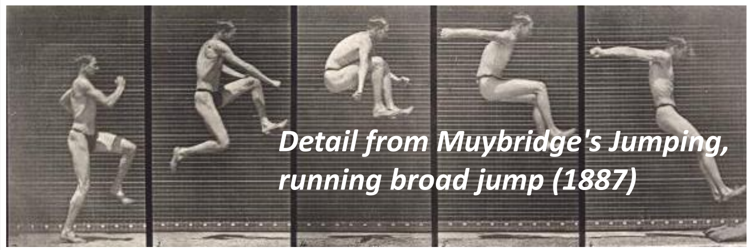
The relative delay between pump and probe pulses needs to be finely and precisely scanned with proper time-resolution.

≈ fs time scale

Reconstruction of system dynamics

≈ ms time scale

Synchronization requirements (FEL vs Pump Laser pulses): ≈ 10 fs rms

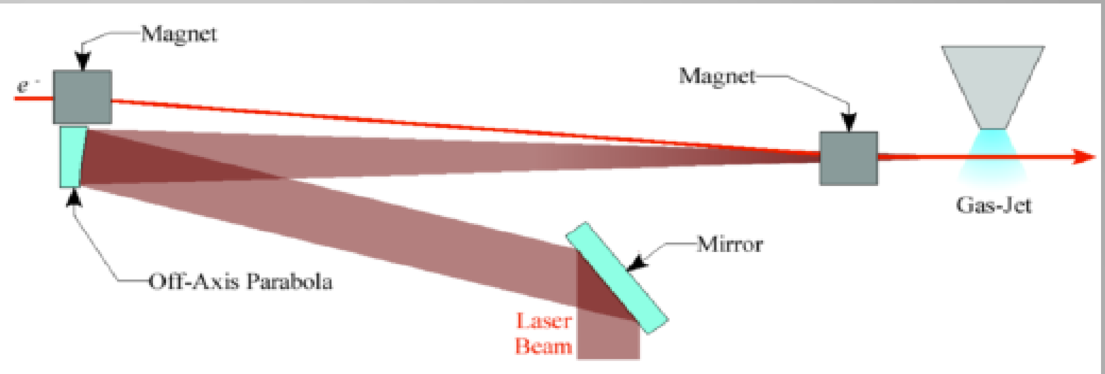


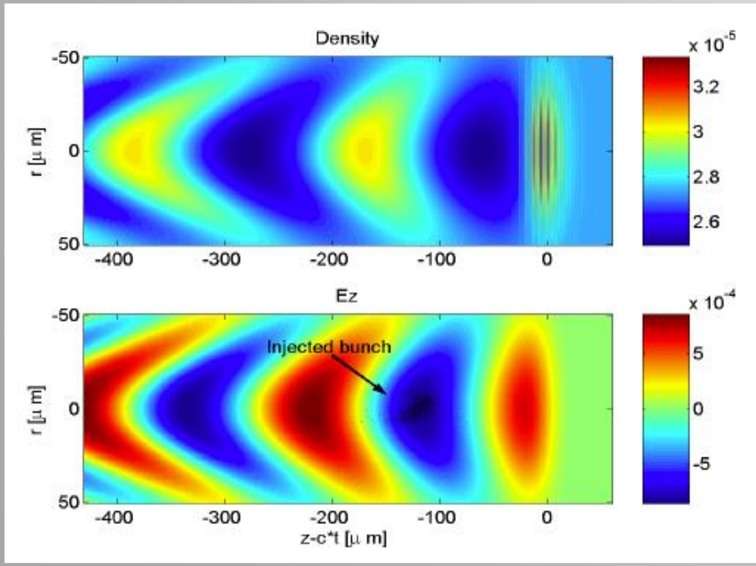
Detail from Muybridge's Jumping, running broad jump (1887)

synchronization of
WLPA of injected bunches

Plasma acceleration is the new frontier in accelerator physics, to overcome the gradient limits of the RF technology in the way to compact, high energy machines.

Wakefield Laser-Plasma Acceleration (WLPA) is a technique using an extremely intense laser pulse on a gas jet to generate a plasma wave with large accelerating gradients (many GV/m).





To produce good quality beams external bunches have to be injected in the plasma wave. The "accelerating buckets" in the plasma wave are typically few 100 µm long.

The injected bunches have to be very short to limit the energy spread after acceleration, and ideally need to be injected constantly in the same position of the plasma wave to avoid shot-to-shot energy fluctuations.

This requires synchronization at the level of a small fraction of the plasma wave period.

Synchronization requirements (external bunch vs laser pulse):
< 10 fs rms

SUMMARY

Circular colliders	*10 ps*	*Low-level RF, beam FBKs, ...*
Future Linear colliders	*1 ps*	*LLRF systems, beam FBKs, ...*
SASE FELs		*Photocathode (PC) laser, LLRF, ...*
Compton sources		*+ Interaction laser*
Seeded FELs	*100 fs*	*Seeding and PC lasers, LLRF, ...*
FELs Pump-Probe	*10 fs*	*FEL (*), Pump laser, ...*
WLPA External injection		*Injected beam (#), Power laser, ...*
	1 fs	
Upcoming ...	*sub fs*	**...**

(#) depends on all RF and laser systems of the injector

(*) depends on beam (LLRFs + PC laser) and laser seed (if any)

BASICS

Glossary:

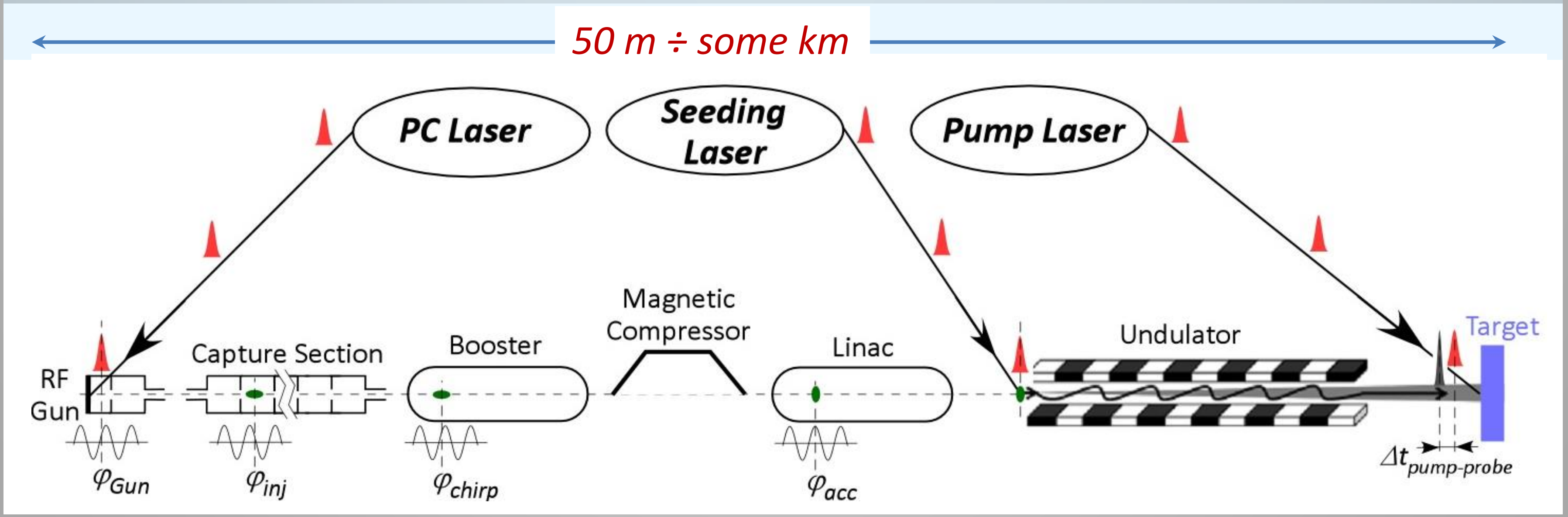
- *Synchronization*
- *Master Oscillator*
- *Drift vs. Jitter*

GLOSSARY:
SYNCHRONIZATION

Every accelerator is built to produce some ***specific physical processes*** (shots of bullet particles, nuclear and sub-nuclear reactions, synchrotron radiation, FEL radiation, Compton photons, ...).

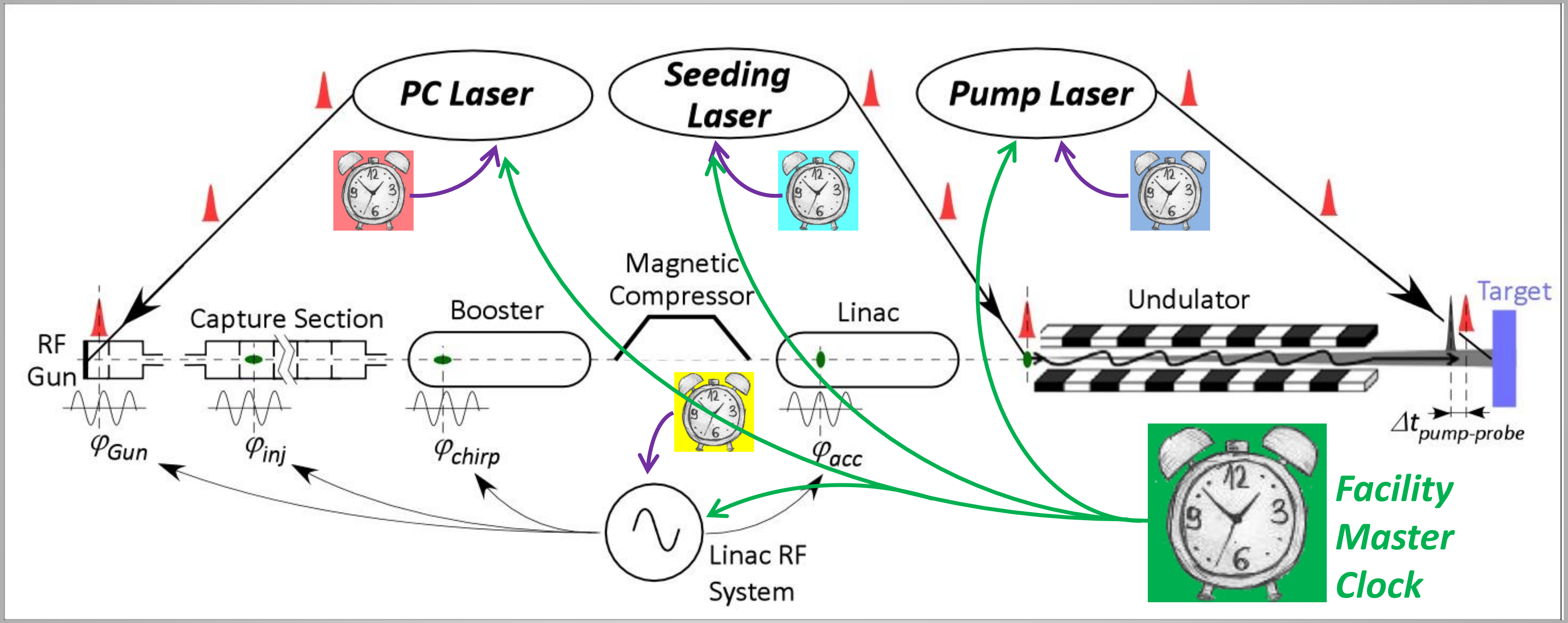
It turns out that a ***necessary condition*** for an efficient and reproducible event production is the ***relative temporal alignment*** of ***all the accelerator sub-systems*** impacting the beam longitudinal phase-space and time-of-arrival (such as RF fields, PC laser system, ...), and of the ***beam bunches*** with ***any other system they have to interact with*** during and after the acceleration (such as RF fields, seeding lasers, pump lasers, interaction lasers, ...).

The ***synchronization system*** is the complex including all the ***hardware***, the ***feedback processes*** and the ***control algorithms*** required to keep ***time-aligned*** the ***beam bunches*** and ***all the machine critical sub-systems*** within the facility specifications.



MASTER OSCILLATOR

Naive approach: can each sub-system be synchronized to a local high-stability clock to have a good global synchronization of the whole facility ?

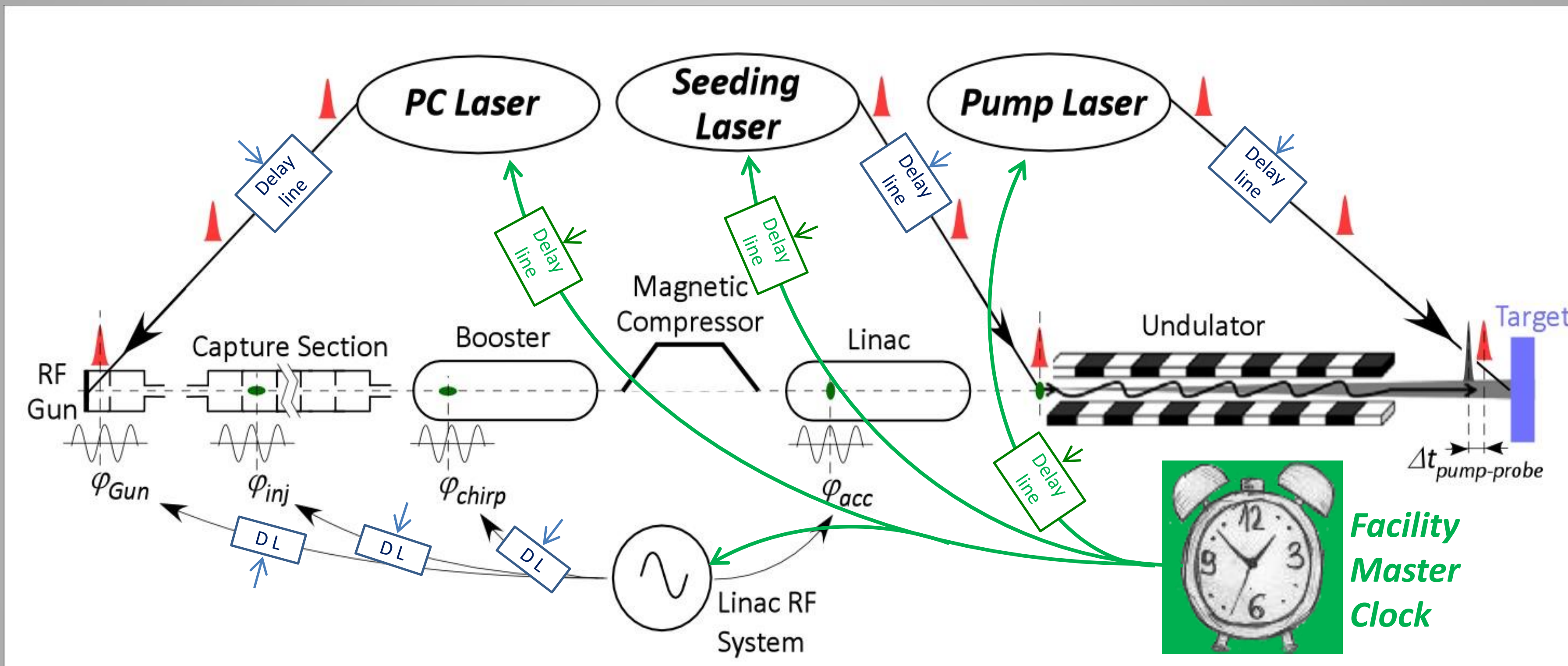


Best optical clocks → $\Delta\omega/\omega \approx 10^{-18}$ → $\Delta T/T \approx 10^{-18}$ → $T \approx 10\, fs/10^{-18} \approx$ ***3 hours !!!***

It is impossible to preserve a tight phase relation over long time scales even with the state-of-the-art technology.

All sub-systems need to be ***continuously re-synchronized*** by a ***common master clock*** that has to be distributed to the all "clients" spread over the facility with a "***star***" network architecture.

MASTER OSCILLATOR



Once the local oscillators have been locked to the reference, they can be shifted in time by means of delay lines of various types – translation stages with mirrors for lasers, trombone-lines or electrical phase shifters for RF signals. This allows setting, correcting, optimizing and changing the working point of the facility synchronization.

Delay lines can be placed either downstream the oscillators or on the reference signal on its path to the client oscillator. The function accomplished is exactly the same.

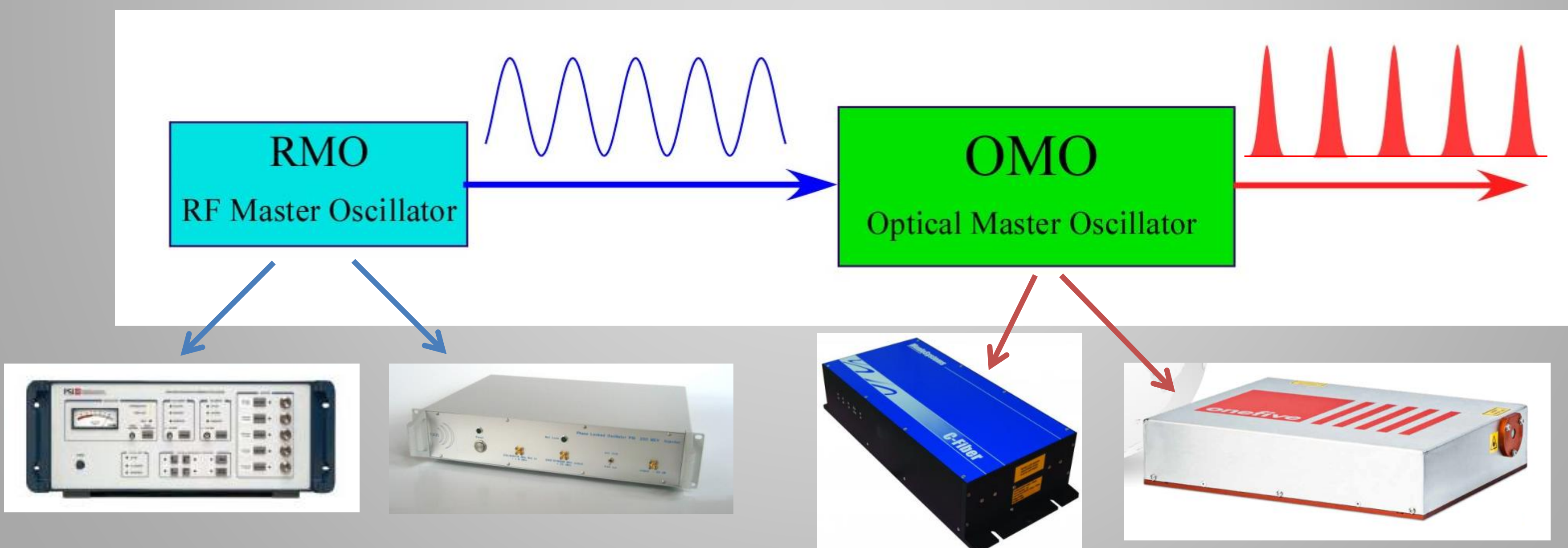
For simplicity, in most of the following sketches the presence of the delay lines will be omitted.

MASTER OSCILLATOR

The ***Master Oscillator*** of a facility based on particle accelerators is typically a ***good***(*), ***low phase noise*** μ-wave generator acting as timing reference for the machine sub-systems. It is often indicated as the ***RMO*** (***RF Master Oscillator***).

The timing reference signal can be distributed straightforwardly as a pure sine-wave voltage through coaxial cables, or through optical-fiber links after being ***encoded in the repetition rate of a pulsed (mode-locked) laser*** (or sometimes in the amplitude modulation of a CW laser).

Optical fibers provide ***less signal attenuation*** and ***larger bandwidths***, so optical technology is definitely preferred for synchronization reference distribution, at least for large facilities.



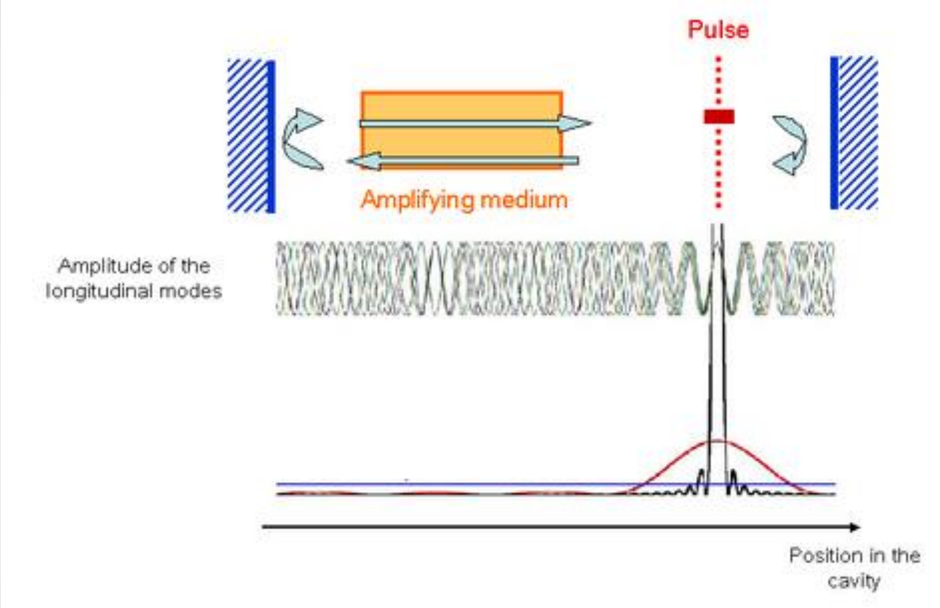
(*) the role of the phase purity of the reference will be discussed later

GLOSSARY:

OPTICAL MASTER OSCILLATOR

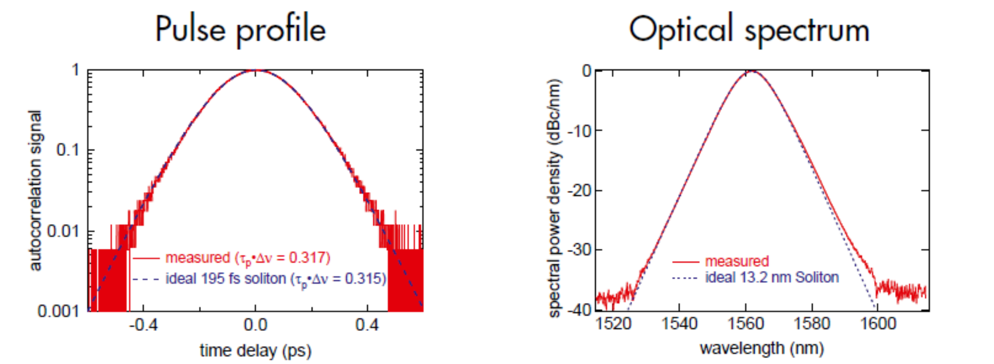
Optical: mode-locked lasers

A ***mode-locked laser*** consists in an ***optical cavity*** hosting an active (amplifying) medium capable of sustaining ***a large number of longitudinal modes*** with frequencies $\nu_k = k\nu_0 = kc/L$ within the bandwidth of the active medium, being L the cavity round trip length and k integer. If the modes are forced to ***oscillate in phase*** and the medium emission BW is wide enough, a ***very short pulse*** (≈100 fs) travels forth and back in the cavity and a sample is coupled out through a leaking mirror.



Origami onefive

Laser specifications	Origami-05	Origami-08	Origami-10	Origami-15
Center wavelength	513 – 535 nm	765 – 785 nm	1025 – 1070 nm	1530 – 1586 nm
Pulse Duration[1,2]	<100 – 230 fs	<60 – 200 fs	<70 – 400 fs	<80 – 500 fs
Avg. output power (up to)[2]	100 mW	30 mW	250 mW	120 mW
Pulse energy (up to)[2]	1.2 nJ	0.7 nJ	5 nJ	2 nJ
Peak power (up to)[2]	10 kW	4.5 kW	30 kW	15 kW
Pulse repetition rate[2]	20 MHz – 1.3 GHz			
Spectral bandwidth	transform-limited ($\tau_p \cdot \Delta\nu \sim 0.32$) → $1/\pi$			
Beam quality	$M^2 < 1.1$, TEM_{00}			
PER	> 23 dB			
Amplitude noise (24 h)	< 0.2% rms, < 0.5% pk-pk			
Center wavelength drift	< 0.1 nm pk-pk			
Laser output	collimated free space (fiber output optional)			



http://www.onefive.com/ds/Datasheet%20Origami%20LP.pdf

GLOSSARY:

JITTER vs. DRIFT

The synchronization error of a client with respect to the reference is identified as ***jitter*** or ***drift*** depending on the ***time scale*** of the involved phenomena.

Jitter = fast variations, caused by inherent residual lack of coherency between oscillators, even if they are locked at the best;

Drift = slow variations, mainly caused by modifications of the environment conditions, such as temperature (primarily) but also humidity, materials and components aging, …

The boundary between the 2 categories is somehow arbitrary. For instance, synchronization errors due to mechanical vibrations can be classified in either category:

Acoustic waves → Jitter

Infrasounds → Drift

For pulsed accelerators, where the beam is produced in the form of a sequence of bunch trains with a certain repetition rate (10 Hz ÷ 120 Hz typically), the ***rep. rate value*** itself can be taken as a reasonable definition of the ***boundary*** between ***jitters*** and ***drifts***.

In this respect, ***drifts*** are phenomena significantly ***slower*** than ***rep. rate*** and will produced effects on the beam that can be ***monitored*** and ***corrected*** pulse-to-pulse.

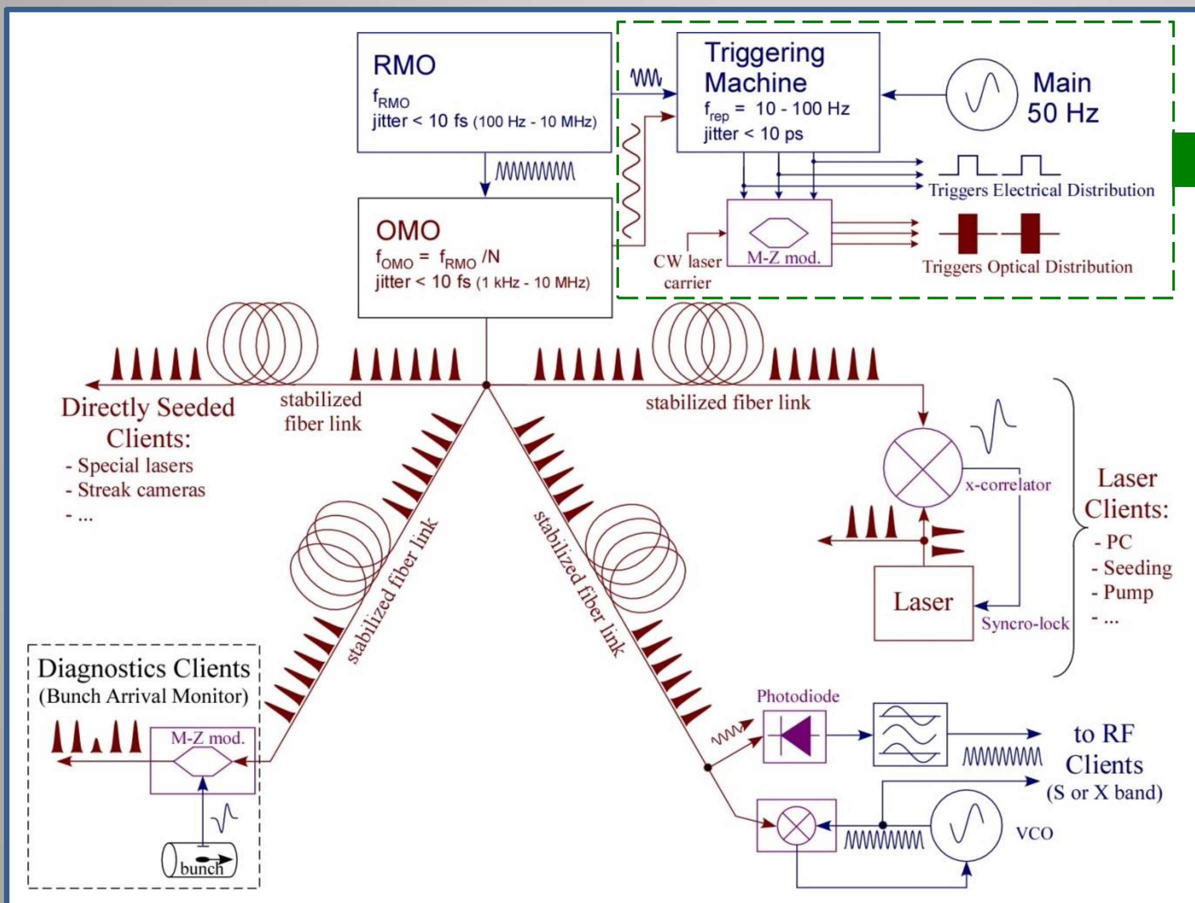
On the contrary, ***jitters*** are ***faster*** than ***rep. rate*** and will result in a pulse-to-pulse ***chaotic scatter*** of the beam characteristics that has to be minimized but that ***can not*** be actively ***corrected***.

Drift → Nasty

Jitter → Killer

Tasks of a Synchronization system:

- ✓ Generate and transport the reference signal to any client local position with constant delay and minimal drifts;
- ✓ Lock the client (laser, RF, ...) fundamental frequency to the reference with minimal residual jitter;
- ✓ Monitor clients and beam, and apply delay corrections to compensate residual (out-of-loop) drifts.



Triggers

Digital signals still in the Timing business but the required precision is orders of magnitude less demanding.

***Not covered in this lecture** (but nevertheless an important aspect of machine operation).*

SECTION II

BASICS

- ***Fourier and Laplace Transforms***
- ***Random Processes***
- ***Phase Noise in Oscillators***

Transforms summary

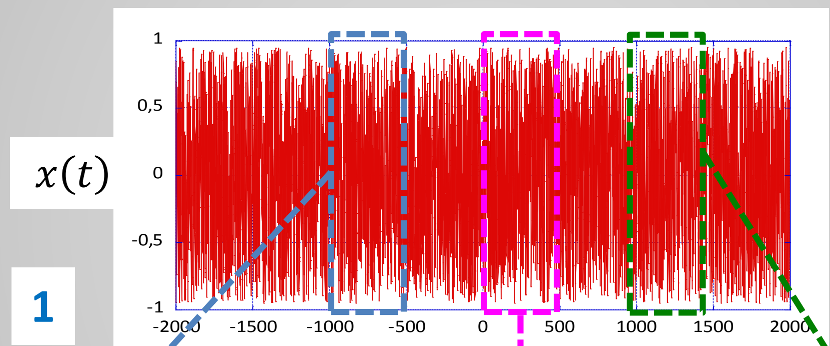
$x_{rms} = 0$

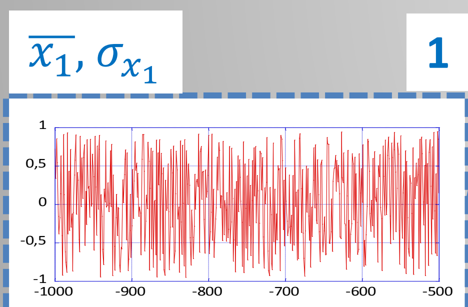
Transforms	Fourier - $\mathcal{F}$	Laplace - $\mathcal{L}$
Definition	$X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t} dt$	$X(s) = \int_{0}^{+\infty} x(t)\, e^{-st} dt$
Inverse transform	$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega)\, e^{j\omega t} d\omega$	$x(t) = \frac{1}{2\pi i} \int_{\gamma - j\cdot\infty}^{\gamma + j\cdot\infty} X(s)\, e^{st} ds$
Transformability conditions	$\int_{-\infty}^{+\infty} \lvert x(t)\rvert^2\, dt \neq \infty$	$x(t) = 0 \text{ if } t < 0;\ x(t) \cdot e^{-\sigma t} \xrightarrow[t \to +\infty]{} 0$
Linearity	$\mathcal{F}\,[a\, x(t) + b\, y(t)] = aX(\omega) + bY(\omega)$	$\mathcal{L}\,[a\, x(t) + b\, y(t)] = aX(s) + bY(s)$
Convolution product	$(x * y)(t) \stackrel{\text{def}}{=} \int_{-\infty}^{+\infty} x(t+\tau) \cdot y(\tau) d\tau$ $\mathcal{F}\,[(x * y)(t)] = X^*(\omega) \cdot Y(\omega)$	$(x * y)(t) \stackrel{\text{def}}{=} \int_{0}^{t} x(t+\tau) \cdot y(\tau) d\tau$ $\mathcal{L}\,[(x * y)(t)] = X^*(s) \cdot Y(s)$
Derivative	$\mathcal{F}\left[\frac{dx}{dt}\right] = j\omega \cdot X(\omega)$	$\mathcal{L}\left[\frac{dx}{dt}\right] = s \cdot X(s)$
Delay	$\mathcal{F}\,[x(t-\tau)] = X(\omega) e^{-j\omega\tau}$	$\mathcal{L}[x(t-\tau)] = X(s) e^{-s\tau}$

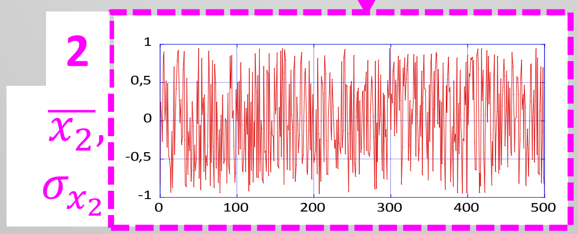
Random process summary

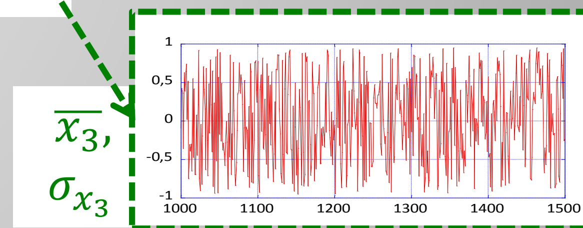
Let's consider a random variable $x(t)$ representing a physical observable quantity.

- Stationary process: statistical properties invariant for a t' time shift $x(t) \rightarrow x(t+t')$









Stationarity:

$$\overline{x_1} \approx \overline{x_2} \approx \overline{x_3} \approx \bar{x}$$

$$\sigma_{x_1} \approx \sigma_{x_2} \approx \sigma_{x_3} \approx \sigma_x$$

- Ergodic process: statistical properties can be estimated by a single process realization

- Uncorrelation: if $x(t)$ and $y(t)$ are 2 random variables completely uncorrelated (statistically independent), then:

$$\sigma_{x+y}^2 = \sigma_x^2 + \sigma_y^2 \qquad with \quad \sigma_x^2 \stackrel{\text{def}}{=} \overline{x^2} - \bar{x}^2$$

Noise power spectrum:

Since $x_{rms} \neq 0$, a real random variable $x(t)$ is in general ***not directly Fourier transformable***. However, if we observe $x(t)$ only for a ***finite time*** ΔT we may truncate the function outside the interval $[-\Delta T/2, \Delta T/2]$ and remove any possible limitation in the function transformability. The truncated function $x_{\Delta T}(t)$ is defined as:

$$x_{\Delta T}(t) = \begin{cases} x(t) & -\Delta T/2 \leq t \leq \Delta T/2 \\ 0 & elsewhere \end{cases}$$

Let $X_{\Delta T}(f)$ be the Fourier transform of the truncated function $x_{\Delta T}(t)$. It might be demonstrated that the rms value of the random variable can be computed on the base of the Fourier transform $X_{\Delta T}(f)$ according to:

$$x_{rms}^2 = \int_0^{+\infty} S_x(f)\, df \qquad with \qquad S_x(f) \stackrel{\text{def}}{=} \lim_{\Delta T \to \infty} 2 \cdot \frac{|X_{\Delta T}(f)|^2}{\Delta T}$$

The function $S_x(f)$ is called "***power spectrum***" or "***power spectral density***" of the random variable $x(t)$. The time duration of the variable observation ΔT sets the minimum frequency $f_{min} \approx 1/\Delta T$ containing meaningful information in the spectrum of $x_{\Delta T}(t)$.

BASICS:
LTI Transfer Functions

Fourier and Laplace transforms are used to compute the response of ***Linear Time Invariant*** (***LTI***) systems:

time domain

$\delta(t) \longrightarrow$ LTI system $\longrightarrow h(t)$ Green's function

$v_{in}(t) \longrightarrow$ LTI system $\longrightarrow v_{in}(t) * h(t)$

Fourier transform

$e^{j\omega t} \longrightarrow$ LTI system $\longrightarrow H(j\omega)e^{j\omega t}$

$V_{in}(j\omega) \longrightarrow$ LTI system $\longrightarrow V_{in}(j\omega) \cdot H(j\omega)$

Laplace transform

$e^{st} \longrightarrow$ LTI system $\longrightarrow H(s)e^{st}$

$V_{in}(s) \longrightarrow$ LTI system $\longrightarrow V_{in}(s) \cdot H(s)$

LTI system Transfer functions

Noise power spectra

$S_{in}(\omega)$

$\downarrow$

LTI system

$\downarrow$

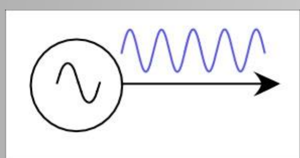
$|H(j\omega)|^2 \cdot S_{in}(\omega)$

Phase Noise in Oscillators

The most important task of a Synchronization system is to ***lock firmly the phase*** of each ***client*** to the ***reference oscillator*** in order to minimize the residual jitter. The clients are basically ***VCOs*** *(Voltage Controlled Oscillators)*, i.e. ***local oscillators*** (electrical for RF systems, optical for laser systems) whose fundamental frequency can be changed by applying a voltage to a control port.

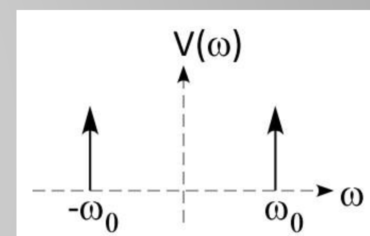
Before discussing the lock schematics and performances, it is worth introducing some ***basic concepts*** on ***phase noise*** in ***real oscillators***.

Ideal oscillator

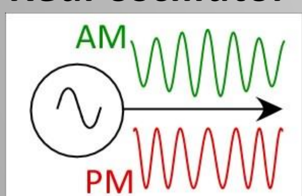


$$V(t) = V_0 \cdot cos(\omega_0 t + \varphi_0)$$

Ideal Spectrum

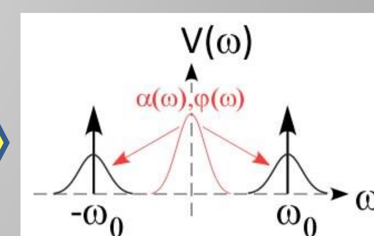


Real oscillator



$$V(t) = V_0 \cdot [1 + \alpha(t)] \cdot cos[\omega_0 t + \varphi(t)]$$

Real Spectrum



In real oscillators the amplitude and phase will always fluctuate in time by a certain amount because of the unavoidable presence of noise. However, by common sense, a well behaving real oscillator has to satisfy the following conditions:

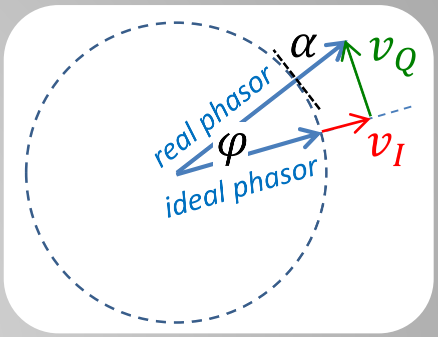
$$|\alpha(t)| \ll 1; \quad \left|\frac{d\varphi}{dt}\right| \ll \omega_0$$

A real oscillator signal can be also represented in ***Cartesian Coordinates*** $(\alpha, \varphi) \rightarrow (v_I, v_Q)$:

$$V(t) = V_0 \cdot cos(\omega_0 t) + v_I(t) \cdot cos(\omega_0 t) - v_Q(t) \cdot sin(\omega_0 t)$$

$if\ v_I(t), v_Q(t) \ll V_0$ $\quad \alpha(t) = v_I(t)/V_0, \quad \varphi(t) = v_Q(t)/V_0 \ll 1$

Cartesian representation only holds for small PM depth



Real oscillator outputs are ***amplitude*** (***AM***) and ***phase*** (***PM***) ***modulated*** carrier signals. In general it turns out that ***close to the carrier*** frequency the contribution of the ***PM noise*** to the signal spectrum ***dominates*** the contribution of the ***AM noise***. For this reason the lecture is mainly focused on phase noise. However, amplitude noise in RF systems directly reflects in energy modulation of the bunches, that may cause bunch arrival time jitter when beam travels through dispersive and bended paths (i.e. when R_{56}≠0 as in magnetic chicanes – see the magnetic compressor example in the next lecture).

Let's consider a real oscillator and neglect the AM component:

$$V(t) = V_0 \cdot cos[\omega_0 t + \varphi(t)] = V_0 \cdot cos\left[\omega_0\left(t + \tau(t)\right)\right] \quad \text{with} \quad \tau(t) \equiv \varphi(t)/\omega_0$$

The statistical properties of $\varphi(t)$ and $\tau(t)$ qualify the oscillator, primarily the values of the standard deviations σ_φ and σ_τ (or equivalently φ_{rms} and τ_{rms} since we may assume a zero average value). As for every noise phenomena they can be computed through the ***phase noise power spectral density*** $S_\varphi(f)$ of the random variable $\varphi(t)$.

Again, for practical reasons, we are only interested in observations of the random variable $\varphi(t)$ for a finite time ΔT. So we may truncate the function outside the interval $[-\Delta T/2, \Delta T/2]$ to recover the function transformability.

$$\varphi_{\Delta T}(t) = \begin{cases} \varphi(t) & -\Delta T/2 \le t \le \Delta T/2 \\ 0 & elsewhere \end{cases}$$

Let $\Phi_{\Delta T}(f)$ be the Fourier transform of the truncated function $\varphi_{\Delta T}(t)$. We have:

$$(\varphi_{rms}^2)_{\Delta T} = \int_{f_{min}}^{+\infty} S_\varphi(f)\,df \;\; with \;\; S_\varphi(f) \stackrel{\text{def}}{=} 2\frac{|\Phi_{\Delta T}(f)|^2}{\Delta T}$$

$S_\varphi(f)$ is the ***phase noise power spectral density***, whose dimensions are rad^2/Hz.

Again, the time duration of the variable observation ΔT sets the minimum frequency $f_{min} \approx 1/\Delta T$ containing meaningful information on the spectrum $\Phi_{\Delta T}(f)$ of the phase noise $\varphi_{\Delta T}(t)$.

IMPORTANT: we might still write

$$\varphi_{rms}^2 = \lim_{\Delta T \to \infty} (\varphi_{rms}^2)_{\Delta T} = \int_0^{+\infty} \left(2 \cdot \lim_{\Delta T \to \infty} \frac{|\Phi_{\Delta T}(f)|^2}{\Delta T}\right) df = \int_0^{+\infty} S_\varphi(f)\,df$$

but we must be aware that in this case φ_{rms} is ***likely to diverge***. This is physically possible since the ***power*** in the carrier does only ***depend*** on ***amplitude*** and ***not*** on ***phase***. In these cases the rms value can better be specified for a given observation time ΔT or equivalently for a given frequency range of integration $[f_1, f_2]$.

BASICS:

Phase Noise in Oscillators

We have:

$$\varphi_{rms}^2\Big|_{\Delta T} = 2 \cdot \int_{f_{min}}^{+\infty} \mathcal{L}(f)\, df \quad with \quad \mathcal{L}(f) = \begin{cases} \dfrac{|\Phi_{\Delta T}(f)|^2}{\Delta T} & f \geq 0 \\ 0 & f < 0 \end{cases}$$

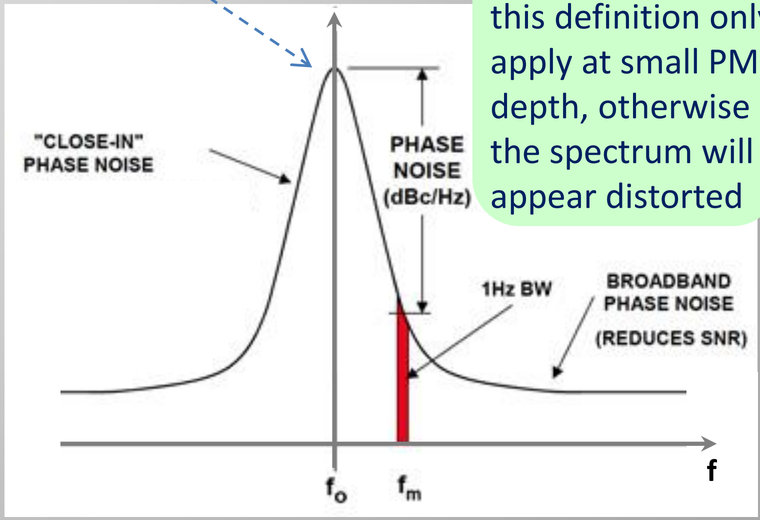
The function $\mathcal{L}(f)$ is defined as the "*Single Sideband Power Spectral Density*" and is called "script-L"

$$\mathcal{L}(f) = \frac{power\ in\ 1\ Hz\ phase\ modulation\ single\ sideband}{total\ signal\ power} = \frac{1}{2} S_\varphi(f) \leftarrow IIIE\ standard\ 1139-1999$$

Linear scale → $\mathcal{L}(f)\ units\ \equiv\ Hz^{-1}\ or\ rad^2/Hz$

Log scale → $\mathbf{10} \cdot \mathrm{Log}[\mathcal{L}(f)]\ units\ \equiv\ dBc/Hz$

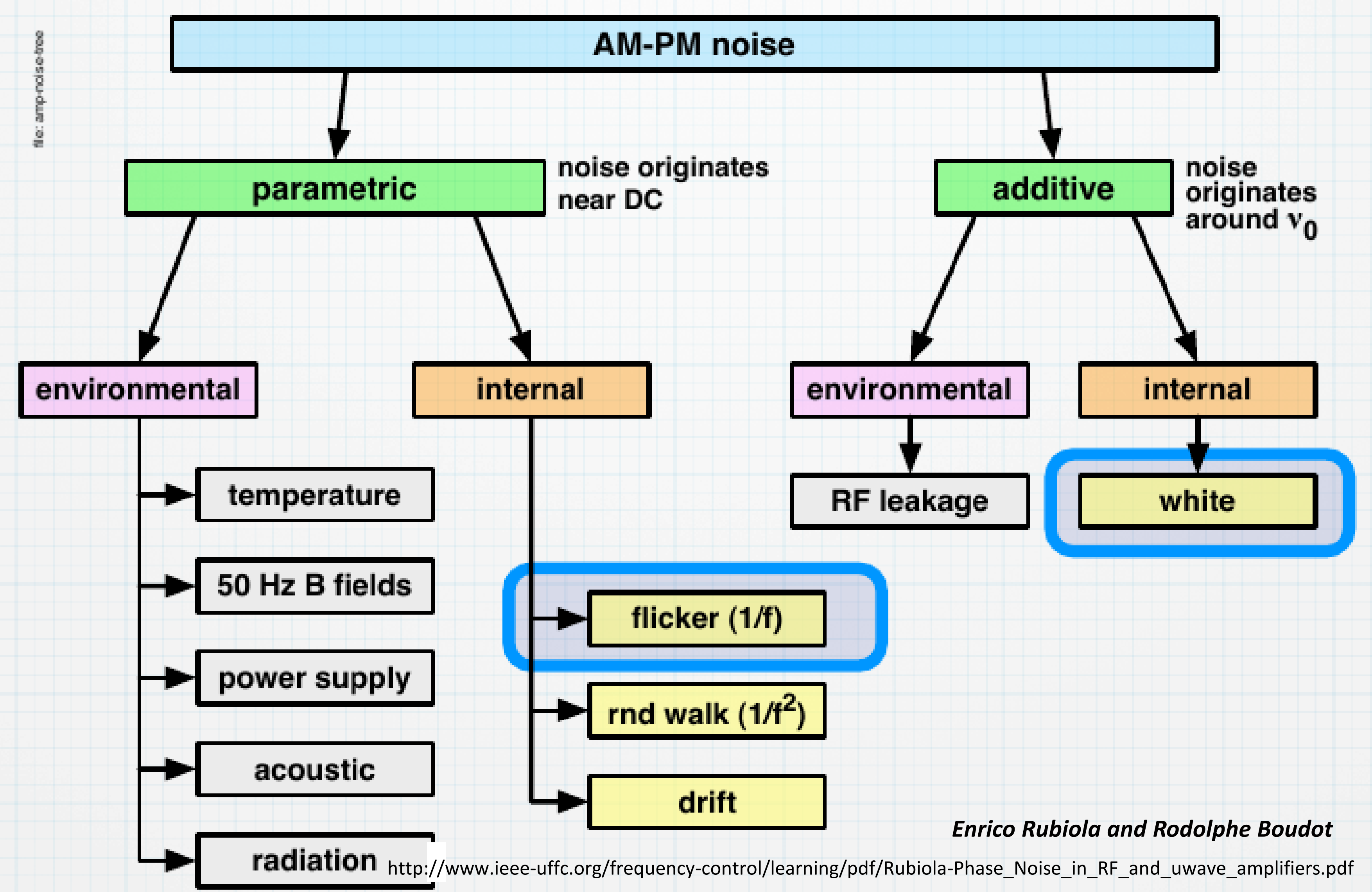
Attention:
this definition only apply at small PM depth, otherwise the spectrum will appear distorted



CONCLUSIONS:

- ✓ Phase (and time) jitters can be computed from the spectrum of $\varphi(t)$ through the $\mathcal{L}(f)$ - or $S_\varphi(f)$ - function;
- ✓ Computed values depend on the integration range, i.e. on the duration ΔT of the observation. Criteria are needed for a proper choice (we will see …).

Phase Noise Nature and Spectra



Enrico Rubiola and Rodolphe Boudot

http://www.ieee-uffc.org/frequency-control/learning/pdf/Rubiola-Phase_Noise_in_RF_and_uwave_amplifiers.pdf

Phase Noise Nature and Spectra

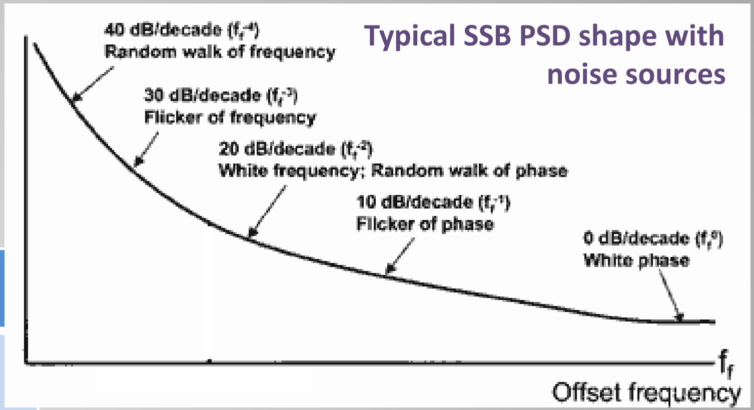
Close-in phase noise:

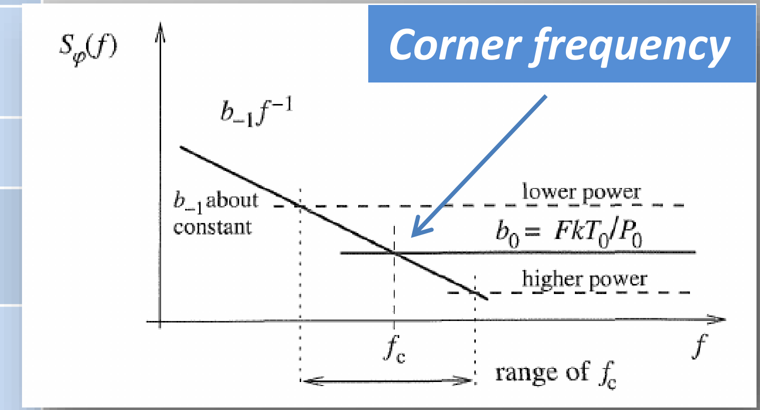
$$S_\varphi(f) = \sum \frac{b_{-k}}{f^k} \quad k = 0,1,2,3,\ldots$$

$$S_{FM}(f) \xrightarrow[\text{transforms}]{F\ or\ L} S_{PM}(f) = S_{FM}(f)/f^2$$

	Type	Origin	$S_\varphi(f)$
f^0	White	*Thermal noise of resistors*	$F \cdot kT/P_0$
	Shot	*Current quantization*	$2q\bar{\imath}R/P_0$
f^{-1}	Flicker	*Flicking PM*	b_{-1}/f
f^{-2}	White FM	*Thermal FM noise*	$b_0^{FM} \cdot \frac{1}{f^2}$
	Random walk	*Brownian motion*	b_{-2}/f^2
f^{-3}	Flicker FM	*Flicking FM*	$\frac{b_{-1}^{FM}}{f} \cdot \frac{1}{f^2}$
f^{-4}	Random walk FM	*Brownian motion → → FM*	$\frac{b_{-2}^{FM}}{f^2} \cdot \frac{1}{f^2}$
f^{-n}	...	*high orders ...*	

$$F \stackrel{\text{def}}{=} SNR_{in}/SNR_{out}$$





$$[b_{-k}] = rad^2 Hz^{k-1}$$

BASICS:

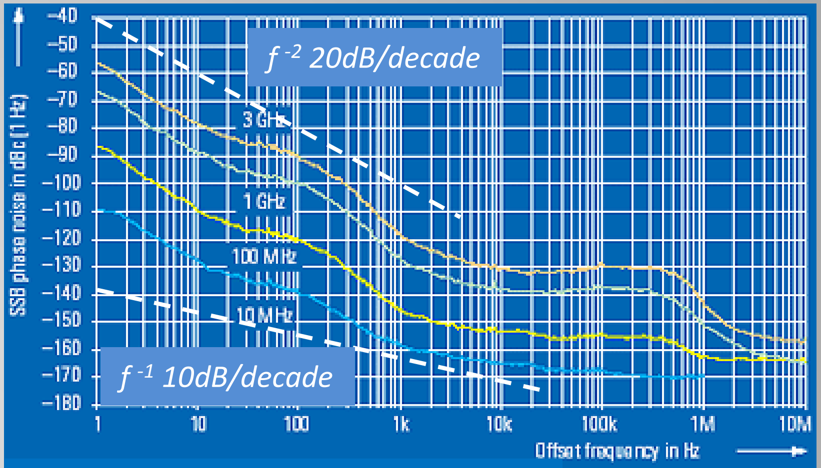
Phase Noise Examples

Time jitter can be computed according to:

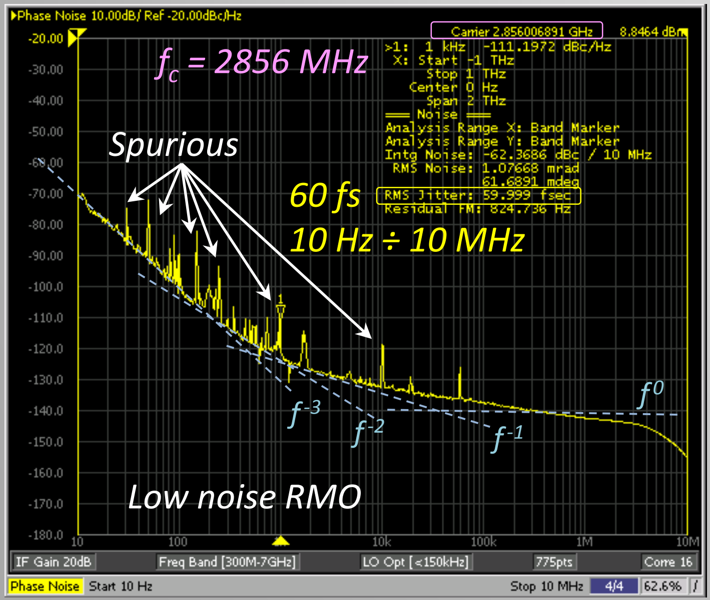
$$\sigma_t^2 = \frac{\sigma_\varphi^2}{\omega_c^2} = \frac{1}{\omega_c^2}\int_{f_{min}}^{+\infty} S_\varphi(f)\,df$$

same time jitter $\rightarrow$ $S_\varphi(f) \div \omega_c^2$

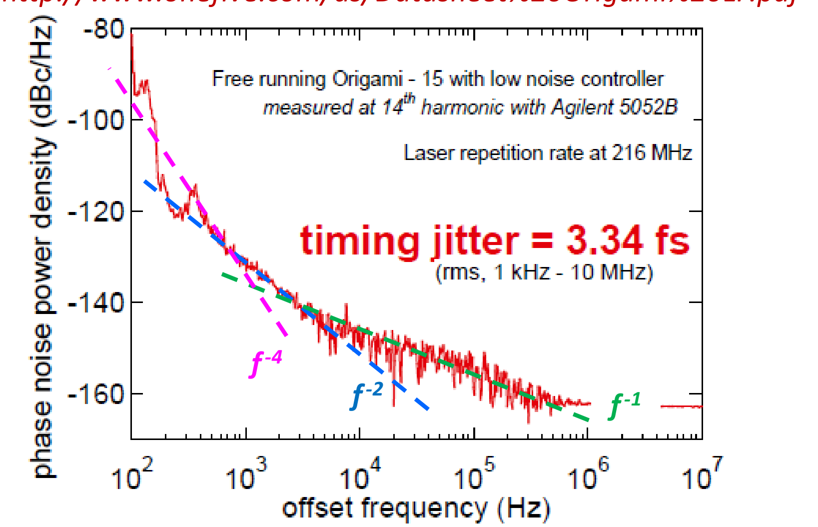
Phase noise spectral densities of different oscillators have to be compared at same carrier frequency ω_c or scaled as ω_c^{-2} before comparison.



Commercial frequency synthesizer



http://www.onefive.com/ds/Datasheet%20Origami%20LP.pdf



OMO – Mode-locked laser – f= 3024 MHz

SECTION III

BASICS

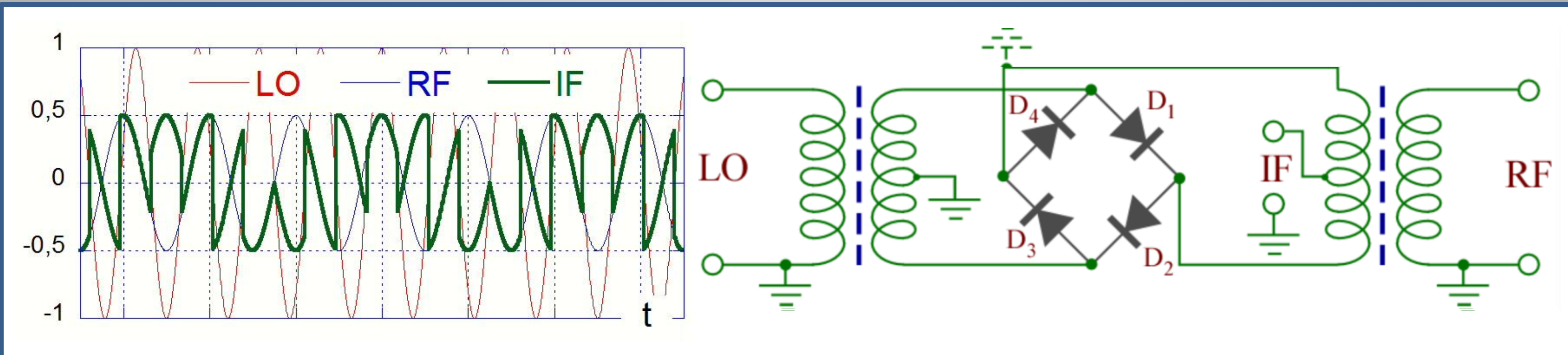
- ***Phase Detectors***
- ***Phase Locked Loops***
- ***Precision phase noise measurements***

Phase Detectors – RF signals

Phase detection on RF signals

The **Double Balanced Mixer** is the **most diffused RF device** for frequency translation (up/down conversion) and detection of the relative phase between 2 RF signals (LO and RF ports). The LO voltage is differentially applied on a diode bridge switching on/off alternatively the D_1-D_2 and D_3-D_4 pairs, so that the voltage at IF is:

$$V_{IF}(t) = V_{RF}(t) \cdot \operatorname{sgn}\left[V_{LO}(t)\right]$$



$$V_{RF}(t) = V_{RF} \cdot \cos\left(\omega_{RF} t\right); \quad V_{LO}(t) = V_{LO} \cdot \cos\left(\omega_{LO} t\right)$$

$$V_{RF} << V_{LO}$$

$$V_{IF}(t) = V_{RF}\cos(\omega_{RF}t) \cdot \operatorname{sgn}\left[\cos(\omega_{LO}t)\right] = V_{RF}\cos(\omega_{RF}t) \cdot \sum_{n=odds} \frac{4}{n\pi}\cos(n\omega_{LO}t) =$$

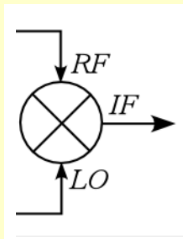
$$= \frac{2}{\pi} V_{RF}\left[\cos((\omega_{LO} - \omega_{RF})t) + \cos((\omega_{LO} + \omega_{RF})t) + intermod \;\; products\right]$$

Phase Detectors – RF signals

Phase detection on RF signals

If $f_{LO} = f_{RF}$ the IF signal has a DC component given by: $V_{IF}\big|_{DC} = \langle V_{IF}(t)\rangle = k_{CL} A_{RF} \cos\varphi$

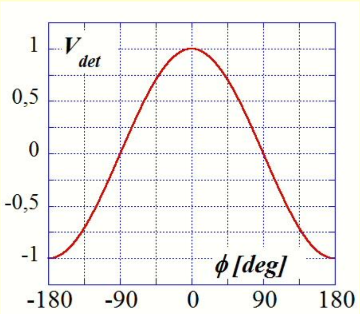
$A_{RF}\cos(\omega t + \varphi)$

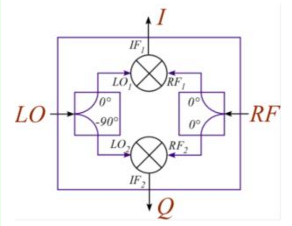


$A_{LO}\cos(\omega t)$

$$V_{det} = V_{IF} = V(\varphi) + \textit{high harm.}$$

$$A_{RF} << A_{LO} \quad \Rightarrow \quad V_{det}(\varphi) = k_{CL} A_{RF} \cos\varphi$$





$$\begin{cases} V_I = k_{CL} A_{RF} \cos(\varphi) & + \textit{high harmonics} \\ V_Q = k_{CL} A_{RF} \sin(\varphi) & + \textit{high harmonics} \end{cases} \Rightarrow \begin{cases} A_{RF} \div \sqrt{V_I^2 + V_Q^2} \\ \varphi = \arctan\left(V_Q / V_I\right) + \dfrac{\pi}{2}\left[1 - \mathrm{sgn}(V_I)\right] \end{cases}$$

$$\left.\frac{dV_{det}}{d\varphi}\right|_{\varphi = \pm\pi/2} = \mu k_{CL} A_{RF} \underset{\substack{CL = 6dB \\ A_{RF} = 1V}}{\approx} 5 \div 10\, mV/Deg \underset{f_c = 10GHz}{\approx} 15 \div 30\, mV/ps$$

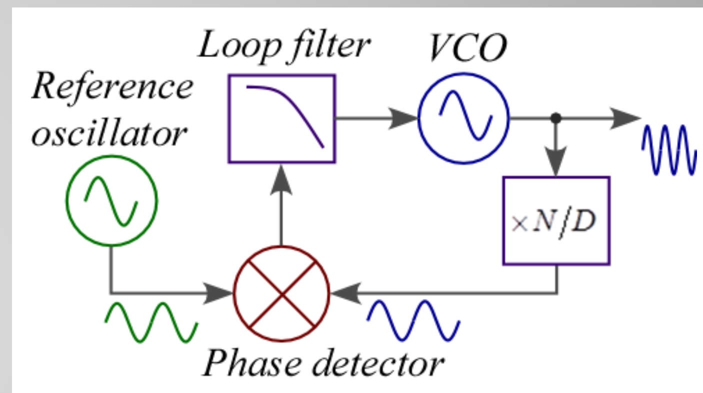
- ✓ Passive
- ✓ Cheap, Robust
- ✓ Wideband

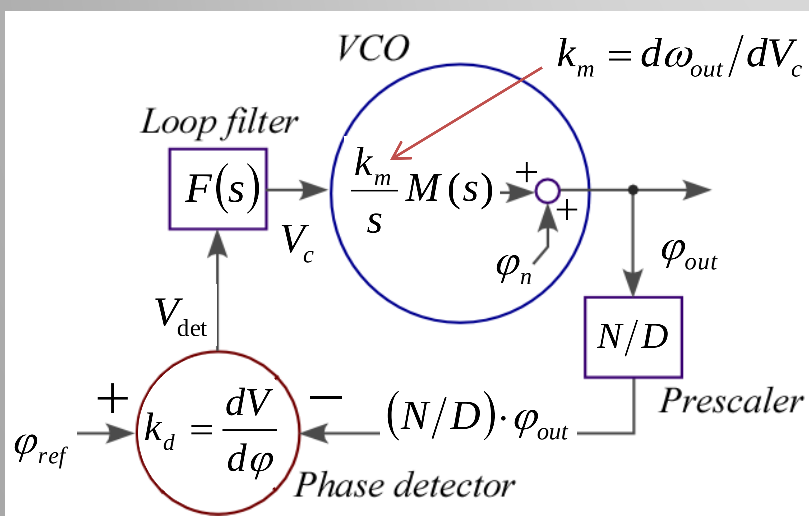
- ✓ Sensitivity proportional to level, AM → PM not fully rejected
- ✓ Noise figure $F \approx CL$
- ✓ Good sensitivity but lower wrt optical devices

Phase Locked Loops

PLLs are a very **general subject** in RF electronics, used **to synchronize oscillators** to a **common reference** or to **extract the carrier** from a **modulated signal** (FM tuning). In our context PLLs are used to ***phase-lock the clients*** of the synchronization system ***to the master clock*** (RMO or OMO). The building blocks are:

- A VCO, whose frequency range includes *(D/N)* f_{ref};
- A phase detector, to compare the scaled VCO phase to the reference;
- A loop filter, which sets the lock bandwidth;
- A prescalers or synthesizer (N/D frequency multiplier, N and D integers) if different frequencies are required.





PLL linear model

PLL transfer function

VCO noise

$$\phi_{out}(s) = \frac{D}{N}\frac{H(s)}{1+H(s)}\phi_{ref}(s) + \frac{1}{1+H(s)}\phi_n(s)$$

$$\text{with } H(s) = \frac{N}{D}\frac{k_d k_m}{s}F(s)M(s)$$

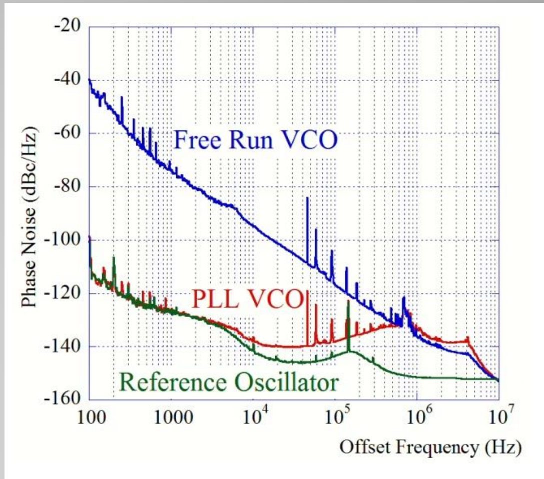
freq-to-phase conversion · *loop filter* · *VCO mod. bandwidth*

Phase Locked Loops

The ***output phase*** spectrum is ***locked*** to the ***reference*** if ***|H(jω)|>>1***, while it returns similar to the ***free run VCO*** if |***H(jω)|<1***.

Loop filters provide **PLL stability**, tailoring the frequency response, and **set loop gain** and **cut-off frequency**. Loop filter optimization can ***improve a lot the PLL performances!***

A flat loop filter $F(s) = F_0$ reflects in a pure integrator loop transfer function thanks to a dc ($f = 0$) pole provided by the frequency-to-phase conversion operated by the VCO + phase detector cascade.



Although this configuration already provides an extremely high (namely infinite) dc loop gain, a residual dc phase error $\Delta\varphi_e = \varphi_{out} - \varphi_{ref}$ is necessary to drive the VCO to the reference oscillator frequency ω_{ref} according to:

$$\Delta\varphi_e \approx -\omega_{ref}/(k_d k_m F_0)$$

This result, that can be deduced by the PLL working principle, can be also rigorously demonstrated as follows:

$$\begin{cases} \omega_{ref}(t) = \omega_{ref} \cdot 1(t) \\ \varphi_{ref}(t) = \omega_{ref} \cdot t \cdot 1(t) \end{cases} \xrightarrow{L\ transf.} \begin{cases} \tilde{\omega}_{ref}(s) = \omega_{ref}/s \\ \tilde{\varphi}_{ref}(s) = \omega_{ref}/s^2 \end{cases}$$

$$\widetilde{\Delta\varphi}_e(s) = \tilde{\varphi}_{out}(s) - \tilde{\varphi}_{ref}(s) = -\frac{1}{1+H(s)}\tilde{\varphi}_{ref}(s) = -\frac{s}{s+k_d k_m F_0} \cdot \frac{\omega_{ref}}{s^2}$$

Limit theorem for the Laplace transform:

$$\lim_{t\to\infty} \Delta\varphi_e(t) = \lim_{s\to 0} s \cdot \widetilde{\Delta\varphi}_e(s) = -\omega_{ref}/(k_d k_m F_0)$$

BASICS:

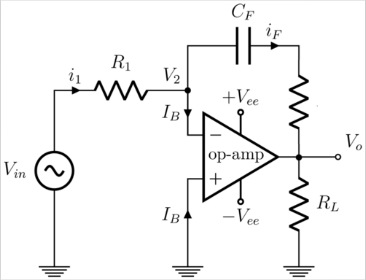
Phase Locked Loops

Non-zero PLL phase offset is in general unwanted. It can result in:

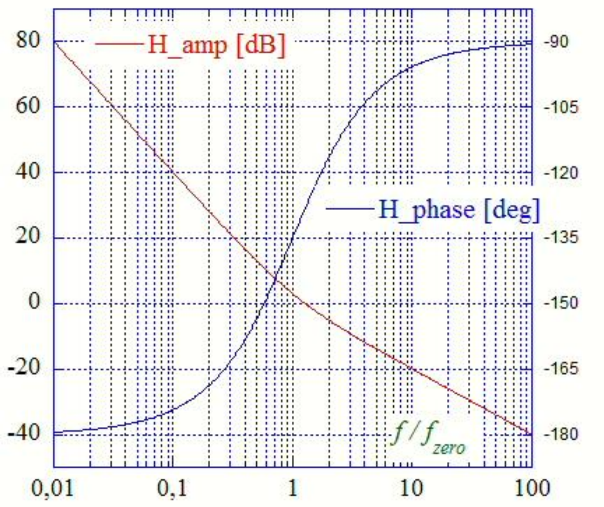
- ✓ *phase variations following the VCO characteristics drifts*
- ✓ *AM-to-PM mixing because of the off-quadrature signals at the phase detector inputs.*

Residual phase offset can be compressed to zero by adding one (or more) dc pole ($s = 0$) and one (or more) compensating zero at a certain $s = \omega_z$ in the loop filter transfer function.

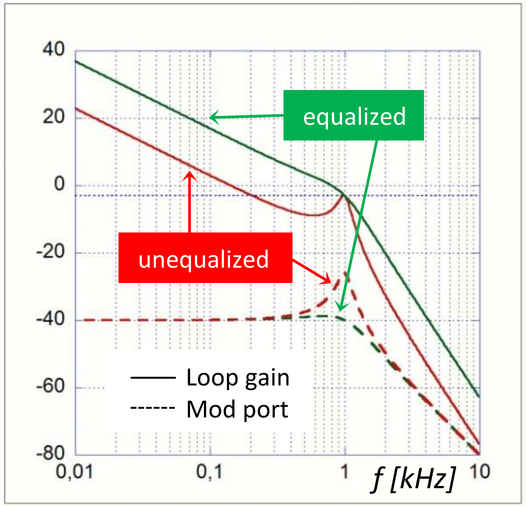
$$F(s) = \frac{\omega_0}{s}(1 - s/\omega_z)$$



Bode plot of the PLL loop gain

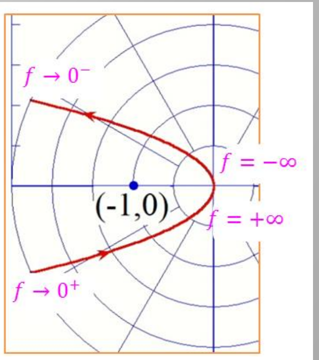


A very steep frequency response can be obtained (slope = 40 dB/decade) in stable conditions (see Nyquist plot).



Loop filters properly designed can also improve the PLL performances by enlarging the PLL BW through equalization of the frequency response of the VCO modulation port.

Equalization of the VCO modulation port frequency response allows increasing the loop gain.

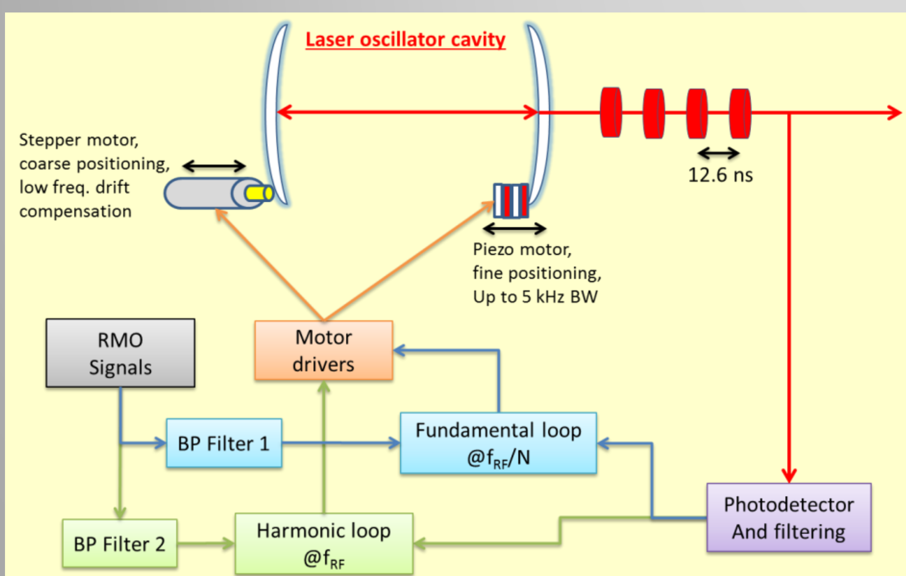


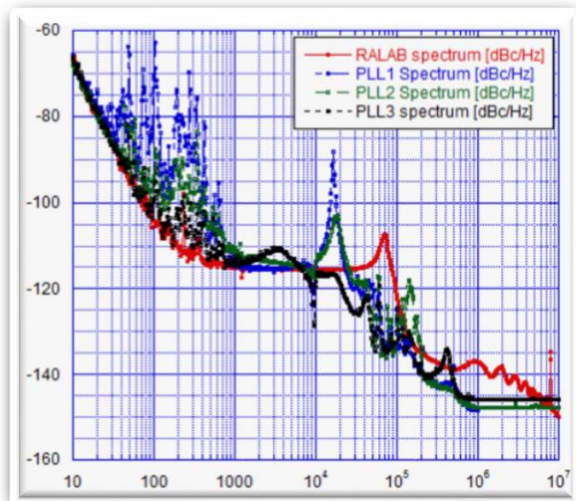
Nyquist locus

BASICS: Phase Locked Loops for
Client Synchronization

What is ***peculiar*** in ***PLLs*** for clients of a ***stabilization system*** of a Particle Accelerator facility ?

- ✓ Both the reference and client oscillators can be either RF VCOs or laser cavities. Phase detectors are chosen consequently;
- ✓ Laser oscillators behave as VCOs by trimming the cavity length through a piezo controlled mirror.
  - ➢ Limited modulation bandwidth (≈ *few kHz typical*);
  - ➢ Limited dynamic range ($\Delta f/f \approx 10^{-6}$), overcome by adding motorized translational stages to enlarge the mirror positioning range;
  - ➢ At frequencies beyond PLL bandwidth ($f > 1$ *kHz*) mode-locked lasers exhibit excellent low-phase noise spectrum.





RMO
$\sigma_t \approx 85$ fs
10 Hz – 10 MHz

Laser:

PLL flat
$\sigma_t \approx 230$ fs

PLL + 1 s=0 pole
$\sigma_t \approx 85$ fs

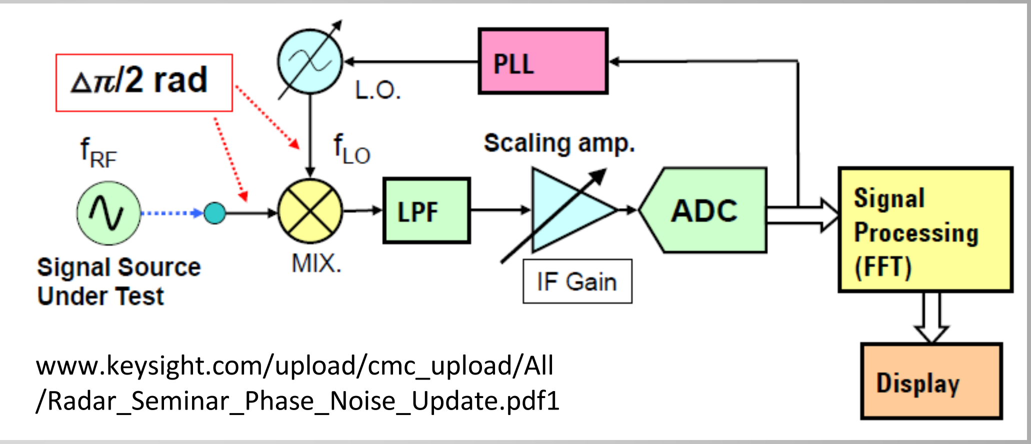
PLL + 2 s=0 poles
$\sigma_t \approx 70$ fs

SSB phase noise of a locked OMO for different loop filters

BASICS: Precision PN Measurements

Phase noise of RF sources can be measured in various ways. The most commonly used technique is the ***PLL with Reference Source***.

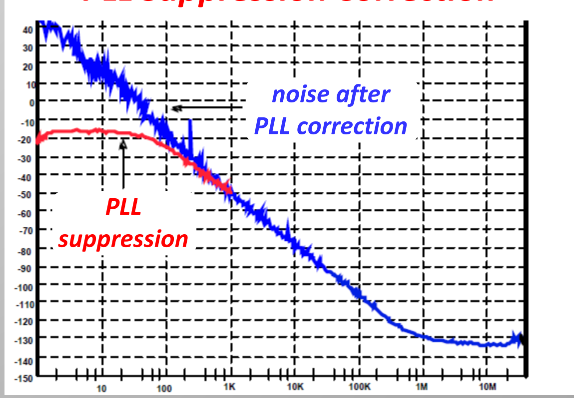
The phase of the ***signal source under test*** is measured ***with respect to*** a tunable reference ***Local Oscillator*** which is locked to the source. The ***baseband signal*** used to drive the PLL is also acquired and processed to extract the ***relative phase error*** information.



$$\Delta\varphi_{meas}(t) = \varphi_{SUT}(t) - \varphi_{LO}(t) \quad \Rightarrow \quad S_{\varphi_{meas}}(\omega) = S_{\varphi_{SUT}}(\omega) + S_{\varphi_{LO}}(\omega)$$

Clearly, the measurements includes a ***contribution from the reference*** source that is possible to ***neglect*** only when it is at least ***15÷20 dB lower*** than the signal. Only sources ***significantly worse (i.e. more noisy) than reference*** can be accurately characterized.

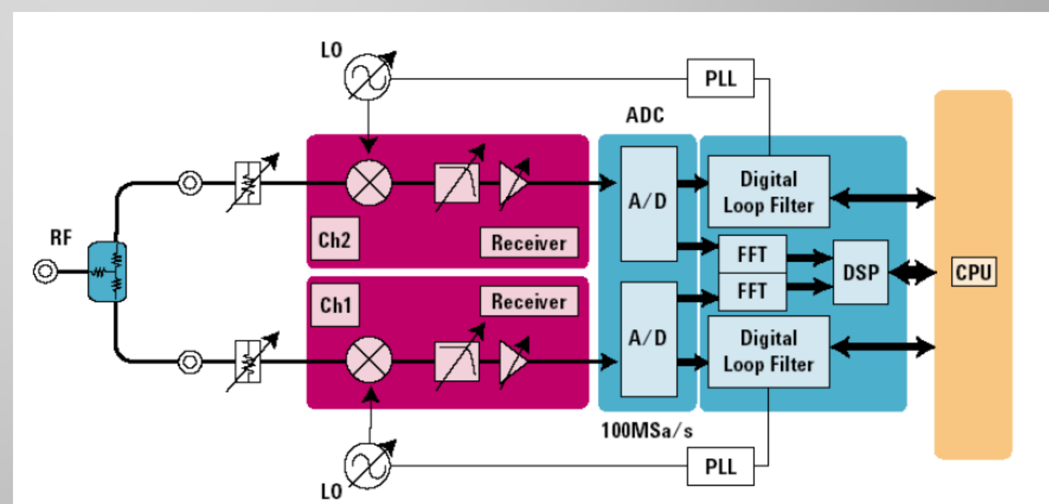
Since the PLL suppresses noise for frequencies within the closed loop bandwidth, measured data need also to be corrected on the base of the PLL characteristics to provide accurate results.



BASICS:
Precision PN Measurements

Signal Source Analyzers SSA are dedicated instruments integrating an optimized set-up for precise phase noise measurements based on PLL with reference source technique.
To overcome the limitation coming from the reference contribution to the measurements, ***two low noise LO oscillators*** are locked to the DUT signal.

The phase noise of the source under test is measured in parallel along two independent channels. The acquired data are Fourier transformed and ***cross-correlated to reduce the contribution of the references*** to the measurement.

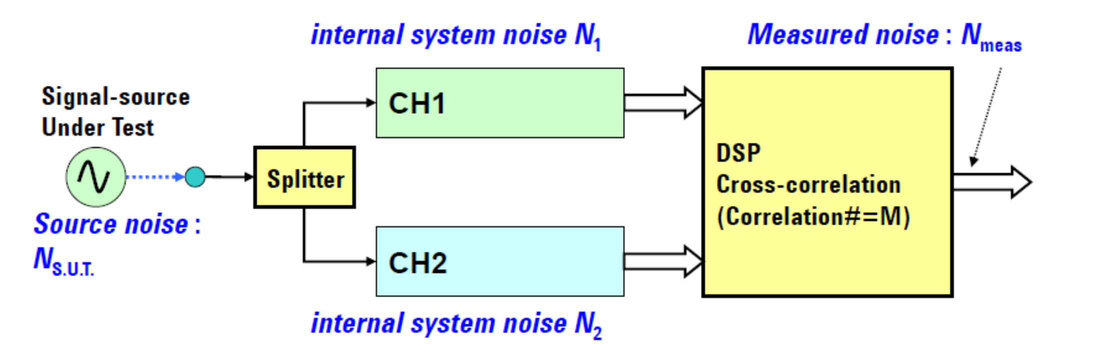


BASICS:
Precision PN Measurements

The SUT phase noise $\varphi_{SUT}(t)$ measured simultaneously wrt the 2 LOs gives:

$$\Delta\varphi_{1,2}(t) = \varphi_{SUT}(t) - \varphi_{LO_{1,2}}(t)$$

$$\Delta\Phi_{1,2}(f) = \Phi_{SUT}(f) - \Phi_{LO_{1,2}}(f)$$



The cross correlation function $r(\tau)$ of $\Delta\varphi_1(t)$ and $\Delta\varphi_2(t)$, and its Fourier transform $R(f)$ are:

$$r(\tau) = \int_{-\infty}^{+\infty} \Delta\varphi_1(t) \cdot \Delta\varphi_2(t+\tau)\, dt \qquad R(f) = \Delta\Phi_1^*(f) \cdot \Delta\Phi_2(f)$$

$$R(f) = |\Phi_{SUT}(f)|^2 - \left[\Phi_{SUT}^*(f) \cdot \Phi_{LO_2}(f) + \Phi_{SUT}(f) \cdot \Phi_{LO_1}^*(f)\right] + \Phi_{LO_1}^*(f) \cdot \Phi_{LO_2}(f)$$

after M averages
("correlations")

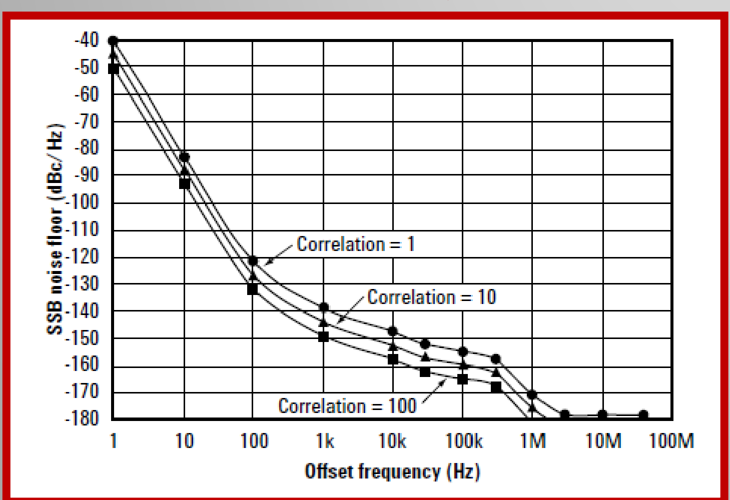
$$S_{\varphi_{meas}}(f) = S_{\varphi_{SUT}}(f) + \left[\sqrt{S_{\varphi_{LO1}}(f)} + \sqrt{S_{\varphi_{LO2}}(f)}\right]\sqrt{\frac{S_{\varphi_{SUT}}(f)}{M}} + \sqrt{\frac{S_{\varphi_{LO1}}(f) S_{\varphi_{LO2}}(f)}{M}}$$

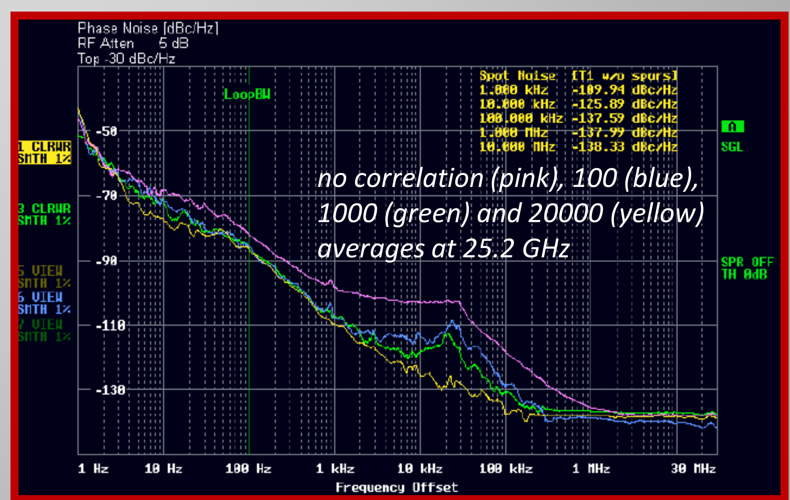
Precision PN Measurements

After ***M averages*** the magnitude of the uncorrelated contributions to the measurement (including the cross product of the phase noises of the 2 reference sources) is expected to ***decrease*** by a ***factor*** $\sqrt{M}$. Measurement accuracy at level of the phase noise of the reference sources or even lower is achieved. Sources of quality ***comparable with references or even better*** can be accurately characterized. The only limit is the ***measurement time*** duration which ***increases*** with the ***number of correlations***.

Random phases of uncorrelated contribution

magnitude $\div \frac{1}{\sqrt{M}}$ **after** M **averages (correlations)**





END LECTURE I

THANKS!

IT CONTINUES …