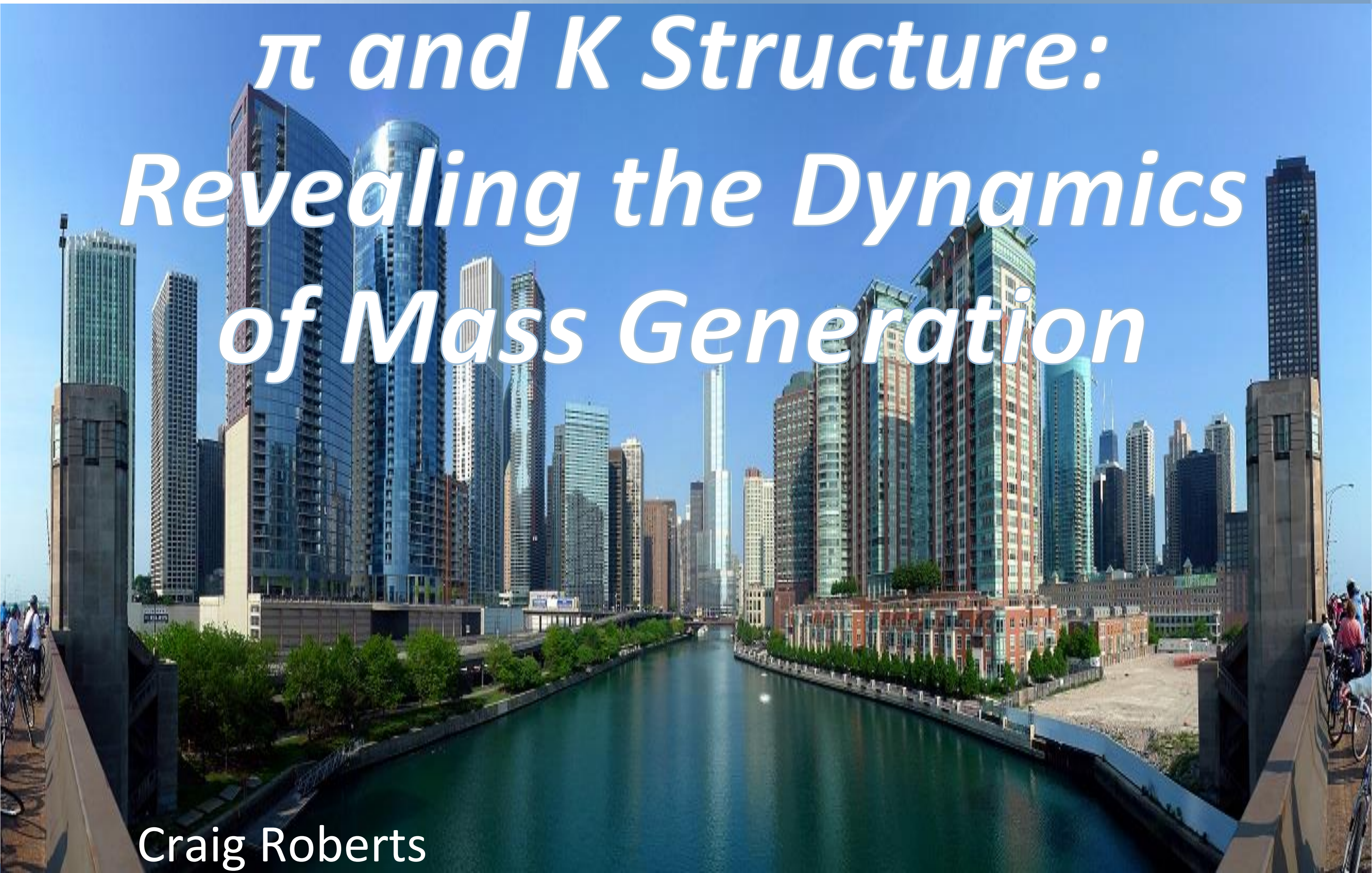
π and K Structure: Revealing the Dynamics of Mass Generation



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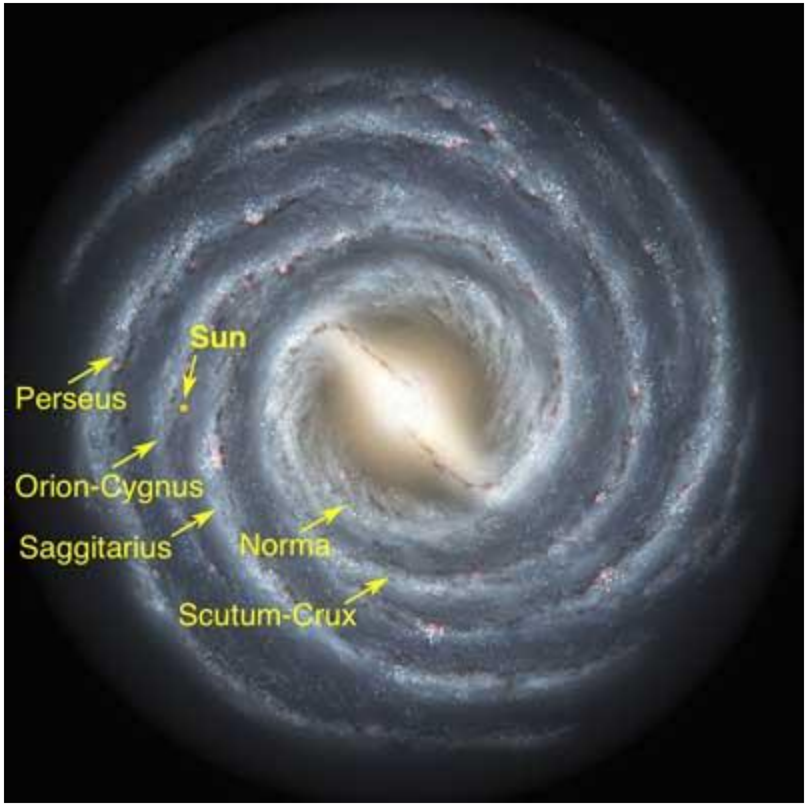
Emergent Phenomena in the Standard Model

Existence of the Universe as we know it depends critically on following empirical facts:

- Proton is massive, *i.e*. the mass-scale for strong interactions is vastly different to that of electromagnetism
- Proton is absolutely stable, despite being a composite object constituted from three valence quarks
- Pion is unnaturally light (but not massless), despite being a strongly interacting composite object built from a valence-quark and valence antiquark

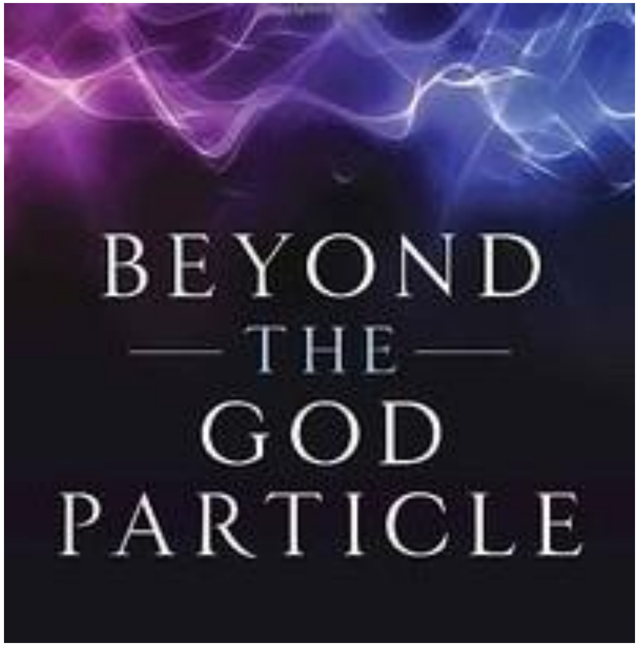


Emergence: low-level rules producing high-level phenomena, with enormous apparent complexity

Overture

- LHC has NOT found the "God Particle" because the Higgs boson is NOT the origin of mass
  - Higgs-boson only produces a little bit of mass
  - Higgs-generated mass-scales explain neither the proton's mass nor the pion's (*near*-)masslessness
  - Hence LHC has, as yet, taught us very little about
    - Origin   Nature   Structure

    of the nuclei whose existence support the Cosmos



- Strong interaction sector of the Standard Model, *i.e*. QCD, is the key to understanding the origin, existence and properties of (almost) all known matter
- Answers are in sight
  - Theoretical tools are reaching point where sound QCD predictions can be made
  - New facilities – in operation or being planned – can validate those predictions

What & where is mass?

Strong Interactions in the Standard Model of Particle Physics

$$\mathcal{L}_{\rm QCD} = \bar{\psi}_i \left(i(\gamma^\mu D_\mu)_{ij} - m\,\delta_{ij}\right)\psi_j - \frac{1}{4}G^a_{\mu\nu}G_a^{\mu\nu}$$

- Only apparent scale in chromodynamics is mass of the quark field
- In connection with everyday matter, that mass is $1/250^{\rm th}$ of the natural (empirical) scale for strong interactions,
  viz. more-than two orders-of-magnitude smaller
- Quark mass is said to be generated by Higgs boson.
- Plainly, however, that mass is very far removed from the natural scale for strongly-interacting matter
- *Nuclear physics mass-scale* – 1 GeV – is an *emergent feature of the Standard Model*
  - No amount of staring at $L_{\rm QCD}$ can reveal that scale
- Contrast with quantum electrodynamics, *e.g*. spectrum of hydrogen levels measured in units of m_e, which appears in $L_{\rm QED}$

$$\mathcal{L}_{\rm QCD} = \bar{\psi}_i\,(i(\gamma^\mu D_\mu)_{ij} \qquad)\,\psi_j - \frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu}$$

Whence Mass?

- Classical chromodynamics … non-Abelian local gauge theory
- Remove the current mass … there's no energy scale left
- *No dynamics in a scale-invariant theory*; only kinematics … the theory looks the same at all length-scales … there can be no clumps of anything … *hence bound-states are impossible*.
- *Our Universe can't exist*
- *Higgs boson doesn't solve this problem* … normal matter is constituted from light-quarks & the mass of protons and neutrons, the kernels of all visible matter, are 100-times larger than anything the Higgs can produce
- *Where did it all begin?*

  … becomes … Where did it all come from?

Whence Mass?

- Poincaré invariance entails that the Energy-Momentum Tensor is divergence-free, *i.e*. it defines a conserved current:

$$\partial_\mu T_{\mu\nu} = 0$$

$T_{\mu\nu}$ can *always* be made symmetric

- Noether current associated with a global scale transformation:

  $x \to e^{-\sigma} x$

  is the dilation current: $D_{\mu\nu}=T_{\mu\nu}\, x_\nu$

- In a scale invariant theory, the dilation current is conserved

$$\partial_\mu \mathcal{D}_\mu = 0 = [\partial_\mu T_{\mu\nu}]x_\nu + T_{\mu\nu}\delta_{\mu\nu} = T_{\mu\mu}\,,$$

- Consequently, in a scale invariant theory

  the *energy-momentum tensor must be traceless:* $T_{\mu\mu} \equiv 0$

Trace Anomaly

- Classical chromodynamics is meaningless … must be quantised
- Regularisation and renormalisation of (ultraviolet) divergences introduces a mass-scale

  … *dimensional transmutation*: mass-dimensionless quantities become dependent on a mass-scale, ζ

- $\alpha \to \alpha(\zeta)$ in QCD's (massless) Lagrangian density, L(m=0) *QCD β function*

  Under a scale transformation $\zeta \to e^{\sigma}\zeta$, then $\alpha \to \sigma\, \alpha\beta(\alpha)$

  $$L \to \sigma\, \alpha\beta(\alpha)\, dL/d\alpha$$

  $$\Rightarrow \partial_\mu D_\mu = \delta L/\delta\sigma = \alpha\beta(\alpha)\, dL/d\alpha = \beta(\alpha)\, \tfrac{1}{4} G_{\mu\nu} G_{\mu\nu} = T_{\rho\rho} =: \Theta_0$$

  Trace anomaly

- Straightforward, nonperturbative derivation, without need for diagrammatic analysis …

  quantisation of renormalisable four-dimensional theory forces nonzero value for trace of energy-momentum tensor

$$\mathcal{L}_{\rm QCD} = \bar{\psi}_i \left(i(\gamma^\mu D_\mu)_{ij} \qquad \right) \psi_j - \frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu}$$

Whence?

- Classical chromodynamics … non-Abelian local gauge theory
- Local gauge invariance; but there is no confinement without a mass-scale
  - Three quarks can still be colour-singlet
  - Colour rotations will keep them colour singlets
  - But they need have no proximity to one another
    … proximity is meaningless in a scale-invariant theory
- Whence mass … equivalent to whence a mass-scale … equivalent to whence a confinement scale
- *Understanding the origin of mass in QCD is quite likely inseparable from the task of understanding confinement.*

Existence alone of a scale anomaly answers neither question

Where is the mass?

$$T_{\mu\mu} = \frac{1}{4}\beta(\alpha(\zeta))G^a_{\mu\nu}G^a_{\mu\nu}$$

Trace Anomaly

- Knowing that a trace anomaly exists does not deliver a great deal … indicates only that a mass-scale exists
- Can one compute and/or understand the magnitude of that scale?
- One can certainly *measure* the magnitude … consider proton:

$$\langle p(P)|T_{\mu\nu}|p(P)\rangle = -P_\mu P_\nu$$

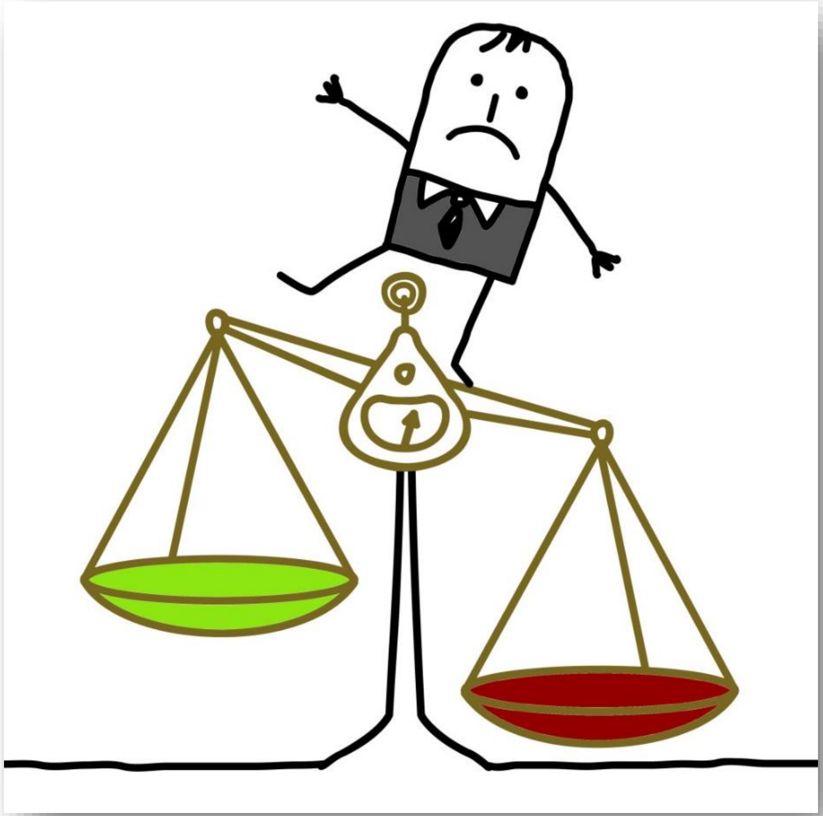
$$\langle p(P)|T_{\mu\mu}|p(P)\rangle = -P^2 = m_p^2$$

$$= \langle p(P)|\Theta_0|p(P)\rangle$$

- In the chiral limit the entirety of the proton's mass is produced by the trace anomaly, Θ_0

  … In QCD, Θ_0 measures the strength of gluon self-interactions

  … so, from one perspective, m_p is completely generated by glue.



On the other hand …

$$T_{\mu\mu} = \frac{1}{4}\beta(\alpha(\zeta))G^a_{\mu\nu}G^a_{\mu\nu}$$

Trace Anomaly

- In the chiral limit

$$\langle\pi(q)|T_{\mu\nu}|\pi(q)\rangle = -q_\mu q_\nu \;\Rightarrow\; \langle\pi(q)|\Theta_0|\pi(q)\rangle = 0$$

- Does this mean that the scale anomaly vanishes trivially in the pion state, *i.e*. gluons contribute nothing to the pion mass?
- That is a difficult way to obtain "zero"
- Easier, perhaps, to imagine that "zero" owes to cancellations between different operator-component contributions to the expectation value of Θ_0.
- Of course, such precise cancellation should not be an accident.

  It could only arise naturally because of some symmetry and/or symmetry-breaking pattern.

Whence "1" and yet "0" ?

$$\langle p(P)|\Theta_0|p(P)\rangle = m_p^2\,, \quad \langle \pi(q)|\Theta_0|\pi(q)\rangle = 0$$

- *No statement of the question*

  "Whence the proton's mass?"

  is complete without the additional clause

  "Whence the **absence** *of a pion mass?"*

- Natural nuclear-physics mass-scale must emerge simultaneously with apparent preservation of scale invariance in related systems
  - Expectation value of Θ_0 in pion is always zero, irrespective of the size of the natural mass-scale for strong interactions = m_p

A New Era for hadro-particle physics

Model independent statement:
There is quark orbital angular momentum in the pion
Probable corollary: What is true in the pion, is true in the proton

A New Era

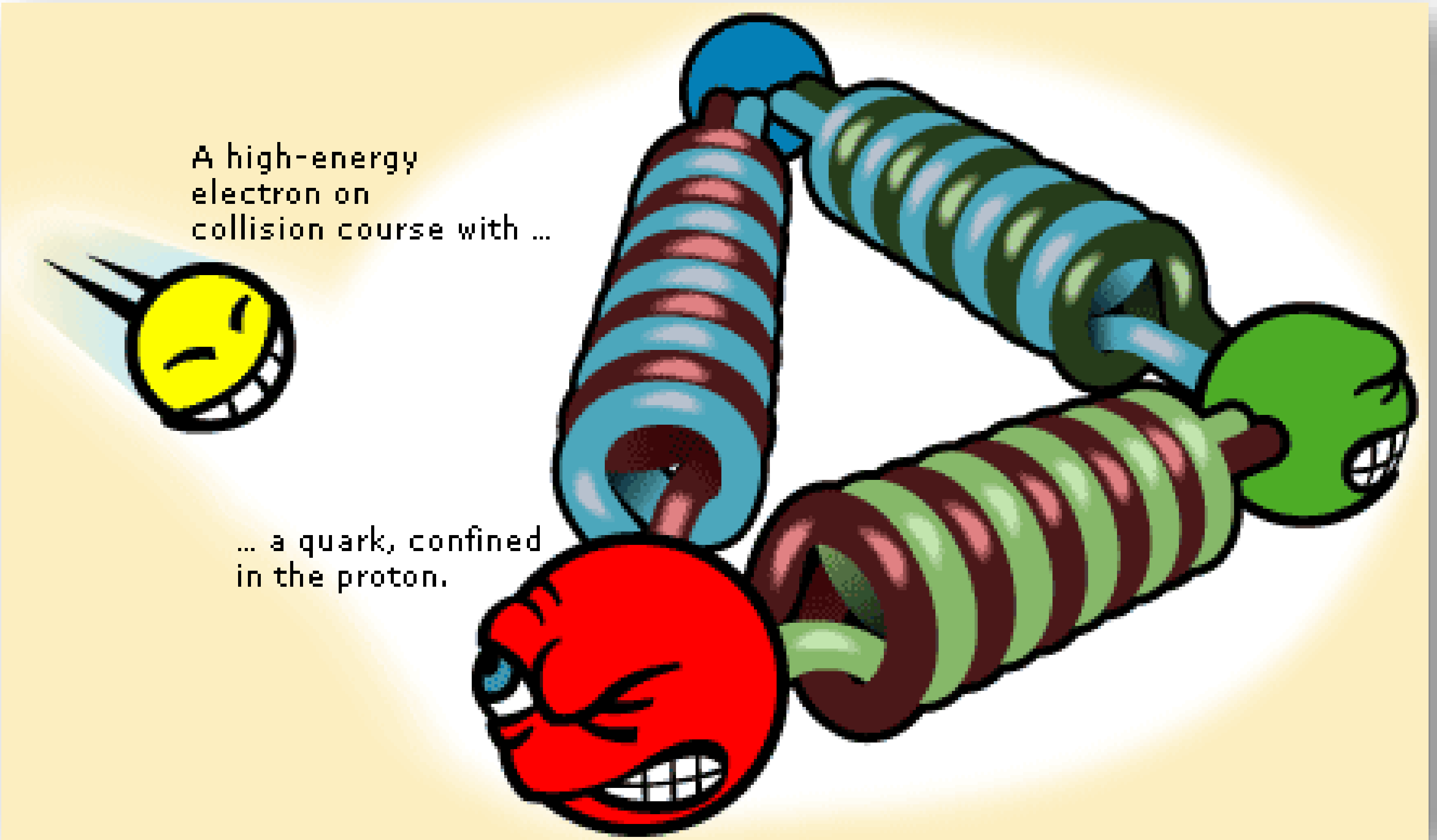
- QCD is the first place that we fully experience the collisions and collusions between relativity and quantum mechanics.
- In attempting to match QCD with Nature, we confront the innumerable complexities of nonperturbative, nonlinear dynamics in relativistic quantum field theory, *e.g*.
  - the loss of particle number conservation,
  - the frame and scale dependence of the explanations and interpretations of observable processes,
  - and the evolving character of the relevant degrees-of-freedom.
- Origin and distribution of mass, momentum, spin within hadrons, *e.g*. where is the proton's spin and how is the pion spinless?
  - Don't forget the latter!
  - How do all the spin-1/2 quarks and spin-1 gluons combine to make a massless, J=0 composite mode?

Overarching Science Challenges for the coming decade

- What is origin of mass in our Universe?
- What is the nature of confinement in real (dynamical-quarks) QCD?
- How are they connected?
- How can any
  - answers,
  - conjectures
  - and/or conclusions

  be empirically verified?

Physics is an Empirical Science



Quantum Chromodynamics

Particle Data Group

Citation: C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C, 40, 100001 (2016) and 2017 update

g or gluon $I(J^P) = 0(1^-)$

SU(3) color octet

Mass $m = 0$. Theoretical value. A mass as large as a few MeV may not be precluded, see YNDURAIN 95.

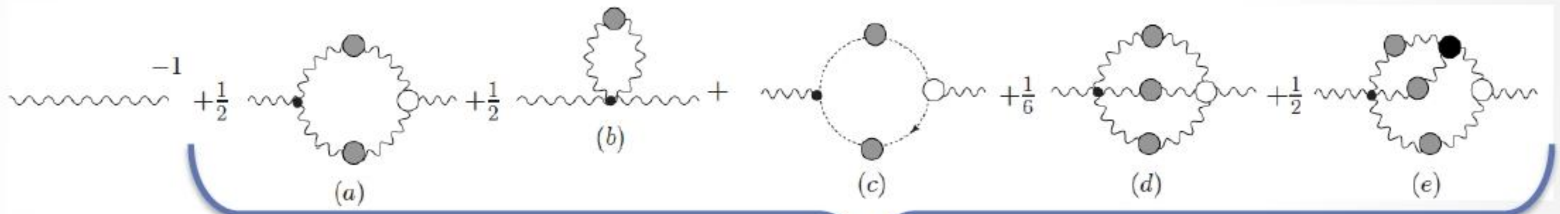
VALUE	*DOCUMENT ID*		*TECN*	*COMMENT*
• • • We do not use the following data for averages, fits, limits, etc. • • •				
	ABREU	92E	DLPH	Spin 1, not 0
	ALEXANDER	91H	OPAL	Spin 1, not 0
	BEHREND	82D	CELL	Spin 1, not 0
	BERGER	80D	PLUT	Spin 1, not 0
	BRANDELIK	80C	TASS	Spin 1, not 0

gluon REFERENCES

YNDURAIN	95	PL B345 524	F.J. Yndurain	(MADU)
ABREU	92E	PL B274 498	P. Abreu *et al.*	(DELPHI Collab.)
ALEXANDER	91H	ZPHY C52 543	G. Alexander *et al.*	(OPAL Collab.)
BEHREND	82D	PL B110 329	H.J. Behrend *et al.*	(CELLO Collab.)
BERGER	80D	PL B97 459	C. Berger *et al.*	(PLUTO Collab.)
BRANDELIK	80C	PL B97 453	R. Brandelik *et al.*	(TASSO Collab.)

Pinch Technique: Theory and Applications
Daniele Binosi & Joannis Papavassiliou
Phys. Rept. 479 (2009) 1-152

$$\Delta^{-1}_{\mu\nu}(q) =$$



$$\Pi_{\mu\nu}(q)$$

$$\Pi_{\mu\nu}(q) = P_{\mu\nu}(q)\Pi(q)$$

$$P_{\mu\nu}(q) = g_{\mu\nu} - q_\mu q_\nu / q^2$$

Gluon Gap Equation

Bridging a gap between continuum-QCD and ab initio predictions of hadron observables, D. Binosi *et al.*, arXiv:1412.4782 [nucl-th], Phys. Lett. B742 (2015) 183-188

In QCD: Gluons become massive!

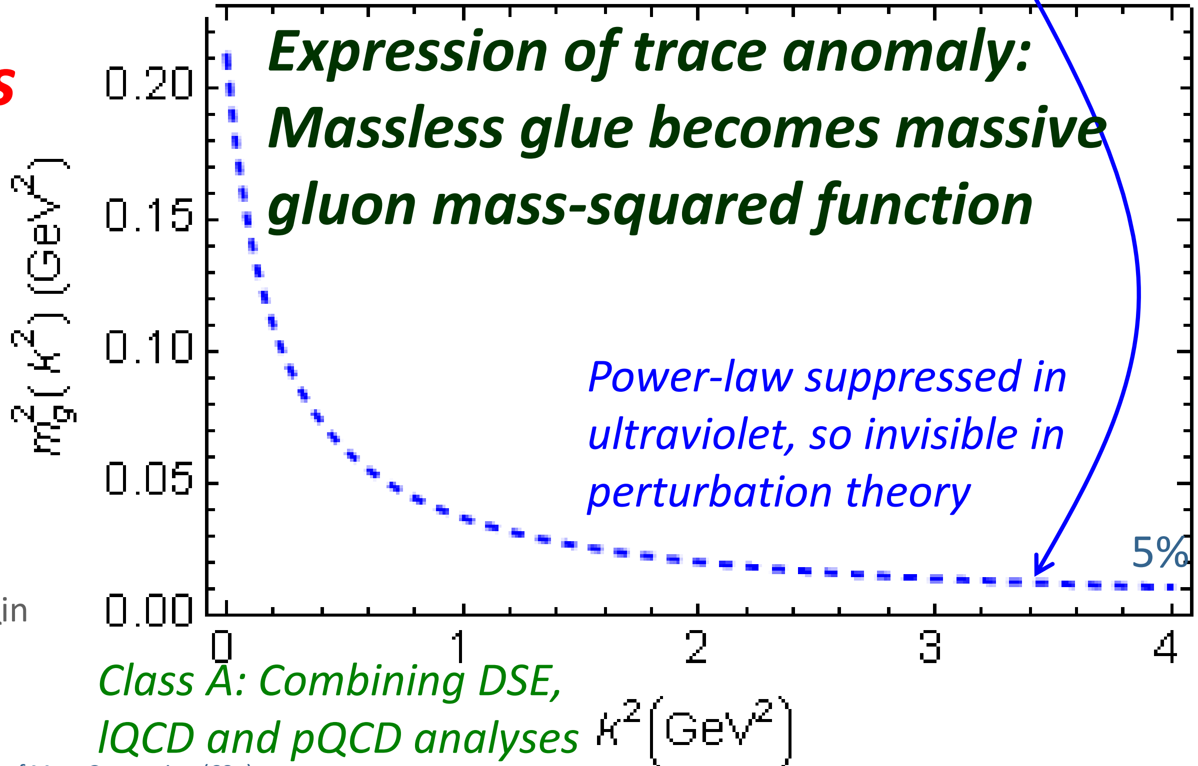
- Running gluon mass

$$d(k^2) = \frac{\alpha(\zeta)}{k^2 + m_g^2(k^2;\zeta)}$$

$$\alpha_s(0) = 2.77 \approx 0.9\pi\,,\ m_g^2(0) = (0.46\,\text{GeV})^2$$

$$m_g^2(k^2) \approx \frac{\mu_g^4}{\mu_g^2 + k^2}$$

- Gluons are ***cannibals*** – a particle species whose members become massive by eating each other!



Class A: Combining DSE, lQCD and pQCD analyses of QCD's gauge sector

Interaction model for the gap equation, S.-x.Qin *et al.*, arXiv:1108.0603 [nucl-th], Phys. Rev. C **84** (2011) 042202(R) [5 pages]

Massive Gauge Bosons!

- Gauge boson cannibalism

  … a new physics frontier … within the Standard Model
- Asymptotic freedom means
  … ultraviolet behaviour of QCD is controllable
- Dynamically generated masses for gluons and quarks means that QCD dynamically generates its own infrared cutoffs
  - Gluons and quarks with

    wavelength $\lambda > 2/\text{mass} \approx 1$ fm

    decouple from the dynamics … ***Confinement?!***
- How does that affect observables?
  - It will have an impact in any continuum study
  - Possibly (probably?) plays a role in gluon saturation ...
    In fact, could be a harbinger of gluon saturation?

Electron Ion Collider:
The Next QCD Frontier

Confined particle

- All QCD solutions for gluon & quark propagators exhibit an inflection point in k^2 … consequence of the running-mass function

$\Rightarrow$ Spectral function is NOT positive

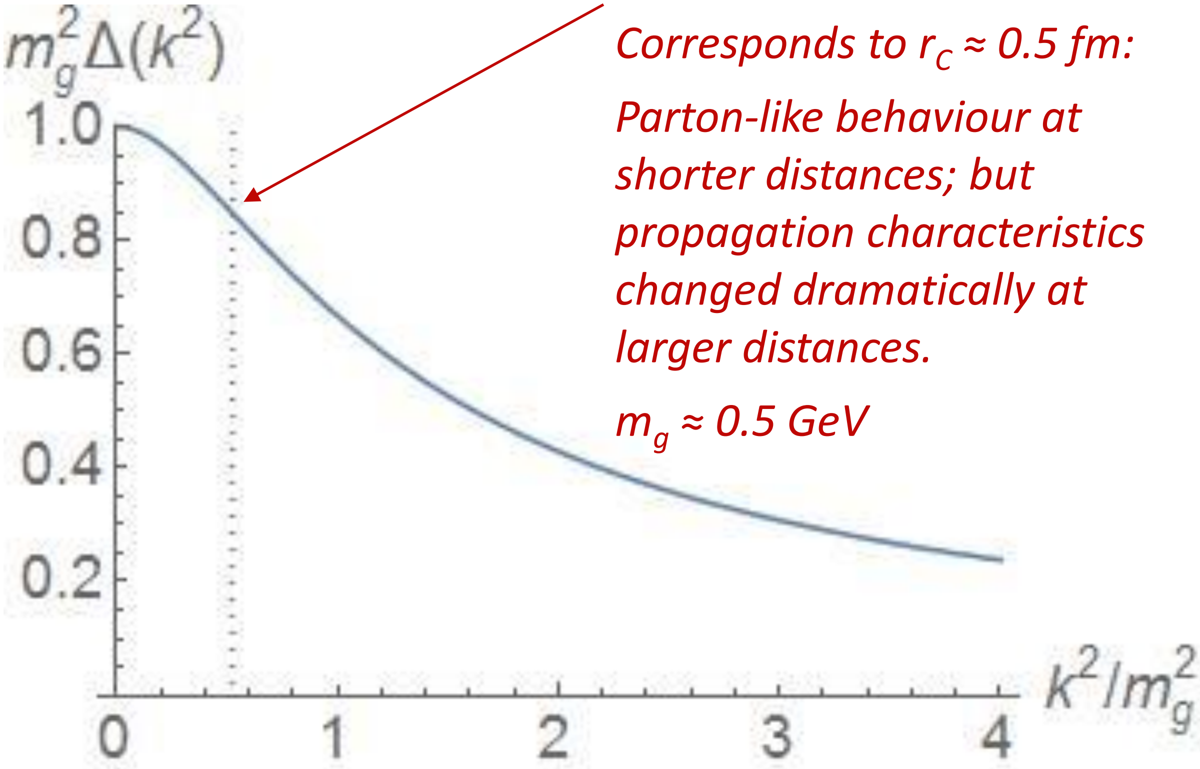
$\Rightarrow$ Such states have negative norm (negative probability)

$\Rightarrow$ Negative norm states are not observable

$\Rightarrow$ This object is confined!

$$\Delta(k^2) = \int_0^\infty ds\, \frac{\rho(s)}{s+k^2}$$

Sum of "probabilities"

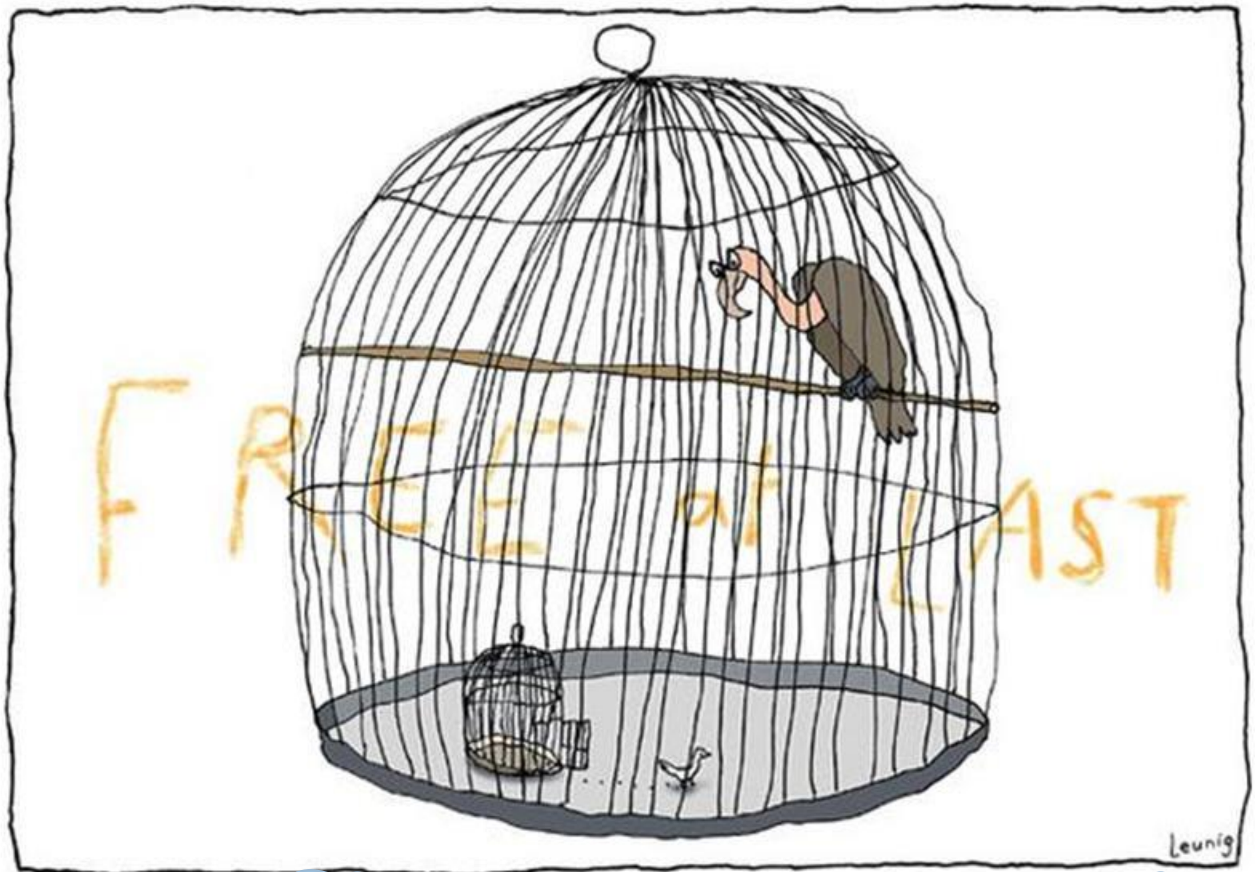


Inflexion point

Corresponds to $r_C \approx 0.5$ fm:

Parton-like behaviour at shorter distances; but propagation characteristics changed dramatically at larger distances.

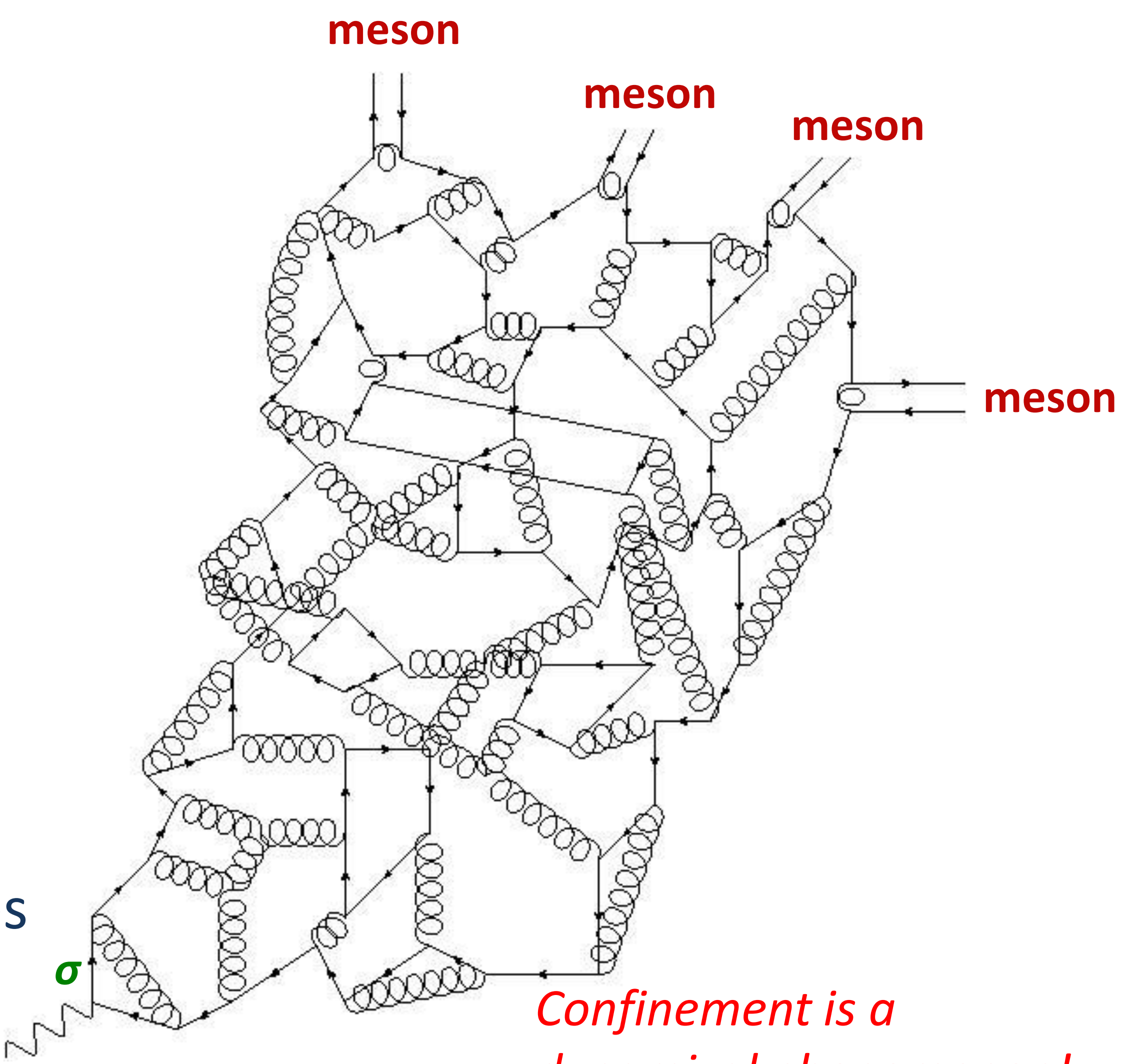
$m_g \approx 0.5$ GeV



Confinement is dynamical

Quark Fragmentation

- A quark begins to propagate
- But after each "step" of length σ, on average, an interaction occurs, so that the quark *loses* its identity, sharing it with other partons
- Finally, a cloud of partons is produced, which coalesces into colour-singlet final states



Confinement is a dynamical phenomenon!

Quark Fragmentation

- A quark begins to propagate
- But after each "step" of length ***σ***, on aver... int... tha... ide... othe...
- Final... parto... which... colour...

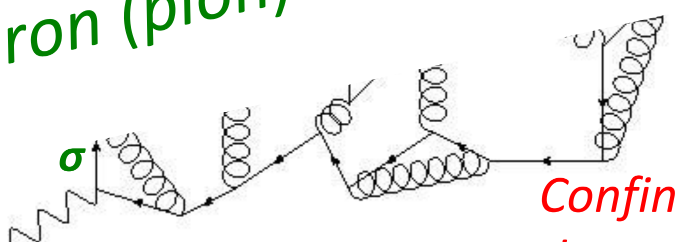
meson

me...

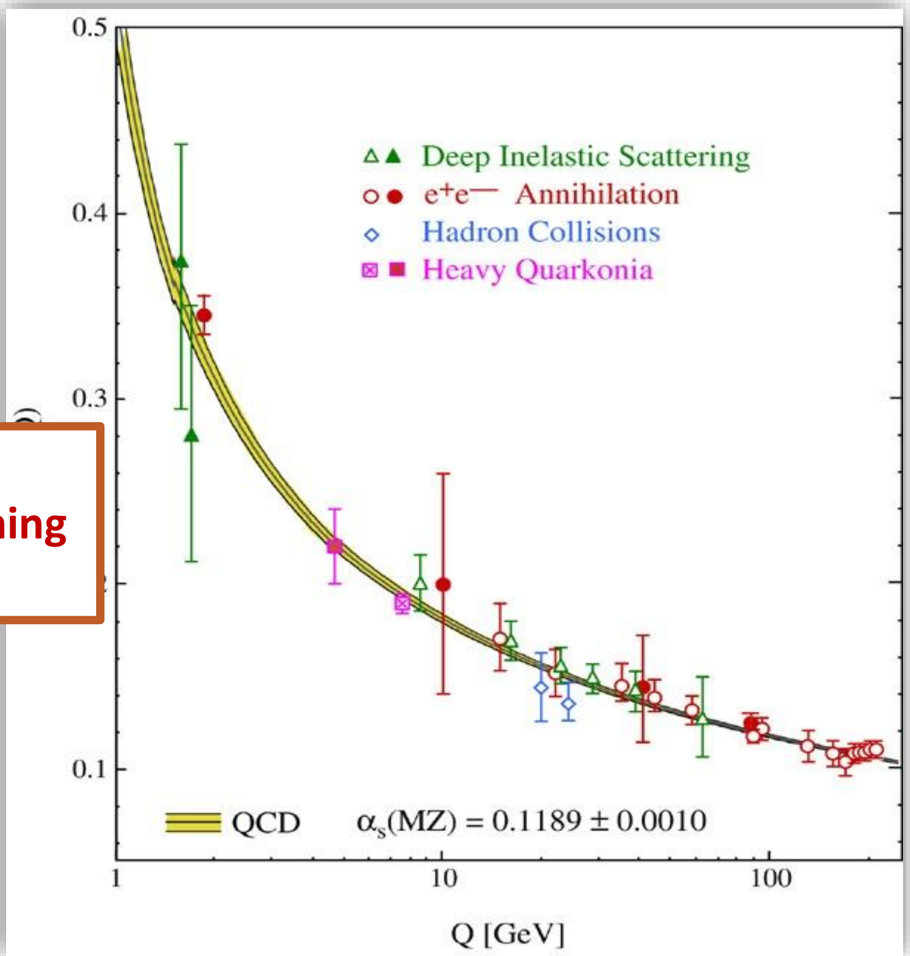
...on

Confinement in hadron physics is largely a dynamical phenomenon, intimately connected with the fragmentation effect.

It is unlikely to be comprehended without simultaneously understanding dynamical chiral symmetry breaking, which is the origin of a near-zero mass hadron (pion).



Confinement is a dynamical phenomenon!

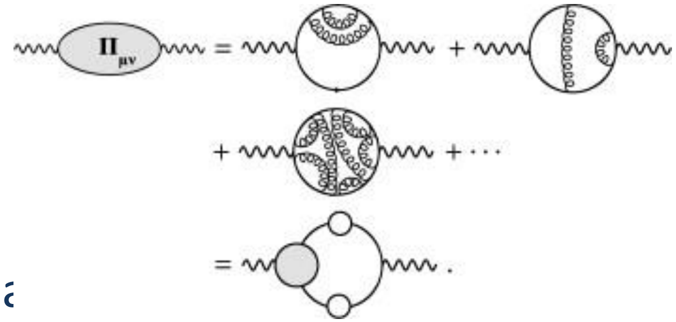


⇐
What's happening out here?!

QCD's Running Coupling

QED Running Coupling

- Quantum gauge field theories in four spacetime dimensions,
  - Lagrangian couplings and masses come to depend on a mass scale
  - Can normally be related to the energy or momentum at which a given process occurs.
- QED, owing to the Ward identity:
  - a single running coupling
  - measures strength of the photon-charged-fermion vertex
  - can be obtained by summing the virtual processes that dress the bare photon, *viz*. by computing the photon vacuum pola

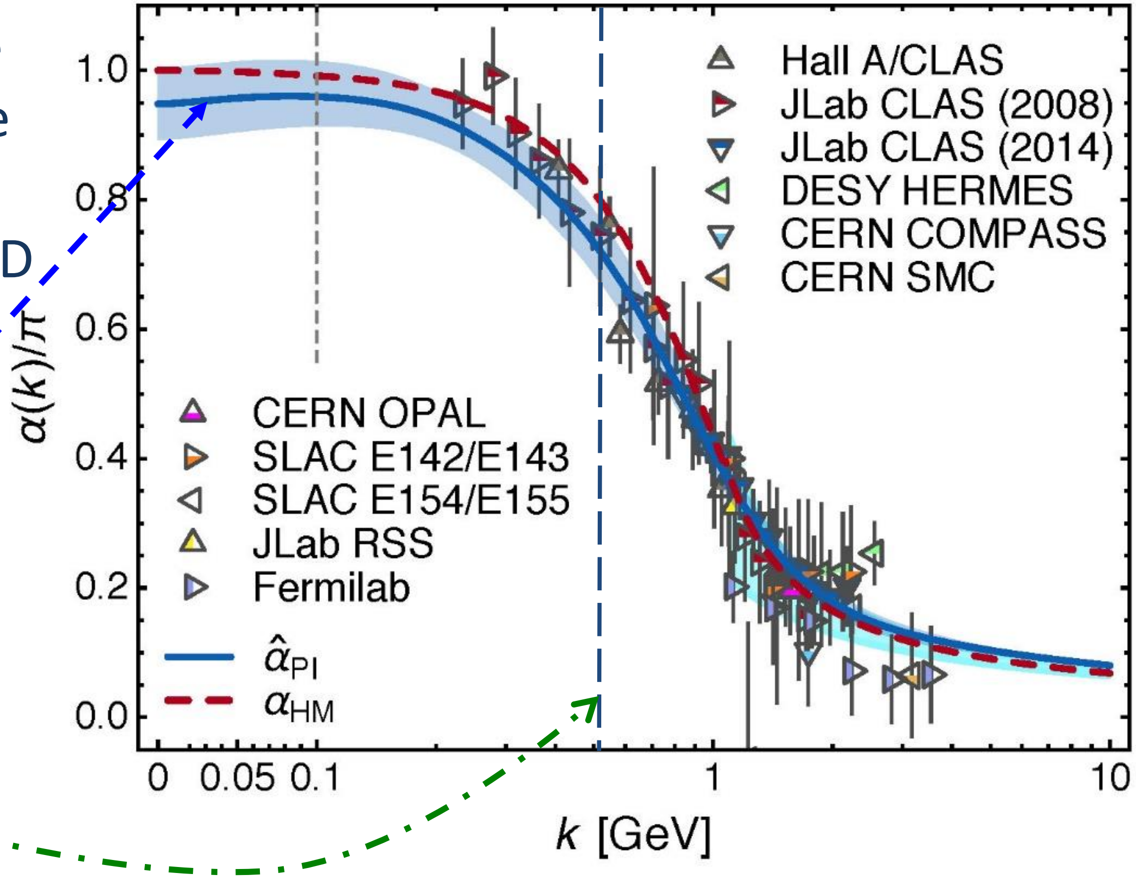


- QED's running coupling is known to great accuracy and the running has been observed directly.

Process independent strong running coupling
Binosi, Mezrag, Papavassiliou, Roberts, Rodriguez-Quintero
arXiv:1612.04835 [nucl-th], Phys. Rev. D 96 (2017) 054026/1-7

Process-independent effective-charge in QCD

- Modern continuum & lattice methods for analysing gauge sector enable analogous quantity to be defined in QCD
- Combined continuum and lattice analysis of QCD's gauge sector yields a parameter-free prediction
- N.B. Qualitative change in $\hat{\alpha}_{PI}(k)$ at $k \approx ½\, m_p$



QCD Effective Charge

Process independent strong running coupling
Binosi, Mezrag, Papavassiliou, Roberts, Rodriguez-Quintero
arXiv:1612.04835 [nucl-th], Phys. Rev. D 96 (2017) 054026/1-7

The QCD Running Coupling,
A. Deur, S. J. Brodsky and G. F. de Teramond,
Prog. Part. Nucl. Phys. **90** (2016) 1-74

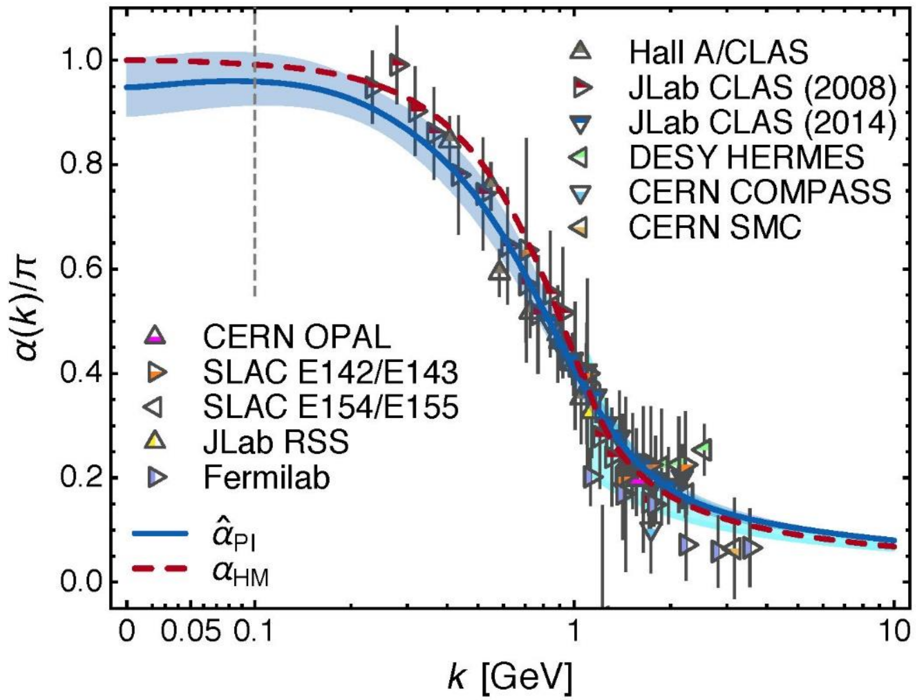
- Near precise agreement between process-independent $\hat{\alpha}_{PI}$ and α_{g1}
  and $\hat{\alpha}_{PI} \approx \alpha_{HM}$

- Perturbative domain:

$$\alpha_{g_1}(k^2) = \alpha_{\overline{MS}}(k^2)(1 + 1.14\,\alpha_{\overline{MS}}(k^2) + \ldots),$$
$$\widehat{\alpha}_{PI}(k^2) = \alpha_{\overline{MS}}(k^2)(1 + 1.09\,\alpha_{\overline{MS}}(k^2) + \ldots),$$

difference = (1/20) $\alpha_{\overline{MS}}^2$

- Parameter-free prediction:
  - curve completely determined by results obtained for gluon & ghost two-point functions using continuum and lattice-regularised QCD.

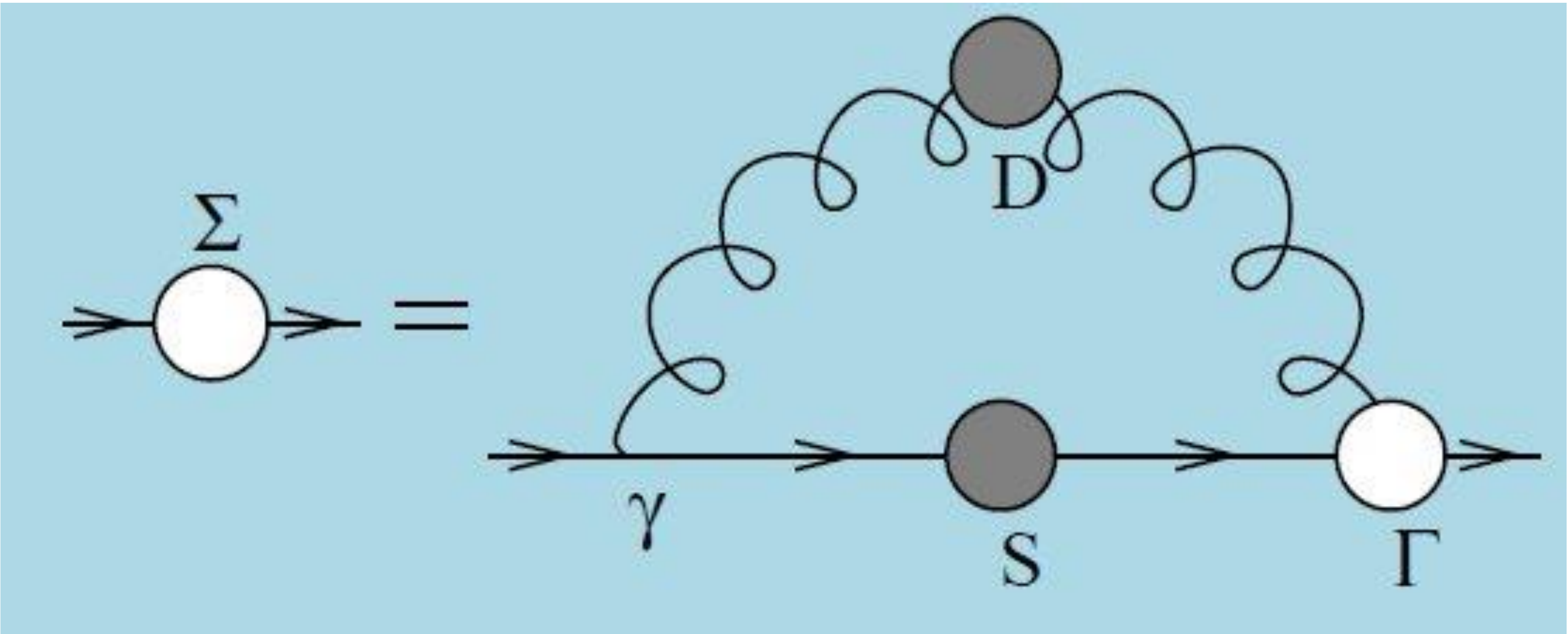


Data: process dependent effective charge: α_{g1} defined via Bjorken Sum Rule

QCD Effective Charge

- $\hat{\alpha}_{PI}$ is a new type of effective charge
  - direct analogue of the Gell-Mann–Low effective coupling in QED, *i.e.* completely determined by the gauge-boson two-point function.
- Prediction for $\hat{\alpha}_{PI}$ is parameter-free
  - Draws best from continuum & lattice results for QCD's gauge sector
- Prediction for $\hat{\alpha}_{PI}$ smoothly unifies the nonperturbative and perturbative domains of the strong-interaction theory.
- $\hat{\alpha}_{PI}$ is
  - process-independent
  - appears in every one of QCD's dynamical equations of motion
  - known to unify a vast array of observables
- $\hat{\alpha}_{PI}$ possesses an infrared-stable fixed-point
  - Nonperturbative analysis demonstrating absence of a Landau pole in QCD
- QCD is IR finite, owing to dynamical generation of gluon mass-scale

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

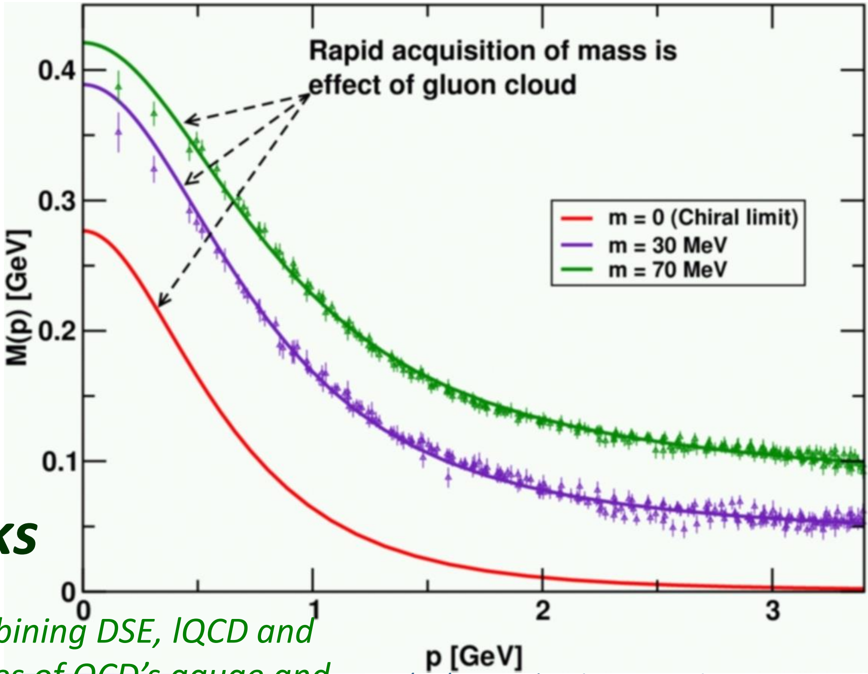


Quark Gap Equation

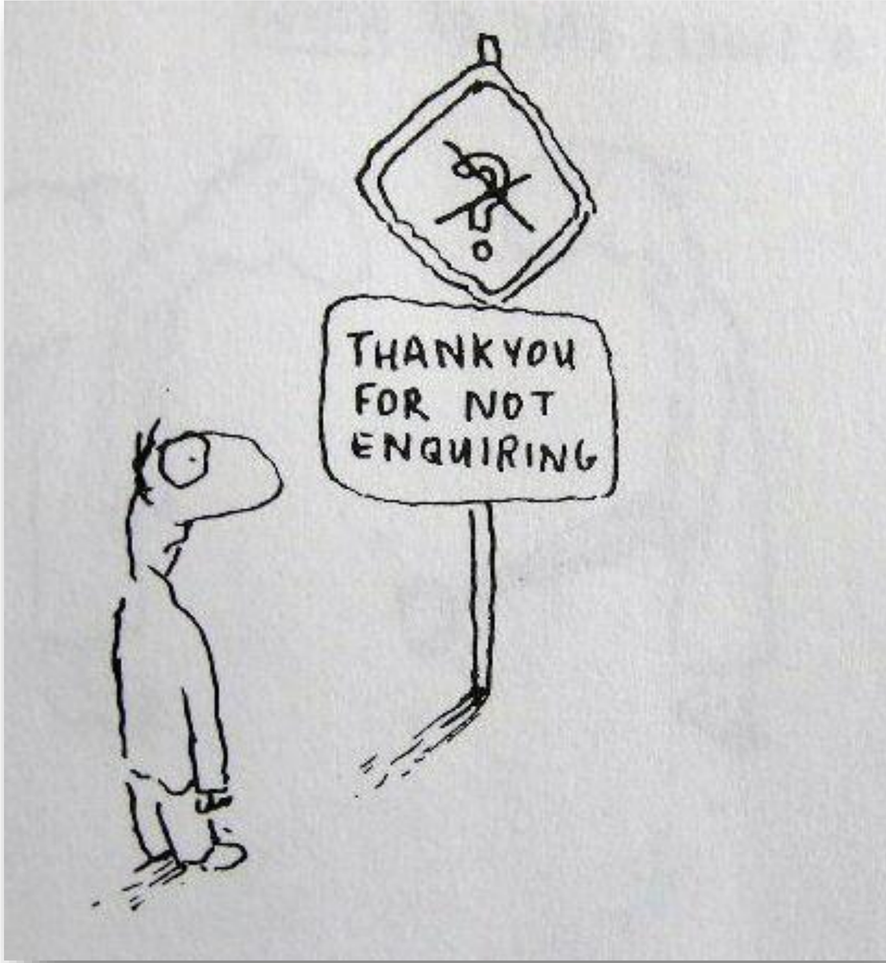
DCSB

- Dynamical chiral symmetry breaking (DCSB) is a critical emergent phenomenon in QCD
- Expressed in hadron wave functions not in vacuum condensates
- Contemporary theory indicates that DCSB is responsible for more than 98% of the visible mass in the Universe; namely, given that classical massless-QCD is a conformally invariant theory, then DCSB is the origin of *mass from nothing*.
- ***Dynamical***, not spontaneous
  - Add nothing to *QCD* ,

    No Higgs field, nothing!
    Effect achieved purely through quark+gluon dynamics.
  - ✓ ***Trace anomaly: massless quarks become massive***



Class A: Combining DSE, lQCD and pQCD analyses of QCD's gauge and matter sectors



Enigma of Mass

Pion's Goldberger -Treiman relation

- Pion's Bethe-Salpeter amplitude

  Solution of the Bethe-Salpeter equation

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\left[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\right]$$

- Dressed-quark propagator $S(p) = \frac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$

- Axial-vector Ward-Takahashi identity entails

$$f_\pi E_\pi(k;P=0) = B(k^2)$$

Owing to DCSB & Exact in Chiral QCD

***Miracle*: two body problem solved, almost completely, once solution of one body problem is known**

Rudimentary version of this relation is apparent in Nambu's Nobel Prize work

Model independent
Gauge independent
Scheme independent

$$f_\pi E_\pi(p^2) = B(p^2)$$

The most fundamental expression of Goldstone's Theorem and DCSB

Rudimentary version of this relation is apparent in Nambu's Nobel Prize work

Model independent
Gauge independent
Scheme independent

$$f_\pi E_\pi(p^2) \Leftrightarrow B(p^2)$$

Pion exists if, and only if, mass is dynamically generated

$f_\pi\, E_\pi(p^2) = B(p^2)$

Enigma of mass

This algebraic identity is why QCD's pion is massless in the chiral limit

- The quark level Goldberger-Treiman relation shows that DCSB has a very deep and far reaching impact on physics within the strong interaction sector of the Standard Model; viz.,

  Goldstone's theorem is fundamentally an expression of equivalence between the one-body problem and the two-body problem in the pseudoscalar channel.

- This emphasises that Goldstone's theorem has a pointwise expression in QCD
- Hence, pion properties are an almost direct measure of the dressed-quark mass function.
- Thus, enigmatically, the properties of the *massless* pion are the cleanest expression of the mechanism that is responsible for almost all the visible mass in the universe.

$$\langle p(P)|\Theta_0|p(P)\rangle = m_p^2\,, \quad \langle \pi(q)|\Theta_0|\pi(q)\rangle = 0$$

Whence "0" ?

$$\langle p(P)|\Theta_0|p(P)\rangle = m_p^2\,, \quad \langle \pi(q)|\Theta_0|\pi(q)\rangle = 0$$

Whence "0" ?

The answer is algebraic

*Munczek, H. J., Phys. Rev. D **52** (1995) pp. 4736-4740*
*Bender, A., Roberts, C.D. and von Smekal, L., Phys. Lett. B **380** (1996) pp. 7-12*
*Maris, P. , Roberts, C.D. and Tandy, P.C., Phys. Lett. B **420** (1998) pp. 267-273*
*Binosi, Chang, Papavassiliou, Qin, Roberts, Phys. Rev. D **93** (2016) 096010/1-7*

Pion masslessness

- Obtain a coupled set of gap- and Bethe-Salpeter equations
  - Bethe-Salpeter Kernel:
    - valence-quarks with a momentum-dependent running mass produced by self-interacting gluons, which have given themselves a running mass
    - Interactions of arbitrary but enumerable complexity involving these "basis vectors"
  - Chiral limit:
    - Algebraic proof
      - at any & each finite order in symmetry-preserving construction of kernels for
        - » the gap (quark dressing)
        - » and Bethe-Salpeter (bound-state) equations,
      - there is a precise cancellation between
        - » mass-generating effect of dressing the valence-quarks
        - » and attraction introduced by the scattering events
    - Cancellation guarantees that
      - simple system, which began massless,
      - becomes a complex system, with
        - » a nontrivial bound-state wave function
        - » attached to a pole in the scattering matrix, which remains at $P^2=0$ …
    - Interacting, bound system remains massless!

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Pion masslessness

- Obtain a coupled set of gap- and Bethe-Salpeter equations

Quantum field theory statement:

In the pseudsocalar channel, the dynamically generated mass of the two fermions is precisely cancelled by the attractive interactions between them – iff –

$$f_\pi E_\pi(p^2) = B(p^2)$$

- Cancellation guarantees that
  - simple system, which began massless,
  - becomes a complex system, with
    - a nontrivial bound-state wave function
    - attached to a pole in the scattering matrix, which remains at $P^2=0$ …
- Interacting, bound system remains massless

$$T_{\mu\mu} = \frac{1}{4}\beta(\alpha(\zeta))G^a_{\mu\nu}G^a_{\mu\nu}$$

Massless Pion

- Chiral limit

$$\langle \pi(q)|T_{\mu\nu}|\pi(q)\rangle = -q_\mu q_\nu \;\Rightarrow\; \langle \pi(q)|\Theta_0|\pi(q)\rangle = 0$$

- "Zero" owes to cancellations between different operator-component contributions to the expectation value of Θ_0.
- The cancellations are precise

  Arising naturally because chiral symmetry – the apparent masslessness of the QCD action – is broken by strong dynamics in a very particular manner.

- In the chiral limit, the pion is massless irrespective of the magnitude of any other hadron's mass

*Munczek, H. J., Phys. Rev. D **52** (1995) pp. 4736-4740*
*Bender, A., Roberts, C.D. and von Smekal, L., Phys. Lett. B **380** (1996) pp. 7-12*
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      - becomes a complex system, with
        - » a nontrivial bound-state wave function
        - » attached to a pole in the scattering matrix, which remains at $P^2=0$ …
      - *i.e.*, interacting, bound system remains massless

Observing *Mass*

Confinement ⇔ DCSB

- Signals for these (equivalent?) emergent phenomena are ubiquitous
- Manifestations do not always appear the same in different systems
- Studying a diverse array of systems and/or observables therefore exposes different aspects of the underlying mechanisms
- Examples … *All the basic subjects of hadro-particle physics*
  - Spectrum of hadrons (level ordering, existence of exotics …)
  - Hadron (meson & baryon) elastic and transition form factors
  - Distribution amplitudes and functions, in all their guises
  - Origin of the nucleon-nucleon interaction and emergence of nuclei
- Complete understanding of QCD – the toughest, most interesting part of the Standard Model – will only be found in a unified description of all these phenomena

Imaging dynamical chiral symmetry breaking: pion wave function on the light front, Lei Chang, et al., arXiv:1301.0324 [nucl-th], Phys. Rev. Lett. **110** (2013) 132001 (2013) [5 pages].

Pion's valence-quark Distribution Amplitude

- Methods have been developed that enable direct computation of the pion's light-front wave function
- $\varphi_\pi(x)$ = twist-two parton distribution amplitude = projection of the pion's Poincaré-covariant wave-function onto the light-front

$$\varphi_\pi(x) = Z_2 \mathrm{tr}_{CD} \int \frac{d^4k}{(2\pi)^4} \delta(n\cdot k - x n\cdot P)\, \gamma_5 \gamma\cdot n\, S(k) \Gamma_\pi(k;P) S(k-P)$$

- Results have been obtained with the DCSB-improved DSE kernel, which unifies matter & gauge sectors

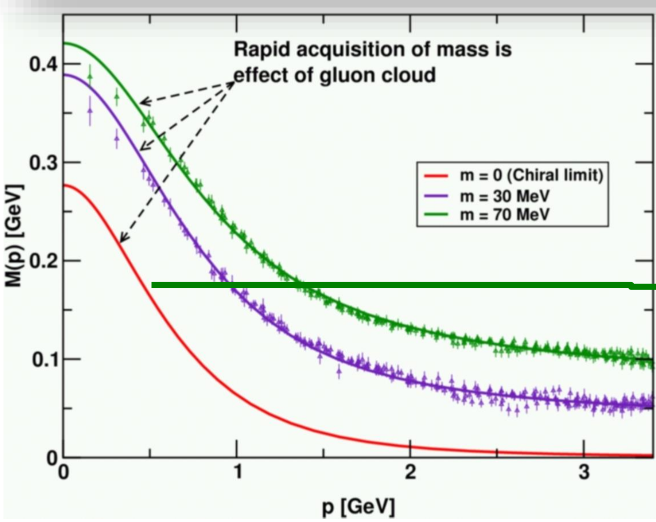
$$\varphi_\pi(x) \propto x^\alpha (1-x)^\alpha, \text{ with } \alpha \approx 0.5$$

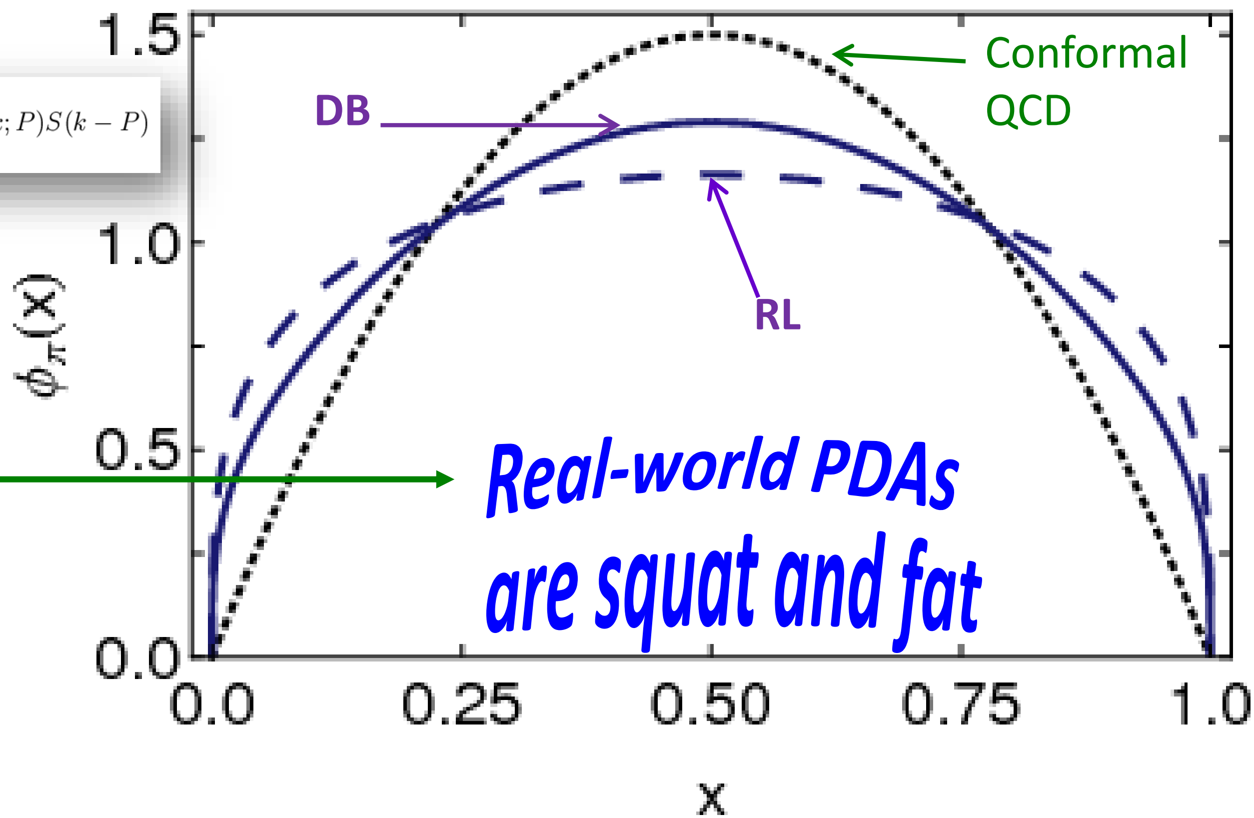
Imaging dynamical chiral symmetry breaking: pion wave function on the light front, Lei Chang, *et al.*, arXiv:1301.0324 [nucl-th], Phys. Rev. Lett. **110** (2013) 132001 (2013) [5 pages].

Pion's valence-quark Distribution Amplitude

- Continuum-QCD prediction:
  marked broadening of $\varphi_\pi(x)$, which owes to DCSB

$$\varphi_\pi(x) = Z_2 \mathrm{tr}_{CD} \int \frac{d^4k}{(2\pi)^4}\, \delta(n\cdot k - x n\cdot P)\, \gamma_5\gamma\cdot n\, S(k)\Gamma_\pi(k;P)S(k-P)$$





Pion's electromagnetic form factor

- PDA Broadening has enormous impact on understanding $F_\pi(Q^2)$

A: Internally-consistent DSE prediction

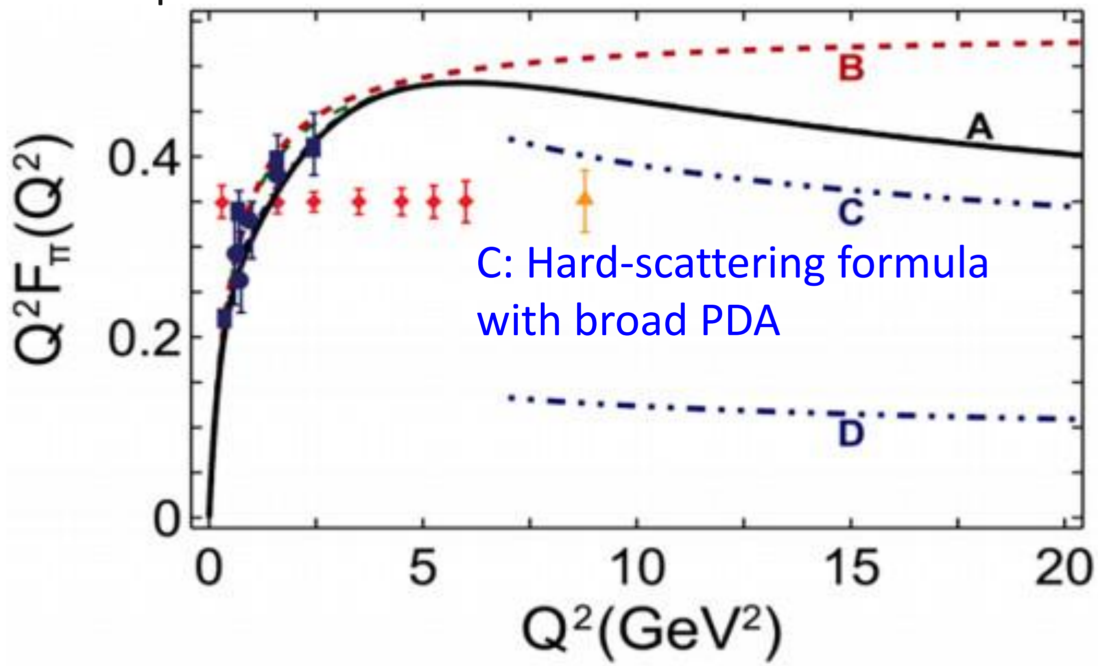
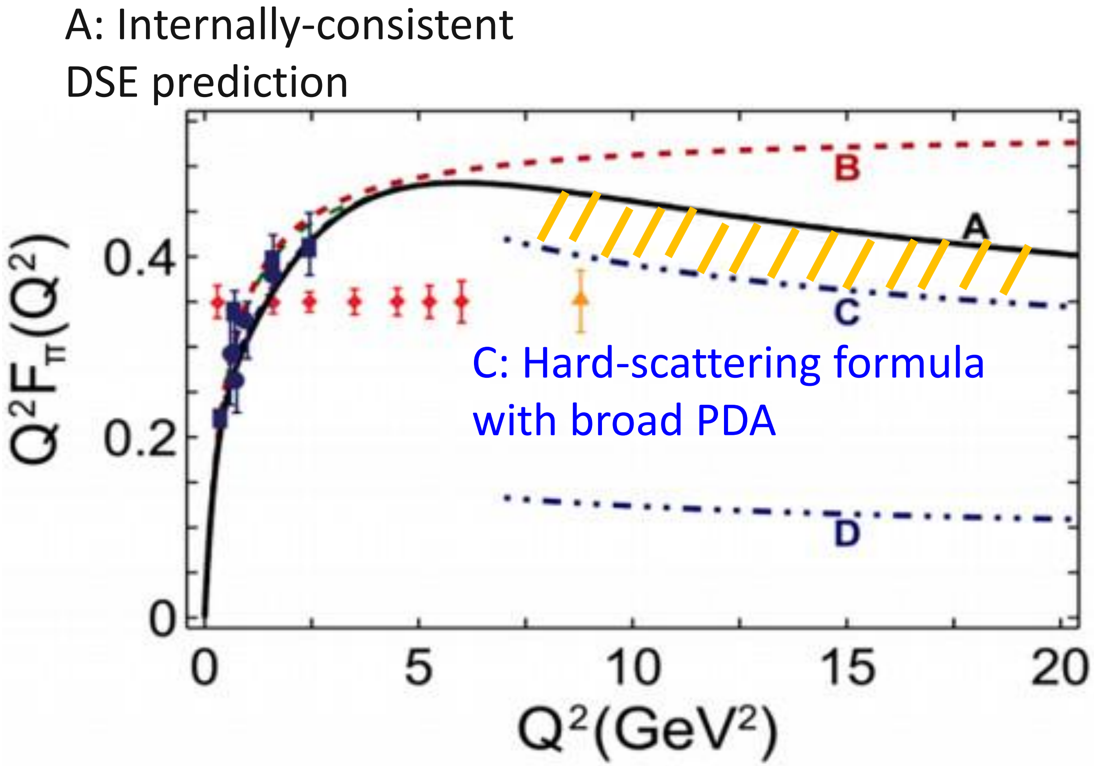


Figure 2.2: *Existing (dark blue) data and projected (red, orange) uncertainties for future data on the pion form factor. The solid curve (A) is the QCD-theory prediction bridging large and short distance scales. Curve B is set by the known long-distance scale—the pion radius. Curves C and D illustrate calculations based on a short-distance quark-gluon view.*

Pion electromagnetic form factor at spacelike momenta
L. Chang et al., arXiv:1307.0026 [nucl-th], Phys. Rev. Lett. 111, 141802 (2013)

Pion's electromagnetic form factor

- PDA Broadening has enormous impact on understanding $F_\pi(Q^2)$
- Appears that JLab12 is within reach of first verification of a QCD hard-scattering formula



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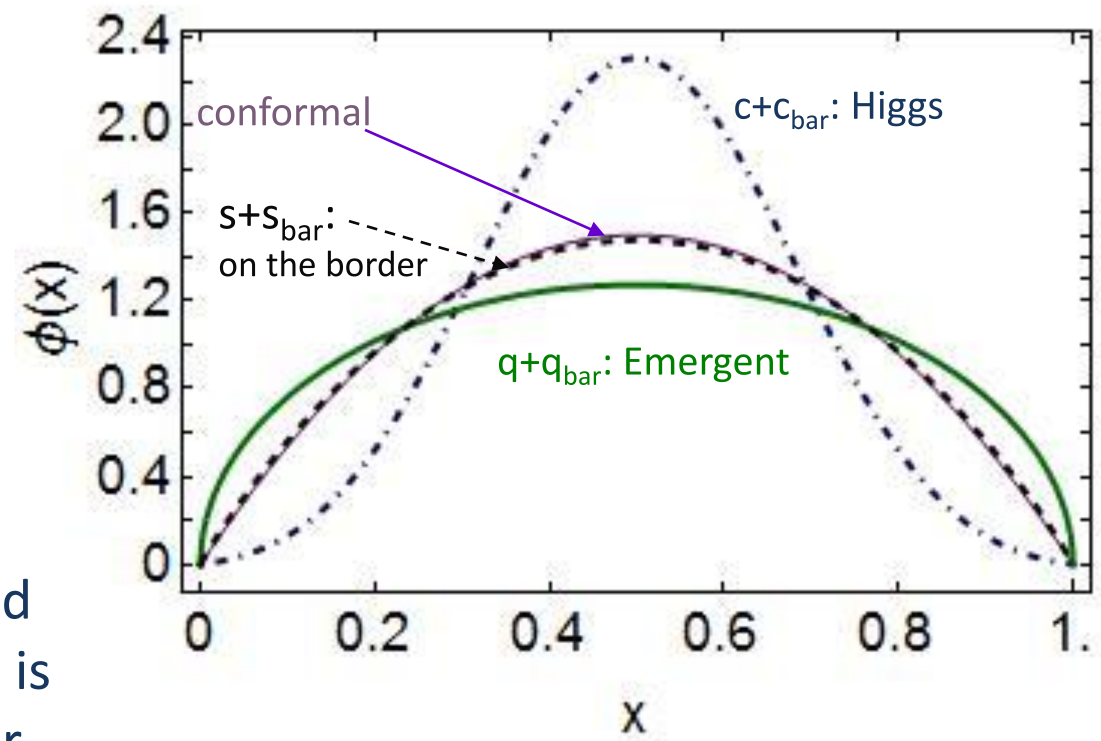
Parton distribution amplitudes of S-wave heavy-quarkonia
Minghui Ding, Fei Gao, Lei Chang, Yu-Xin Liu and Craig D. Roberts
arXiv:1511.04943 [nucl-th], Phys. Lett. B **753** (2016) pp. 330-335

Emergent Mass *vs.* Higgs Mechanism

- When does Higgs mechanism begin to influence mass generation?
- limit $m_{quark} \to \infty$

  $\varphi(x) \to \delta(x-½)$
- limit $m_{quark} \to 0$

  $\varphi(x) \sim (8/\pi)\,[x(1-x)]^{½}$
- Transition boundary lies just above $m_{strange}$
- Comparison between distributions of light-quarks and those involving strange-quarks is obvious place to find signals for strong-mass generation



Exposing strangeness: projections for kaon electromagnetic form factors, Fei Gao, Lei Chang, Yu-Xin Liu, Craig D. Roberts and Peter C. Tandy, arXiv:1703.04875 [nucl-th], Phys. Rev. D **96** (2017) 034024

Kaon electromagnetic form factor

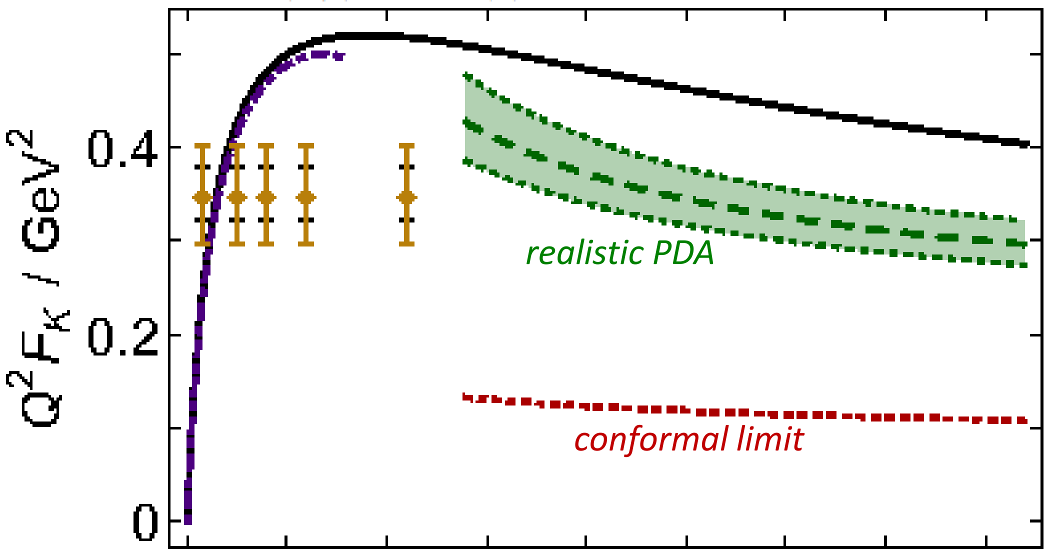
$$\exists \bar{Q}_0 > \Lambda_{\rm QCD} \mid Q^2 F_K(Q^2) \stackrel{Q^2 > \bar{Q}_0^2}{\approx} 16\pi\alpha_s(Q^2) f_K^2 w_K^2(Q^2)$$

with [41] $f_K = 0.110\,\text{GeV}$ and, for the K^+:

$$w_K^2 = e_{\bar{s}} w_{\bar{s}}^2 + e_u w_u^2,$$

$$w_{\bar{s}} = \frac{1}{3}\int_0^1 dx\, \frac{1}{1-x}\varphi_K(x)\,, \quad w_u = \frac{1}{3}\int_0^1 dx\, \frac{1}{x}\varphi_K(x)$$

- Dashed purple curve = DSE (Maris-Tandy 2000) prediction
- Solid black curve = modern DSE prediction
- Hard-scattering formula
  - Long-dashed green curve, with error band marking uncertainty in dilation of the PDA
- Skewing is not the issue: 12%-15%, DSE- and lattice-QCD agree
- It's extent of the broadening that generates the uncertainty
- JLab 12 can reach to the point at which VMD fails … as for the pion, can potentially validate general hard scattering formula



- Two successful comparisons of experiment and theory couldn't be an accident!

*Exposing strangeness: projections for kaon electromagnetic form factors, Fei Gao, Lei Chang, Yu-Xin Liu, Craig D. Roberts and Peter C. Tandy, arXiv:1703.04875 [nucl-th], Phys. Rev. D **96** (2017) 034024*

Kaon form factor - flavour separation

$$\exists \bar{Q}_0 > \Lambda_{\rm QCD} \mid Q^2 F_K(Q^2) \overset{Q^2>\bar{Q}_0^2}{\approx} 16\pi\alpha_s(Q^2) f_K^2 w_K^2(Q^2)$$

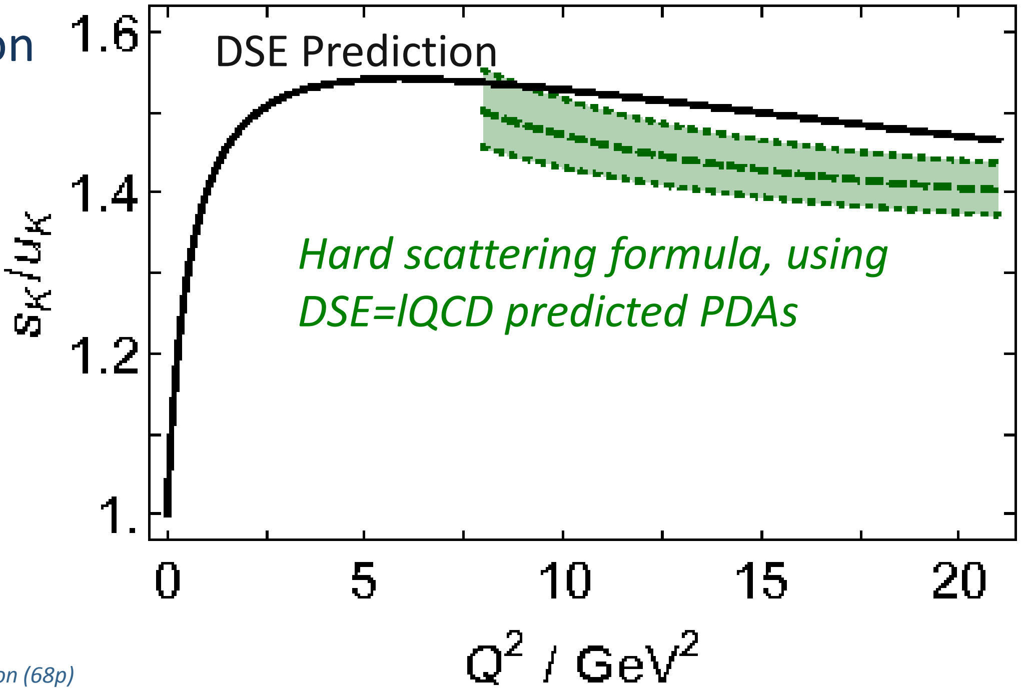
with [41] $f_K = 0.110\,\text{GeV}$ and, for the K^+:

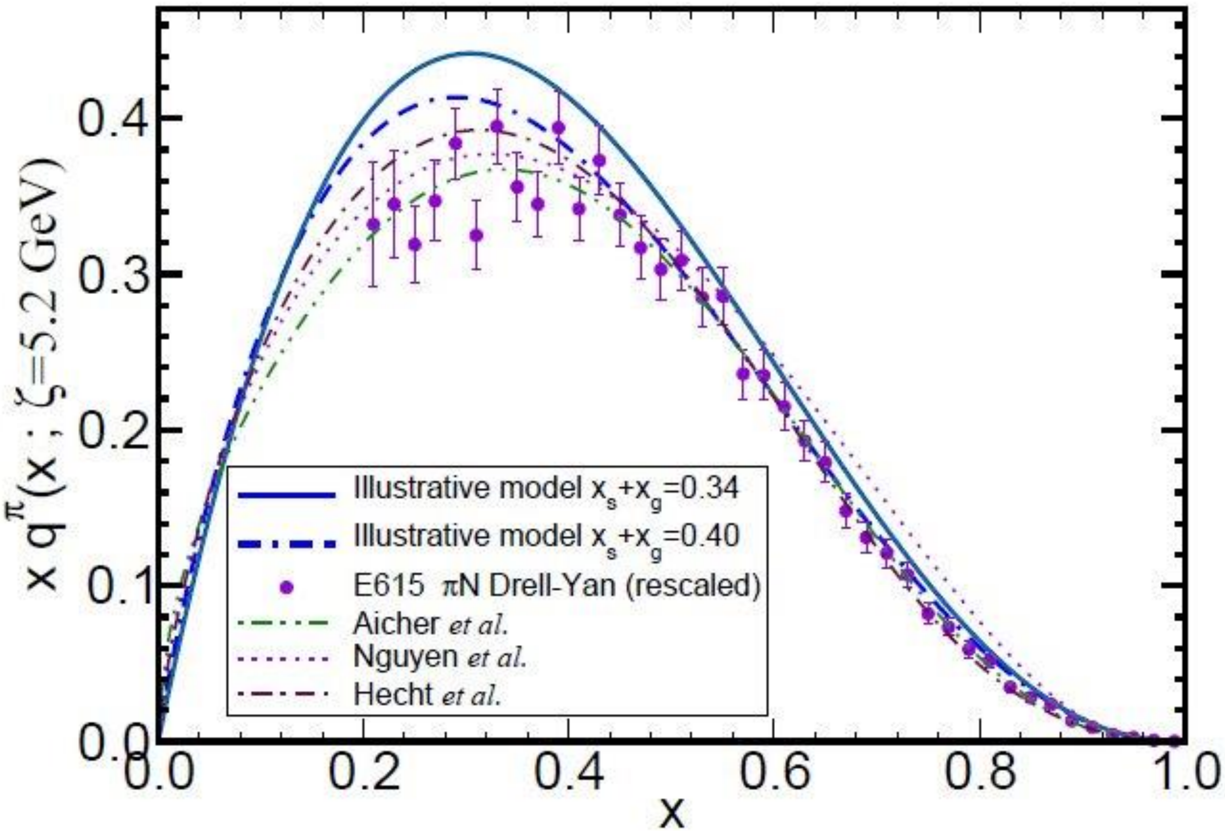
$$w_K^2 = e_{\bar{s}} w_{\bar{s}}^2 + e_u w_u^2,$$

$$w_{\bar{s}} = \frac{1}{3}\int_0^1 dx\, \frac{1}{1-x}\varphi_K(x)\,, \quad w_u = \frac{1}{3}\int_0^1 dx\, \frac{1}{x}\varphi_K(x)$$

- Current conservation
  $F_{uss}(0) = F_{uus}(0)$
- Under evolution:
  $\varphi_K \to 6\,x\,(1-x) \Rightarrow \omega_{\bar{s}} \to \omega_u \Rightarrow$ Ratio $\to 1$
- Agreement between direct calculation and hard-scattering formula, using consistent PDA
- Ratio never exceeds 1.5 and logarithmic approach to unity
- Typical signal of DCSB-dominance in flavour-symmetry breaking:
  - $M_s(0) \sim 1.25\, M_u(0)$
  - but this scale difference becomes irrelevant under evolution

$[\, \bar{s}\,\gamma\, s\, u_{spectator} \,/\, \bar{u}\,\gamma\, u\, s_{spectator} \,]^2 \leq 1.5$





π & *K* Valence-quark Distribution Functions

π & K PDFs

- Experimental data on π & K PDFs obtained in mesonic Drell-Yan scattering from nucleons in heavy nuclei; but it's old: 1980-1989
- Newer data would be welcome:
  - persistent doubts about the Bjorken-$x \simeq 1$ behaviour of the pion's valence-quark PDF
  - single modest-quality measurement of $u^K(x)/u^\pi(x)$ cannot be considered definitive.
- Approved experiment, using tagged DIS at JLab 12, should contribute to a resolution of pion question; and a similar technique might also serve for the kaon.
- Future:
  - new mesonic Drell-Yan measurements at modern facilities could yield valuable information on π and K PDFs,
  - as could two-jet experiments at the large hadron collider;
  - EIC would be capable of providing access to π and K PDFs through measurements of forward nucleon structure functions.
- Gribov-Lipatov reciprocity (crossing symmetry) entails connection between PDFs and fragmentation functions on $z \simeq 1$ $(z \geq 0.75)$

$$D_{H/q}(z) \approx z\, q^H(z)$$

Reliable information on meson fragmentation functions is critical if the worldwide programme aimed at determining TMDs is to be successful

Basic features of the pion valence-quark distribution function,
Lei Chang, Cédric Mezrag, et al.,
arXiv:1406:5450 [nucl-th], Phys. Lett. B **737** (2014)pp. 23–29

Valence-quark distribution functions in the kaon and pion,
Chen Chen, Lei Chang et al.
arXiv:1602.01502 [nucl-th], Phys. Rev. D**93** (2016) 074021/1-11

Valence-quark PDFs within mesons

- Compute PDFs from imaginary part of virtual-photon – pion forward Compton scattering amplitude:

  $\gamma\,\pi \rightarrow \gamma\,\pi$

- Handbag diagram is insufficient. Doesn't even preserve global symmetries. Exists a class of leading-twist corrections that remedies this defect ⇒

Projection onto light-front

$$u_V^\pi(x) = N_c \text{tr} \int_{dk} \delta_n^x(k_\eta^\pi)$$

Partial derivative wrt relative momentum

$$\times\, \partial_{k_\eta^\pi} \left[\Gamma_\pi(k_\eta^\pi, -P_\pi) S(k_\eta^\pi)\right] \Gamma_\pi(k_{\bar\eta}^\pi, P_\pi)\, S(k_{\bar\eta}^\pi)\,,$$

Similar expressions for $u_V^K(x)$, $s_V^K(x)$

Measurable quantities
Directly related to
dynamically generated quark masses
& bound-state wave functions

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Valence-quark distribution functions in the kaon and pion,
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Valence-quark PDFs within mesons

- Formulae guarantee that valence-quark PDFs satisfy, independent of any and all structural details:

$$\langle x \rangle_u^0 = \int_0^1 dx\, x\, u_V^0(x) = \frac{1}{2}$$

$$\int_0^1 dx\, x[u_V^K(x) + \bar{s}_V^K(x)] = 1$$

- Algebraic proof that at an hadronic scale $\zeta \approx 0.5$ GeV

  $q_V^M(x \simeq 1) \propto (1-x)^{2n}$

  in any theory with $(1/k^2)^n$ vector-boson exchange interaction

Realistic distributions

- Pion distribution at ζ_H
  - DSE prediction, following from analysis of leptonic decay: π contains 5% sea
  - Assume GRV analysis of πN Drell-Yan is reliable, then 30% of π momentum carried by glue

  Adopting standard PDF parametrisations, this additional information is sufficient to completely fix realistic $u_\pi(x;\zeta_H) = valence + sea + glue$

- Kaon distribution at ζ_H
  - Owing to heavier mass of intermediate states that can introduce sea-quarks, safe to assume sea-quark content of kaon is effectively zero
  - Treat momentum fraction carried by glue as a parameter to be used in understanding $u^K(x)/u^\pi(x)$

    … owing to heavier mass of s-quark,

    expect $\langle x\rangle_g^K < \langle x\rangle_g^\pi$;

    … but how much less?

Pion: DSE comparison with lQCD moments

- All lQCD studies agree with each other, within errors

  [66]: Brommel et al. (2007)

  [67]: Best *et al*. (1997)

  [68]: Detmold *et al*. (2003)

- DSE and lQCD agree, within errors; and DSE at level of 4% with lQCD-average

- *On light-front, just 52% of the pion's momentum is carried by valence-quarks at ζ_2 = 2GeV, down from 65% at ζ_H= 0.51GeV*

moments. Such results are available for $u^\pi(x)$, *e.g.* a contemporary simulation [66], using two dynamical fermion flavours, $m_\pi \gtrsim 0.34\,\text{GeV}$ and nonperturbative renormalisation at $\zeta_2 = 2\,\text{GeV}$, produces the first row here:

	$\langle x\rangle_u^\pi$	$\langle x^2\rangle_u^\pi$	$\langle x^3\rangle_u^\pi$
[66]	0.27(1)	0.13(1)	0.074(10)
[67]	0.28(8)	0.11(3)	0.048(20)
[68]	0.24(2)	0.09(3)	0.053(15)
average	0.26(8)	0.11(4)	0.058(27)
herein	0.26	0.11	0.052

(37)

The results in Ref. [66] agree with those obtained in earlier estimates based on simulations of quenched lQCD [67, 68] and are consistent with the values obtained from our computed distribution, which are reported in the last row of Eq. (37).

Pion PDF

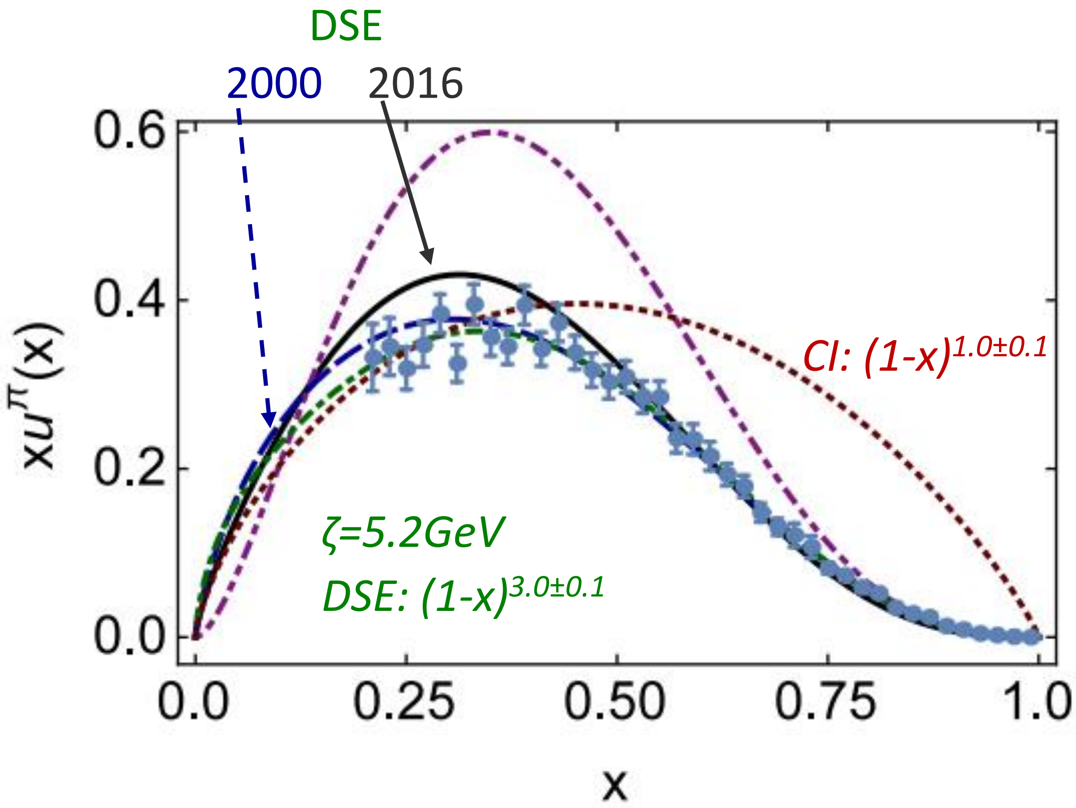


FIG. 3. $xu^{\pi}(x;\zeta_{5.2})$. Solid (black) curve, our prediction, expressed in Eqs. (32), (33); dot-dot-dashed (purple) curve, result obtained when sea-quark and gluon contributions are neglected at ζ_H, *i.e.* using $u_V^{\pi}(x)$ from Eqs. (14), (17); dashed (blue) curve first DSE prediction [38]; and data, Ref. [4], rescaled according to the reanalysis described in Ref. [40], from which the dot-dashed (green) curve is drawn. The dotted (red) curve is the result obtained using a Poincaré-covariant regularisation of a contact interaction, Eq. (36).

- Purple dot-dot-dash = prediction at ζ_H
- Data = modern reappraisal of E615: NLO analysis plus soft-gluon resummation (ASV)
- Solid black curve, prediction evolved to ζ=5.2GeV, the scale associated with the experiments
- Blue dashed curve = first DSE prediction, in 2000 (ζ=5.2GeV)
- Dotted red curve = result obtained with momentum-independent gluon exchange (contact interaction, ζ=5.2GeV)

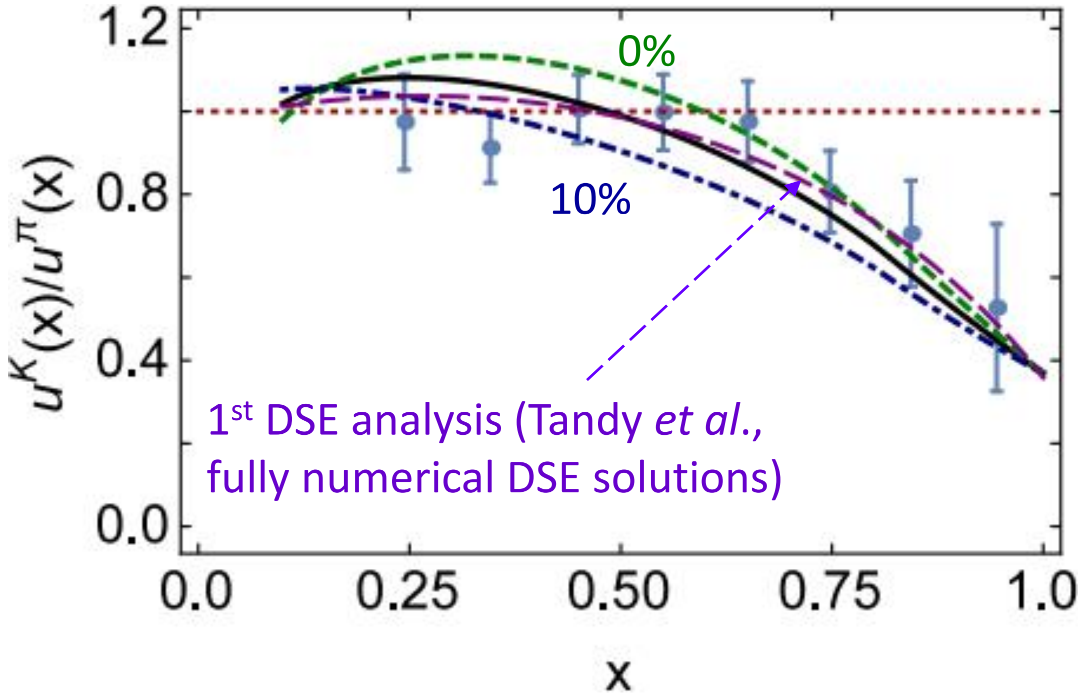
Kaon's gluon content

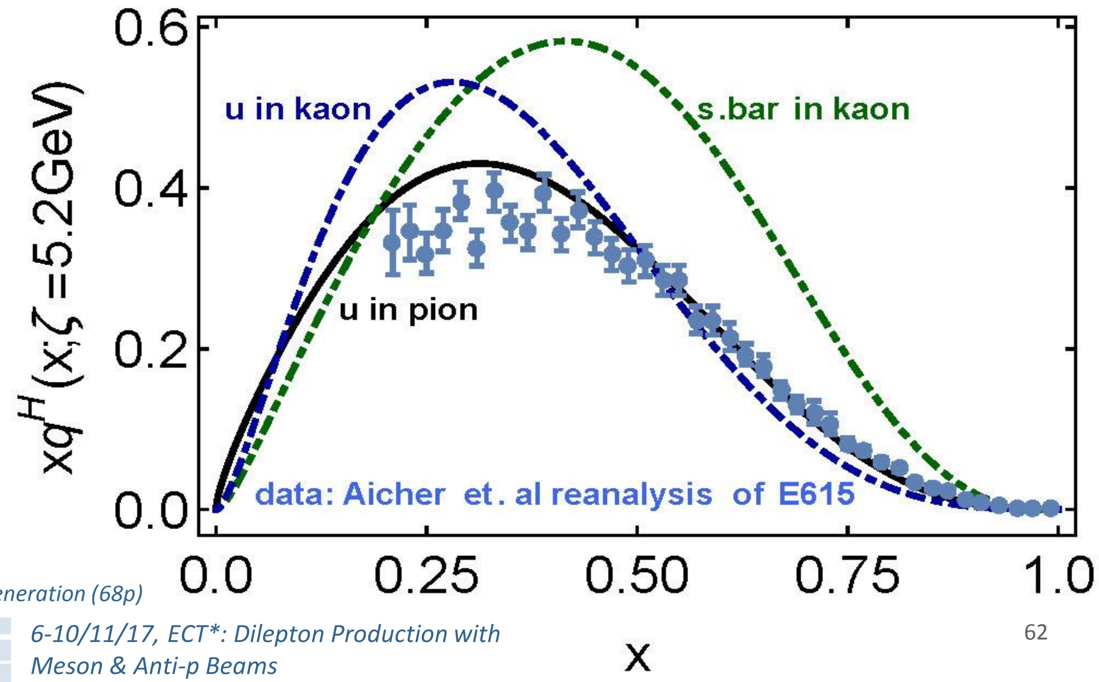
- $\langle x \rangle_g^K(\zeta_H) = 0.05 \pm 0.05$
  $\Rightarrow$ Valence quarks carry 95% of kaon's momentum at ζ_H
- Evolved to ζ_2

q	$\langle x \rangle_q^K$	$\langle x^2 \rangle_q^K$	$\langle x^3 \rangle_q^K$
u	0.28	0.11	0.048
$\bar{s}$	0.36	0.17	0.092

Valence-quarks carry ⅔ of kaon's light-front momentum

Only ½ for the pion

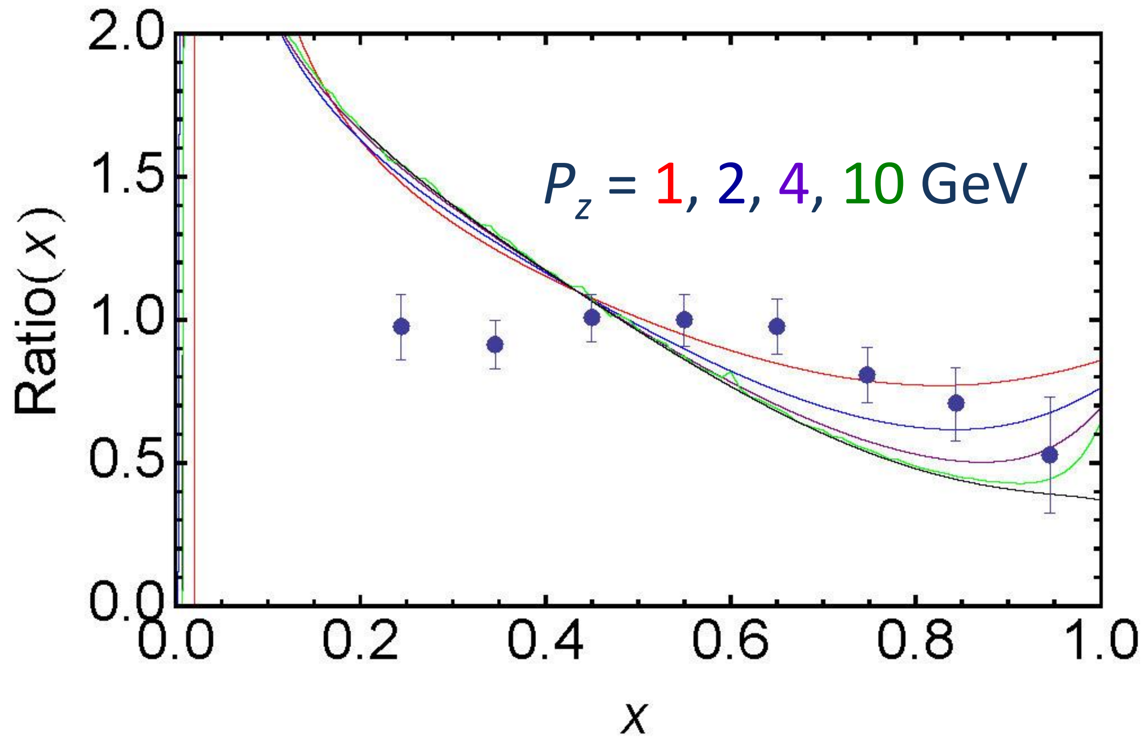




Aside: quasi-PDF

- Continuum computation of valence-quark π & K quasi-PDFs
- Individually, accuracy increases with increasing P_z
- Ratio, however, is *fairly* accurate, within empirical errors, for

  $P_z > 2$ GeV



π & K PDFs

- Marked differences between π & K gluon content
  - ζ_H:
    - Whilst $\frac{1}{3}$ of pion's light-front momentum carried by glue
    - *Only* $\frac{1}{20}$ of the kaon's light-front momentum lies with glue
  - $\zeta_2^{\,2} = 4\ \mathrm{GeV}^2$
    - Glue carries $\frac{1}{2}$ of pion's momentum and $\frac{2}{3}$ of kaon's momentum
  - Evident in differences between large-x behaviour of valence-quark distributions in these two mesons
- Signal of Nambu-Goldstone boson character of π
  - Nearly complete cancellation between one-particle dressing and binding attraction in this almost massless pseudoscalar system

$$2\,M + U \approx 0$$

Valence-quark distribution functions in the kaon and pion,
Chen Chen, Lei Chang *et al*.
arXiv:1602.01502 [nucl-th], Phys. Rev. D**93** (2016) 074021/1-11

π & *K* PDFs

- Existing textbook description of Goldstone's theorem via pointlike modes is *outdated* and *simplistic*

Valence-quark distribution functions in the kaon and pion,
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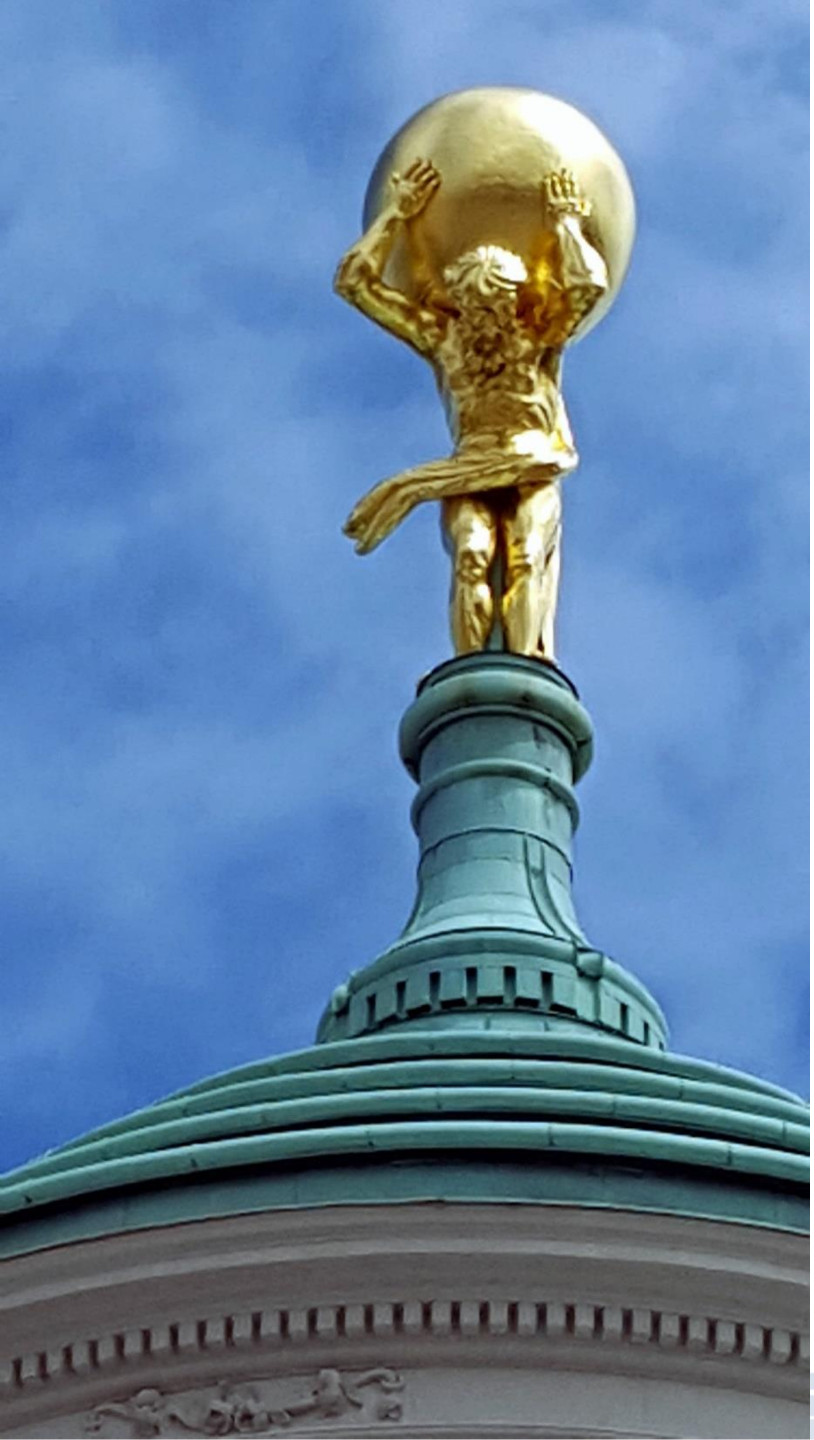
arXiv:1602.01502 [nucl-th], Phys. Rev. D**93** (2016) 074021/1-11

π & K PDFs

- The appearance of Nambu-Goldstone modes in the Standard Model is far more interesting
  - Nambu-Goldstone modes are nonpointlike!
  - Intimately connected with origin of mass!
  - Quite probably inseparable from expression of confinement!
- Difference between gluon content of π & K is measurable … using well-designed EIC
- Write a definitive new chapter in future textbooks on the Standard Model



Epilogue

Epilogue

- Challenge: Explain and Understand
  the Origin of the vast bulk of Visible Mass
- Current Paradigm: Quantum Chromodynamics
- QCD is plausibly a mathematically well-defined quantum field theory,
  The only one we've ever produced
  - Consequently, it is a worthwhile paradigm for developing Beyond-SM theories
- Challenge is to reveal the content of QCD
- Parton distributions are measurable, but for QCD validation
  - Unsound model computations and rough sketches are inadequate
  - *Need precise QCD-connected predictions of their pointwise behavior*
  - *Need empirical validation of QCD-connected predictions*
- Comparison – light-quark vs. s-quark observables/distributions
  - Potentially hold cleanest signals for the origin of vast bulk of visible mass
  - Reveal the *glue* that binds us all