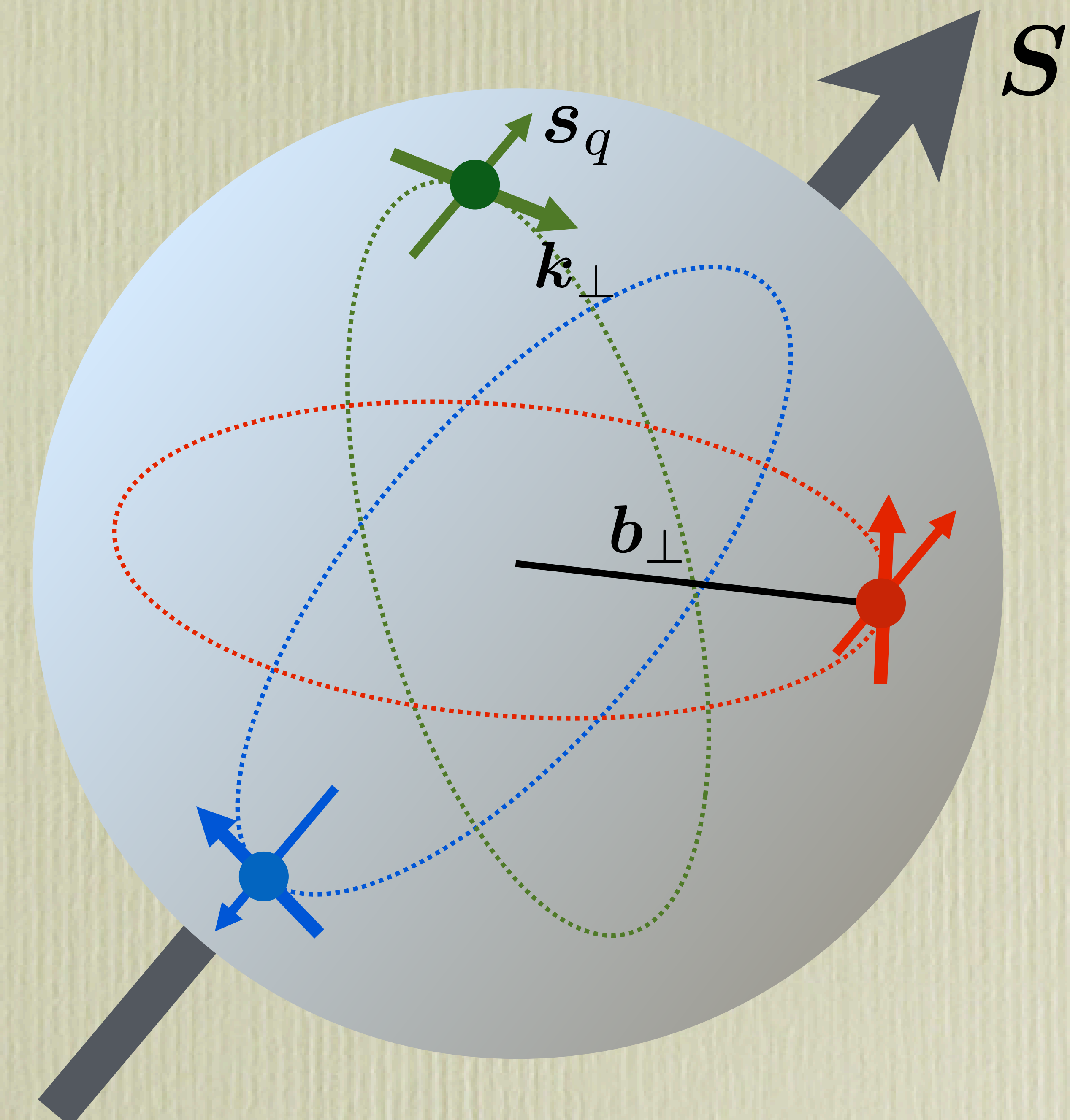
What Do We Learn from the Sivers Effect?



Dilepton Production with Meson and Antiproton Beams

Trento, November 6-10, 2017

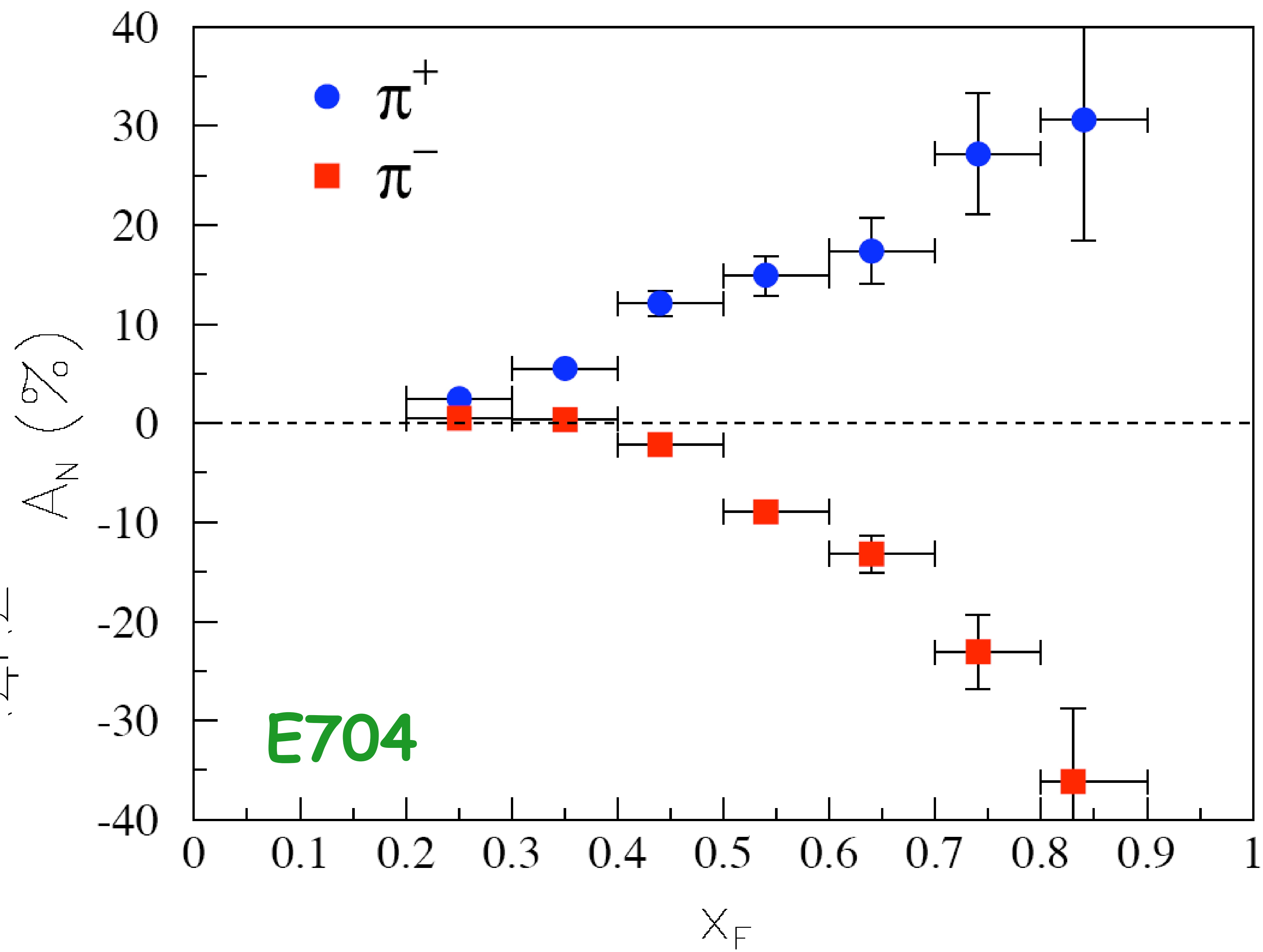
where it all started from ... (~1991)

large P_T

$$p^{\uparrow} p \rightarrow \pi X$$

Single Spin Asymmetry

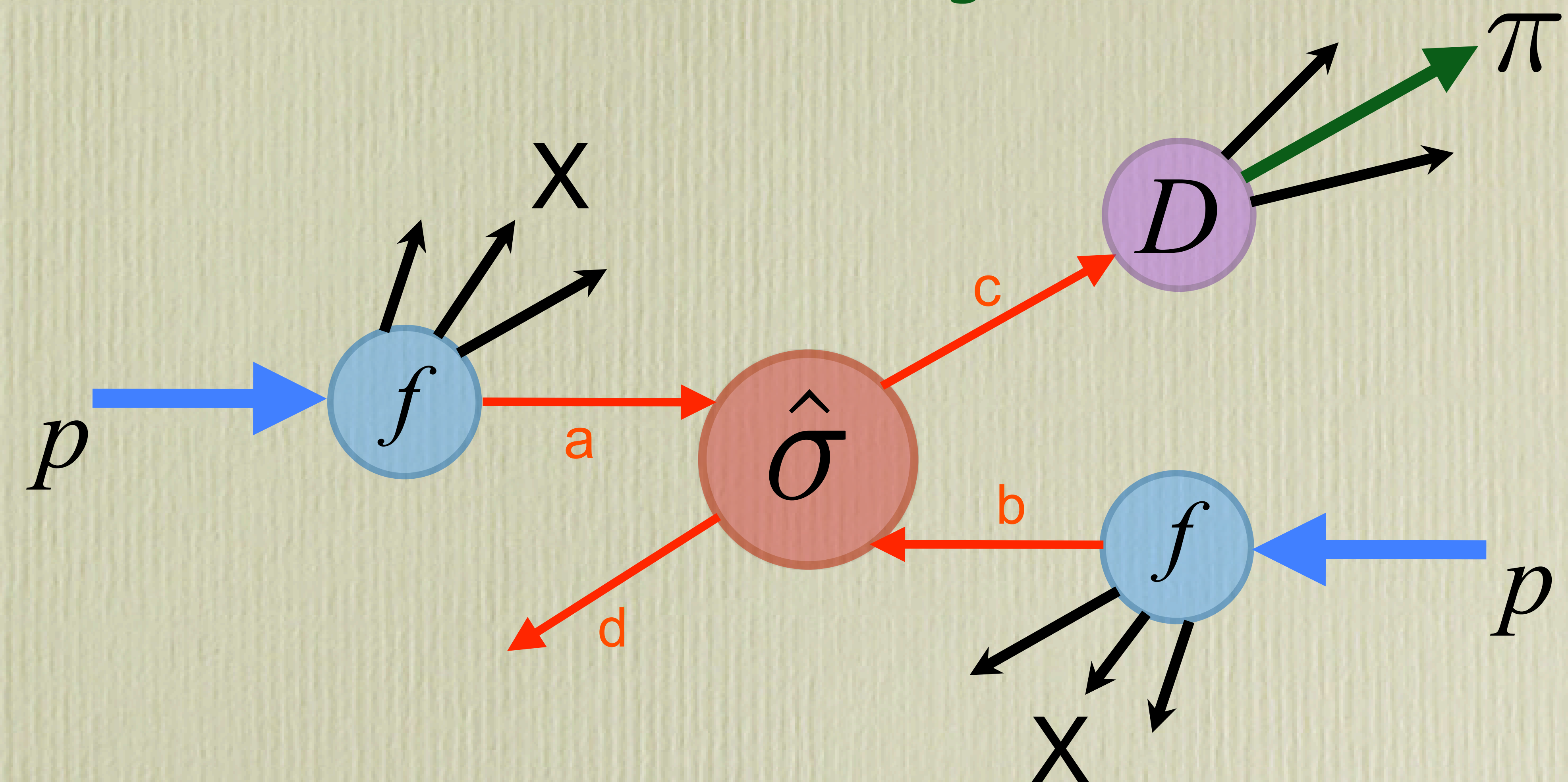
$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$



E704 $\sqrt{s}$ = 20 GeV   0.7 < p_T < 2.0

Cross section for $p\,p \to \pi\,X$ in pQCD

based on factorization theorem
(in collinear configuration)



$$d\sigma = \sum_{a,b,c,d=q,\bar{q},g} \underbrace{f_{a/p}(x_a) \otimes f_{b/p}(x_b)}_{\text{PDF}} \otimes d\hat{\sigma}^{ab\to cd} \otimes \underbrace{D_{\pi/c}(z)}_{\text{FF}}$$

pQCD elementary interactions

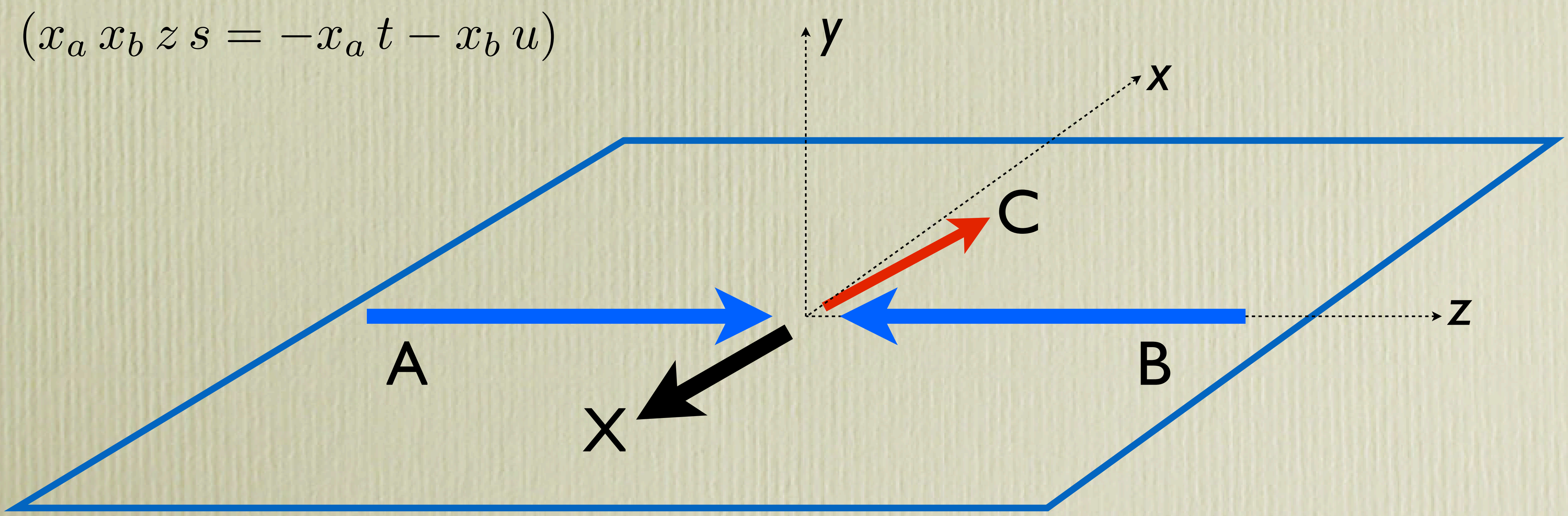
$$\frac{E_C\, d\sigma^{AB\to CX}}{d^3\boldsymbol{p}_C} = \sum_{a,b,c,d} \int dx_a\, dx_b\, dz\, f_{a/A}(x_a, Q^2)\, f_{b/B}(x_b, Q^2)$$

$$\times \quad \frac{\hat{s}}{\pi z^2}\, \frac{d\hat{\sigma}^{ab\to cd}}{d\hat{t}}(\hat{s}, \hat{t}, \hat{u}, x_a, x_b)\, \delta(\hat{s}+\hat{t}+\hat{u})\, D_{C/c}(z, Q^2)$$

$$= \sum_{a,b,c,d} \int dx_a\, dx_b\, f_{a/A}(x_a, Q^2)\, f_{b/B}(x_b, Q^2)$$

$$\times \quad \frac{1}{\pi z}\, \frac{d\hat{\sigma}^{ab\to cd}}{d\hat{t}}(\hat{s}, \hat{t}, \hat{u}, x_a, x_b)\, D_{C/c}(z, Q^2)$$

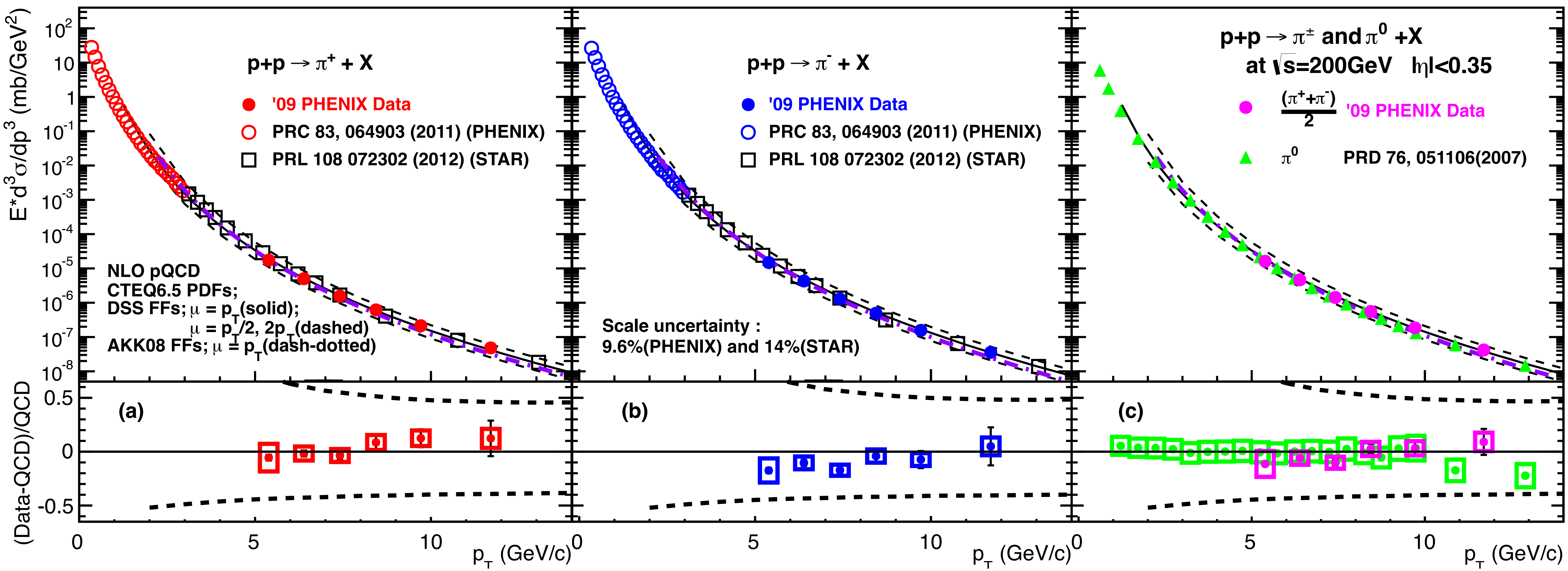
$(x_a\, x_b\, z\, s = -x_a\, t - x_b\, u)$



mid-rapidity RHIC data, unpolarised cross sections

(arXiv:1409.1907 [hep-ex], Phys. Rev. D91 (2015) 3, 032001)

large P_T single pion production $p\,p \to \pi\,X$



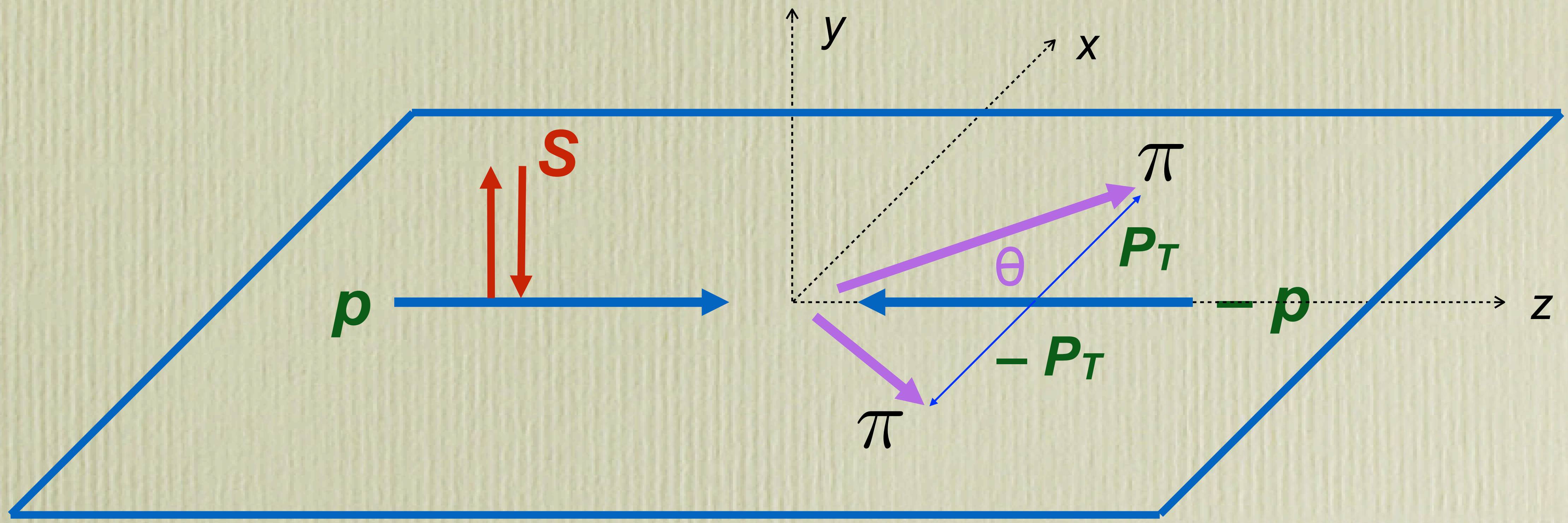
good agreement between RHIC data and collinear pQCD calculations

(maybe x_T scaling not quite correct, Arleo-Brodsky)

but there are problems with spin dependent data …

A_N = simple left-right asymmetry

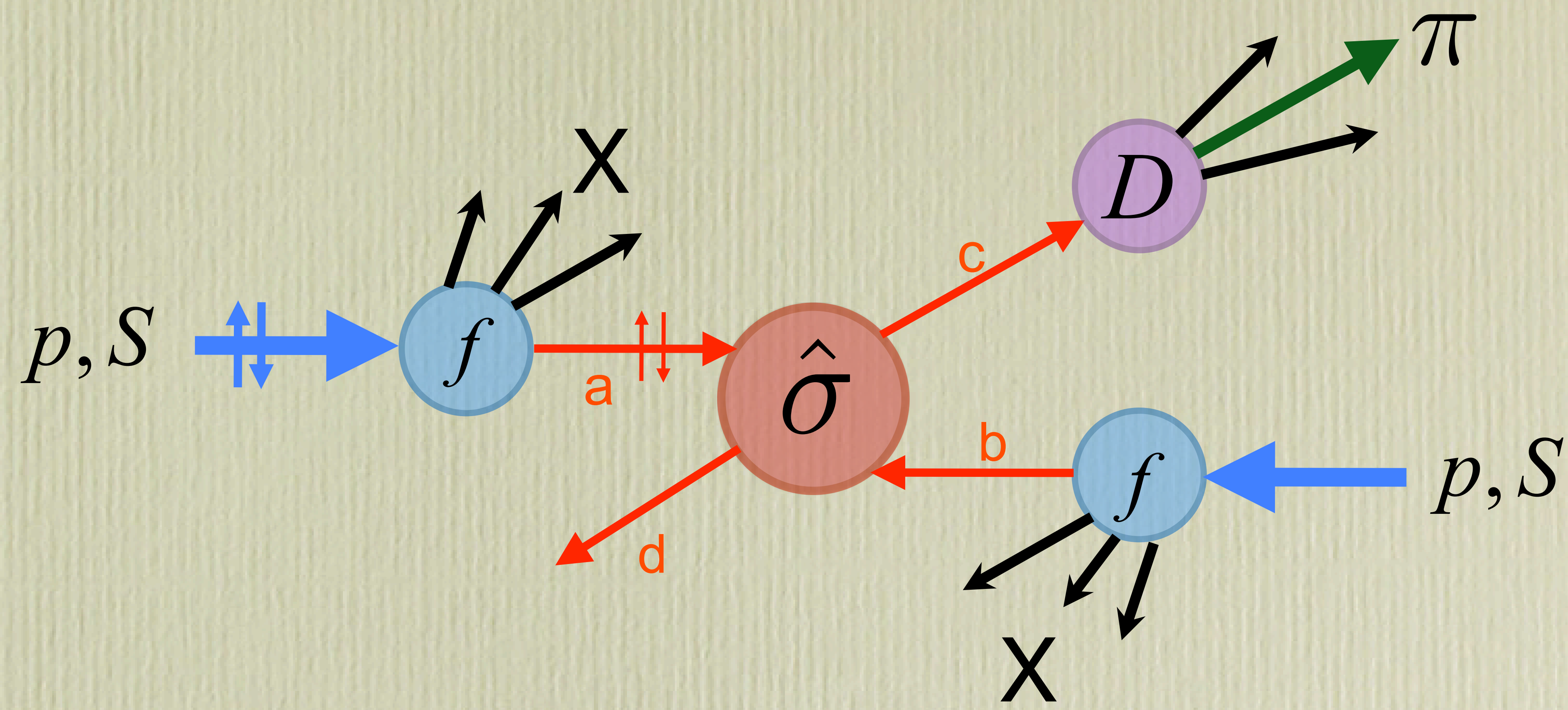
$$A_N = \frac{d\sigma^\uparrow(\boldsymbol{P}_T) - d\sigma^\downarrow(\boldsymbol{P}_T)}{d\sigma^\uparrow(\boldsymbol{P}_T) + d\sigma^\downarrow(\boldsymbol{P}_T)} = \frac{d\sigma^\uparrow(\boldsymbol{P}_T) - d\sigma^\uparrow(-\boldsymbol{P}_T)}{2\, d\sigma^{\rm unp}(P_T)}$$



$$A_N \equiv \frac{\mathrm{d}\sigma^\uparrow - \mathrm{d}\sigma^\downarrow}{\mathrm{d}\sigma^\uparrow + \mathrm{d}\sigma^\downarrow} \propto \boldsymbol{S}\cdot(\boldsymbol{p}\times\boldsymbol{P}_T) \propto \sin\theta$$

transverse Single Spin Asymmetry (SSA)

SSA in $p\,p \to \pi\,X$?

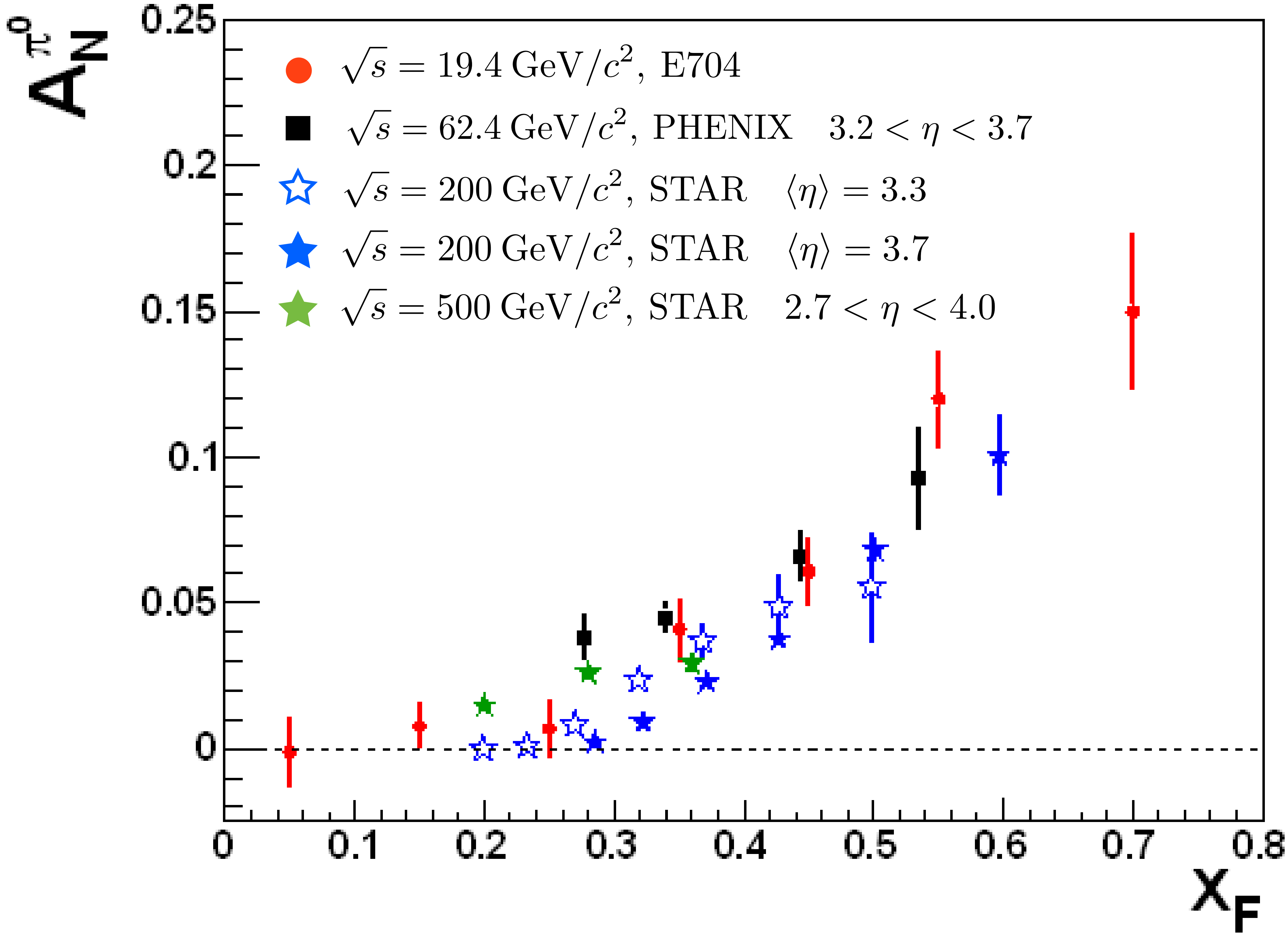


$$d\sigma^{\uparrow} - d\sigma^{\downarrow} = \sum_{a,b,c,d=q,\bar{q},g} \underbrace{\Delta_T f_a}_{\text{transversity}} \otimes f_b \otimes \underbrace{[d\hat{\sigma}^{\uparrow} - d\hat{\sigma}^{\downarrow}]}_{\text{pQCD elementary SSA}} \otimes \underbrace{D_{\pi/c}}_{\text{FF}}$$

$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} \propto \hat{a}_N \propto \frac{m_q}{E_q}\,\alpha_s$$

was considered almost a theorem

A_N large and persistent at high energies ….



The birth of TMDs: D. Sivers
PRD 41 (1990) 83

$$G_{a/p}(x;\mu^2) \to G_{a/p}(x,\boldsymbol{k}_T;\mu^2)$$

The relevance of the transverse momentum for the asymmetry can be seen from the venerable Chou-Yang[1] model of the constituent structure of a transversely polarized proton. If we assume a correlation between the spin of the proton and the orbital motion of its constituents, Chou and Yang showed the existence of a nontrivial A_N in elastic scattering. *The coherent dynamics which correlates the spin of the proton with the orbital angular momentum of the quarks and gluons can also produce a constituent-level asymmetry in transverse momentum:*

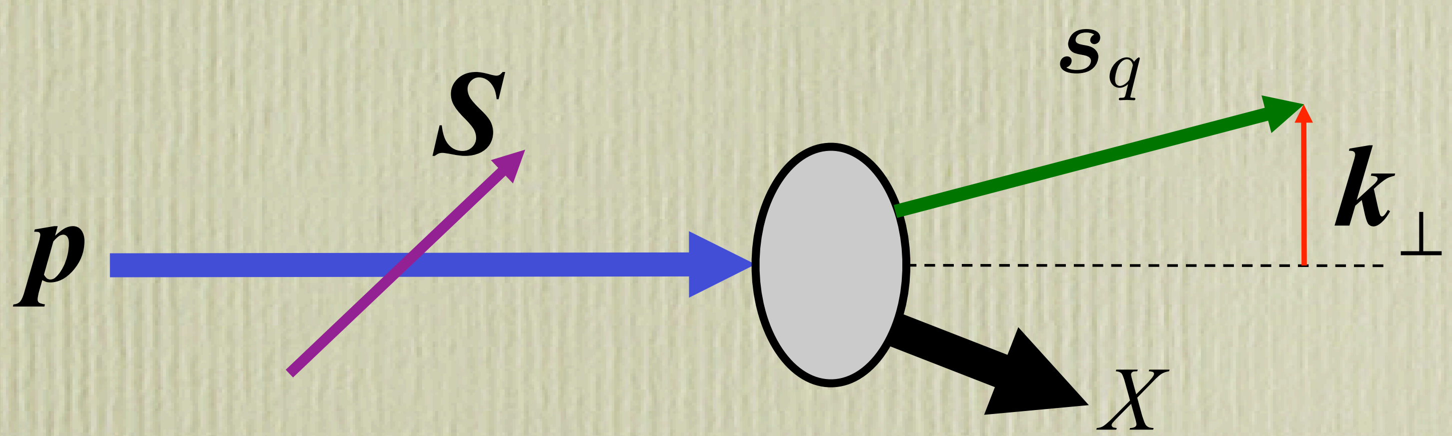
$$\begin{aligned}\Delta^N G_{a/p(\uparrow)}(x,\boldsymbol{k}_T;\mu^2) &= \sum_h \left[G_{a(h)/p(\uparrow)}(x,\boldsymbol{k}_T;\mu^2) - G_{a(h)/p(\downarrow)}(x,\boldsymbol{k}_T;\mu^2)\right] \\ &= \sum_h \left[G_{a(h)/p(\uparrow)}(x,\boldsymbol{k}_T;\mu^2) - G_{a(h)/p(\uparrow)}(x,-\boldsymbol{k}_T;\mu^2)\right]\end{aligned}$$

1 T. T. Chou and C. N. Yang, Nucl. Phys. B107, 1 (1976)

$$A_N\left[E\frac{d^3\sigma}{d^3p}(pp_\uparrow \to mX)\right] \simeq \sum_{ab\to cd}\int d^2\boldsymbol{k}_T^a\,dx_a\int d^2\boldsymbol{k}_T^b\,dx_b\int d^2\boldsymbol{k}_{TC}\frac{dx_c}{x_c^2}\Delta^N G_{a/p_\uparrow}(x_a,k_T^a;\mu^2)$$

$$\times\, G_{b/p}(x_b,k_T^b;\mu^2)\,D_{m/c}(x_c,k_T^c:\mu^2)\times\tilde{s}\,\frac{d\sigma}{d\tilde{t}}(ab\to cd)\,\delta(\tilde{s}+\tilde{t}+\tilde{u})$$

.... this equation corresponds to a probabilistic formula in the original spirit of the parton model where the unknown soft nonperturbative dynamics have been absorbed into the specification of the density Δ^N G ...



Sivers function

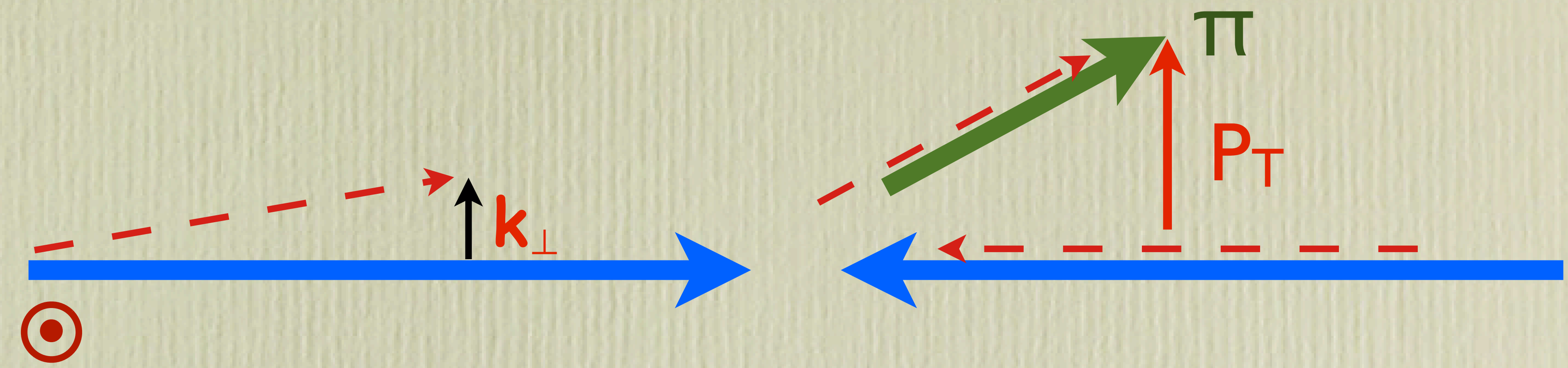
$$f_{q/p,\boldsymbol{S}}(x,\boldsymbol{k}_\perp) = f_{q/p}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q/p^\uparrow}(x,k_\perp)\,\boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$

$$= f_{q/p}(x,k_\perp) - \frac{k_\perp}{M}f_{1T}^{\perp q}(x,k_\perp)\,\boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$

simple physical picture for Sivers effect

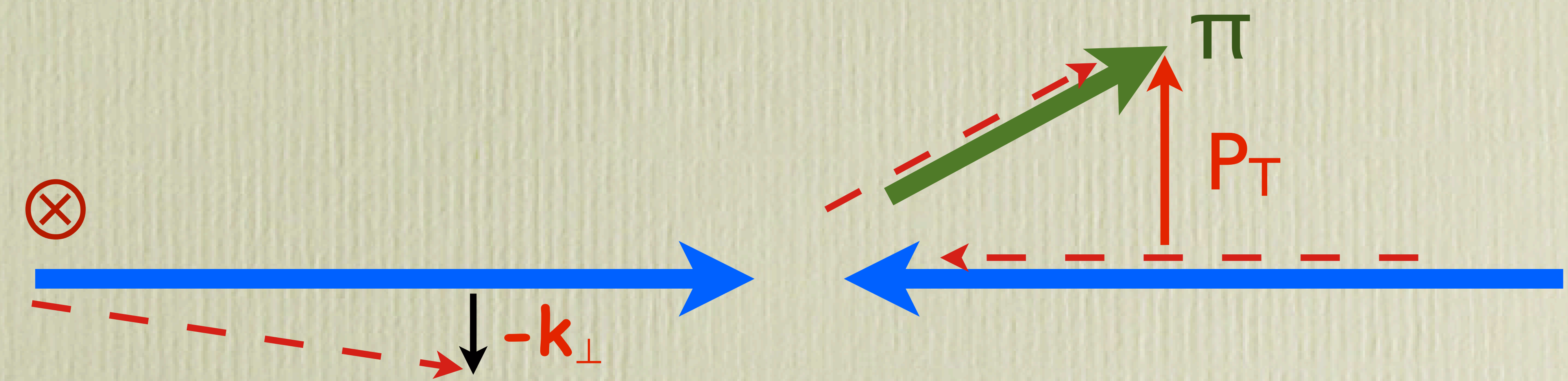
(correlation between **S** and **k**$_\perp$)

$d\sigma^\uparrow$



partonic smaller scattering angle

$d\sigma^\downarrow$



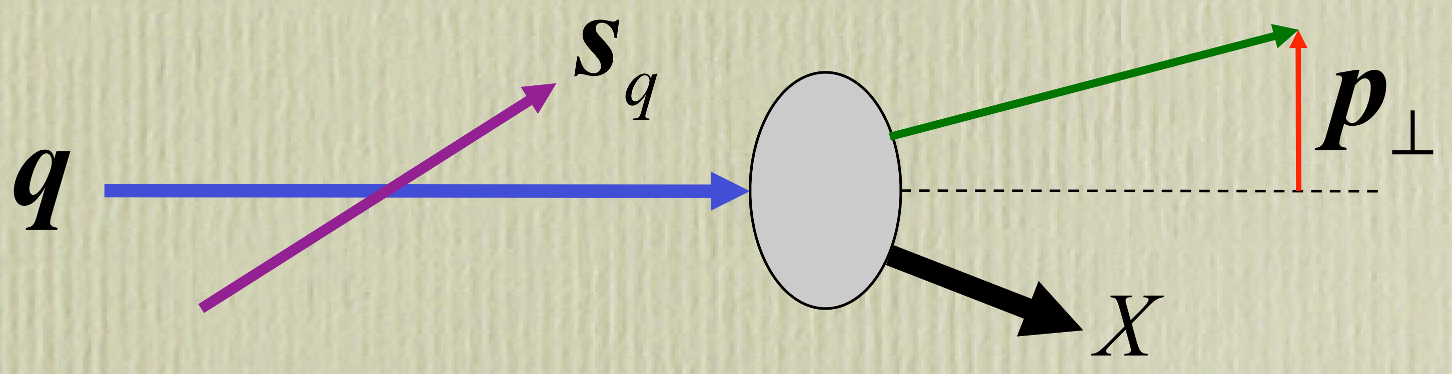
partonic larger scattering angle

$$d\sigma^\uparrow \neq d\sigma^\downarrow$$

Collins fragmentation function

Nucl. Phys. B396 (1993) 161

It is shown that the azimuthal dependence of the distribution of hadrons in a quark jet is a probe of the transverse spin of the quark initiating the jet. This results in a new spin-dependent fragmentation function that acts at the twist-2 level.



Collins function

$$
\begin{aligned}
D_{h/q,\boldsymbol{s}_q}(z,\boldsymbol{p}_\perp) &= D_{h/q}(z,p_\perp) + \frac{1}{2}\Delta^N D_{h/q^\uparrow}(z,p_\perp)\,\boldsymbol{s}_q\cdot(\hat{\boldsymbol{p}}_q\times\hat{\boldsymbol{p}}_\perp) \\
&= D_{h/q}(z,p_\perp) + \frac{p_\perp}{zM_h} H_1^{\perp q}(z,p_\perp)\,\boldsymbol{s}_q\cdot(\hat{\boldsymbol{p}}_q\times\hat{\boldsymbol{p}}_\perp)
\end{aligned}
$$

Collins, Nucl. Phys. B396 (1993) 161

It follows from the parity and time-reversal invariance of QCD that the number density of quarks is independent of the spin state of the initial hadron, so that we have

$$\hat{f}_{a/A}(x, |k_\perp|) \equiv \int \frac{\mathrm{d}y^- \, \mathrm{d}^2 y_\perp}{(2\pi)^3} \mathrm{e}^{-ixp^+y^- + ik_\perp \cdot y_\perp} \langle p| \, \bar{\psi}_i(0, y^-, y_\perp) \, \frac{\gamma^+}{2} \, \psi_i(0) \, |p\rangle$$

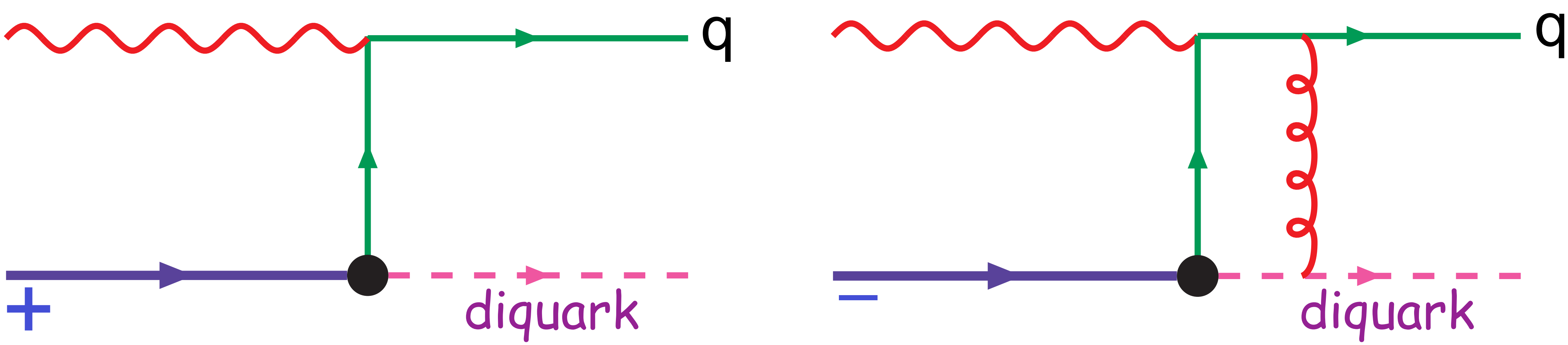
We have ignored here the subtleties needed to make this a gauge invariant definition: an appropriate path ordered exponential of the gluon field is needed [18].

Sivers suggested that the $k_\perp$ distribution of the quark could have an azimuthal asymmetry when the initial hadron has transverse polarization. However, such an asymmetry is prohibited because QCD is time-reversal invariant....

premature death of Sivers effect?

gauge links have physical consequences; quark models for non vanishing Sivers function,

SIDIS final state interactions



Brodsky, Hwang, Schmidt, PL B530 (2002) 99 - Collins, PL B536 (2002) 43

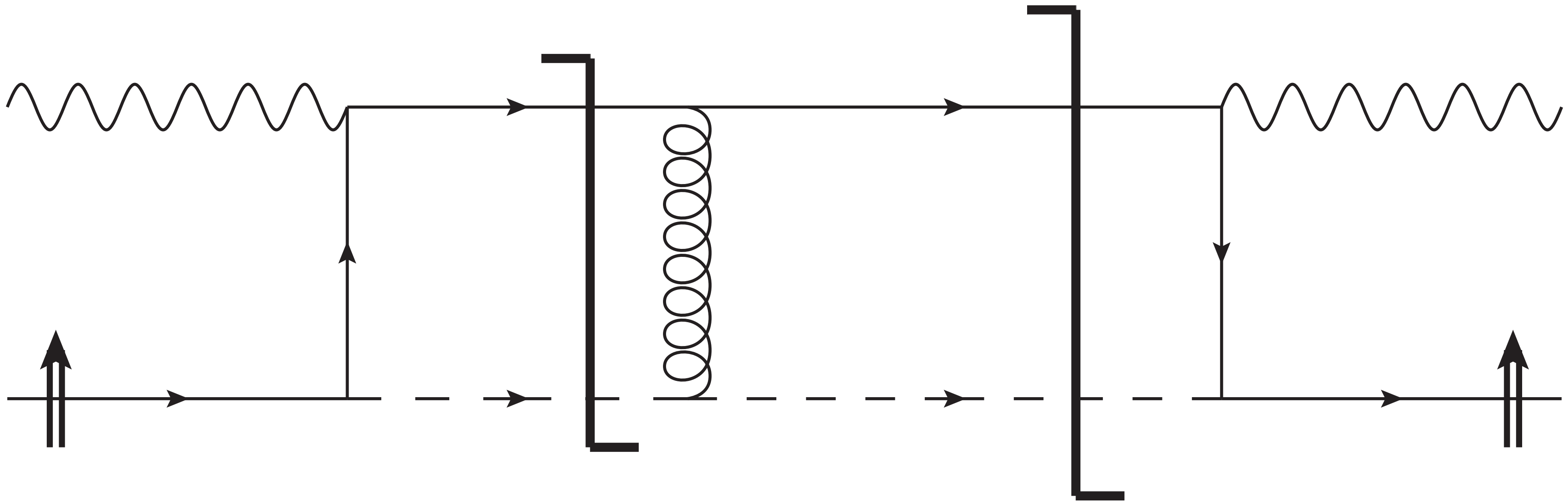
An earlier proof that the Sivers asymmetry vanishes because of time-reversal invariance is invalidated by the path-ordered exponential of the gluon field in the operator definition of parton densities. Instead, the time-reversal argument shows that the Sivers asymmetry is reversed in sign in hadron-induced hard processes (e.g., Drell-Yan), thereby violating naive universality of parton densities. Previous phenomenology with time-reversal-odd parton densities is therefore validated.

$$[f_{1T}^{q\perp}]_{\rm SIDIS} = -[f_{1T}^{q\perp}]_{\rm DY}$$

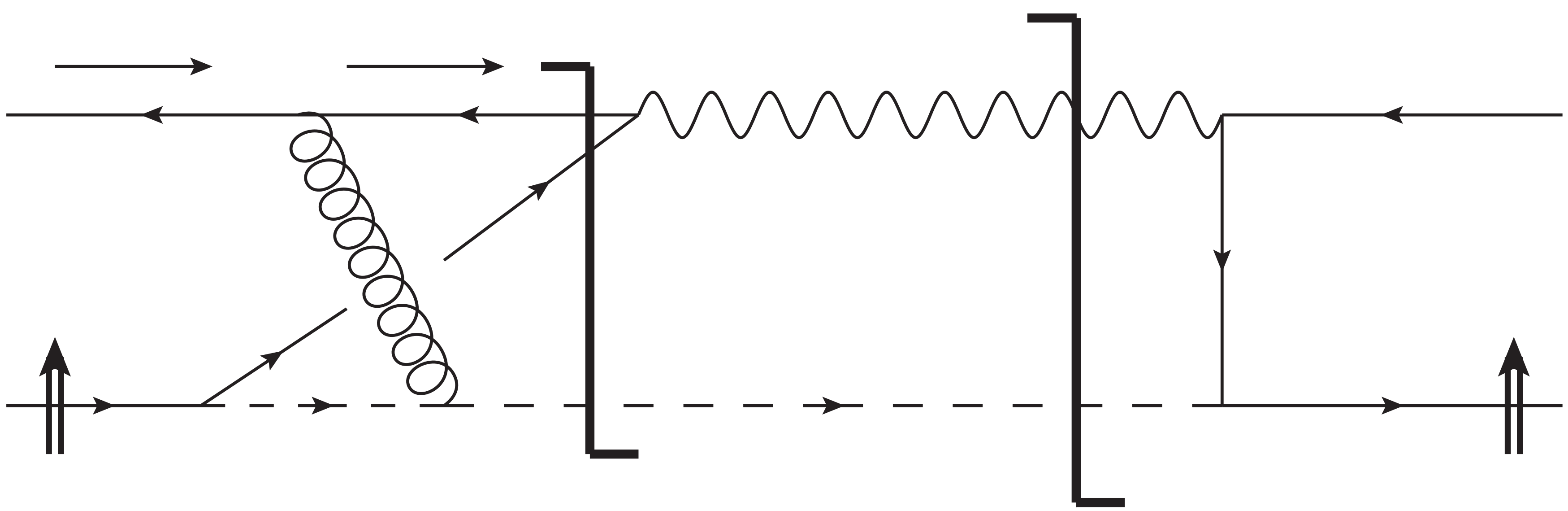
models of Sivers effect and gauge links, process dependence

$$\boxed{[f_{1T}^{q\perp}]_{\mathrm{SIDIS}} = -[f_{1T}^{q\perp}]_{\mathrm{DY}}}$$

SIDIS final state interactions ($\Rightarrow A_N$)



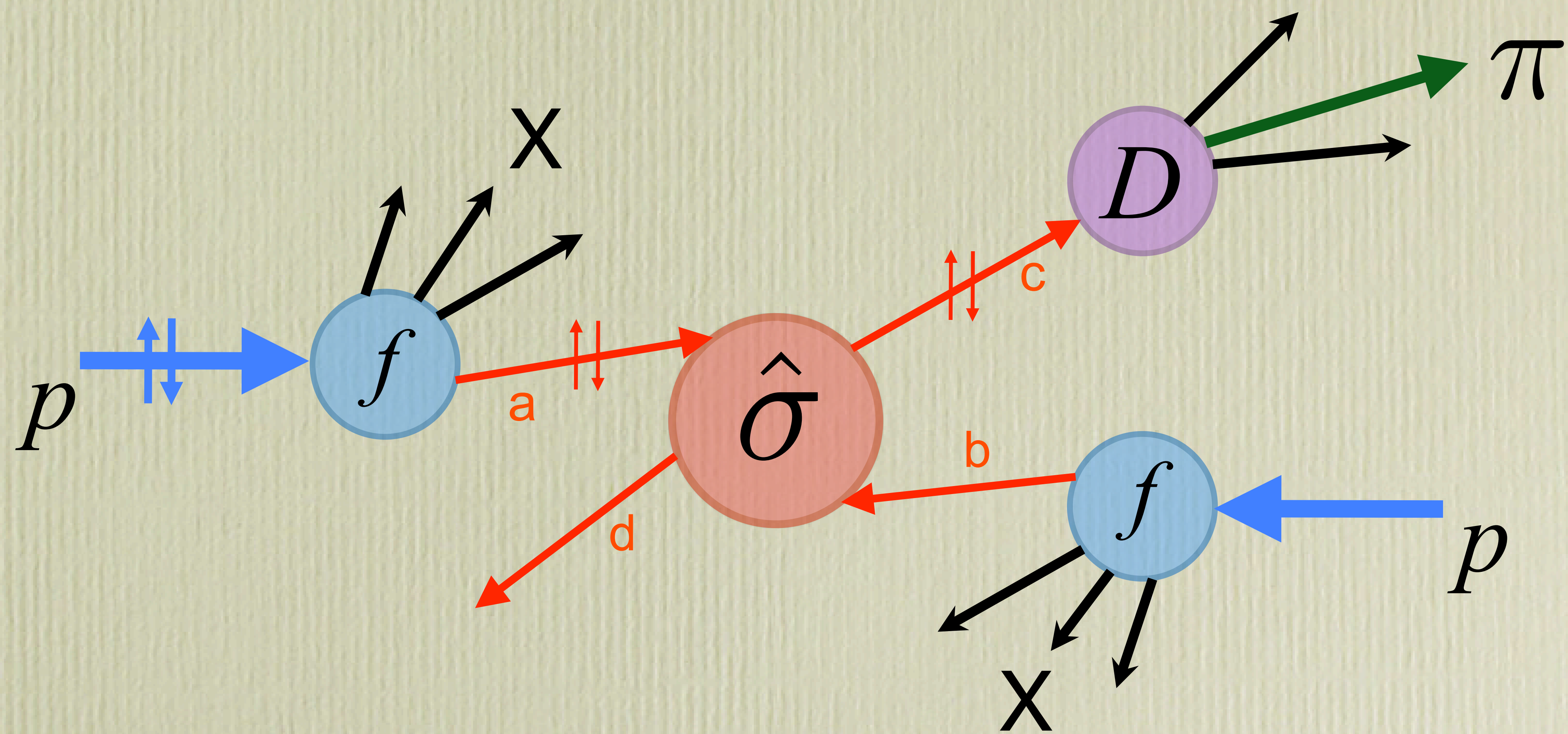
D-Y initial state interactions ($\Rightarrow -A_N$)



Brodsky, Hwang, Schmidt, PL B530 (2002) 99; NP B642 (2002) 344
Brodsky, Hwang, Kovchegov, Schmidt, Sievert, PR D88 (2013) 014032

SSA in hadronic processes: TMDs, a possible explanation

Generalization of collinear scheme (GPM)
(assuming factorization)



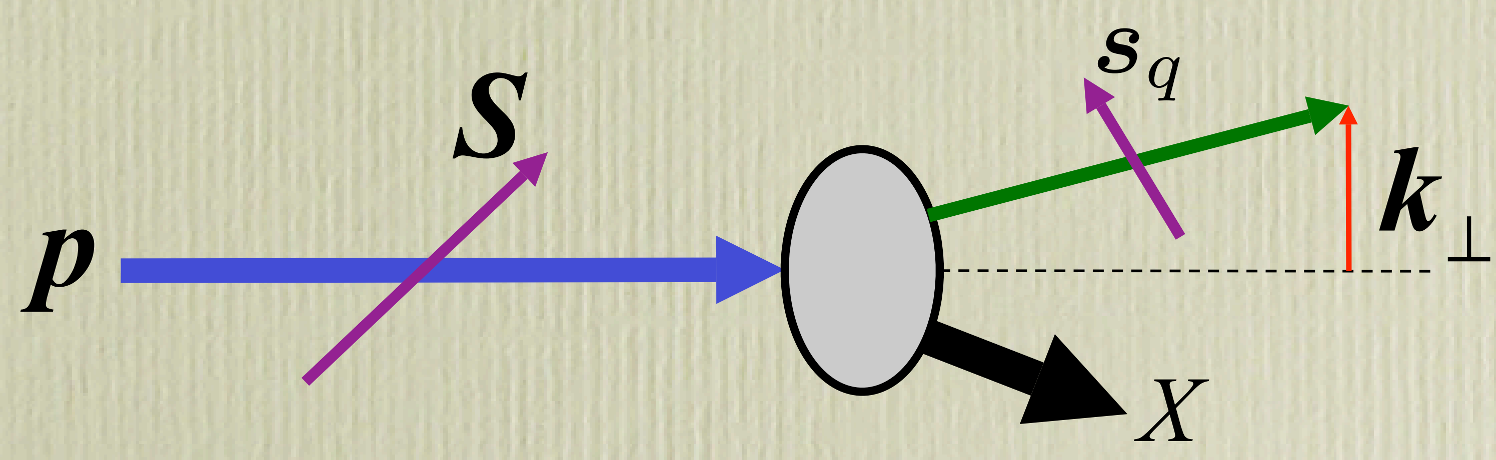
$$d\sigma^{\uparrow} = \sum_{a,b,c=q,\bar{q},g} \underbrace{f_{a/p^{\uparrow}}(x_a, \boldsymbol{k}_{\perp a})} \otimes \underbrace{f_{b/p}(x_b, \boldsymbol{k}_{\perp b})} \otimes d\hat{\sigma}^{ab\to cd}(\boldsymbol{k}_{\perp a}, \boldsymbol{k}_{\perp b}) \otimes \underbrace{D_{\pi/c}(z, \boldsymbol{p}_{\perp\pi})}$$

single spin effects in TMDs

TMDs in simple parton model

TMDs = Transverse Momentum Dependent Parton Distribution Functions (TMD-PDF) or Transverse Momentum Dependent Fragmentation Functions (TMD-FF)

TMD-PDFs give the number density of partons, with their intrinsic motion and spin, inside a fast moving proton, with its spin.



$\boldsymbol{S} \cdot (\boldsymbol{p} \times \boldsymbol{k}_\perp)$ $\boldsymbol{s}_q \cdot (\boldsymbol{p} \times \boldsymbol{k}_\perp)$ $\boldsymbol{S} \cdot \boldsymbol{s}_q$ $\cdots$

"Sivers effect"   "Boer-Mulders effect"

there are 8 independent TMD-PDFs

$f_1^q(x, \boldsymbol{k}_\perp^2)$ unpolarized quarks in unpolarized protons
unintegrated unpolarized distribution

$g_{1L}^q(x, \boldsymbol{k}_\perp^2)$ correlate s_L of quark with S_L of proton
unintegrated helicity distribution

$h_{1T}^q(x, \boldsymbol{k}_\perp^2)$ correlate s_T of quark with S_T of proton
unintegrated transversity distribution

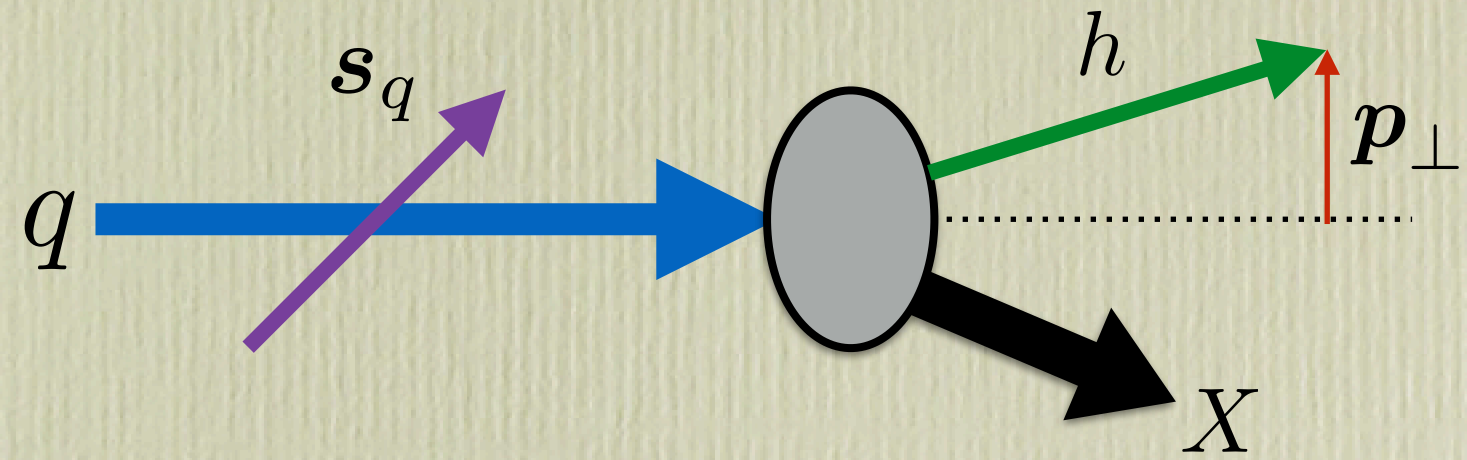
$$\underbrace{\phantom{f_1^q(x, \boldsymbol{k}_\perp^2)\quad g_{1L}^q(x, \boldsymbol{k}_\perp^2)\quad h_{1T}^q(x, \boldsymbol{k}_\perp^2)}}_{\text{only these survive in the collinear limit}}$$

$f_{1T}^{\perp q}(x, \boldsymbol{k}_\perp^2)$ correlate $k_\perp$ of quark with S_T of proton (Sivers)

$h_1^{\perp q}(x, \boldsymbol{k}_\perp^2)$ correlate $k_\perp$ and s_T of quark (Boer-Mulders)

$$\underbrace{g_{1T}^{\perp q}(x, \boldsymbol{k}_\perp^2) \qquad h_{1L}^{\perp q}(x, \boldsymbol{k}_\perp^2)}_{\text{worm-gears}} \qquad \underbrace{h_{1T}^{\perp q}(x, \boldsymbol{k}_\perp^2)}_{\text{pretzelosity}}$$

TMD-FFs give the number density of hadrons, with their momentum, originated in the fragmentation of a fast moving parton, with its spin.



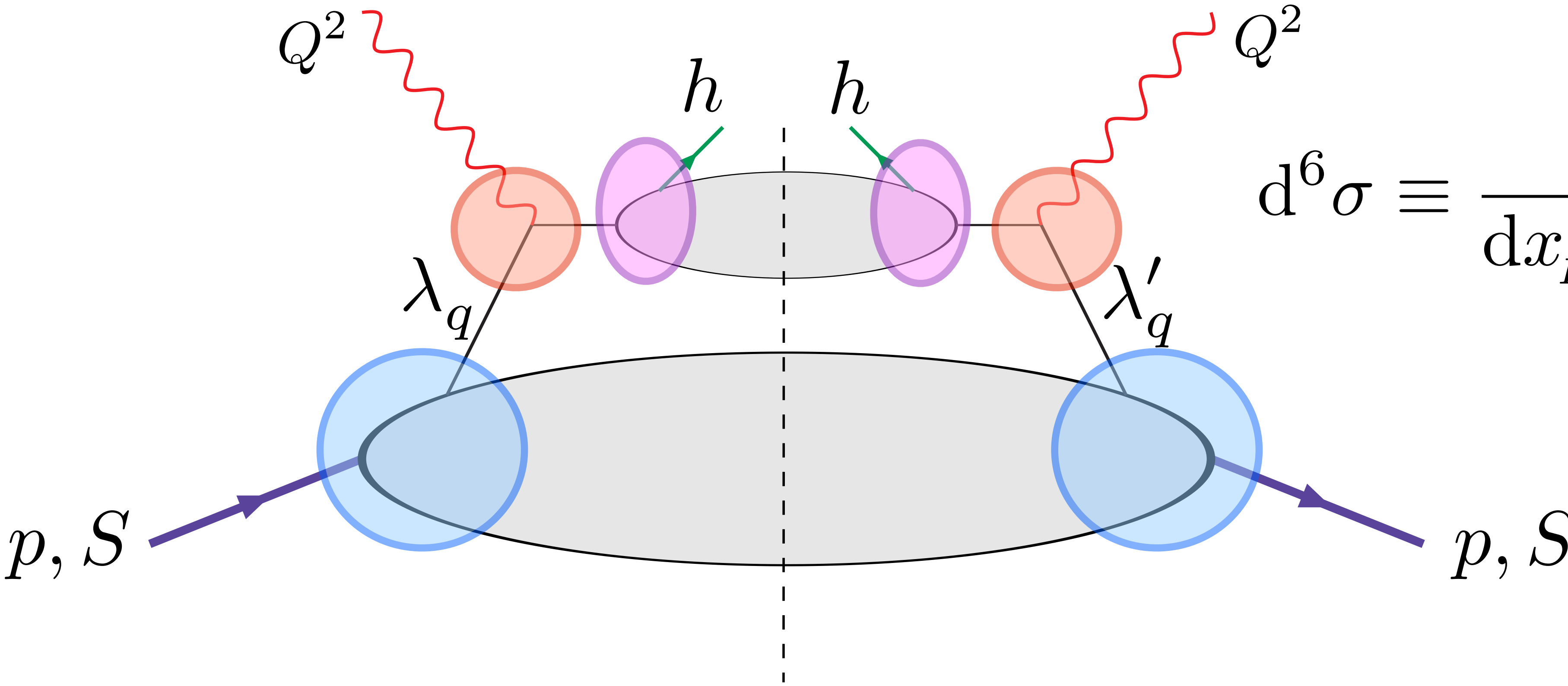
$\boldsymbol{s}_q \cdot (\boldsymbol{p}_q \times \boldsymbol{p}_\perp)$ "Collins effect"

there are 2 independent TMD-FFs for spinless hadrons

$D_1^q(z, \boldsymbol{p}_\perp^2)$ unpolarized hadrons in unpolarized quarks
unintegrated fragmentation function

$H_1^{\perp q}(z, \boldsymbol{p}_\perp^2)$ correlate $p_\perp$ of hadron with s_T of quark (Collins)

TMDs in SIDIS



$$\mathrm{d}^6\sigma \equiv \frac{\mathrm{d}^6\sigma^{\ell p^\uparrow \to \ell h X}}{\mathrm{d}x_B \, \mathrm{d}Q^2 \, \mathrm{d}z_h \, \mathrm{d}^2\boldsymbol{P}_T \, \mathrm{d}\phi_S}$$

$$\boldsymbol{P}_T \simeq \boldsymbol{p}_\perp + z\,\boldsymbol{k}_\perp$$

$$x_B \simeq x \qquad z_h \simeq z$$

TMD factorization holds at large Q^2, and $P_T \approx k_\perp \approx \Lambda_{QCD}$

Two scales: $P_T \ll Q^2$

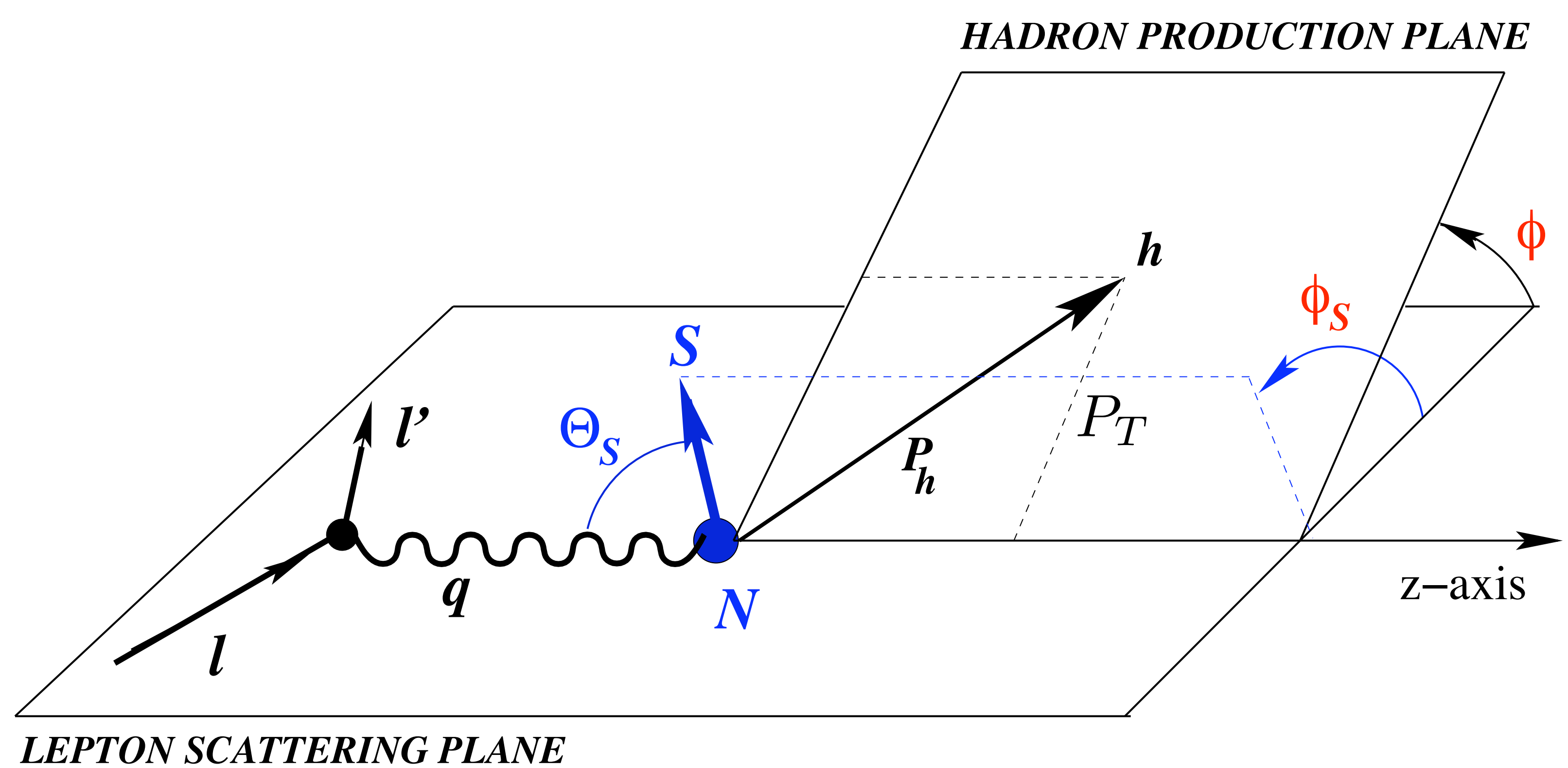
TMD-PDFs | hard scattering | TMD-FFs

$$\mathrm{d}\sigma^{\ell p \to \ell h X} = \sum_q f_q(x, \boldsymbol{k}_\perp; Q^2) \otimes \mathrm{d}\hat{\sigma}^{\ell q \to \ell q}(y, \boldsymbol{k}_\perp; Q^2) \otimes D_q^h(z, \boldsymbol{p}_\perp; Q^2)$$

(Collins, Soper, Ji, J.P. Ma, Yuan, Qiu, Vogelsang, Collins, Metz...)

$$
\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}\phi} \; = \; & F_{UU} + \cos(2\phi)\, F_{UU}^{\cos(2\phi)} + \frac{1}{Q}\, \cos\phi\, F_{UU}^{\cos\phi} + \lambda\, \frac{1}{Q}\, \sin\phi\, F_{LU}^{\sin\phi} \\
+ \; & S_L \left\{ \sin(2\phi)\, F_{UL}^{\sin(2\phi)} + \frac{1}{Q}\, \sin\phi\, F_{UL}^{\sin\phi} + \lambda \left[F_{LL} + \frac{1}{Q}\, \cos\phi\, F_{LL}^{\cos\phi} \right] \right\} \\
+ \; & S_T \Big\{ \underset{\text{Sivers}}{\sin(\phi-\phi_S)\, F_{UT}^{\sin(\phi-\phi_S)}} + \underset{\text{Collins}}{\sin(\phi+\phi_S)\, F_{UT}^{\sin(\phi+\phi_S)}} + \sin(3\phi-\phi_S)\, F_{UT}^{\sin(3\phi-\phi_S)} \\
+ \; & \frac{1}{Q} \left[\sin(2\phi-\phi_S)\, F_{UT}^{\sin(2\phi-\phi_S)} + \sin\phi_S\, F_{UT}^{\sin\phi_S} \right] \\
+ \; & \lambda \left[\cos(\phi-\phi_S)\, F_{LT}^{\cos(\phi-\phi_S)} + \frac{1}{Q} \left(\cos\phi_S\, F_{LT}^{\cos\phi_S} + \cos(2\phi-\phi_S)\, F_{LT}^{\cos(2\phi-\phi_S)} \right) \right] \Big\}
\end{aligned}
$$

the $F^{(\cdots)}_{S_B\; S_T}$ cont
the TMDs; plen
of Spin
Asymmetries



at leading twist there are 8 independent azimuthal modulations, with different combinations of TMDs.

$$F_{UU} \sim \sum_a e_a^2 f_1^a \otimes D_1^a \qquad F_{LT}^{\cos(\phi-\phi_S)} \sim \sum_a e_a^2 g_{1T}^{\perp a} \otimes D_1^a$$

$$F_{LL} \sim \sum_a e_a^2 g_{1L}^a \otimes D_1^a \qquad F_{UT}^{\sin(\phi-\phi_S)} \sim \sum_a e_a^2 f_{1T}^{\perp a} \otimes D_1^a$$

chiral-even TMDs

$$F_{UU}^{\cos(2\phi)} \sim \sum_a e_a^2 h_1^{\perp a} \otimes H_1^{\perp a} \qquad F_{UT}^{\sin(\phi+\phi_S)} \sim \sum_a e_a^2 h_{1T}^a \otimes H_1^{\perp a}$$

$$F_{UL}^{\sin(2\phi)} \sim \sum_a e_a^2 h_{1L}^{\perp a} \otimes H_1^{\perp a} \qquad F_{UT}^{\sin(3\phi-\phi_S)} \sim \sum_a e_a^2 h_{1T}^{\perp a} \otimes H_1^{\perp a}$$

chiral-odd TMDs

D_1^a is unpolarized fragmentation function

$H_1^{\perp a}$ is Collins fragmentation function

integrated $f_1^q(x)$ and $g_{1L}^q(x)$ can be measured in usual DIS

origin of Sivers effect in SIDIS - $F_{UT}^{\sin(\phi-\phi_S)}$

$$\mathrm{d}\sigma^{\uparrow,\downarrow} = \sum_q f_{q/p^{\uparrow,\downarrow}}(x, \boldsymbol{k}_\perp; Q^2) \otimes \mathrm{d}\hat{\sigma}(y, \boldsymbol{k}_\perp; Q^2) \otimes D_{h/q}(z, \boldsymbol{p}_\perp; Q^2)$$

$$f_{q/p^{\uparrow,\downarrow}}(x, \boldsymbol{k}_\perp) = f_{q/p}(x, k_\perp) \pm \frac{1}{2}\Delta^N f_{q/p^\uparrow}(x, k_\perp)\, \boldsymbol{S} \cdot (\hat{\boldsymbol{p}} \times \hat{\boldsymbol{k}}_\perp)$$

$$\left(\Delta^N f_{q/p^\uparrow} = -\frac{2k_\perp}{M} f_{1T}^{\perp q}\right)$$

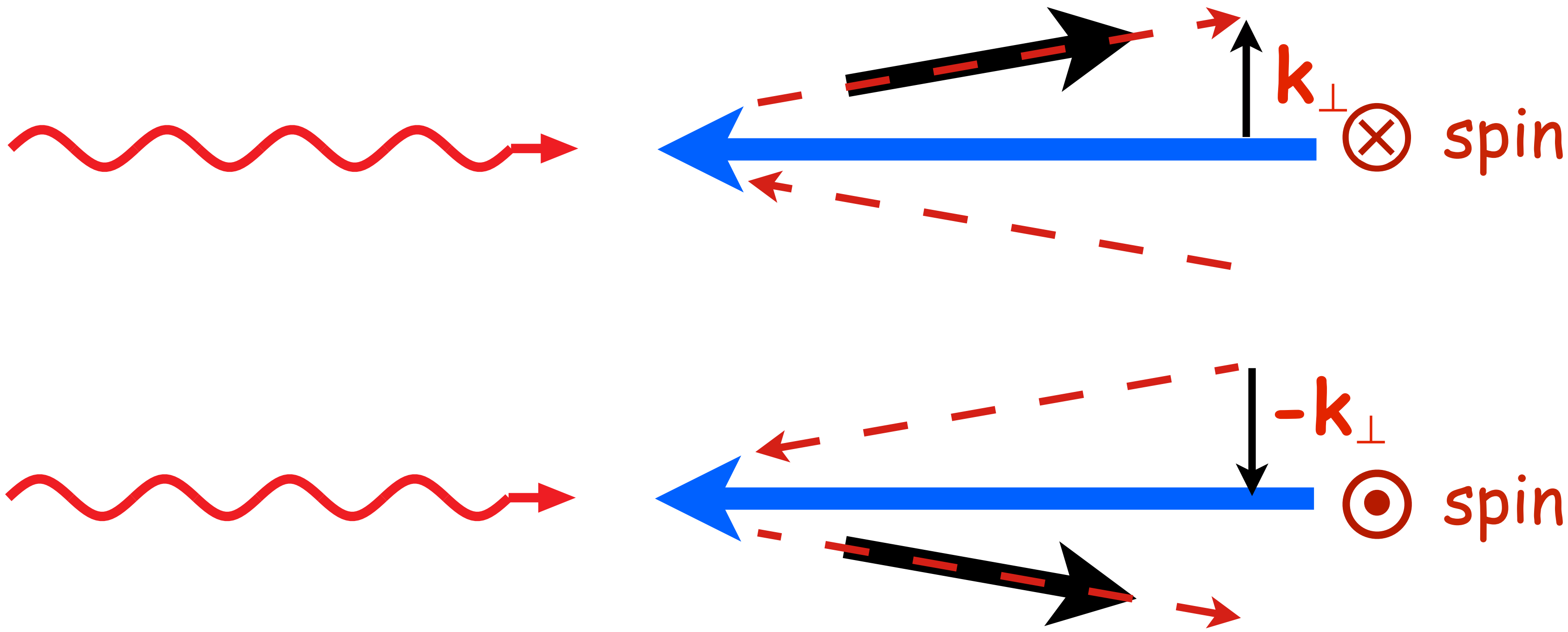
$$\mathrm{d}\sigma^\uparrow - \mathrm{d}\sigma^\downarrow = \sum_q \Delta^N f_{q/p^\uparrow}(x, k_\perp)\, \underbrace{\boldsymbol{S} \cdot (\hat{\boldsymbol{p}} \times \hat{\boldsymbol{k}}_\perp)}_{\sin(\varphi - \phi_S)} \otimes \mathrm{d}\hat{\sigma}(y, \boldsymbol{k}_\perp) \otimes D_{h/q}(z, \boldsymbol{p}_\perp)$$

no SSA if $\mathbf{k}_\perp = 0$!

measured quantity

$$\left\{\begin{array}{l} 2\langle \sin(\phi - \phi_S)\rangle = A_{UT}^{\sin(\phi-\phi_S)} \\ \equiv 2\,\dfrac{\int \mathrm{d}\phi\, \mathrm{d}\phi_S\, [\mathrm{d}\sigma^\uparrow - \mathrm{d}\sigma^\downarrow]\, \sin(\phi - \phi_S)}{\int \mathrm{d}\phi\, \mathrm{d}\phi_S\, [\mathrm{d}\sigma^\uparrow + \mathrm{d}\sigma^\downarrow]} \end{array}\right.$$

the Sivers effect has a simple physical picture...



$$P_T \sim k_\perp$$

$$f_{q/p,\boldsymbol{S}}(x,\boldsymbol{k}_\perp) = f_{q/p}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q/p^\uparrow}(x,k_\perp)\,\boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$

$$= f_{q/p}(x,k_\perp) - \frac{k_\perp}{M} f_{1T}^{\perp q}(x,k_\perp)\,\boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$

left-right spin asymmetry for the process $\gamma^* q \to q$

the spin-$\mathbf{k}_\perp$ correlation is an intrinsic property of the nucleon; it should be related to the parton orbital motion

extraction of u and d Sivers functions - first phase

measured quantity

$$A_{UT}^{\sin(\phi_h-\phi_S)} = 2\,\frac{\int d\phi_S\, d\phi_h\, [d\sigma^\uparrow - d\sigma^\downarrow]\,\sin(\phi_h-\phi_S)}{\int d\phi_S\, d\phi_h\, [d\sigma^\uparrow + d\sigma^\downarrow]}$$

TMD factorization at $\mathcal{O}(k_\perp/Q)$

$$\frac{d\sigma^{\ell p^\uparrow\to \ell h X}}{dx_{_B}\, dQ^2\, dz_h\, d^2\boldsymbol{P}_T} = \sum_q \boxed{e_q^2} \int d^2\boldsymbol{k}_\perp\, f_{q/p^\uparrow}(x,\boldsymbol{k}_\perp)\, \boxed{\frac{2\pi\alpha^2}{x^2 s^2}\,\frac{\hat{s}^2+\hat{u}^2}{Q^4}}\, D_{h/q}(z,\boldsymbol{p}_\perp)$$

$$f_{q/p^\uparrow}(x,\boldsymbol{k}_\perp) = f_{q/p}(x,k_\perp) + \frac{1}{2}\boxed{\Delta^N f_{q/p^\uparrow}(x,k_\perp)\, \boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)}$$

$$A_{UT}^{\sin(\phi_h-\phi_S)} = \frac{\sum_q \int d\phi_S\, d\phi_h\, d^2\boldsymbol{k}_\perp\, \boxed{\Delta^N f_{q/p^\uparrow}(x,k_\perp)\sin(\varphi-\phi_S)}\, \boxed{\frac{d\hat{\sigma}^{\ell q\to\ell q}}{dQ^2}}\, D_q^h(z,p_\perp)\sin(\phi_h-\phi_S)}{\sum_q \int d\phi_S\, d\phi_h\, d^2\boldsymbol{k}_\perp\, f_{q/p}(x,k_\perp)\, \frac{d\hat{\sigma}^{\ell q\to\ell q}}{dQ^2}\, D_q^h(z,p_\perp)}$$

two different notations $\quad \Delta^N f_{q/p^\uparrow} = -\frac{2\,k_\perp}{M_p}\, f_{1T}^{\perp q}$

simple parameterisations

$$\Delta^N f_{q/p^\uparrow}(x, k_\perp, Q) = 2\,\mathcal{N}(x)\, h(k_\perp)\, \underbrace{f_{q/p}(x, Q)\, \frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi\langle k_\perp^2\rangle}}_{f_{q/p}(x,k_\perp)}$$

$$\mathcal{N}_q(x) = N_q\, x^{\alpha_q}(1-x)^{\beta_q}\, \frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\,\beta_q^{\beta_q}}$$

$$h(k_\perp) = \sqrt{2e}\,\frac{k_\perp}{M_1}\, e^{-k_\perp^2/M_1^2}$$

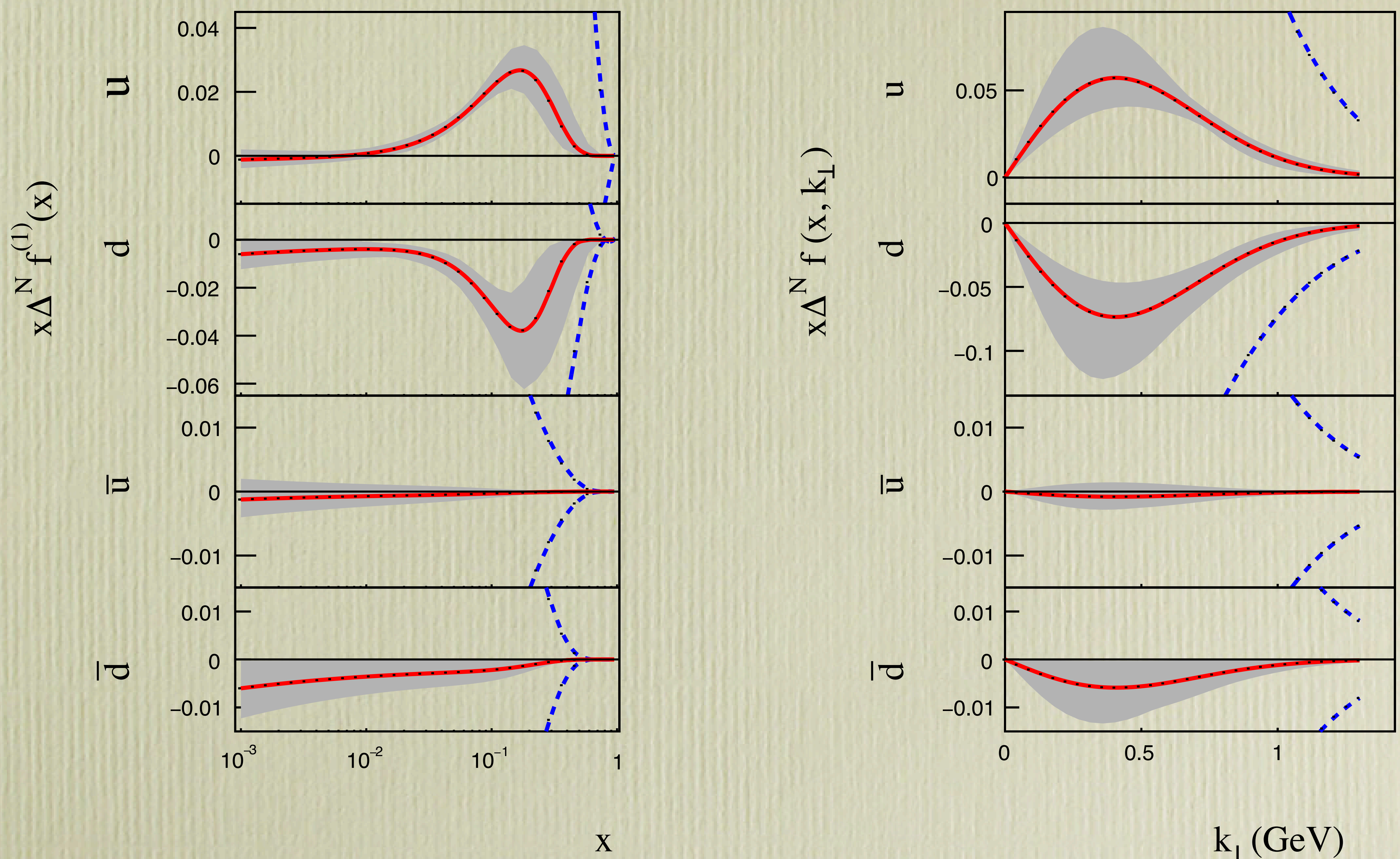
$$D_{h/q}(z, p_\perp) = D_{h/q}(z)\, \frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle}$$

Q^2 evolution only taken into account in the collinear part
(usual DGLAP PDF evolution)

M.A, M. Boglione, U. D'Alesio, A. Kotzinian, S. Melis, F. Murgia, A. Prokudin,
Phys. Rev. D71 (2005) 074006; Eur. Phys. J. A39 (2009) 89
(results in agreement with those of several other groups)

most recent extraction of the Sivers functions

M.A, M. Boglione, U. D'Alesio, F. Murgia, A. Prokudin, JHEP 1704 (2017) 046



$$\Delta^N f_q^{(1)}(x,Q) = \int d^2\boldsymbol{k}_\perp \, \frac{k_\perp}{4M_p} \, \Delta^N f_{q/p^\uparrow}(x,k_\perp;Q) = -f_{1T}^{\perp(1)}(x,Q)$$

TMDs and QCD - TMD evolution

how does gluon emission affect the parton transverse motion?

TMD phenomenology - phase 2

Different TMD evolution schemes and different implementations within the same scheme

it needs non perturbative inputs

dedicated workshops, QCD Evolution
2011, 2012, 2013, 2014, 2015, 2016, 2017

dedicated tools:

TMDlib and TMDplotter: library and plotting tools for transverse-momentum-dependent parton distributions

study of the QCD evolution of TMDs and TMD factorisation in rapid development

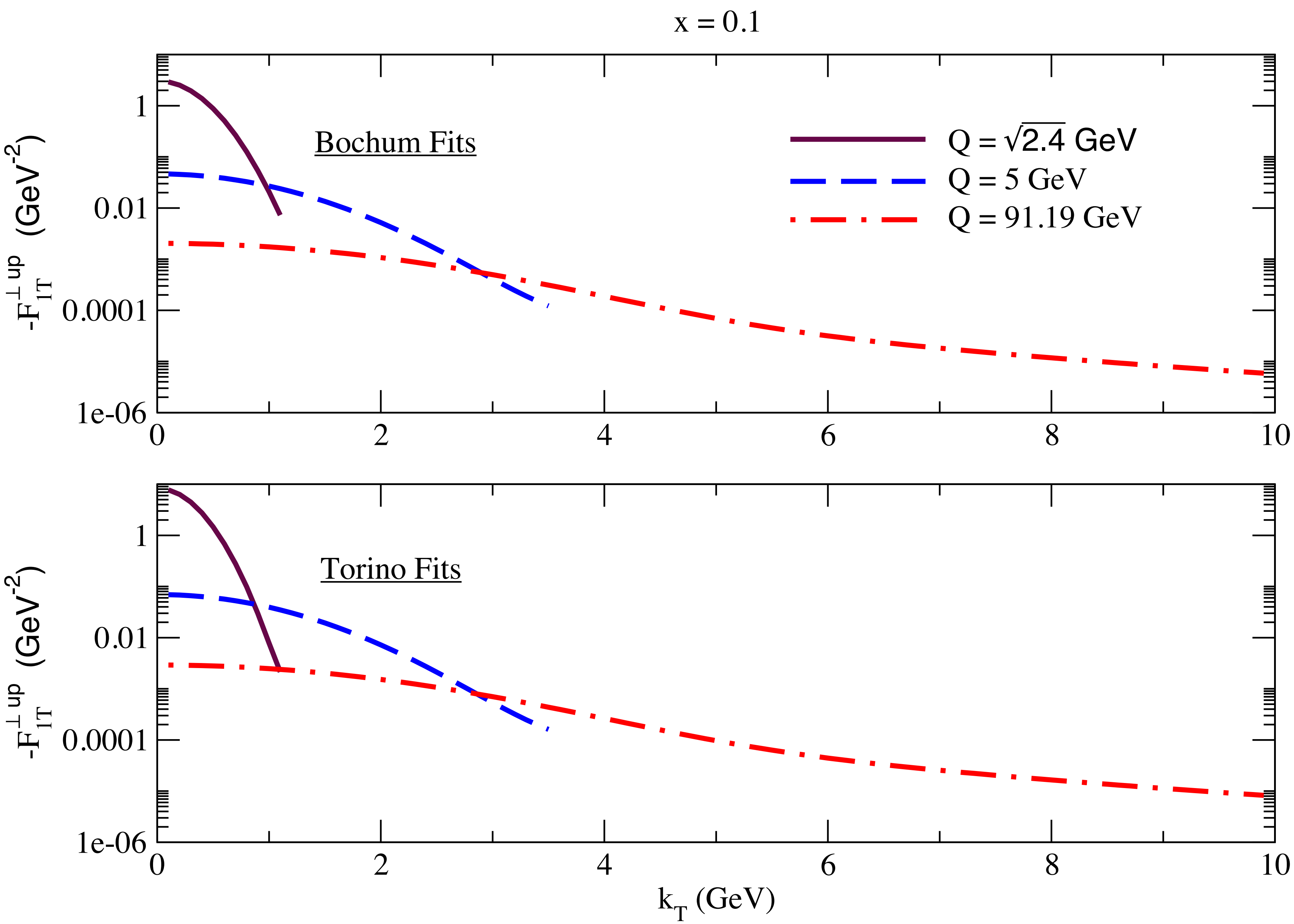
Collins, "Foundations of perturbative QCD", Cambridge University Press (2011)

TMD phenomenology - phase 2

how does gluon emission affect the transverse motion?

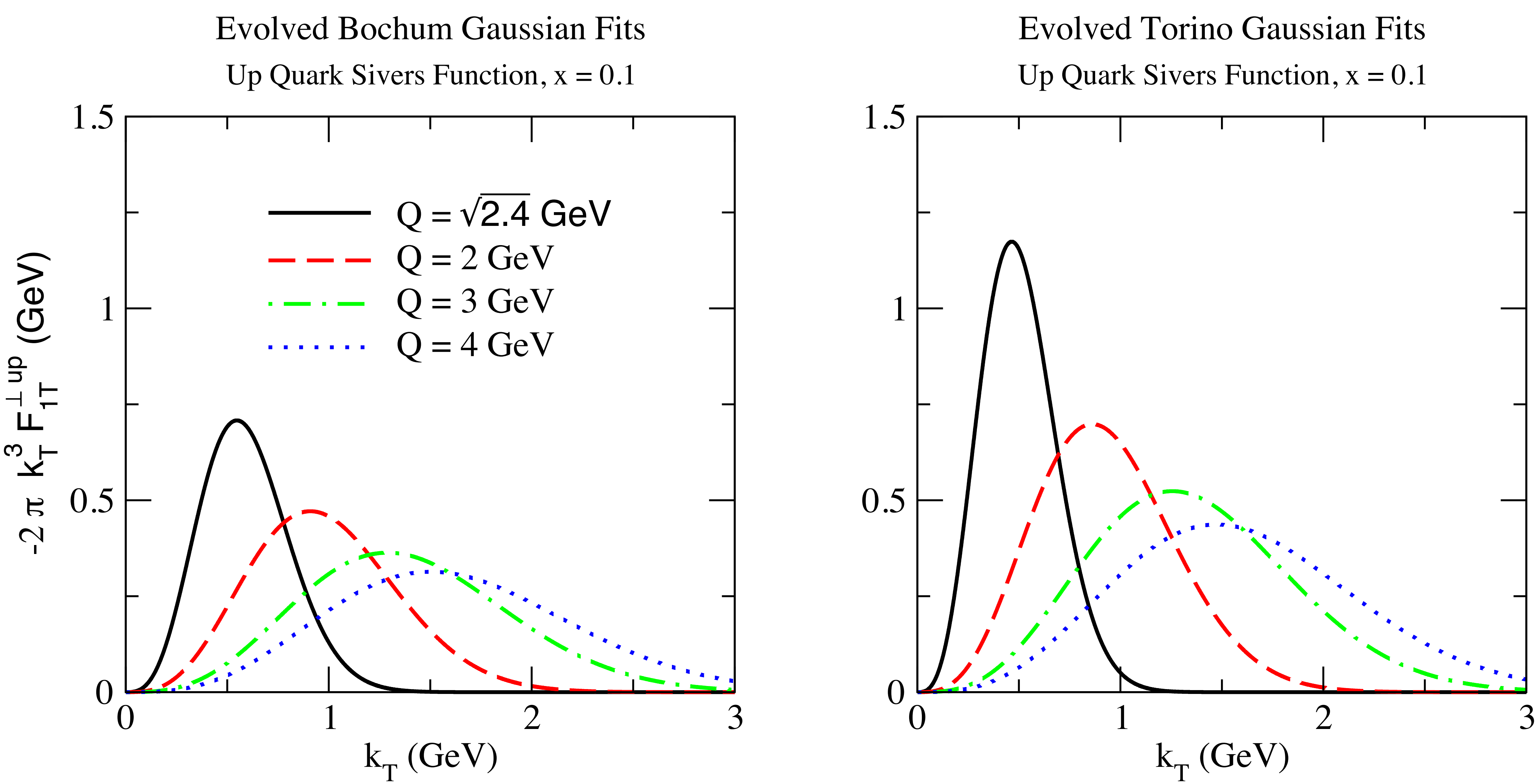
a few selected results, examples

TMD evolution of up quark Sivers function



Aybat, Collins, Qiu, Rogers, Phys. Rev. D85 (2012) 034043

TMD evolution of up quark Sivers function



Aybat, Collins, Qiu, Rogers, Phys.Rev. D85 (2012) 034043

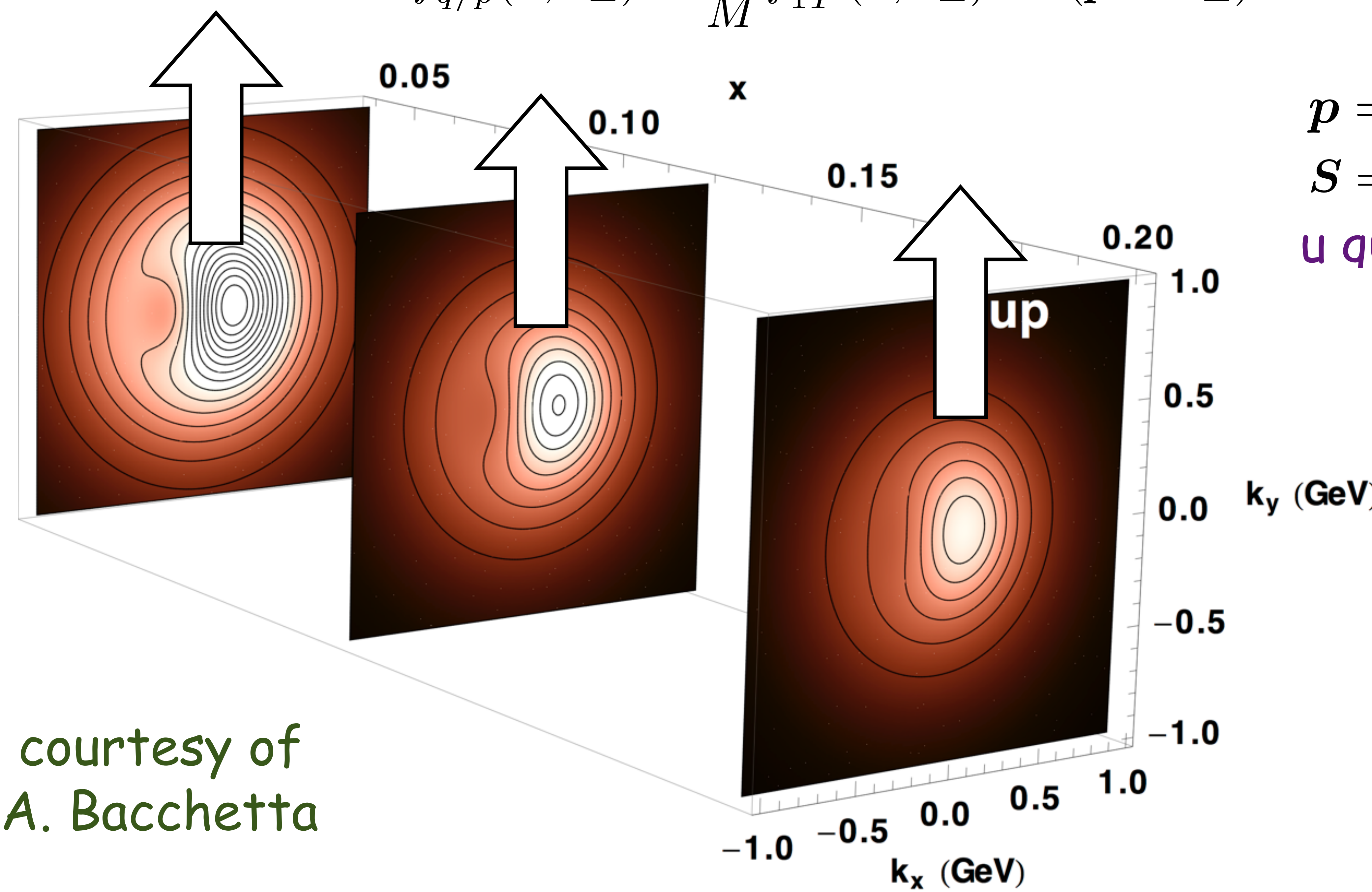
TMD evolution of Sivers function studied also by
Echevarria, Idilbi, Kang, Vitev, Phys. Rev. D89 (2014) 074013

more on the Sivers effect, what does it teach us?

it induces distortions in the parton distributions

$$f_{q/p,\boldsymbol{S}}(x,\boldsymbol{k}_\perp) = f_{q/p}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q/p^\uparrow}(x,k_\perp)\,\boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$

$$= f_{q/p}(x,k_\perp) - \frac{k_\perp}{M} f_{1T}^{\perp q}(x,k_\perp)\,\boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$



$\boldsymbol{p} = p\hat{\boldsymbol{z}}$

$\boldsymbol{S} = S\hat{\boldsymbol{y}}$

u quark

courtesy of A. Bacchetta

Sivers function and orbital angular momentum

Ji's sum rule

forward limit of GPDs

$$J^q = \frac{1}{2}\int_0^1 dx\, x\, [H^q(x,0,0) + E^q(x,0,0)]$$

usual PDF $q(x)$

cannot be measured directly

anomalous magnetic moments

$$\kappa^p = \int_0^1 \frac{dx}{3}\, \left[2E^{u_v}(x,0,0) - E^{d_v}(x,0,0) - E^{s_v}(x,0,0)\right]$$

$$\kappa^n = \int_0^1 \frac{dx}{3}\, \left[2E^{d_v}(x,0,0) - E^{u_v}(x,0,0) - E^{s_v}(x,0,0)\right]$$

$$(E^{q_v} = E^q - E^{\bar{q}})$$

Sivers function and orbital angular momentum

assume

$$f_{1T}^{\perp(0)a}(x;Q_L^2) = -L(x)E^a(x,0,0;Q_L^2)$$

$$f_{1T}^{\perp(0)a}(x,Q) = \int d^2\boldsymbol{k}_\perp\, \widehat{f}_{1T}^{\perp a}(x,k_\perp;Q)$$

L(x) = lensing function
(unknown, can be computed in models)

parameterize Sivers and lensing functions

fit SIDIS and magnetic moment data

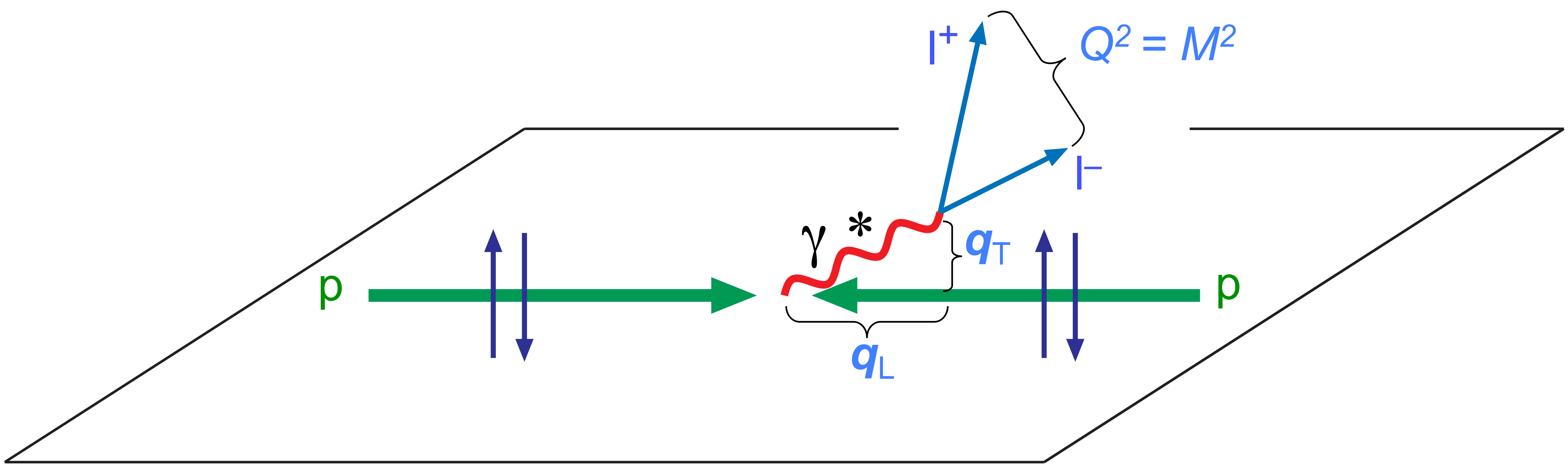
obtain E^q and estimate orbital angular momentum

results at $Q^2 = 4\ \text{GeV}^2$: $J^u \approx 0.23$, $J^{q\neq u} \approx 0$

Bacchetta, Radici, PRL 107 (2011) 212001

TMDs in Drell-Yan processes

COMPASS, RHIC, Fermilab, NICA, AFTER...



factorization holds, two scales, M^2, and $q_T \ll M$

$$\mathrm{d}\sigma^{D-Y} = \sum_a f_q(x_1, \boldsymbol{k}_{\perp 1}; Q^2) \otimes f_{\bar{q}}(x_2, \boldsymbol{k}_{\perp 2}; Q^2)\, \mathrm{d}\hat{\sigma}^{q\bar{q}\to \ell^+\ell^-}$$

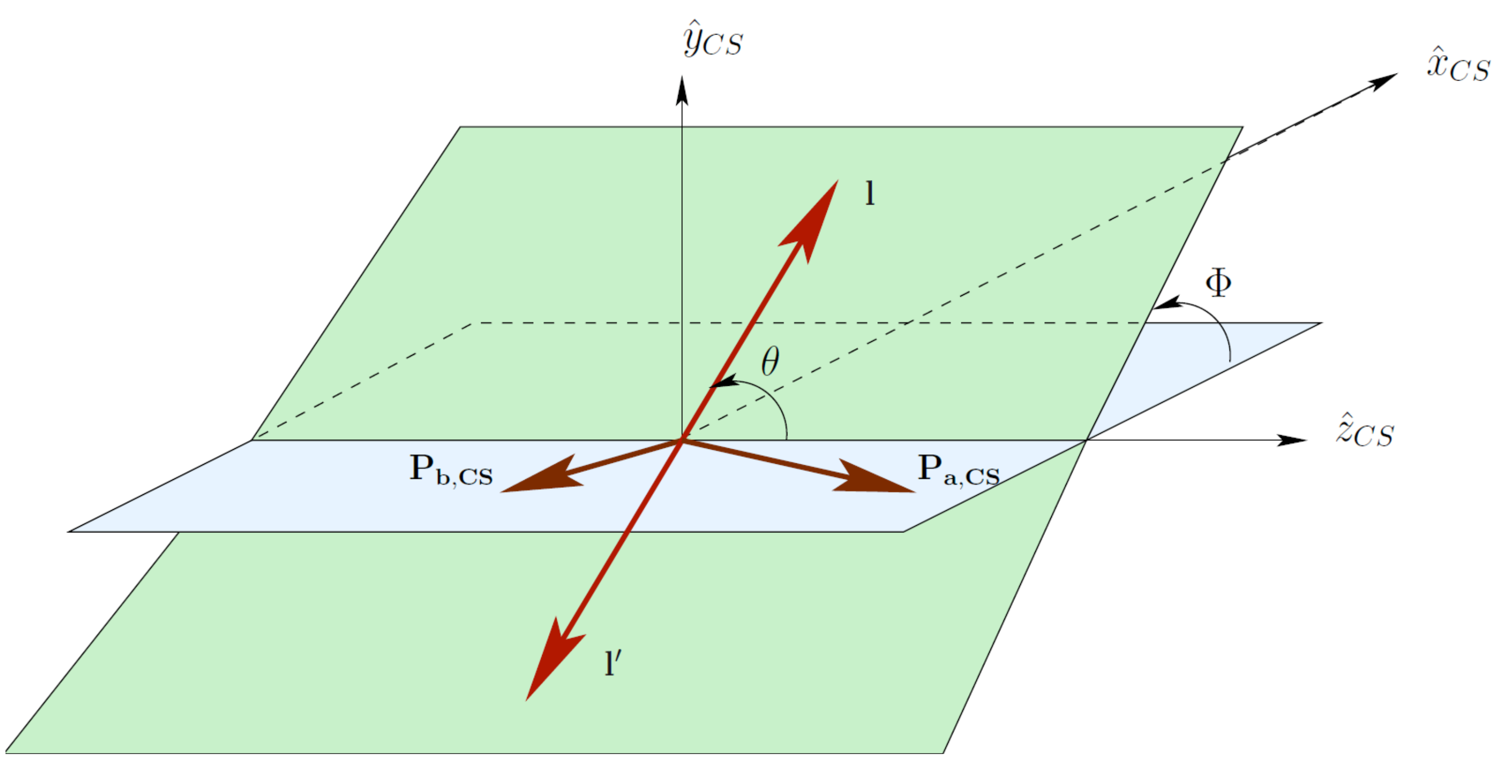
direct product of TMDs, no fragmentation process

Case of one polarized nucleon only

$$\frac{d\sigma}{d^4q\, d\Omega} = \frac{\alpha^2}{\Phi\, q^2} \Big\{ (1+\cos^2\theta)\, F_U^1 + (1-\cos^2\theta)\, F_U^2 + \sin 2\theta \cos\phi\, F_U^{\cos\phi} + \sin^2\theta \cos 2\phi\, F_U^{\cos 2\phi}$$
$$+\; S_L \left(\sin 2\theta \sin\phi\, F_L^{\sin\phi} + \sin^2\theta \sin 2\phi\, F_L^{\sin 2\phi} \right)$$
$$+\; S_T \Big[\left(F_T^{\sin\phi_S} + cos^2\theta\, \tilde{F}_T^{\sin\phi_S} \right) \sin\phi_S + \sin 2\theta \Big(\sin(\phi+\phi_S)\, F_T^{\sin(\phi+\phi_S)}$$
$$+\; \sin(\phi-\phi_S)\, F_T^{\sin(\phi-\phi_S)} \Big)$$

Sivers effect

$$+\; \sin^2\theta \left(\sin(2\phi+\phi_S)\, F_T^{\sin(2\phi+\phi_S)} + \sin(2\phi-\phi_S)\, F_T^{\sin(2\phi-\phi_S)} \right) \Big] \Big\}$$



Collins-Soper frame

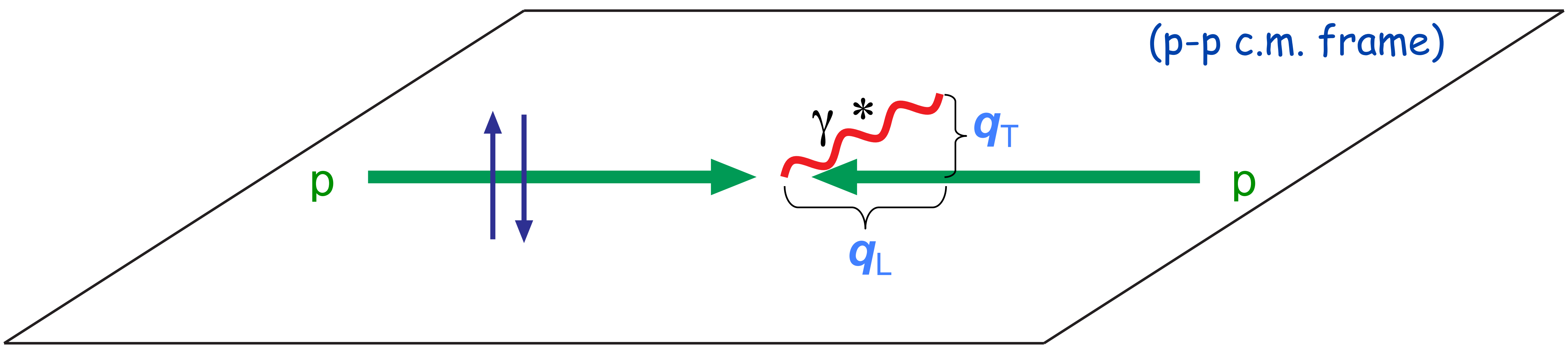
origin of Sivers effect in DY processes

By looking at the $d^4\sigma/d^4q$ cross section one can single out the Sivers effect in D-Y processes

$$d\sigma^\uparrow - d\sigma^\downarrow \propto \sum_q \Delta^N f_{q/p^\uparrow}(x_1, \boldsymbol{k}_{\perp 1}) \otimes f_{\bar{q}/p}(x_2, k_{\perp 2}) \otimes d\hat{\sigma}$$

$q = u, \bar{u}, d, \bar{d}, s, \bar{s}$

$$A_N^{\sin(\phi_S - \phi_\gamma)} \equiv \frac{2\int_0^{2\pi} d\phi_\gamma \, [d\sigma^\uparrow - d\sigma^\downarrow] \sin(\phi_S - \phi_\gamma)}{\int_0^{2\pi} d\phi_\gamma \, [d\sigma^\uparrow + d\sigma^\downarrow]}$$



with the simple parameterization of the unpolarized and Sivers distributions one has:

$$A_N^{\sin(\phi_\gamma - \phi_S)}(x_F, M, q_T) = \frac{\int d\phi_\gamma \, [N(x_F, M, q_T, \phi_\gamma)] \, \sin(\phi_\gamma - \phi_S)}{\int d\phi_\gamma \, [D(x_F, M, q_T)]}$$

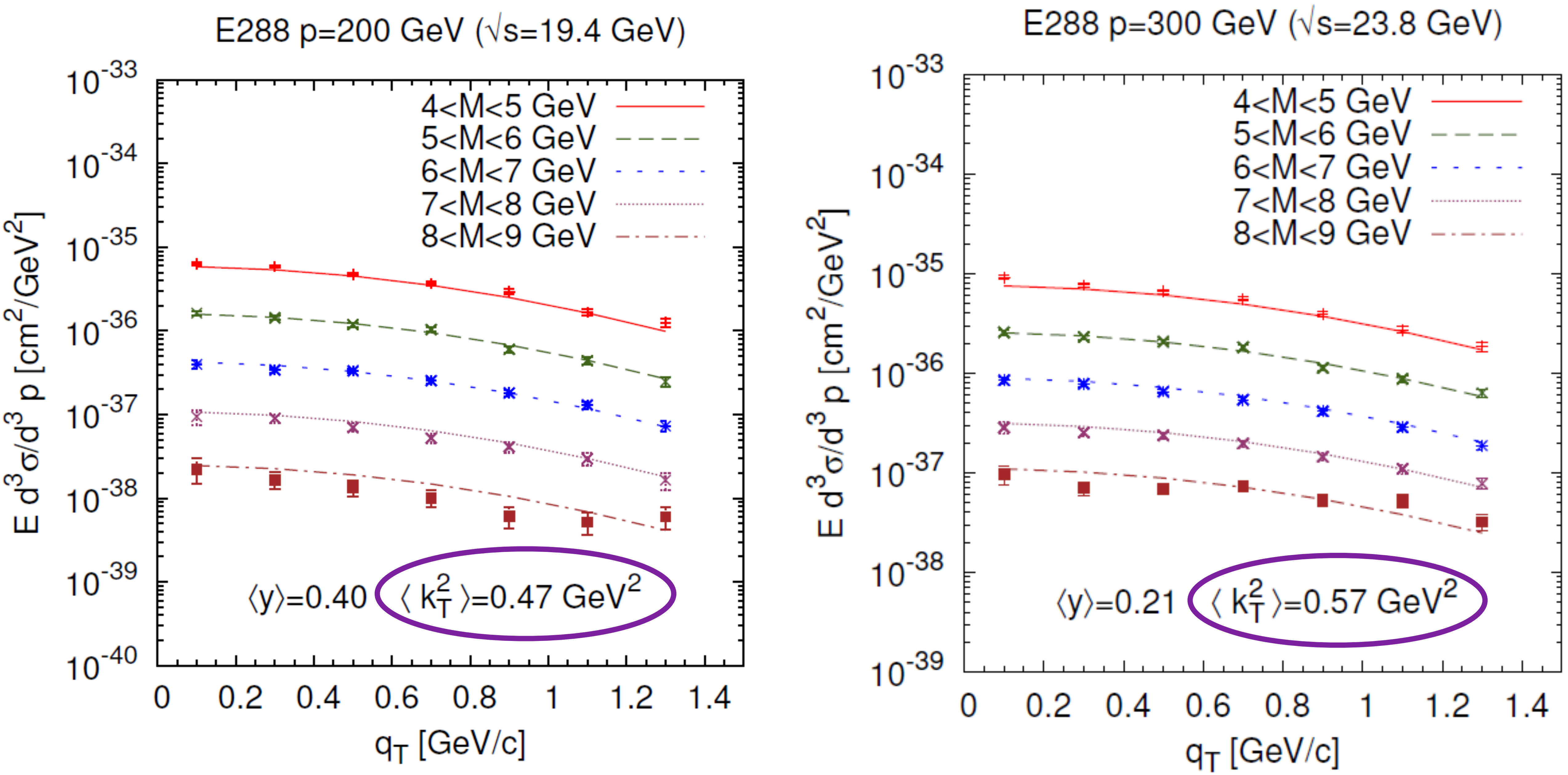
$$\begin{aligned} N(x_F, M, q_T, \phi_\gamma) \; &\equiv \; \frac{d^4\sigma^\uparrow}{dx_F \, dM^2 \, d^2\boldsymbol{q}_T} - \frac{d^4\sigma^\downarrow}{dx_F \, dM^2 \, d^2\boldsymbol{q}_T} \\ &= \frac{4\,\pi\,\alpha^2}{9\,M^2\,s} \sum_q \frac{e_q^2}{x_1 + x_2} \, \Delta^N f_{q/A^\uparrow}(x_1) \, f_{\bar{q}/B}(x_2) \, \sqrt{2e} \, \frac{q_T}{M_1} \, \frac{\langle k_S^2 \rangle^2 \, \exp\left[-q_T^2/\left(\langle k_S^2\rangle + \langle k_{\perp 2}^2\rangle\right)\right]}{\pi \left[\langle k_S^2\rangle + \langle k_{\perp 2}^2\rangle\right]^2 \langle k_{\perp 2}^2\rangle} \, \sin(\phi_S - \phi_\gamma) \end{aligned}$$

$$\begin{aligned} D(x_F, M, q_T) \; &\equiv \; \frac{1}{2}\left[\frac{d^4\sigma^\uparrow}{dx_F \, dM^2 \, d^2\boldsymbol{q}_T} + \frac{d^4\sigma^\downarrow}{dx_F \, dM^2 \, d^2\boldsymbol{q}_T}\right] = \frac{d^4\sigma^{unp}}{dx_F \, dM^2 \, d^2\boldsymbol{q}_T} \\ &= \frac{4\,\pi\,\alpha^2}{9\,M^2\,s} \sum_q \frac{e_q^2}{x_1 + x_2} f_{q/A}(x_1) \, f_{\bar{q}/B}(x_2) \, \frac{\exp\left[-q_T^2/\left(\langle k_{\perp 1}^2\rangle + \langle k_{\perp 2}^2\rangle\right)\right]}{\pi \left[\langle k_{\perp 1}^2\rangle + \langle k_{\perp 2}^2\rangle\right]} \end{aligned}$$

the unpolarized cross section has a simple q_T gaussian dependence

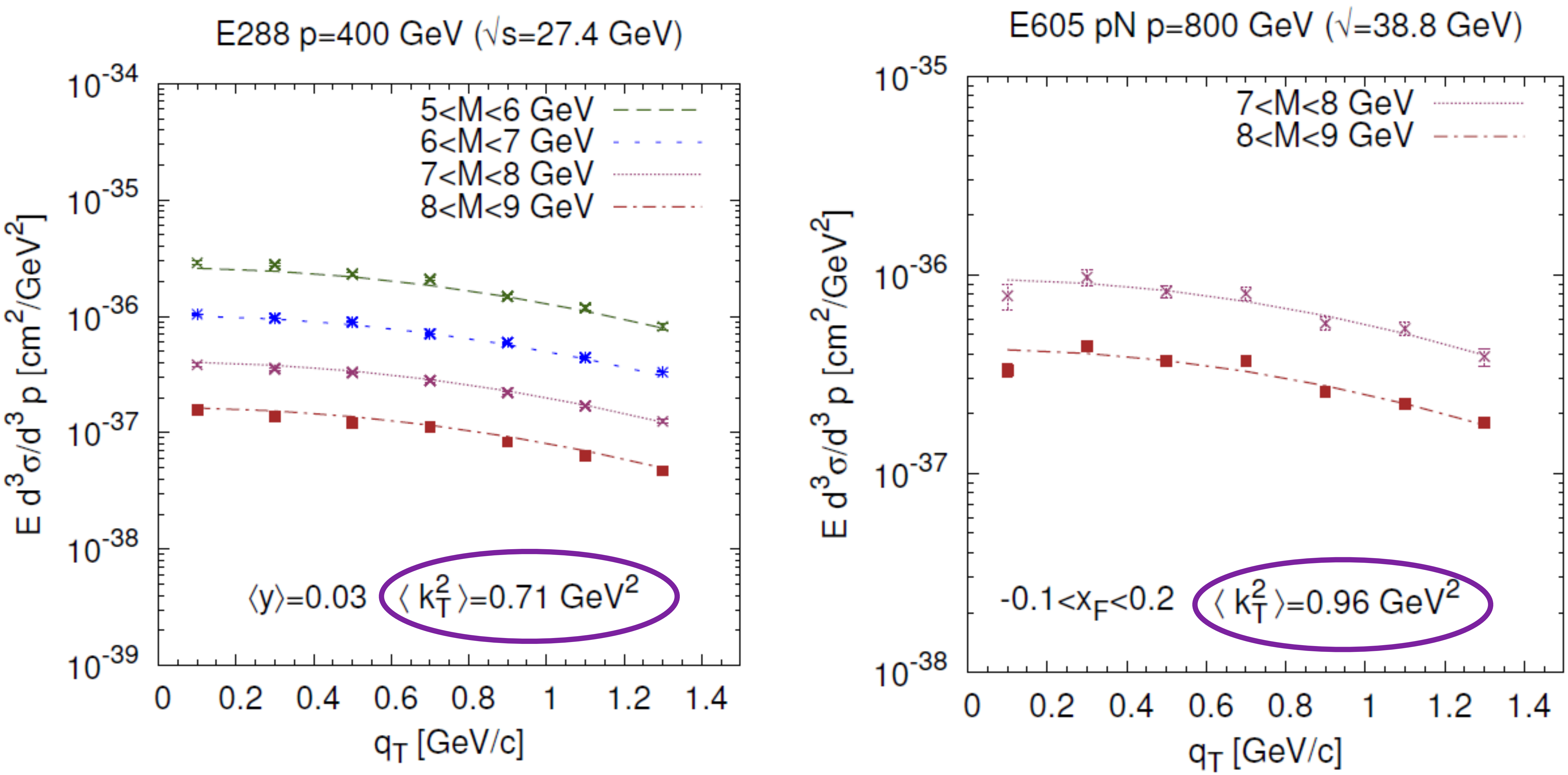
$$d\sigma \sim \frac{\exp[-q_T^2/(2\,\langle k_\perp^2\rangle)}{2\,\pi\,\langle k_\perp^2\rangle} \qquad (\langle k_{\perp 1}^2\rangle = \langle k_{\perp 2}^2\rangle)$$

fit of unpolarized D-Y data, S. Melis

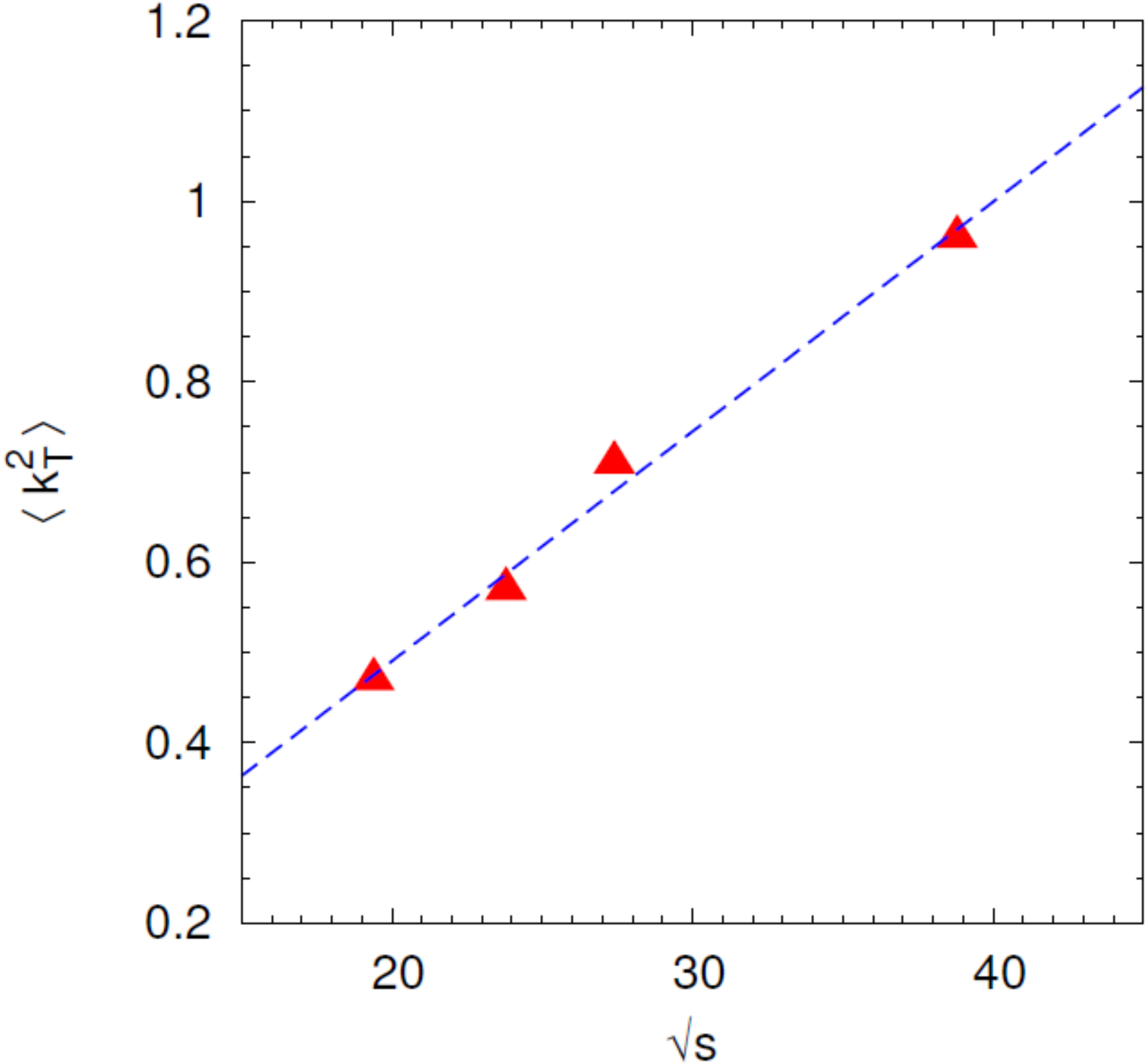


$$f_{q/p}(x, k_\perp) = f_q(x)\, \frac{1}{\pi \langle k_\perp^2 \rangle}\, e^{-k_\perp^2/\langle k_\perp^2 \rangle}$$

a different $\langle k_\perp^2 \rangle$ for each set of data

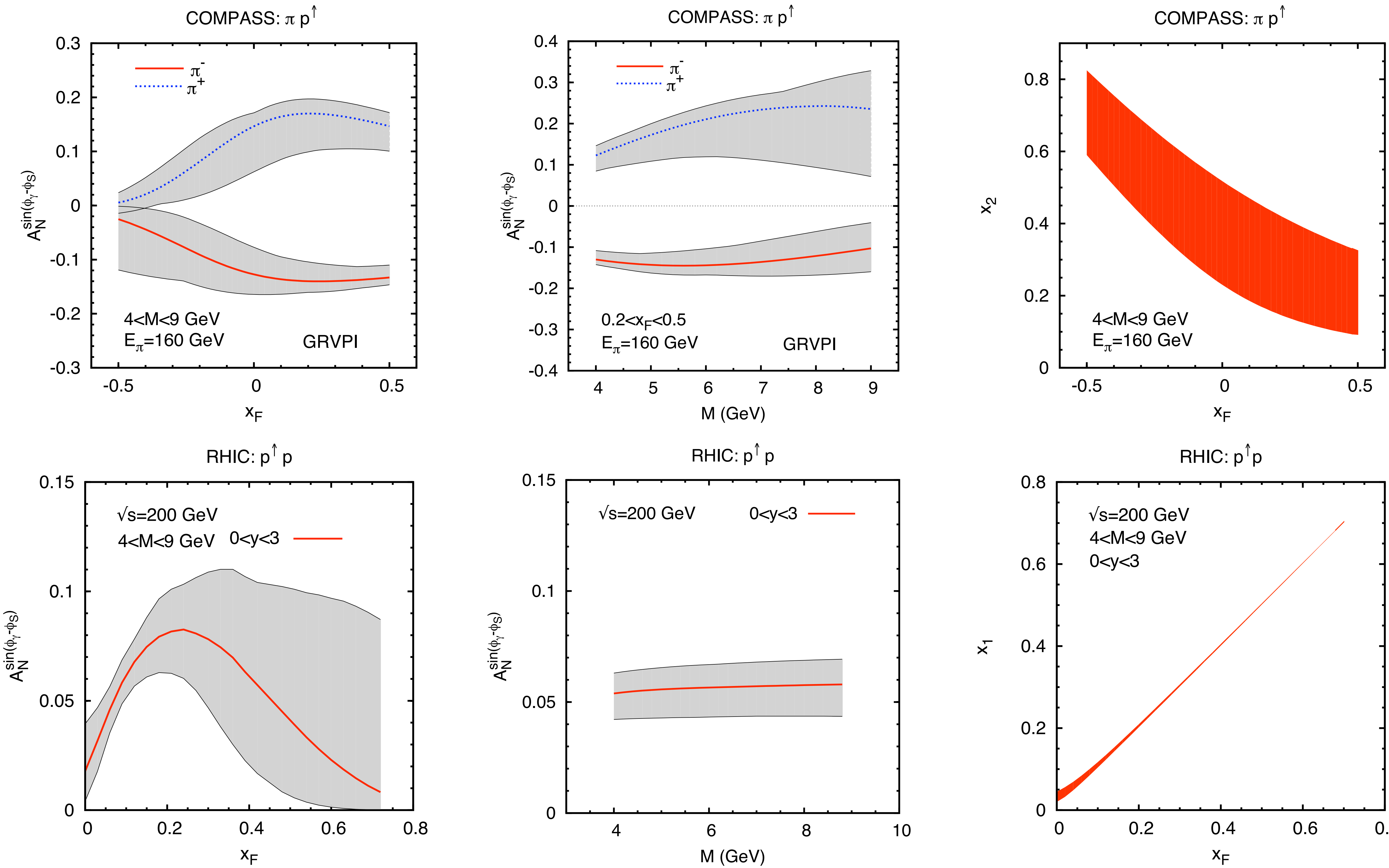


dependence of $\langle k_\perp^2 \rangle$ with energy?


1.2
1
0.8
0.6
0.4
0.2
$\langle k_T^2 \rangle$
20
30
40
$\sqrt{s}$

Predictions for A_N - no TMD evolution

Sivers functions as extracted from SIDIS data, **with opposite sign**

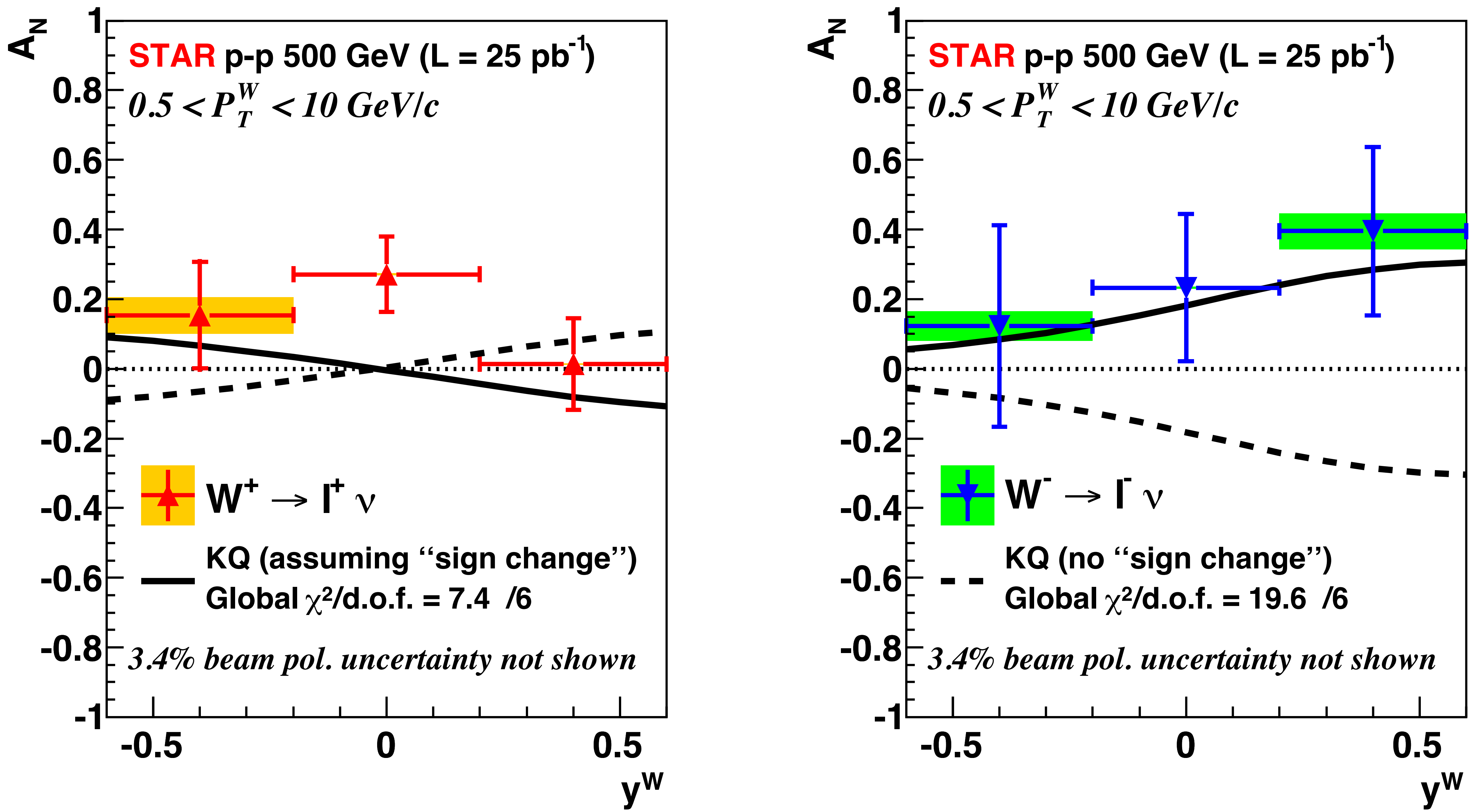


M.A., M. Boglione, U. D'Alesio, S. Melis, F. Murgia, A. Prokudin, PR D79 (2009) 054010

what about the sign change ….?

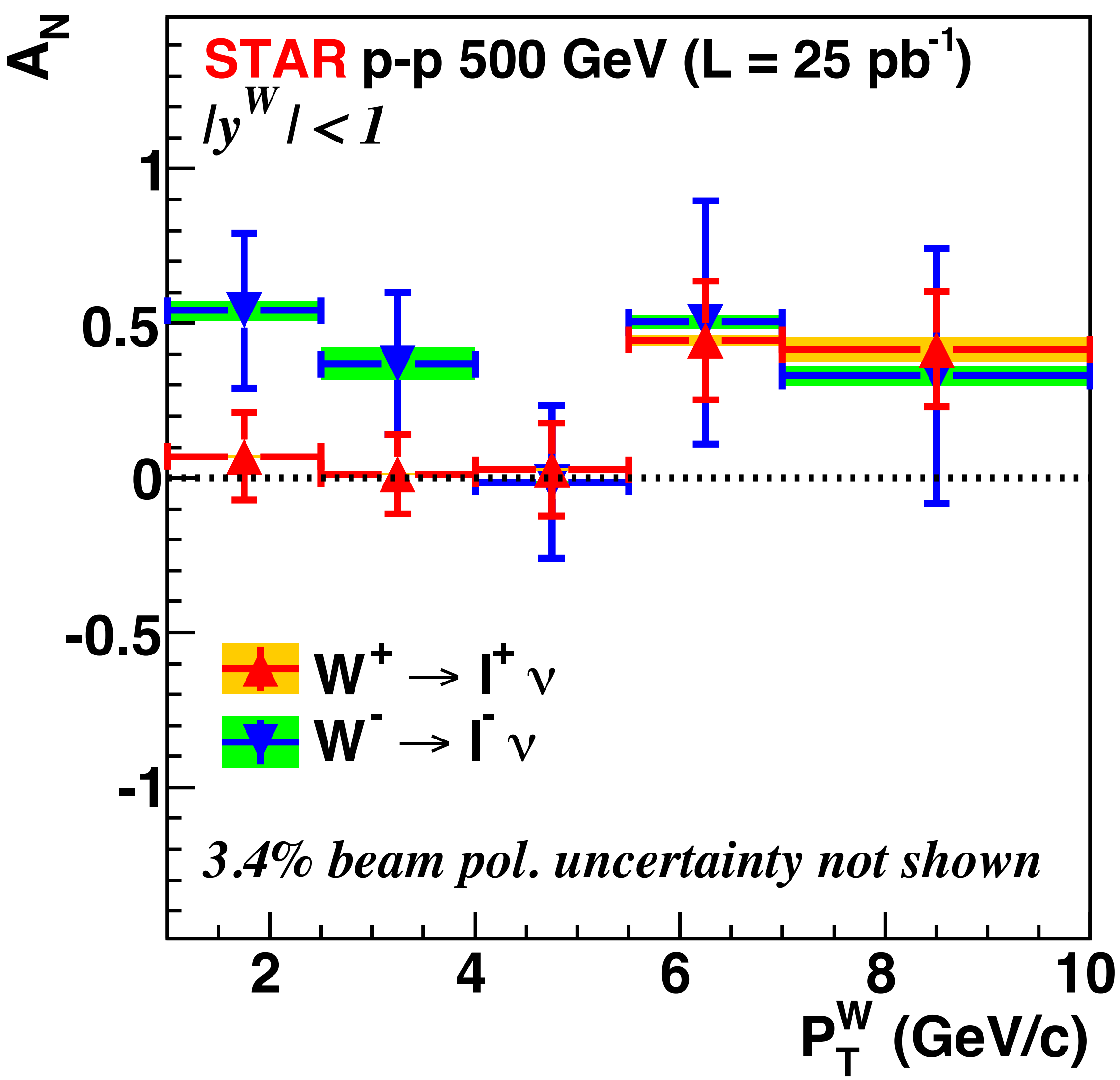
First results from RHIC, $p^{\uparrow}p \to W^{\pm}X$

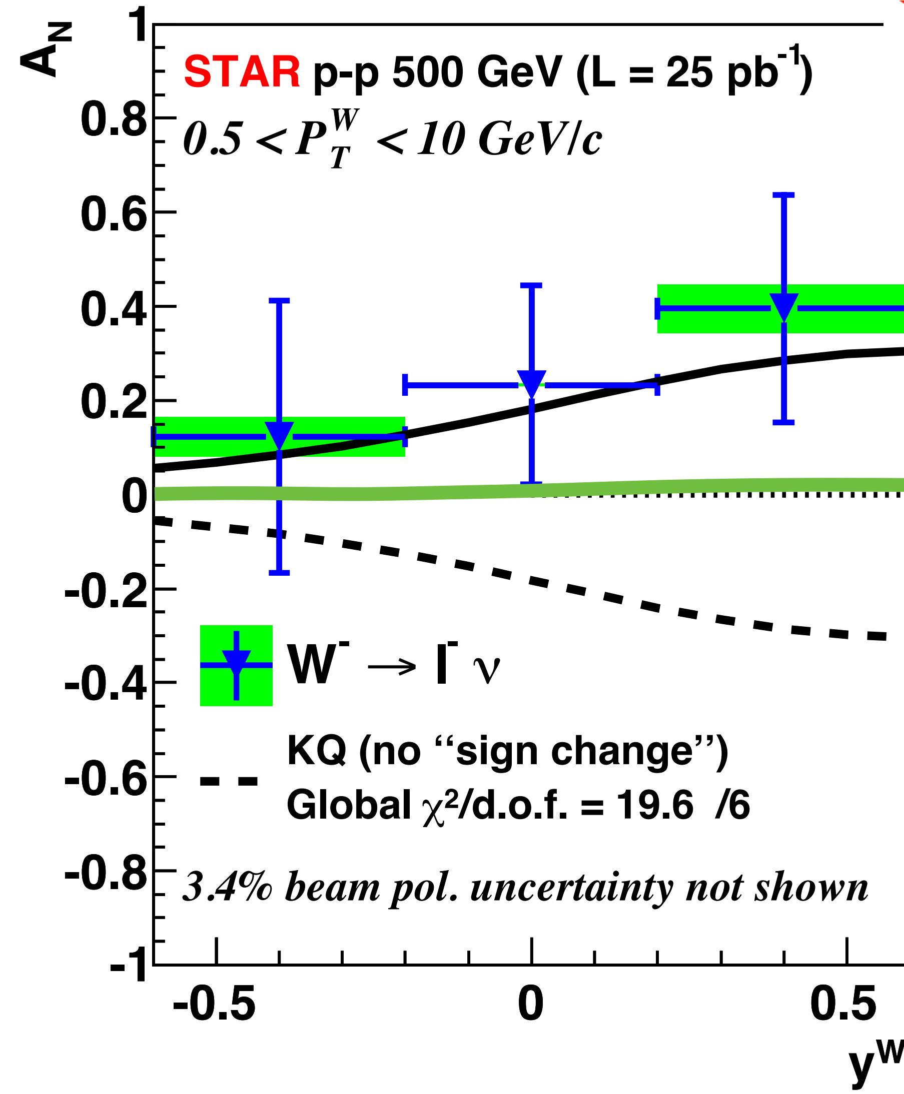
STAR Collaboration, PRL 116 (2016) 132301



some hints at a sign change of the Sivers function…..

some caution still necessary …

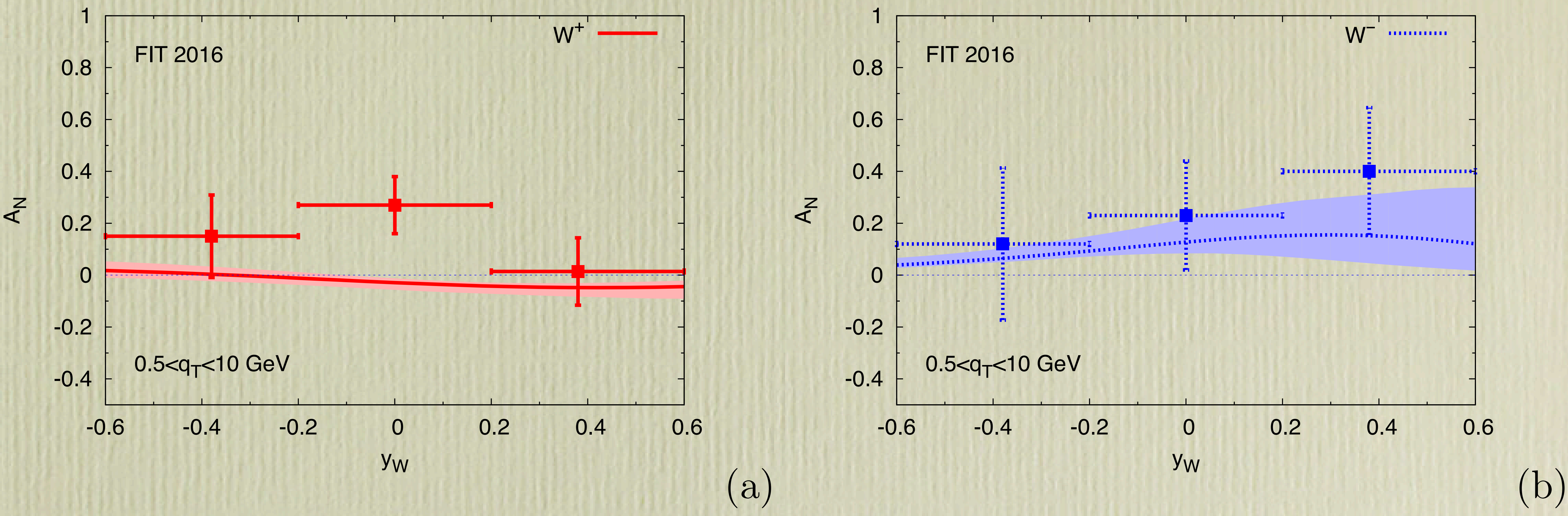




predictions with TMD evolution equation

experimental data up to large p_T values, beyond the validity of TMD factorization. TMD evolution might strongly suppress the asymmetry

M.A, M. Boglione, U. D'Alesio, F. Murgia, A. Prokudin, JHEP 1704 (2017) 046



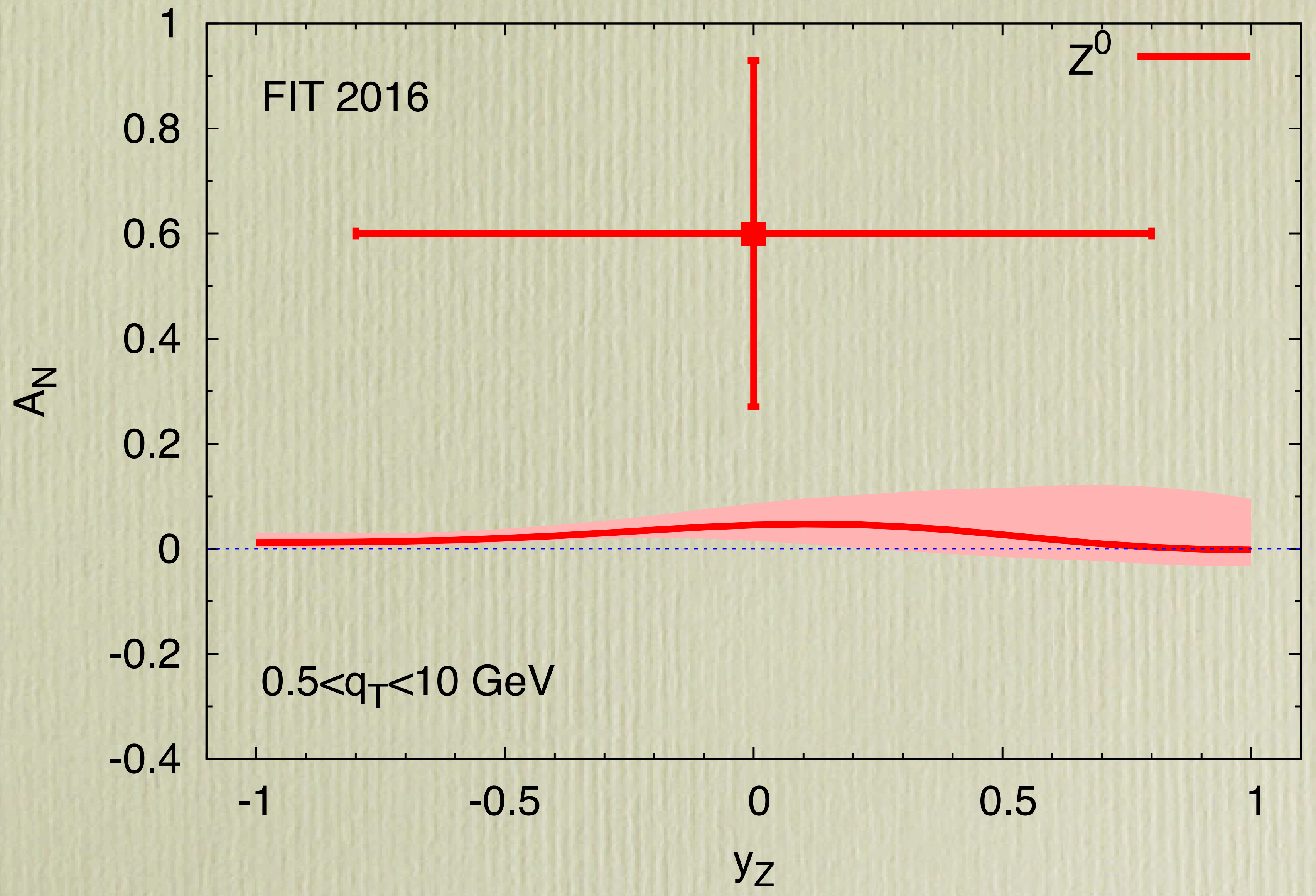
estimates of the Sivers asymmetry A_N for W^+(a) and W^-(b) production, assuming a sign change of the SIDIS Sivers functions, compared with the experimental data as function of y_W

$\langle \chi^2/\mathrm{n.o.d.} \rangle = 1.63$ with sign change

$\langle \chi^2/\mathrm{n.o.d.} \rangle = 2.35$ with no sign change

First results from RHIC, $p^{\uparrow}p \to Z^0\, X$

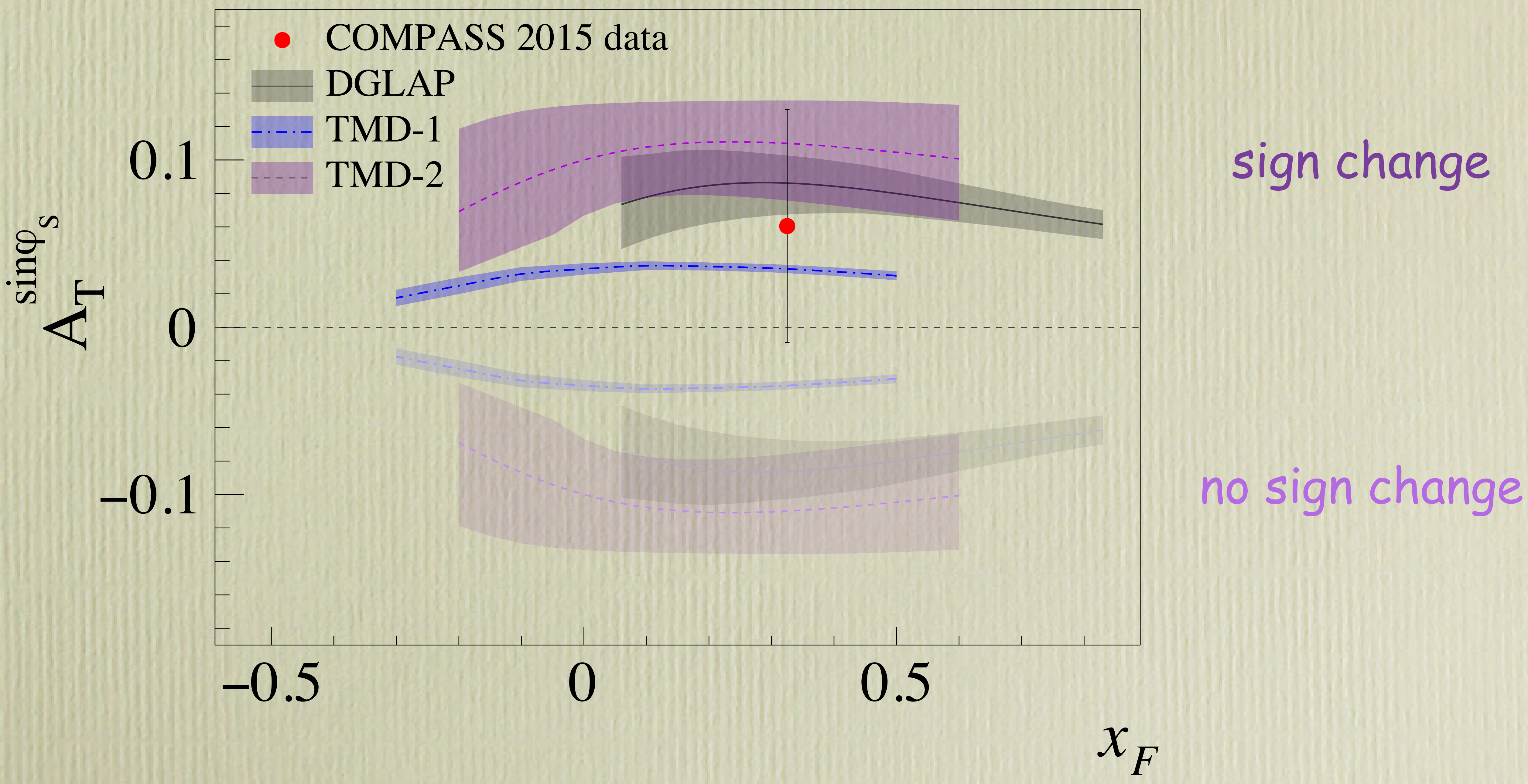
STAR Collaboration, PRL 116 (2016) 132301



prediction with sign change

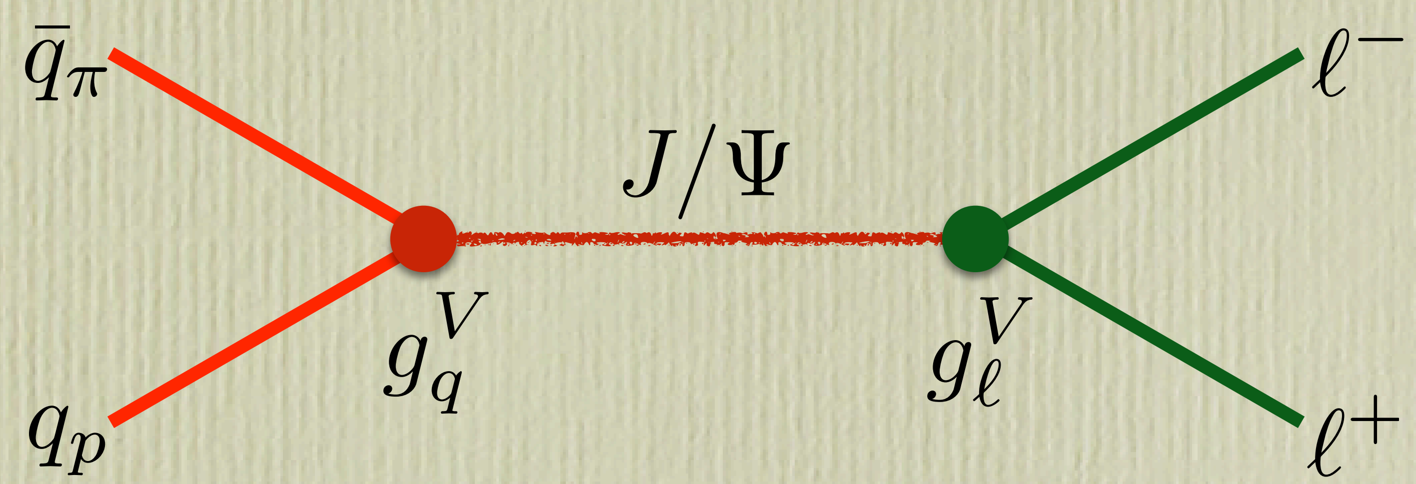
Sivers asymmetry in DY at COMPASS

arXiv:1704.00488



Sivers asymmetry in $\pi\, p^\uparrow \to J/\Psi\, X \to e^+ e^- X$ at COMPASS

M.A., V. Barone, M. Boglione, PLB 770 (2017) 302



$$16\pi^2\alpha^2 e_q^2 \to (g_q^V)^2\,(g_\ell^V)^2$$

$$\frac{1}{M^4} \to \frac{1}{(M^2 - M_V^2)^2 + M_V^2\,\Gamma_V^2}$$

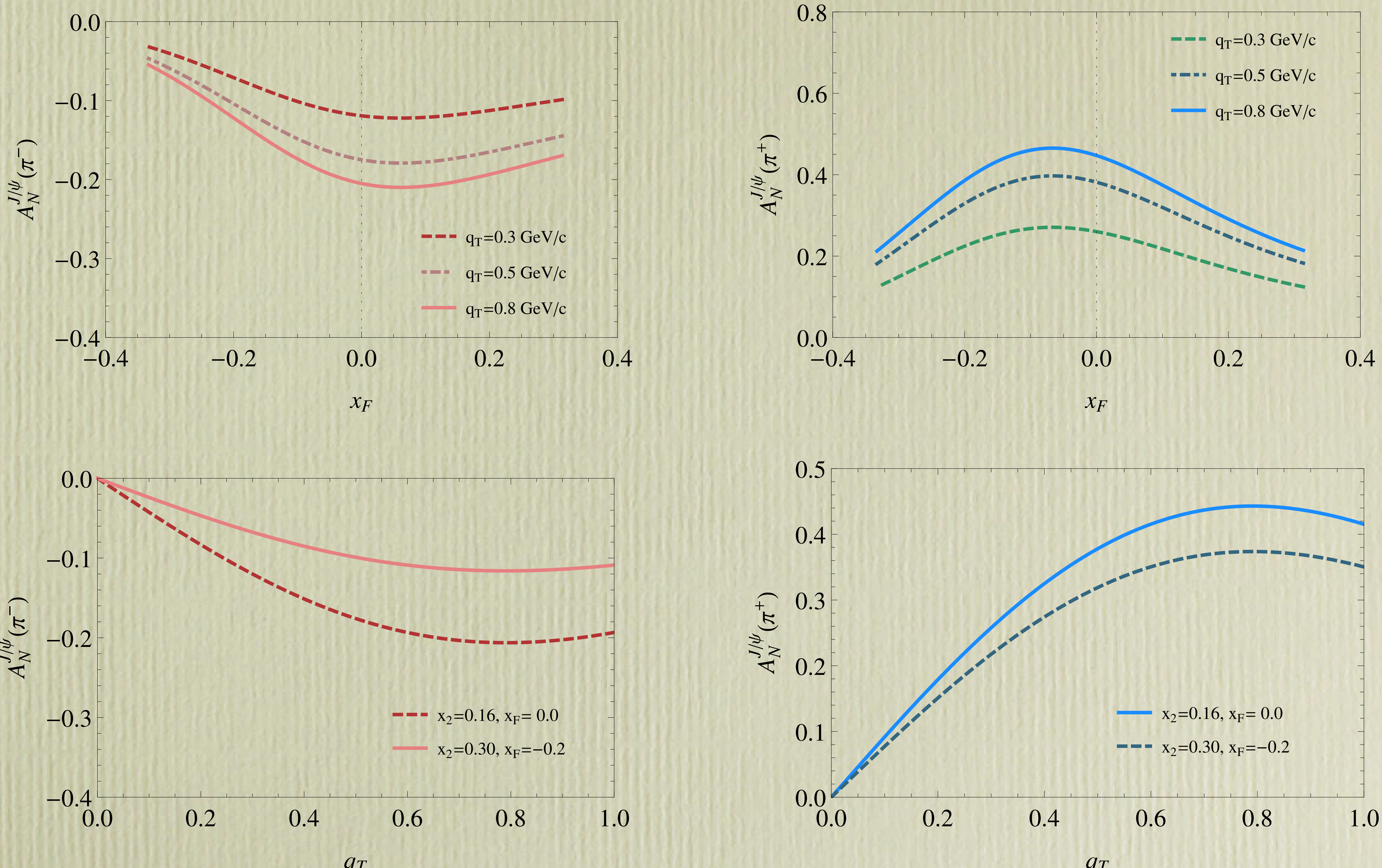
in central region $x_1 \simeq x_2 \simeq 0.16$

same as usual D-Y with

$$x_{1,2} = \frac{M}{\sqrt{s}}\, e^{\pm y} \qquad x_F = \frac{2\,q_L}{\sqrt{s}} = x_1 - x_2 = \left(x_1 - \frac{M^2}{s\,x_1}\right) = \left(\frac{M^2}{s\,x_2} - x_2\right)$$

$$A_N^{J/\Psi}(\boxed{\pi^-}\,x_1, x_2, \boldsymbol{q}_T) \simeq \frac{\int d^2\boldsymbol{k}_{\perp 1}\, d^2\boldsymbol{k}_{\perp 2}\, \delta^2(\boldsymbol{k}_{\perp 1} + \boldsymbol{k}_{\perp 2} - \boldsymbol{q}_T)\, \boldsymbol{S}\cdot(\hat{\boldsymbol{p}}_2 \times \hat{\boldsymbol{k}}_{\perp 2})\, f_{\bar{u}/\pi^-}(x_1, k_{\perp 1})\, \boxed{\Delta^N f_{u/p^\uparrow}}(x_2, k_{\perp 2})}{2\int d^2\boldsymbol{k}_{\perp 1}\, d^2\boldsymbol{k}_{\perp 2}\, \delta^2(\boldsymbol{k}_{\perp 1} + \boldsymbol{k}_{\perp 2} - \boldsymbol{q}_T)\, f_{\bar{u}/\pi^-}(x_1, k_{\perp 1})\, f_{u/p}(x_2, k_{\perp 2})}$$

$$A_N^{J/\Psi}(\boxed{\pi^+}\,x_1, x_2, \boldsymbol{q}_T) \simeq \frac{\int d^2\boldsymbol{k}_{\perp 1}\, d^2\boldsymbol{k}_{\perp 2}\, \delta^2(\boldsymbol{k}_{\perp 1} + \boldsymbol{k}_{\perp 2} - \boldsymbol{q}_T)\, \boldsymbol{S}\cdot(\hat{\boldsymbol{p}}_2 \times \hat{\boldsymbol{k}}_{\perp 2})\, f_{\bar{d}/\pi^+}(x_1, k_{\perp 1})\, \boxed{\Delta^N f_{d/p^\uparrow}}(x_2, k_{\perp 2})}{2\int d^2\boldsymbol{k}_{\perp 1}\, d^2\boldsymbol{k}_{\perp 2}\, \delta^2(\boldsymbol{k}_{\perp 1} + \boldsymbol{k}_{\perp 2} - \boldsymbol{q}_T)\, f_{\bar{d}/\pi^+}(x_1, k_{\perp 1})\, f_{d/p}(x_2, k_{\perp 2})}$$



predictions with SIDIS extracted Sivers functions, with sign change

large asymmetries, worth measuring

about the Sivers effect:

it deeply probes the internal momentum structure of the nucleon; it is experimentally well established with first extraction of the Sivers function ...

it must be related to (valence) parton orbital motion ...

it might be related to QCD gauge links and our current understanding of TMD factorization ...

it is crucial to test its sign change and its universality

Thank you !