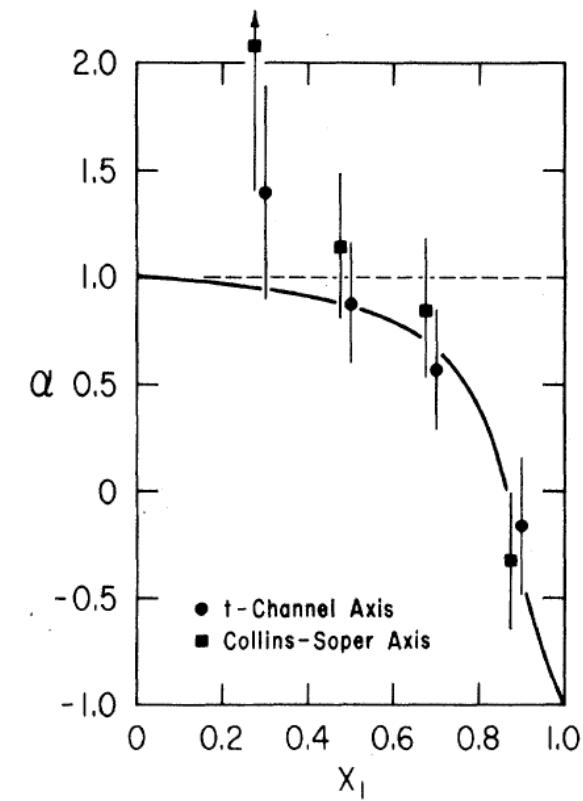
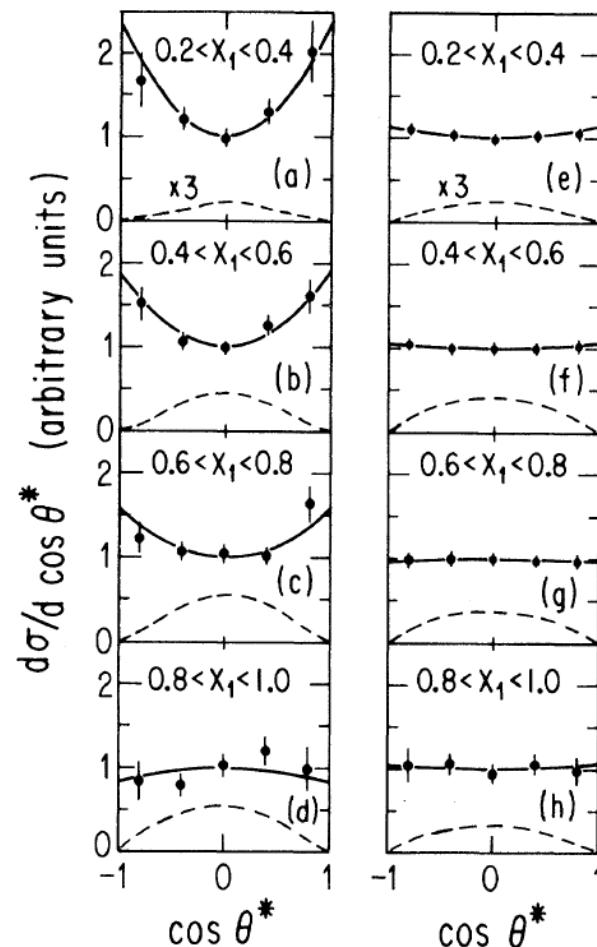


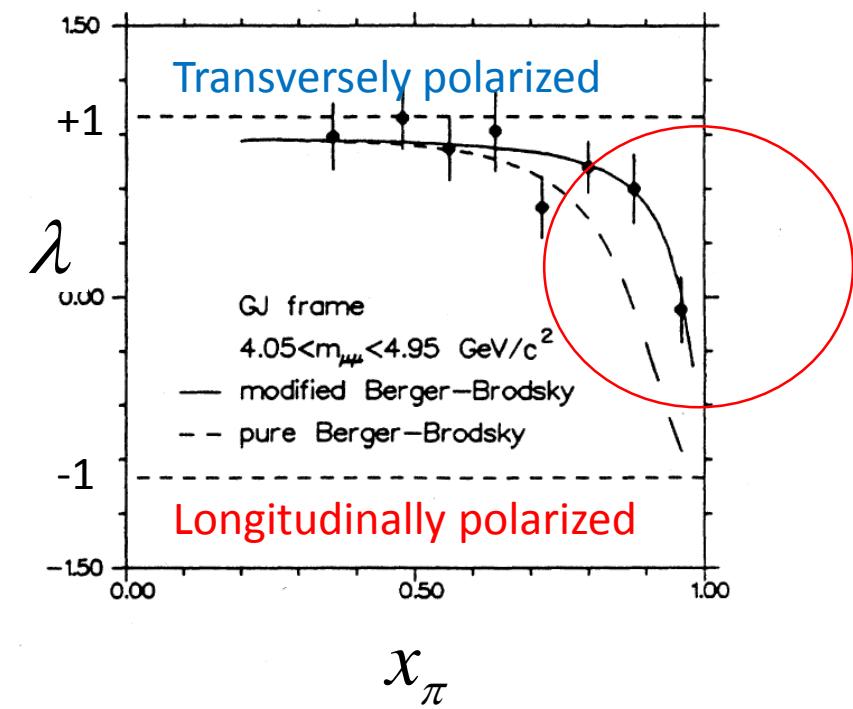
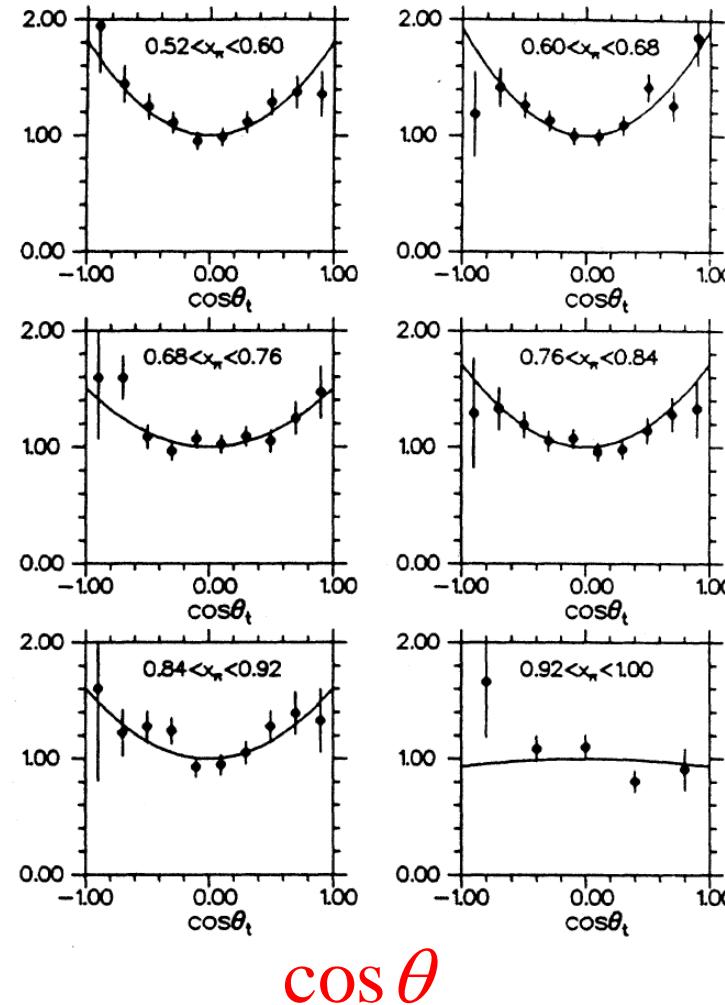
# Discussion Session: When “**Inclusive**” meets “**Exclusive**” - Exclusive Hard Process

November 8, 2017

# CIP (PRL 43, 1219, (1979)) : Longitudinally Polarized Photon at large $x_1$



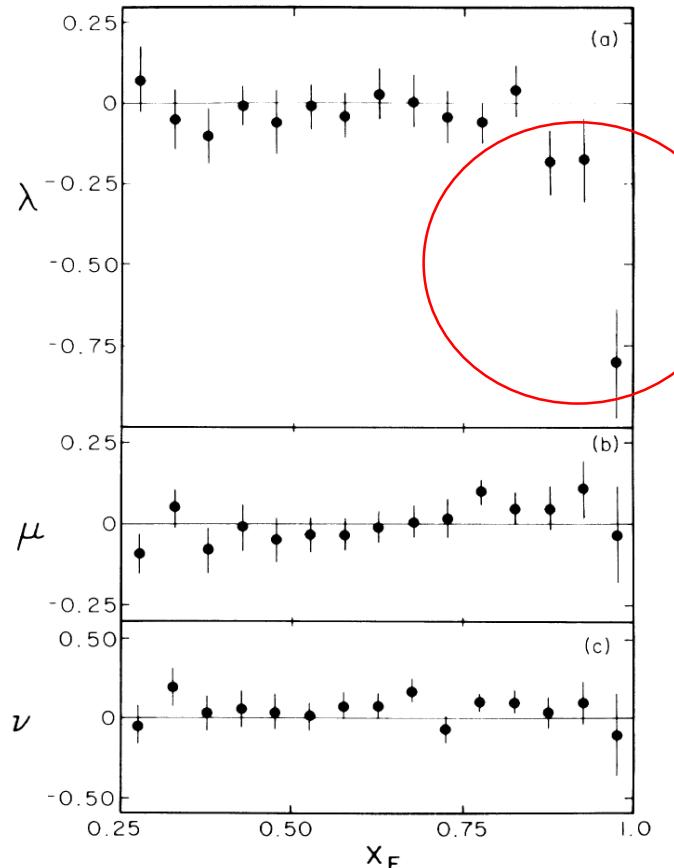
# E615 (PRD 39, 92 (1989)): Higher Twist Effect at large $x_\pi$



# CIP (PRL 58, 2523 (1987))

$\pi W \rightarrow J/\psi X$

Sign of  $q\bar{q}$  annihilation dominating?



$$d^2\sigma/d\cos\theta d\phi \propto 1 + \lambda \cos^2\theta + \mu \sin 2\theta \cos\phi + \frac{1}{2} \nu \sin^2\theta \cos 2\phi.$$

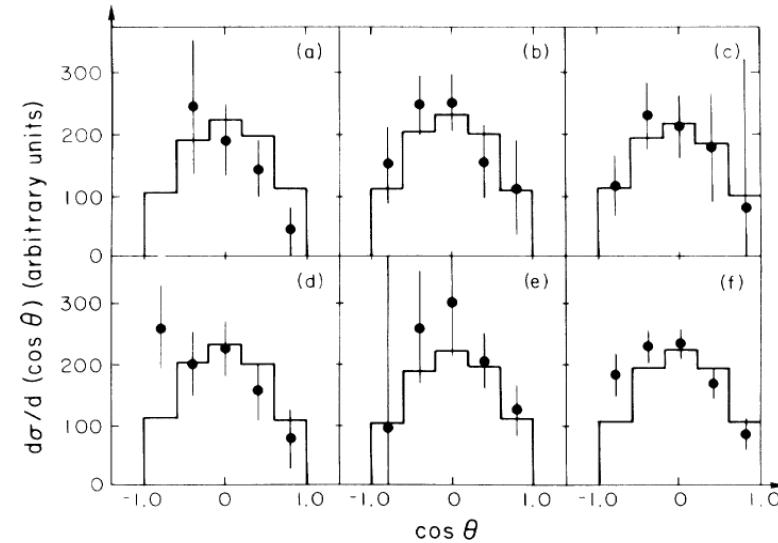
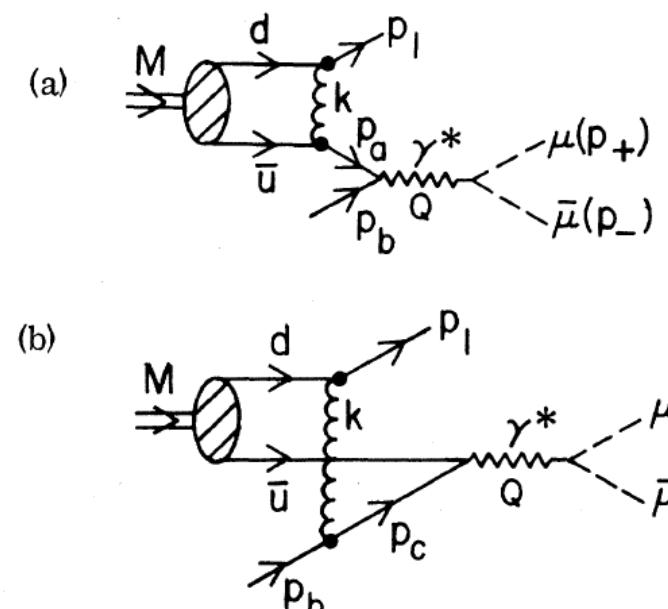
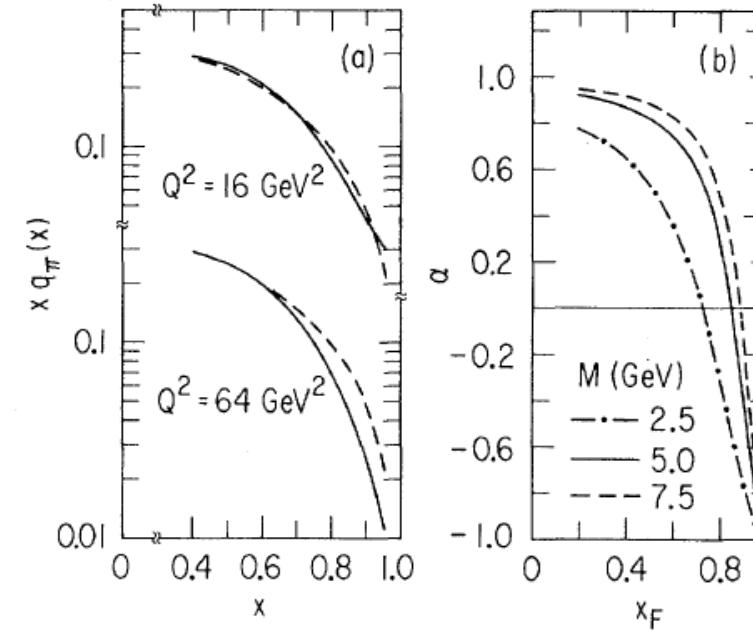


FIG. 4. The  $J/\psi$  decay angular distribution vs  $\cos\theta$  for the five regions of  $\phi$ , and summed over all  $\phi$  in the highest  $x_F$  bin,  $0.95 < x_F < 1.0$ . The histograms are the result of the fit described in the text. (a)  $-\pi < \phi < -0.6\pi$ , (b)  $-0.6\pi < \phi < -0.2\pi$ , (c)  $-0.2\pi < \phi < 0.2\pi$ , (d)  $0.2\pi < \phi < 0.6\pi$ , (e)  $0.6\pi < \phi < \pi$ , (f)  $-\pi < \phi < \pi$ .

Berger and Brodsky (PRL 42, 940, (1979)) :  
 Higher Twist Effect at large  $x_\pi$



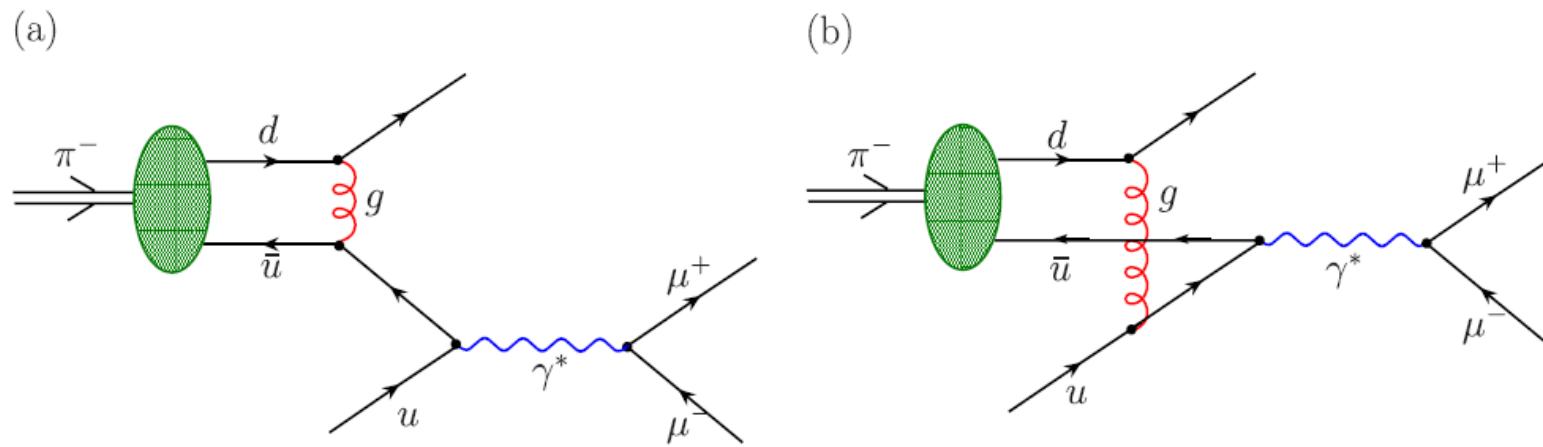
$$d\sigma \propto (1 + \alpha \cos^2 \theta)$$



$$d\sigma \propto (1 - x_\pi)^2 (1 + \cos^2 \theta) + \frac{4x_\pi^2 \langle k_T^2 \rangle}{9m_{\mu\mu}^2} \sin^2 \theta$$

# Brandenburg et al. (PRL 73, 939 (1994))

## Higher-twist Effect & Pion Distribution Amplitude



$$\frac{Q^2 d\sigma(\pi^- N \rightarrow \mu^+ \mu^- X)}{dQ^2 dQ_T^2 dx_L d\Omega} = \frac{1}{(2\pi)^4} \frac{1}{64} \int_0^1 dx_u G_{u/N}(x_u) \int_0^1 dx_{\bar{u}} \frac{x_{\bar{u}}}{1 - x_{\bar{u}} + Q_T^2/Q^2} |M|^2$$

$$\times \delta(x_L - x_{\bar{u}} + x_u - Q_T^2 s^{-1} (1 - x_{\bar{u}})^{-1})$$

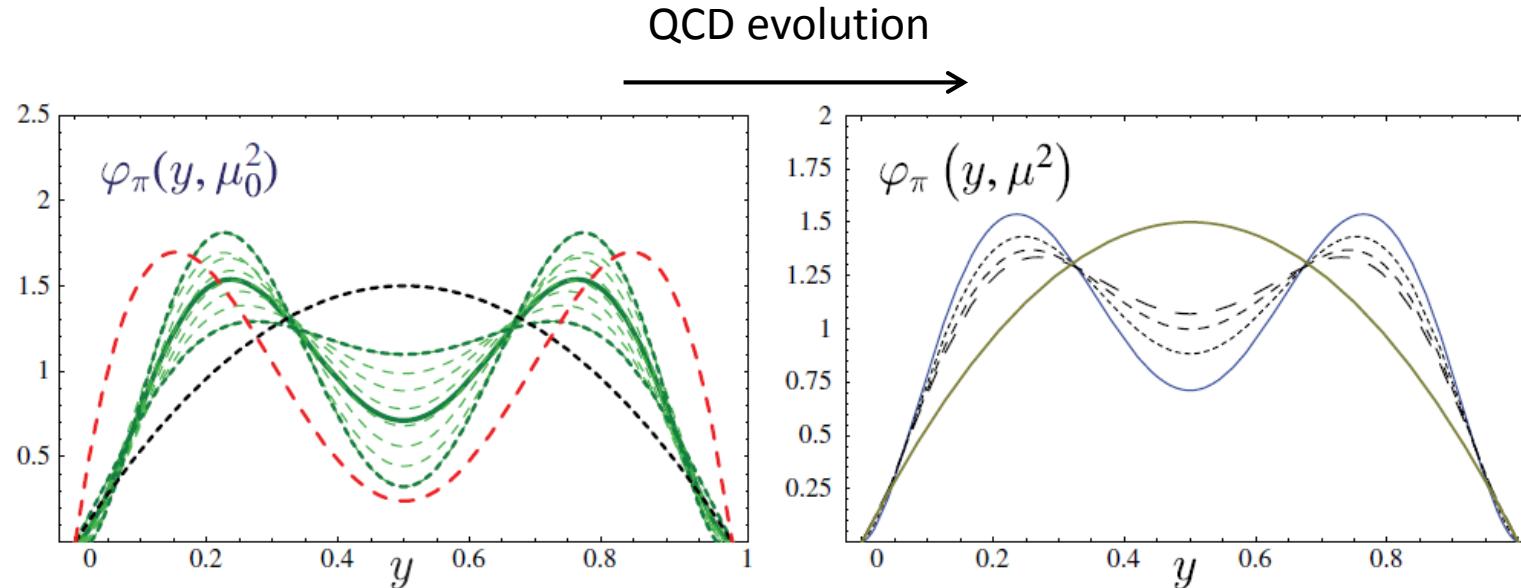
$$\times \delta(Q^2 - sx_u x_{\bar{u}} + Q_T^2 (1 - x_{\bar{u}})^{-1}) + \{u \rightarrow \bar{d}, \bar{u} \rightarrow d\}.$$

$\nearrow$

**Pion distribution amplitude:** distribution of LC momentum fractions in the lowest-particle number valence Fock state.

A. P. Bakulev, N. G. Stefanis, and O. V. Teryaev  
(Phys. Rev. D 76, 074032 (2007))

# Pion Distribution Amplitude

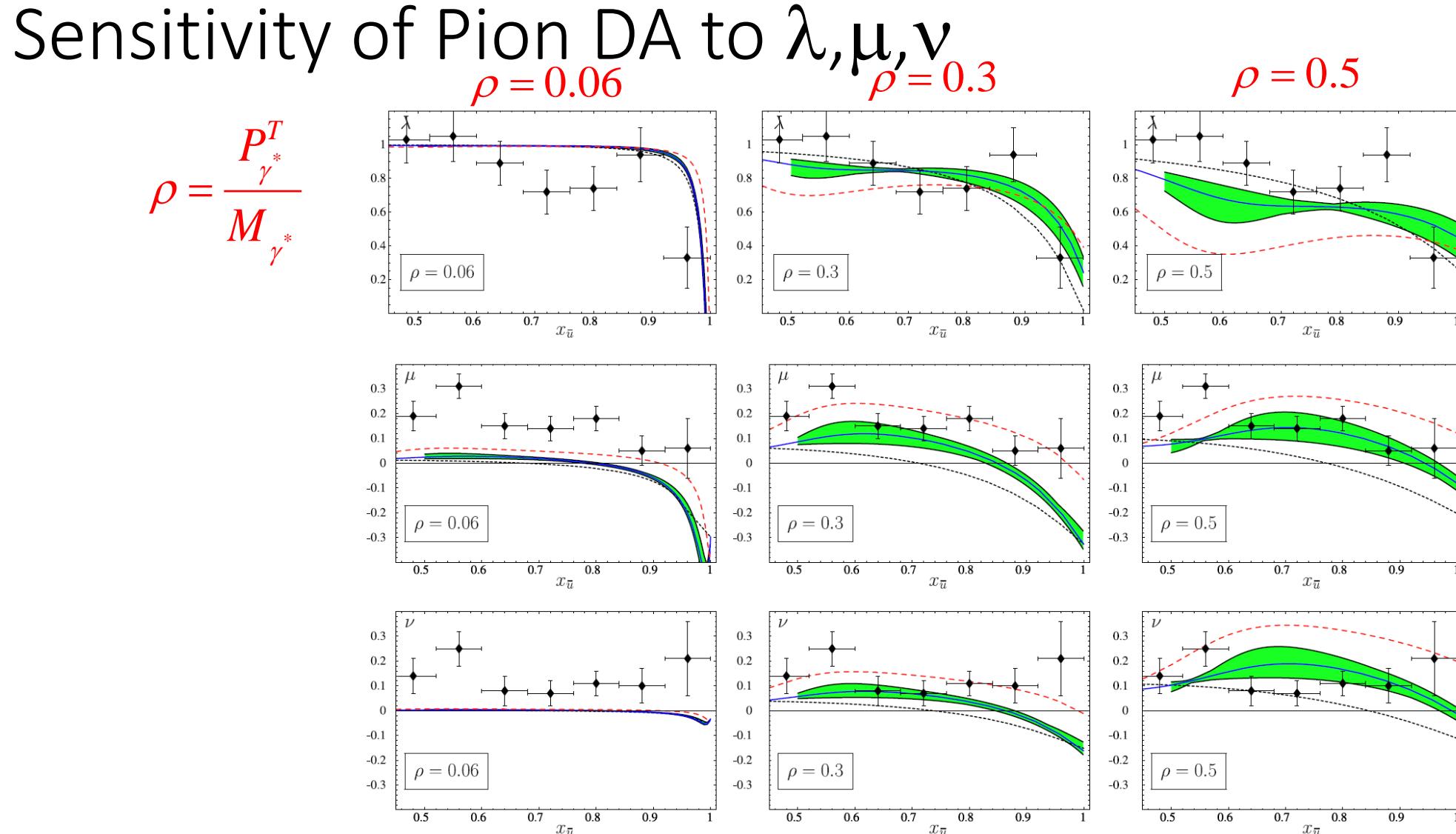


asymptotic-like form =  $6y(1-y)$

Chernyak-Zhitnitsky (CZ)-like form

Nonlocal QCD sum rules

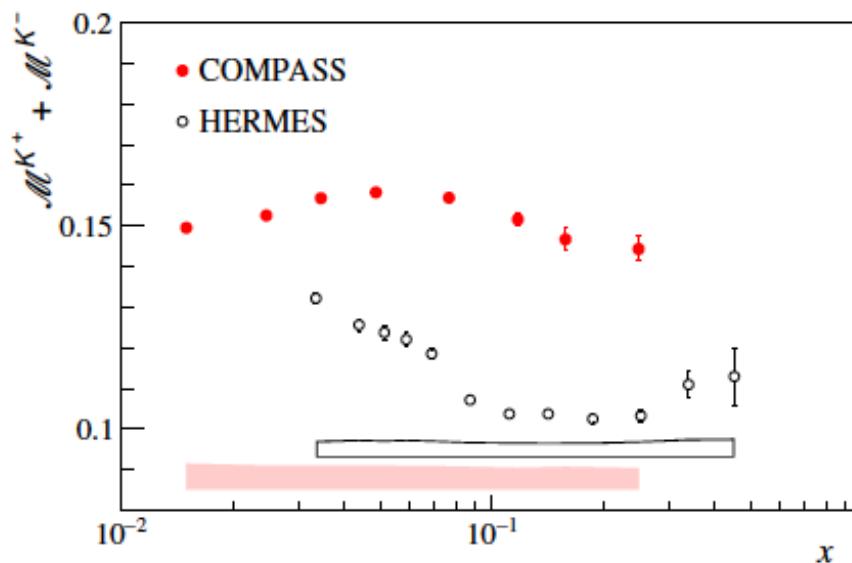
A. P. Bakulev, N. G. Stefanis, and O. V. Teryaev  
 (Phys. Rev. D 76, 074032 (2007))



# SIDIS (kaon) multiplicities

Charged kaon multiplicities (2006 160 GeV  ${}^6\text{LiD}$ ) – published in [PLB 767 \(2017\) 133](#)

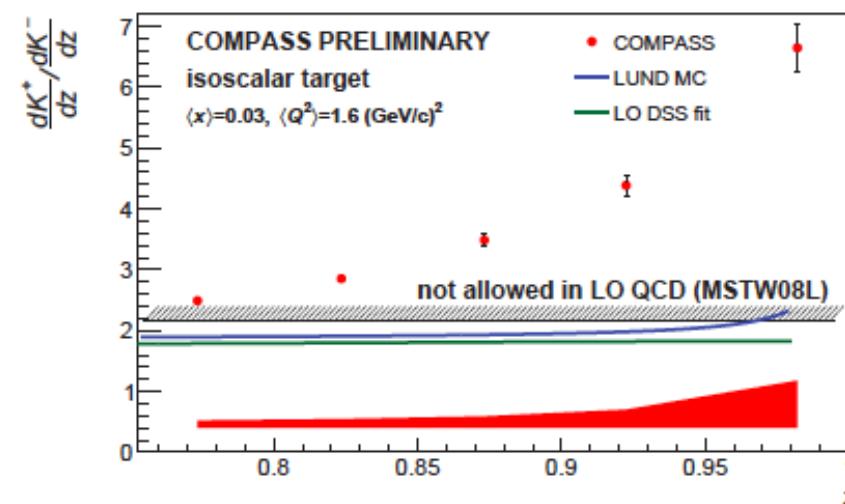
The 3-dimensional data set ( $x$ ,  $y$  and  $z$ ) → an important input for future NLO pQCD analyses of world data in terms of FFs.

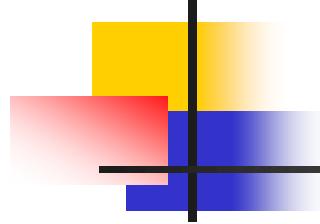


Recently new results were produced on the kaon multiplicity ratio  $K^+/K^-$ , at high  $z$ ,  $0.75 < z < 1$ . Surprisingly our data go far beyond the LO upper boundary value of  $(u+d)/(\bar{u}+\bar{d})$  calculated at  $x=0.03$  using [MSTW08L](#) as well as beyond the actual predictions of the  $K^+/K^-$  multiplicity ratio using Lund model or LO [DSS](#) fit.

Important message – HERMES and COMPASS data are in tension.  
Can not be explained only by different  $Q^2$  range, the discussion is going on.

**Large  $z$ ,  $\mu p \rightarrow \mu K \Lambda$**



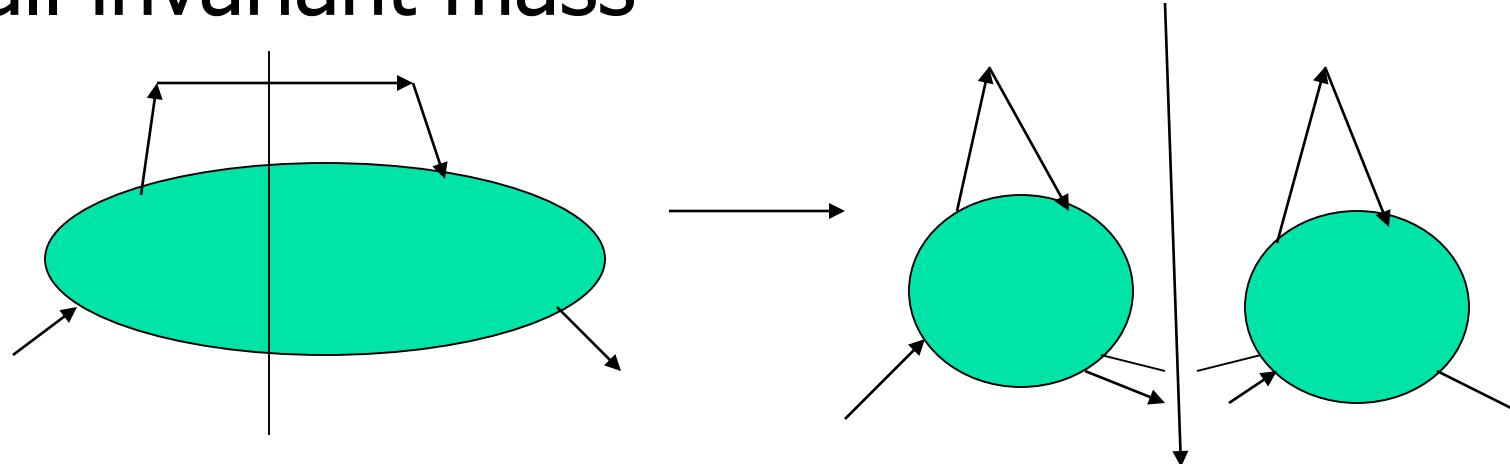


# Exclusive large x limit

- Consider the dilepton carrying the most of collision energy; small number of hadrons in the central region; correspond to large x of pdf's
- DIS – Bloom Gilman duality, Drell-Yan-West relations
- Is there any analog for DY?

# Exclusive limit : DIS and space-like (transitional and elastic) FFs

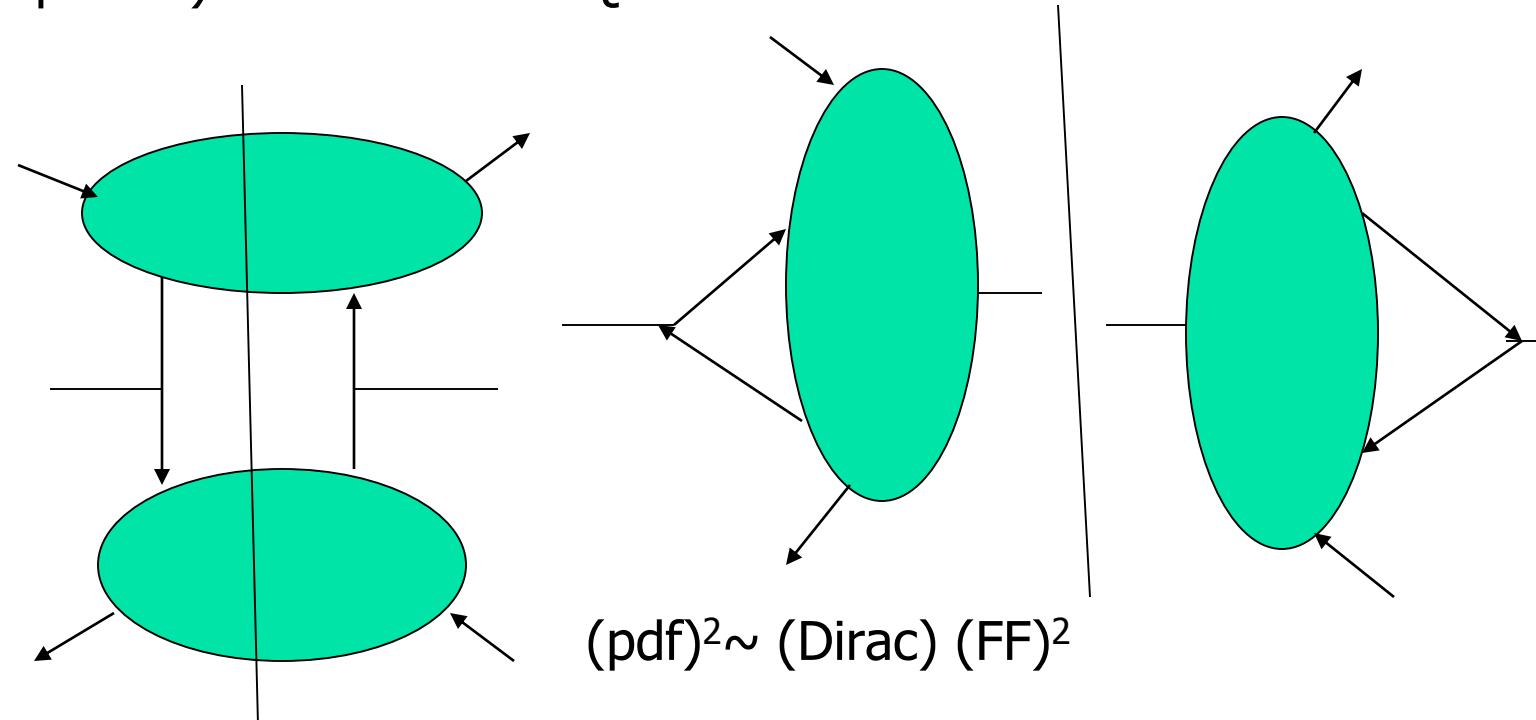
- Small invariant mass



- BG duality and DYW relation
- Relation between  $x \rightarrow 1$  and large  $Q^2$
- $\text{pdf} \sim (\text{FF})^2$

# Exclusive limit of DY and time-like FFs (OT'14)

- (Proton-antiproton) DY at small  $s - Q^2$

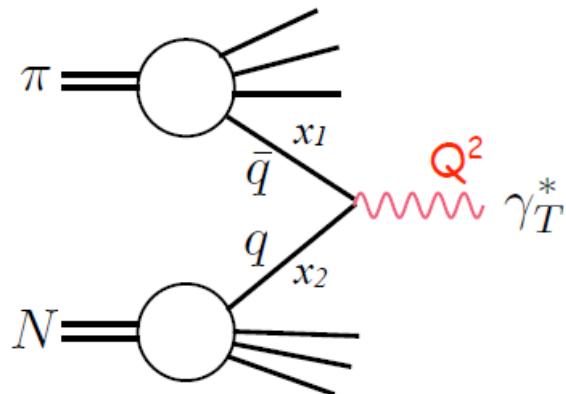


- Other beams – baryon number conservation – time-like transition FFs

P. Hoyer, M. Jrvinen and S. Kurki,  
JHEP 0810 (2008) 086, arXiv: 0808.0626

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Drell-Yan in the Bj limit:  $Q^2 \rightarrow \infty$  at fixed  $x$



$$Q^2 = x_1 x_2 s \rightarrow \infty$$

$$x_1, x_2; x_F = x_1 - x_2 \quad \text{fixed}$$

Transversely polarized photon,  
since quarks are  $\sim$  on-shell

Leading twist: One active parton in beam and target hadrons

Spectators are incoherent with the hard subprocess

$$\text{Factorization: } \sigma = f_{\bar{q}/\pi}(x_1) f_{q/N}(x_2) \hat{\sigma}(\bar{q}q \rightarrow \gamma^*)$$

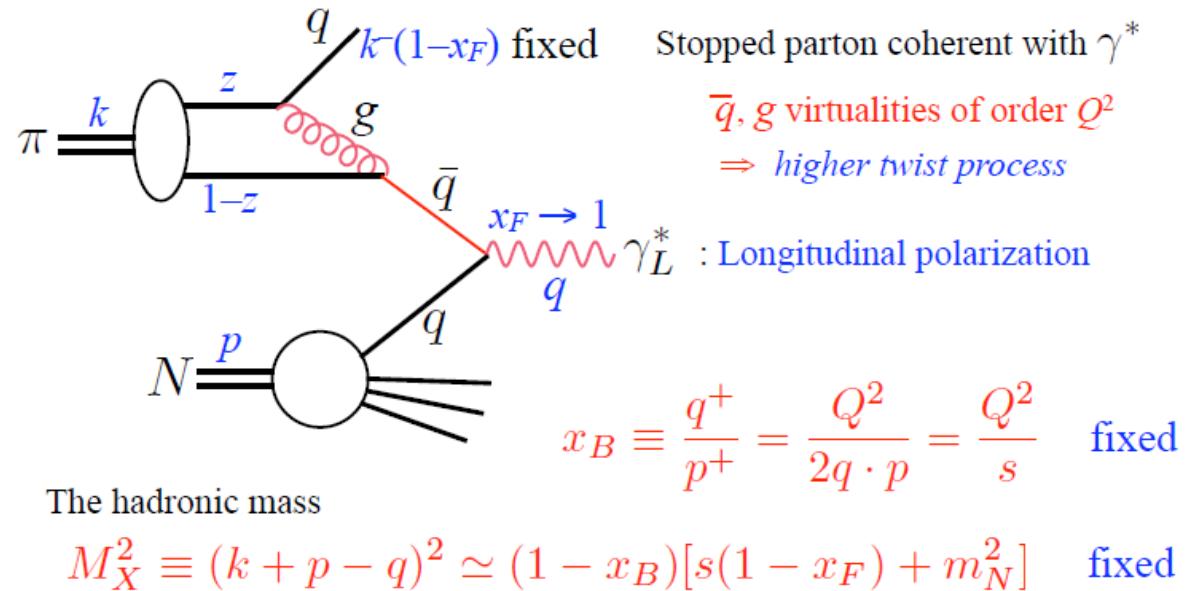
$$\text{Higher twist corrections are of order } \frac{1}{Q^2} \frac{1}{1-x}$$

Paul Hoyer Krakow January 6, 2009

P. Hoyer, M. Jrvinen and S. Kurki,  
JHEP 0810 (2008) 086, arXiv: 0808.0626

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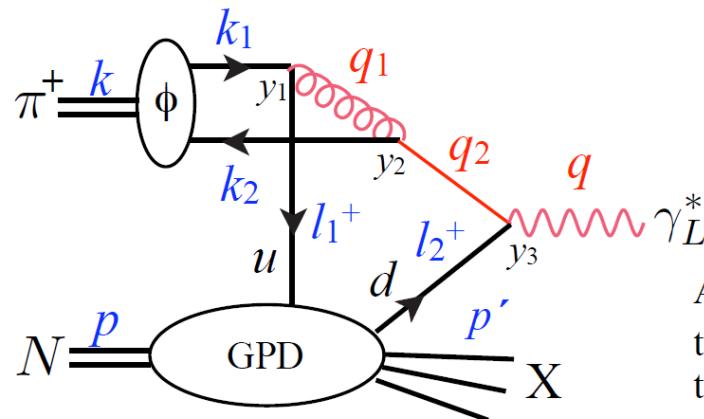
Drell-Yan in the BB limit:  $Q^2 \rightarrow \infty$  at fixed  $Q^2(1-x_F)$



Stopped quark is comoving with the target.  
Its interactions in the target affect the hard subprocess.

# P. Hoyer, M. Jrvinen and S. Kurki, JHEP 0810 (2008) 086, arXiv: 0808.0626

Hence the stopped quark should  
be connected to the target:



For each final state X the target matrix element is given by a GPD with skewness

$$l_2^+ - l_1^+ = q^+ = x_B p^+$$

$$\begin{aligned} k_1 &= (0^+, zk^-, \mathbf{k}_\perp) \\ k_2 &= (0^+, (1-z)k^-, -\mathbf{k}_\perp) \end{aligned}$$

Since  $q_1^2 \approx -zk^- l_1^+ \rightarrow \infty$   
the pion wave function contributes  
through its *distribution amplitude*  $\phi$

Also  $q_2^2, q_1^-, q_2^- \rightarrow \infty$ , hence  
the space-time separation of the  
target interaction points  $y_1, y_3$  is

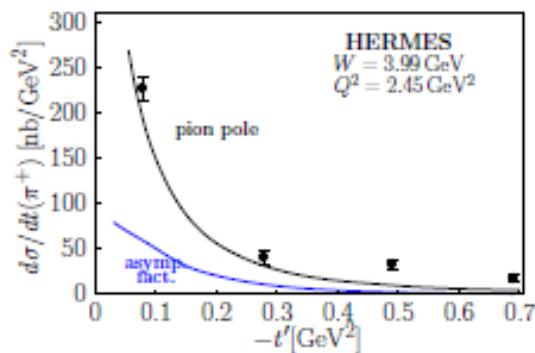
$$\begin{aligned} |y_{1\perp} - y_{3\perp}| &= \mathcal{O}(1/Q) \rightarrow 0 \\ |y_1^+ - y_3^+| &= \mathcal{O}(1/Q^2) \rightarrow 0 \\ |y_1^- - y_3^-| &= \mathcal{O}(1/\ell_1^+) \text{ finite} \end{aligned}$$

Using perturbative propagators for the gluon  $q_1$   
and  $d$ -quark  $q_2$  and adding three more diagrams we get

# The pion pole

For  $\pi^+$  production - pion pole:

(Mankiewicz et al (98), Penttinen et al (99))



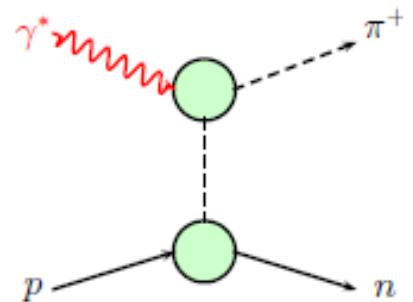
$$\tilde{E}_{\text{pole}}^u = -\tilde{E}_{\text{pole}}^d = \Theta(|x| \leq \xi) \frac{m f_\pi g_{\pi NN}}{\sqrt{2}\xi} \frac{F_{\pi NN}(t)}{m_\pi^2 - t} \Phi_\pi\left(\frac{x + \xi}{2\xi}\right)$$

$$\Rightarrow \frac{d\sigma_L^{\text{pole}}}{dt} \sim \frac{-t}{Q^2} \left[ \sqrt{2} e_0 g_{\pi NN} \frac{F_{\pi NN}(t)}{m_\pi^2 - t} Q^2 F_\pi^{\text{pert}}(Q^2) \right]^2$$

underestimates cross section (blue line)

$$F_\pi^{\text{pert.}} \simeq 0.3 - 0.5 F_\pi^{\text{exp.}}$$

(note:  $F_\pi$  measured in  $\pi^+$  electroproduction at Jlab)



Goloskokov-K(09):  $F_\pi^{\text{pert}} \rightarrow F_\pi^{\text{exp}}$

as one-pion-exchange contr.

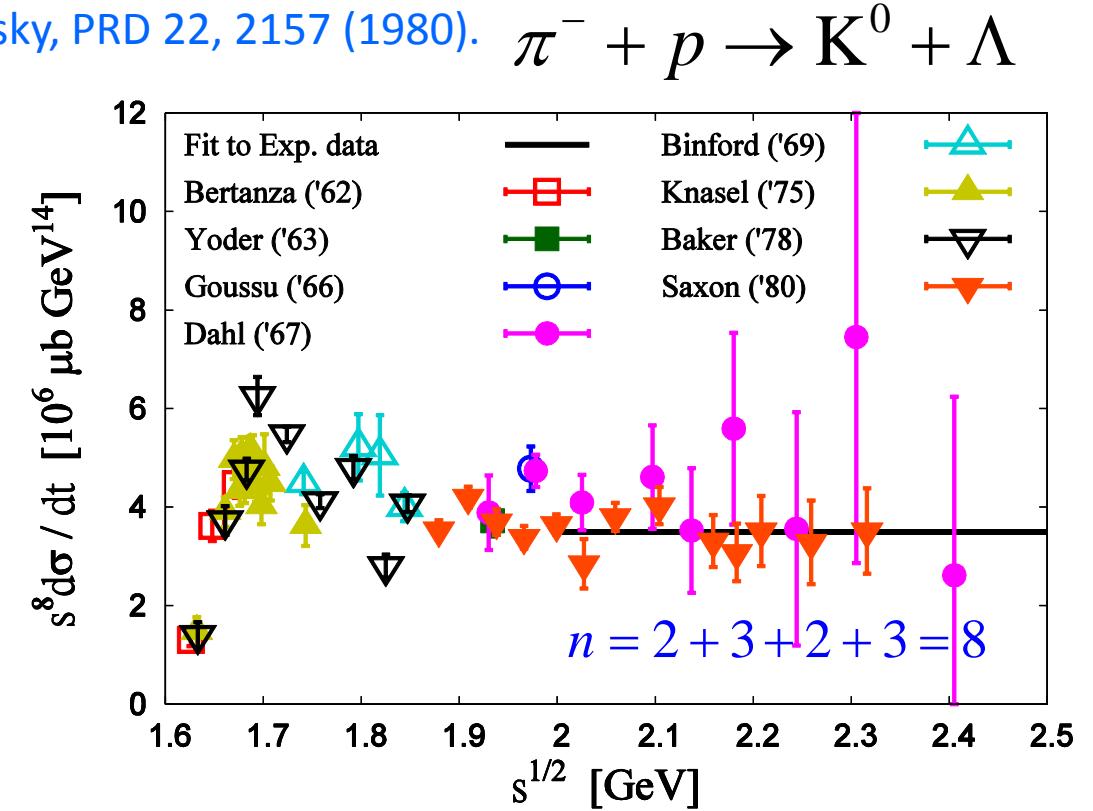
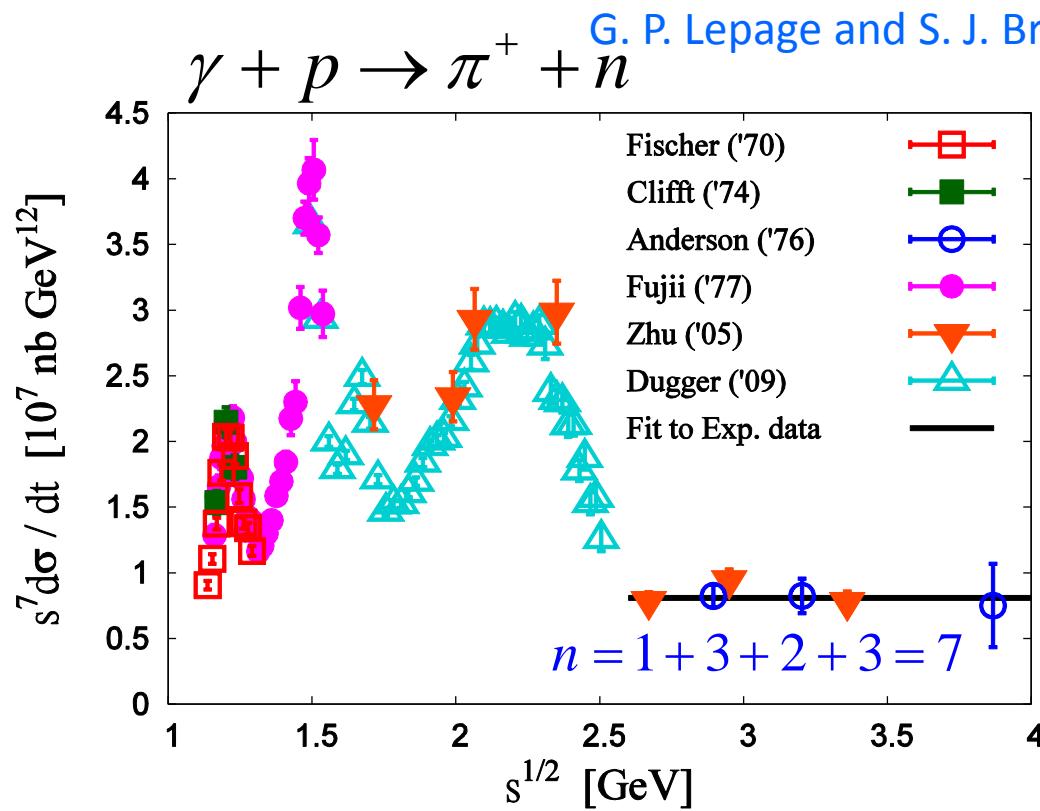
knowledge of the sixties suffices to explain

$\pi^+$  data at small  $-t$

(detailed comparison Favart et al (16))

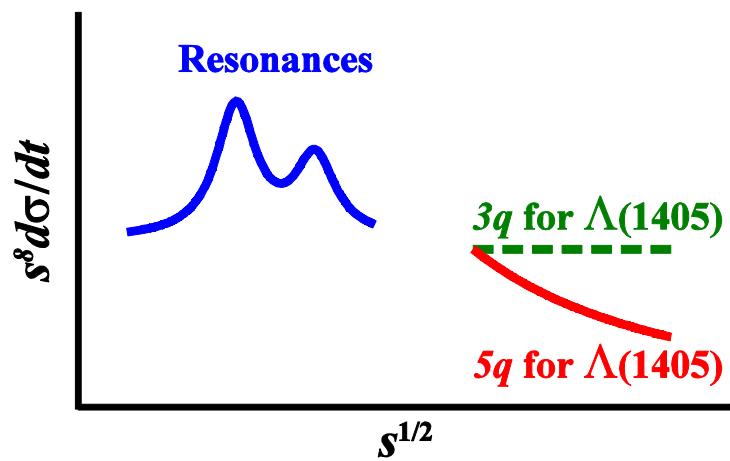
# Constituent-Counting Rule in Hard Exclusive Process

$$\frac{d\sigma}{dt}(a+b \rightarrow c+d) = \frac{1}{s^{n-2}} f(\theta_{CM}) \quad n = n_a + n_b + n_c + n_d$$

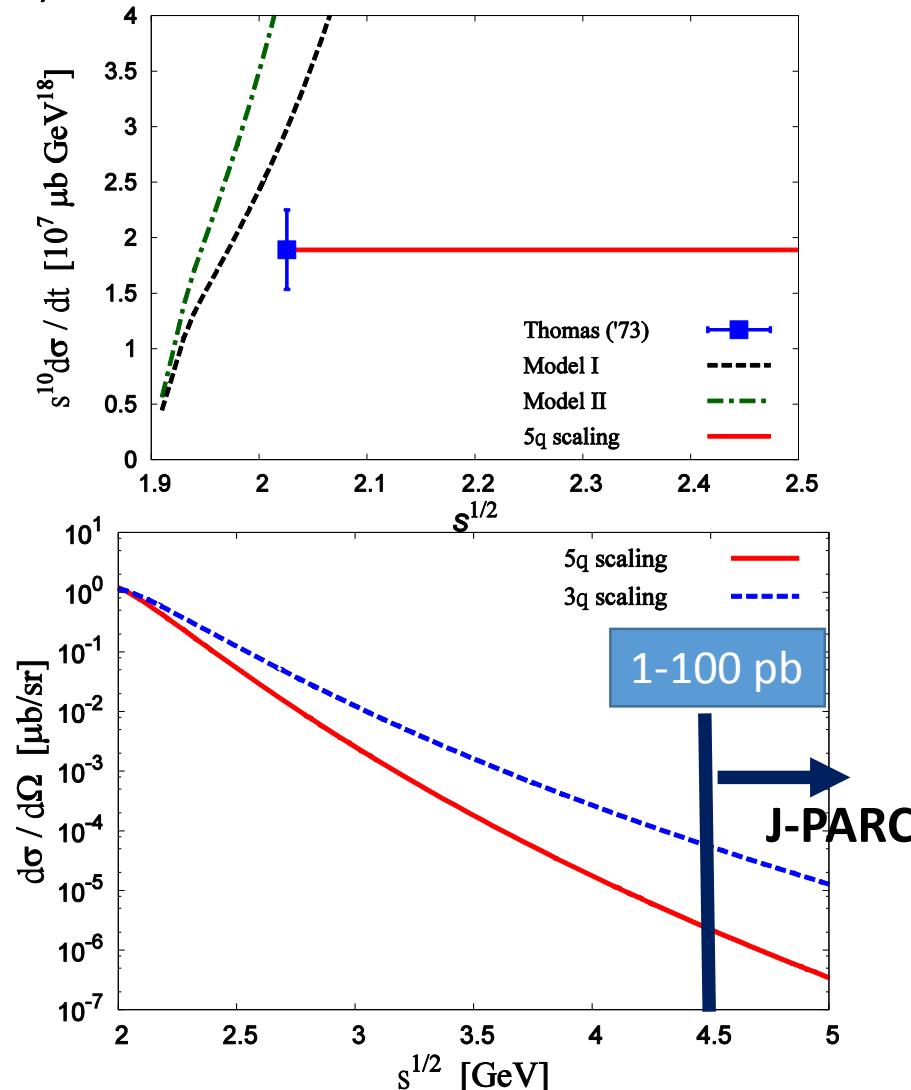


# Quark Degrees of $\Lambda(1405)$

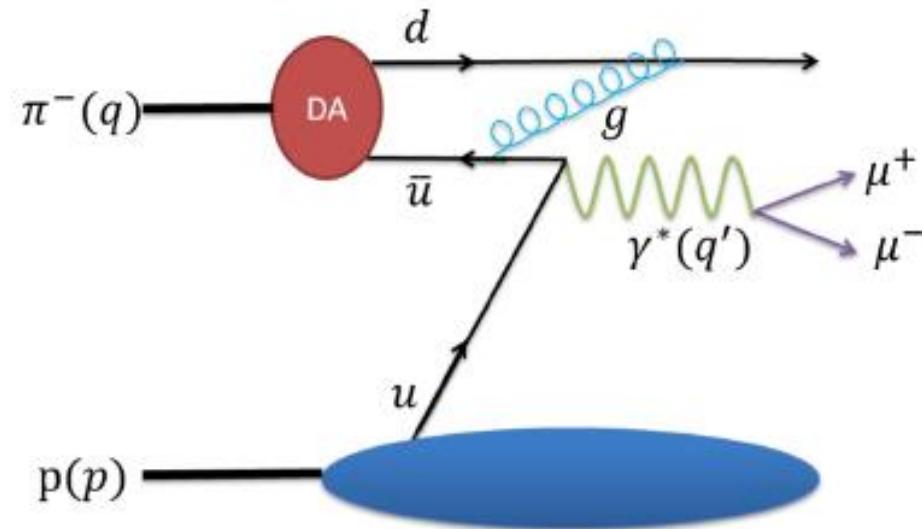
Kawamura et al., PRD 88, 034010 (2013)



T. Sekihara's talk



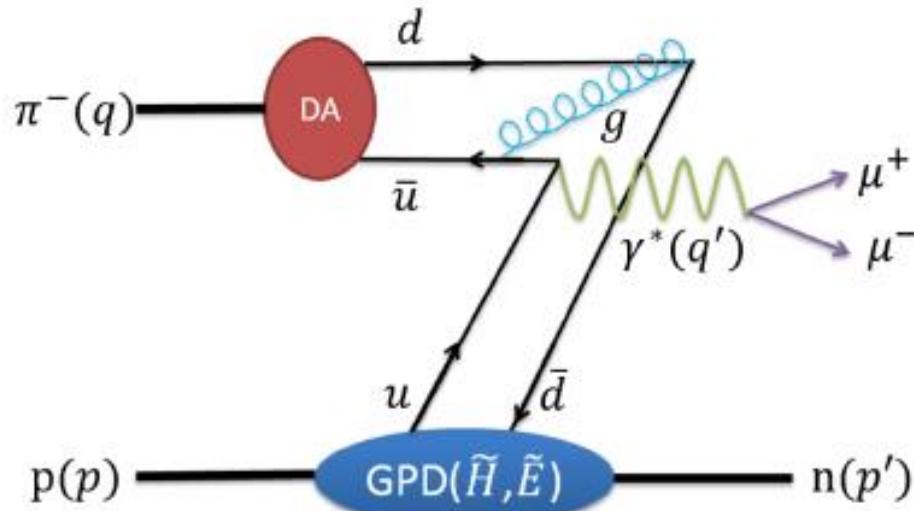
# From Inclusive to Exclusive



(a)

(a) Semi-exclusive pion-induced Drell-Yan process at large  $x_\pi$

$$s, Q^2, x_1(x_F), P_T$$



(b)

(b) Exclusive pion-induced Drell-Yan process.

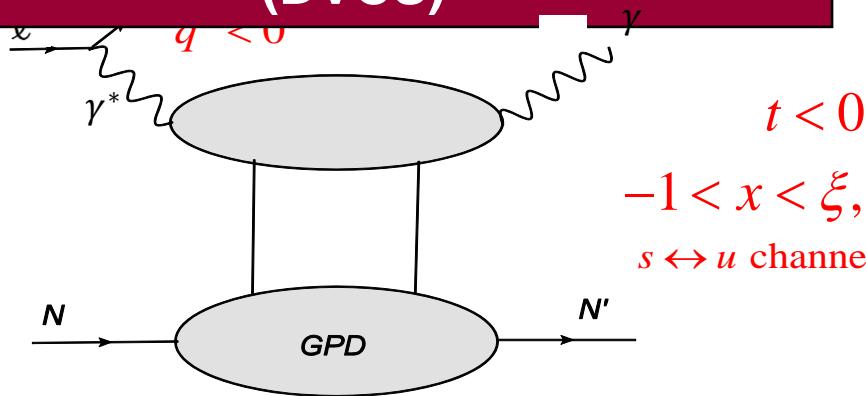
$$s, Q^2, x_1(x_F)=1, P_T \quad s, Q^2, \tau, t$$

# Extraction of GPDs

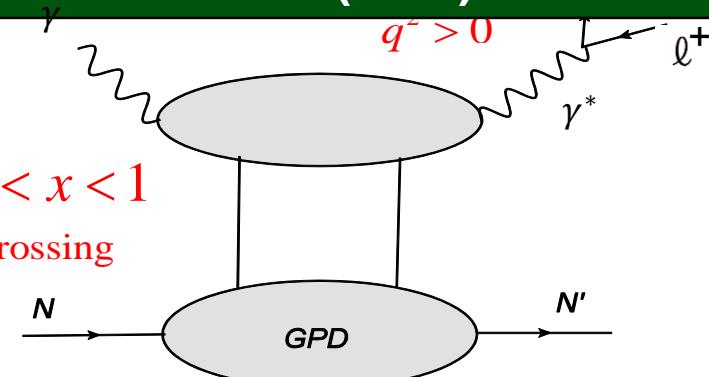
## Space-like vs. Time-like Processes

Muller et al., PRD 86 031502(R) (2012)

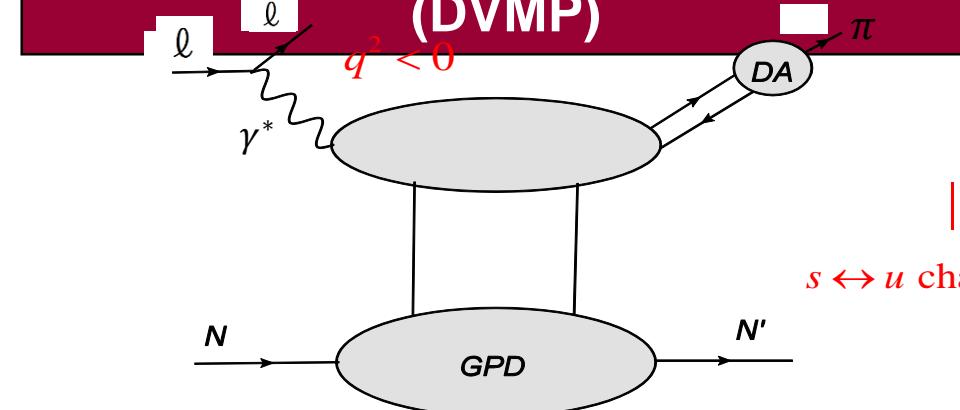
### Deeply Virtual Compton Scattering (DVCS)



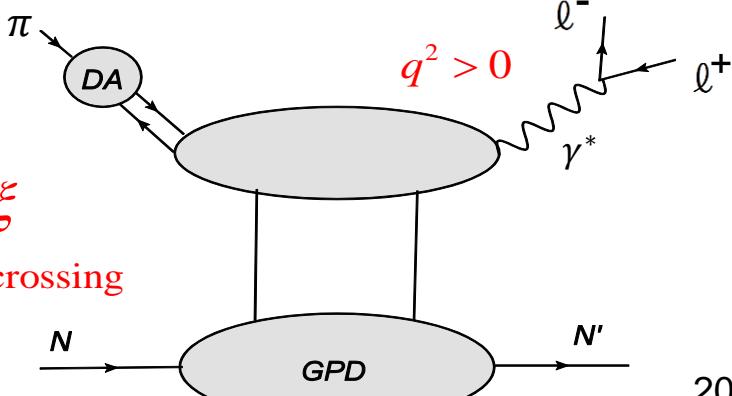
### Time-like Compton Scattering (TCS)



### Deeply Virtual Meson Production (DVMP)



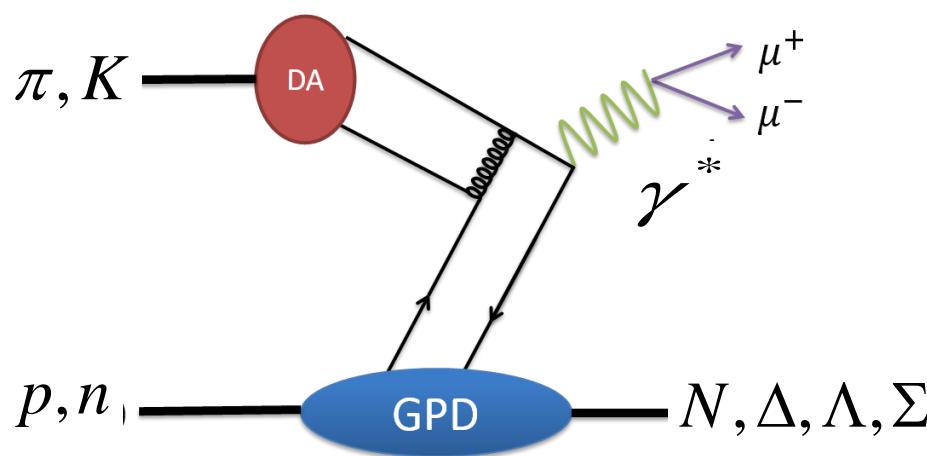
### Exclusive meson-induced DY



# “GPD” and “Transition GPD”

“Transition GPD”: L. L. Frankfurt et al., PRD 60, 014010 (1999)

- $\pi^- p \rightarrow \gamma^* n$
- $\pi^- p \rightarrow \gamma^* \Delta^0$
- $\pi^- n \rightarrow \gamma^* \Delta^-$
- $\pi^+ n \rightarrow \gamma^* p$
- $\pi^+ p \rightarrow \gamma^* \Delta^{++}$
- $\pi^+ n \rightarrow \gamma^* \Delta^+$
- $K^- p \rightarrow \gamma^* \Lambda$
- $K^- p \rightarrow \gamma^* \Lambda(1405)$
- $K^- p \rightarrow \gamma^* \Lambda(1520)$
- $K^- n \rightarrow \gamma^* \Sigma^-$
- $K^+ n \rightarrow \gamma^* \Theta^+$



# A Few Simple Questions

- Should we consider the finite quark mass,  $kT$  or  $pT$  in the exclusive limit?
- Can we learn something from the measurements of semi-inclusive DY events which match those with exclusive DY ones?
- ...