Color flow dependence of azimuthal asymmetries in Drell-Yan

Daniël Boer

Trento, November 10, 2017

university of groningen

Color flow in high energy scattering processes

Factorization and color flow

Theoretical description of high-energy scattering cross sections is based on **factorization** of the perturbative scattering of partons and the nonperturbative distributions of partons

Higgs production: $pp \rightarrow HX$

Color treatment is simple at high energies: separate traces, not dependent on kinematics

But in the actual process there are no colored final states and there are many soft gluons exchanged to balance the color
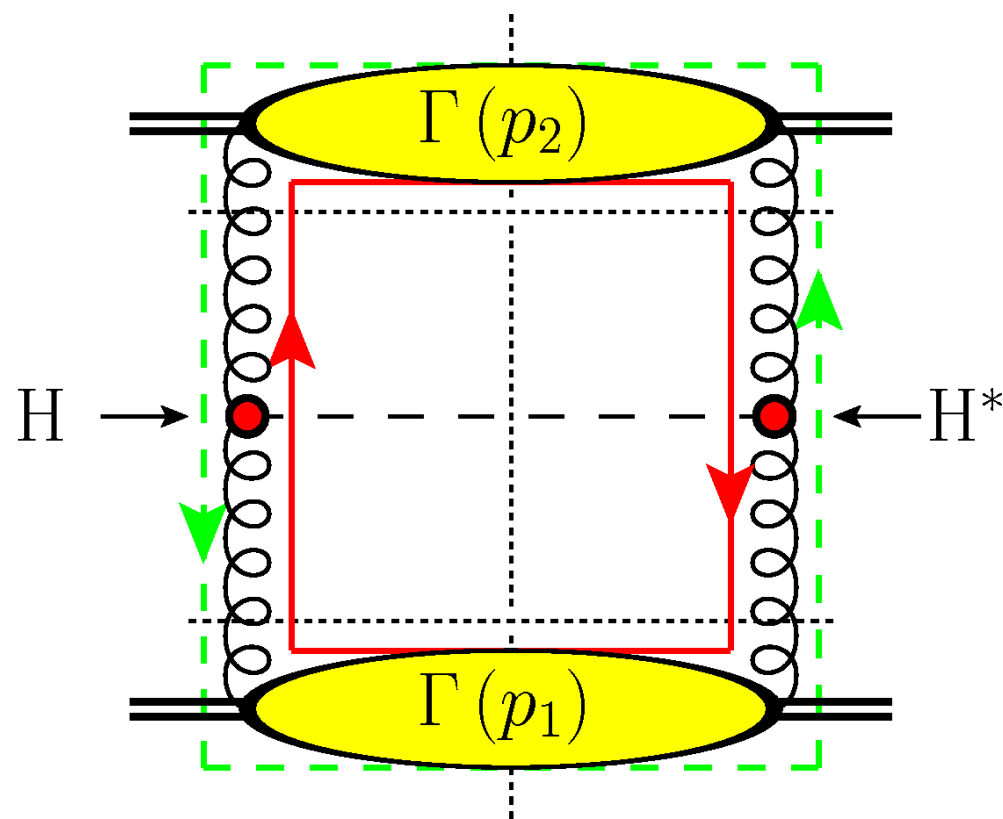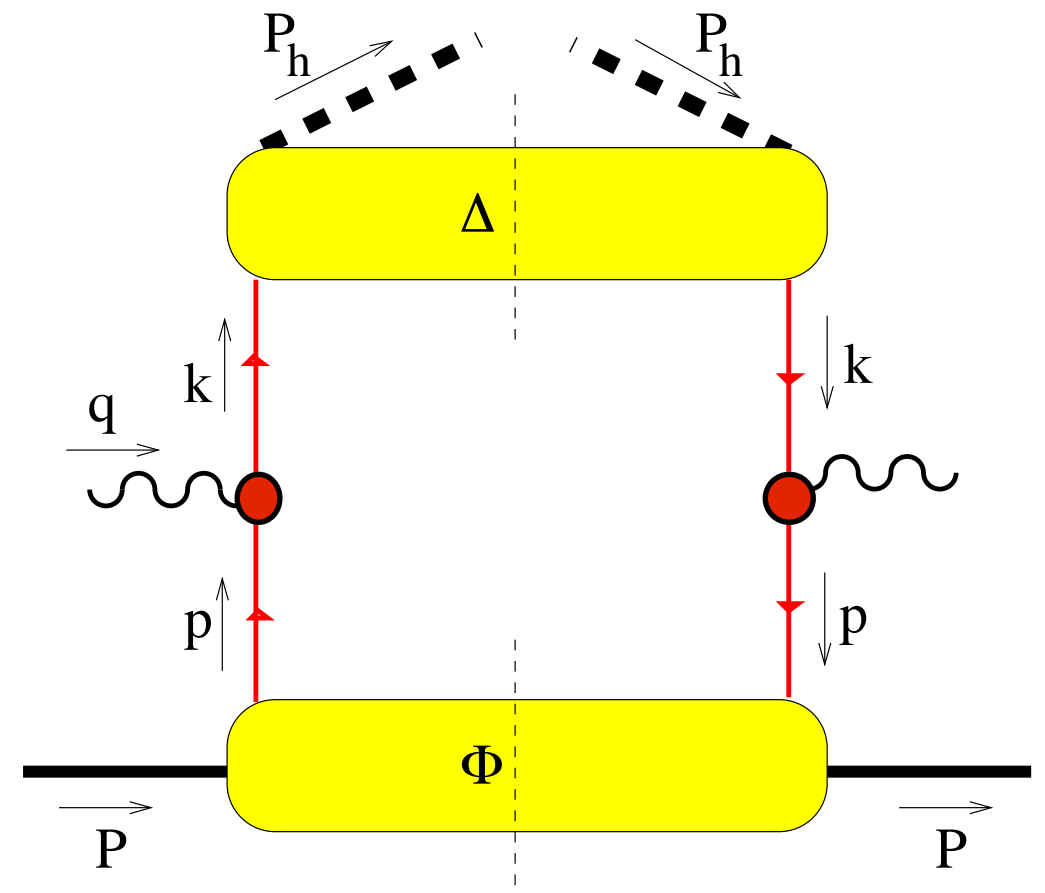

The cartoon version of the color flow works fine in most cases, first and foremost, when collinear factorization applies

Factorization in terms of correlators

Similarly, one would expect that the following two processes involve the same color trace and that the dynamics is unaffected by the color flow
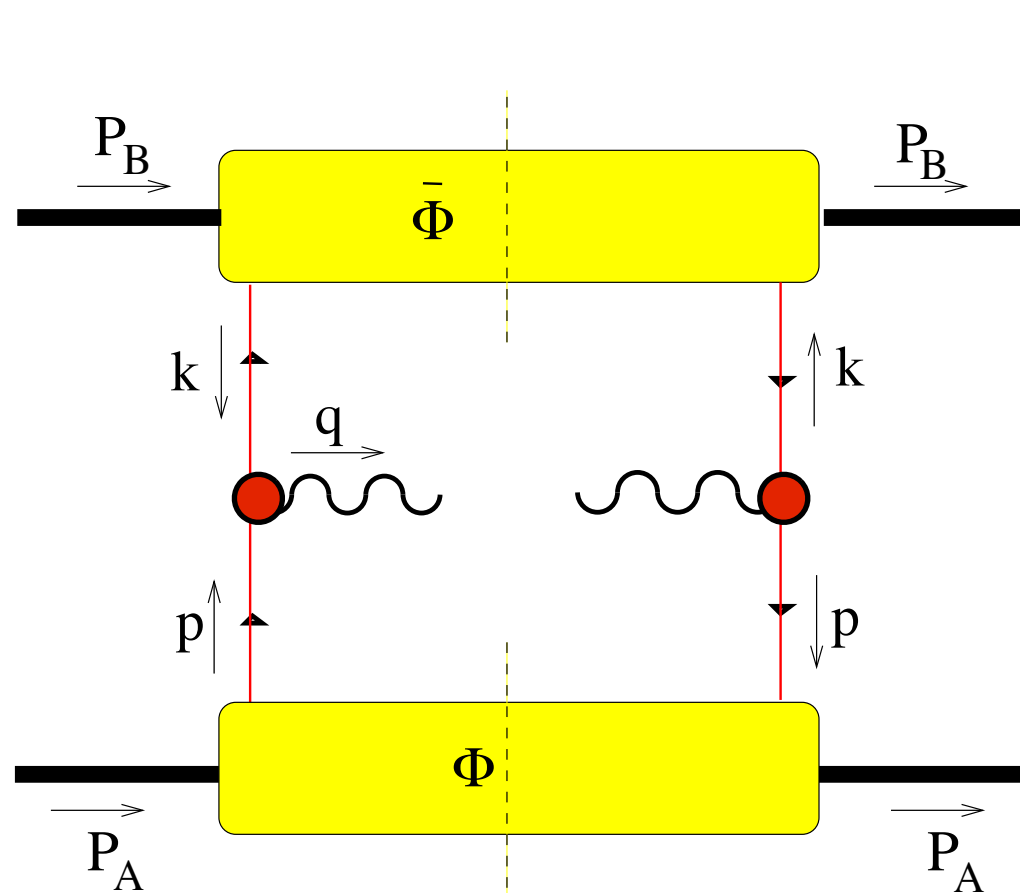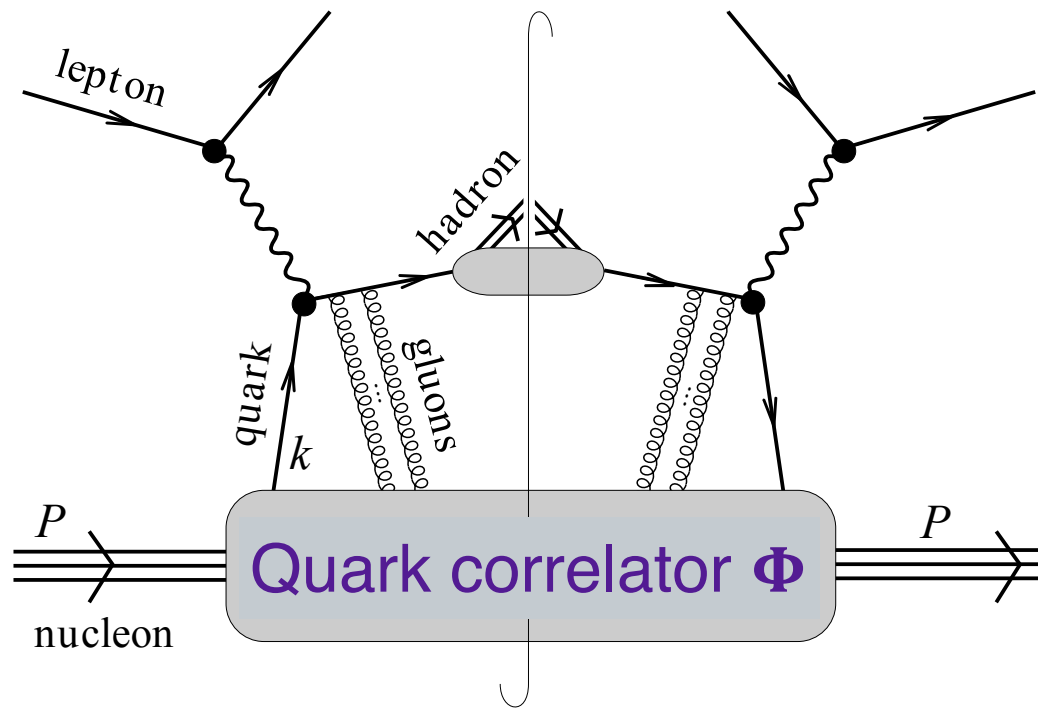

$pp \rightarrow \gamma^*X$ (Drell-Yan)

$\gamma^*p \rightarrow h\,X$ (SIDIS)

However, this is not always the case, e.g. for certain differential cross sections, that are sensitive to the transverse momentum of the partons

Gauge invariance of correlators

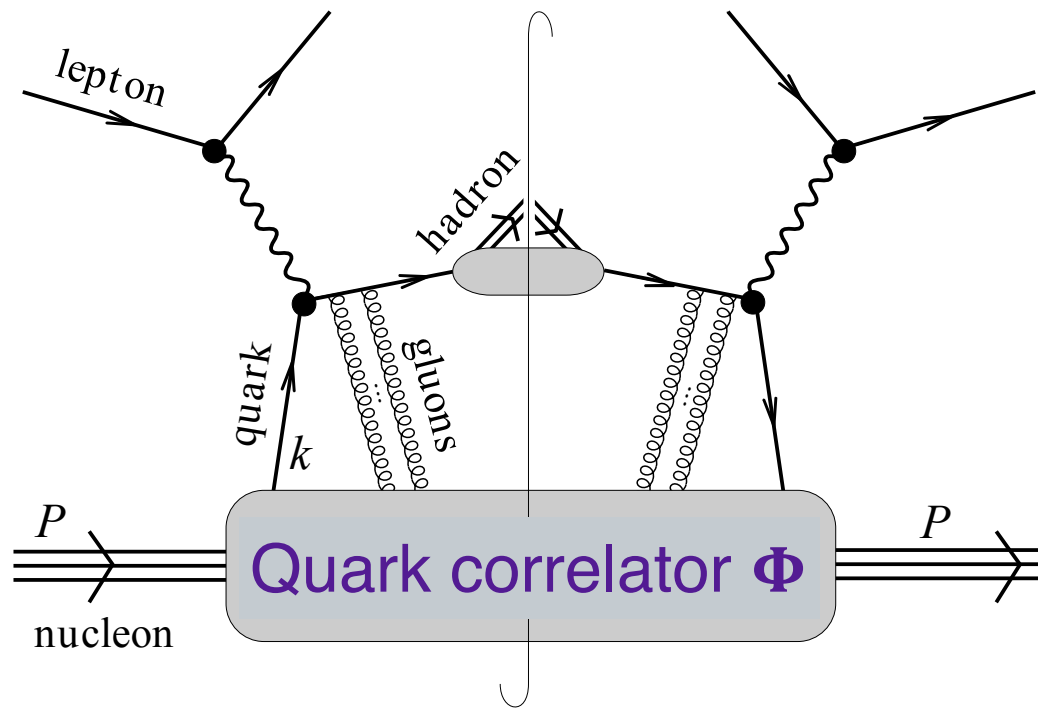summation of all gluon exchanges leads to *path-ordered exponentials* in the correlators

$$\mathcal{L}_{\mathcal{C}}[0, \xi] = \mathcal{P} \exp \left(-ig \int_{\mathcal{C}[0,\xi]} ds_\mu \, A^\mu(s) \right)$$

$$\Phi \propto \langle P | \overline{\psi}(0) \mathcal{L}_{\mathcal{C}}[0, \xi] \psi(\xi) | P \rangle$$

Efremov & Radyushkin, Theor. Math. Phys. 44 ('81) 774

Gauge invariance of correlators

summation of all gluon exchanges leads to *path-ordered exponentials* in the correlators

$$\mathcal{L}_{\mathcal{C}}[0,\xi] = \mathcal{P} \exp\left(-ig \int_{\mathcal{C}[0,\xi]} ds_\mu \, A^\mu(s)\right)$$

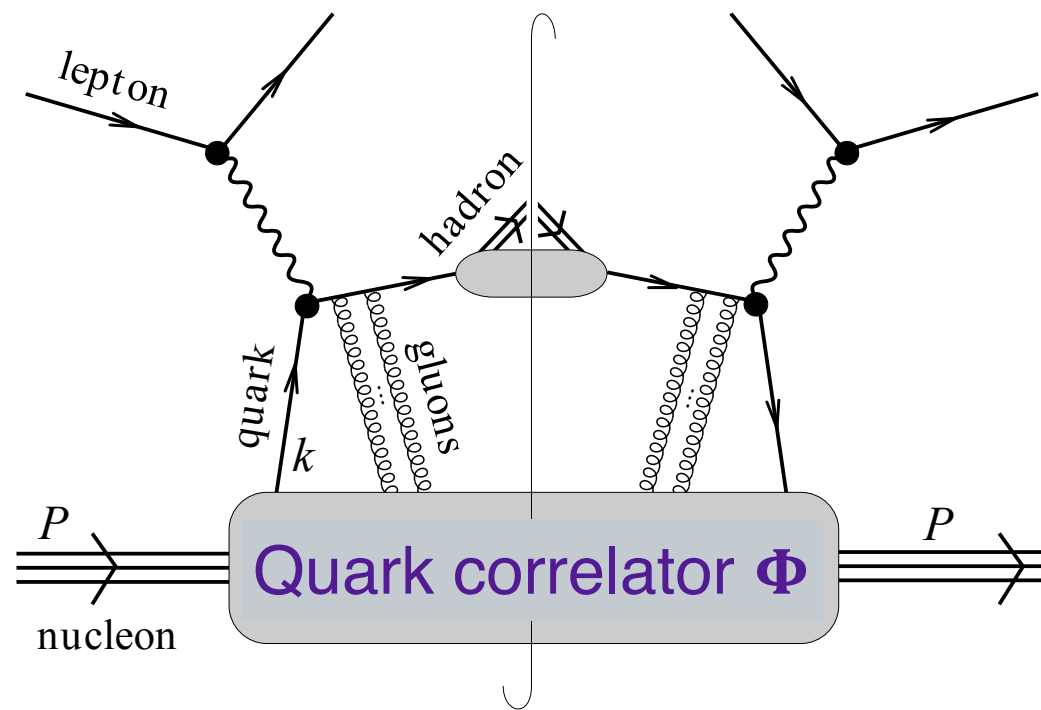$$\Phi \propto \langle P|\overline{\psi}(0)\mathcal{L}_{\mathcal{C}}[0,\xi]\psi(\xi)|P\rangle$$

Efremov & Radyushkin, Theor. Math. Phys. 44 ('81) 774

The path C depends on whether the color interactions are with an incoming or outgoing color charge, yielding different paths for different processes

[Collins & Soper, 1983; Boer & Mulders, 2000; Brodsky, Hwang & Schmidt, 2002; Collins, 2002; Belitsky, Ji & Yuan, 2003; Boer, Mulders & Pijlman, 2003]

Gauge invariance of correlators

summation of all gluon exchanges leads to *path-ordered exponentials* in the correlators

$$\mathcal{L}_{\mathcal{C}}[0,\xi] = \mathcal{P} \exp\left(-ig \int_{\mathcal{C}[0,\xi]} ds_\mu \, A^\mu(s)\right)$$

$$\Phi \propto \langle P|\overline{\psi}(0)\mathcal{L}_{\mathcal{C}}[0,\xi]\psi(\xi)|P\rangle$$

Efremov & Radyushkin, Theor. Math. Phys. 44 ('81) 774

The path C depends on whether the color interactions are with an incoming or outgoing color charge, yielding different paths for different processes

[Collins & Soper, 1983; Boer & Mulders, 2000; Brodsky, Hwang & Schmidt, 2002; Collins, 2002; Belitsky, Ji & Yuan, 2003; Boer, Mulders & Pijlman, 2003]

These *gauge links* may or may not affect observables and it turns out that they do in certain cases sensitive to the transverse momentum
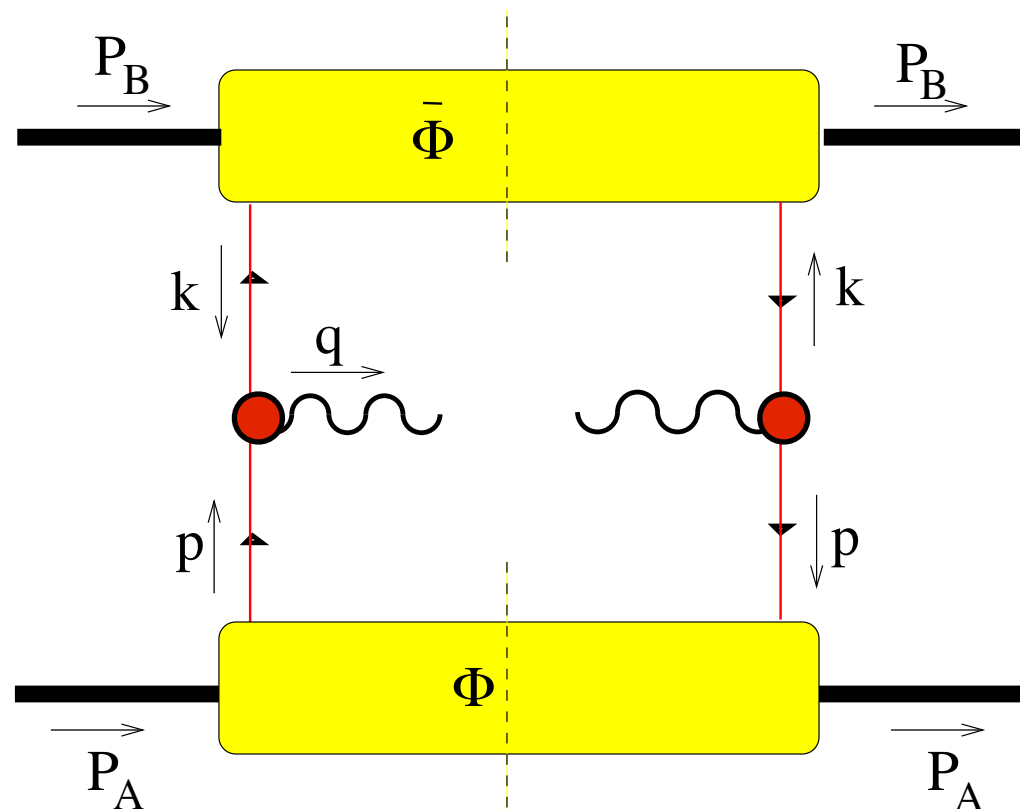
Then the path has extent ξ_T in the transverse direction (ξ_T conjugate to k_T) which can be located at different places along the lightfront
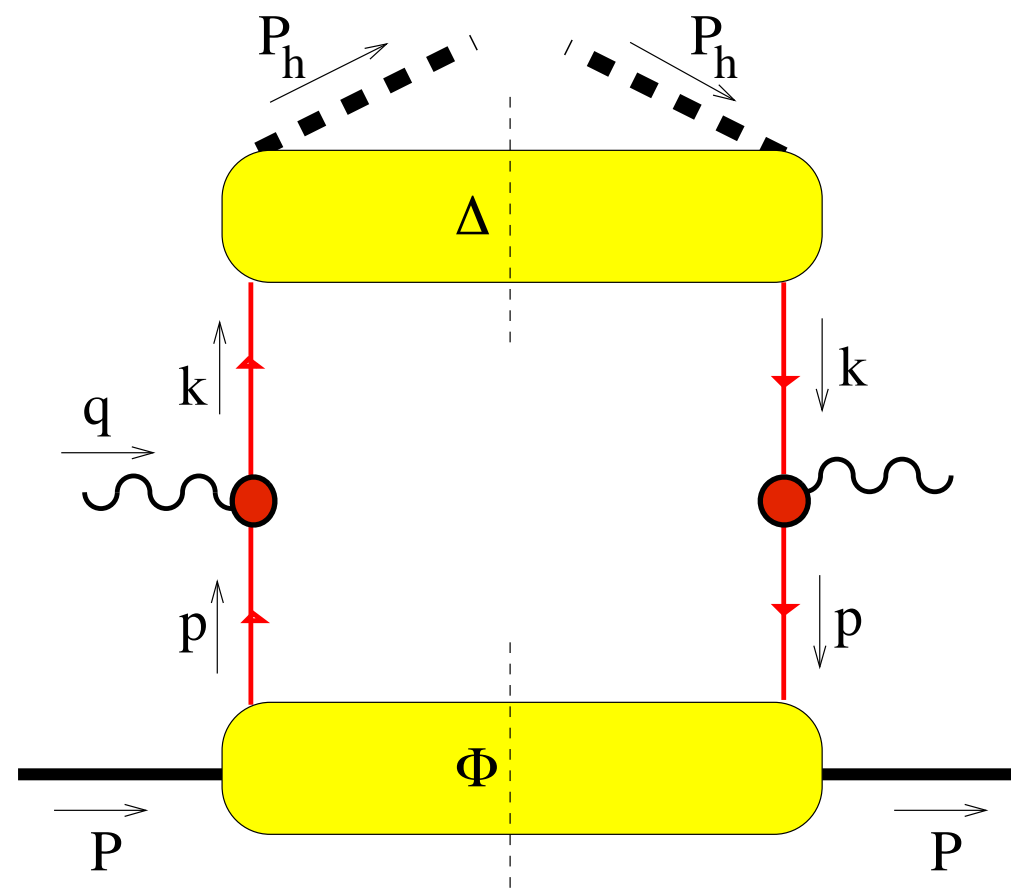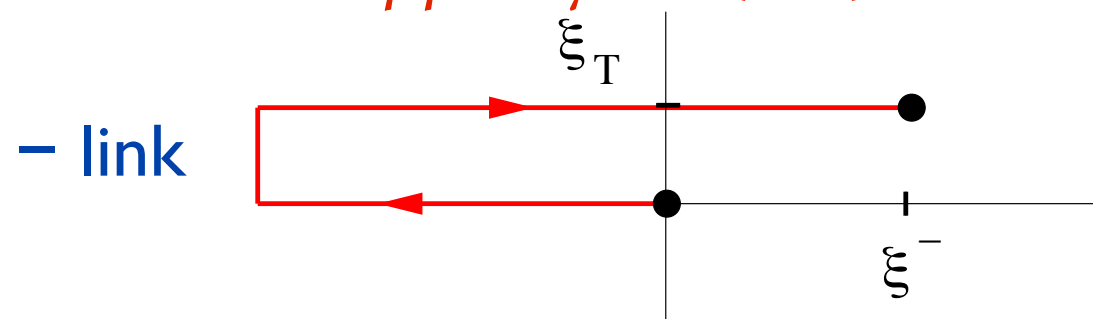
Process dependence of gauge links

Gauge invariant definition of TMDs in semi-inclusive DIS contains a future pointing Wilson line, whereas in Drell-Yan (DY) it is past pointing [Belitsky, Ji & Yuan '03]

$pp \rightarrow \gamma^* X$ (DY)

$\gamma^* p \rightarrow h\, X$ (SIDIS)

− link

+ link

Process dependence of gauge links

Gauge invariant definition of TMDs in semi-inclusive DIS contains a future pointing Wilson line, whereas in Drell-Yan (DY) it is past pointing [Belitsky, Ji & Yuan '03]
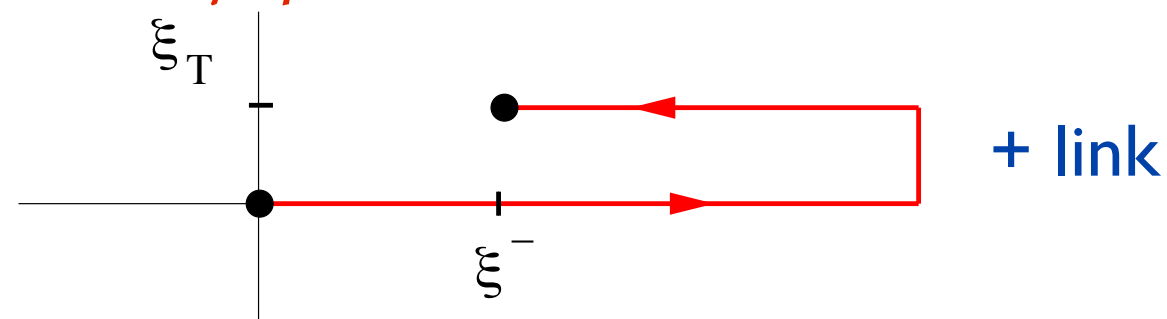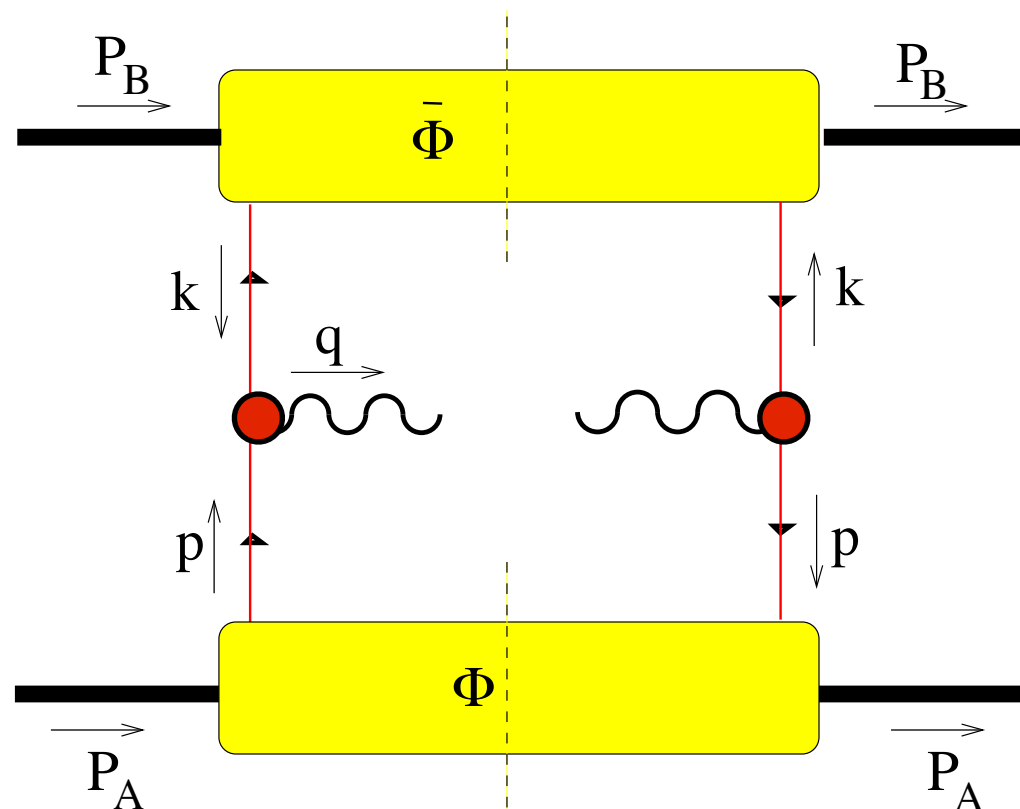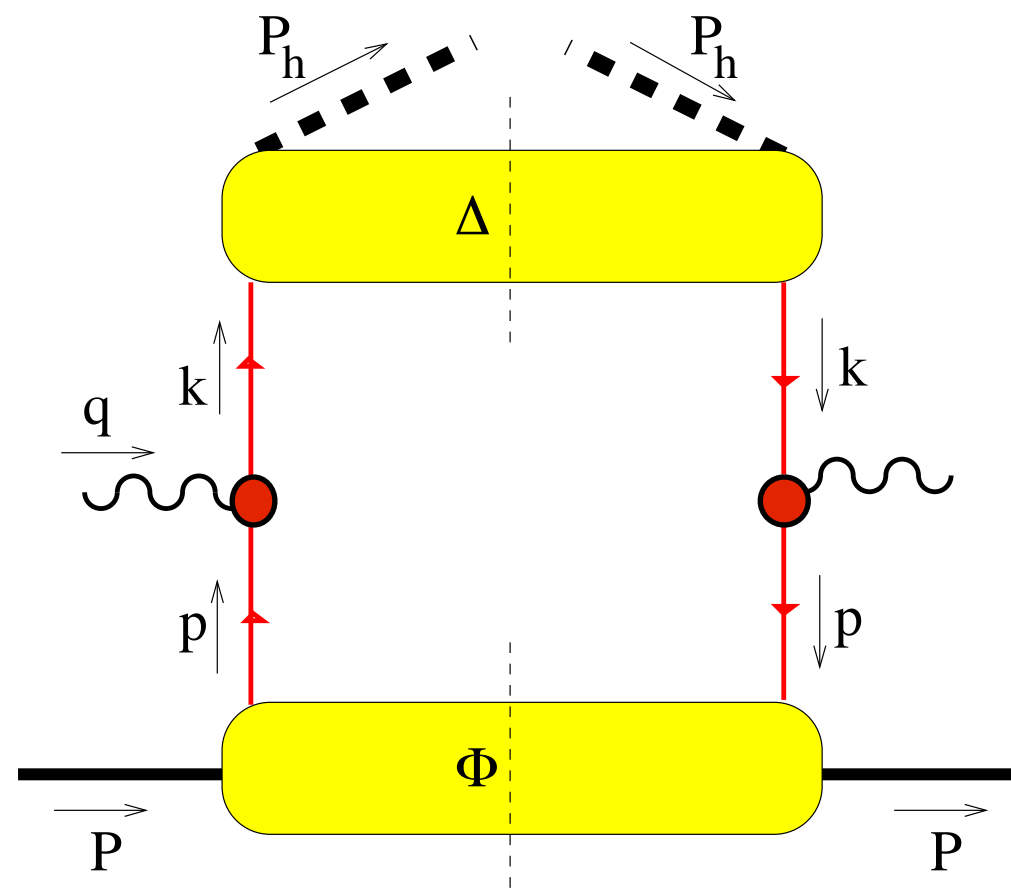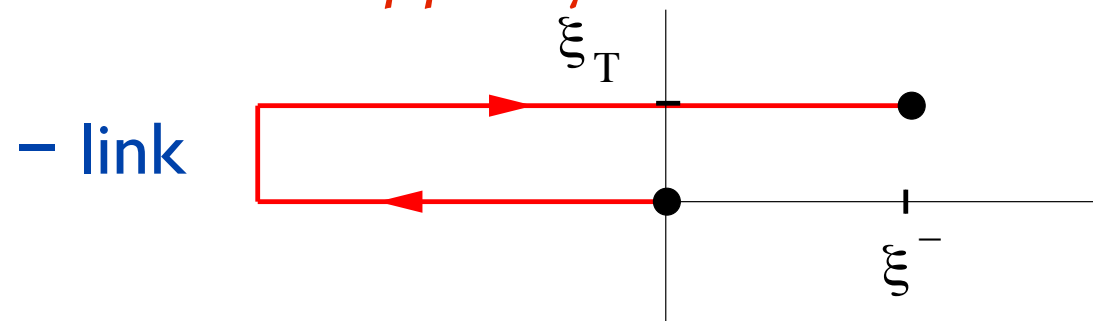
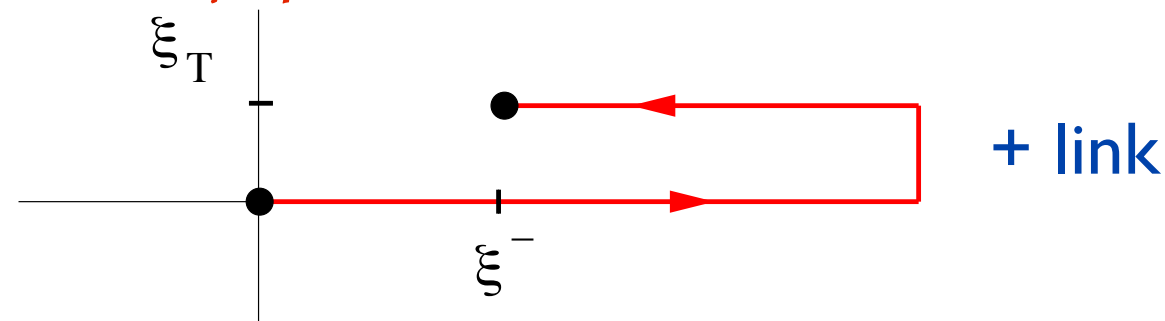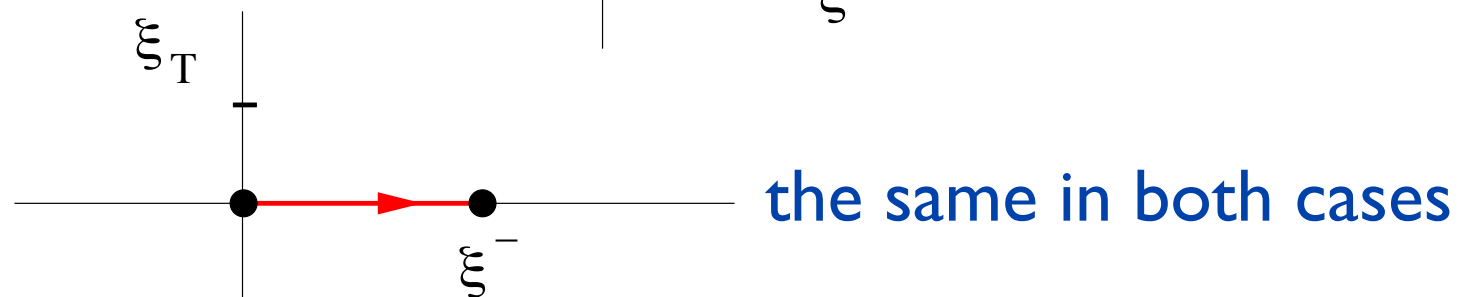

$pp \rightarrow \gamma^* X$ (DY)

$\gamma^* p \rightarrow h\, X$ (SIDIS)

− link

+ link

$$\int dk_T \longrightarrow \xi_T = 0 \longrightarrow$$

the same in both cases

Transversely polarized protons

The link dependence yields the famous sign change relation for the Sivers function

$$f_{1T}^{\perp q[\mathrm{SIDIS}]}(x, k_T^2) = -f_{1T}^{\perp q[\mathrm{DY}]}(x, k_T^2)$$ [Collins '02]

Transversely polarized protons

The link dependence yields the famous sign change relation for the Sivers function

$$f_{1T}^{\perp q[\mathrm{SIDIS}]}(x, k_T^2) = -f_{1T}^{\perp q[\mathrm{DY}]}(x, k_T^2)$$

[Collins '02]

The transverse momentum dependence can be correlated with the spin, e.g.

D. Sivers ('90):

$$k_T \times S_T$$

Transversely polarized protons

The link dependence yields the famous sign change relation for the Sivers function

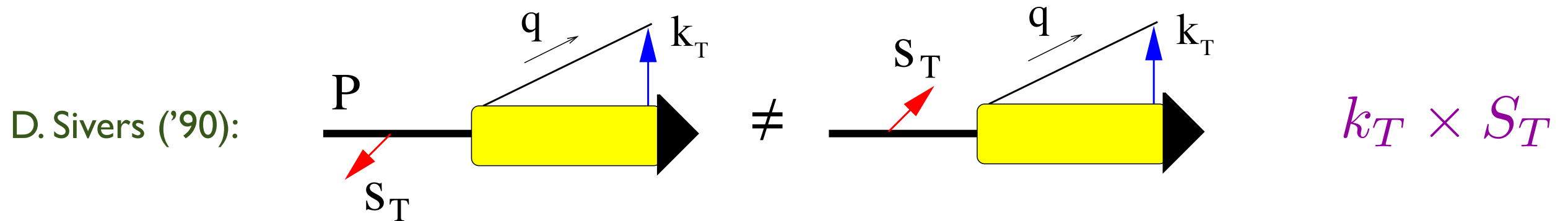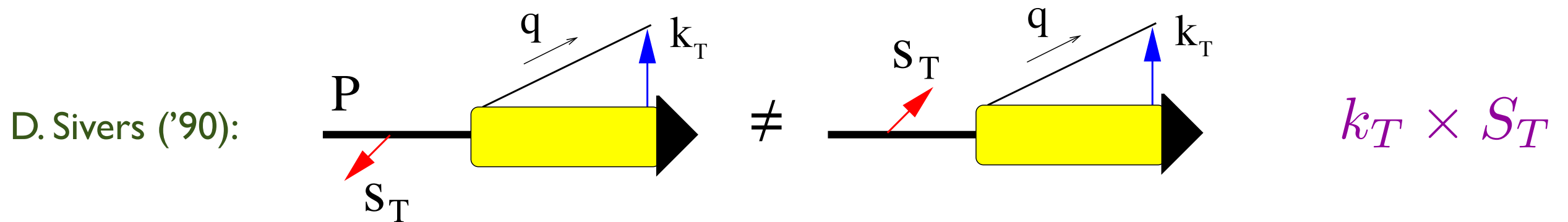$$f_{1T}^{\perp q[\text{SIDIS}]}(x, k_T^2) = -f_{1T}^{\perp q[\text{DY}]}(x, k_T^2)$$

[Collins '02]

The transverse momentum dependence can be correlated with the spin, e.g.

D. Sivers ('90):

$$k_T \times S_T$$

Sivers function

$$\Phi(x, \boldsymbol{k}_T) = \frac{M}{2}\left\{ f_1(x, \boldsymbol{k}_T^2) \frac{\not{P}}{M} + f_{1T}^{\perp}(x, \boldsymbol{k}_T^2) \frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu P^\nu k_T^\rho S_T^\sigma}{M^2} + g_{1s}(x, \boldsymbol{k}_T^2) \frac{\gamma_5 \not{P}}{M} \right.$$

$$\left. + h_{1T}(x, \boldsymbol{k}_T^2) \frac{\gamma_5 \not{S}_T \not{P}}{M} + h_{1s}^{\perp}(x, \boldsymbol{k}_T^2) \frac{\gamma_5 \not{k}_T \not{P}}{M^2} + h_1^{\perp}(x, \boldsymbol{k}_T^2) \frac{i \not{k}_T \not{P}}{M^2} \right\}$$

[Ralston, Soper '79; Sivers '90; Collins '93; Kotzinian '95; Mulders, Tangerman '95; D.B., Mulders '98]
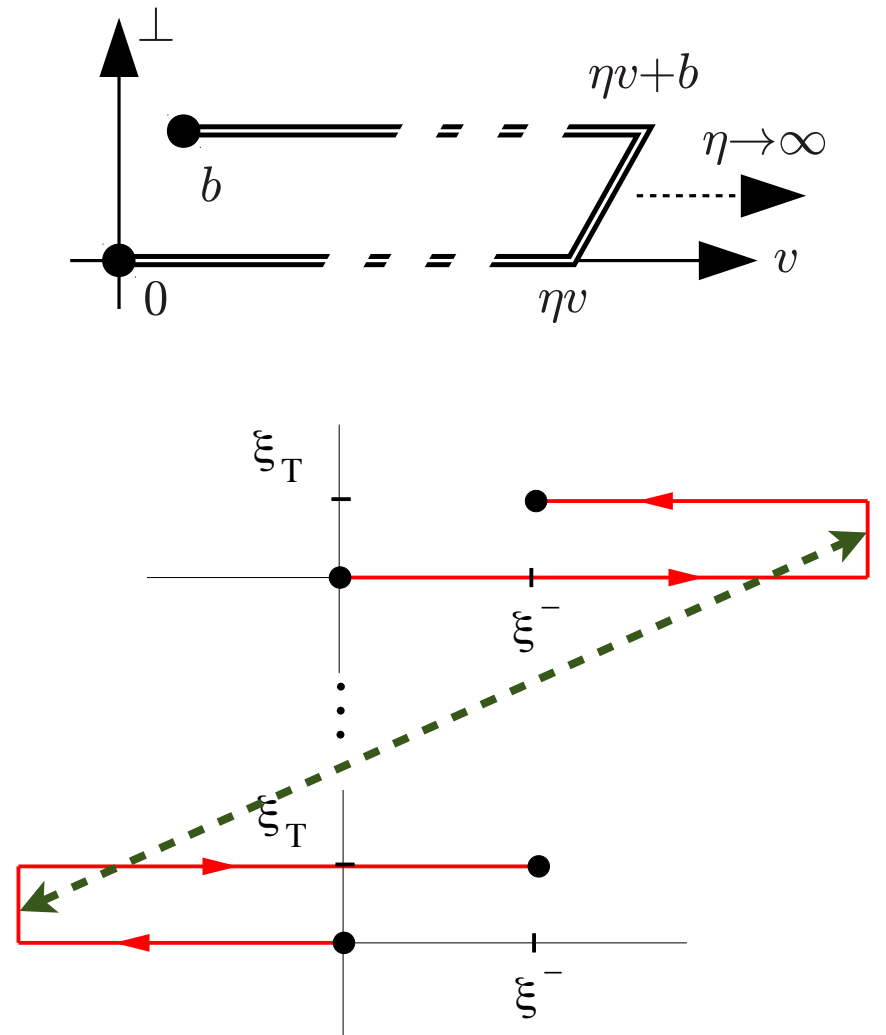
Sivers function on the lattice

By taking specific x and k_T integrals one can define the "Sivers shift" $<k_T \times S_T>(n, b_T)$: the average transverse momentum shift orthogonal to transverse spin S_T
[Boer, Gamberg, Musch, Prokudin, 2011]

This well-defined quantity can be evaluated on the lattice
[Musch, Hägler, Engelhardt, Negele & Schäfer, 2012]

Sivers−Shift, u−d − quarks

$\hat{\zeta} = 0.39,$

$|\mathbf{b}_T| = 0.12$ fm,

$m_\pi = 518$ MeV

$\longleftarrow$ DY
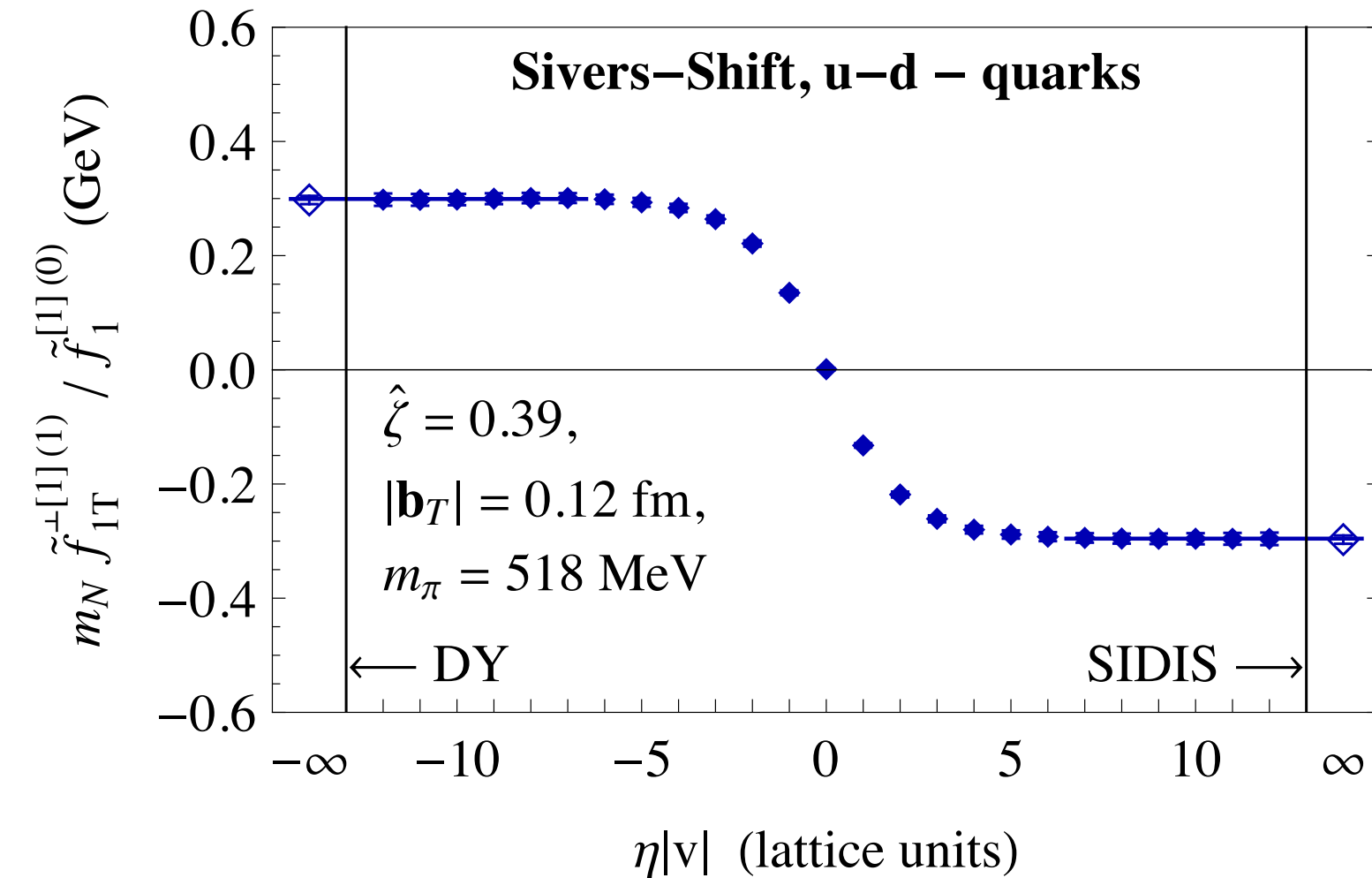
SIDIS $\longrightarrow$

$\eta |v|$ (lattice units)

Sivers function on the lattice

By taking specific x and k_T integrals one can define the "Sivers shift" $\langle k_T \times S_T \rangle(n, b_T)$: the average transverse momentum shift orthogonal to transverse spin S_T
[Boer, Gamberg, Musch, Prokudin, 2011]

This well-defined quantity can be evaluated on the lattice
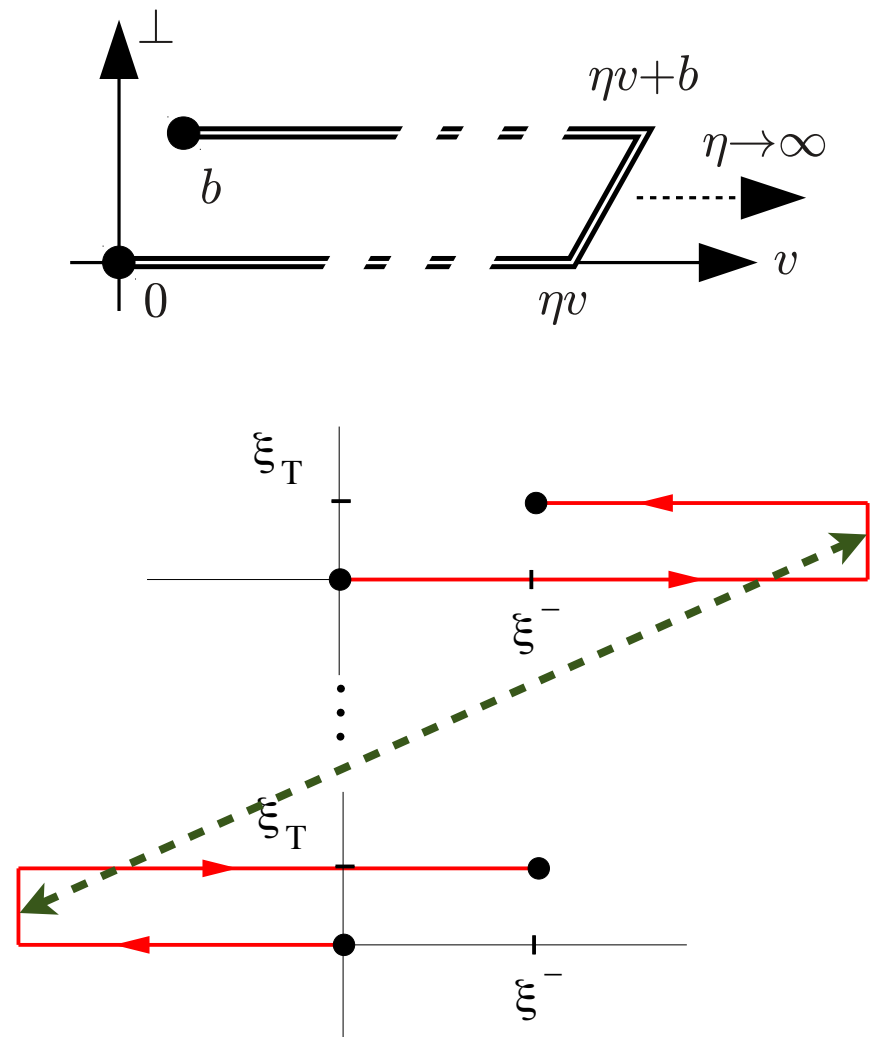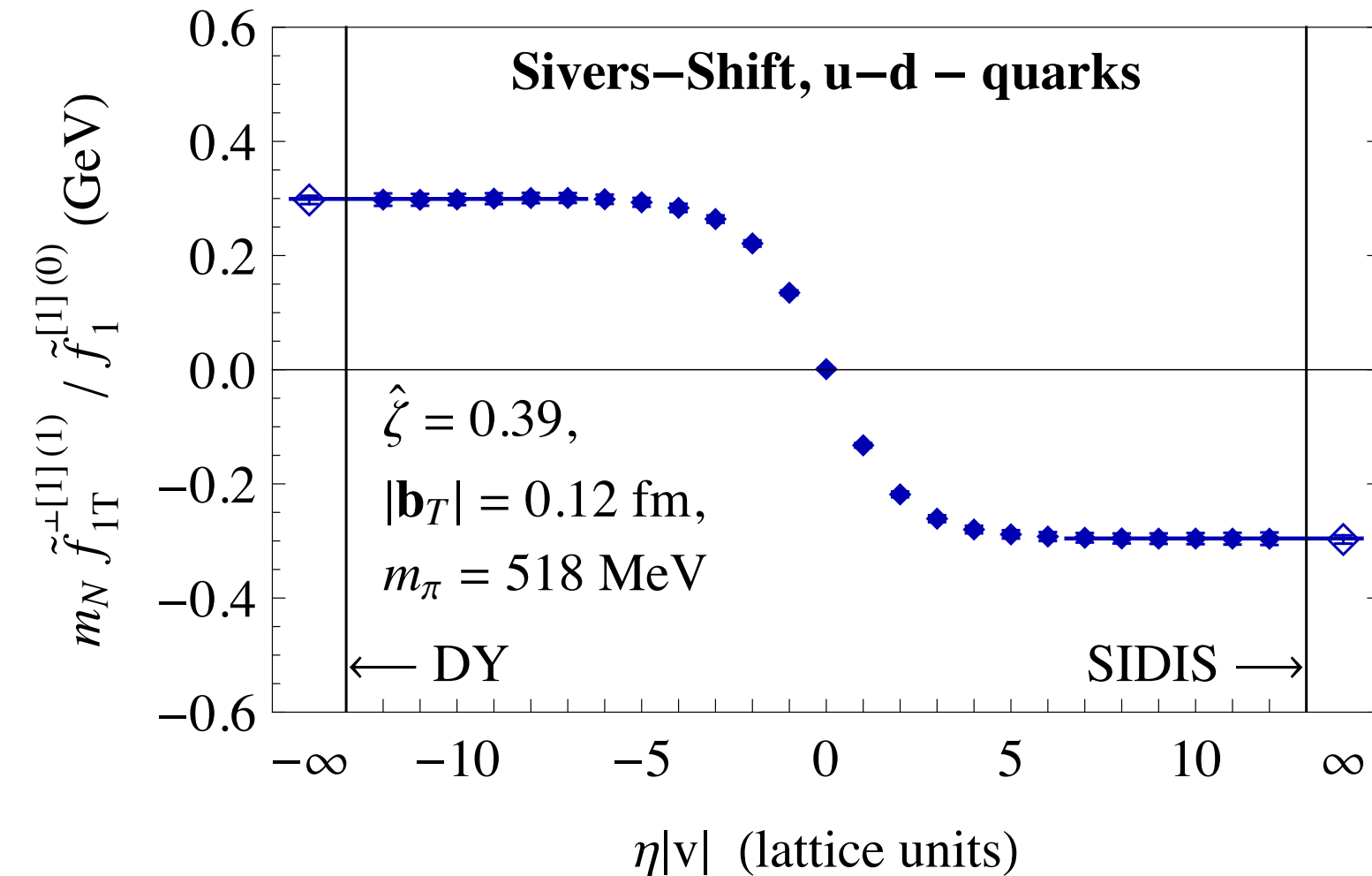[Musch, Hägler, Engelhardt, Negele & Schäfer, 2012]

This is the first `first-principle' demonstration that the Sivers function is nonzero for staple-like links. It clearly corroborates the sign change relation (as it should)

Measurements of the Sivers TMD

The Sivers effect in SIDIS has been clearly observed by HERMES at DESY (PRL 2009) & COMPASS at CERN (PLB 2010)

The corresponding DY experiments are investigated at CERN (COMPASS), Fermilab (SeaQuest) & RHIC (W-boson production rather) & planned at NICA (Dubna) & IHEP (Protvino)

The first data is compatible with the sign-change prediction of the TMD formalism
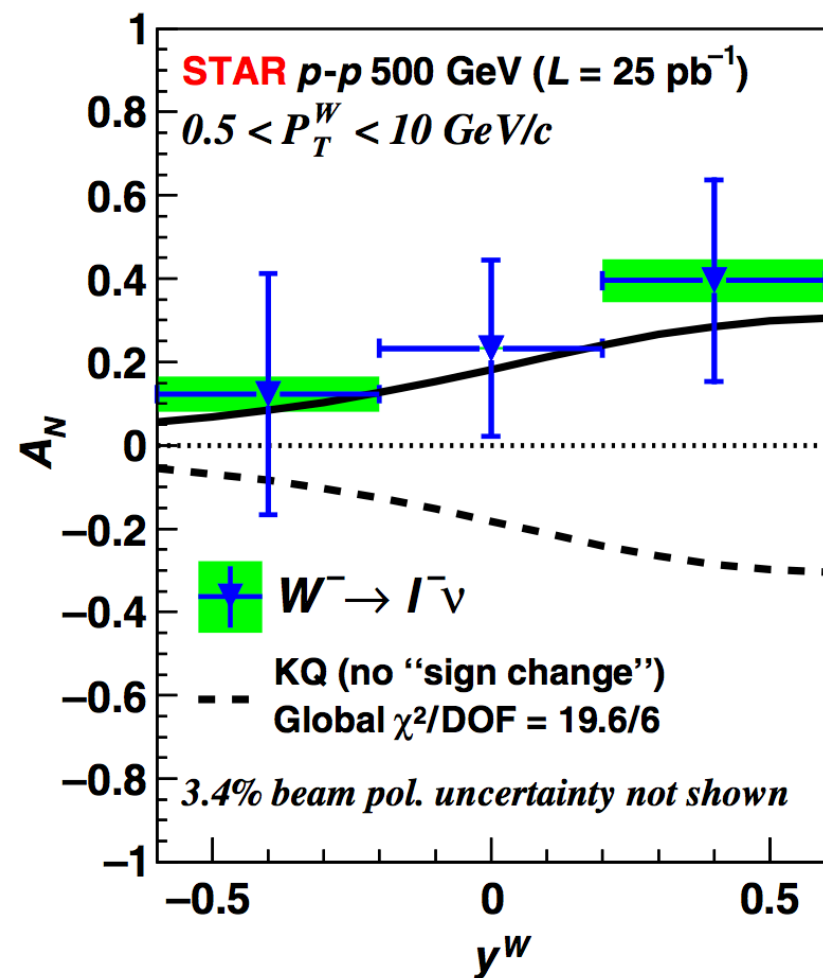
STAR, PRL 2016
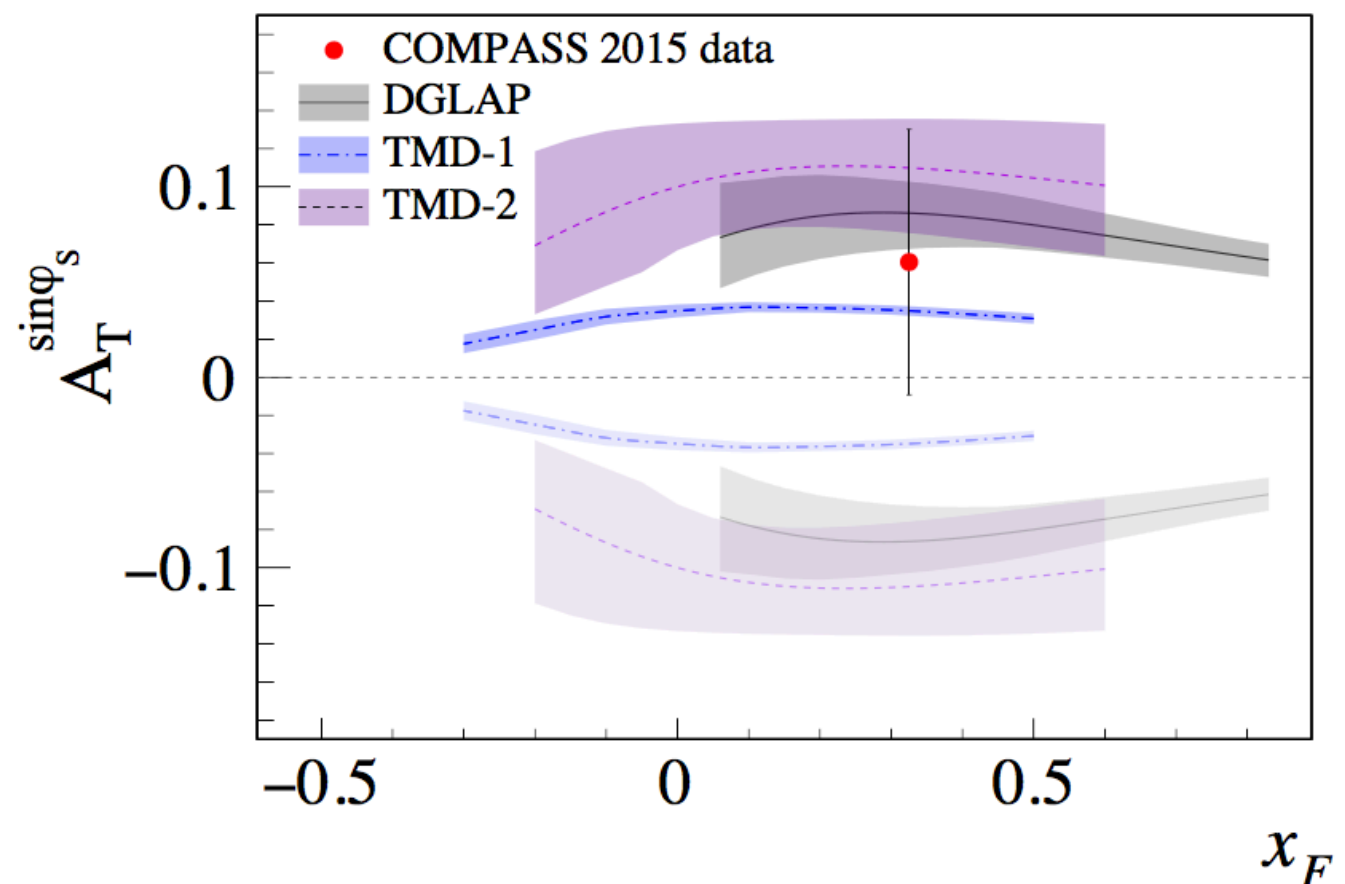
COMPASS, PRL 2017

cos(2ϕ) asymmetry in DY and Lam-Tung relation

Spin averaged scattering of protons

Drell-Yan

lepton plane (cm)
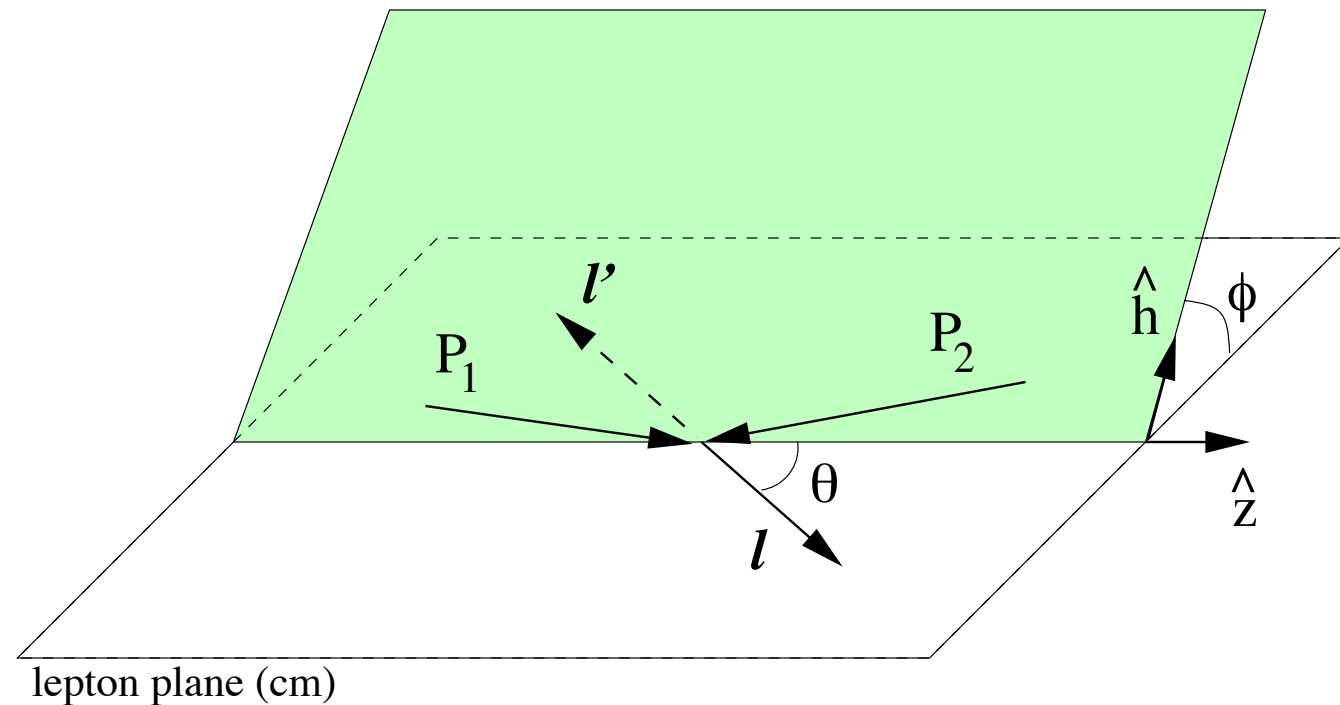
$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} \propto \left(1 + \lambda\cos^2\theta + \mu\sin 2\theta\,\cos\phi + \frac{\nu}{2}\sin^2\theta\cos 2\phi\right)$$

The $O(\alpha_s)$ Lam-Tung relation:

$$1 - \lambda - 2\nu = 0$$

Large deviations from the Lam-Tung relation were observed in DY [NA10 ('86/'88) & E615 ('89)]
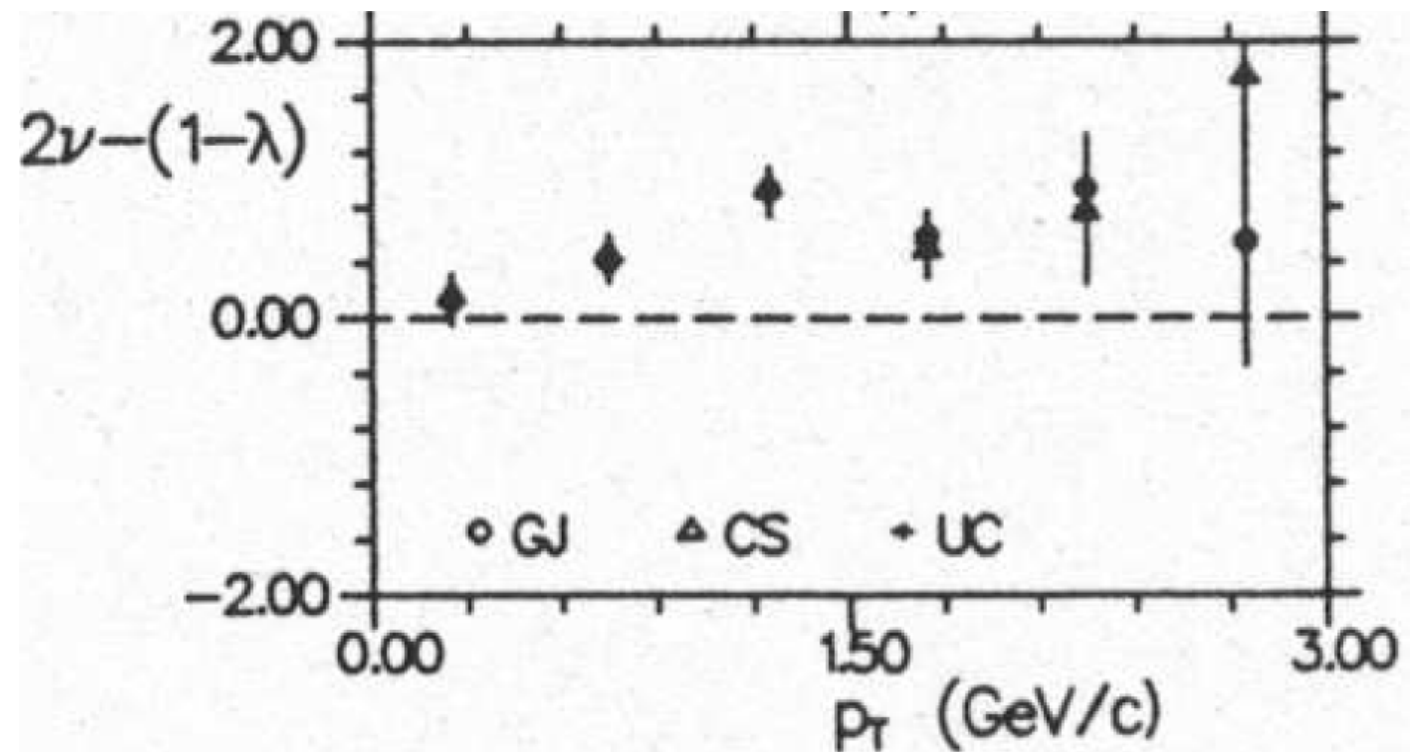
Failure of collinear pQCD treatment

With collinear parton densities, only higher order gluon emission can generate deviations from Lam-Tung
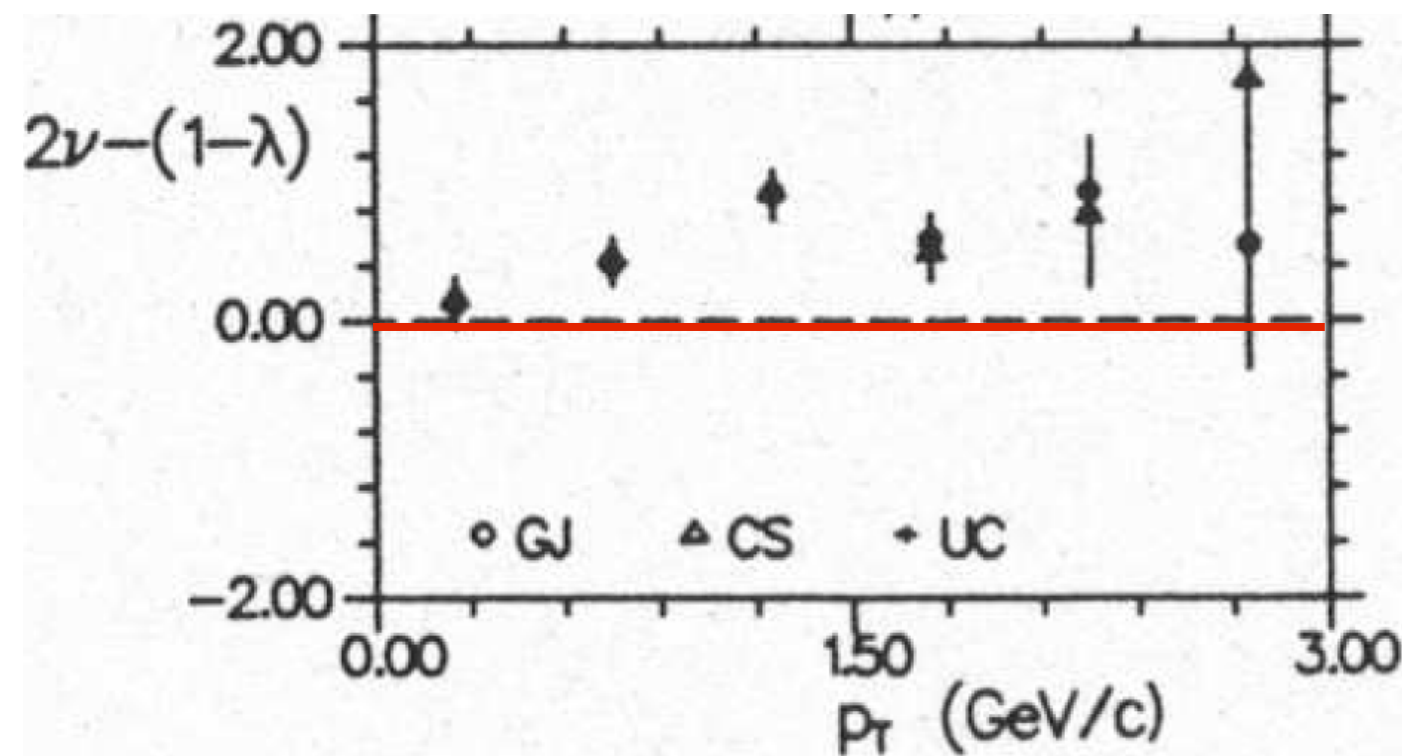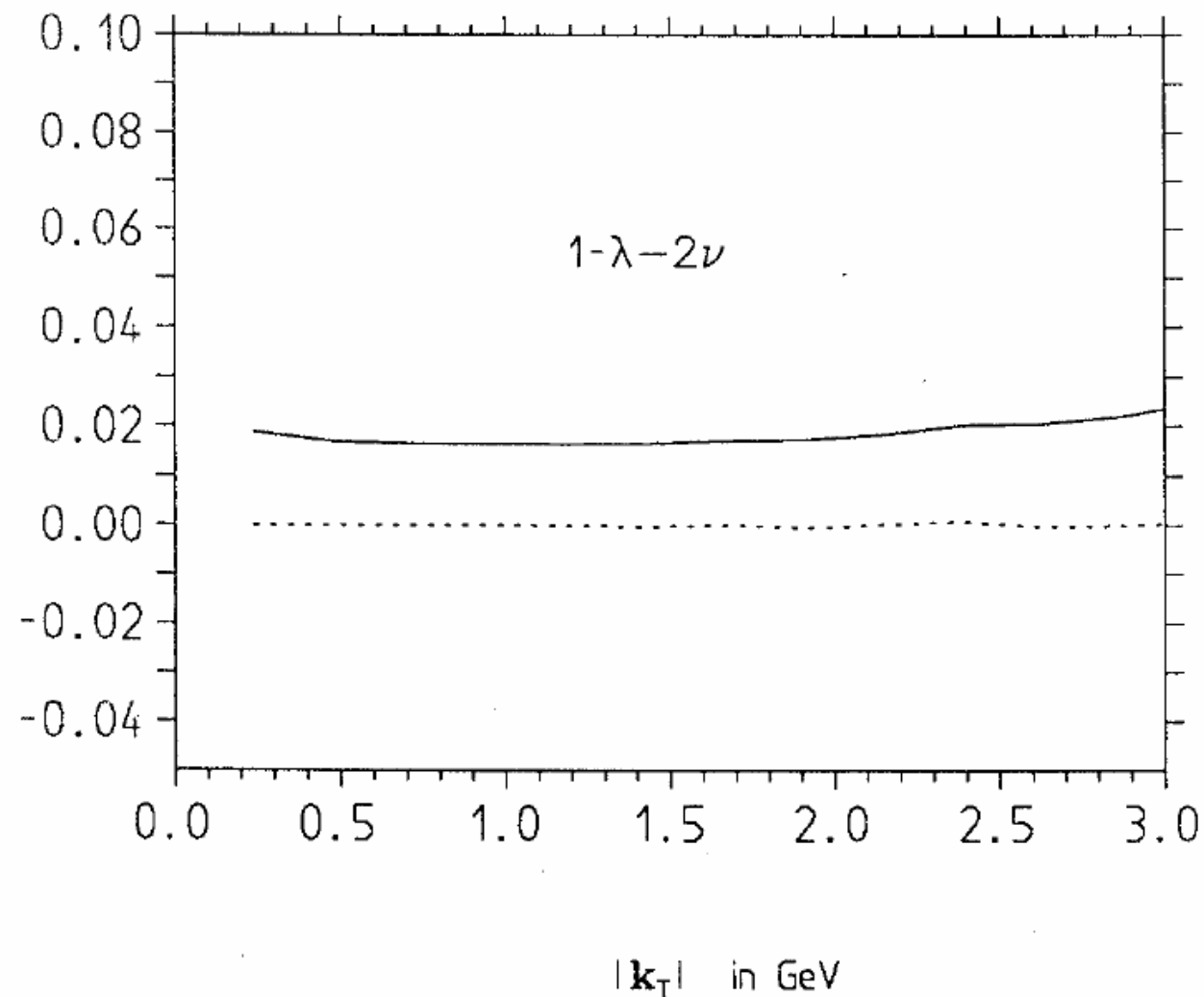
Deviation from Lam-Tung relation in NNLO $O(\alpha_s^2)$ pQCD is (at least) an order of magnitude smaller and of opposite sign (for the π induced fixed-target data)

[Brandenburg, Nachtmann & Mirkes '93; Mirkes & Ohnemus '95]

Lam-Tung $1 - \lambda - 2\nu$

E615

E866
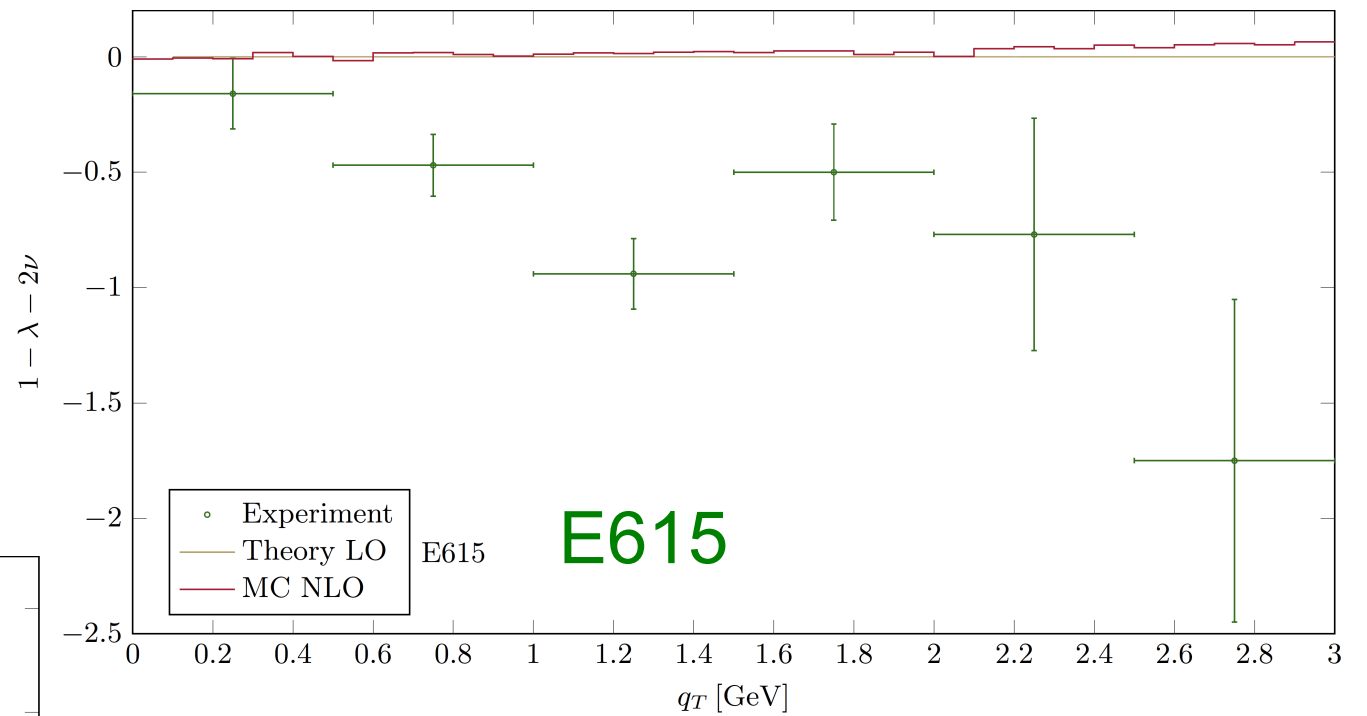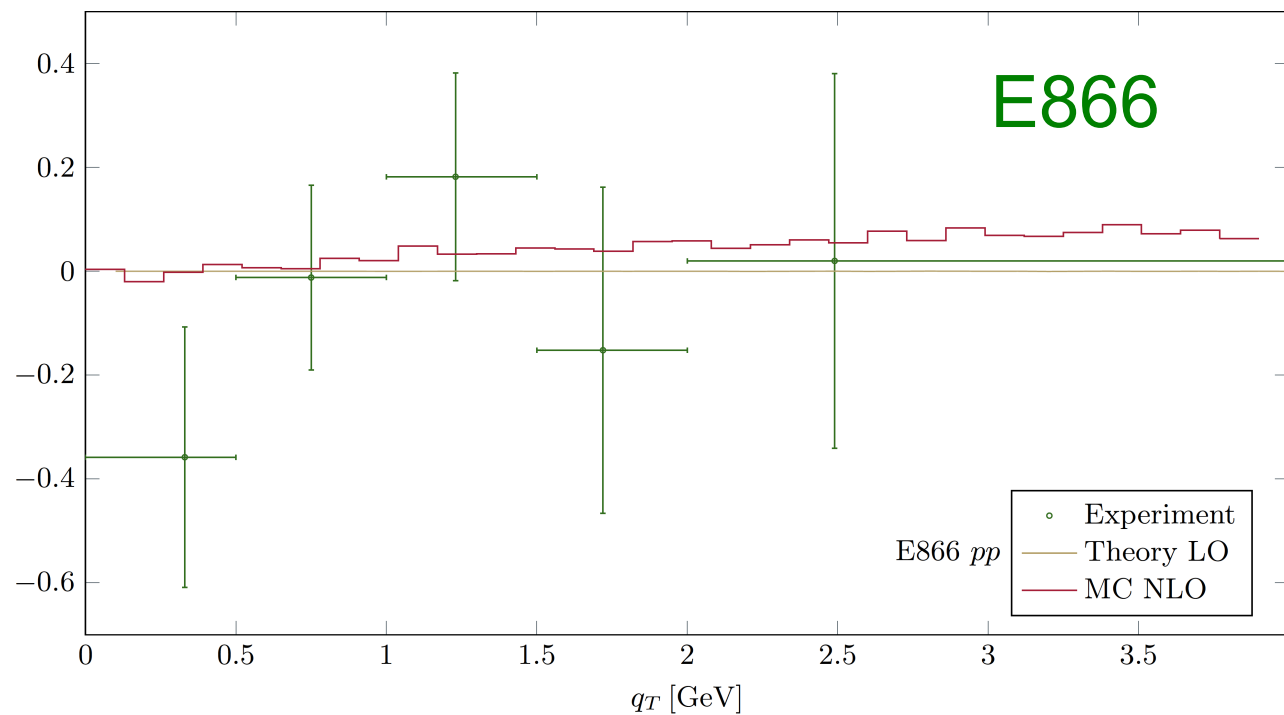
LT violation in π-W DY is incompatible with NLO results

NA10

Results by Lambertsen & Vogelsang, presented by Vogelsang at "3D parton distributions: path to the LHC", Frascati, dec 2016
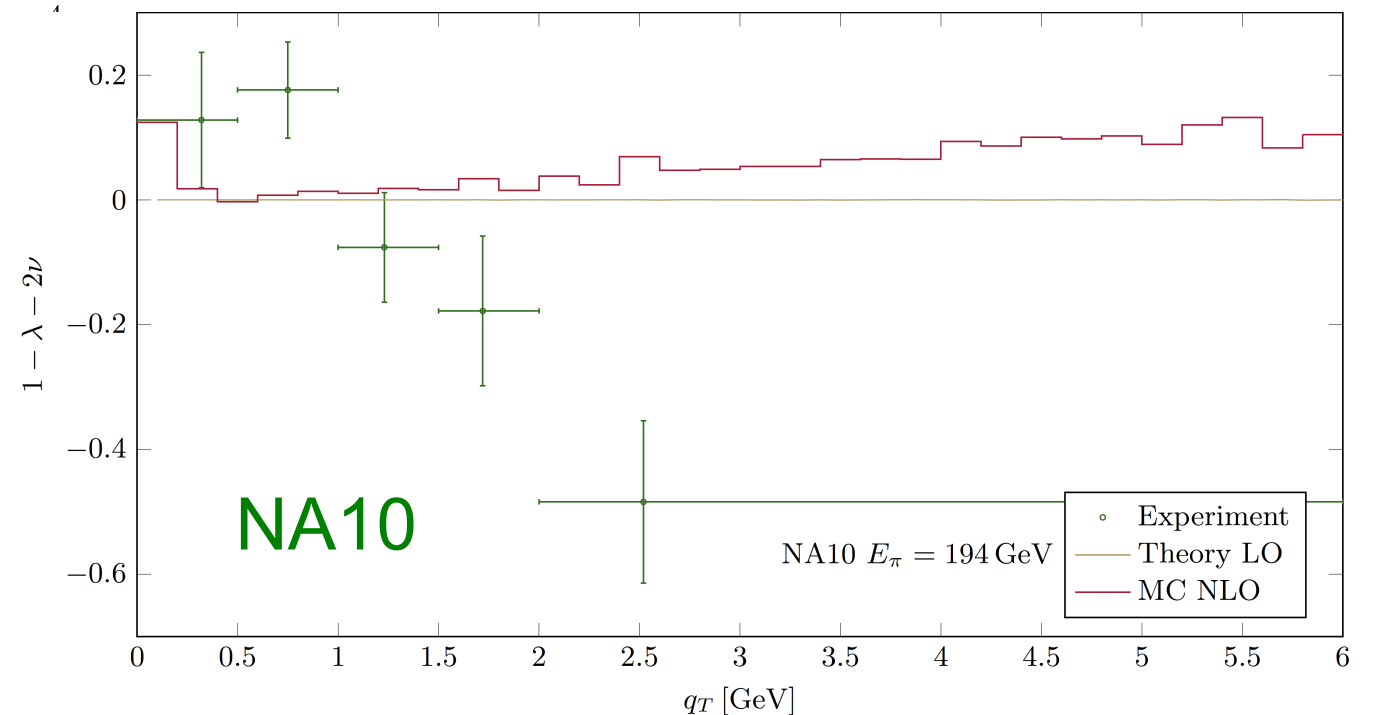
LT violation in fixed-target data questioned
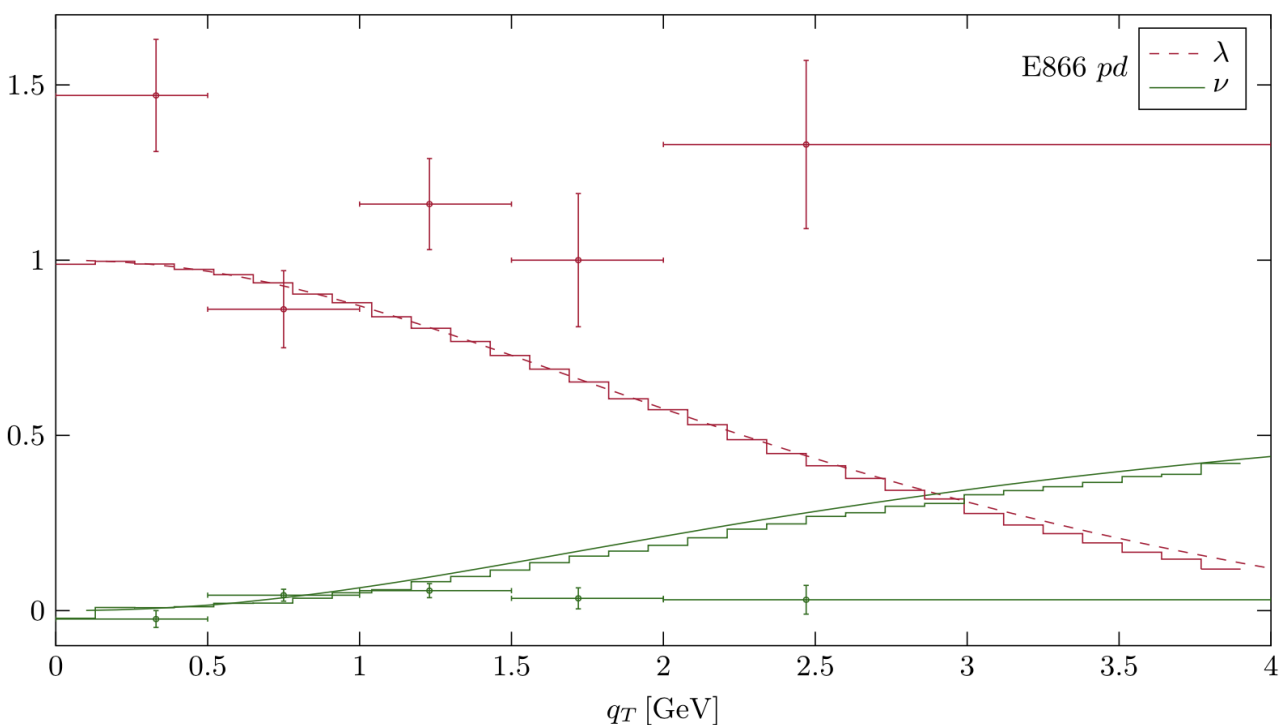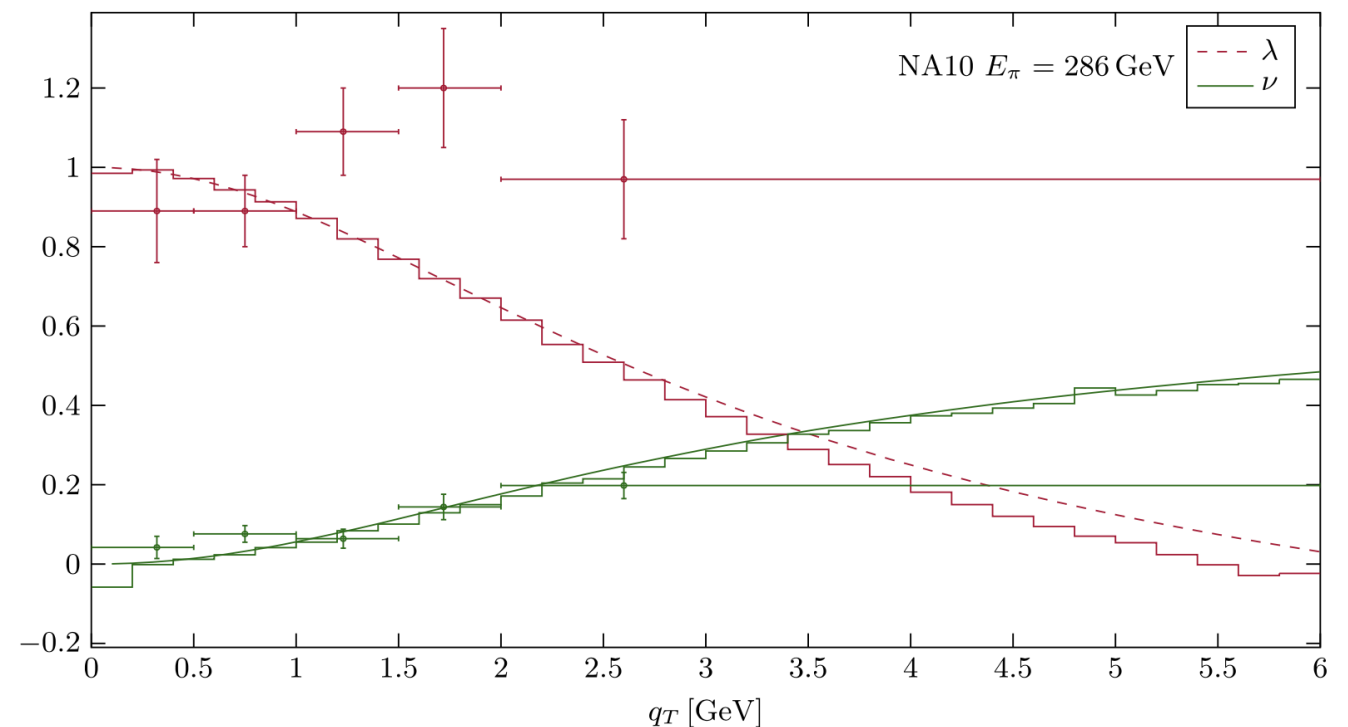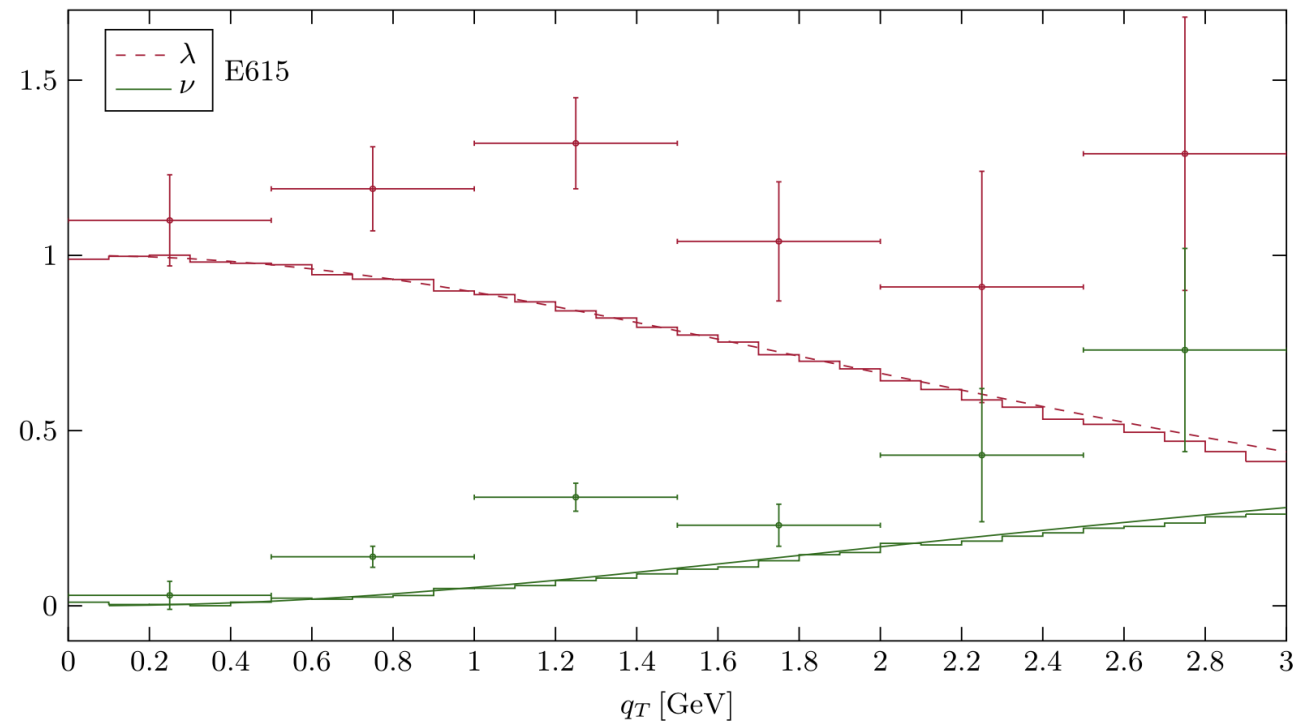
Lambertsen & Vogelsang (PRD 2016): something is wrong with data of fixed target experiments because $\lambda \leq 1$ not satisfied

D.B. @ QCD evolution 2016: it may just be a problem of going to the Collins-Soper frame

By using the rotation invariant LT violation parameter $\mathcal{F}$ this issue can be avoided

$1-\lambda-2\nu$ is not rotationally invariant, but if it is zero, it is zero in all rotated frames

Rotation invariant LT violation parameter

Rotation invariant measure of LT violation (in the dilepton c.o.m. frame):
[Faccioli, Lourenço, Seixas, Wöhri, PRD 83 (2011) 056008]

$$\mathcal{F} = \frac{1 + \lambda + \nu}{3 + \lambda} \quad \xrightarrow{1-\lambda=2\nu} \quad \frac{1}{2}$$
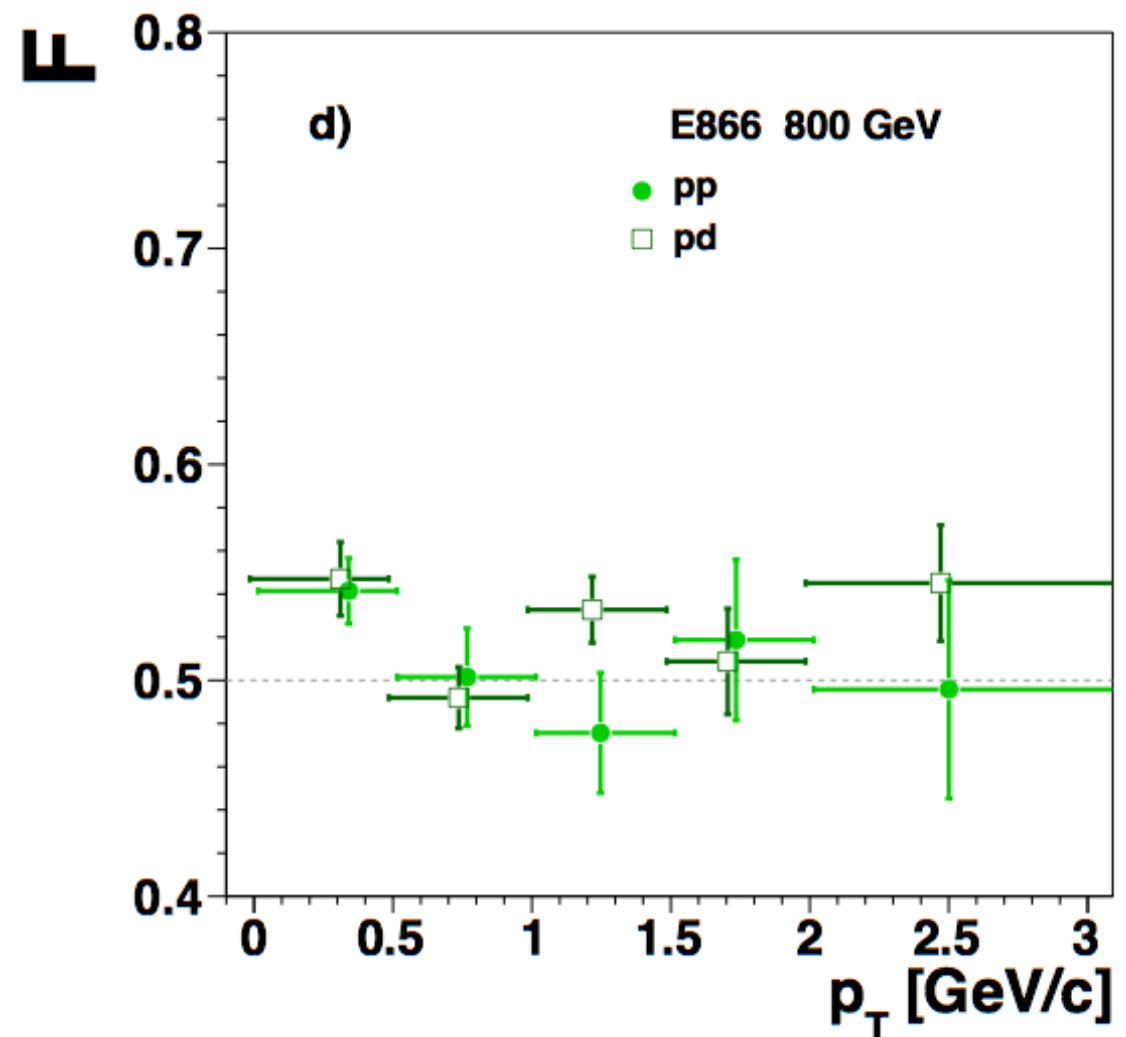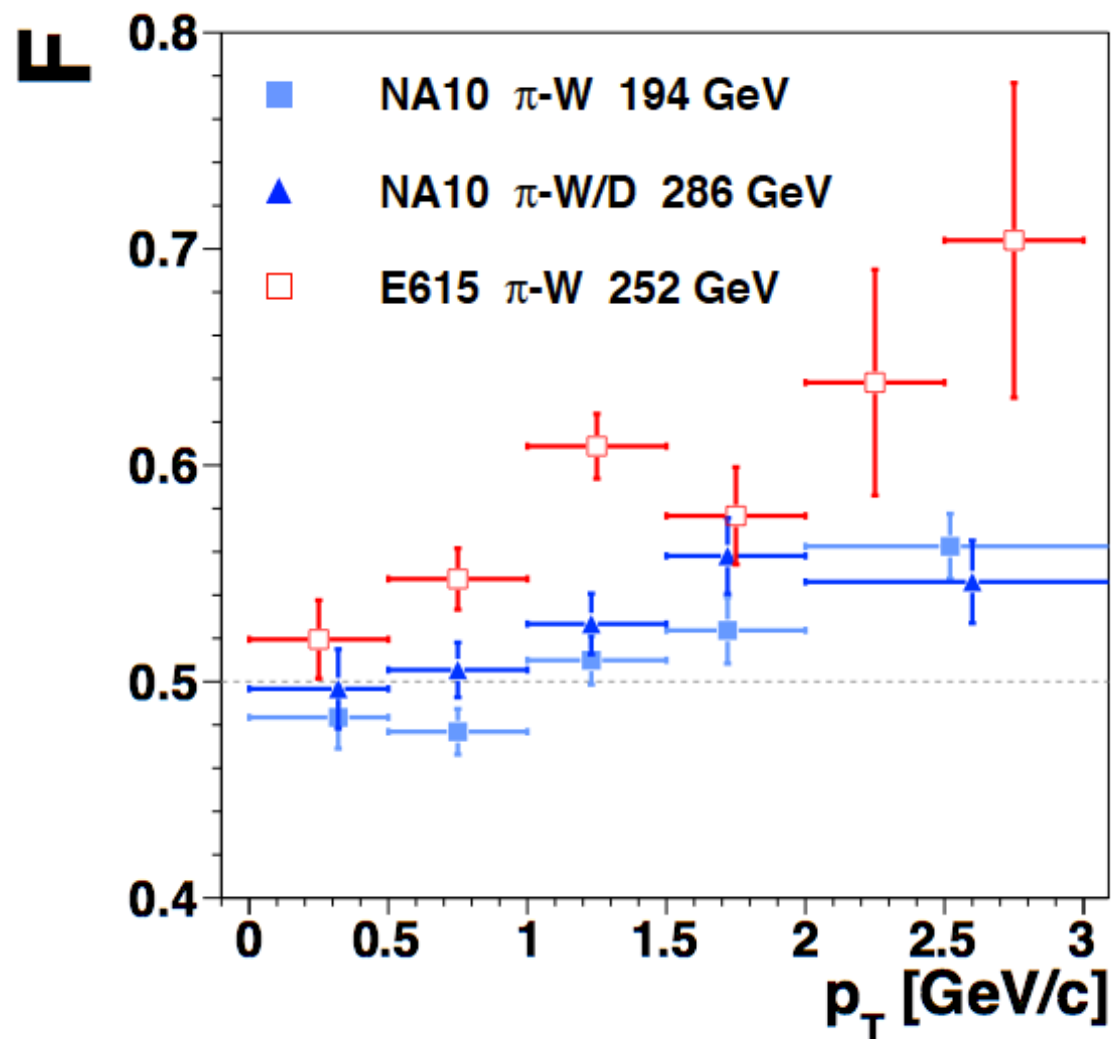
No significant violation observed in pp and pd DY (no valence anti-quarks)
[FNAL-E866/NuSea Collaboration, L.Y. Zhu et al. PRL '07 & '09]

Quark polarization inside unpolarized hadrons

LT violation naturally explained
within TMD framework
[DB, PRD 1999; DB, Brodsky & Hwang, PRD 2003]

Partonic transverse momentum allows
for transversely polarized quarks
inside an unpolarized hadron:
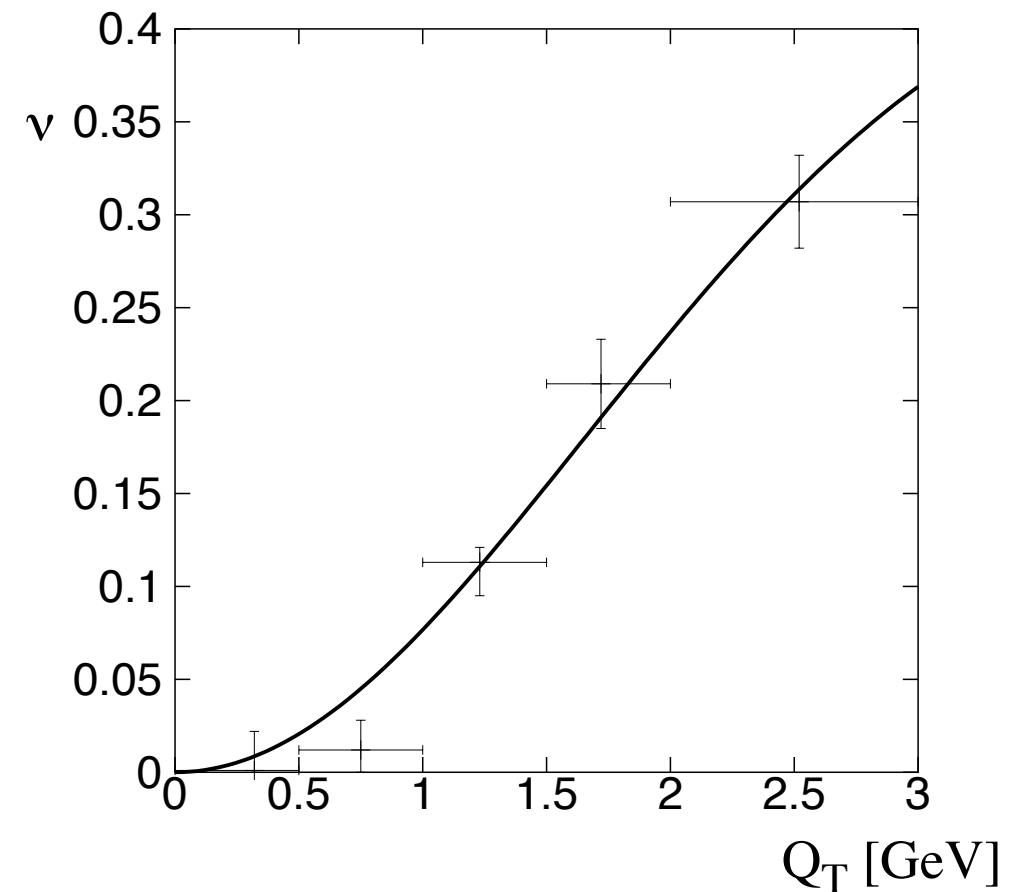
$$\Phi(x, \boldsymbol{k}_T) = \frac{M}{2}\left\{ f_1(x, \boldsymbol{k}_T^2)\,\frac{\slashed{P}}{M} + h_1^{\perp}(x, \boldsymbol{k}_T^2)\,\frac{i\,\slashed{k}_T\,\slashed{P}}{M^2} \right\}$$

$$h_1^{\perp} = \text{(diagram)} - \text{(diagram)}$$
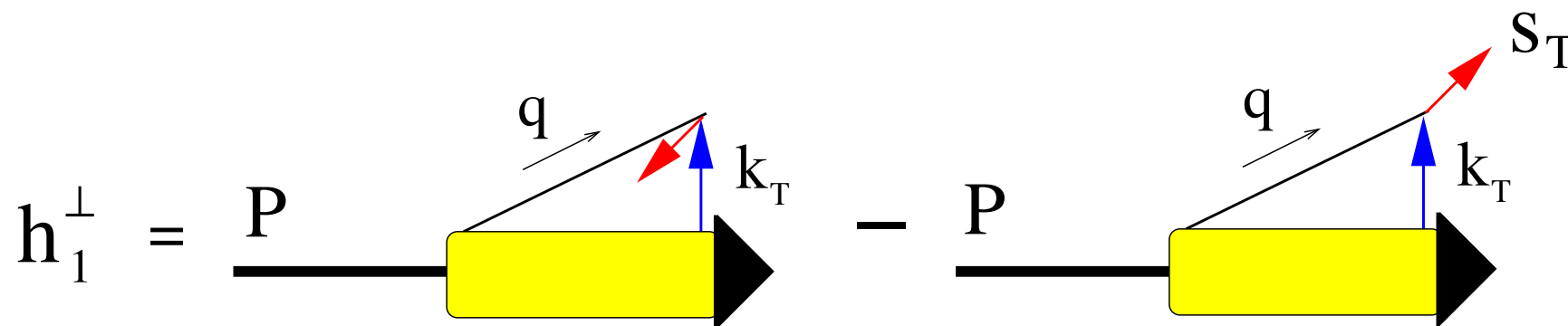
DB & Mulders ('98)

Angular asymmetry requires helicity flip

The $\cos 2\phi$ asymmetry arises from an interference between $+1$ and -1 photon helicities
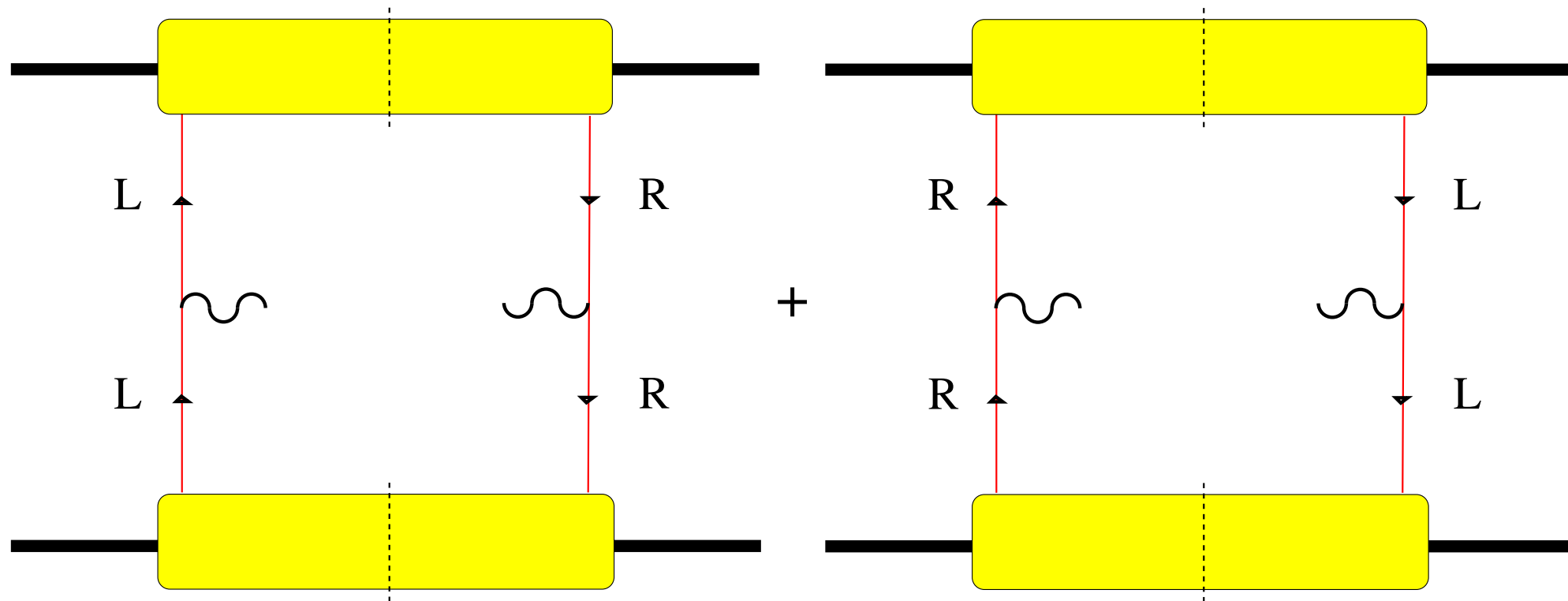
This requires transversely polarized quark-antiquark annihilation

Anomalous asymmetry could be a hadronic effect

Possibly originates from chiral symmetry breaking
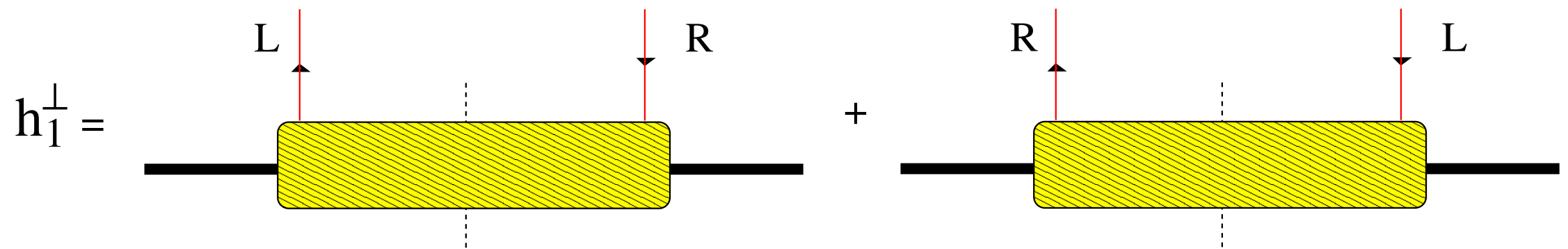
$$h_1^\perp \neq 0 \quad \Longrightarrow \quad \text{deviation from Lam-Tung relation}$$

$$\kappa \equiv -\frac{1}{4}(1 - \lambda - 2\nu) \approx \frac{\nu}{2} \propto h_1^\perp(\pi)\, h_1^\perp(N)$$

Simple model for k_T dependence of $h_1^\perp$ allows for a good description of the data

Flavor dependence of $h_1^\perp$

$$\left(f_1^{(\pm)}\right)_{t_1 t_2} = 4MN_c\delta_{t_2 t_1}V_1^{(\pm)} = O(N_c^2)\,,$$

$$\left(g_{1T}^{(\pm)}\right)_{t_1 t_2} = -\frac{8}{3}I_3(\tau^3)_{t_2 t_1}M^2 N_c V_2^{(\pm)} = O(N_c^3)\,,$$

$$\left(g_{1L}^{(\pm)}\right)_{t_1 t_2} = \frac{8}{3}I_3(\tau^3)_{t_2 t_1}MN_c V_3^{(\pm)} = O(N_c^2)\,,$$

$$\left(f_{1T}^{\perp(\pm)}\right)_{t_1 t_2} = -\frac{8}{3}I_3(\tau^3)_{t_2 t_1}M^2 N_c V_4^{(\pm)} = O(N_c^3)\,,$$ $\longleftarrow$ according to large N_c arguments u and d Sivers have opposite sign

$$\left(h_1^\perp\right)_{t_1 t_2} = -4M^2 N_c\delta_{t_2 t_1}V_5^{(\pm)} = O(N_c^3)\,,$$ $\longleftarrow$ $h_1^\perp$ has the same sign for u and d

$$\left(h_{1L}^{\perp(\pm)}\right)_{t_1 t_2} = -\frac{8}{3}I_3(\tau^3)_{t_2 t_1}M^2 N_c V_6^{(\pm)} = O(N_c^3)\,,$$

$$\left(h_{1T}^{\perp(\pm)}\right)_{t_1 t_2} = \frac{8}{3}I_3(\tau^3)_{t_2 t_1}M^3 N_c V_7^{(\pm)} = O(N_c^4)\,,$$

$$\left(h_{1T}^{(\pm)}\right)_{t_1 t_2} = \frac{8}{3}I_3(\tau^3)_{t_2 t_1}MN_c V_8^{(\pm)} = O(N_c^2)\,.$$

Also expected from GPD calculations on lattice, from models and s-p interference (Burkardt-Hannafious, 2007)

Pobylitsa, hep-ph/0301236 (first paper that obtained same sign for up and down BM functions)

Lattice calculation

After taking Mellin moments *and* Bessel transverse moments of the Sivers function, one has a well-defined quantity $<k_T \times s_T>(n, \mathcal{B}_T)$, that can be evaluated on the lattice

[Musch, Hägler, Engelhardt, Negele & Schäfer, PRD 85 (2012) 094510]
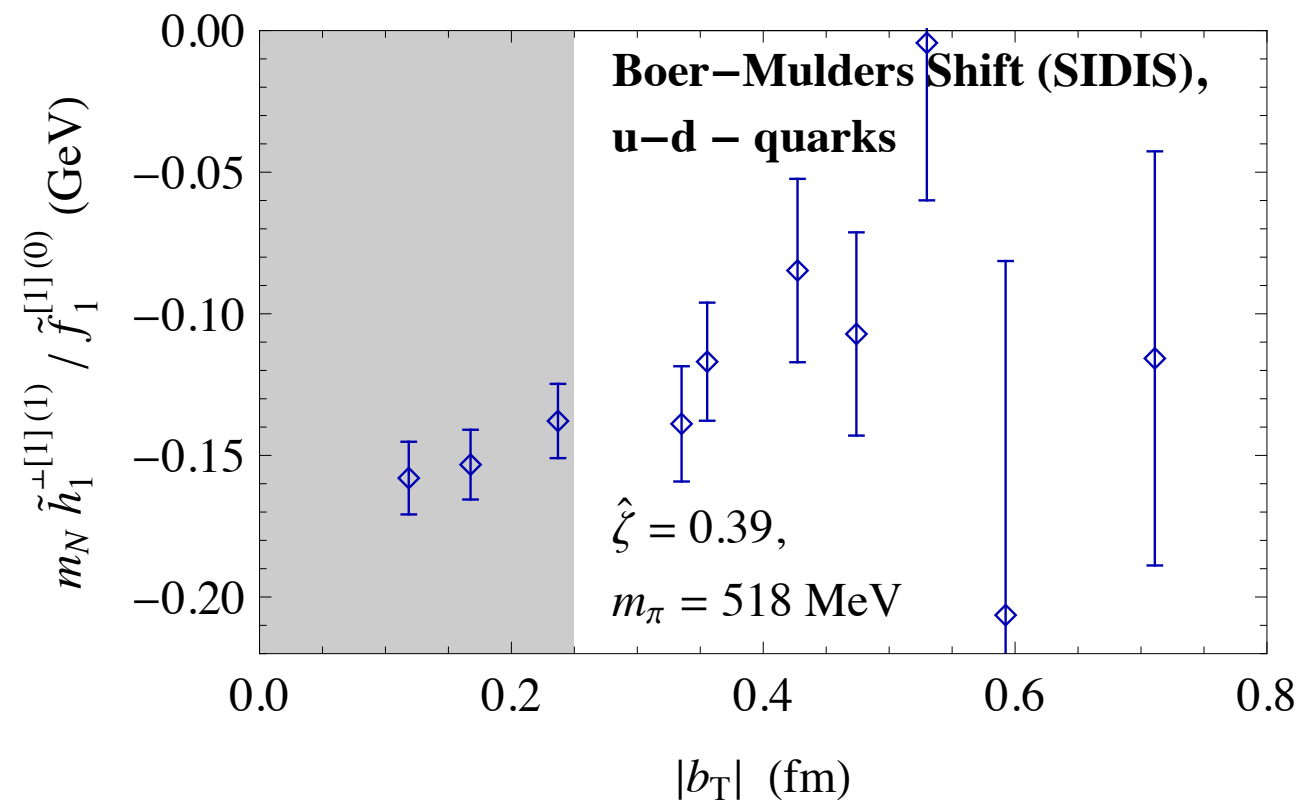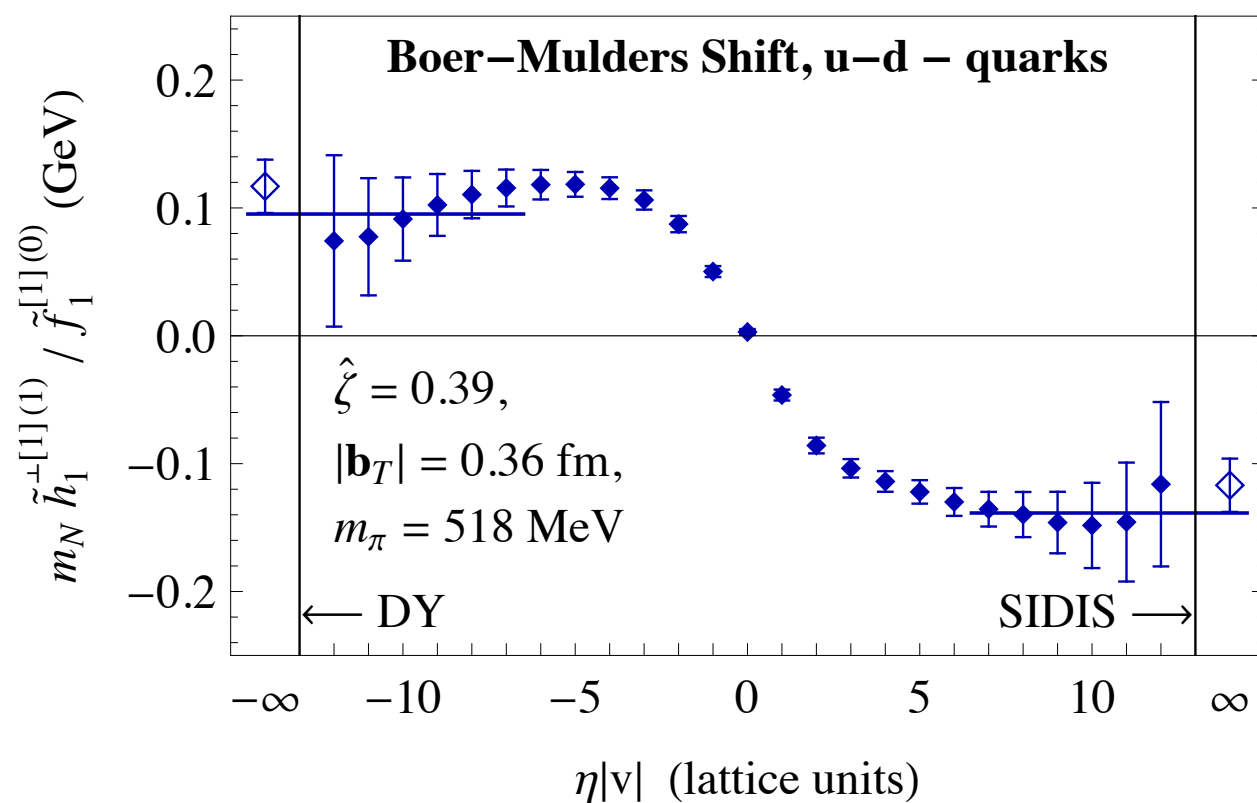
Compatible with $h_1^{\perp u,d}$ both negative in SIDIS and $|h_1^{\perp u}|$ significantly larger than $|h_1^{\perp d}|$ (comes in addition to u-quark dominance due to the electric charge squared)

Lattice calculation

After taking Mellin moments *and* Bessel transverse moments of the Sivers function, one has a well-defined quantity $<k_T \times s_T>(n, \mathcal{B}_T)$, that can be evaluated on the lattice

[Musch, Hägler, Engelhardt, Negele & Schäfer, PRD 85 (2012) 094510]
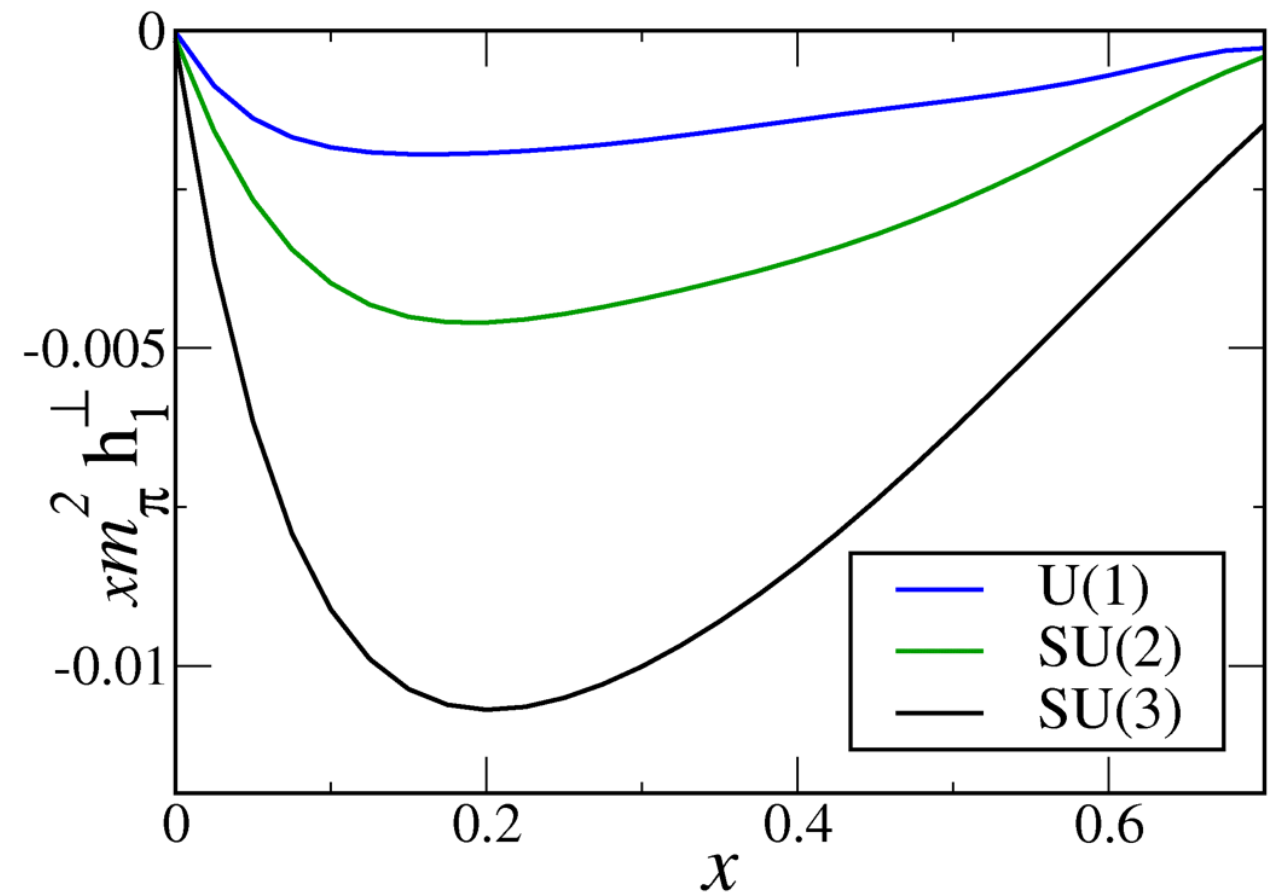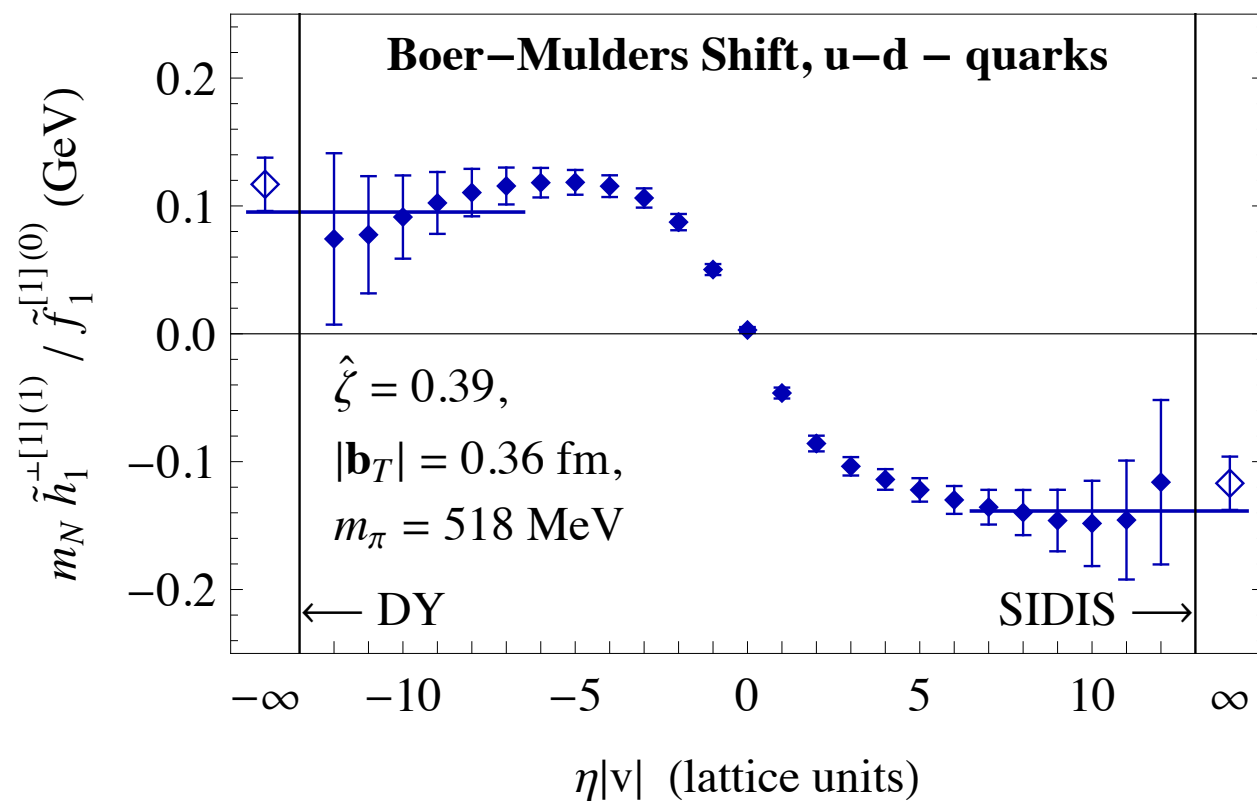
Compatible with $h_1^{\perp u,d}$ both negative in SIDIS and $|h_1^{\perp u}|$ significantly larger than $|h_1^{\perp d}|$ (comes in addition to u-quark dominance due to the electric charge squared)

Using the lensing function to get from GPDs to TMDs, also yields a negative $h_1^{\perp}$

[Gamberg & Schlegel '09]

Bacchetta, Conti, Radici, PRD 78 (2008) 074010: diquark spectator model

$$x\, h_1^{\perp(1)}(x)$$
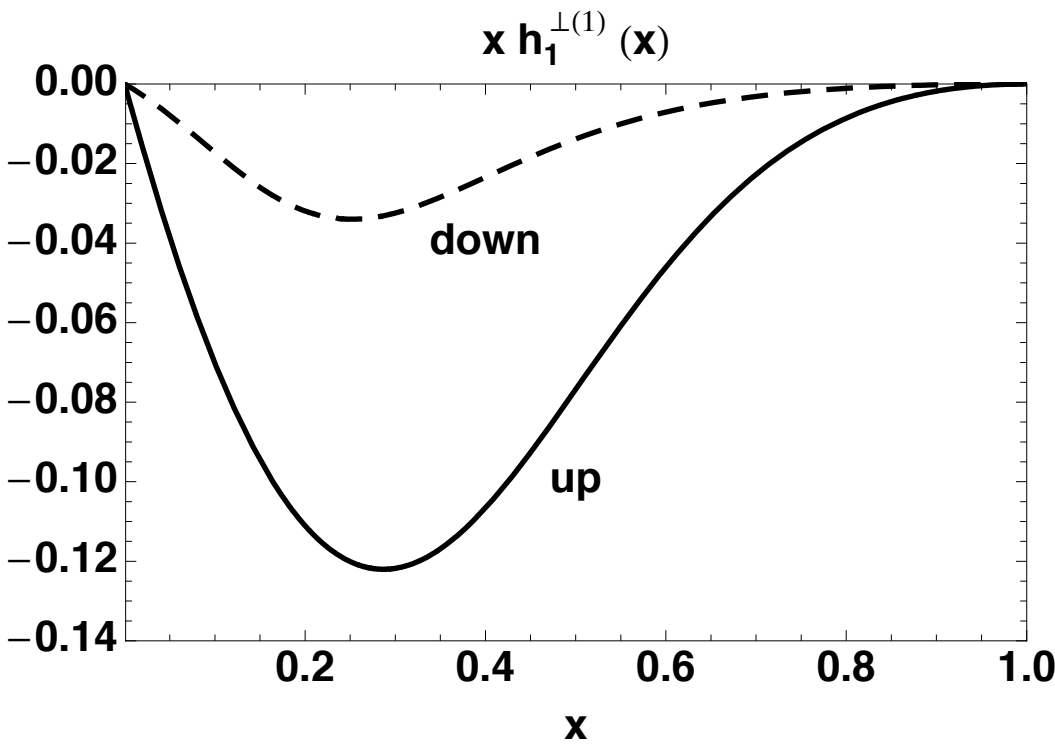
Pasquini, Yuan, PRD 78 (2008) 074010: lightcone quark model, S-P interference (dashed), P-D interference (dotted)

$$h_1^{\perp(1)u}$$

$$h_1^{\perp(1)d}$$
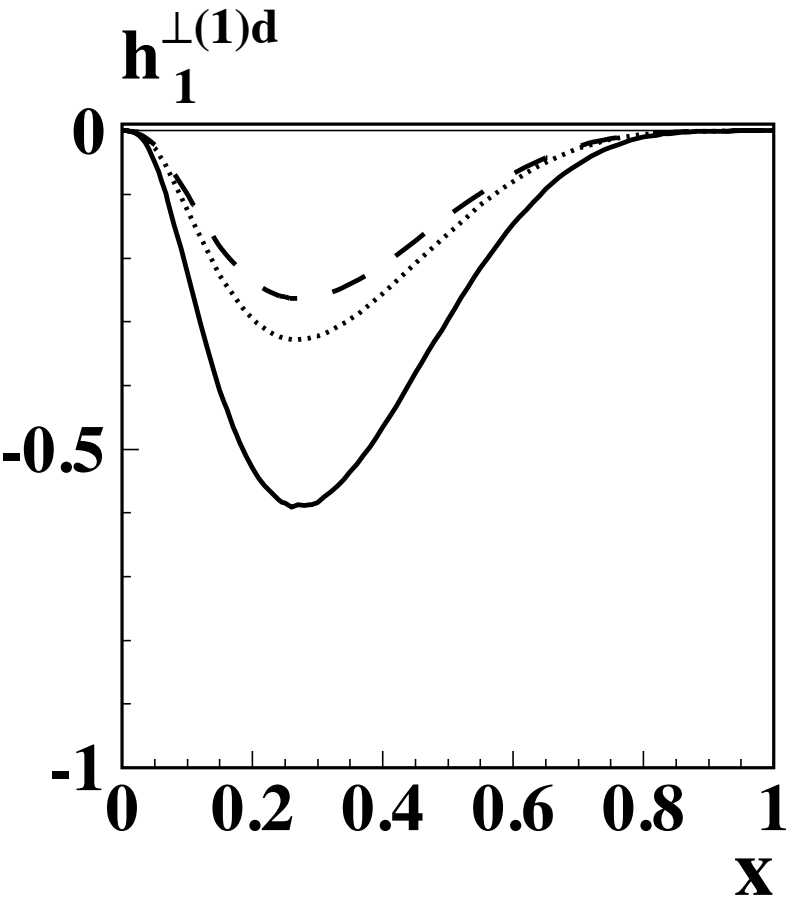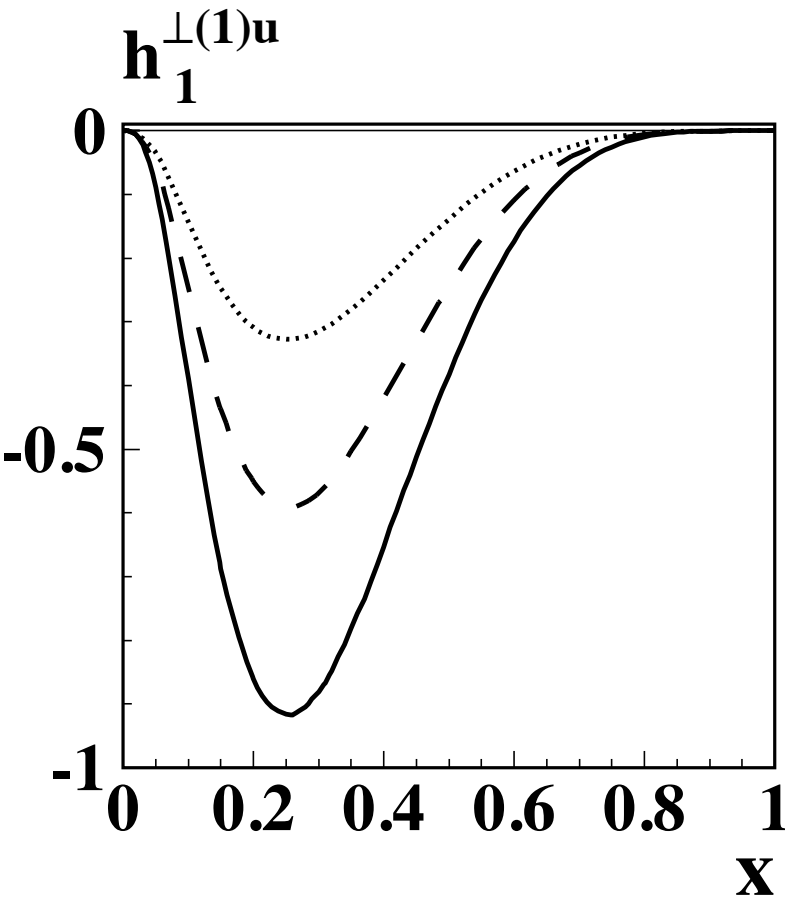
Alternative explanations

- QCD vacuum effect [Nachtmann & Reiter, ZPC 1984; Brandenburg, Nachtmann & Mirkes, ZPC1993; Botz, Haberl & Nachtmann, ZPC 1994]

- Higher twist effect [Brandenburg, Brodsky, Khoze & D. Mueller, PRL 1994; Eskola, Hoyer, Vanttinen & R. Vogt, PLB 1994]

- Nuclear effect (deuterium - tungsten comparison rules this out)

- Nuclear enhanced higher twist effect [Fries, Schäfer, Stein, B. Müller, NPB 2000]

- Fixed-target experiments problem [Lambertsen & Vogelsang, PRD 2016]

- Resummation [yes: Chiapetta & Le Bellac, ZPC 1986; no: D.B. & Vogelsang, PRD 2006]

Lam-Tung at LHC: CMS

CMS data of Z production at $\sqrt{s} = 8\,\text{TeV}$

Roughly 60% qG and 40% qqbar

LT violation due to two-gluon emission
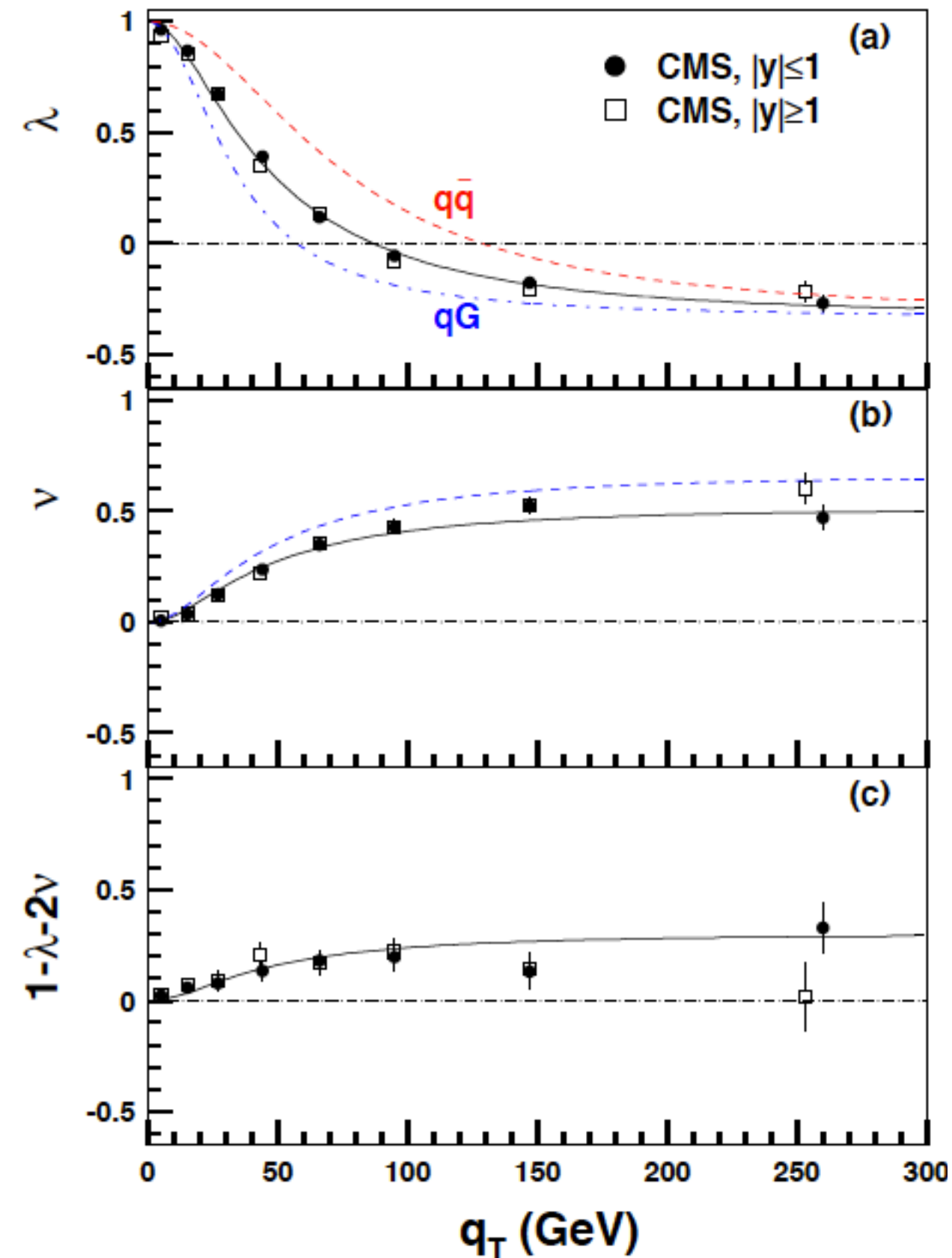
[Peng, Chang, McClellan, Teryaev, 2015]   [Lambertsen & Vogelsang, PRD 2016]

Lam-Tung at LHC: ATLAS

ATLAS data of Z production at $\sqrt{s} = 8$ TeV show a large LT violation at large q_T that is puzzling

$$\frac{dN}{d\Omega} = \frac{3}{16\pi}\left[1 + \cos^2\theta + \frac{A_0}{2}(1 - 3\cos^2\theta)\right.$$

$$\left. + A_1 \sin 2\theta \cos\phi + \frac{A_2}{2}\sin^2\theta \cos 2\phi\right]$$
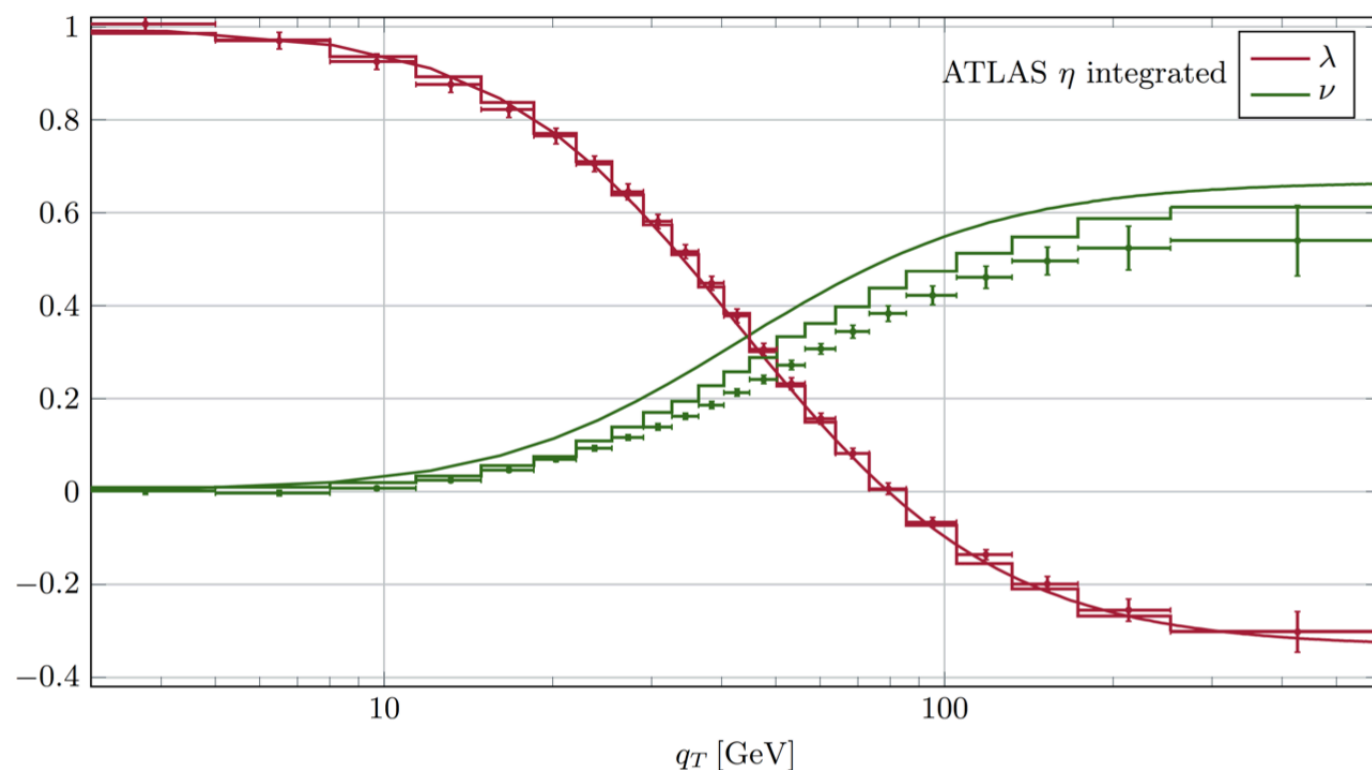
$$1 - \lambda - 2\nu = (A_0 - A_2)\frac{4}{2 + A_0}$$

[Lambertsen & Vogelsang, PRD 2016]

Lam-Tung at LHC: ATLAS
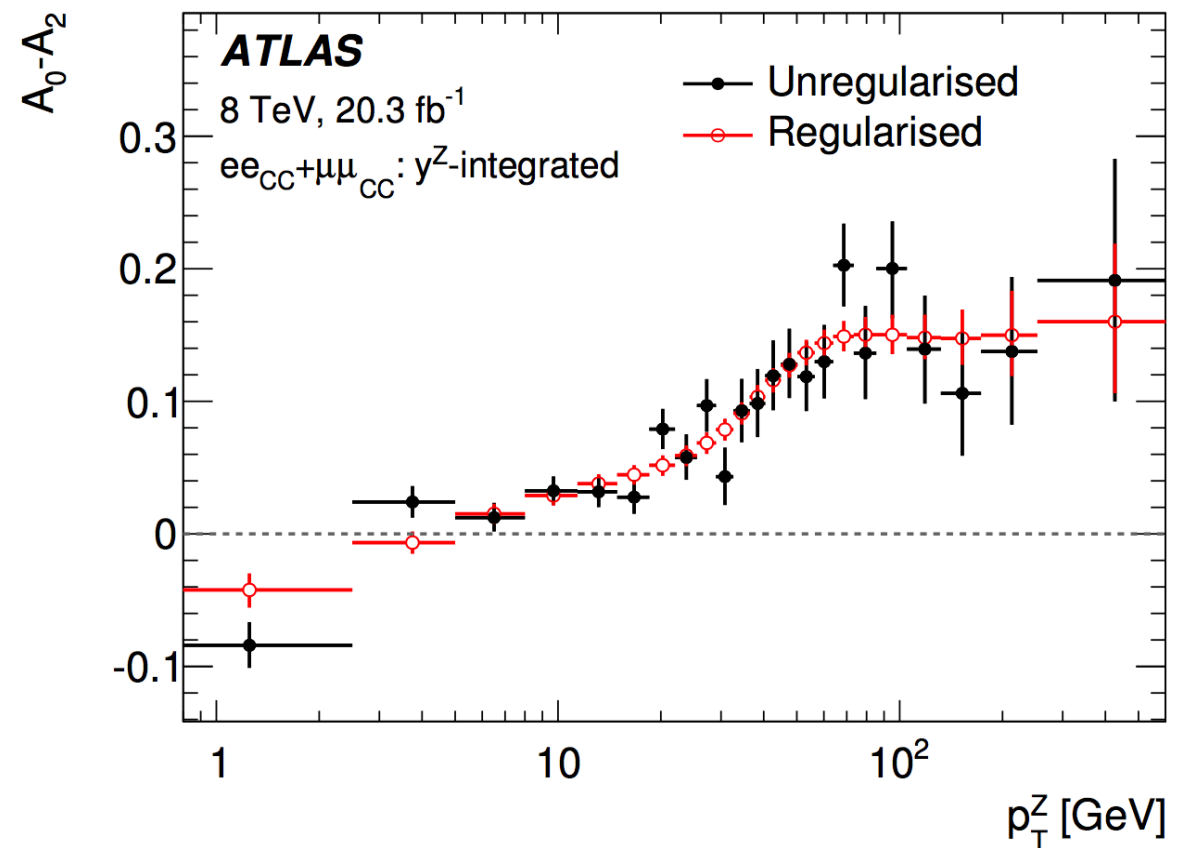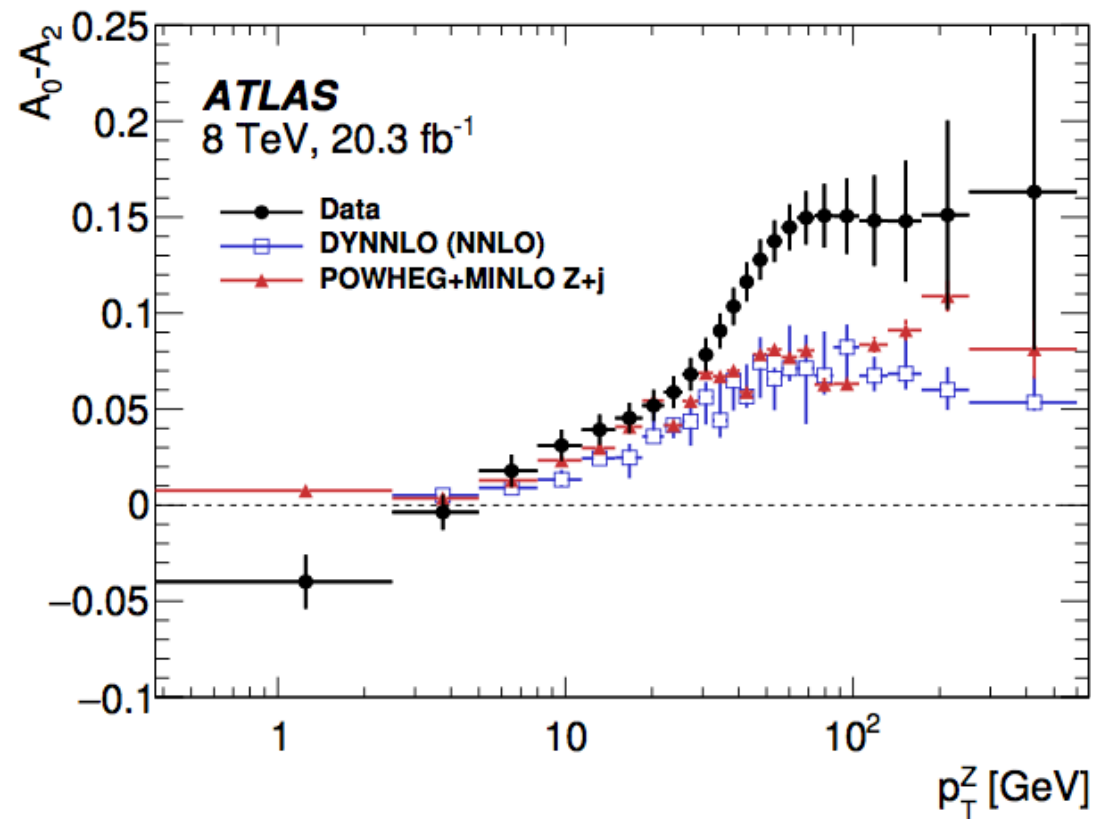
ATLAS data of Z production at $\sqrt{s} = 8\,\text{TeV}$ show a large LT violation at large q_T that is puzzling

$$\frac{dN}{d\Omega} = \frac{3}{16\pi}\left[1 + \cos^2\theta + \frac{A_0}{2}(1 - 3\cos^2\theta)\right.$$

$$\left. + A_1 \sin2\theta \cos\phi + \frac{A_2}{2}\sin^2\theta \cos2\phi\right]$$

$$1 - \lambda - 2\nu = (A_0 - A_2)\frac{4}{2 + A_0}$$

[Lambertsen & Vogelsang, PRD 2016]

NNLO

NLO (ATLAS): $\quad \chi^2/N_{\mathrm{data}} = 185.8/38 = 4.89$

NNLO (ATLAS): $\quad \chi^2/N_{\mathrm{data}} = 68.3/38 = 1.80$.

Gauld, Gehrmann-De Ridder, Gehrmann, Glover & Huss, 2017

NLO (CMS): $\quad \chi^2/N_{\mathrm{data}} = 24.5/14 = 1.75$

NNLO (CMS): $\quad \chi^2/N_{\mathrm{data}} = 14.2/14 = 1.01$
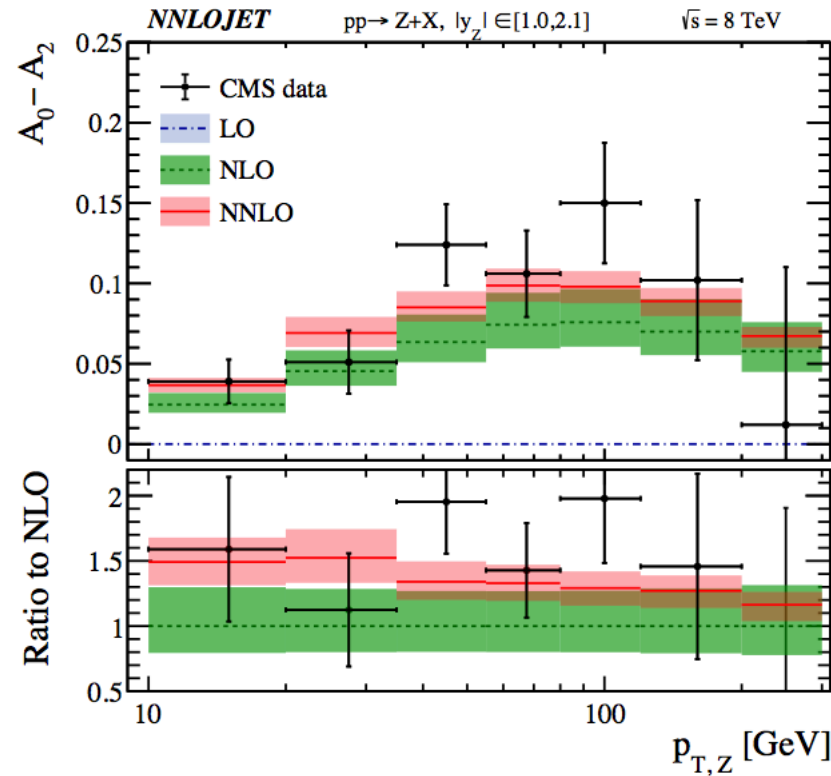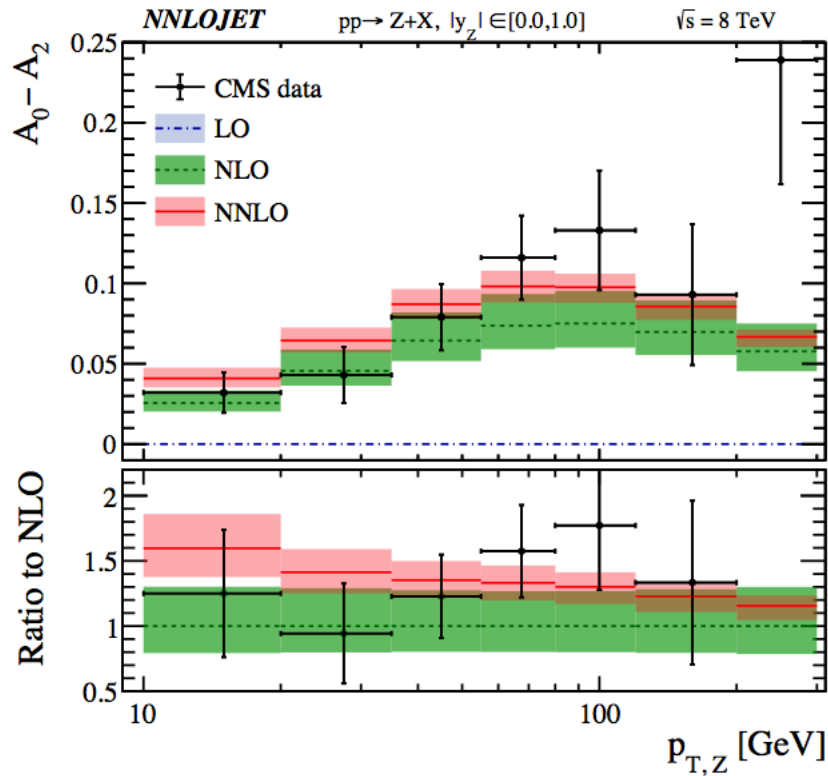
NNLO
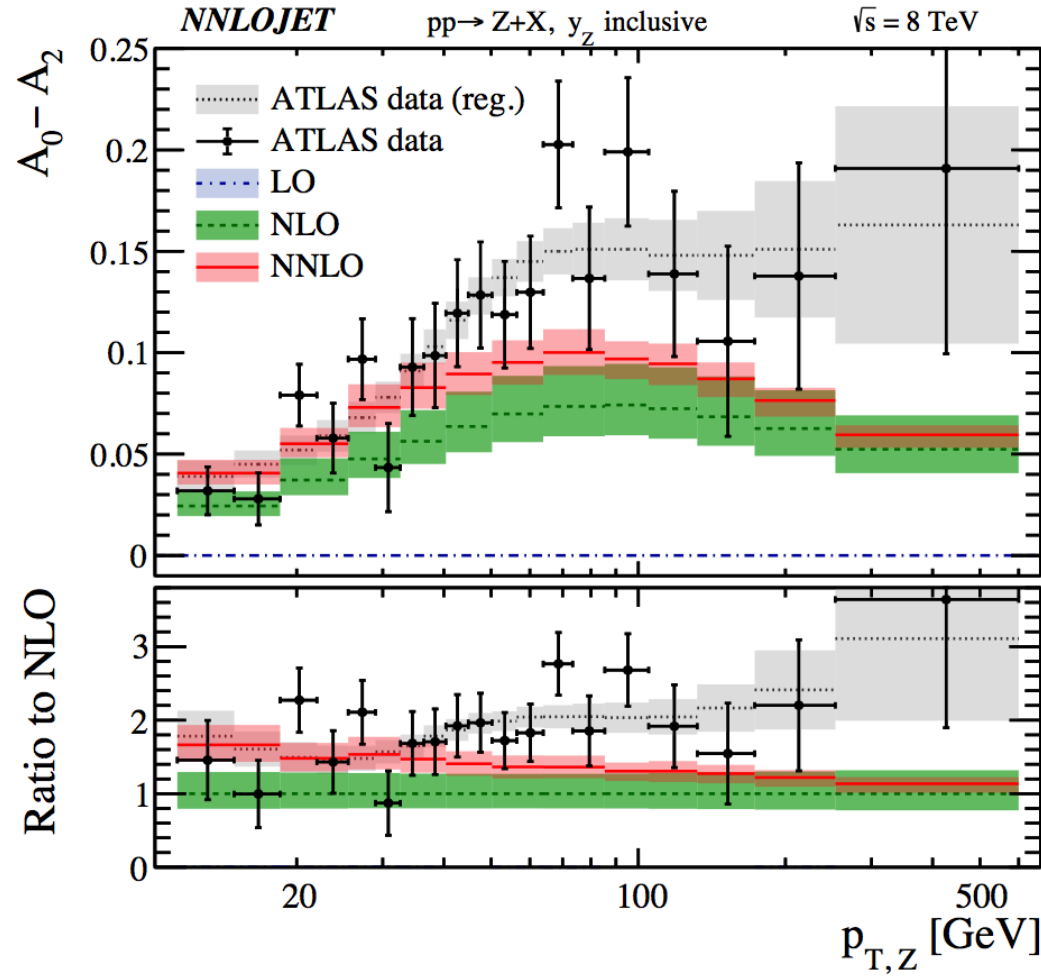
NLO (ATLAS): $\chi^2/N_{\text{data}} = 185.8/38 = 4.89$

NNLO (ATLAS): $\chi^2/N_{\text{data}} = 68.3/38 = 1.80$.

Gauld, Gehrmann-De Ridder, Gehrmann, Glover & Huss, 2017

NLO (CMS): $\chi^2/N_{\text{data}} = 24.5/14 = 1.75$

NNLO (CMS): $\chi^2/N_{\text{data}} = 14.2/14 = 1.01$
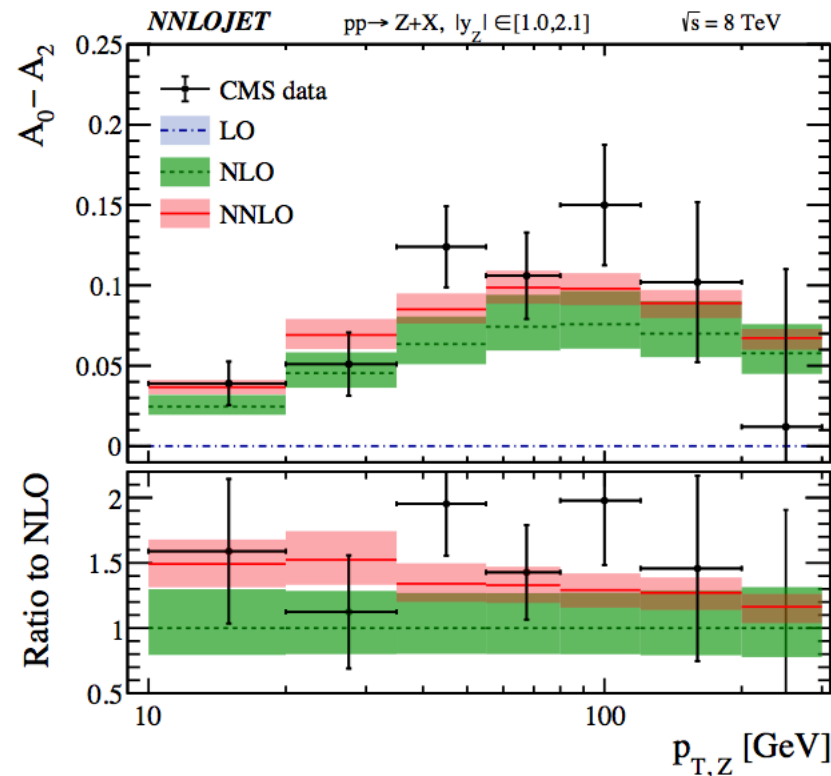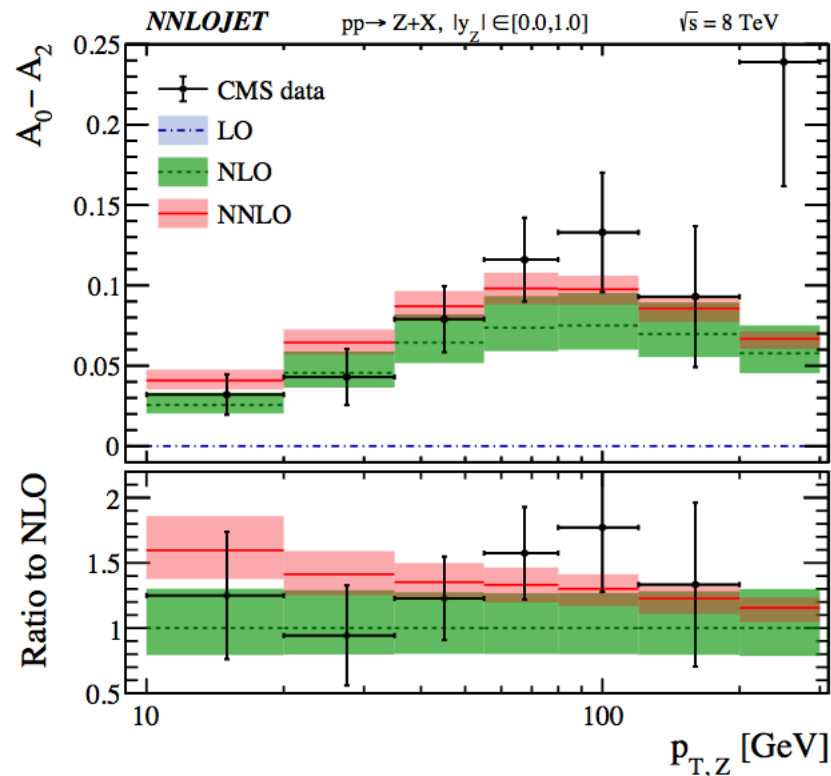
$$A_0 - A_2 = 2(1 - 2\mathcal{F})$$

A0-A2 is rotationally invariant

NNLO
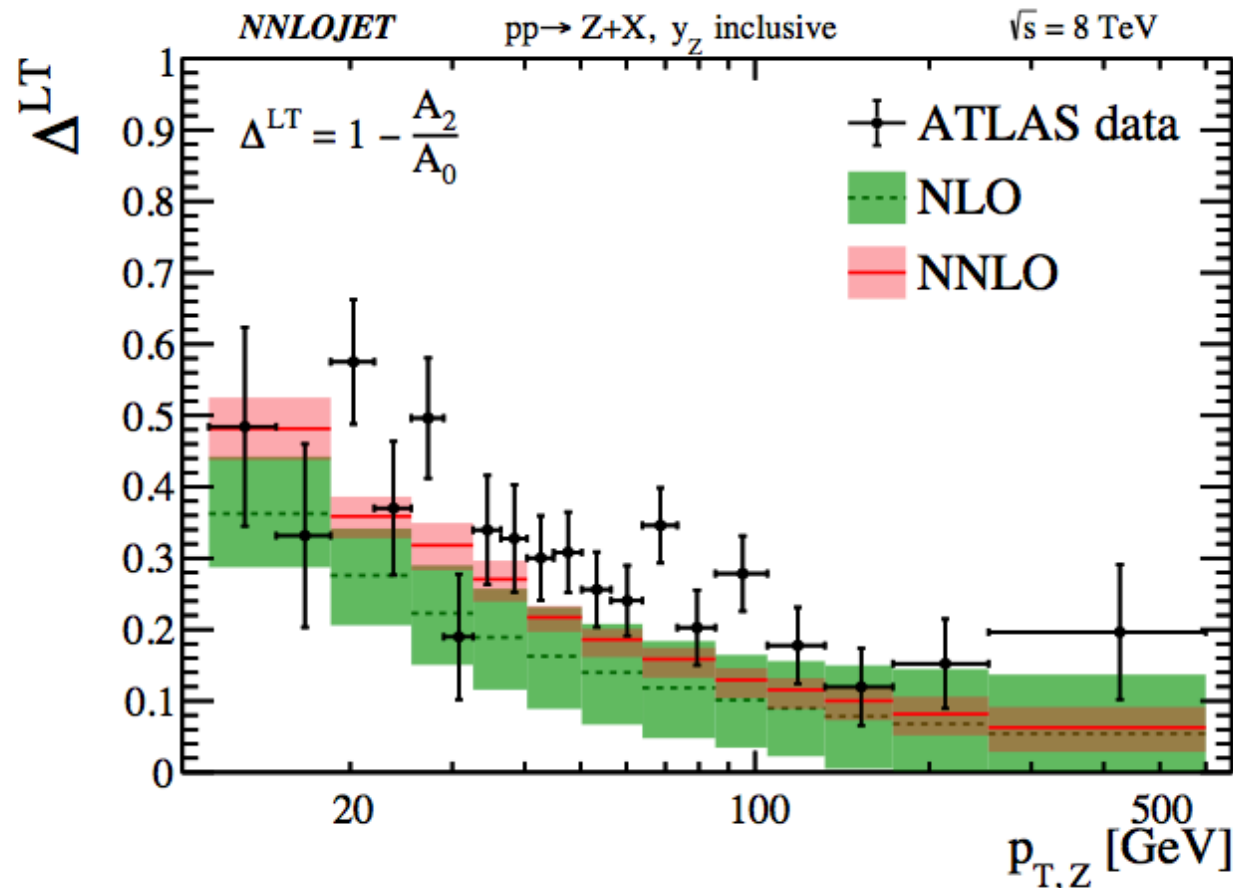
Gauld, Gehrmann-De Ridder, Gehrmann, Glover & Huss, 2017:

"While there is some tendency for the data to prefer a stronger Lam–Tung violation for $p_{T,Z} > 40$ GeV, more precise data is required to confirm this behaviour"
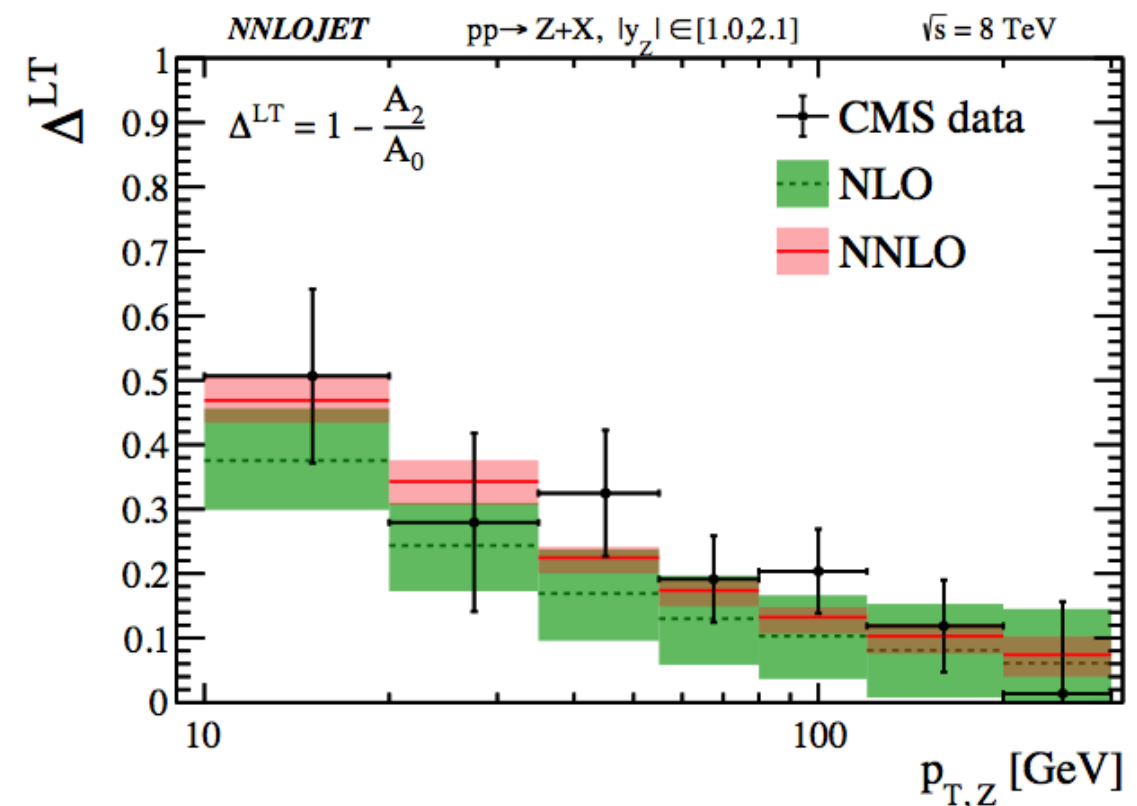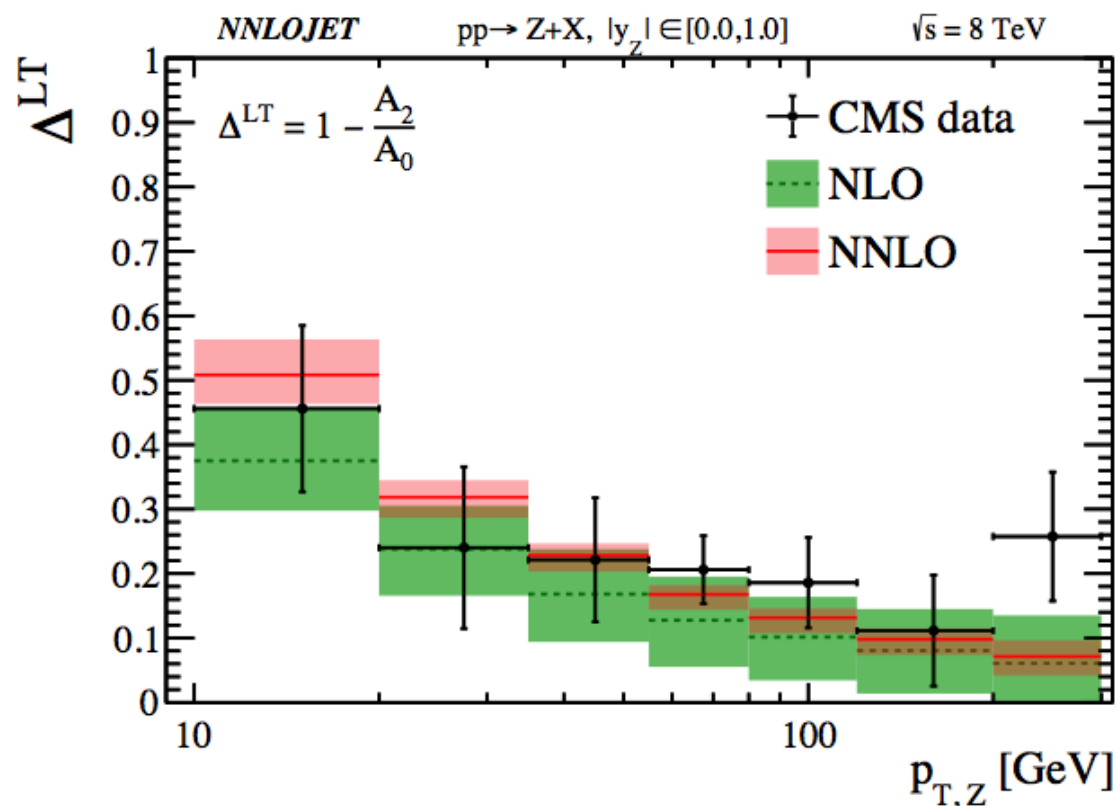
Color flow dependence of cos(2φ)

Color entanglement in DY

In 2014 Buffing & Mulders claimed that the double Boer-Mulders (dBM) term suffers from color-entanglement and is actually of opposite sign and suppressed by an additional color factor

$$d\sigma_{\text{DY}} = \text{Tr}_c[U_-^\dagger[p_2]\Phi(x_1, p_{1T})U_-[p_2]\Gamma^*$$
$$\times U_-^\dagger[p_1]\bar{\Phi}(x_2, p_{2T})U_-[p_1]\Gamma]$$
$$\neq \frac{1}{N_c}\Phi^{[-]}(x_1, p_{1T})\Gamma^*\bar{\Phi}^{[-^\dagger]}(x_2, p_{2T})\Gamma$$

$$\sigma(x_1, x_2, q_T) \sim \frac{1}{N_c} f_{R_{G1}R_{G2}}^{[U_1, U_2]}\Phi^{[U_1]}(x_1, p_{1T})$$
$$\otimes \bar{\Phi}^{[U_2]}(x_2, p_{2T})\hat{\sigma}(x_1, x_2),$$

starting point based on Bomhof, Mulders & Pijlman, EPJC 2006

[Buffing & Mulders, PRL 2014]

TABLE I. The factor $f_{R_{G1}R_{G2}}^{[-,-^\dagger]}$ as a function of the gluonic pole ranks of both the quark and antiquark correlator in the Drell-Yan process.

R_G for $\bar{\Phi}^{[-^\dagger]}$	R_G for $\Phi^{[-]}$		
	0	1	2
0	1	1	1
1	1	$-\frac{1}{N_c^2-1}$	$\frac{N_c^2+2}{(N_c^2-2)(N_c^2-1)}$
2	1	$\frac{N_c^2+2}{(N_c^2-2)(N_c^2-1)}$	$\frac{3N_c^4-8N_c^2-4}{(N_c^2-2)^2(N_c^2-1)}$

Color disentanglement in DY

Recently it was shown in a (sufficiently rich) model context that at the first potentially problematic order the gauge links *do* in fact disentangle

- Consider all possible final-state cuts (for the Glauber region):

[DB, Brodsky & Hwang, 2003]

color entangled diagram

$$\propto \mathrm{Tr}_c\left(t^a t^a t^b t^b\right) = \frac{1}{N_c}\mathrm{Tr}_c\left(t^a t^a\right)\mathrm{Tr}_c\left(t^b t^b\right)$$

$$\propto \mathrm{Tr}_c\left(t^a t^b t^a t^b\right) = -\frac{1}{N_c^2-1}\frac{1}{N_c}\mathrm{Tr}_c\left(t^a t^a\right)\mathrm{Tr}_c\left(t^b t^b\right)$$

Buffing-Mulders color factor

Triple gluon vertex saves the day!

$$\propto \mathrm{Tr}_c\left(if^{abc}t^a t^b t^c\right) = -\frac{N_c^2}{N_c^2-1}\frac{1}{N_c}\mathrm{Tr}_c\left(t^a t^a\right)\mathrm{Tr}_c\left(t^b t^b\right)$$

After taking sum over all cut diagrams the regular color factor results
[DB, Van Daal, Gaunt, Kasemets & Mulders, 2017]

Sign change of $h_1^\perp$

How about measuring the overall sign change of $h_1^\perp$?

$$h_1^{\perp[\mathrm{SIDIS}]}(x, k_T^2) = -h_1^{\perp[\mathrm{DY}]}(x, k_T^2)$$

Chiral-odd functions always appear in pairs, hence not straightforward

If one restricts to valence quarks and assumes up-quark dominance and LO,
it is possible:

- $(e\, p^\uparrow \to e'\, h\, X)/(e\, p \to e'\, h\, X) \propto h_1^u/h_1^{\perp\, u\, [\mathrm{SIDIS}]}$

- $(\pi^-\, p^\uparrow \to \ell\bar{\ell}X)/(\pi^-\, p \to \ell\bar{\ell}X) \propto h_1^u/h_1^{\perp\, u\, [\mathrm{DY}]}$ (or $\bar{p}\, p^\uparrow \to \ell\bar{\ell}X \propto h_1^{\perp\, u\, [\mathrm{DY}]}h_1^u$)

h_1: distribution of transversely polarized quarks inside transversely polarized hadrons

Including *d*-quarks requires many more observables, using e⁺e⁻ and pp collisions and
exploiting Λ↑ and dihadron (DiFF) final states, to achieve a closed system

$\gamma^{(*)}$-jet production

Azimuthal asymmetry in γ-jet production
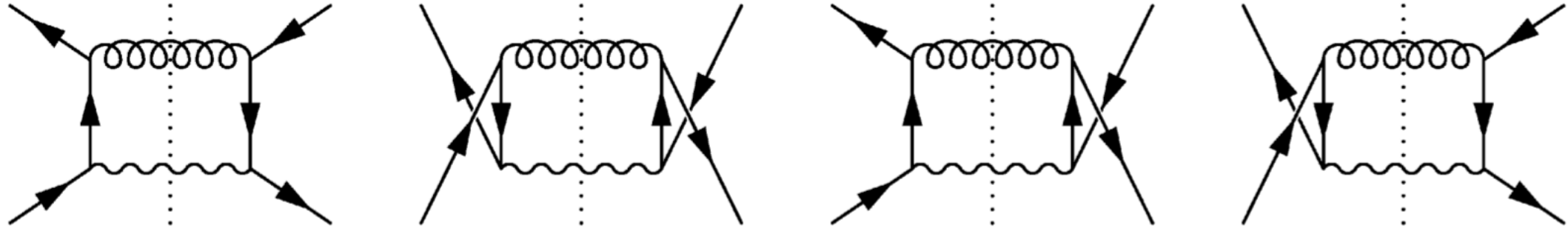
$q\bar{q} \to \gamma g$: $\quad \dfrac{N^2}{N^2-1} \Phi_q^{[+(\square^{\dagger})]} \otimes \bar{\Phi}_q^{[+^{\dagger}(\square)]} \otimes |H|^2 \otimes \Delta_g^{[-][-^{\dagger}]} - \dfrac{1}{N^2-1} \Phi_q^{[-]} \otimes \bar{\Phi}_q^{[-^{\dagger}]} \otimes |H|^2 \otimes \Delta_g^{[-][-^{\dagger}]}$

If the Wilson loop ($\square$) does not matter, then link dependence is irrelevant in general and the naive result is recovered:

$$\frac{d\sigma^{h_1 h_2 \to \gamma \mathrm{jet} X}}{d\eta_\gamma \, d\eta_j \, d^2 \boldsymbol{K}_{\gamma\perp} \, d^2 \boldsymbol{q}_\perp} = \frac{1}{\pi^2} \frac{d\sigma^{h_1 h_2 \to \gamma \mathrm{jet} X}}{d\eta_\gamma \, d\eta_j \, d K_{\gamma\perp}^2 \, d q_\perp^2} \left(1 + \mathcal{A}(y, x_1, x_2, \boldsymbol{q}_\perp^2) \cos 2(\phi_\perp - \phi_\gamma) \right)$$

$$\mathcal{A}(y, x_1, x_2, \boldsymbol{q}_\perp^2) = v(x_1, x_2, \boldsymbol{q}_\perp^2) R(y, x_1, x_2, \boldsymbol{q}_\perp^2)$$

0.1-0.5 in mid-rapidity region
at Tevatron kinematics in p$\bar{\mathrm{p}}$

for p$\bar{\mathrm{p}}$ same as in DY

[D.B., Mulders, Pisano, PBL 2008]   gluon contribution to $\mathcal{A}$ power suppressed for real γ

Process dependence of unpolarized quark TMD

$$f_1^{[+]}(x, p_T^2) = f_1^{[-]}(x, p_T^2)$$

$$f_1^{[\square+]}(x, p_T^2) \neq f_1^{[+]}(x, p_T^2)$$

[D.B., Buffing, Mulders, JHEP 2015]

Irrespective of whether one can isolate the function with an additional loop from experiment, one can study particular Mellin-Bessel moments of it on the lattice:

$$\frac{\tilde{f}_1^{[1](1)[\square+]}(\boldsymbol{b}_T^2; \mu, \zeta)}{\tilde{f}_1^{[1](1)[+]}(\boldsymbol{b}_T^2; \mu, \zeta)} = \frac{\langle P | \overline{\psi}(0, 0_T) \gamma^+ U_{[0,b]}^{[+]} U_{[b,0]}^{[-]} U_{[0,b]}^{[+]} \psi(0, b_T) | P \rangle}{\langle P | \overline{\psi}(0, 0_T) \gamma^+ U_{[0,b]}^{[+]} \psi(0, b_T) | P \rangle}$$

This will give us information on how important the flux of $F^{\mu\nu}$ through the loop is and hence how important the process dependence effects are or can be

This especially relevant to know for gluon TMD effects

E.g. the gluon Sivers TMD with [+,−] link structure at small-x is entirely determined by the loop and in turn fully determines the SSA in certain processes

D.B., Echevarria, Mulders, Jian Zhou, PRL 2015

Gluons TMDs

The gluon correlator:

$$\Gamma_g^{\mu\nu[\mathcal{U},\mathcal{U}']}(x,k_T) \equiv \text{F.T.}\langle P|\text{Tr}_c\left[F^{+\nu}(0)\,\mathcal{U}_{[0,\xi]}\,F^{+\mu}(\xi)\,\mathcal{U}'_{[\xi,0]}\right]|P\rangle$$

For unpolarized protons:

$$\Gamma_U^{\mu\nu}(x,\boldsymbol{p}_T) = \frac{x}{2}\left\{-g_T^{\mu\nu}\,f_1^g(x,\boldsymbol{p}_T^2) + \left(\frac{p_T^\mu p_T^\nu}{M_p^2} + g_T^{\mu\nu}\frac{\boldsymbol{p}_T^2}{2M_p^2}\right)h_1^{\perp\,g}(x,\boldsymbol{p}_T^2)\right\}$$

Gluons TMDs

The gluon correlator:

$$\Gamma_g^{\mu\nu[\mathcal{U},\mathcal{U}']}(x, k_T) \equiv \text{F.T.}\langle P|\text{Tr}_c \left[F^{+\nu}(0)\,\mathcal{U}_{[0,\xi]}\,F^{+\mu}(\xi)\,\mathcal{U}'_{[\xi,0]} \right]|P\rangle$$

For unpolarized protons:

$$\Gamma_U^{\mu\nu}(x, \boldsymbol{p}_T) = \frac{x}{2}\left\{ -g_T^{\mu\nu}\,f_1^g(x, \boldsymbol{p}_T^2) + \left(\frac{p_T^\mu p_T^\nu}{M_p^2} + g_T^{\mu\nu}\frac{\boldsymbol{p}_T^2}{2M_p^2} \right) h_1^{\perp\,g}(x, \boldsymbol{p}_T^2) \right\}$$

unpolarized gluon TMD

Gluons TMDs

The gluon correlator:

$$\Gamma_g^{\mu\nu[\mathcal{U},\mathcal{U}']}(x,k_T) \equiv \text{F.T.}\langle P|\text{Tr}_c\left[F^{+\nu}(0)\,\mathcal{U}_{[0,\xi]}\,F^{+\mu}(\xi)\,\mathcal{U}'_{[\xi,0]}\right]|P\rangle$$

For unpolarized protons:

$$\Gamma_U^{\mu\nu}(x,\boldsymbol{p}_T) = \frac{x}{2}\left\{-g_T^{\mu\nu}\,f_1^g(x,\boldsymbol{p}_T^2) + \left(\frac{p_T^\mu p_T^\nu}{M_p^2} + g_T^{\mu\nu}\frac{\boldsymbol{p}_T^2}{2M_p^2}\right)h_1^{\perp\,g}(x,\boldsymbol{p}_T^2)\right\}$$

unpolarized gluon TMD

linearly polarized gluon TMD

Gluons inside *unpolarized* protons can be polarized!

[Mulders, Rodrigues, 2001]

Gluons TMDs

The gluon correlator:

$$\Gamma_g^{\mu\nu[\mathcal{U},\mathcal{U}']}(x,k_T) \equiv \mathrm{F.T.}\langle P|\mathrm{Tr}_c\left[F^{+\nu}(0)\,\mathcal{U}_{[0,\xi]}\,F^{+\mu}(\xi)\,\mathcal{U}'_{[\xi,0]}\right]|P\rangle$$

For unpolarized protons:

$$\Gamma_U^{\mu\nu}(x,\boldsymbol{p}_T) = \frac{x}{2}\left\{-g_T^{\mu\nu}\,f_1^g(x,\boldsymbol{p}_T^2) + \left(\frac{p_T^\mu p_T^\nu}{M_p^2} + g_T^{\mu\nu}\frac{\boldsymbol{p}_T^2}{2M_p^2}\right)h_1^{\perp\,g}(x,\boldsymbol{p}_T^2)\right\}$$

unpolarized gluon TMD

linearly polarized gluon TMD

Gluons inside *unpolarized* protons can be polarized!

[Mulders, Rodrigues, 2001]

$h_1^{\perp g}$ is k_T-even, chiral-even and T-even,
still it is process dependent
([+,+] = [−,−] and [+,−] = [−,+])
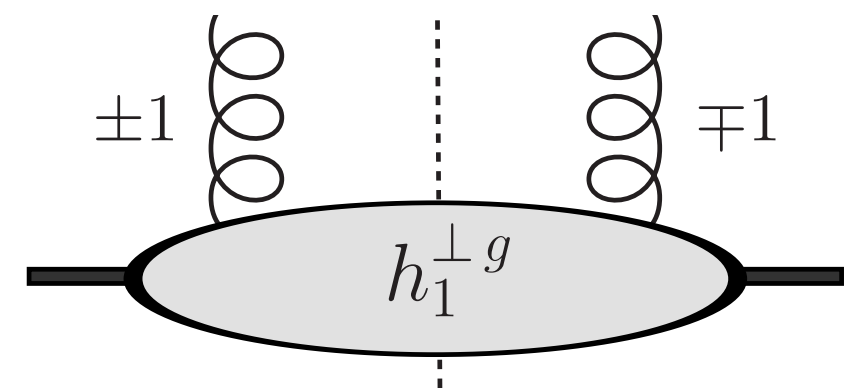
an interference between
±1 helicity gluon states

Azimuthal asymmetry in γ^*-jet production

Linear gluon polarization *not* suppressed in pp→γ^* jet X for $Q^2 \sim P_{\perp,\text{jet}}^2$ leading to a $\cos(2\varphi)$ asymmetry, where $\varphi=\varphi_T-\varphi_\perp$

In a hybrid factorization approach (assumed to be applicable at small x):

see e.g. Mueller, Xiao, Yuan, 2013

$$\frac{d\sigma^{pA\to\gamma^* qX}}{dP.S} = \sum_q x_p f_1^q(x_p)\left\{ xf_1^g(x,k_\perp)H_{\text{Born}} + \cos(2\phi)\, xh_1^{\perp g}(x,k_\perp)H_{\text{Born}}^{\cos(2\phi)}\right\}$$

[D.B., Mulders, Jian Zhou & Ya-jin Zhou, 2017]

Azimuthal asymmetry in γ*-jet production

Linear gluon polarization *not* suppressed in pp→γ* jet X for $Q^2 \sim P_{\perp,\text{jet}}^2$
leading to a cos(2φ) asymmetry, where φ=φ$_T$-φ$_\perp$

In a hybrid factorization approach (assumed to be applicable at small x):

see e.g. Mueller, Xiao, Yuan, 2013

$$\frac{d\sigma^{pA\to\gamma^*qX}}{dP.S} = \sum_q x_p f_1^q(x_p)\left\{xf_1^g(x,k_\perp)H_{\text{Born}} + \cos(2\phi)\,xh_1^{\perp g}(x,k_\perp)H_{\text{Born}}^{\cos(2\phi)}\right\}$$

[D.B., Mulders, Jian Zhou & Ya-jin Zhou, 2017]

This process probes the [+,−] link structure (at small x referred to as DP for
`dipole')

At high gluon density (large A and/or small x) the linear gluon polarization can
become maximal, as was first shown in the MV model for the CGC

$$xh_{1,DP}^{\perp g}(x,k_\perp) = 2xf_{1,DP}^g(x,k_\perp)$$

[Metz & Jian Zhou, 2011]

Linear gluon polarization at small x

There is no theoretical reason why $h_1^{\perp g}$ should be small, especially at small x

Linear gluon polarization at small x

There is no theoretical reason why $h_1^{\perp g}$ should be small, especially at small x

DGLAP evolution: $h_1^{\perp g}$ has the same 1/x growth as f_1

$$\tilde{h}_1^{\perp g}(x, b^2; \mu_b^2, \mu_b) = \frac{\alpha_s(\mu_b)C_A}{2\pi} \int_x^1 \frac{d\hat{x}}{\hat{x}} \left(\frac{\hat{x}}{x} - 1\right) f_{g/P}(\hat{x}; \mu_b) + \mathcal{O}(\alpha_s^2)$$

Linear gluon polarization at small x

There is no theoretical reason why $h_1^{\perp g}$ should be small, especially at small x

DGLAP evolution: $h_1^{\perp g}$ has the same $1/x$ growth as f_1

$$\tilde{h}_1^{\perp g}(x, b^2; \mu_b^2, \mu_b) = \frac{\alpha_s(\mu_b) C_A}{2\pi} \int_x^1 \frac{d\hat{x}}{\hat{x}} \left(\frac{\hat{x}}{x} - 1 \right) f_{g/P}(\hat{x}; \mu_b) + \mathcal{O}(\alpha_s^2)$$

The small-x limit of the DP correlator in the TMD formalism:

$$\Gamma^{[+,-]\, ij}(x, \boldsymbol{k}_T) \xrightarrow{x \to 0} \frac{k_T^i k_T^j}{2\pi L} \Gamma_0^{[\square]}(\boldsymbol{k}_T) \qquad U^{[\square]} = U_{[0,y]}^{[+]} U_{[y,0]}^{[-]}$$

$$\Gamma_U^{ij}(x, \boldsymbol{k}_T) = \frac{x}{2} \left[-g_T^{ij}\, f_1(x, \boldsymbol{k}_T^2) + \frac{k_T^{ij}}{M^2}\, h_1^\perp(x, \boldsymbol{k}_T^2) \right] \xrightarrow{x \to 0} \frac{k_T^i k_T^j}{2M^2}\, e(\boldsymbol{k}_T^2)$$

$$\lim_{x \to 0} x f_1(x, \boldsymbol{k}_T^2) = \frac{\boldsymbol{k}_T^2}{2M^2} \lim_{x \to 0} x h_1^\perp(x, \boldsymbol{k}_T^2) = \frac{\boldsymbol{k}_T^2}{2M^2}\, e(\boldsymbol{k}_T^2)$$

In the TMD formalism the DP $h_1^{\perp g}$ becomes maximal when x → 0

D.B., Cotogno, van Daal, Mulders, Signori & Jian Zhou, JHEP 2016

Sudakov suppression of linear gluon polarization

$P_\perp = 6\,\text{GeV}$
$y_{\gamma^*} : 1.5 - 2$
$y_q : 0.5 - 1.5$

$Q^2 : 30 - 55\,\text{GeV}^2$
$Q^2 : 55 - 80\,\text{GeV}^2$

y-axis: $-\cos(2\varphi)$

x-axis: $k_\perp[\text{GeV}]$
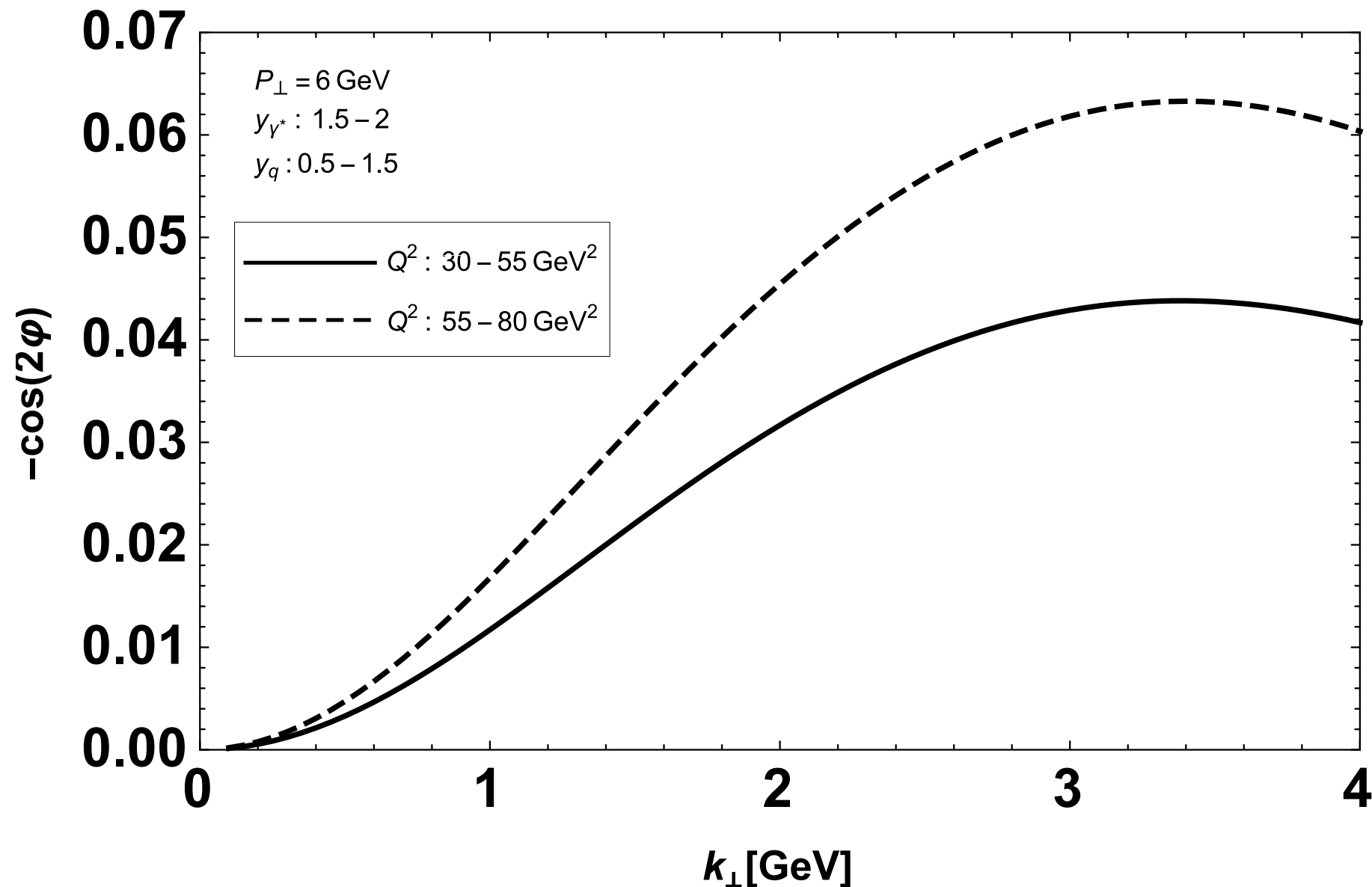
Despite the maximal DP linear gluon polarization at small x, there is Sudakov suppression of the cos(2φ) asymmetry in pA→γ^* jet X: ~5% asymmetry at RHIC

D.B., Mulders, Jian Zhou & Ya-jin Zhou, 2017

TMD evolution suppresses linear gluon polarization

The relative contribution of linearly polarized gluons in $pp \to HX$:

$$\mathcal{R}(Q_T) \equiv \frac{\mathcal{C}[w_H \, h_1^{\perp g} \, h_1^{\perp g}]}{\mathcal{C}[f_1^g \, f_1^g]} \qquad w_H = \frac{(\boldsymbol{p}_T \cdot \boldsymbol{k}_T)^2 - \frac{1}{2}\boldsymbol{p}_T^2 \boldsymbol{k}_T^2}{2M^4}$$

TMD evolution suppresses linear gluon polarization

The relative contribution of linearly polarized gluons in $pp \rightarrow HX$:
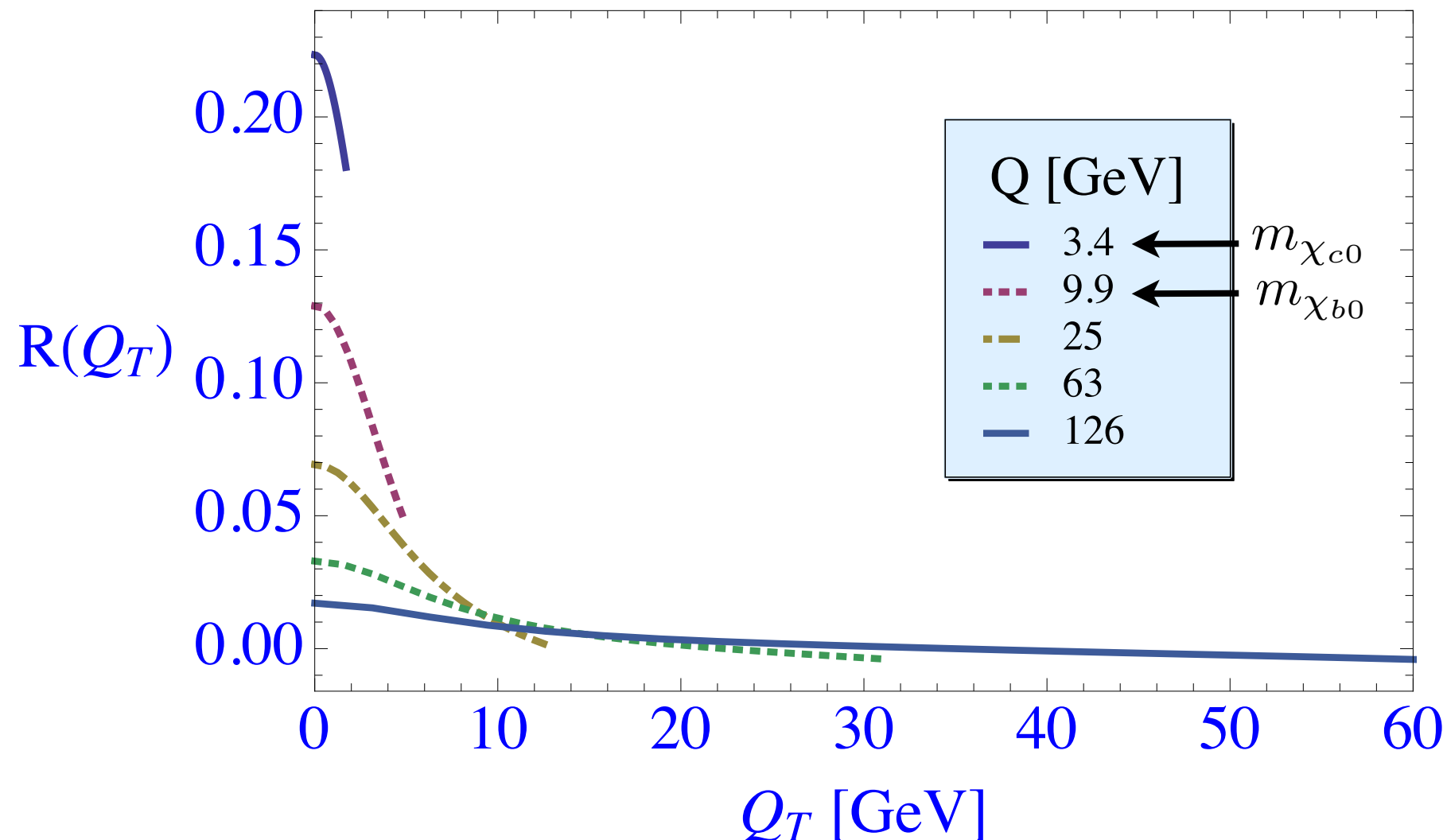
$$\mathcal{R}(Q_T) \equiv \frac{\mathcal{C}[w_H\, h_1^{\perp g}\, h_1^{\perp g}]}{\mathcal{C}[f_1^g\, f_1^g]}$$

$$w_H = \frac{(\boldsymbol{p}_T \cdot \boldsymbol{k}_T)^2 - \frac{1}{2}\boldsymbol{p}_T^2 \boldsymbol{k}_T^2}{2M^4}$$

TMD evolution suppresses this ratio with increasing energy

D.B. & Den Dunnen
2014

Conclusions

Conclusions

- All TMDs are process dependent, with observable and testable effects

- The process dependence can be studied through azimuthal asymmetries, e.g. in DY

- The Drell-Yan process is TMD factorizing and has no color entanglement problem

- The dBM effect can offer an explanation for the violation of the Lam-Tung relation

- Because the BM effect is larger for u-quarks than d-quarks and thereby enhances the u-quark dominance in DY and SIDIS, it allows for a sign change test

- In $p\bar{p} \rightarrow \gamma$ jet X an asymmetry arises that is a calculable factor times ν of DY. The process dependence (involving a Wilson loop) is expected to modify this result.

- In $pp \rightarrow \gamma^*$ jet X the dipole gluon TMDs dominate at small x, where they become Wilson loop matrix elements, yielding naively a maximal azimuthal asymmetry
Evolution suppresses the result considerably however (Sudakov suppression)

- Importance of the loop (and hence of the links) can be studied on the lattice

Back-up slides

Transverse momentum averaged LT violation
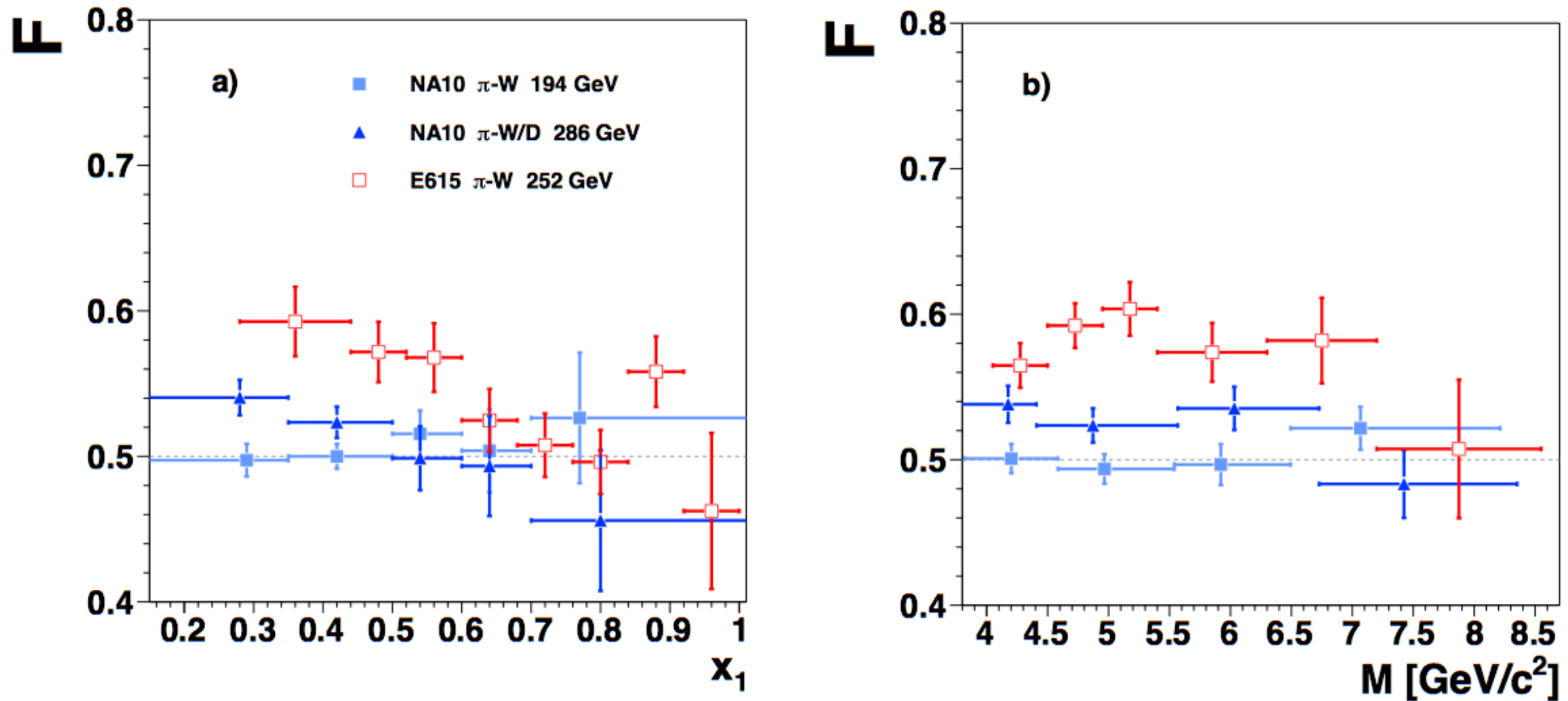
Absence of clear violation for 194 GeV data as function of x_1 or M, simply corresponds to the small pt-average of $2\nu+\lambda-1$: 0.01 ± 0.04

[FNAL-E866/NuSea Collaboration, L.Y. Zhu et al. PRL '09]

Lam-Tung on resonance

Usually Drell-Yan data is taken in the safe region Q=4-12 GeV, cutting out resonances

But vector particles yield same asymmetries
in the q q-bar channel
[Anselmino, Barone, Drago & Nikolaev, 2004]

NA10 data (1986) at 194 GeV on the Υ
is compatible with data above/below it,
but inconclusive about LT violation

In the gg channel no LT relation expected &
no (unsuppressed) contribution from $h_1^{\perp g}$
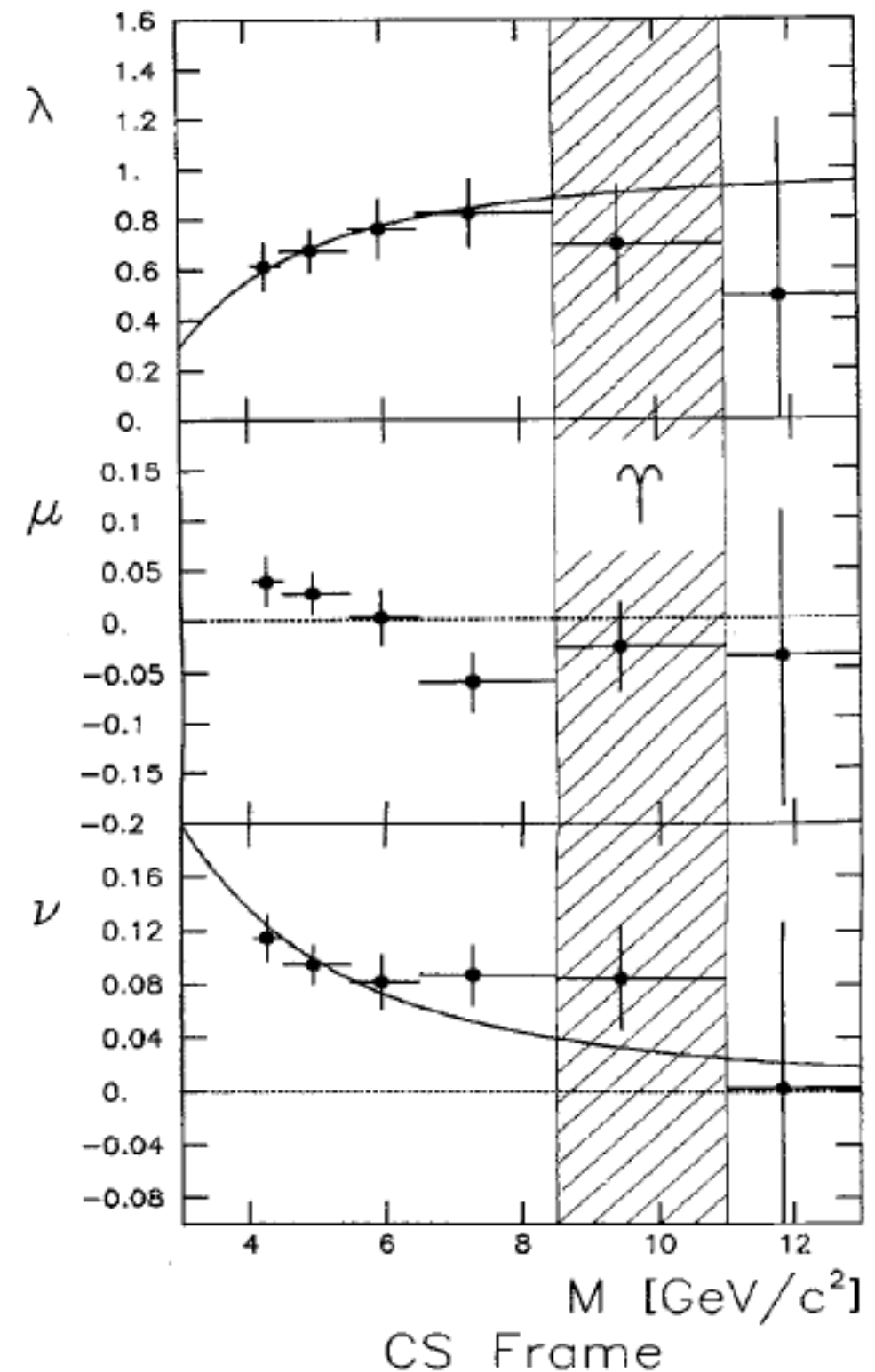
Lam-Tung on resonance

FNAL-E866/NuSea Collaboration, L.Y. Zhu et al.,
PRL 100 (2008) 062301
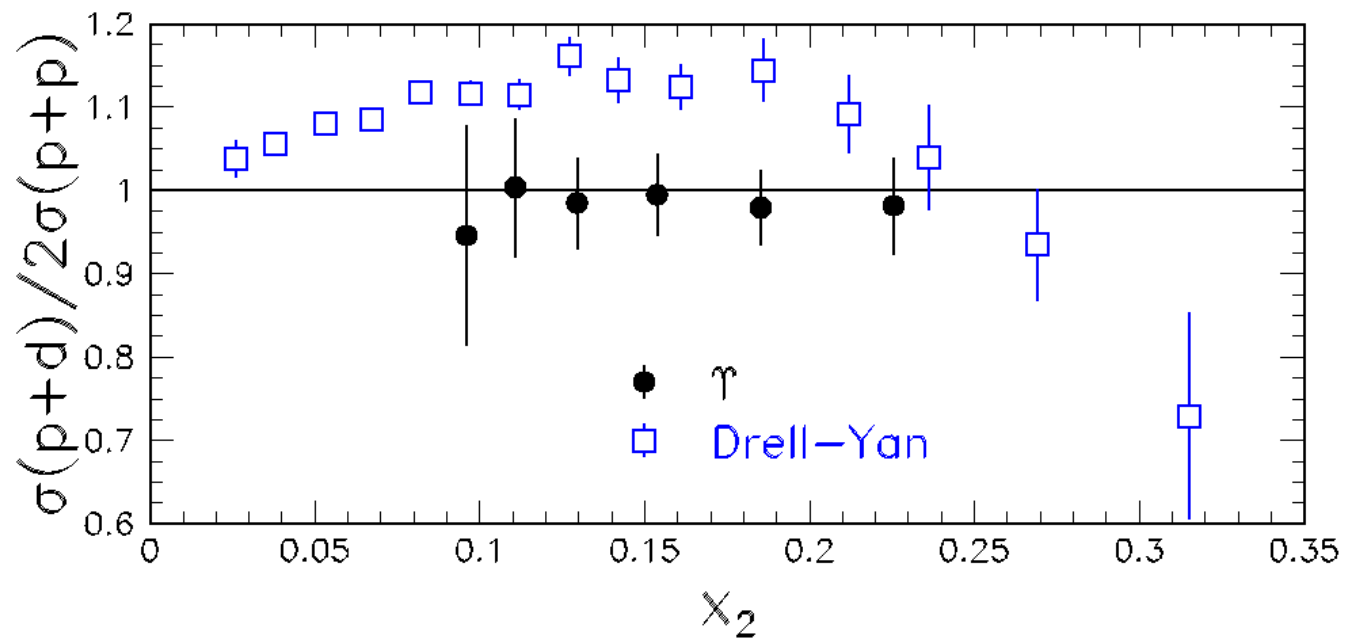
800 GeV p d data indicates Y produced
from gg mainly
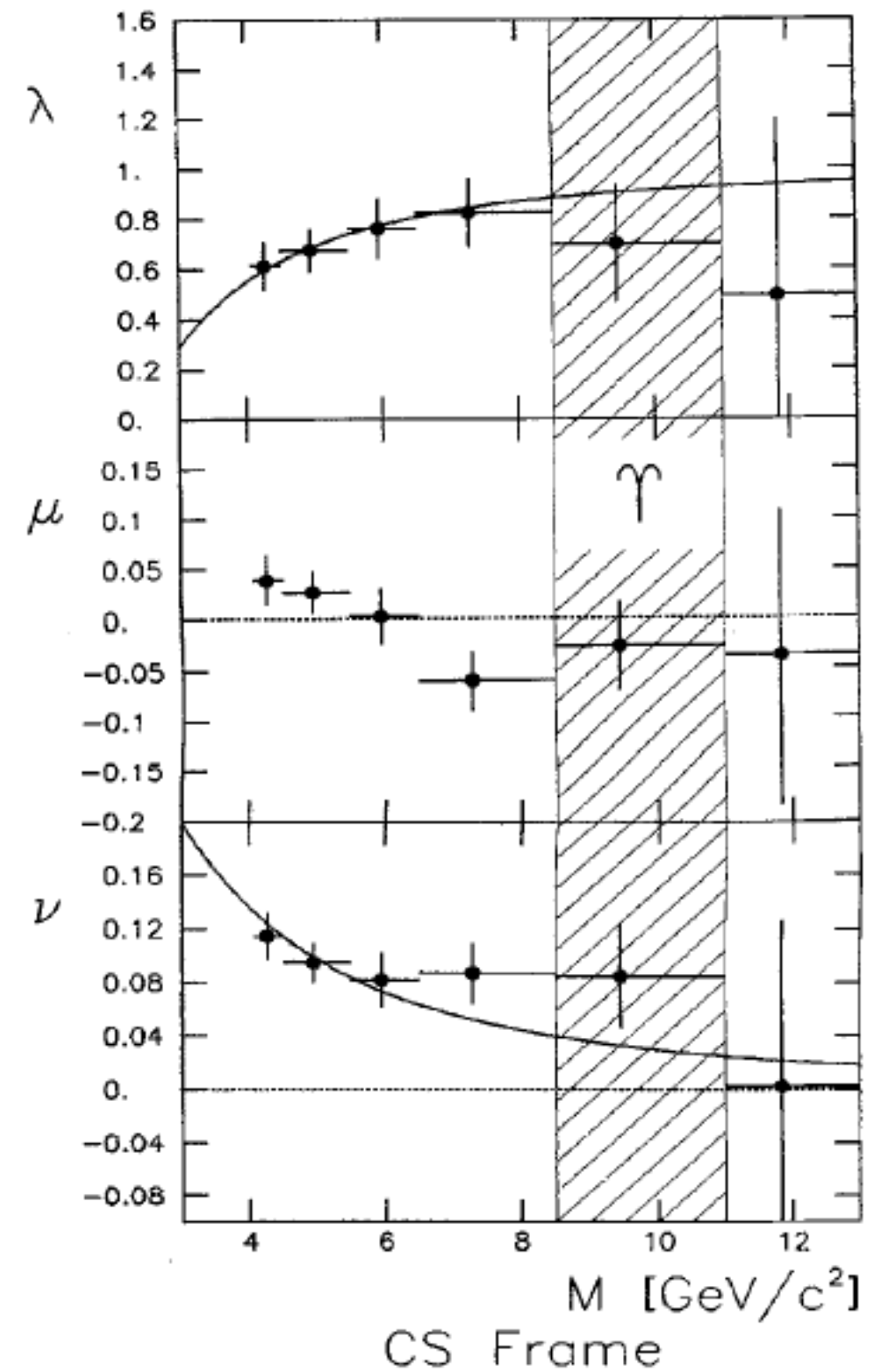
Lam-Tung on resonance

FNAL-E866/NuSea Collaboration, L.Y. Zhu et al.,
PRL 100 (2008) 062301
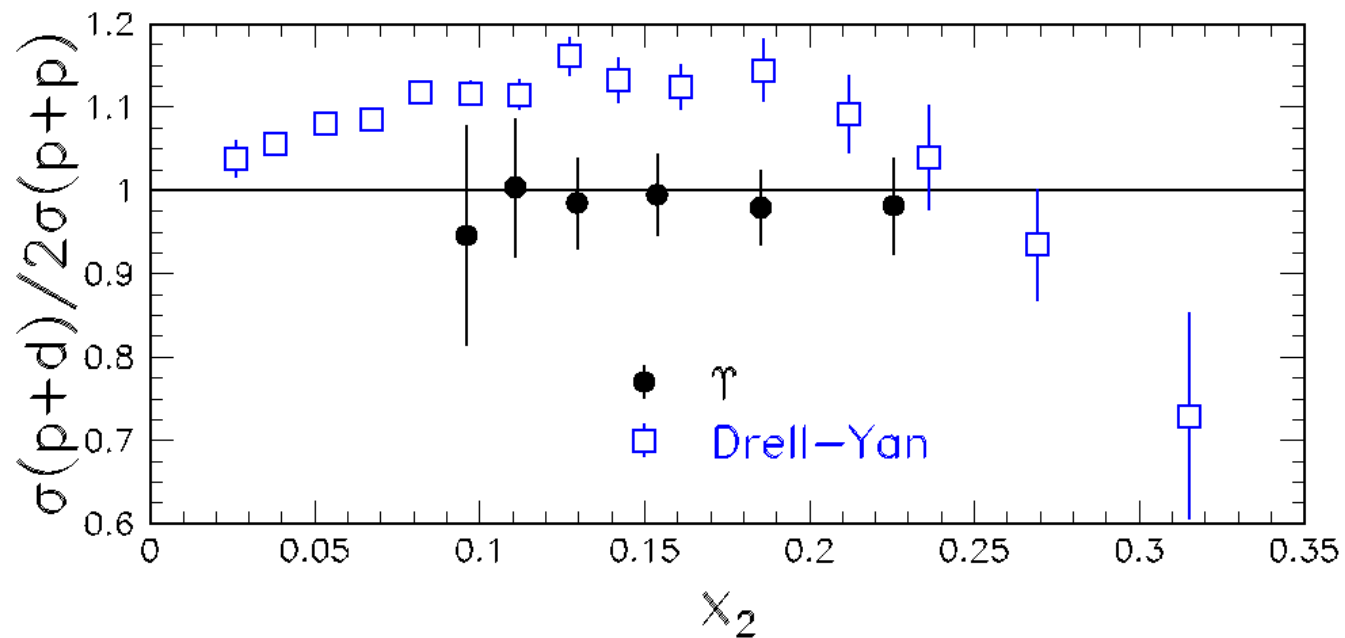
800 GeV p d data indicates Υ produced
from gg mainly
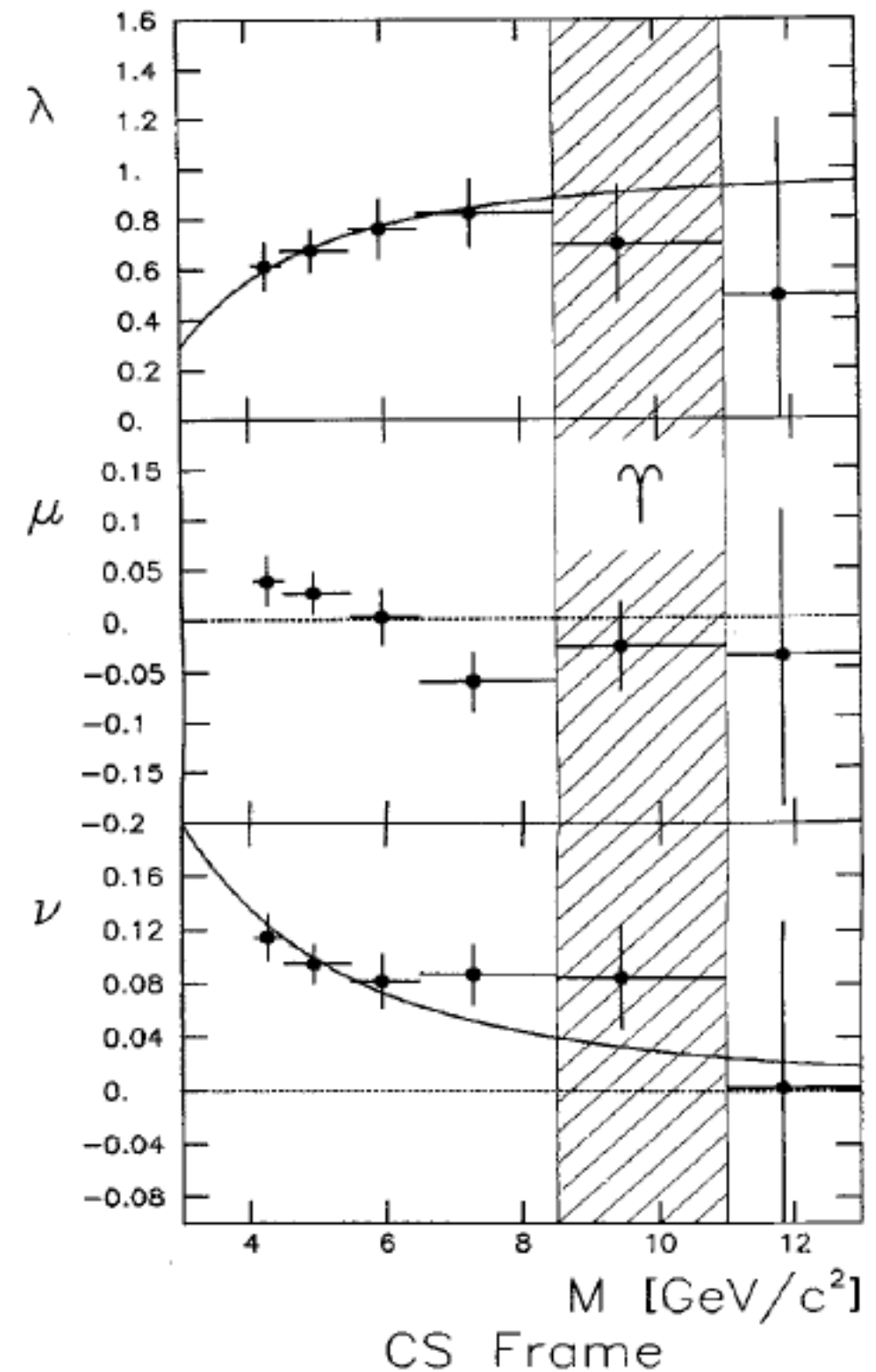
$\sqrt{s}$~40 GeV, Q~10 GeV: Q/$\sqrt{s}$~0.25 $\sqrt{s}$~20 GeV, Q~10 GeV: Q/$\sqrt{s}$~0.5

Lam-Tung on resonance
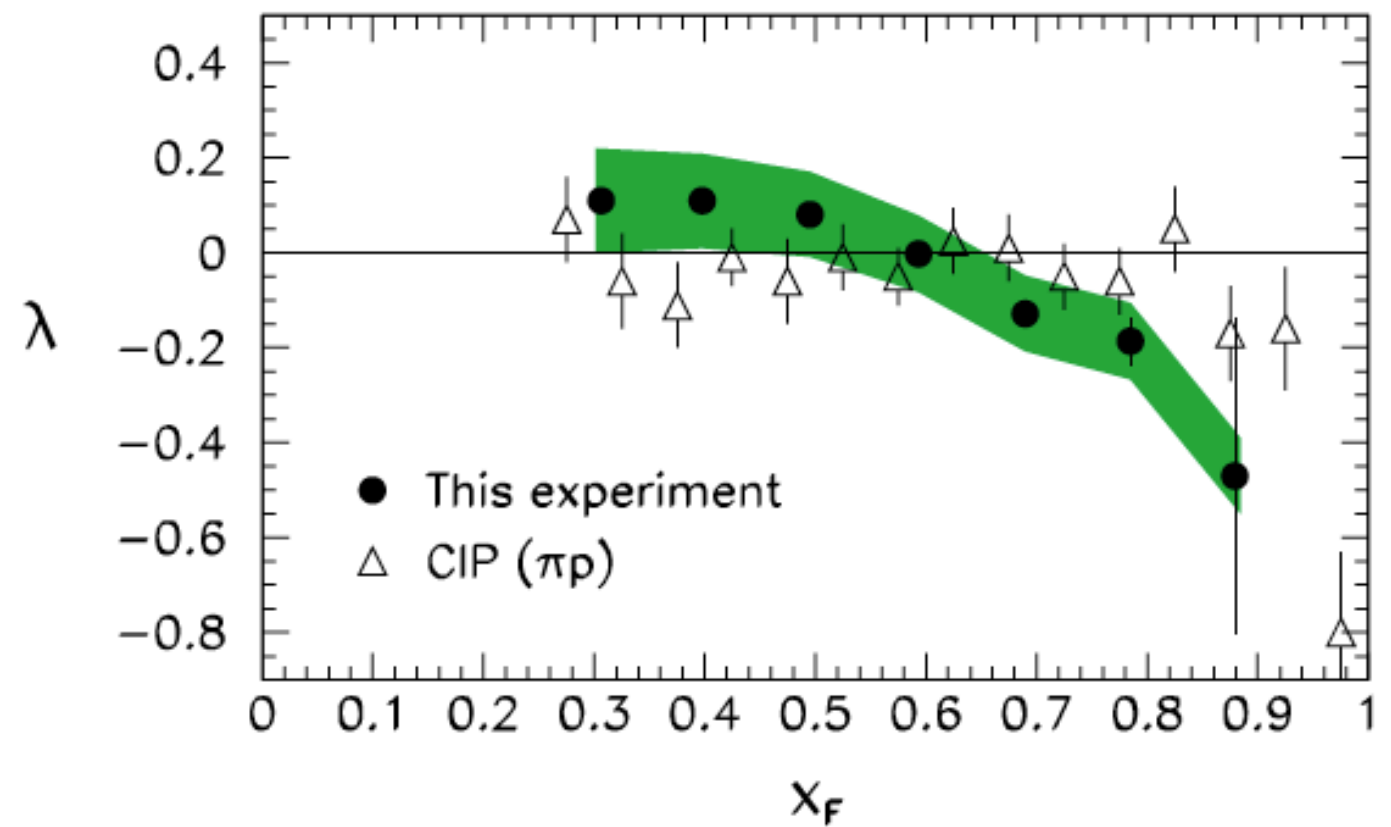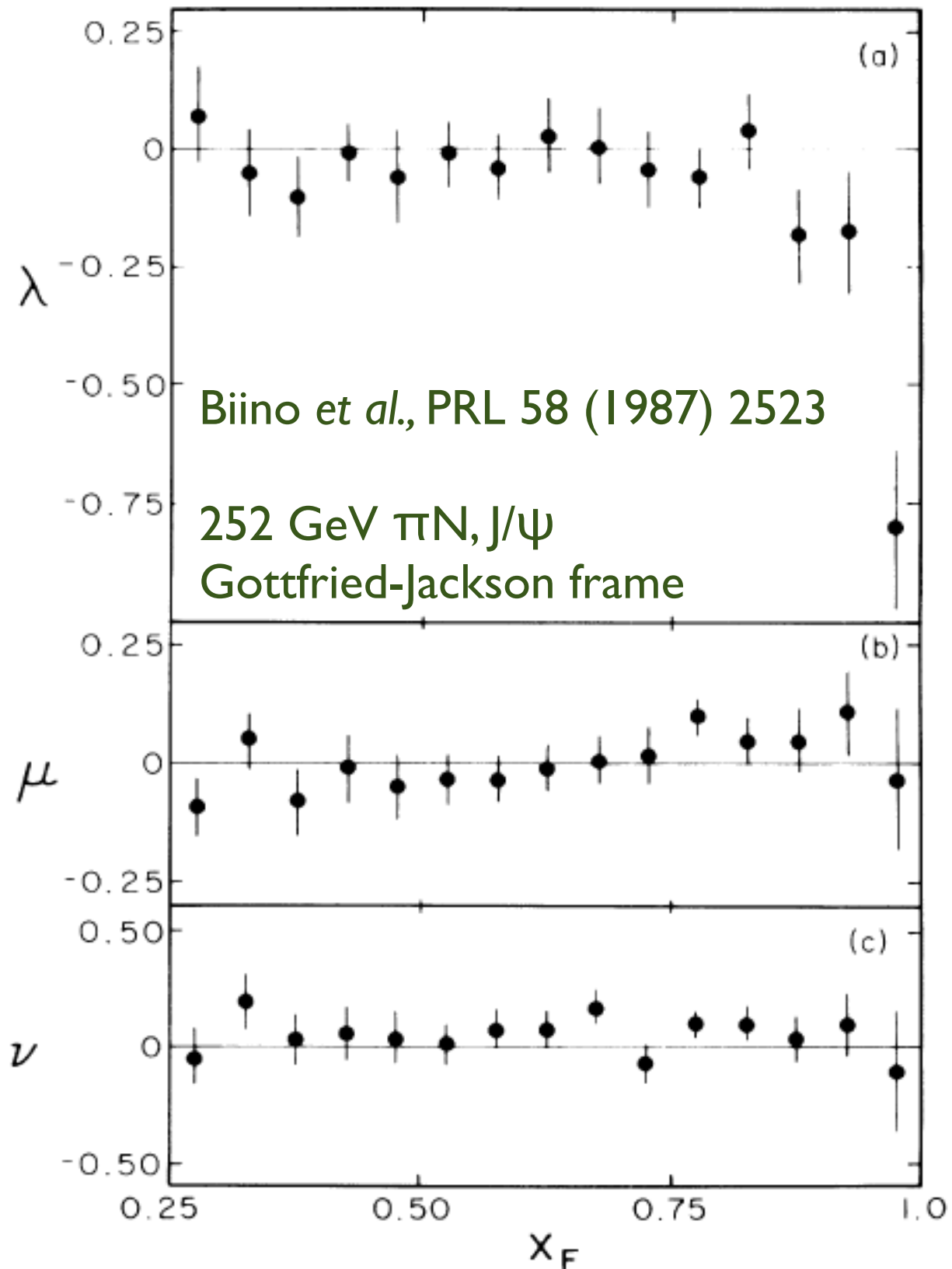
Biino *et al.,* PRL 58 (1987) 2523

252 GeV πN, J/ψ
Gottfried-Jackson frame

FNAL E866/NuSea Collaboration,
Chang *et al.,* PRL 91 (2003) 211801
800 GeV p Cu, Collins-Soper frame

$$\mathcal{F} = \frac{1 + \lambda + \nu}{3 + \lambda} \quad \xrightarrow{\lambda=0,\nu=0} \quad \frac{1}{3}$$

Lam-Tung on resonance
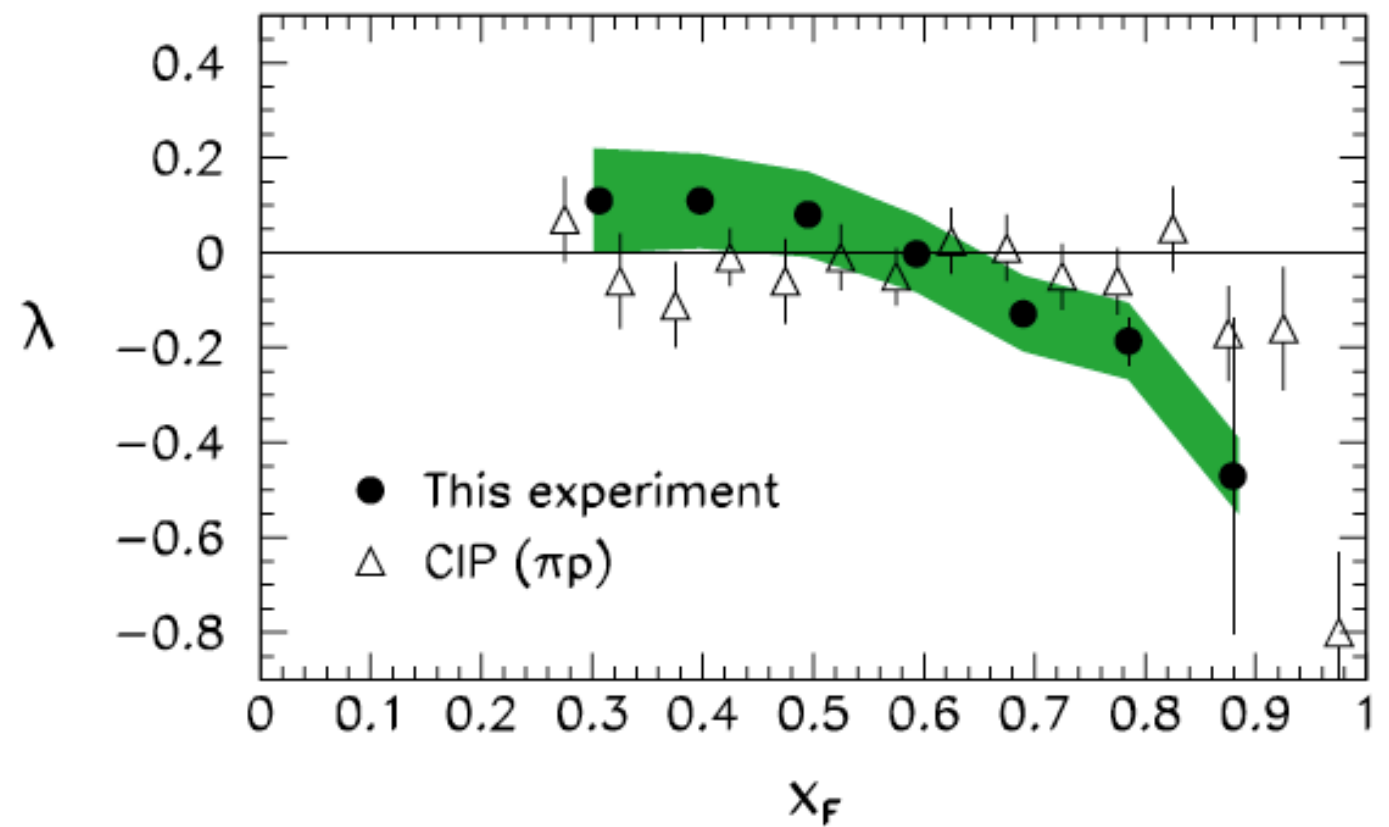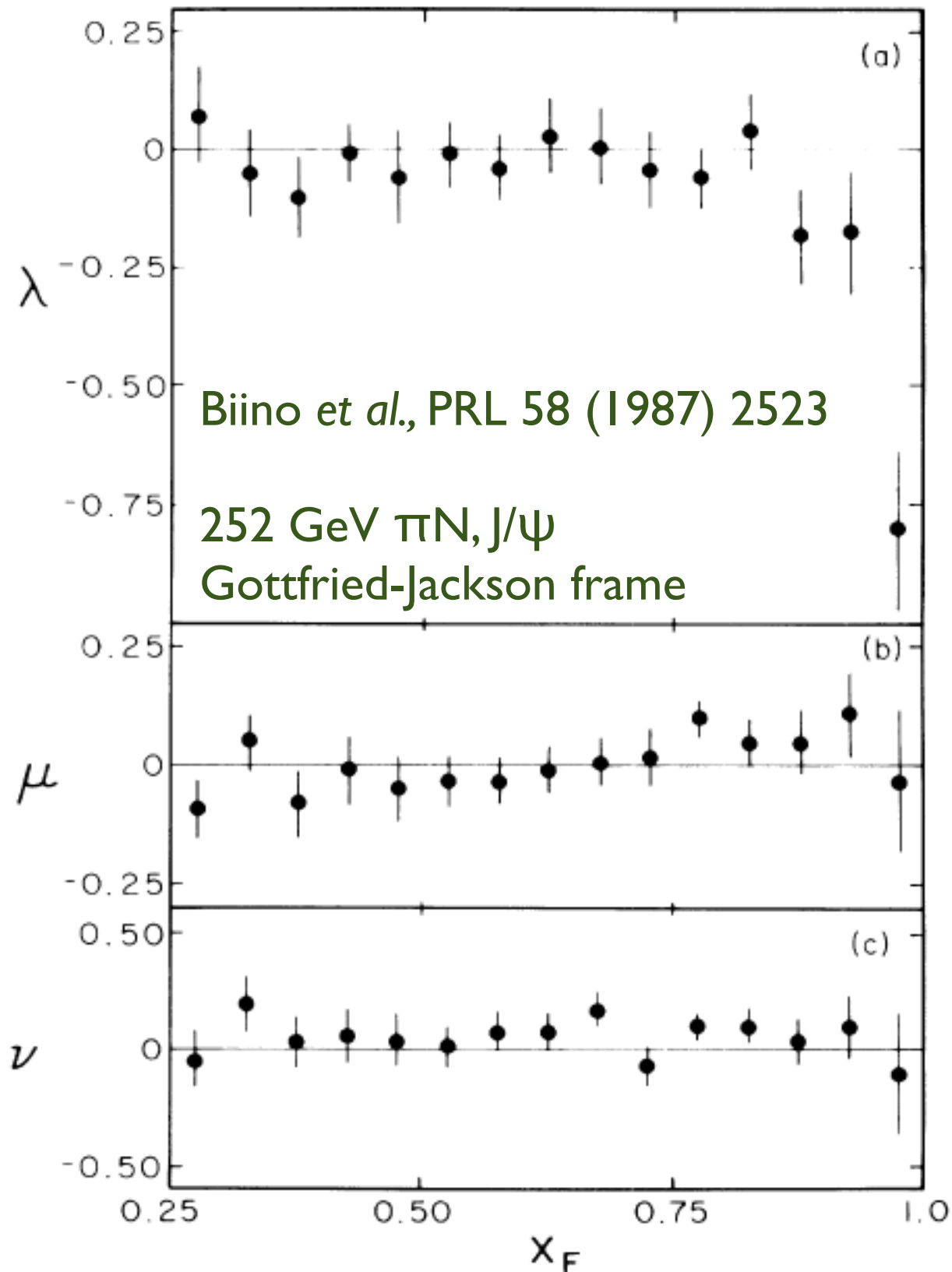
Biino *et al.,* PRL 58 (1987) 2523

252 GeV πN, J/ψ
Gottfried-Jackson frame

FNAL E866/NuSea Collaboration,
Chang *et al.,* PRL 91 (2003) 211801
800 GeV p Cu, Collins-Soper frame

$$\mathcal{F} = \frac{1 + \lambda + \nu}{3 + \lambda} \quad \xrightarrow{\lambda=0, \nu=0} \quad \frac{1}{3}$$

√s~22 GeV, Q~3 GeV: Q/√s~0.14   √s~40 GeV, Q~3 GeV: Q/√s~0.075

Lam-Tung on resonance
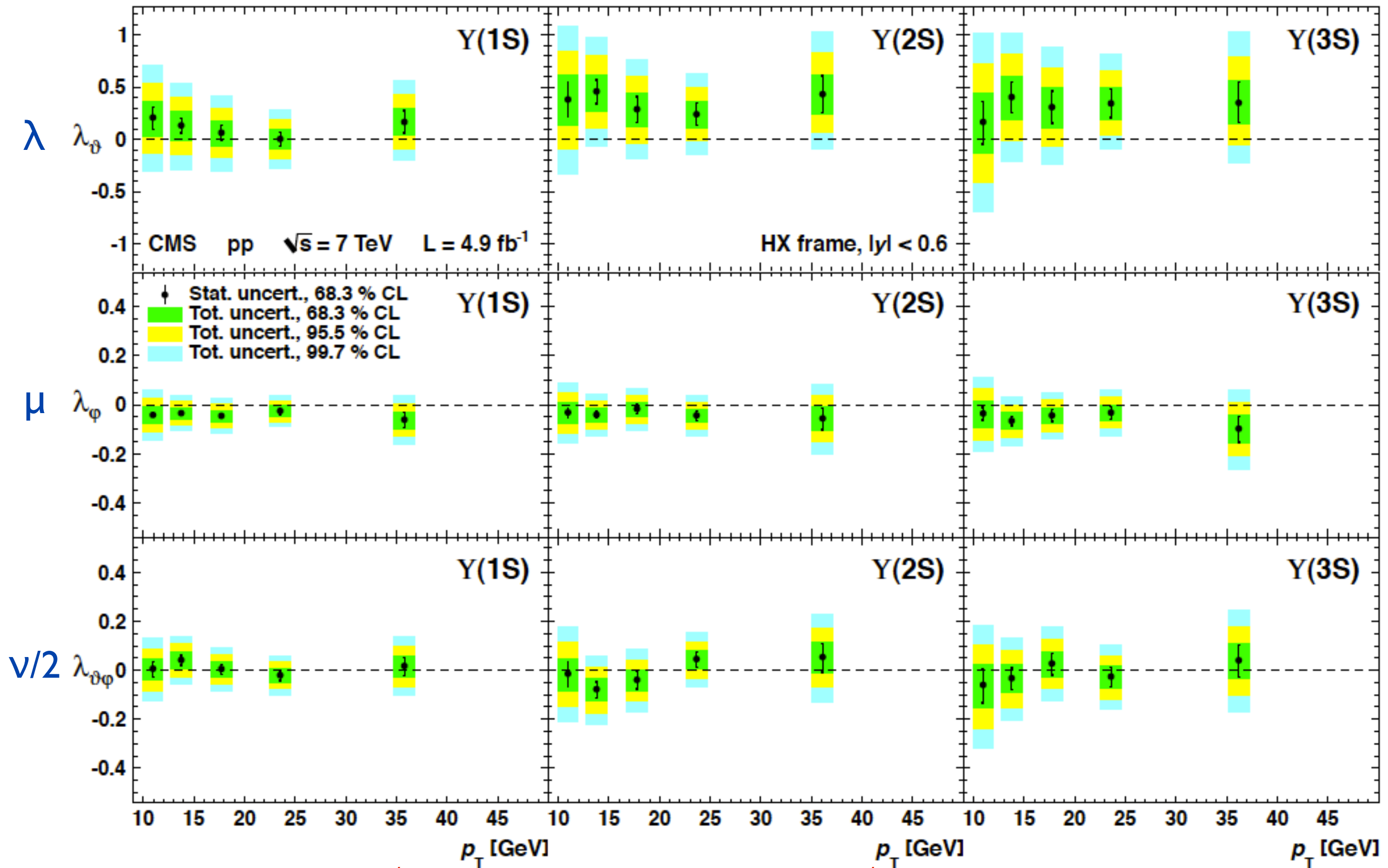
√s~7000 GeV, Q~10 GeV: Q/√s~0.0014

J/Ψ Production at 30 GeV

[Slide by Jen-Chieh Peng]

At 800 GeV, J/Ψ production is dominated by gluon-gluon fusion
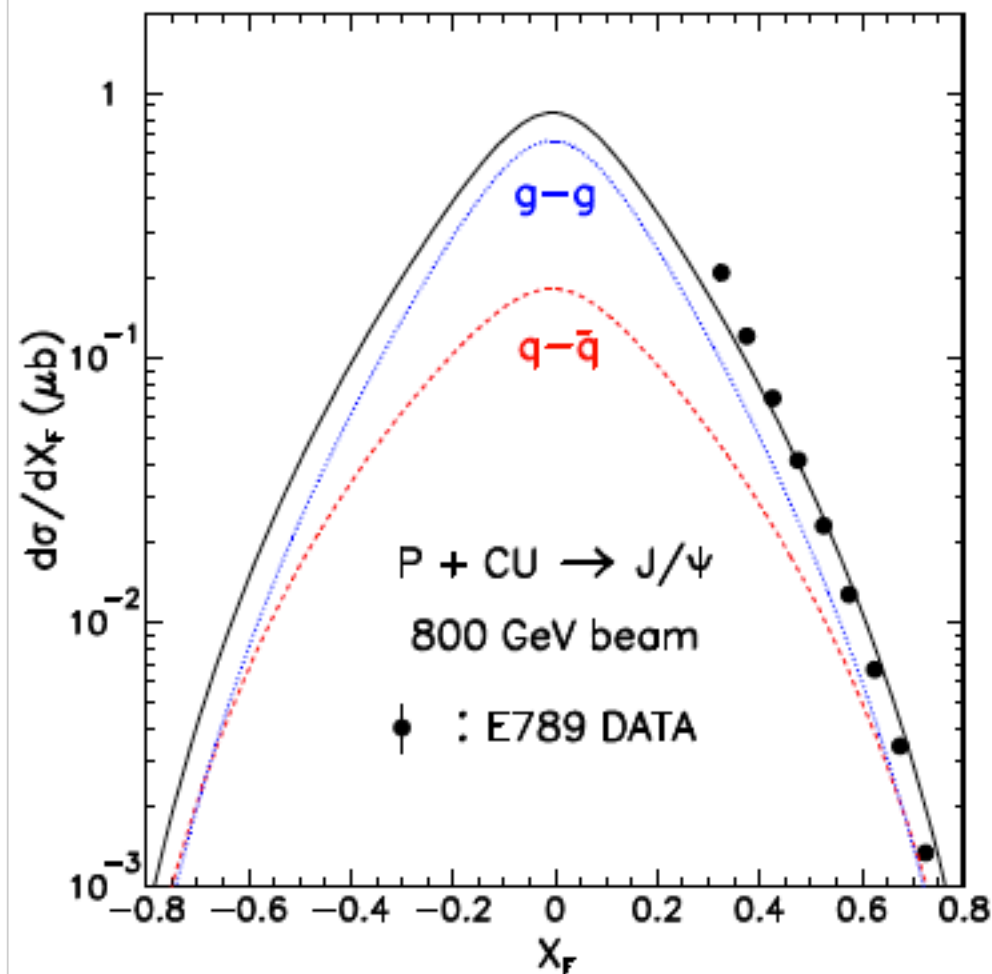
At 30 GeV J/Ψ production is dominated by quark-antiquark annihilation
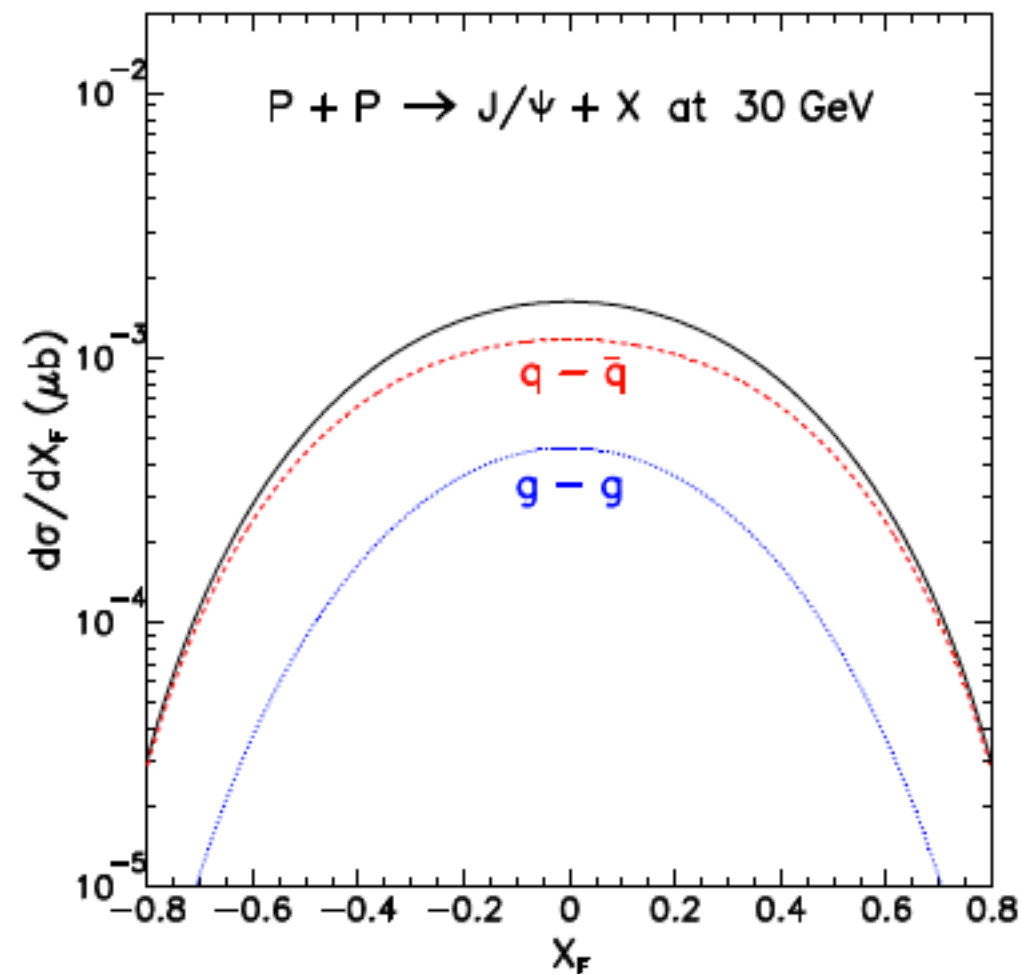
J/Ψ production at 30 GeV is sensitive to quark and antiquark distributions

41

$\sqrt{s}$~40 GeV, Q~3 GeV: Q/$\sqrt{s}$~0.075 $\sqrt{s}$~7.7 GeV, Q~3 GeV: Q/$\sqrt{s}$~0.4

Using Lam-Tung violation

Instead of looking only at λ for the polarization or λ,μ,ν individually, parameters like $\mathcal{F}$ or $\kappa=1-\lambda-2\nu$ convey more information about the partonic subprocess

$\kappa=1-\lambda-2\nu$ serves as a probe of q q-bar (< 0) versus gg (~1) channel
$Q/\sqrt{s}$ is a rough indicator of what to expect, except for large x_F

Using Lam-Tung violation

Instead of looking only at λ for the polarization or λ,μ,ν individually, parameters like $\mathcal{F}$ or $\kappa=1-\lambda-2\nu$ convey more information about the partonic subprocess

$\kappa=1-\lambda-2\nu$ serves as a probe of q q-bar (< 0) versus gg (~1) channel
$Q/\sqrt{s}$ is a rough indicator of what to expect, except for large x_F

Faccioli, Lourenço, Seixas, Wöhri, PRL 102, 151802 (2009)
λ for J/Ψ as function of total momentum:
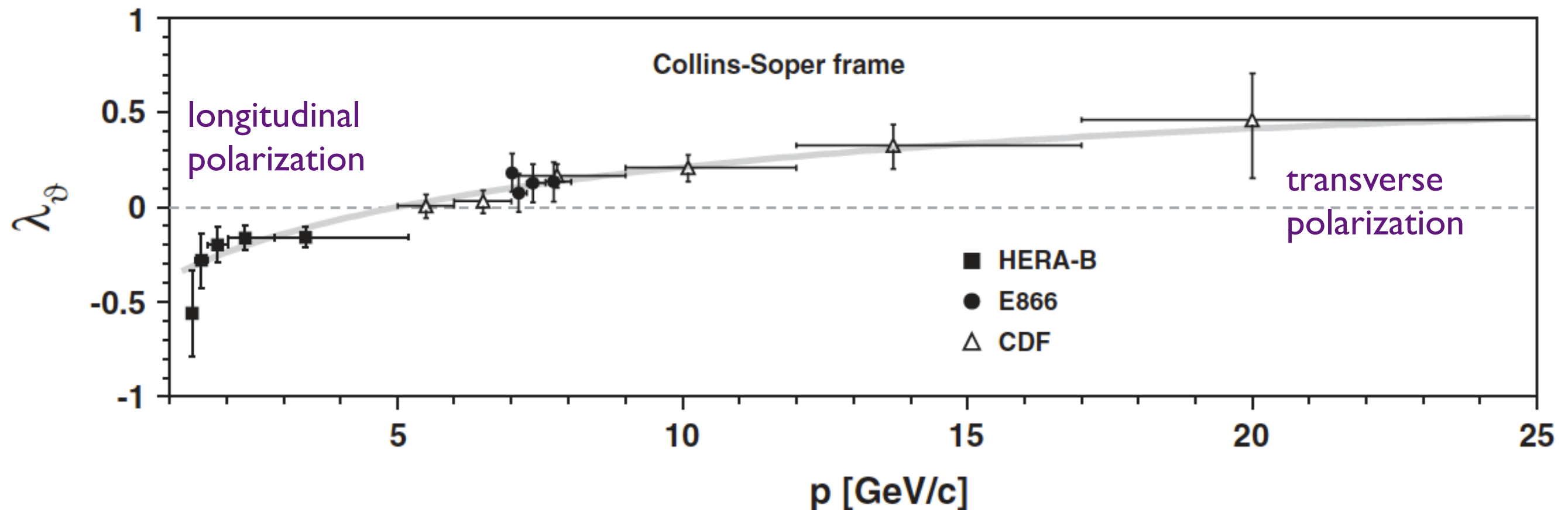
NB: what is longitudinal or transverse polarization depends on the frame

cos 2φ in SIDIS

The cos 2φ asymmetry has different high and low Q_T contributions

At low Q_T: $\sim h_1^\perp H_1^\perp$, with M^2/Q_T^2 suppressed high-Q_T tail
At high Q_T: $\sim f_1 D_1$, which is Q_T^2/Q^2 suppressed at low Q_T

The two contributions both need to be included, which is not double counting

[Bacchetta, DB, Diehl, Mulders, JHEP 0808 (2008) 023]
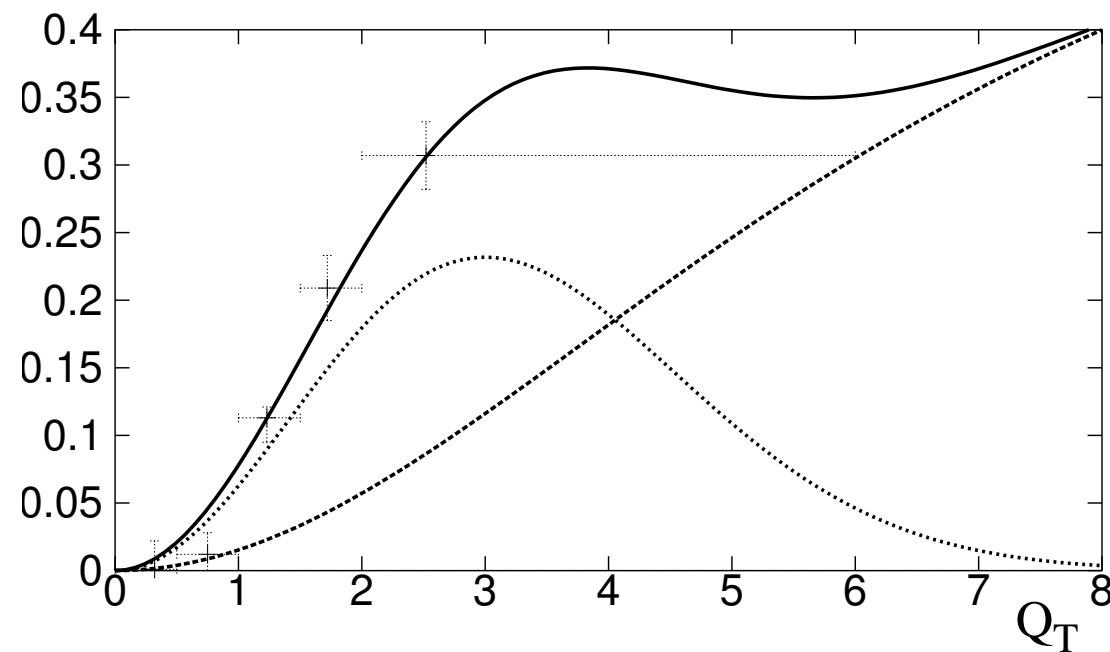
At low Q^2 the twist-4 Cahn effect ($\sim M^2/Q^2$) also enters

cos 2φ in SIDIS

The cos 2φ asymmetry has different high and low Q_T contributions

At low Q_T: ~ $h_1^\perp H_1^\perp$, with M^2/Q_T^2 suppressed high-Q_T tail
At high Q_T: ~ $f_1 D_1$, which is Q_T^2/Q^2 suppressed at low Q_T

The two contributions both need to be included, which is not double counting
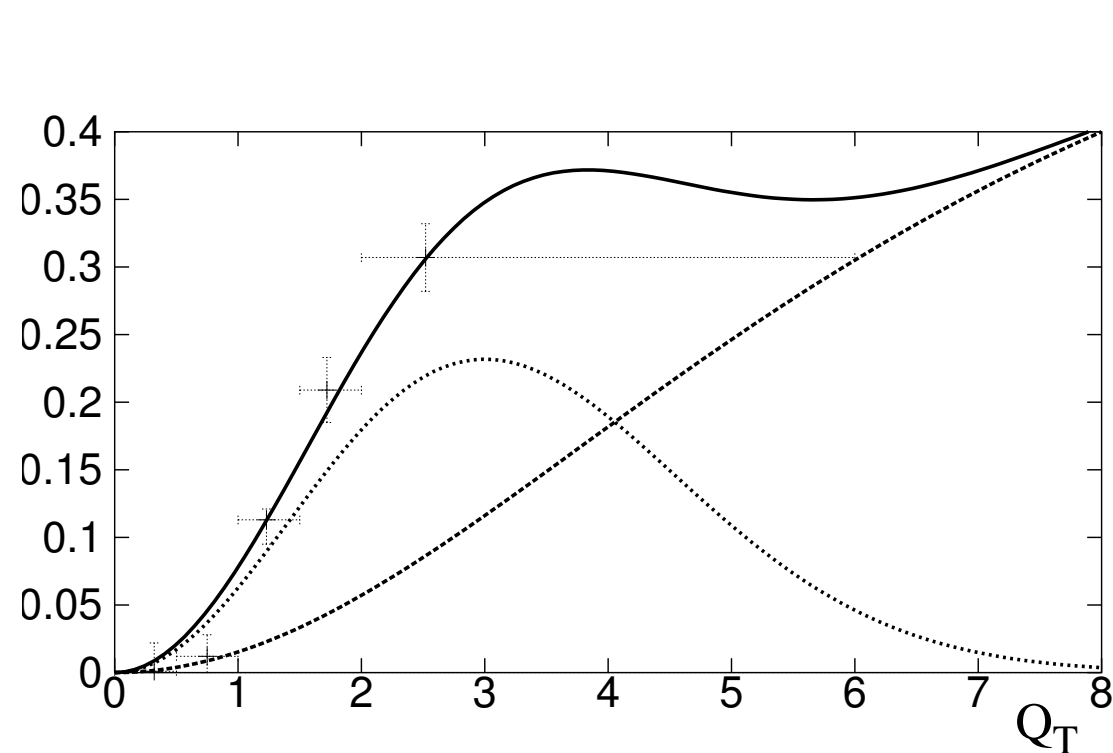
$$\nu = \nu_{h_1^\perp} + \nu_{\text{pert}} + \mathcal{O}(\frac{Q_T^2}{Q^2} \text{ or } \frac{M^2}{Q_T^2})$$

Nontrivial since a ratio of sums becomes approximately a sum of ratios

Bacchetta, DB, Diehl, Mulders, JHEP 0808 (2008) 023

At low Q^2 the twist-4 Cahn effect (~M^2/Q^2) also enters

Fits in DY mainly u-quark distribution, fits in SIDIS problematic (Cahn effect)

cos 2φ in unpolarized SIDIS at low Q²

COMPASS Deuteron
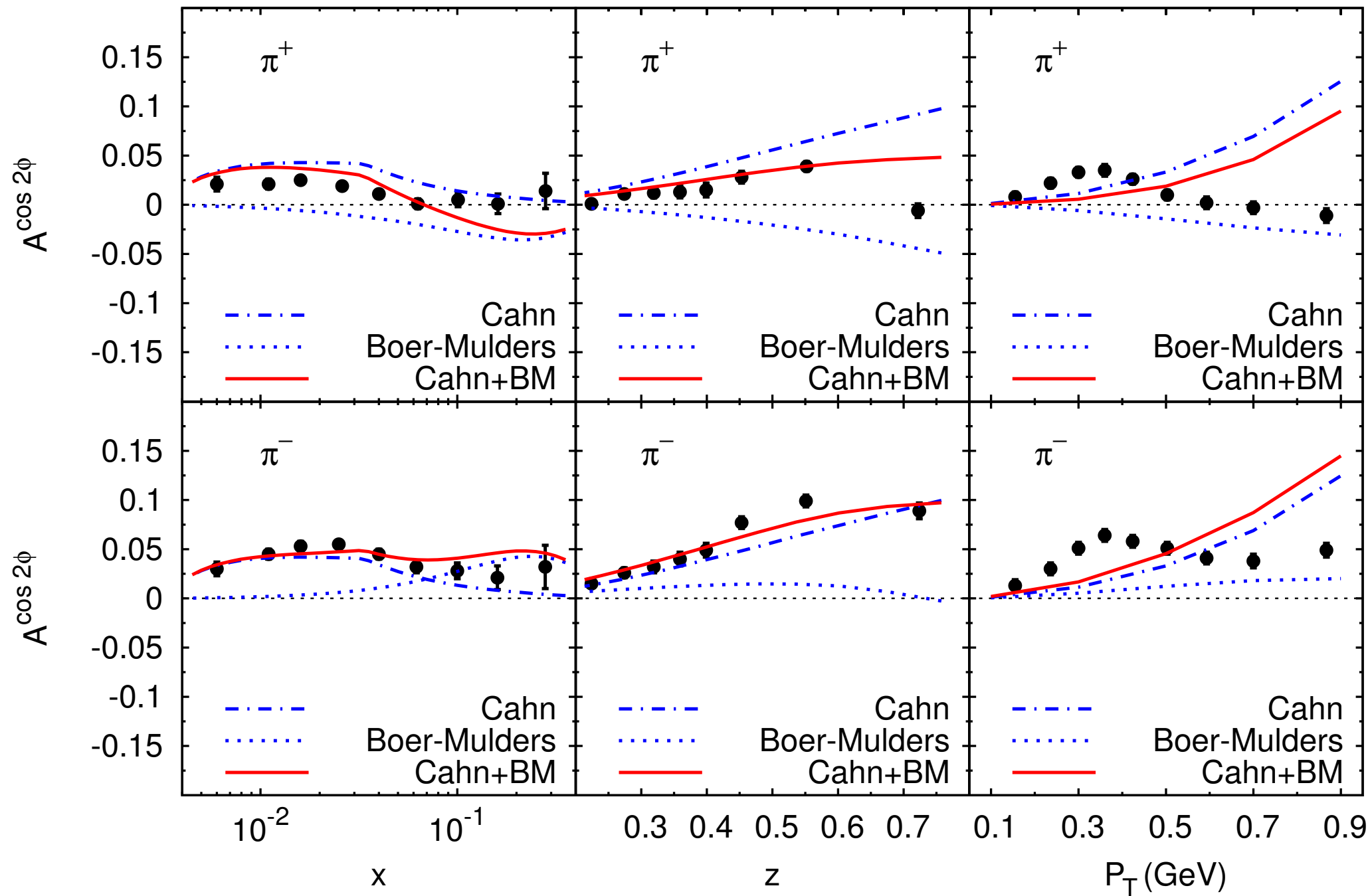
Barone, Melis, Prokudin, PRD 81 (2010) 114026

If twist-4 Cahn effect becomes equally important or larger, then extraction of BM function becomes questionable

cos 2φ in unpolarized SIDIS at high Q^2

ZEUS data for charged hadrons at $<Q^2>$ = 750 GeV^2 is consistent with LO pQCD
ZEUS Collaboration, PLB 481 (2000) 199 & EPJC 51 (2007) 289
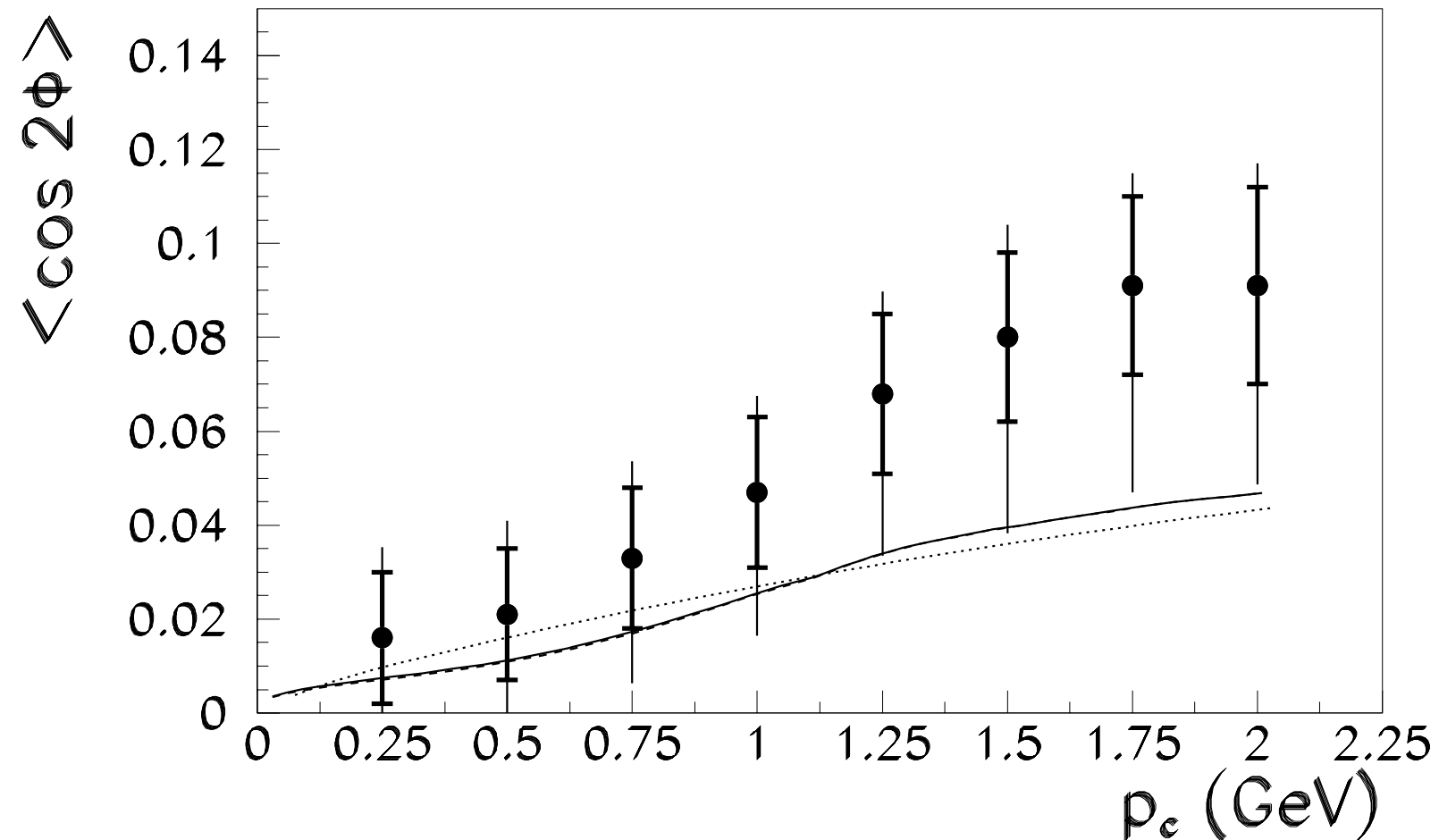
Note: p_c is a lower cut on observed p_T

Cahn effect is negligible at high Q^2, but also $h_1^\perp$ contribution is suppressed, just like the double Collins effect asymmetry (effectively twist-3)

SIDIS data inconclusive about $h_1^\perp$

cos φ

In the low Q_T TMD region a cos φ asymmetry is generated at twist-3: $\sim f^\perp D_1 + f_1 D^\perp$

$$F_{UU}^{\cos\phi_h} = \frac{2M}{Q} \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\left(xh\,H_1^\perp + \frac{M_h}{M}\,f_1\frac{\tilde{D}^\perp}{z}\right) - \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\left(xf^\perp D_1 + \frac{M_h}{M}\,h_1^\perp\frac{\tilde{H}}{z}\right)\right]$$

TMD factorization beyond leading twist is however not established

There is only one hint that it may work out for the $f^\perp D_1$ cos φ term:
the low Q_T expression *almost* matched onto the fixed order collinear factorization
result at high Q_T

DB, Vogelsang, PRD 74 (2006) 014004; Bacchetta, DB, Diehl, Mulders, JHEP 0808 (2008) 023

A slightly modified TMD factorization beyond leading twist may hold

A similar thing may apply to the beam-spin asymmetry A_{LU}:

$$F_{LU}^{\sin\phi_h} = \frac{2M}{Q} \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\left(xe\,H_1^\perp + \frac{M_h}{M}\,f_1\frac{\tilde{G}^\perp}{z}\right) + \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\left(xg^\perp D_1 + \frac{M_h}{M}\,h_1^\perp\frac{\tilde{E}}{z}\right)\right]$$

A-dependence of cos φ
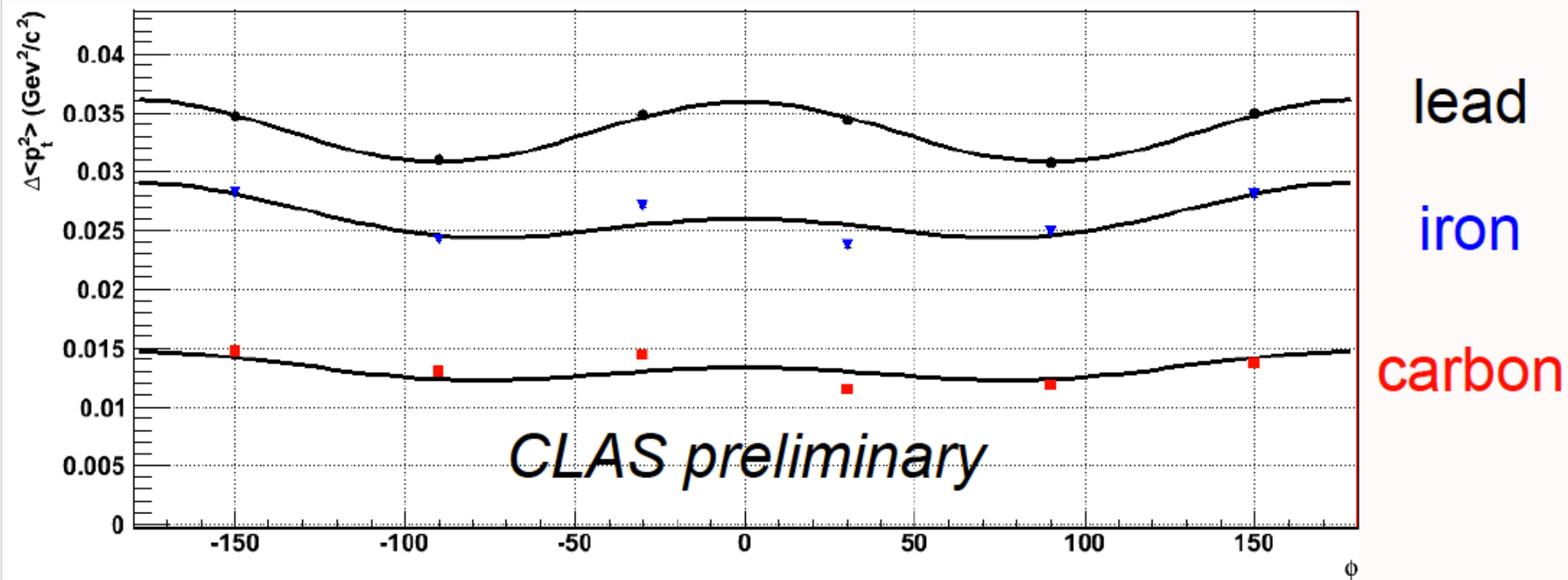
New: dependence of p_T broadening on φ_{pq}

$\Delta\langle p_t^2\rangle$ (Gev²/c²)

lead

iron

carbon

CLAS preliminary

curves shown contain terms in $\cos(\varphi_{pq})$ and $\cos(2\varphi_{pq})$
only statistical uncertainties shown

K. Hicks at J-Parc
workshop at KEK
January 17, 2013

A-dependence of cos φ

New: dependence of p_T broadening on φ_{pq}

lead

iron

carbon

CLAS preliminary

curves shown contain terms in $\cos(\varphi_{pq})$ and $\cos(2\varphi_{pq})$
only statistical uncertainties shown

K. Hicks at J-Parc
workshop at KEK
January 17, 2013

Gauge link dependence of $f^\perp$ implies process dependence, but likely also A-dependence

Gauge links arise from rescattering, consequently (anti-)shadowing can be A-dependent
[Brodsky, Hoyer, Marchal, Peigné, Sannino, PRD 65 (2002) 114025]

A-dependence of cos φ

New: dependence of p_T broadening on φ_{pq}

lead

iron

carbon

CLAS preliminary

curves shown contain terms in $\cos(\varphi_{pq})$ and $\cos(2\varphi_{pq})$
only statistical uncertainties shown

K. Hicks at J-Parc
workshop at KEK
January 17, 2013

Gauge link dependence of $f^{\perp}$ implies process dependence, but likely also A-dependence

Gauge links arise from rescattering, consequently (anti-)shadowing can be A-dependent
[Brodsky, Hoyer, Marchal, Peigné, Sannino, PRD 65 (2002) 114025]

p_T broadening involves a $p_T{}^2$ weighting, which theoretically yields divergent quantities, hence usually it is defined as a (finite) difference:

$$\Delta p_T^2 \equiv \langle p_T^2 \rangle_A - \langle p_T^2 \rangle_p$$

A-dependence of cos φ
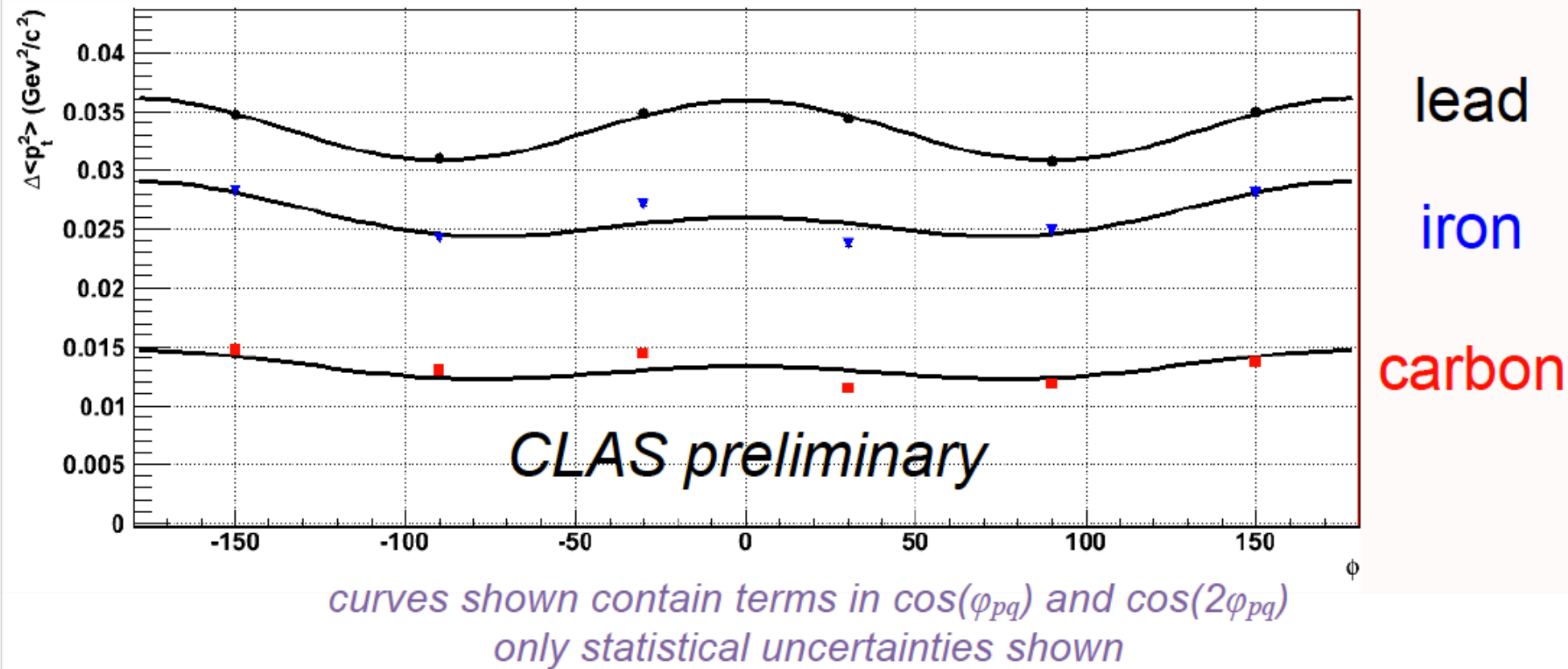
New: dependence of p_T broadening on φ_{pq}

lead

iron

carbon

CLAS preliminary

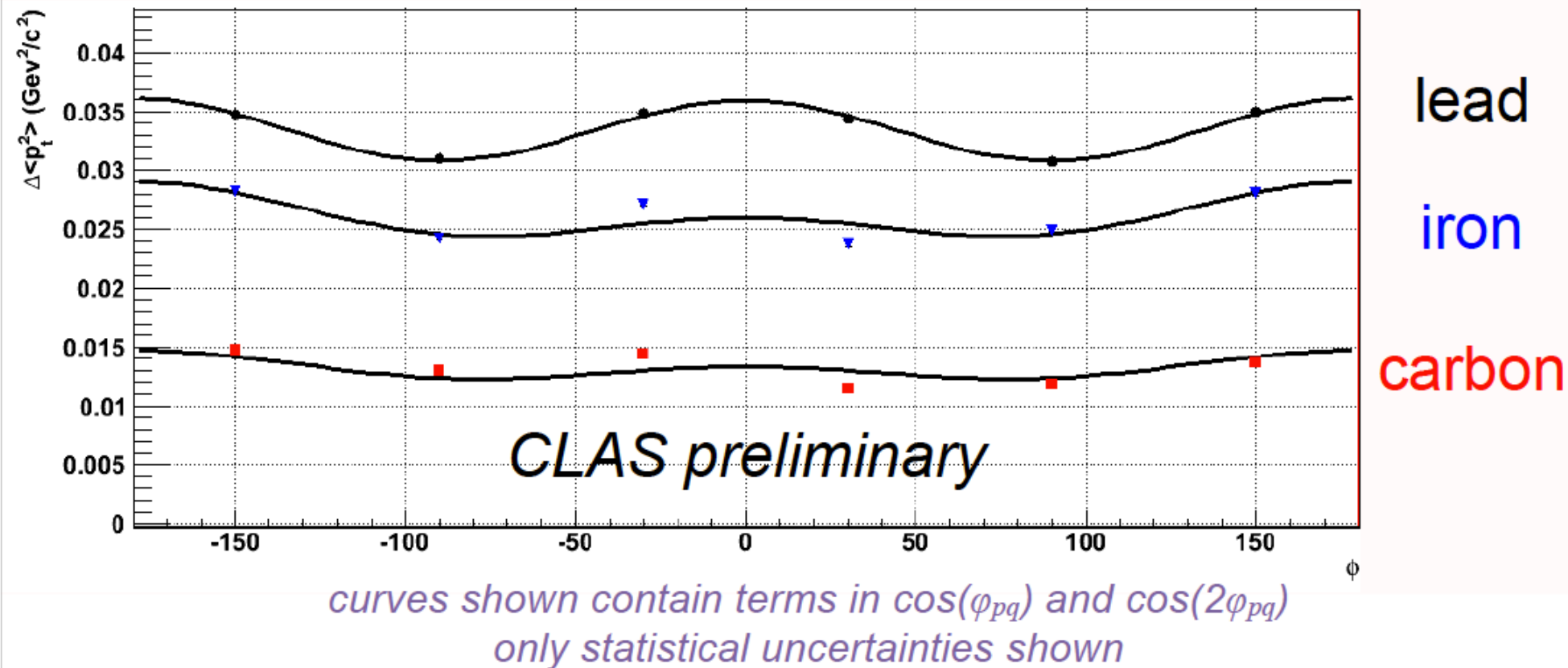$\Delta \langle p_t^2 \rangle$ (Gev^2/c^2)

φ

curves shown contain terms in $\cos(\varphi_{pq})$ and $\cos(2\varphi_{pq})$
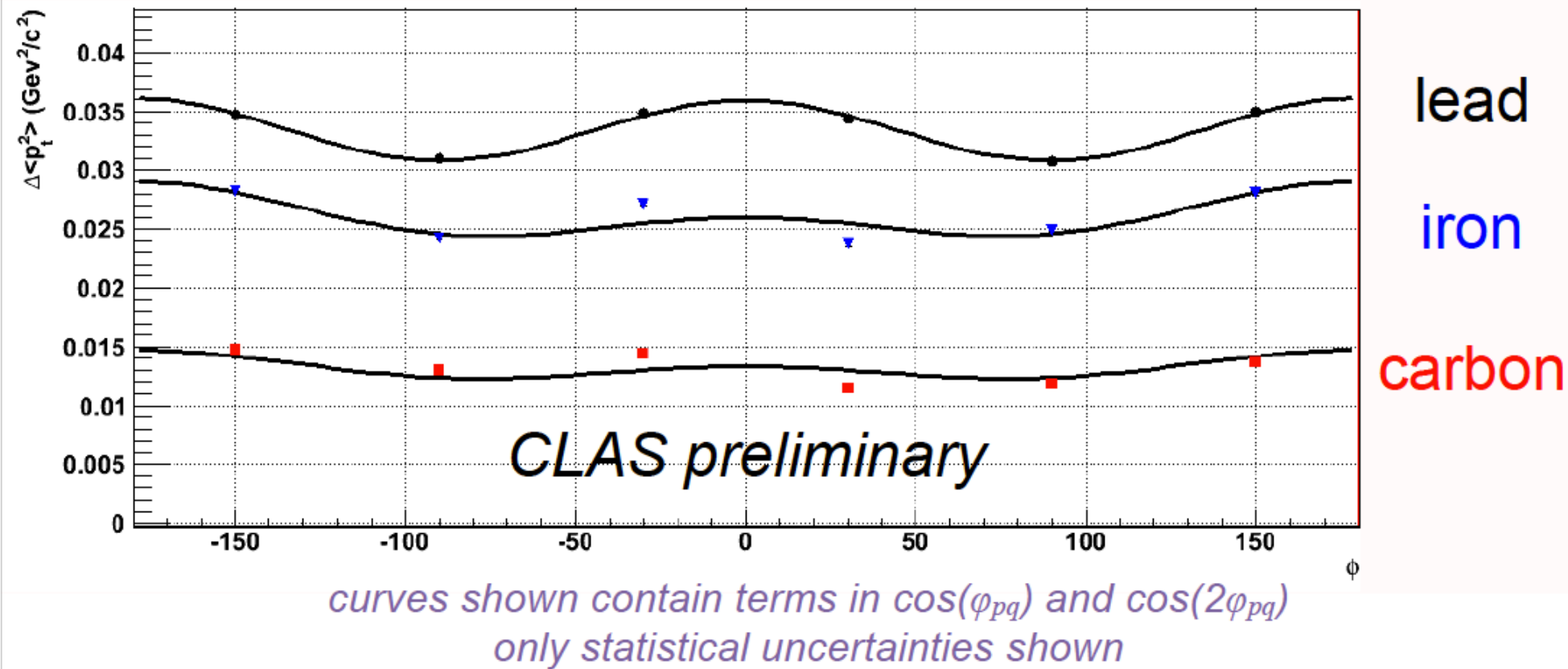only statistical uncertainties shown

K. Hicks at J-Parc
workshop at KEK
January 17, 2013

Gauge link dependence of $f^\perp$ implies process dependence, but likely also A-dependence

Gauge links arise from rescattering, consequently (anti-)shadowing can be A-dependent
[Brodsky, Hoyer, Marchal, Peigné, Sannino, PRD 65 (2002) 114025]

p_T broadening involves a p_T^2 weighting, which theoretically yields divergent quantities, hence usually it is defined as a (finite) difference:

$$\Delta p_T^2 \equiv \langle p_T^2 \rangle_A - \langle p_T^2 \rangle_p$$

An alternative is to consider Bessel weighting
[DB, Gamberg, Musch, Prokudin, JHEP 10 (2011) 021]

Linear gluon polarization at small x

$h_1^{\perp g}$ is more difficult to extract, as it cannot be probed in DIS, DY, SIDIS, nor in inclusive hadron or γ+jet production in pp or pA collisions

Linear gluon polarization at small x

$h_1^{\perp g}$ is more difficult to extract, as it cannot be probed in DIS, DY, SIDIS, nor in inclusive hadron or γ+jet production in pp or pA collisions

Selection of processes that probe the WW or DP linearly polarized gluon TMD:

	$pp \to \gamma\,\gamma\,X$	$pA \to \gamma^*\,\mathrm{jet}\,X$	$e\,p \to e'\,Q\,\overline{Q}\,X$ $e\,p \to e'\,j_1\,j_2\,X$	$pp \to \eta_{c,b}\,X$ $pp \to H\,X$	$pp \to J/\psi\,\gamma\,X$ $pp \to \Upsilon\,\gamma\,X$
$h_1^{\perp\,g\,[+,+]}$ (WW)	$\checkmark$	$\times$	$\checkmark$	$\checkmark$	$\checkmark$
$h_1^{\perp\,g\,[+,-]}$ (DP)	$\times$	$\checkmark$	$\times$	$\times$	$\times$

Linear gluon polarization at small x

$h_1^{\perp g}$ is more difficult to extract, as it cannot be probed in DIS, DY, SIDIS, nor in inclusive hadron or γ+jet production in pp or pA collisions

Selection of processes that probe the WW or DP linearly polarized gluon TMD:

	$pp \to \gamma\,\gamma\,X$	$pA \to \gamma^*\,\mathrm{jet}\,X$	$e\,p \to e'\,Q\,\overline{Q}\,X$ $e\,p \to e'\,j_1\,j_2\,X$	$pp \to \eta_{c,b}\,X$ $pp \to H\,X$	$pp \to J/\psi\,\gamma\,X$ $pp \to \Upsilon\,\gamma\,X$
$h_1^{\perp g\,[+,+]}$ (WW)	$\checkmark$	$\times$	$\checkmark$	$\checkmark$	$\checkmark$
$h_1^{\perp g\,[+,-]}$ (DP)	$\times$	$\checkmark$	$\times$	$\times$	$\times$

Higgs and $0^{\pm+}$ quarkonium production allows to measure the linear gluon polarization using the angular independent p_T distribution

All other suggestions use angular modulations

Linear gluon polarization at small x

$h_1^{\perp g}$ is more difficult to extract, as it cannot be probed in DIS, DY, SIDIS, nor in inclusive hadron or γ+jet production in pp or pA collisions

Selection of processes that probe the WW or DP linearly polarized gluon TMD:

	$pp \to \gamma\,\gamma\,X$	$pA \to \gamma^*\,\mathrm{jet}\,X$	$e\,p \to e'\,Q\,\overline{Q}\,X$ $e\,p \to e'\,j_1\,j_2\,X$	$pp \to \eta_{c,b}\,X$ $pp \to H\,X$	$pp \to J/\psi\,\gamma\,X$ $pp \to \Upsilon\,\gamma\,X$
$h_1^{\perp\,g\,[+,+]}$ (WW)	$\checkmark$	$\times$	$\checkmark$	$\checkmark$	$\checkmark$
$h_1^{\perp\,g\,[+,-]}$ (DP)	$\times$	$\checkmark$	$\times$	$\times$	$\times$

Higgs and $0^{\pm+}$ quarkonium production allows to measure the linear gluon polarization using the angular independent p_T distribution

All other suggestions use angular modulations

EIC can probe the WW $h_1^{\perp g}$, while RHIC/LHC can probe both the WW and DP one

Qiu, Schlegel, Vogelsang, 2011; Jian Zhou , 2016; D.B., Brodsky, Pisano, Mulders, 2011; D.B., Pisano, 2012; Sun, Xiao, Yuan, 2011; D.B., den Dunnen, Pisano, Schlegel, Vogelsang, 2012; den Dunnen, Lansberg, Piano, Schlegel, 2014