RECENT RESULTS ON THE PROPERTIES OF STRONGLY INTERACTING MATTER FROM FIRST PRINCIPLES

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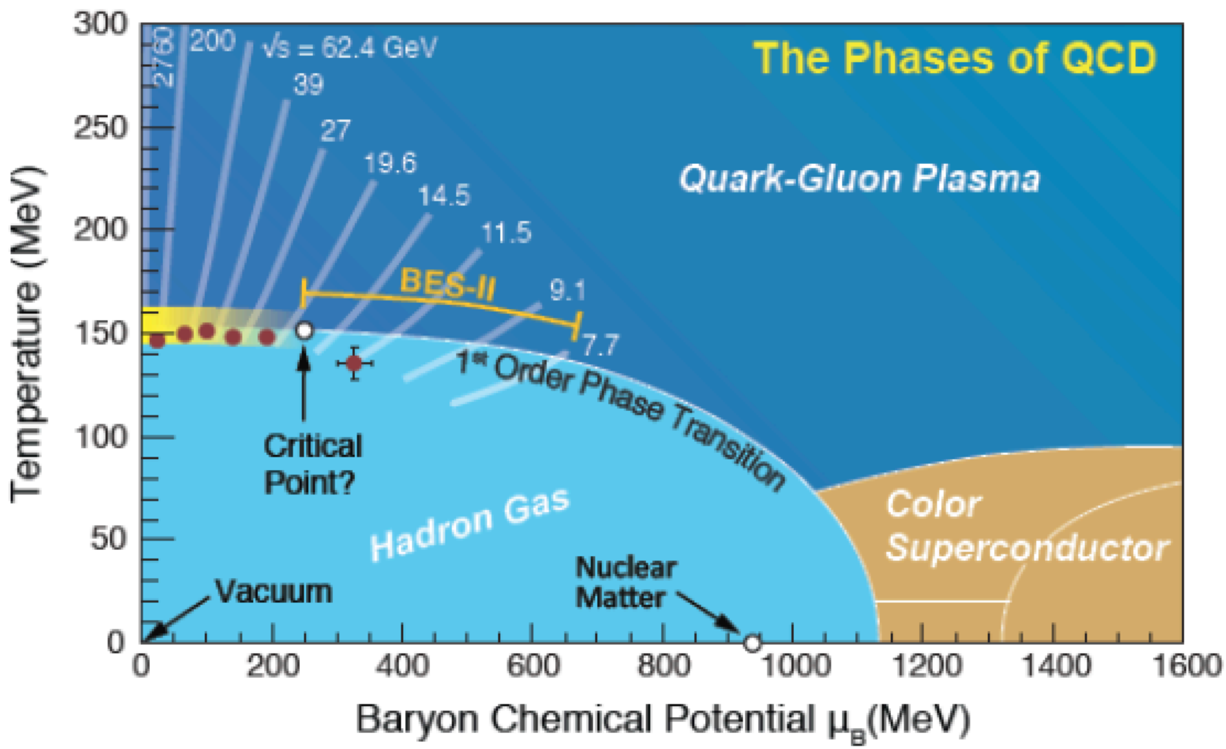
Ultimate goals

- Map the phase diagram of Quantum Chromodynamics
- Locate the critical point
- Find it in experiments



Sign problem

- The QCD path integral is computed by Monte Carlo algorithms which samples field configurations with a weight proportional to the exponential of the action

$$Z(\mu_B, T) = \mathrm{Tr}\left(e^{-\frac{H_{\mathrm{QCD}} - \mu_B N_B}{T}}\right) = \int \mathcal{D}U e^{-S_G[U]} \det M[U, \mu_B]$$

- $\det M[\mu_B]$ complex → Monte Carlo simulations are not feasible

- We can rely on a few approximate methods, viable for small μ_B/T:
  - Taylor expansion of physical quantities around $\mu_B=0$ (Bielefeld-Swansea collaboration 2002; R. Gavai, S. Gupta 2003)
  - Simulations at imaginary chemical potentials (plus analytic continuation) (Alford, Kapustin, Wilczek, 1999; de Forcrand, Philipsen, 2002; D'Elia, Lombardo 2003)

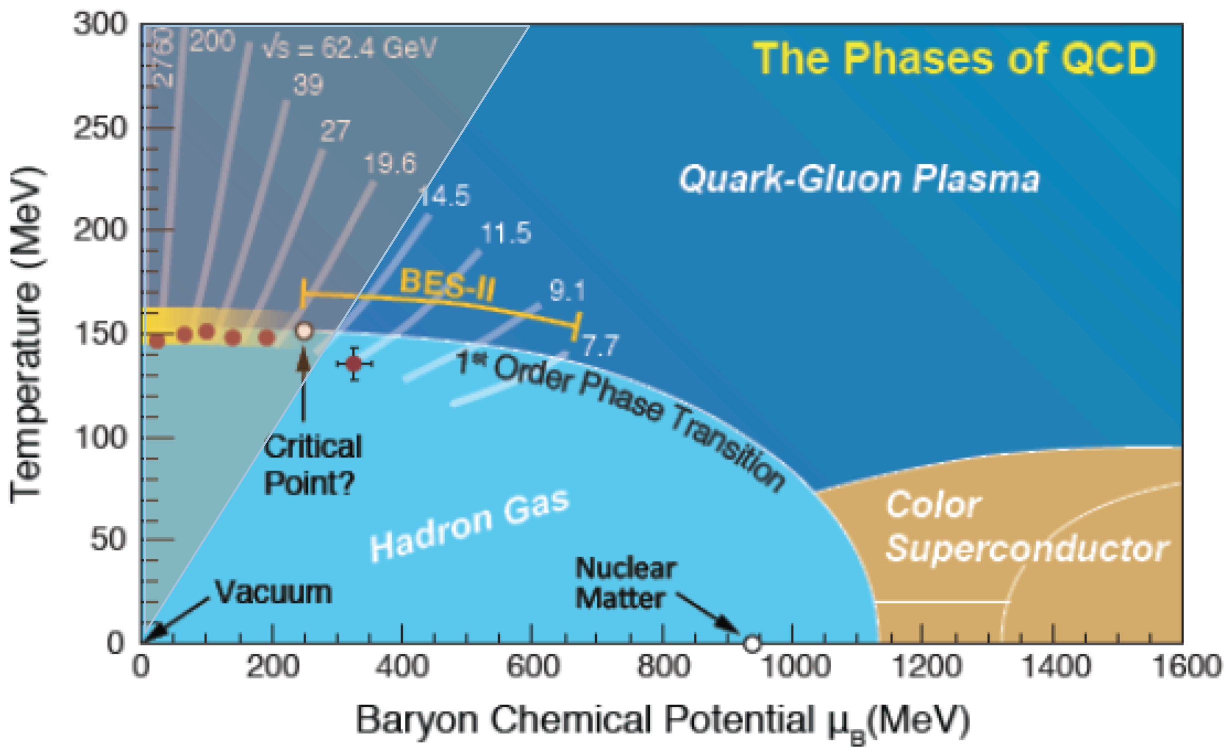
Equation of state as a Taylor expansion in μ_B

- Notation:

$$\hat{\mu}_B \equiv \mu_B/T \quad \hat{p} \equiv p/T^4 \quad \hat{n} \equiv n_B/T^3 \quad \hat{s} \equiv s/T^3$$

- Taylor expansion for the pressure:

$$\hat{p} = c_0(T) + c_2(T) \cdot \hat{\mu}_B^2 + c_4(T) \cdot \hat{\mu}_B^4 + c_6(T) \cdot \hat{\mu}_B^6 + \ldots$$



Physics at imaginary μ

- At imaginary μ there is no sign problem
- The partition function is periodic in μ_I with period $2\pi T$

$$Z = \mathrm{Tr}\left(e^{-\beta\hat{H}+i\beta\mu_I\hat{N}}\right)$$

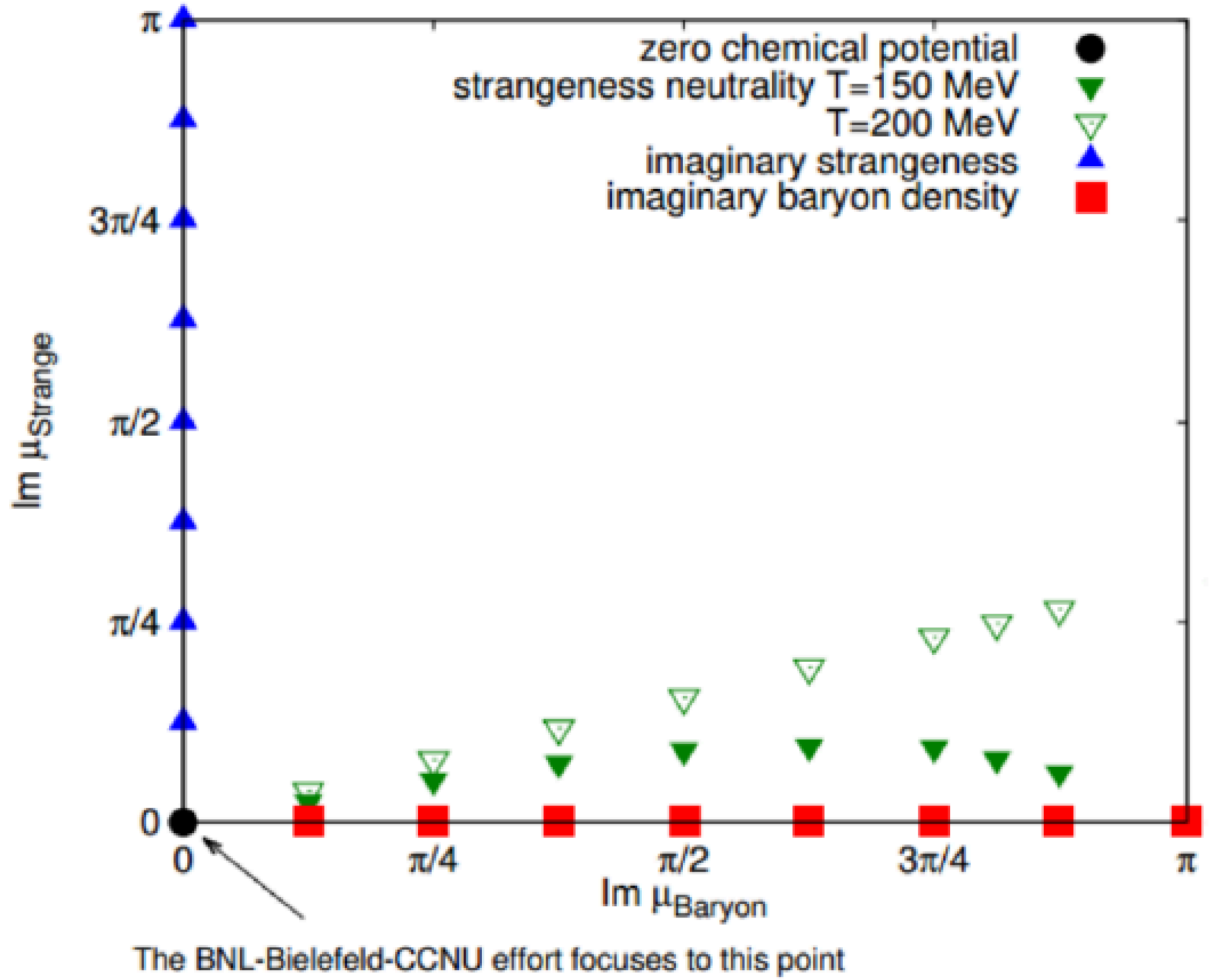
- For more chemical potentials: μ_B, μ_Q, μ_S, several trajectories are possible → useful for different physics
  - Here we use:

$\langle n_S\rangle=0$ $\qquad\qquad$ $\langle n_Q\rangle=0.4\langle n_B\rangle$

  - Other choices are possible, e.g.:

$\mu_S=0$ $\qquad\qquad$ $\mu_Q=0$

Simulation landscape

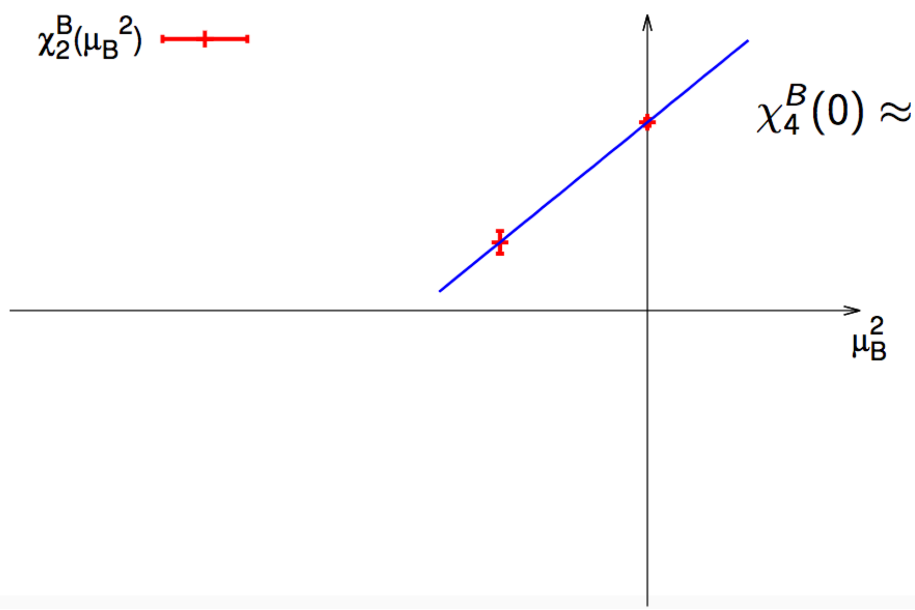


The imaginary μ_B approach

The standard method to calculate higher order fluctuations is to calculate the non-gaussianity of the baryon number distribution at $\mu_B = 0$.

$$\chi_4^B(0) = \frac{1}{VT^3}\left(\langle N_B^4\rangle - 3\langle N_B^2\rangle^2\right)$$

Alternatively one measures the variance $\chi_2^B(\mu_B^2)$ at non-zero chemical potentials.



$$\chi_4^B(0) \approx 2\frac{\chi_2^B(0) - \chi_2^B(-\Delta\mu_B^2)}{\Delta\mu_B^2}$$

The imaginary μ_B approach

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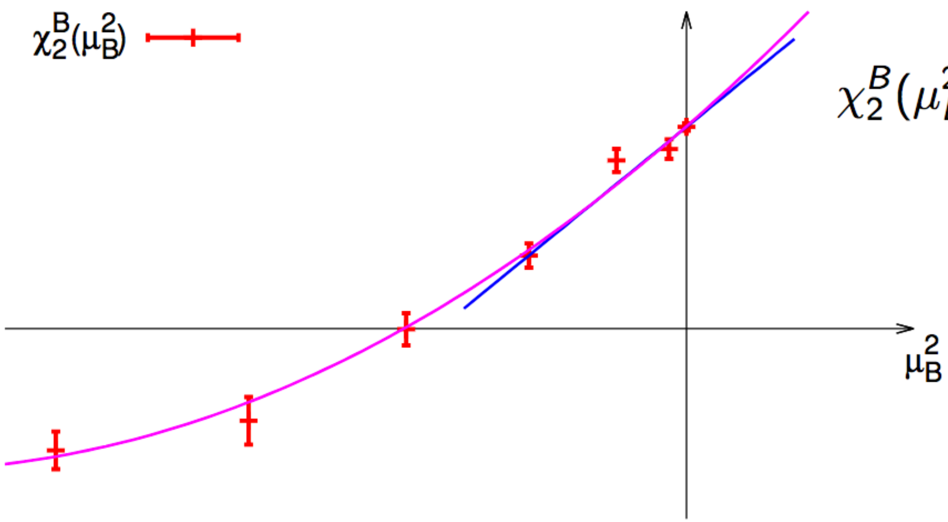


$$\chi_2^B(\mu_B^2) \approx \chi_2^B(0) + \frac{1}{2}\mu_B^2\chi_4^B(0) + \frac{1}{24}\mu_B^4\chi_6^B(0)$$

See also: [Wuppertal-Budapest 1607.02493, D'Elia et al 1611.08285]

Thermodynamic identities

- For the pressure we measure:

$$\frac{n}{\mu_B T^2} = \frac{T}{\mu_B} \frac{d\left(p/T^4\right)}{d\left(\mu_B/T\right)}\bigg|_{\langle n_S\rangle=0,\langle n_Q\rangle=0.4\langle n_B\rangle,T=\text{const.}}$$

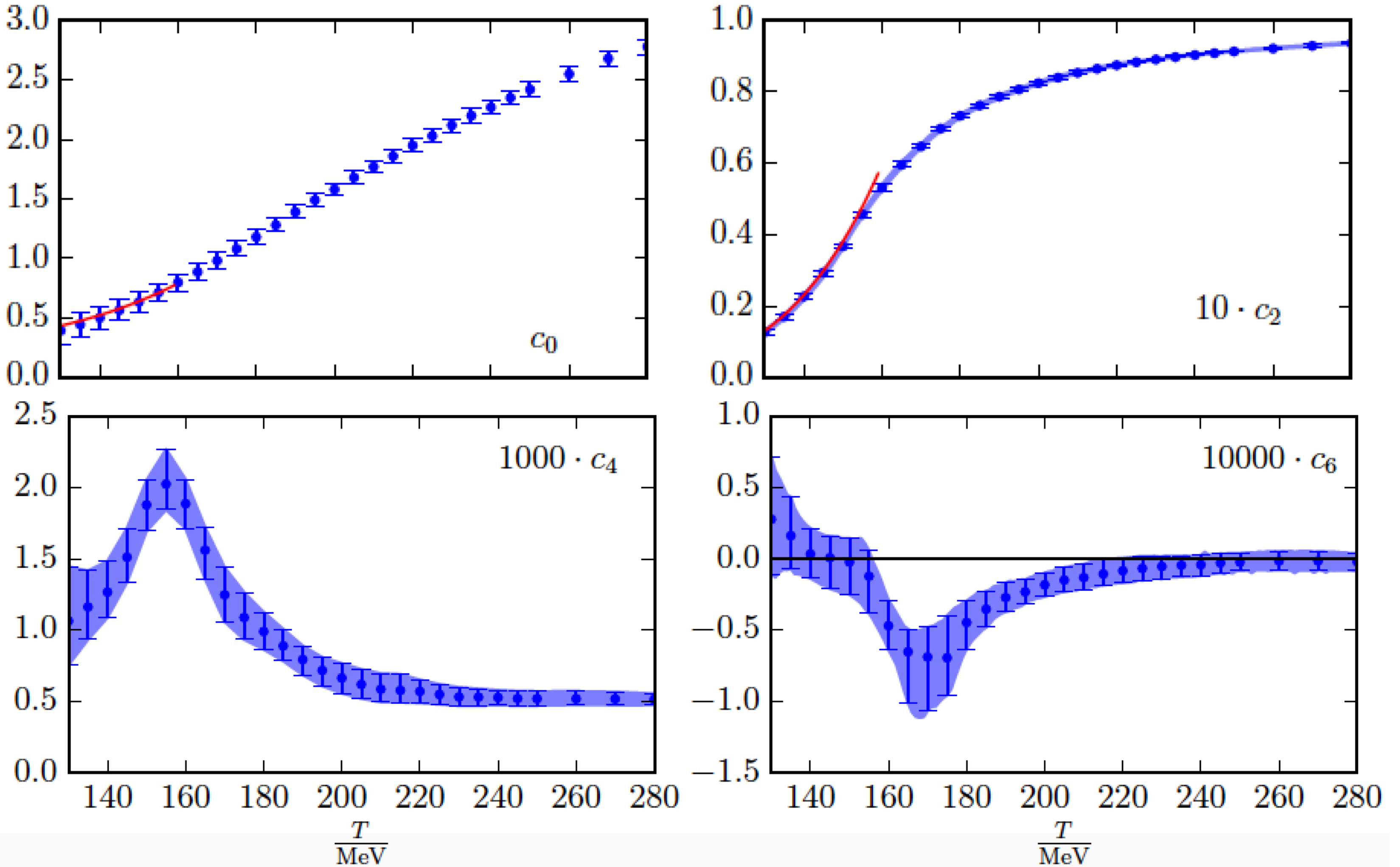
$$= n_B\left(1+0.4\frac{d\mu_Q}{d\mu_B}\right) = 2c_2 + 4c_4\left(\frac{\mu_B}{T}\right)^2 + 6c_6\left(\frac{\mu_B}{T}\right)^4 + \ldots$$

- For the entropy and energy:

$$s = [T^4\partial/\partial T + 4T^3](p/T^4)$$

$$\hat{\epsilon} = \hat{s} - \hat{p} + \hat{\mu}_Q\hat{n}_Q + \hat{\mu}_B\hat{n}_B$$

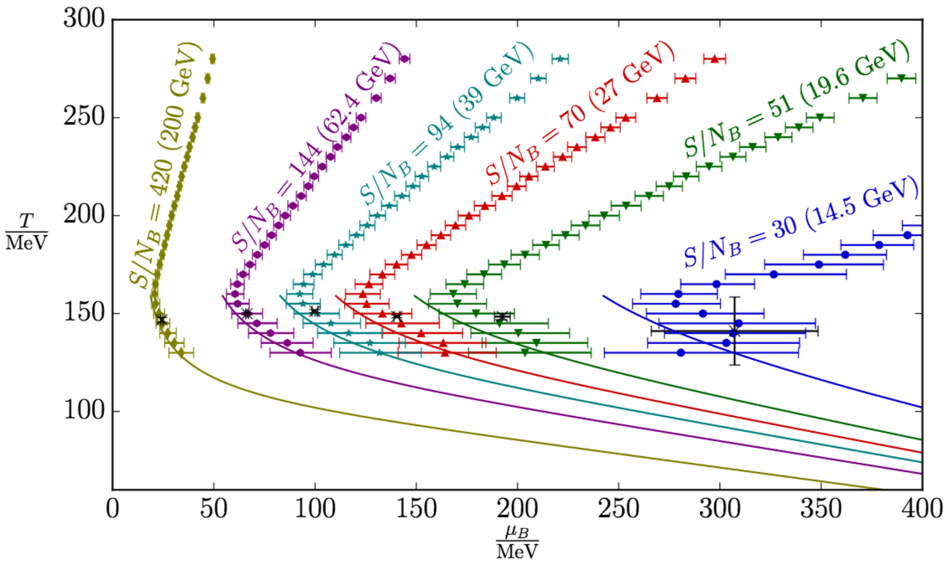
Taylor expansion of the pressure for <n_S>=0

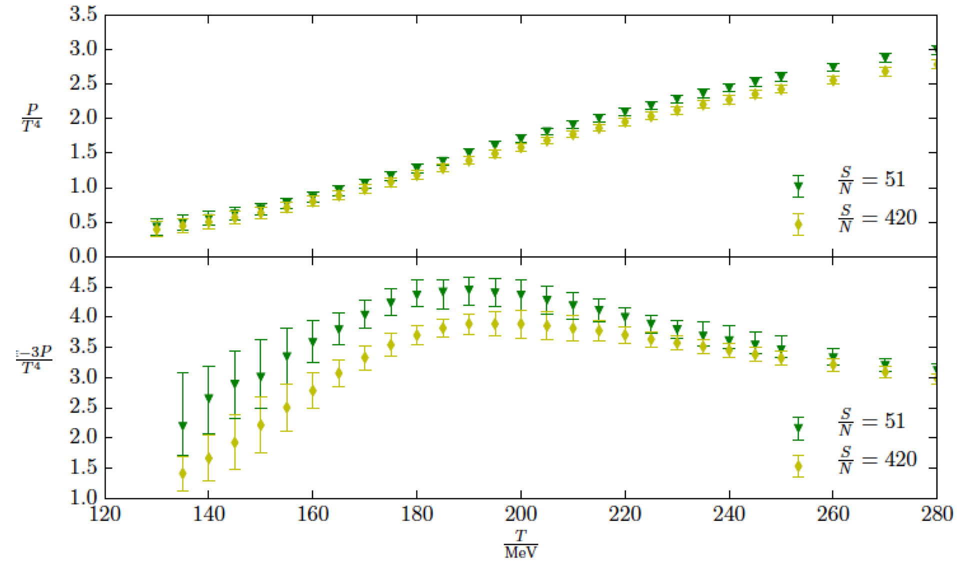


WB: S. Borsanyi et al. 1607.02493 (2016)

Equation of state at μ_B>0

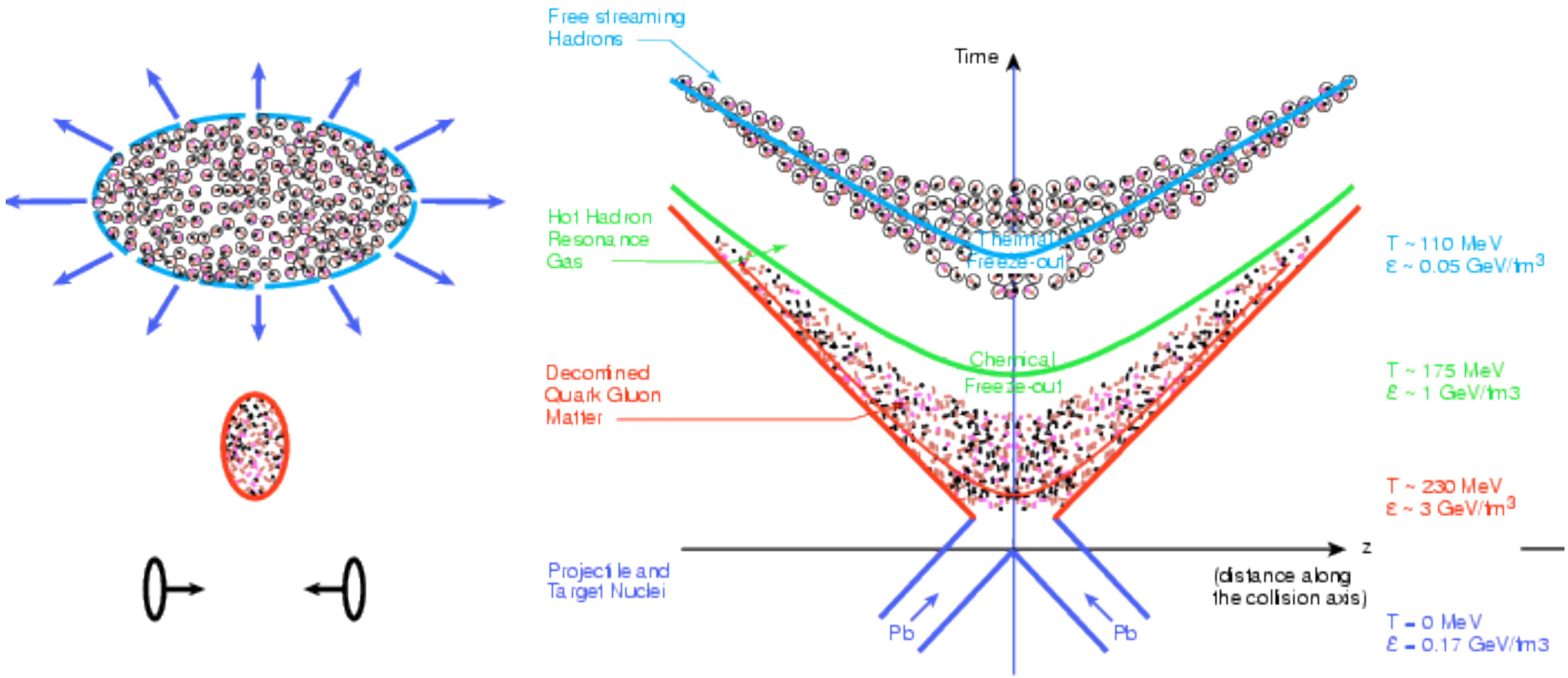
- Extract the isentropic trajectory that the system follows in the absence of dissipation
- Calculate the EoS along these constant S/N trajectories





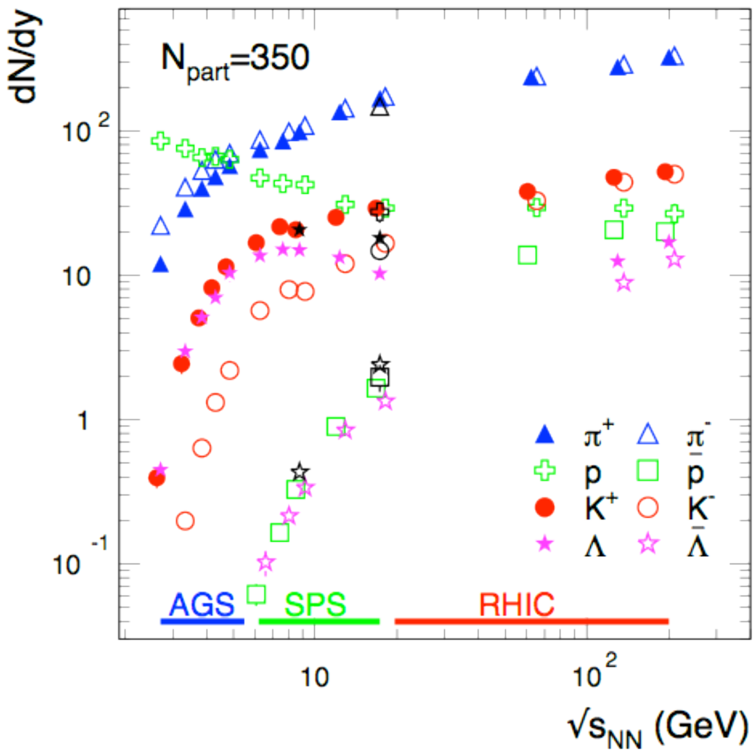
WB: S. Borsanyi et al. 1607.02493, (2016)

Evolution of a Heavy Ion Collision



- Chemical freeze-out: inelastic reactions cease: the chemical composition of the system is fixed (particle yields and fluctuations)
- Kinetic freeze-out: elastic reactions cease: spectra and correlations are frozen (free streaming of hadrons)
- Hadrons reach the detector

Hadron yields

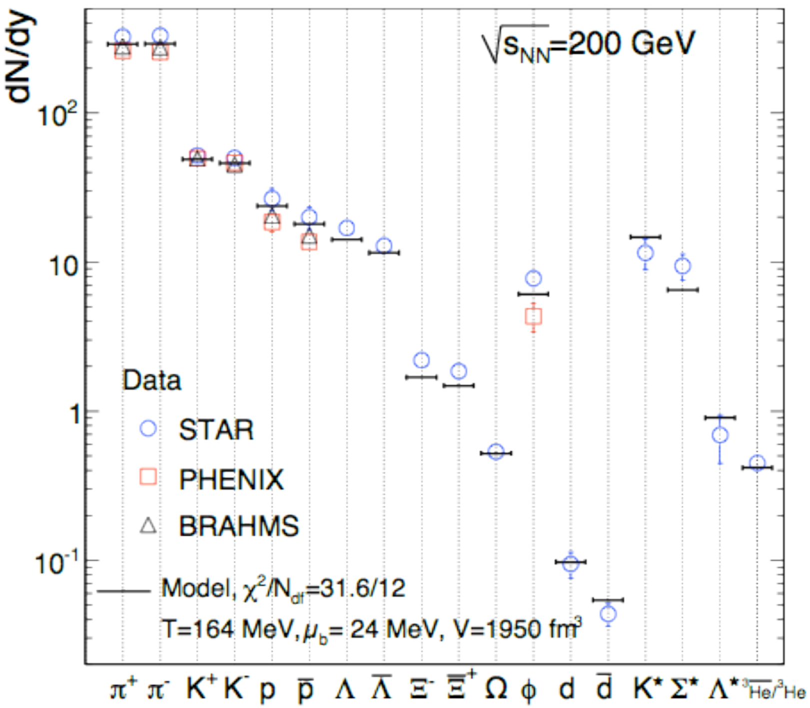


- $E=mc^2$: lots of particles are created
- Particle counting (average over many events)
- Take into account:
  - detector inefficiency
  - missing particles at low p_T
  - decays

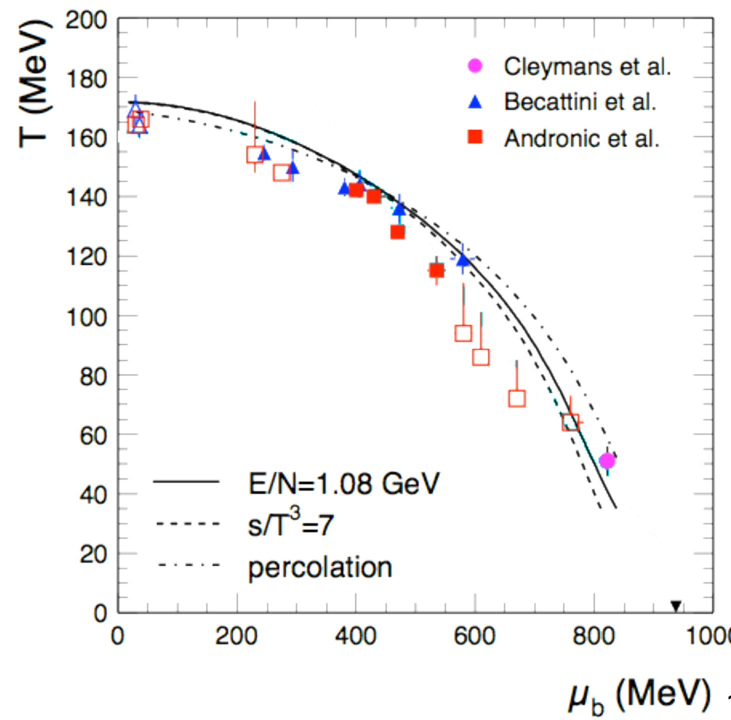
- HRG model: test hypothesis of hadron abundancies in equilibrium

$$N_i = -T\frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i V}{2\pi^2}\int_0^\infty \frac{p^2 dp}{\exp[(E_i - \mu_i)/T] \pm 1}$$

The thermal fits



- Fit is performed minimizing the χ^2
- Fit to yields: parameters T, μB, V
- Fit to ratios: the volume V cancels out



- Changing the collision energy, it is possible to draw the freeze-out line in the T, μB plane

Fluctuations of conserved charges

- Definition:

$$\chi_{lmn}^{BSQ} = \frac{\partial^{l+m+n} p/T^4}{\partial(\mu_B/T)^l \partial(\mu_S/T)^m \partial(\mu_Q/T)^n}.$$

- Relationship between chemical potentials:

$$\begin{aligned} \mu_u &= \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q; \\ \mu_d &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q; \\ \mu_s &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S. \end{aligned}$$

- They can be calculated on the lattice and compared to experiment

Connection to experiment

- Fluctuations of conserved charges are the cumulants of their event-by-event distribution

$$\text{mean}: M = \chi_1 \qquad \text{variance}: \sigma^2 = \chi_2$$

$$\text{skewness}: S = \chi_3/\chi_2^{3/2} \qquad \text{kurtosis}: \kappa = \chi_4/\chi_2^2$$

$$S\sigma = \chi_3/\chi_2 \qquad \kappa\sigma^2 = \chi_4/\chi_2$$

$$M/\sigma^2 = \chi_1/\chi_2 \qquad S\sigma^3/M = \chi_3/\chi_1$$

F. Karsch: Centr. Eur. J. Phys. (2012)

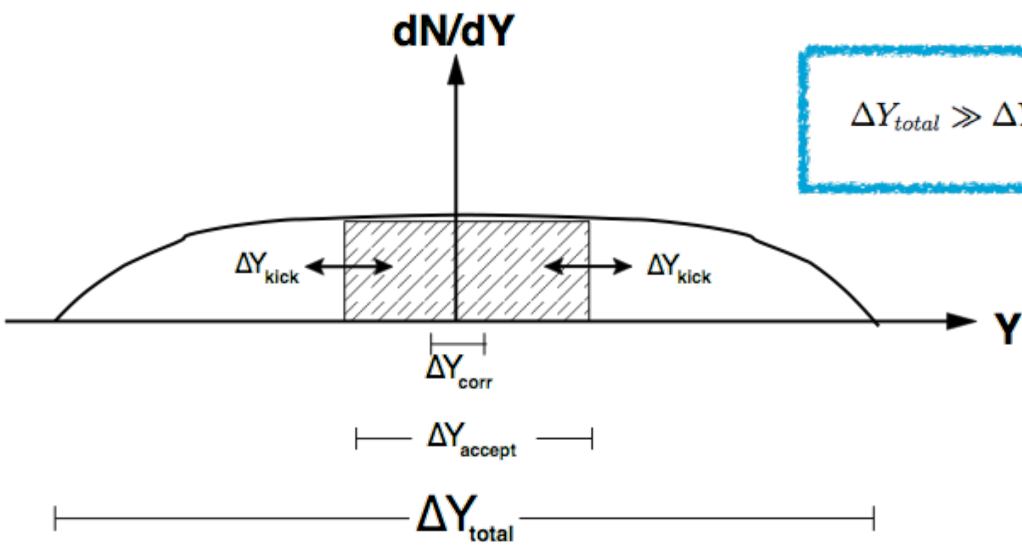
- The chemical potentials are not independent: fixed to match the experimental conditions:

$$\langle n_S \rangle = 0 \qquad \langle n_Q \rangle = 0.4 \langle n_B \rangle$$

Fluctuations of conserved charges

V. Koch (2008)

- ✱ If we look at the entire system, none of the conserved charges will fluctuate
- ✱ By studying a sufficiently small subsystem, the fluctuations of conserved quantities become meaningful



$$\Delta Y_{total} \gg \Delta Y_{accept} \gg \Delta Y_{kick}$$

- ❑ ΔYtotal: range for total charge multiplicity distribution
- ❑ ΔYaccept: interval for the accepted charged particles
- ❑ ΔYkick: rapidity shift that charges receive during and after hadronization

Things to keep in mind

- Effects due to volume variation because of finite centrality bin width
  - Experimentally corrected by centrality-bin-width correction method

V. Skokov et al., PRC (2013), P. Braun-Munzinger et al., NPA (2017), V. Begun and M. Mackowiak-Pawlowska (2017)

- Finite reconstruction efficiency
  - Experimentally corrected based on binomial distribution A.Bzdak,V.Koch, PRC (2012)
- Spallation protons
  - Experimentally removed with proper cuts in p_T
- Canonical vs Gran Canonical ensemble
  - Experimental cuts in the kinematics and acceptance V. Koch, S. Jeon, PRL (2000)
- Proton multiplicity distributions vs baryon number fluctuations
  - Recipes for treating proton fluctuations

M. Asakawa and M. Kitazawa, PRC(2012), M. Nahrgang et al., 1402.1238

- Final-state interactions in the hadronic phase
  - Consistency between different charges = fundamental test

J.Steinheimer et al., PRL (2013)

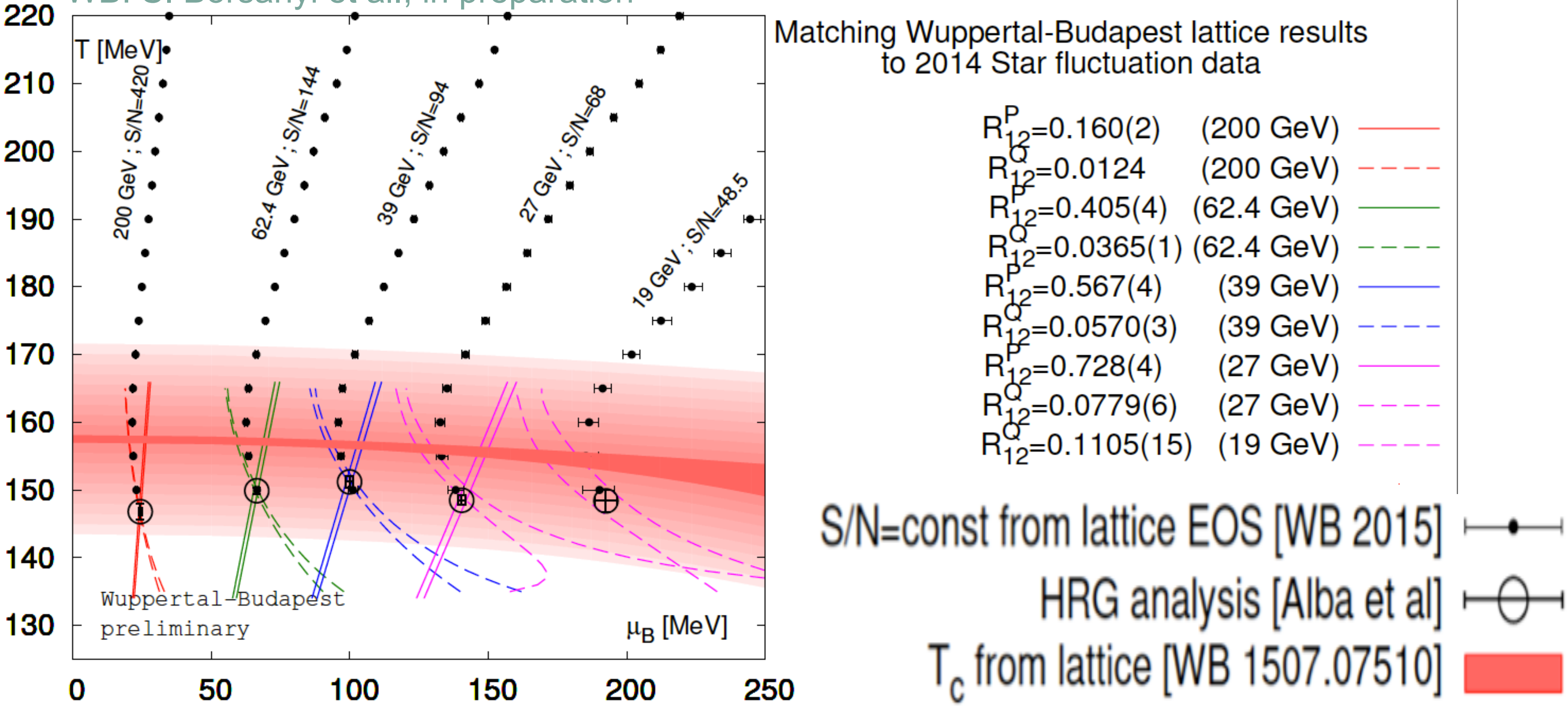
Freeze-out line from first principles

- Use T- and μ_B-dependence of R_{12}^Q and R_{12}^B for a combined fit:

$$R_{12}^Q(T,\mu_B) = \frac{\chi_1^Q(T,\mu_B)}{\chi_2^Q(T,\mu_B)} = \frac{\chi_{11}^{QB}(T,0) + \chi_2^Q(T,0)q_1(T) + \chi_{11}^{QS}(T,0)s_1(T)}{\chi_2^Q(T,0)}\frac{\mu_B}{T} + \mathcal{O}(\mu_B^3).$$

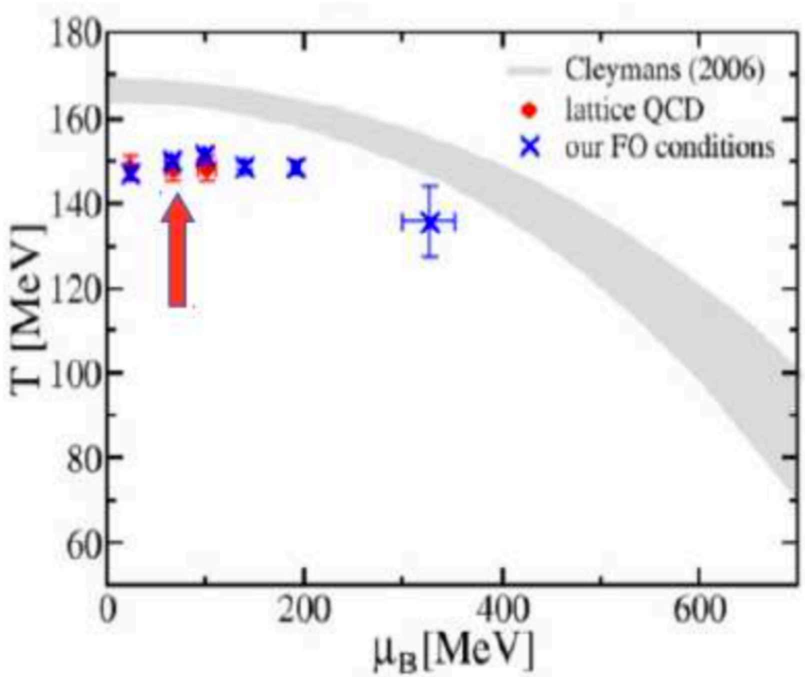
$$R_{12}^B(T,\mu_B) = \frac{\chi_1^B(T,\mu_B)}{\chi_2^B(T,\mu_B)} = \frac{\chi_2^B(T,0) + \chi_{11}^{BQ}(T,0)q_1(T) + \chi_{11}^{BS}(T,0)s_1(T)}{\chi_2^B(T,0)}\frac{\mu_B}{T} + \mathcal{O}(\mu_B^3)$$

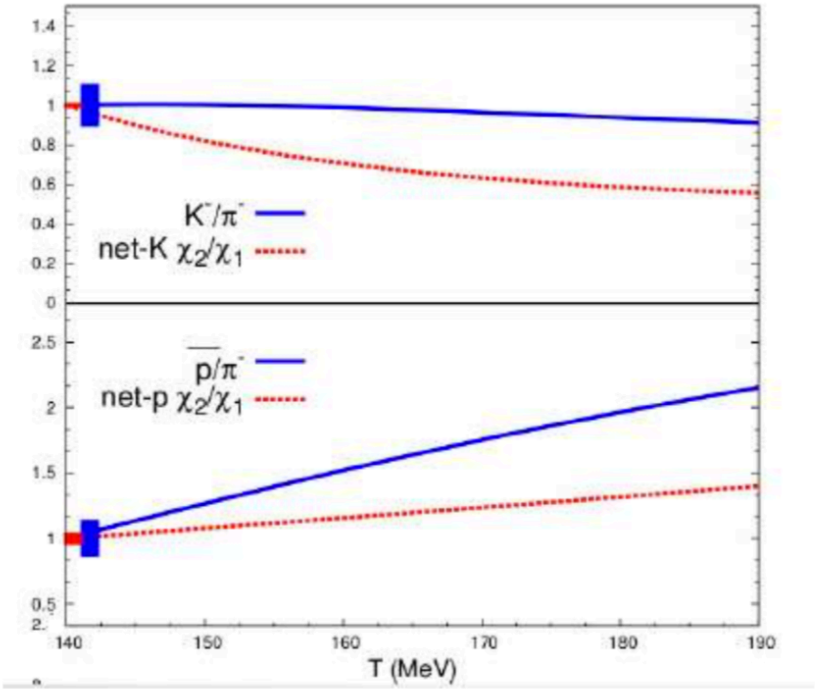
WB: S. Borsanyi et al., in preparation



Freeze-out in the HRG model

- Experimental cuts in acceptance and momentum
- Resonance decay and regeneration



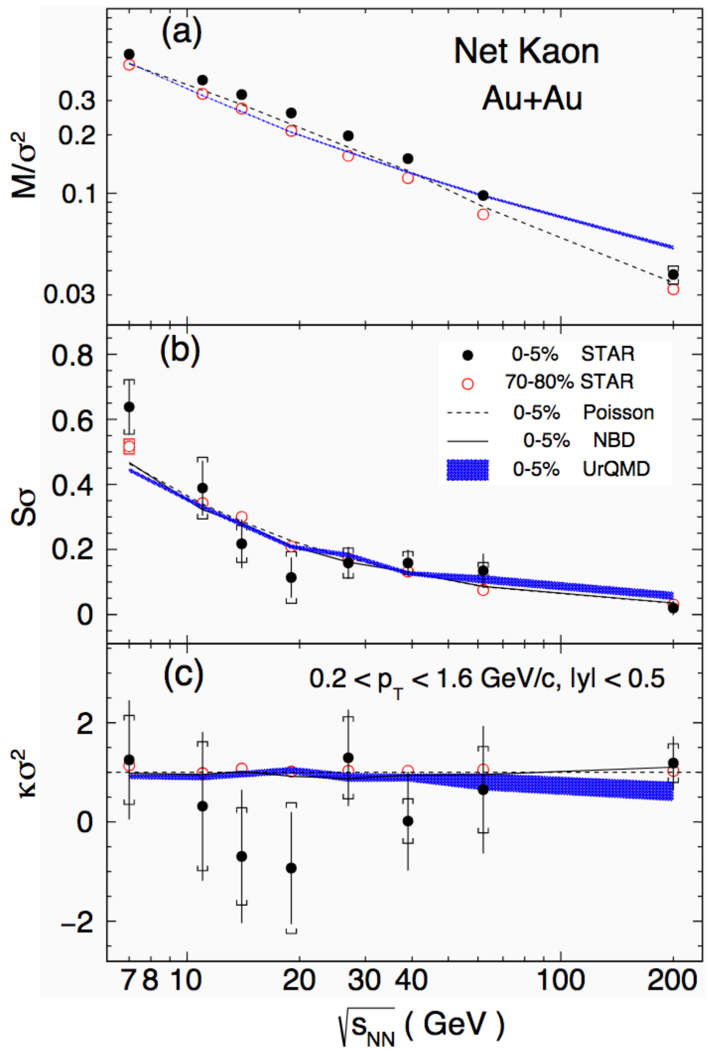


P. Alba et al., PLB 2014

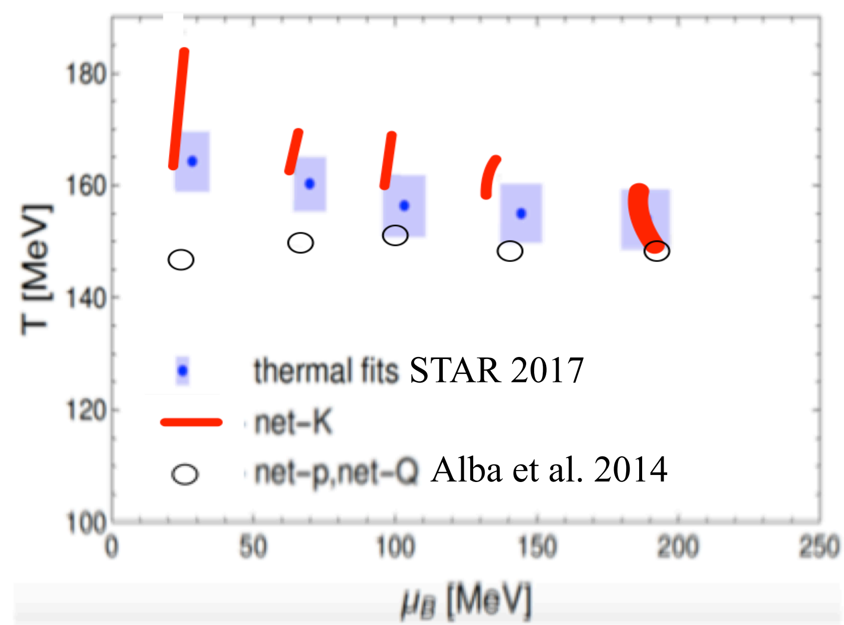
P. Alba et al., PRC 2016

Freeze-out of kaons in the HRG model

STAR Collaboration (2017)



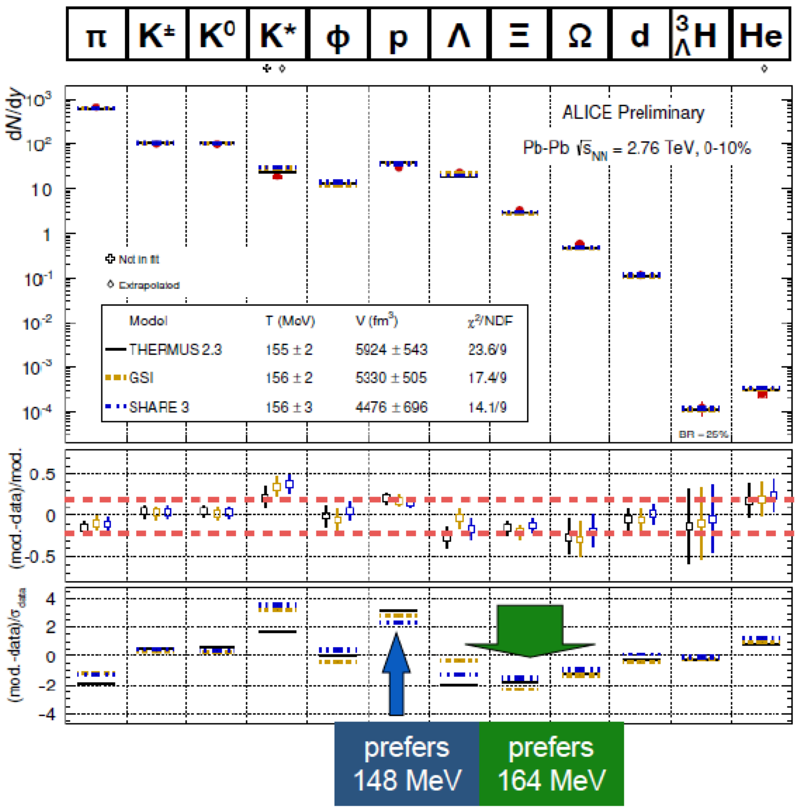
- Calculate χ_1/χ_2 for kaons in the HRG model, including resonance decays and acceptance cuts
- Calculate it along the isentropes



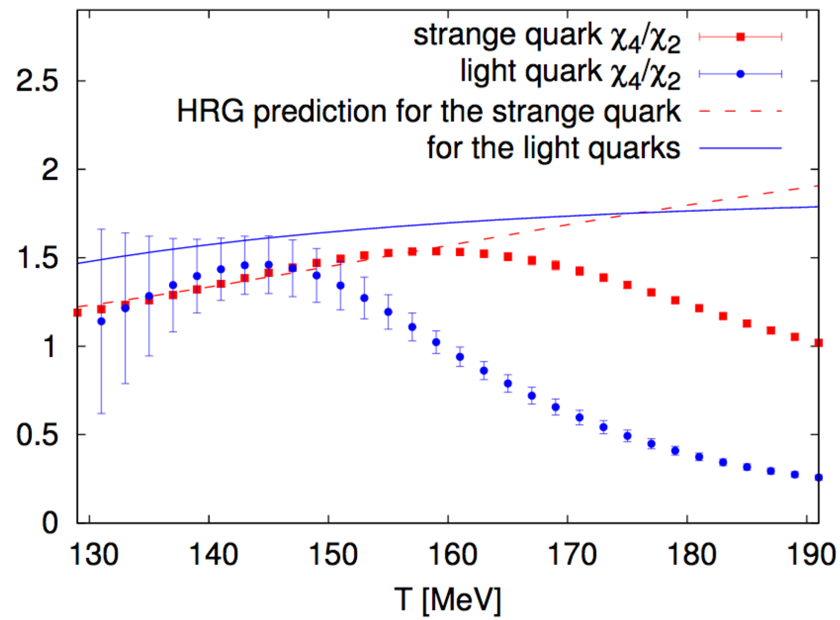
J. Noronha-Hostler. et al., in preparation

Flavor-dependent freeze-out temperature?

- Also yield fits hint at a higher temperature for strange particles



M. Floris: QM 2014

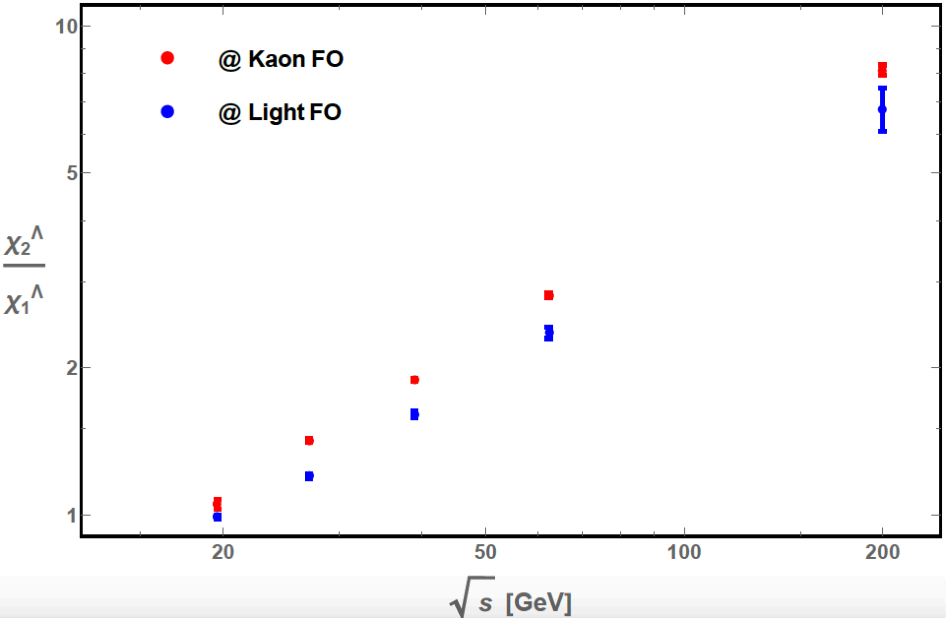


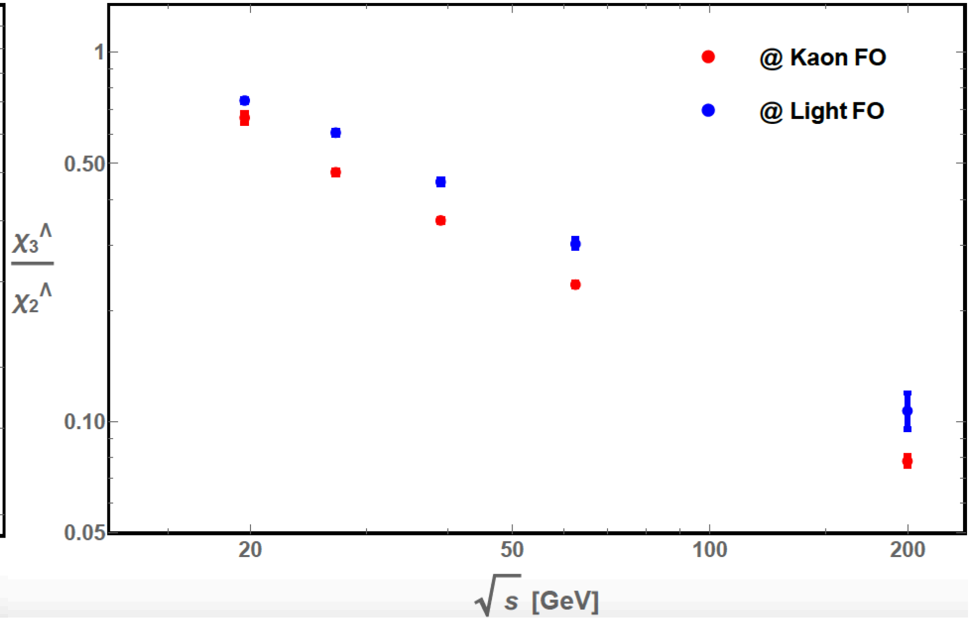
- Similar behavior found in lattice QCD results

R. Bellwied et al. (WB Collaboration): PRL2013

Prediction: Λ fluctuations

- Calculate χ_1/χ_2 for Λ at the light vs kaon freeze-out parameters
- Clear separation between the two scenarios
- Future measurements will be able to distinguish





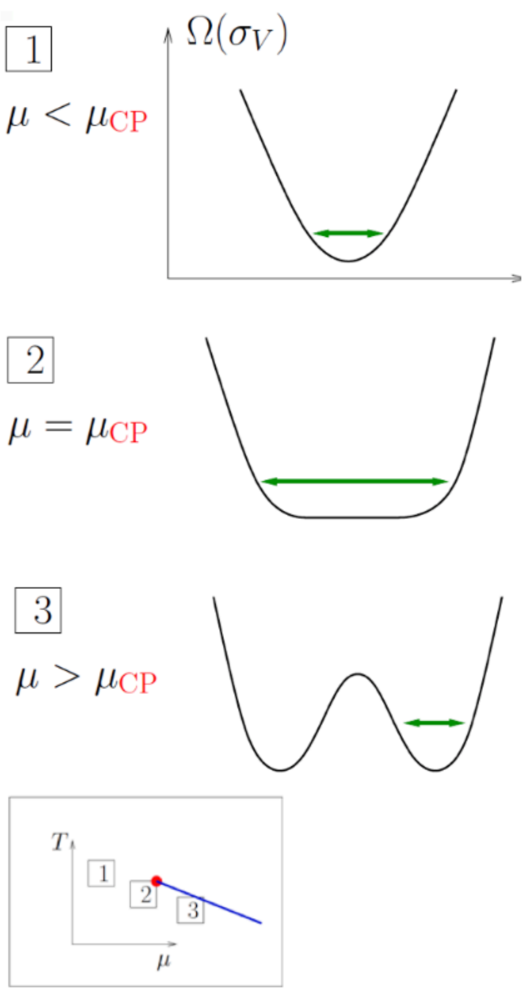
J. Noronha-Hostler. et al., in preparation

Conclusions

- Unprecedented precision in lattice QCD data allows a direct comparison to experiment for the first time
- QCD thermodynamics at μ_B=0 can be simulated with high accuracy
- Extensions to finite density are under control up to $O(\mu_B^6)$
- Comparison with experiment allows to determine properties of strongly interacting matter (freeze-out parameters)
- Kaons seem to prefer a larger freeze-out temperature
- Future Λ fluctuations measurements will help clarify this issue

Fluctuations at the critical point

M. Stephanov, PRL (2009).



The probability distribution for the order parameter

$$P[\sigma] \sim \exp\{-\Omega[\sigma]/T\}$$

$$\Omega = \int d^3x \left[\frac{1}{2}(\nabla\sigma)^2 + \frac{m_\sigma^2}{2}\sigma_2 + \frac{\lambda_3}{3}\sigma^3 + \cdots\right]$$

The correlation length ($\xi = m_\sigma^{-1}$)

$\xi \sim |T - T_c|^{-\nu}$ where $\nu > 0$

$$\chi_2 = VT\xi^2$$

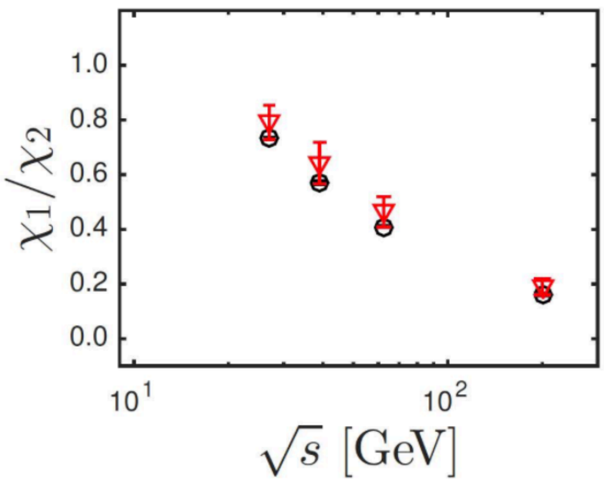
$$\chi_3 = 2VT^{3/2}\hat{\lambda}_3\xi^{9/2}$$

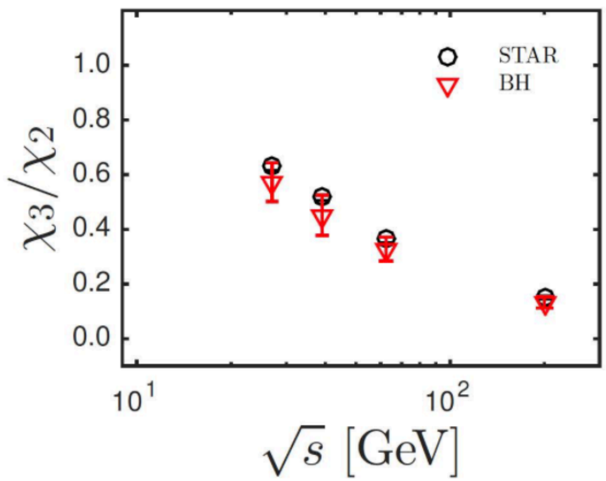
$$\chi_4 = 6VT^2[2\hat{\lambda}_3^2 - \hat{\lambda}_4]\xi^7$$

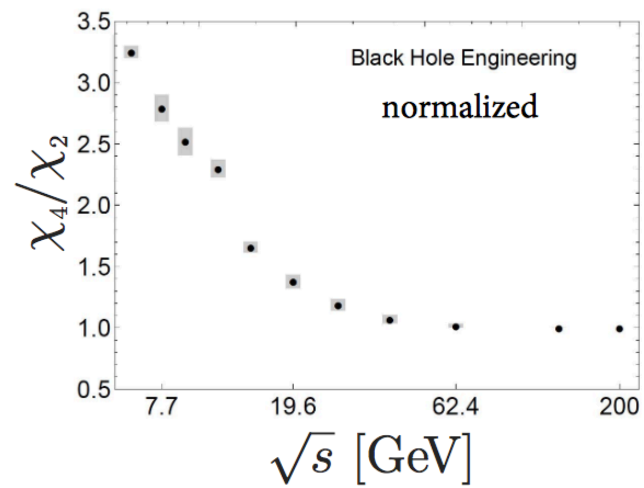
Connection to experiment

R. Critelli, C. R. et al., (2017)

- We want to estimate the collision energy we need to find the critical point in experiments
- We compare the baryonic BH susceptibilities ratios with the net-proton moments measured at STAR
- Freeze-out parameters are extracted by fitting the experimental values for χ1/χ2 and χ3/χ2







[STAR] Phys. Rev. Lett. **112** (2014)

Testing the Taylor expansion

R. Critelli, C. R. et al., forthcoming

Taylor expansion of observables in terms of susceptibilities $\chi_n = \chi_n^B(T, \mu_B = 0)$

- Pressure

$$\frac{p(T,\mu_B) - p(T,\mu_B = 0)}{T^4} = \sum_{n=1}^{\infty} \frac{\chi_{2n}}{(2n!)} \left(\frac{\mu_B}{T}\right)^{2n}$$

- Baryonic density

$$\frac{\rho_B(T,\mu_B)}{T^3} = \sum_{n=1}^{\infty} \frac{\chi_{2n}}{(2n-1)!} \left(\frac{\mu_B}{T}\right)^{2n-1}$$

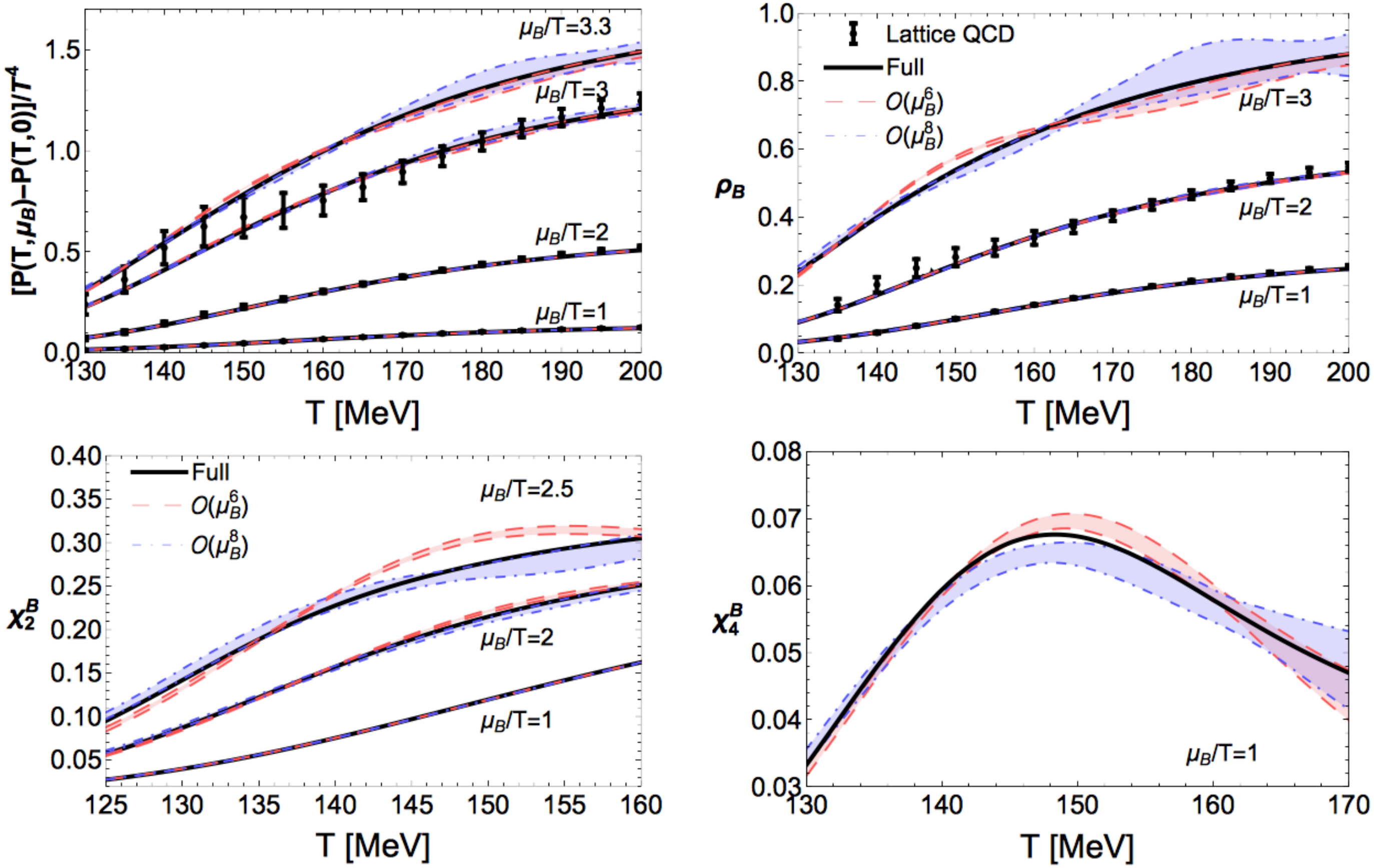
- Susceptibilities χ_2 and χ_4

$$\chi_2(T,\mu_B) = \sum_{n=0}^{\infty} \frac{\chi_{2n+2}}{(2n!)} \left(\frac{\mu_B}{T}\right)^{2n} \qquad \chi_4(T,\mu_B) = \sum_{n=0}^{\infty} \frac{\chi_{2n+4}}{(2n!)} \left(\frac{\mu_B}{T}\right)^{2n}$$

Testing the Taylor expansion

R. Critelli, C. R. et al., forthcoming

Reconstruction of thermodynamic quantities at different values of μ_B/T via Taylor series from calculations at $\mu_B = 0$.



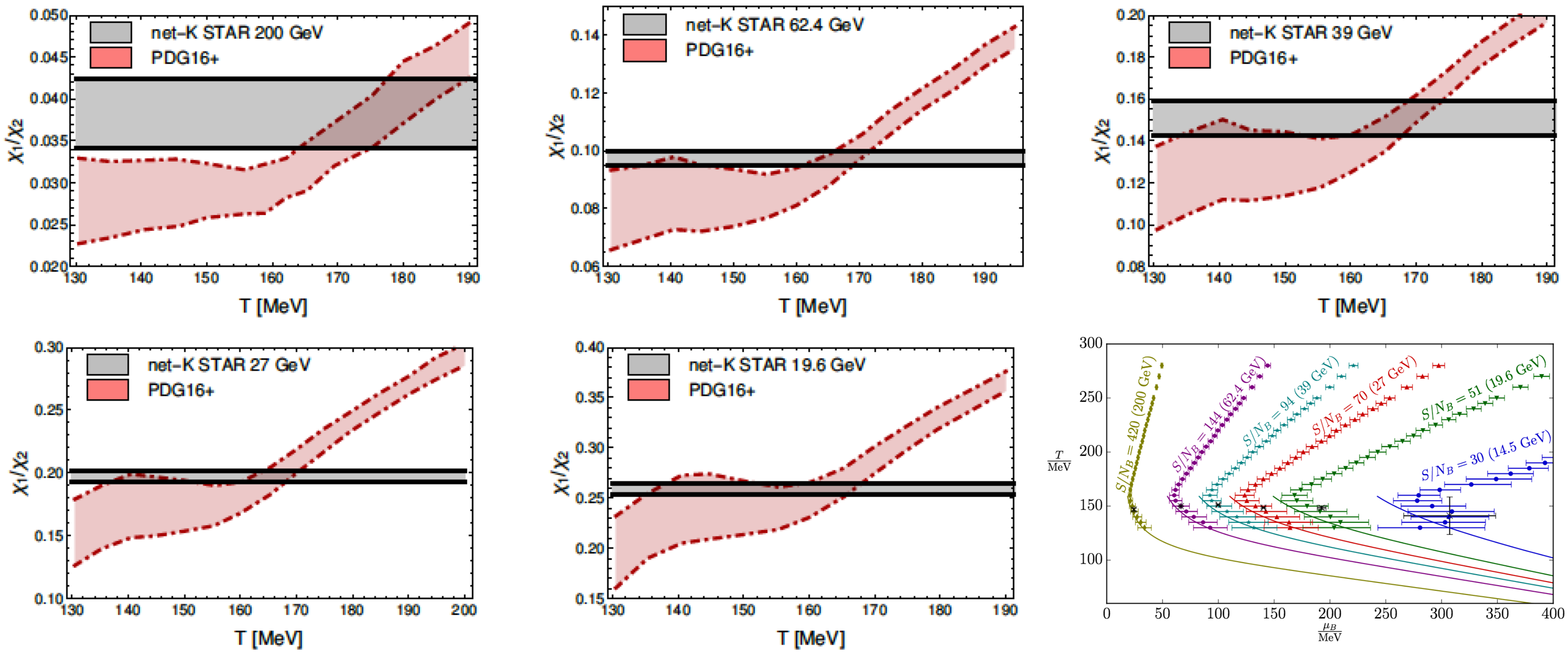
Lattice results: [HotQCD] Phys. Rev. D, **95**, 054504 (2017).

Lattice details

- **The 4stout staggered action**
  - 2+1+1 dynamical flavors
  - 4 levels of stout smearing in the fermionic action
  - The masses are set by bracketing both the pion and the kaon masses within a few percent, keeping $m_c/m_s=11.85$
  - The scale is set in two ways: f_π and w_0 (with Wilson flow). The scale setting procedure is one of the source of the systematic error in all of the plots

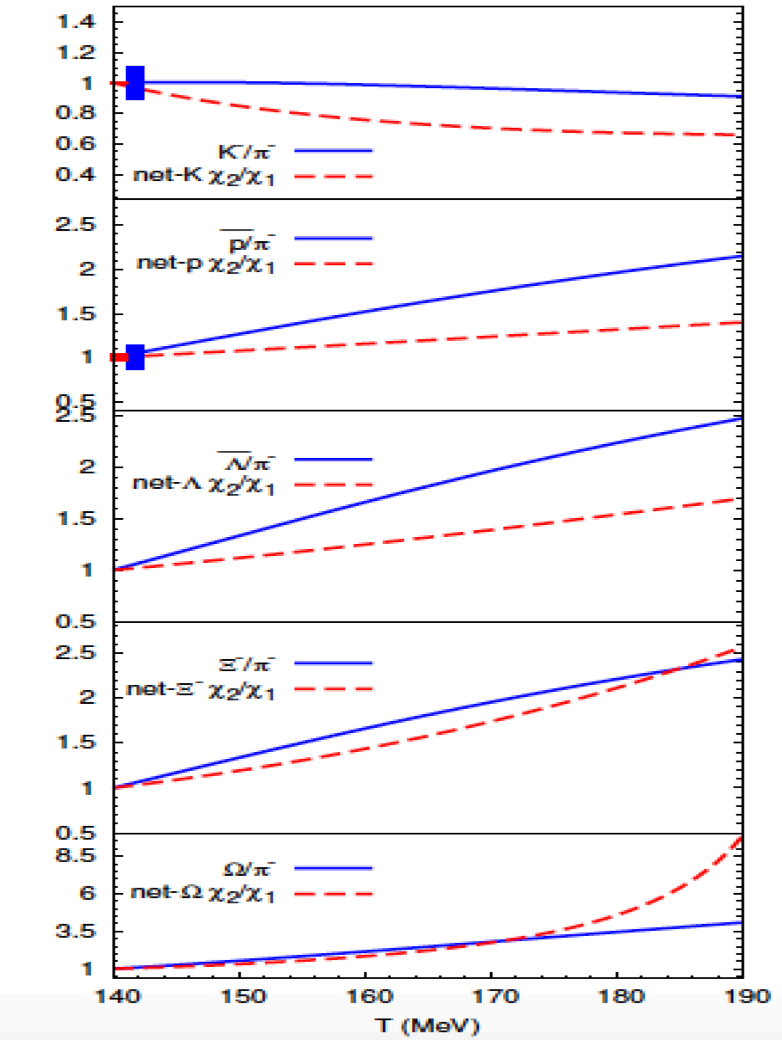
- **Ensembles**
  - Continuum limit from $N_t=10, 12, 16$
  - For imaginary μ we have $\mu_B=iT\pi j/8$, with $j=3, 4, 5, 6, 6.5, 7$

Kaon freeze-out fit



C. R. et al., in preparation

Sensitivity of kaon fluctuations to freeze-out parameters

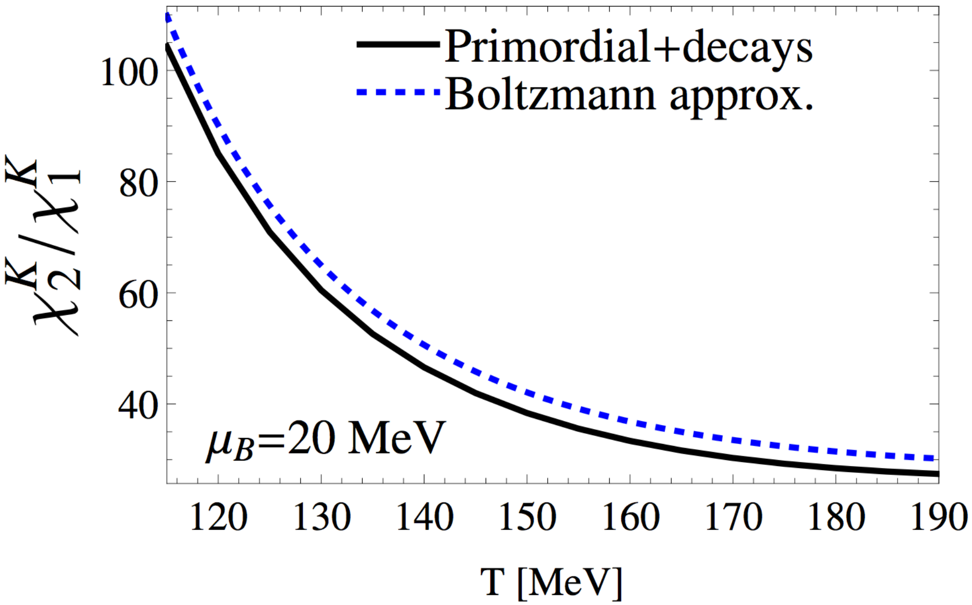


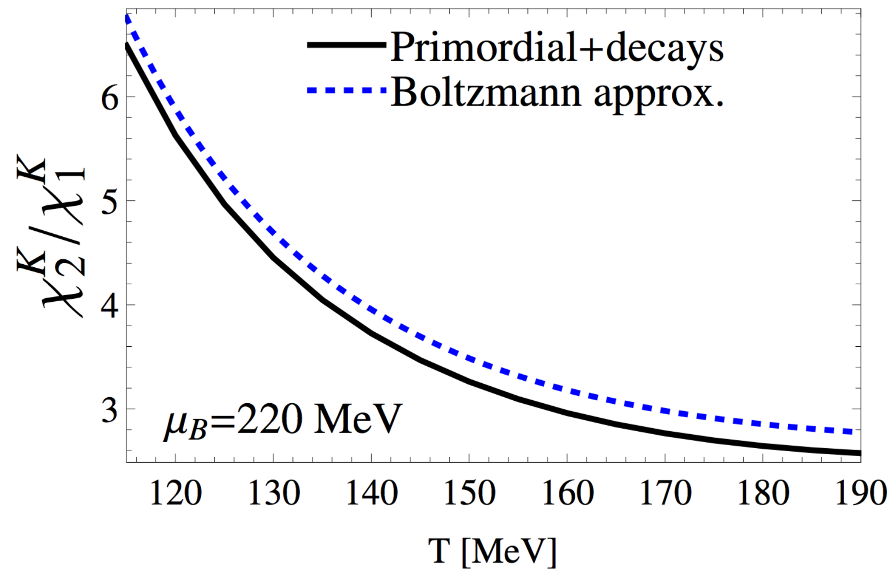
Kaon fluctuations are more sensitive to the freeze-out parameters, compared to yields

P. Alba et al., PRC (2015)

Kaon fluctuations on the lattice

J. Noronha-Hostler, C.R. et al., 1607.02527





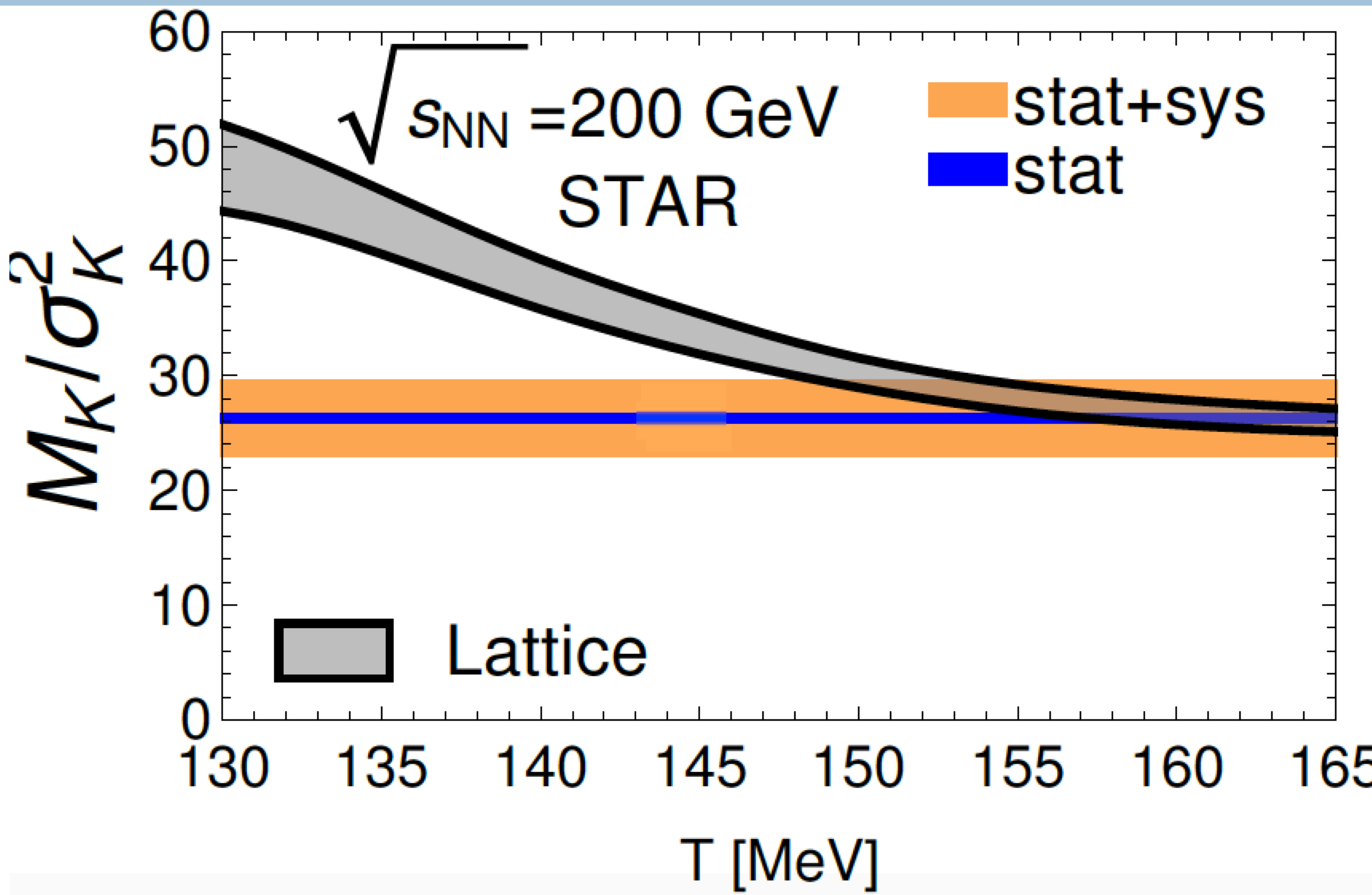
- Boltzmann approximation works well for lower order kaon fluctuations

$$\frac{\chi_2^K}{\chi_1^K} = \frac{\cosh(\hat{\mu}_S + \hat{\mu}_Q)}{\sinh(\hat{\mu}_S + \hat{\mu}_Q)}$$

- χ_2^K/χ_1^K from primordial kaons + decays is very close to the one in the Boltzmann approximation

Kaon fluctuations on the lattice

J. Noronha-Hostler, C.R. et al., 1607.02527

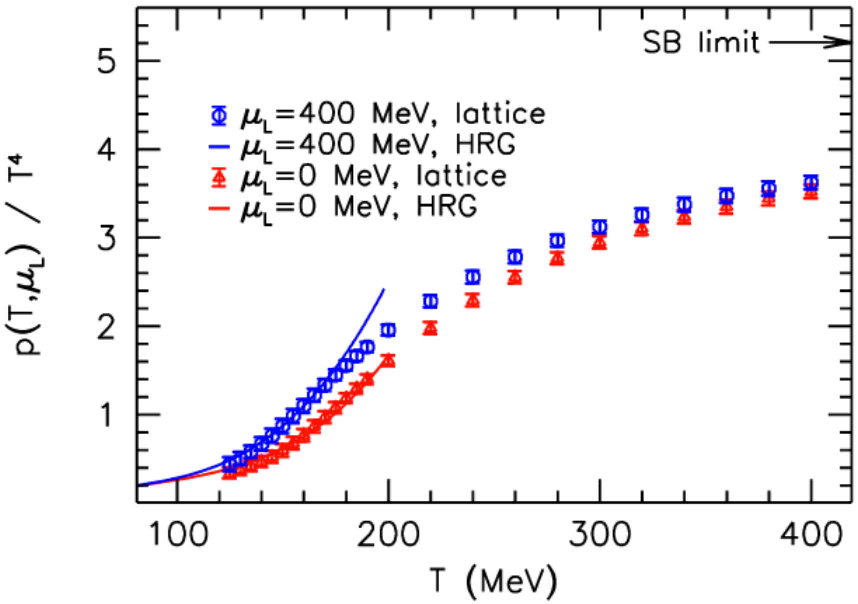


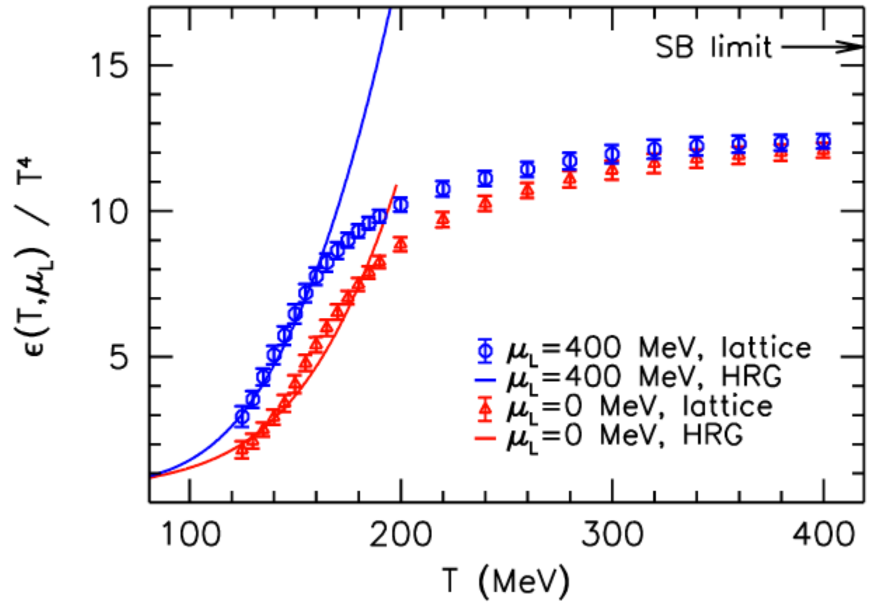
- Experimental uncertainty does not allow a precise determination of T_f^K
- It looks like T_f^K>150 MeV

Equation of state at $\mu_B>0$

- Expand the pressure in powers of μ_B (or $\mu_L=3/2(\mu_u+\mu_d)$)

$$\frac{p(T,\{\mu_i\})}{T^4}=\frac{p(T,\{0\})}{T^4}+\frac{1}{2}\sum_{i,j}\frac{\mu_i\mu_j}{T^2}\chi_2^{ij} \quad \text{with} \quad \chi_2^{ij}\equiv\frac{T}{V}\frac{1}{T^2}\left.\frac{\partial^2\log\mathcal{Z}}{\partial\mu_i\partial\mu_j}\right|_{\mu_i=\mu_j=0}$$





S. Borsanyi et al., JHEP (2012)

- Continuum extrapolated results at the physical mass

Analytical continuation – illustration of systematics