Classical Langevin Approach to Heavy Quarkonia

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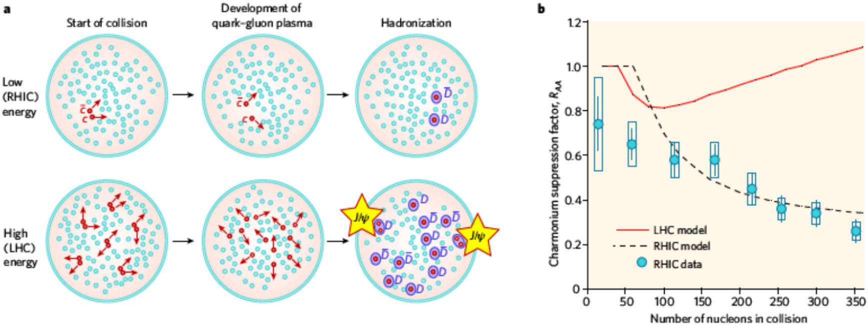
34th Winter Workshop on Nuclear Dynamics (WWND 2018), Guadeloupe March 25-31, 2018

March 25-31, 2018

Outline

Motivation

- J/ψ suppression in heavy-ion collisions as signal for deconfinement [MS86]
- heavy quarkonia produced in primordial hard collisions
- melt/dissociate in QGP due to color screening/collisions
- at higher beam energies: also regeneration



[from J. Stachel, Talk at EMMI workshop Feb/13/18]

Langevin Equation for a $Q\bar{Q}$ pair

Langevin Equation for a $Q\bar{Q}$ pair

- Fokker-Planck equation for c and c̄ quarks
- single as well as many pairs

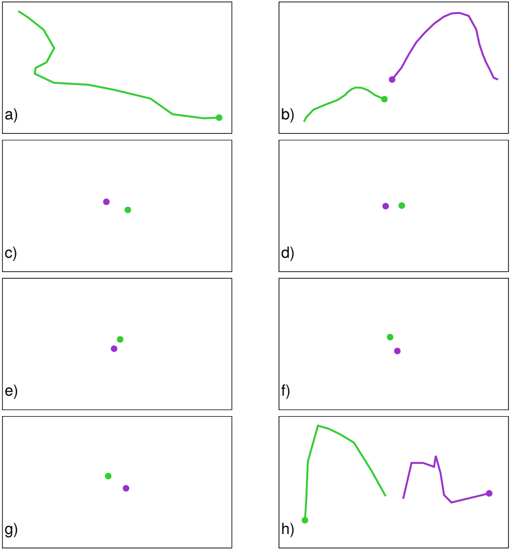
$$\dot{\boldsymbol{r}} = \frac{1}{2M}\boldsymbol{p},$$
$$\dot{\boldsymbol{p}} = \boldsymbol{F}(\boldsymbol{r}-\bar{\boldsymbol{r}}) - \gamma\boldsymbol{p} + \sqrt{2MT\gamma\Delta t}\,\boldsymbol{\rho}$$

and analogous for c̄

- γ: drag coefficient from [BDBFG16] (Abelian plasma model)
- $\boldsymbol{\rho}$: Gaussian-distributed white noise
- $\boldsymbol{F} = -\nabla V$ from same model!

Langevin Equation for a $Q\bar{Q}$ pair

Time evolution



(a) trajectory of single c quark

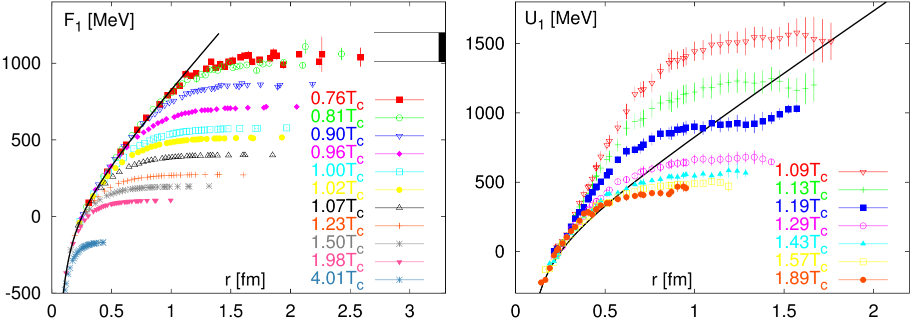
(b) trajectory of a $c\bar{c}$ pair

(c-g) time sequence of a bound $c\bar{c}$ pair

(h) trajectory of a dissociating bound state

In-medium lQCD heavy-quark potentials

- free energy F_1
- internal energy $U_1 = F_1 + TS$

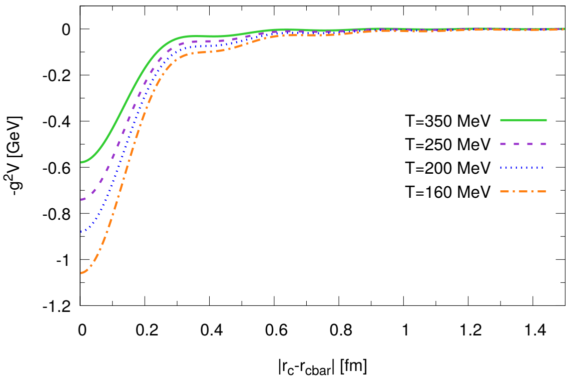


[KZ06]

Langevin Equation for $Q\bar{Q}$ pair

- UV-regularize screened Coulomb potential: $\Lambda = 4\,\text{GeV}$
- running coupling

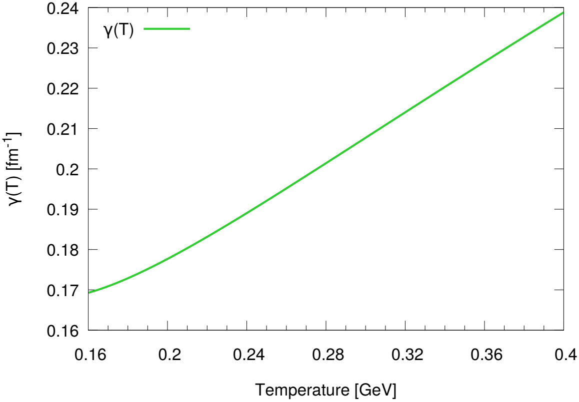
$$g^2 = 4\pi\alpha_s = \frac{4\pi\alpha_s(T_c)}{1 + C\ln(T/T_c)}, \quad C = 0.76, \quad T_c = 160\,\text{MeV}, \quad \alpha_s(T_c) = 0.5$$



Langevin Equation for $Q\bar{Q}$ pair

- UV-regularize screened Coulomb potential: $\Lambda = 4\,\text{GeV}$
- drag coefficient

$$\gamma = \frac{m_\text{D}^2}{24\pi M}\left[\ln\left(1+\frac{\Lambda^2}{m_\text{D}^2}\right) - \frac{\Lambda^2/m_\text{D}^2}{1+\Lambda^2/m_\text{D}^2}\right]$$

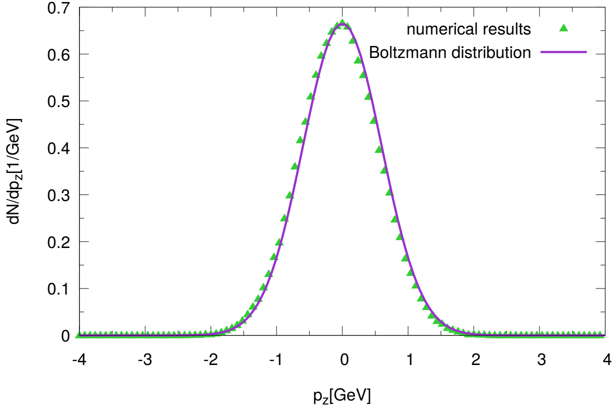


Numerical tests

Equilibrium limit: single-quark distribution

- single $c\bar{c}$ pair, $T = 200\,\text{MeV}$, $M = 1.8\,\text{GeV}$ open system
- equlibrium limit: momentum distribution

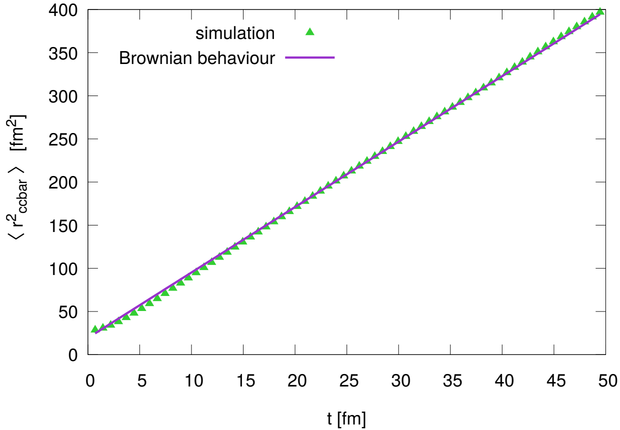
$$f_{\text{eq}}(\boldsymbol{p}) \propto \exp\left(-\frac{\boldsymbol{p}^2}{2MT}\right)$$



Diffusion behavior

- single $c\bar{c}$ pair, $T = 200\,\text{MeV}$, $M = 1.8\,\text{GeV}$, open system
- Brownian behavior of rms distance between c and $\bar{c}$

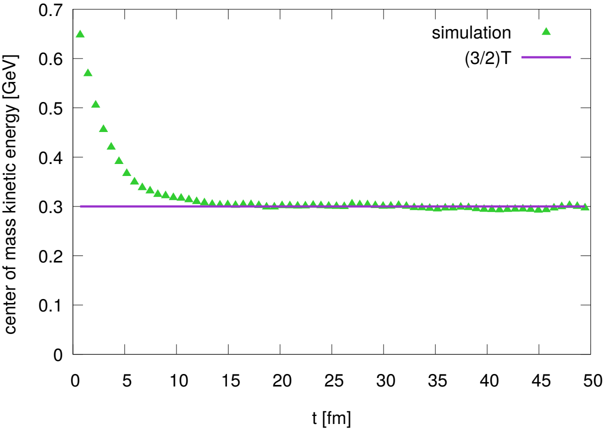
$$\left\langle \left(\boldsymbol{r}_{c\bar{c}}^2(t) \right\rangle \underset{t \to \infty}{\cong} 2 \cdot 6 \cdot D_s t, \quad D_s = \frac{T}{M\gamma}$$



Equipartition theorem

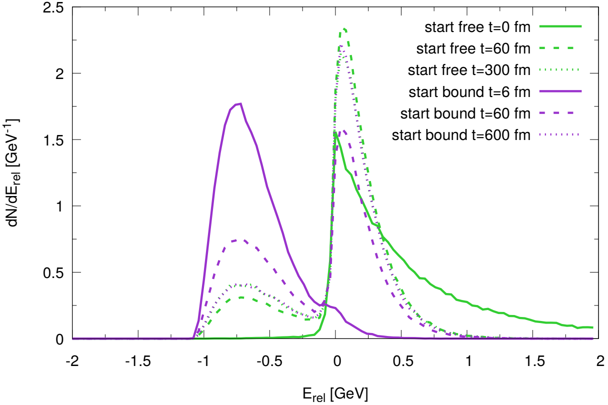
- single $c\bar{c}$ pair, $T = 200\,\text{MeV}$, $M = 1.8\,\text{GeV}$, open system
- equipartition theorem for center-mass-pair energy

$$\langle E_{\text{cm}} \rangle \underset{t\to\infty}{\cong} \frac{3}{2} T$$



Time evolution: Distribution of $E_{\rm rel}$

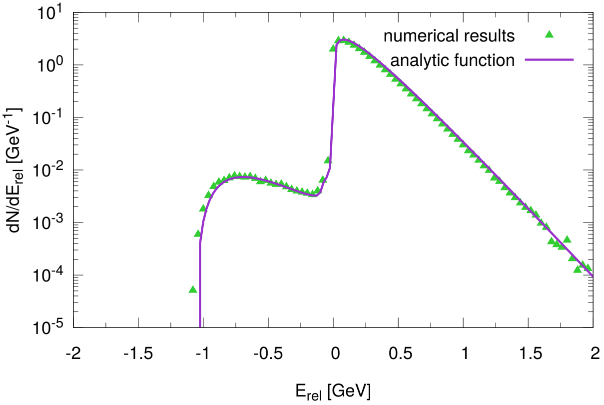
- single $c\bar{c}$ pair, $T = 160\,\text{MeV}$ $M = 1.8\,\text{GeV}$, $(2\,\text{fm})^3$ rigid box
- $E_{\rm rel}$ distribution
- time evolution starting with free (green) or bound (violet) $c\bar{c}$ pair
- long-time limit: reaches equilibrium expectation!



Equilibrium limit: Distribution of E_{rel}

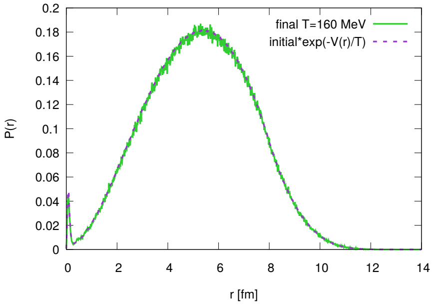
- single $c\bar{c}$ pair, $T = 160\,\text{MeV}$, $M = 1.8\,\text{GeV}$, $(8\,\text{fm})^3$ rigid box
- E_{rel} distribution in equilibrium limit

$$\frac{\mathrm{d}N}{\mathrm{d}E_{\text{rel}}} = (4\pi)^2 (2\mu)^{3/2} C \exp\left(-\frac{E_{\text{rel}}}{T}\right) \int_0^R \mathrm{d}r\,\mathrm{d}r^2 \sqrt{E_{\text{rel}} - V(r)}$$



Equilibrium limit: $c\bar{c}$-distance distribution

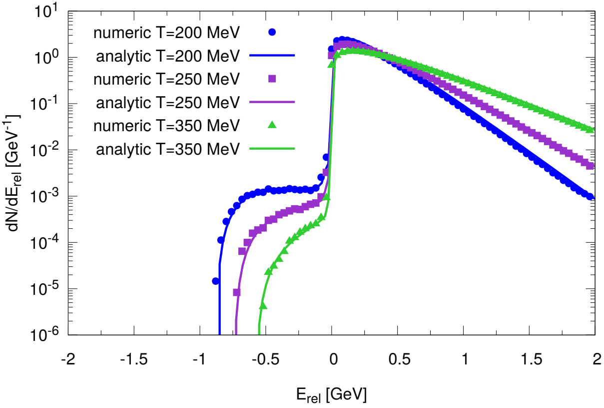
- single $c\bar{c}$ pair, $T = 160\,\text{MeV}$, $M = 1.8\,\text{GeV}$, $(8\,\text{fm})^3$ rigid box
- $P(r) = \mathrm{d}N/\mathrm{d}r$



Equilibrium limit: Distribution of E_{rel}

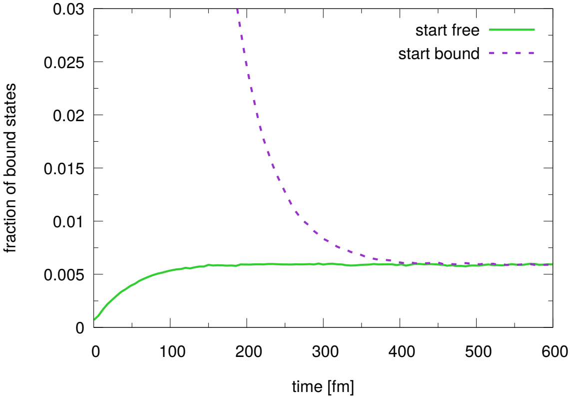
- single $c\bar{c}$ pair, $T = 200$, 250, 300 MeV $M = 1.8$ GeV, $(8\,\text{fm})^3$ rigid box
- E_{rel} distribution in equilibrium limit

$$\frac{\mathrm{d}N}{\mathrm{d}E_{\text{rel}}} = (4\pi)^2 (2\mu)^{3/2} C \exp\left(-\frac{E_{\text{rel}}}{T}\right) \int_0^R \mathrm{d}r\, \mathrm{d}r^2 \sqrt{E_{\text{rel}} - V(r)}$$



Time evolution: fraction of bound states

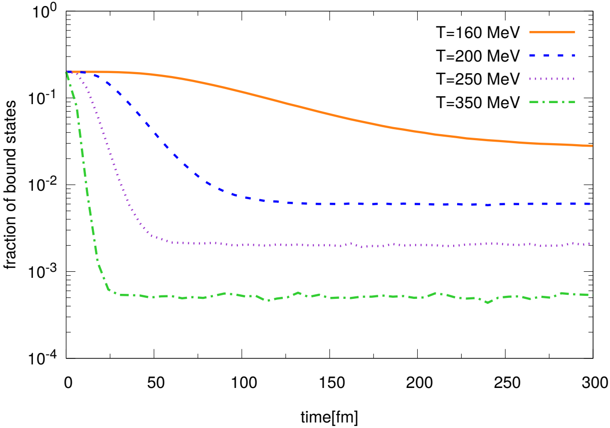
- single $c\bar{c}$ pair, $T = 160\,\mathrm{MeV}$, $M = 1.8\,\mathrm{GeV}$, $(8\,\mathrm{fm})^3$ rigid box
- fraction of bound states



- long time scales...

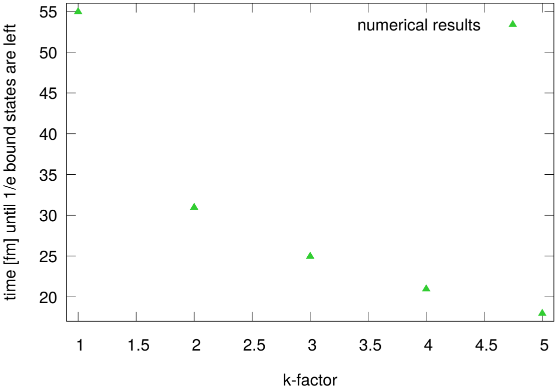
Time evolution: fraction of bound states

- five $c\bar{c}$ pairs, $M = 1.8\,\text{GeV}$, $(8\,\text{fm})^3$ rigid box
- fraction of bound states



“Chemical relaxation time”

- single $c\bar{c}$ pair, $T = 160\,\text{MeV}$, $M = 1.8\,\text{GeV}$, $(8\,\text{fm})^3$ rigid box
- relaxation time for bound-state formation/dissociation
- “k factor”: $\gamma \rightarrow k\gamma$



- still long time scales...

Comparison to statistical hadronization model

- single $c\bar{c}$ pair, $T = 160\,\text{MeV}$, rigid boxes
- fraction of bound states:
  Langevin equilibrium limit vs. (grand-)canonical ensemble

Volume	$(8\,\text{fm})^3$	$(10\,\text{fm})^3$	$(12\,\text{fm})^3$
grand canonical fraction of J/ψ	0.0066	0.0035	0.002
Langevin fraction of J/ψ	0.0059	0.0029	0.0017

Summary and Outlook

- Summary
  - Langevin simulation of charm-anticharm quarks in QGP
  - based on potential and drag coefficients from Abelian gauge model [BDBFG16]
  - passes all equilibration "box tests"
  - bound-state properties in finite box $\Leftrightarrow$ GC ensemble
- Outlook
  - generalize to expanding fireballs mimicking a heavy-ion collision
  - explore different heavy-quark potentials
  - using quantum Wigner function to analyze phase-space distribution functions [YS09] in terms of quantum bound states
  - long-time goal: full in-medium quantum Langevin treatment

Nadja Krenz

Backup Slides

Langevin Equation for $Q\bar{Q}$ pair

- heavy quarks in an Abelian plasma [BDBFG16]
- $Q\bar{Q}$ pair described by set of Langevin equations

$$M\ddot{\boldsymbol{r}} + \frac{\beta g^2}{2}[\mathscr{H}(0)\dot{\boldsymbol{r}} - \mathscr{H}(\boldsymbol{s})\dot{\bar{\boldsymbol{r}}} - g^2\nabla V(\boldsymbol{s}) = \xi(\boldsymbol{s}, t)$$

$$M\ddot{\bar{\boldsymbol{r}}} + \frac{\beta g^2}{2}[\mathscr{H}(0)\dot{\bar{\boldsymbol{r}}} - \mathscr{H}(\boldsymbol{s})\dot{\boldsymbol{r}}] + g^2\nabla V(\boldsymbol{s} = \bar{\xi}(\boldsymbol{s}, t).$$

- Drag and diffusion coefficients derived from complex potential

$$\mathscr{V}(\boldsymbol{s}) = -g^2 V(\boldsymbol{r}) - \mathrm{i}g^2[W(\boldsymbol{s}) - W(0)]$$

$$= -\frac{g^2}{4\pi}\frac{\exp(-m_{\mathrm{D}} s)}{s} - \mathrm{i}\frac{g^2 T}{4\pi}\phi(m_{\mathrm{D}} s),$$

$$\phi(x) = 2\int_0^\infty \mathrm{d}z \frac{z}{(z^2+1)^2}\left[1 - \frac{\sin(zx)}{zx}\right],$$

$$\mathscr{H}_{\alpha\beta}(\boldsymbol{s}) = \frac{\partial^2 W(\boldsymbol{s})}{\partial r_\alpha \partial r_\beta}$$

Langevin Equation for $Q\bar{Q}$ pair

- heavy quarks in an Abelian plasma [BDBFG16]
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$$M\ddot{\boldsymbol{r}} + \frac{g^2}{2T}[\mathscr{H}(0)\dot{\boldsymbol{r}} - \mathscr{H}(\boldsymbol{s})\dot{\bar{\boldsymbol{r}}} - g^2\nabla V(\boldsymbol{s}) = \xi(\boldsymbol{s}, t)$$
$$M\ddot{\bar{\boldsymbol{r}}} + \frac{g^2}{2T}[\mathscr{H}(0)\dot{\bar{\boldsymbol{r}}} - \mathscr{H}(\boldsymbol{s})\dot{\boldsymbol{r}}] + g^2\nabla V(\boldsymbol{s} = \bar{\xi}(\boldsymbol{s}, t).$$

- "Random force" as white noise

$$\left\langle \xi_\alpha(\boldsymbol{s}, t)\xi_\beta(\boldsymbol{s}, t')\right\rangle = \left\langle \bar{\xi}_\alpha(\boldsymbol{s}, t)\bar{\xi}_\beta(\boldsymbol{s}, t')\right\rangle = g^2\mathscr{H}(0)\delta_{\alpha\beta}\delta(t-t'),$$
$$\left\langle \xi_\alpha(\boldsymbol{s}, t)\bar{\xi}_\beta(\boldsymbol{s}, t')\right\rangle = -g^2\mathscr{H}_{\alpha\beta}(\boldsymbol{s})\delta(t-t'),$$
$$g^2\mathscr{H}_{\alpha\beta}(0) = 2MT\gamma\delta_{\alpha\beta}$$

- for $m_{\mathrm{D}} s \gg 1$ center-mass coordinate $\boldsymbol{\rho} = (\boldsymbol{r} + \bar{\boldsymbol{r}})/2$ moves as Brownian particle with mass $2M$, drag γ, diffusion coefficient $D = 2M\gamma T$

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