Stony Brook University

Venezia 2018 Quark Matter

Hydro and flow in nuclear (A+A) collisions

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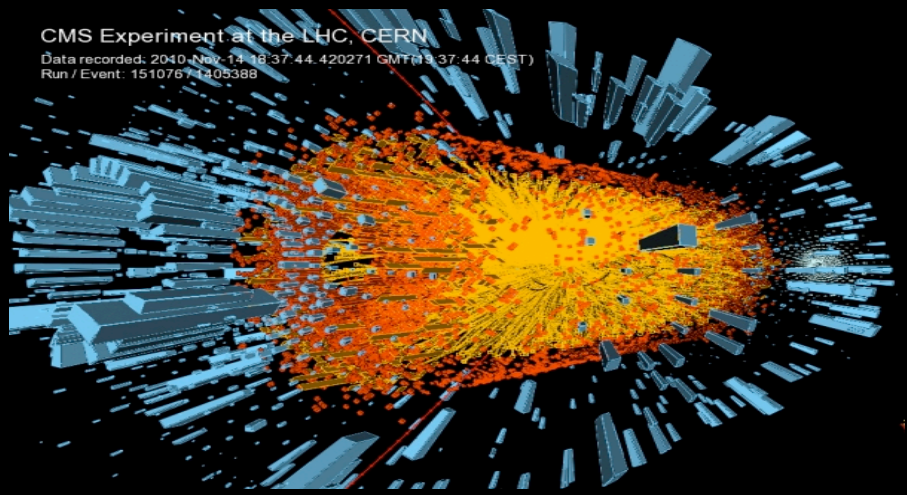
- Space-time dynamics and hydro
- Flow observables and how to measure them
- Flow fluctuations from event to event
- Flow fluctuations within the same event
- Roads toward precision.

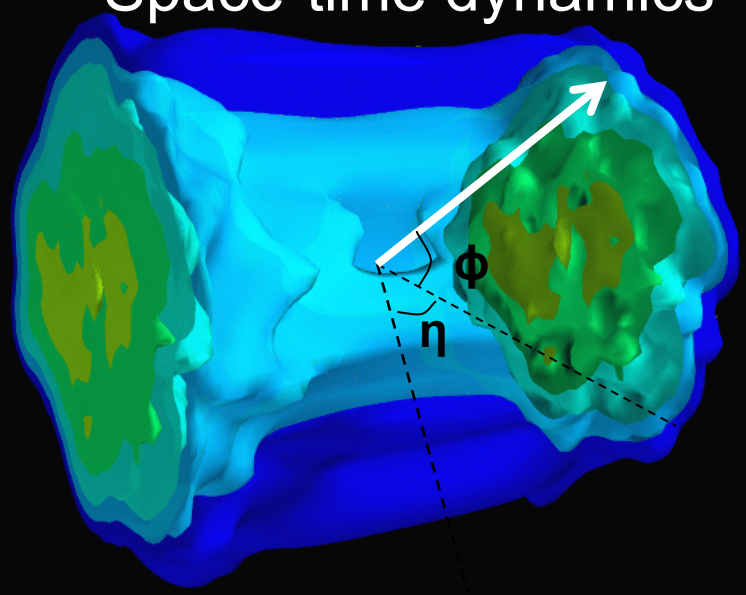
Seen by detector

$t \sim 10^{-8}$ s



Space-time dynamics

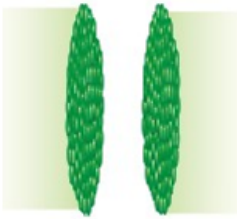
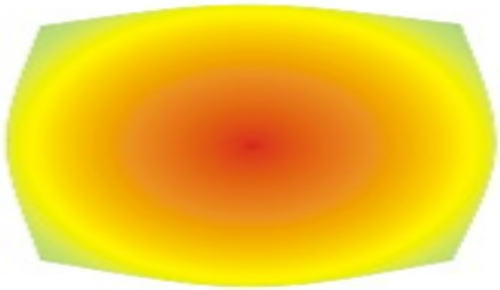
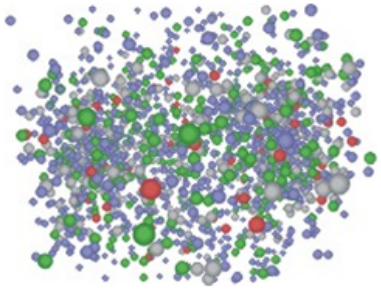
$t \sim 10 fm/c = 10^{-22}$ s



Model by 3D relativistic viscous hydrodynamics

Credit: Bjoern Schenke

Space-time dynamics

$\tau<0$	$0<\tau<0.5$fm/c	$0.5<\tau<6$ fm/c	$6<\tau<10$ fm/c
			
initial state	**pre-equilibrium**	**QGP & expansion**	**Phase transition&freeze-out**
Glauber CGC	Field theory +kinetic	Hydrodynamics	Hydro+kinetic

Use hydro to unfold the space-time dynamics

Basics of hydrodynamics: ideal

Energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0$$

Charge conservation

$$\partial_\mu N^{\mu} = 0$$

System always in local equilibrium: ideal hydrodynamics

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu - P g^{\mu\nu}$$

$$N^{\mu} = n u^{\mu}$$

Six unknown: ε, P, u^μ, and n, only five equations-of-motion

Closed by the equation-of-state (EOS) :* ε=ε(P)

* zero chemical potential

Hydro-response controlled by QCD EoS.

Basics of hydrodynamics: viscous

Energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0$$

Charge conservation

$$\partial_\mu N^{\mu} = 0$$

Include near-equilibrium corrections: viscous hydrodynamics

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - (P + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

Bulk pressure

Shear tensor

$$N^{\mu} = n u^{\mu} + n^{\mu}$$

Charge diffusion

Basics of hydrodynamics: 1st order

Energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0$$

Charge conservation

$$\partial_\mu N^{\mu} = 0$$

Include near-equilibrium corrections: viscous hydrodynamics

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - (P + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

Bulk pressure

Shear tensor

$$N^{\mu} = n u^{\mu} + n^{\mu}$$

Charge diffusion

Include 1st-order gradient expansion:

$$\pi^{\mu\nu} = -\eta\sigma^{\mu\nu}$$

η: shear viscosity coefficient

$$\Pi = -\zeta\nabla^{\perp}_{\lambda} u^{\lambda}$$

ζ : bulk viscosity coefficient

Basics of hydrodynamics: 2nd order

Energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0$$

Charge conservation

$$\partial_\mu N^\mu = 0$$

Include near-equilibrium corrections: viscous hydrodynamics

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - (P + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

Bulk pressure

Shear tensor

$$N^\mu = n u^\mu + n^\mu$$

Charge diffusion

Include up to 2nd-order gradient expansion

$$\pi^{\mu\nu} = -\eta\sigma^{\mu\nu} + \eta\tau_\pi\left[{}^{<}D\sigma^{\mu\nu>} + \frac{\nabla_\lambda^\perp u^\lambda}{3}\sigma^{\mu\nu}\right] + \kappa\left[R^{<\mu\nu>} - 2u_\lambda u_\rho R^{\lambda<\mu\nu>\rho}\right] + \lambda_1\sigma^{<\mu}{}_\lambda\sigma^{\nu>\lambda}$$
$$+\lambda_2\sigma^{<\mu}{}_\lambda\Omega^{\nu>\lambda} + \lambda_3\Omega^{<\mu}{}_\lambda\Omega^{\nu>\lambda} + \kappa^* 2u_\lambda u_\rho R^{\lambda<\mu\nu>\rho} + \eta\tau_\pi^*\frac{\nabla_\lambda^\perp u^\lambda}{3}\sigma^{\mu\nu} + \bar{\lambda}_4\nabla_\perp^{<\mu}\ln\epsilon\nabla_\perp^{\nu>}\ln\epsilon$$

$$\Pi = -\zeta\left(\nabla_\lambda^\perp u^\lambda\right) + \zeta\tau_\Pi D\left(\nabla_\lambda^\perp u^\lambda\right) + \xi_1\sigma^{\mu\nu}\sigma_{\mu\nu} + \xi_2\left(\nabla_\lambda^\perp u^\lambda\right)^2$$
$$+\xi_3\Omega^{\mu\nu}\Omega_{\mu\nu} + \bar{\xi}_4\nabla_\mu^\perp\ln\epsilon\nabla_\perp^\mu\ln\epsilon + \xi_5 R + \xi_6 u^\lambda u^\rho R_{\lambda\rho}\,.$$

Taken from 1712.05815

Many transport coeff.→ probe microscopic theory, QCD

Hydro to decipher the QGP properties?

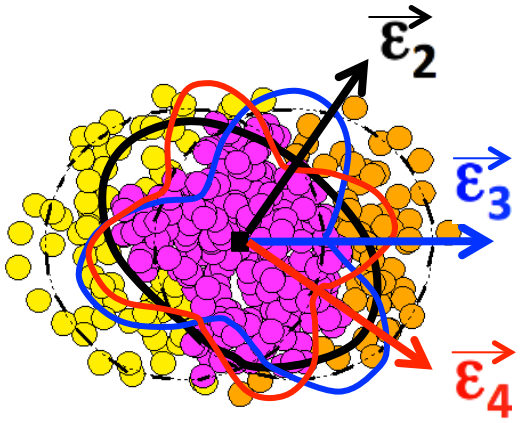
	Gauge/Gravity	Kinetic (BGK)	pQCD	Lattice QCD
$\epsilon(P)$	3 P	Eq. (3.30)	3 P	Eq. (3.125)
η	$\frac{\epsilon+P}{4\pi T}$	$\frac{(\epsilon+P)\tau_R}{5}$	$\frac{3.85(\epsilon+P)}{g^4 \ln(2.765 g^{-1})T}$	$0.10(6)\frac{\epsilon+P}{T}$
τ_π	$\frac{2-\ln 2}{2\pi T}$	τ_R	$\frac{5.9\eta}{\epsilon+P}$	
λ_1	$\frac{\eta}{2\pi T}$	$\frac{5}{7}\eta\tau_R$	$\frac{5.2\eta^2}{\epsilon+P}$	
λ_2	$2\eta\tau_\pi - 4\lambda_1$	$-2\eta\tau_R$	$-2\eta\tau_\pi$	
λ_3	0	0	$\frac{30(\epsilon+P)}{8\pi^2 T^2}$	
κ	$\frac{\epsilon+P}{4\pi^2 T^2}$	0	$\frac{5(\epsilon+P)}{8\pi^2 T^2}$	$0.36(15)T^2$
Refs.	[19, 28, 29] [128, 129]	[28, 119, 120]	[121–123] [130]	[124–127] [131, 132]

Table 2.1: Compilation of leading-order results for transport coefficients in various calculational approaches, see text for details. table from 1712.05815

ab.initio calc. for QGP not easy, relies on model/data comparison

Connecting the initial and final state

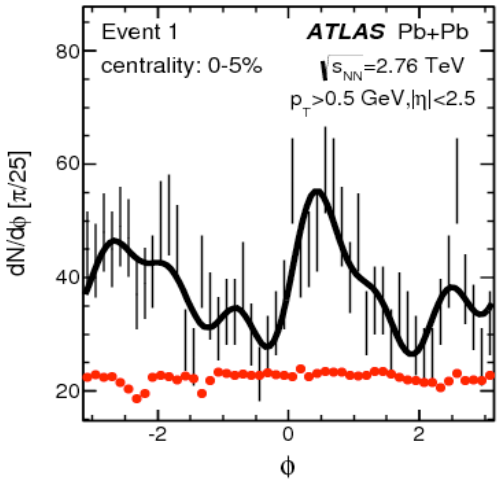
Initial state



Hydro-response

Space-time dynamics

Particle flow

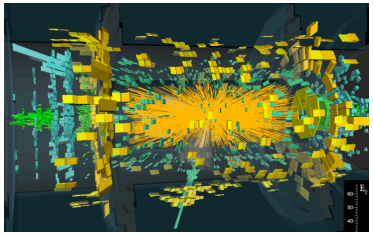


$$\vec{\epsilon}_n \equiv \epsilon_n e^{in\Phi_n^*} \equiv -\frac{\langle r^n e^{in\phi} \rangle}{\langle r^n \rangle}$$

$$\frac{dN}{d\phi} \propto 1 + 2\sum_n v_n \cos n\left(\phi - \Phi_n\right)$$

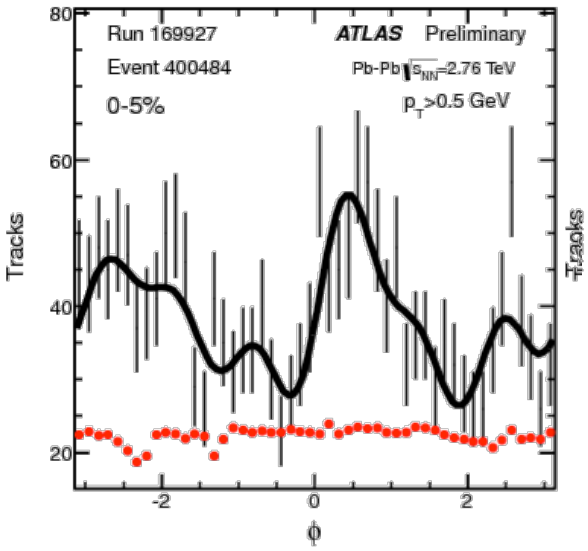
- What is the nature of the initial state fluctuation ?
- What is the space-time evolution of the produced matter ?
  - How are (ε_n,Φ_n^*) transferred to (v_n, Φ_n) event-by-event?
- What are the properties of the produced matter ?

Hydrodynamic behavior in each event

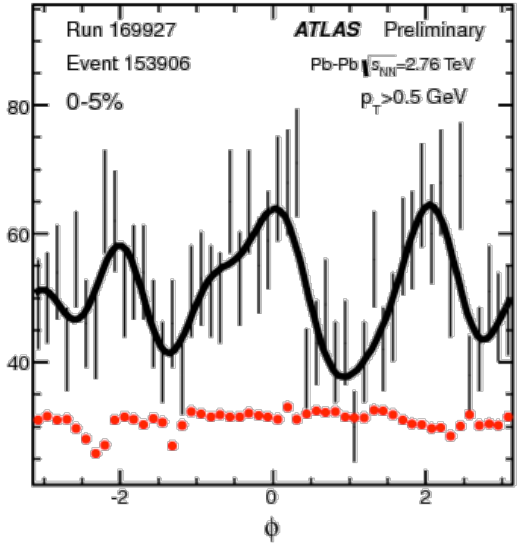


$$\frac{dN}{d\phi} \propto 1 + 2\sum_n v_n \cos n\left(\phi - \Phi_n\right)$$

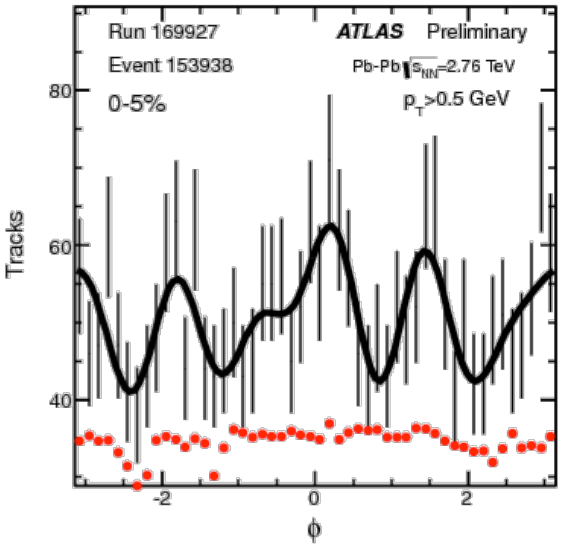
Event 1



Event 2



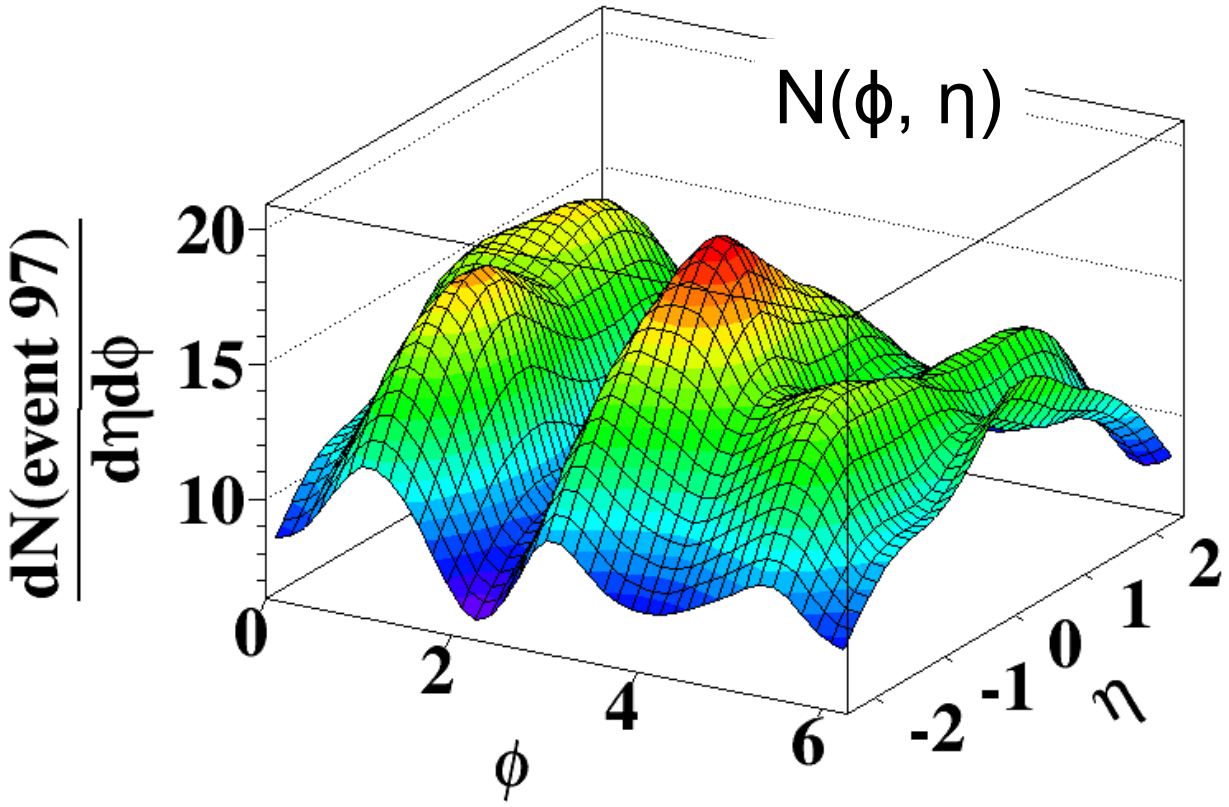
Event 3



- v_n sensitive to initial perturbation and viscosity.
  - Bigger initial fluctuation lead to bigger v_n
  - Small viscosity ensure efficient transfer of initial fluctuation to final state flow.

Richness of flow fluctuations

Curtsey of L.Pang and X.N Wang, EbyE 3D hydro+AMPT condition



Fluctuation from event to event

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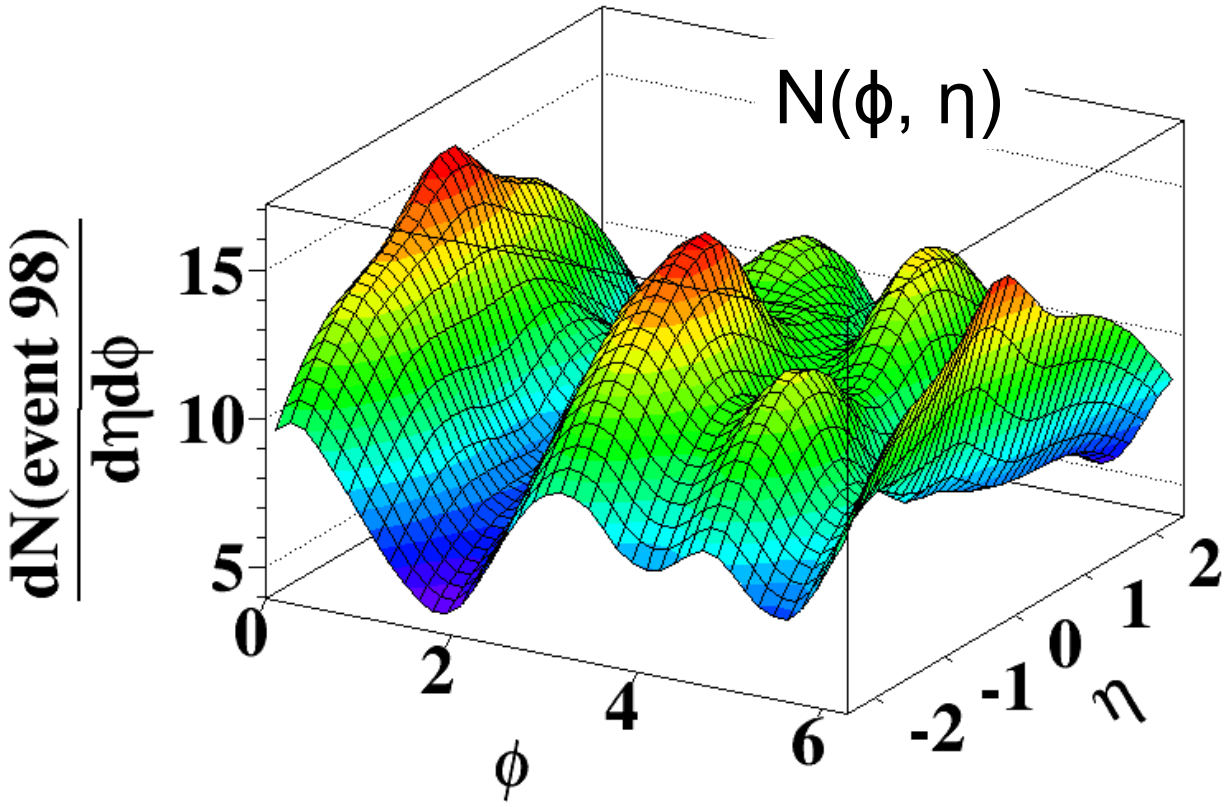

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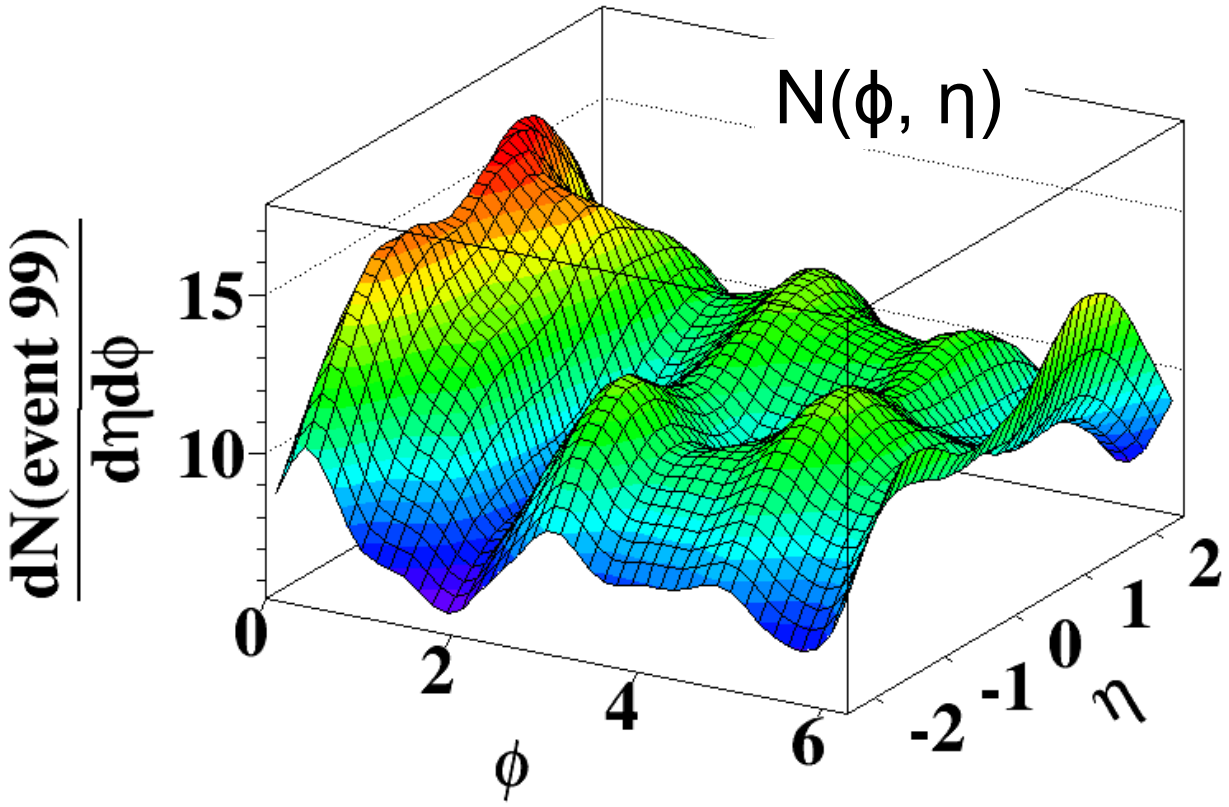

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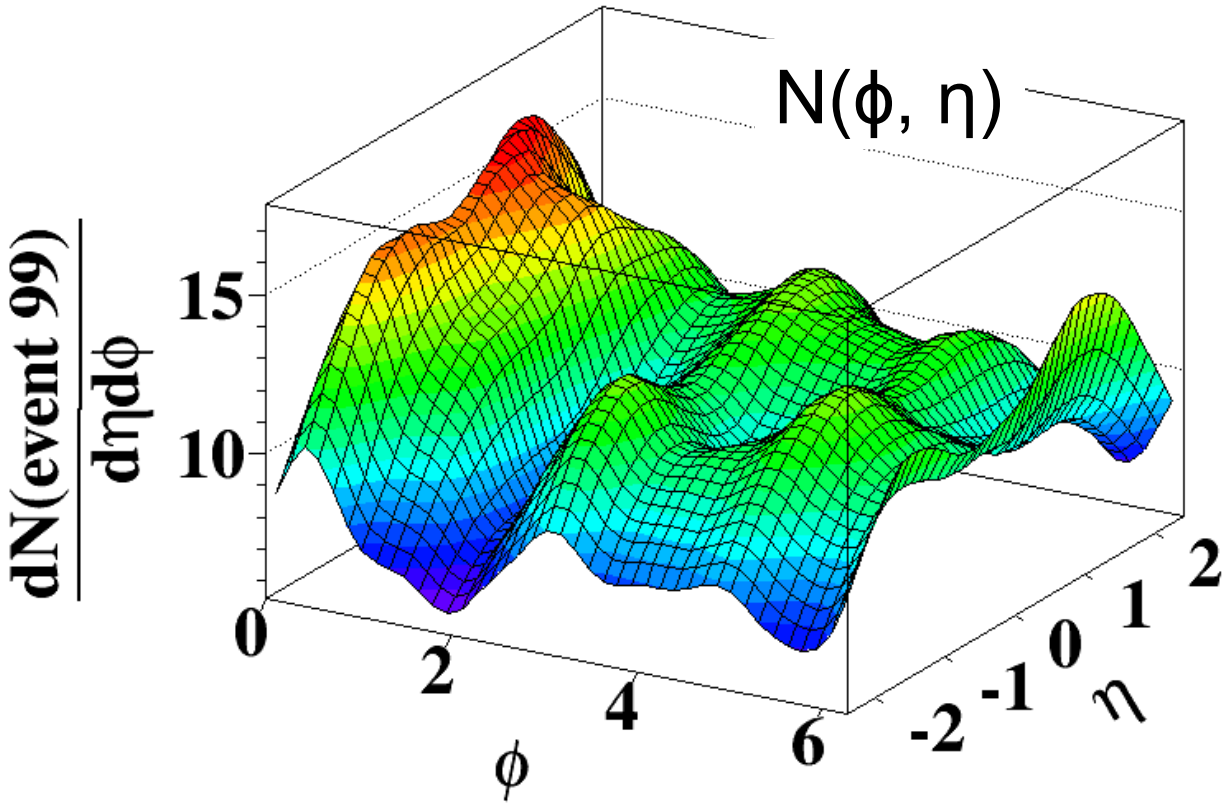

Fluctuation within a single event

$$\frac{dN}{d\phi} \propto 1 + 2\sum_n \mathrm{v}_n(p_T,\eta,\ldots)\cos n\left(\phi - \Phi_n(p_T,\eta,\ldots)\right)$$

Experimental flow observables

- Single particle distribution

$$\frac{dN}{d\phi d\eta dp_T}$$

- Two-particle correlation function

$$\left\langle \frac{dN_1}{d\phi d\eta dp_T} \frac{dN_2}{d\phi d\eta dp_T} \right\rangle$$

- Multi-particle correlation function

$$\left\langle \frac{dN_1}{d\phi d\eta dp_T} .. \frac{dN_m}{d\phi d\eta dp_T} \right\rangle$$

Experimental flow observables

- Single particle distribution

Flow vector: $\boldsymbol{V}_n = v_n e^{in\Phi_n}$

$$\frac{dN}{d\phi d\eta dp_T} = N(p_T,\eta)\left[1+2\sum_n \mathrm{v}_\mathrm{n}(p_T,\eta)\cos n\left(\phi-\Phi_n(p_T,\eta)\right)\right]$$

$$= N(p_T,\eta)\left[\sum_{n=-\infty}^{\infty} \boldsymbol{V}_\mathrm{n}(p_T,\eta)e^{in\phi}\right]$$

Radial flow

Anisotropic flow

- Two-particle correlation function

$$\left\langle \frac{dN_1}{d\phi d\eta dp_T}\frac{dN_2}{d\phi d\eta dp_T} \right\rangle$$

- Multi-particle correlation function

$$\left\langle \frac{dN_1}{d\phi d\eta dp_T}\cdot\cdot\frac{dN_m}{d\phi d\eta dp_T} \right\rangle$$

Experimental flow observables

- Single particle distribution

Flow vector: $\boldsymbol{V}_n = v_n e^{in\Phi_n}$

$$\frac{dN}{d\phi d\eta dp_T} = N(p_T,\eta)\left[1+2\sum_n \mathrm{v}_n(p_T,\eta)\cos n\left(\phi-\Phi_n(p_T,\eta)\right)\right]$$

$$= N(p_T,\eta)\left[\sum_{n=-\infty}^{\infty} \boldsymbol{V}_n(p_T,\eta)e^{in\phi}\right]$$

Radial flow

Anisotropic flow

- Two-particle correlation function

$$\left\langle \frac{dN_1}{d\phi d\eta dp_T}\frac{dN_2}{d\phi d\eta dp_T}\right\rangle \Rightarrow \left\langle \boldsymbol{V}_n(p_{T1},\eta_1)\boldsymbol{V}_n^*(p_{T2},\eta_2)\right\rangle$$

v_n from 2PC

- Multi-particle correlation function

$$\left\langle \frac{dN_1}{d\phi d\eta dp_T}..\frac{dN_m}{d\phi d\eta dp_T}\right\rangle \Rightarrow \left\langle \boldsymbol{V}_{n_1}\boldsymbol{V}_{n_2}...\boldsymbol{V}_{n_m}\right\rangle \quad n_1+n_2+...+n_m=0$$

$$\downarrow$$

$$\langle v_{n_1}v_{n_2}...v_{n_m}\cos(n_1\Phi_{n_1}+n_2\Phi_{n_2}+...+n_m\Phi_{n_m})\rangle$$

Experimental flow observables

- Single particle distribution

Flow vector: $\boldsymbol{V}_n = v_n e^{in\Phi_n}$

$$\frac{dN}{d\phi d\eta dp_T} = N(p_T,\eta)\left[1+2\sum_n \mathrm{v}_n(p_T,\eta)\cos n\left(\phi-\Phi_n(p_T,\eta)\right)\right]$$

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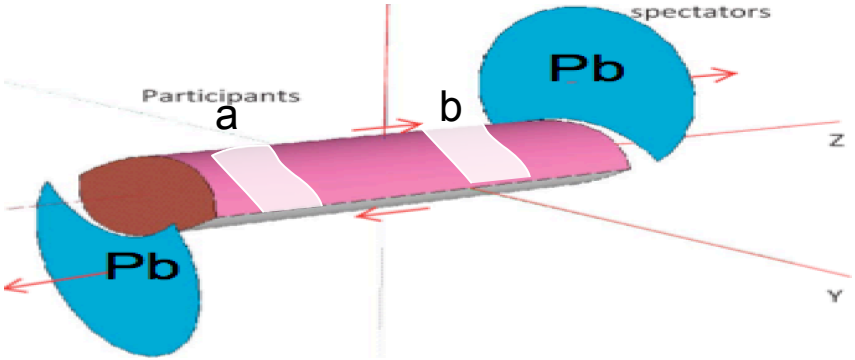
$$p(v_n, v_m, ..., \Phi_n, \Phi_m, ...) = \frac{1}{N_{\mathrm{evts}}}\frac{dN_{\mathrm{evts}}}{dv_n dv_m ... d\Phi_n d\Phi_m ...}$$

How to measure flow? $V_n = v_n e^{in\Phi_n}$

By particle correlations!



Determine flow vector in one subevent:

$$q_n = \frac{\Sigma_i e^{in\phi_i}}{\Sigma_i} = v_n e^{in\Phi_n} + \delta$$

Statistical noise

Noise uncorrelated between two subevents, average over events:

$$\left\langle q_n^a q_n^{b*} \right\rangle = \left\langle (v_n^a e^{in\Phi_n^a} + \delta^a)(v_n^b e^{-in\Phi_n^b} + \delta^{b*}) \right\rangle = \left\langle V_n^a V_n^{b*} \right\rangle$$

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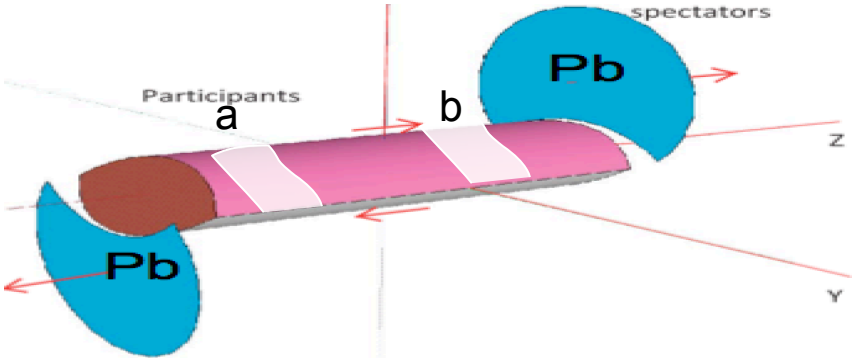

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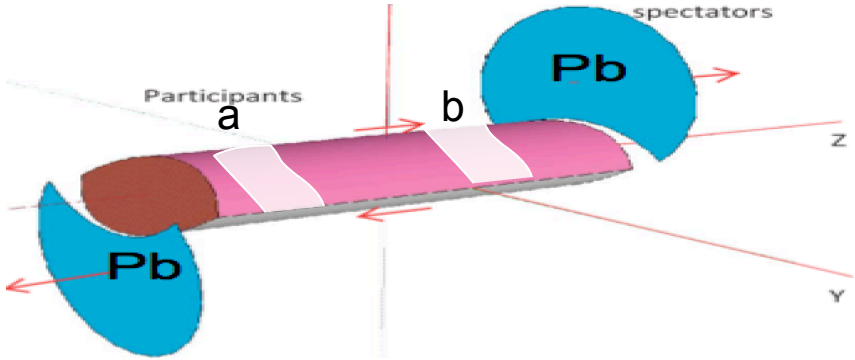
We often assume $p(v_n)$ independent of p_T and η, i.e. ignoring intra-event fluctuation $p(V_n) = f(p_T, \eta) p(\bar{v}_n)$

$$\left\langle V_n^a V_n^{b*} \right\rangle = f(p_T^a, \eta^a) f(p_T^b, \eta^b) \left\langle v_n^2 \right\rangle$$

How to measure flow? $V_n = v_n e^{in\Phi_n}$

By particle correlations!



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$$\left\langle V_n^a V_n^{b*} \right\rangle = f(p_T^a,\eta^a) f(p_T^b,\eta^b) \left\langle v_n^2 \right\rangle$$

Event-plane or scalar-product methods, e.g. measure flow in subevent **c** wrt symmetric subevents **a**&**b**:

$$f(p_T^a,\eta^a) = f(p_T^b,\eta^b)$$

$$v_n^{meas} = \frac{\left\langle q_n^c q_n^{a*} \right\rangle}{\sqrt{\left\langle q_n^a q_n^{b*} \right\rangle}} = \frac{f(p_T^c,\eta^c)\cancel{f(p_T^a,\eta^a)}\left\langle \bar{v}_n^2 \right\rangle}{\sqrt{\cancel{f(p_T^a,\eta^a) f(p_T^b,\eta^b)}\left\langle \bar{v}_n^2 \right\rangle}} = f(p_T^c,\eta^c)\sqrt{\left\langle \bar{v}_n^2 \right\rangle} = \sqrt{\left\langle v_n^c v_n^c \right\rangle}$$

Lessons: 1) We often report RMS value of v_n, 2) relies on factorization assumption!

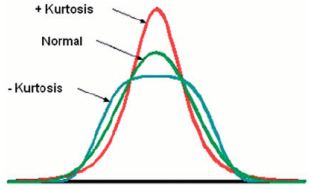
How to measure EbyE flow fluctuations?

Multi-particle correlations ➔ moments, cumulants

$$C_2 = \langle \delta X^2 \rangle \qquad \delta X = X - \langle X \rangle$$
$$C_3 = \langle \delta X^3 \rangle$$
$$C_4 = \langle \delta X^4 \rangle - 3 \langle \delta X^2 \rangle^2$$
$$C_5 = \langle \delta X^5 \rangle - 10 \langle \delta X^3 \rangle \langle \delta X^2 \rangle$$

Quantifies the shape of p(x)

C_2 variance, C_3 Skewness, C_4 Kurtosis



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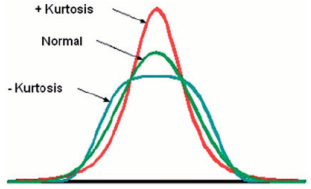

Replace with harmonics $X=v_n e^{in\Phi_n}$ ➔ cumulants for 2D functions
Simplification by symmetry➔$\langle X \rangle = 0$, $\langle X^n \rangle = 0$, $\langle XX^* \rangle = \langle v_n^2 \rangle$…

How to measure EbyE flow fluctuations?

Multi-particle correlations ➔ moments, cumulants

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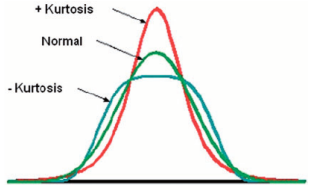

Replace with harmonics $X=v_n e^{in\Phi_n}$ ➔ cumulants for 2D functions

Simplification by symmetry➔$\langle X \rangle = 0$, $\langle X^n \rangle = 0$, $\langle XX^* \rangle = \langle v_n^2 \rangle$…

Cumulant for a single flow harmonic:

N.Borghini, P.Dinh, J.Ollitrault nucl-th/0007063

$$c_n\{4\} = \langle\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} \rangle\rangle - \langle\langle e^{in(\phi_1-\phi_3)} \rangle\rangle \langle\langle e^{in(\phi_2-\phi_4)} \rangle\rangle - \langle\langle e^{in(\phi_1-\phi_4)} \rangle\rangle \langle\langle e^{in(\phi_2-\phi_3)} \rangle\rangle - \langle\langle e^{in(\phi_1+\phi_2)} \rangle\rangle \langle\langle e^{in(\phi_3+\phi_4)} \rangle\rangle$$
$$= \langle\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} \rangle\rangle - 2 \langle\langle e^{in(\phi_1-\phi_2)} \rangle\rangle^2$$
$$= \langle v_n^4 \rangle - 2 \langle v_n^2 \rangle^2 + \text{non-flow}$$

Vanish by symmetry

Suppress 2PC non-flow

Probe the shape of $p(v_n)$, e.g. non-Gaussianity

How to measure EbyE flow fluctuations?

Four-particle symmetric cumulants:

A.Bilandzic, C.Christensen, K.Gulbrandsen, A.Hansen, Y.Zhou 1312.3572

$$
\begin{aligned}
\mathrm{sc}_{n,m}\{4\} &= \left\langle\left\langle e^{i n(\phi_1-\phi_2)+m(\phi_3-\phi_4)}\right\rangle\right\rangle - \left\langle\left\langle e^{i n(\phi_1-\phi_2)}\right\rangle\right\rangle\left\langle\left\langle e^{i m(\phi_3-\phi_4)}\right\rangle\right\rangle \\
&\quad - \left\langle\left\langle e^{i(n\phi_1+m\phi_3)}\right\rangle\right\rangle\left\langle\left\langle e^{i(n\phi_2+m\phi_4)}\right\rangle\right\rangle - \left\langle\left\langle e^{i(n\phi_1-m\phi_4)}\right\rangle\right\rangle\left\langle\left\langle e^{i(-n\phi_2+m\phi_3)}\right\rangle\right\rangle \\
&= \left\langle\left\langle e^{i n(\phi_1-\phi_2)+m(\phi_3-\phi_4)}\right\rangle\right\rangle - \left\langle\left\langle e^{i n(\phi_1-\phi_2)}\right\rangle\right\rangle\left\langle\left\langle e^{i m(\phi_3-\phi_4)}\right\rangle\right\rangle \\
&= \left\langle v_n^2 v_m^2\right\rangle - \left\langle v_n^2\right\rangle\left\langle v_m^2\right\rangle + \text{non-flow}
\end{aligned}
$$

How to measure EbyE flow fluctuations?

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A.Bilandzic, C.Christensen, K.Gulbrandsen, A.Hansen, Y.Zhou 1312.3572

$$\begin{aligned}
\mathrm{sc}_{n,m}\{4\} &= \left\langle\left\langle e^{in(\phi_1-\phi_2)+m(\phi_3-\phi_4)}\right\rangle\right\rangle - \left\langle\left\langle e^{in(\phi_1-\phi_2)}\right\rangle\right\rangle\left\langle\left\langle e^{im(\phi_3-\phi_4)}\right\rangle\right\rangle \\
&\quad - \left\langle\left\langle e^{i(n\phi_1+m\phi_3)}\right\rangle\right\rangle\left\langle\left\langle e^{i(n\phi_2+m\phi_4)}\right\rangle\right\rangle - \left\langle\left\langle e^{i(n\phi_1-m\phi_4)}\right\rangle\right\rangle\left\langle\left\langle e^{i(-n\phi_2+m\phi_3)}\right\rangle\right\rangle \\
&= \left\langle\left\langle e^{in(\phi_1-\phi_2)+m(\phi_3-\phi_4)}\right\rangle\right\rangle - \left\langle\left\langle e^{in(\phi_1-\phi_2)}\right\rangle\right\rangle\left\langle\left\langle e^{im(\phi_3-\phi_4)}\right\rangle\right\rangle \\
&= \left\langle v_n^2 v_m^2\right\rangle - \left\langle v_n^2\right\rangle\left\langle v_m^2\right\rangle + \text{non-flow}
\end{aligned}$$

Three-particle asymmetric cumulants (event-plane correlators):

$$\begin{aligned}
\mathrm{ac}_{n,m}\{3\} &= \left\langle\left\langle e^{i(n\phi_1+m\phi_2-(n+m)\phi_3)}\right\rangle\right\rangle - \text{terms involving } \left\langle\left\langle e^{in\phi}\right\rangle\right\rangle, \left\langle\left\langle e^{im\phi}\right\rangle\right\rangle, \left\langle\left\langle e^{i(n+m)\phi}\right\rangle\right\rangle \\
&= \left\langle\left\langle e^{i(n\phi_1+m\phi_2-(n+m)\phi_3)}\right\rangle\right\rangle \\
&= \left\langle v_n v_m v_{n+m}\cos\left(n\Phi_n + m\Phi_m - (n+m)\Phi_{n+m}\right)\right\rangle + \text{non-flow}
\end{aligned}$$

ATLAS 1403.0489

Probe the shape of $p(v_n, v_m)$ and $p(\Phi_n, \Phi_m)$

How to measure EbyE flow fluctuations?

Four-particle symmetric cumulants:

A.Bilandzic, C.Christensen, K.Gulbrandsen, A.Hansen, Y.Zhou 1312.3572

$$\begin{aligned}
\mathrm{sc}_{n,m}\{4\} &= \left\langle\left\langle e^{in(\phi_1-\phi_2)+m(\phi_3-\phi_4)}\right\rangle\right\rangle - \left\langle\left\langle e^{in(\phi_1-\phi_2)}\right\rangle\right\rangle\left\langle\left\langle e^{im(\phi_3-\phi_4)}\right\rangle\right\rangle \\
&\quad - \left\langle\left\langle e^{i(n\phi_1+m\phi_3)}\right\rangle\right\rangle\left\langle\left\langle e^{i(n\phi_2+m\phi_4)}\right\rangle\right\rangle - \left\langle\left\langle e^{i(n\phi_1-m\phi_4)}\right\rangle\right\rangle\left\langle\left\langle e^{i(-n\phi_2+m\phi_3)}\right\rangle\right\rangle \\
&= \left\langle\left\langle e^{in(\phi_1-\phi_2)+m(\phi_3-\phi_4)}\right\rangle\right\rangle - \left\langle\left\langle e^{in(\phi_1-\phi_2)}\right\rangle\right\rangle\left\langle\left\langle e^{im(\phi_3-\phi_4)}\right\rangle\right\rangle \\
&= \left\langle v_n^2 v_m^2\right\rangle - \left\langle v_n^2\right\rangle\left\langle v_m^2\right\rangle + \text{non-flow}
\end{aligned}$$

Three-particle asymmetric cumulants (event-plane correlators):

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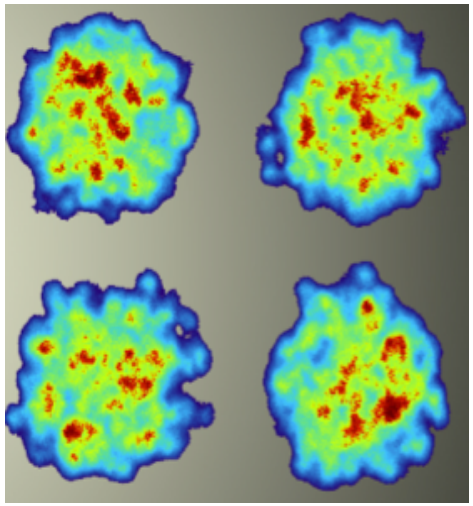
&= \left\langle\left\langle e^{i(n\phi_1+m\phi_2-(n+m)\phi_3)}\right\rangle\right\rangle \qquad \text{ATLAS 1403.0489} \\
&= \left\langle v_n v_m v_{n+m}\cos(n\Phi_n + m\Phi_m - (n+m)\Phi_{n+m})\right\rangle + \text{non-flow}
\end{aligned}$$

Probe the shape of $p(v_n, v_m)$ and $p(\Phi_n, \Phi_m)$

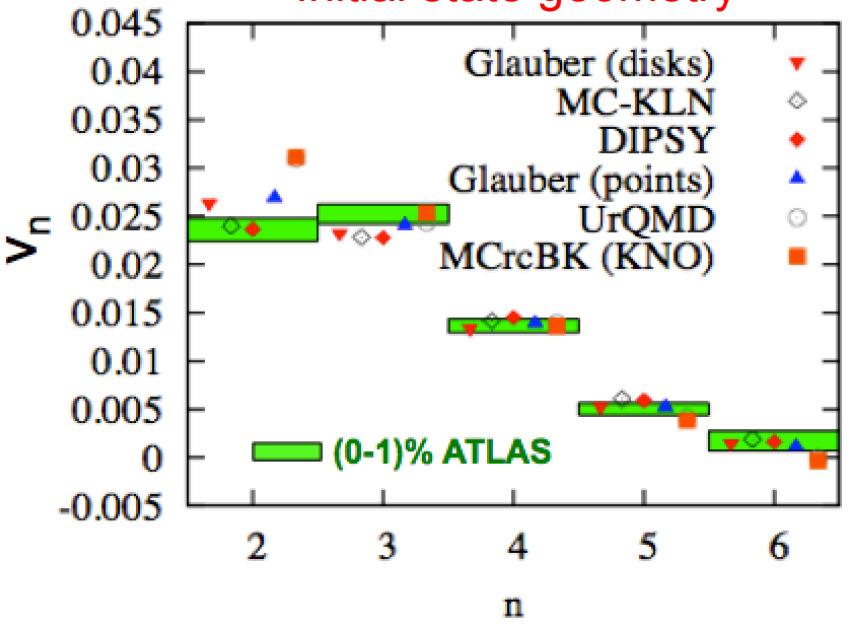
Generalize to more particles, e.g. $\langle\cos 12(\Phi_2 - \Phi_4)\rangle$ is 9-particle correlator

Data/hydro comparison: two-particle correlation

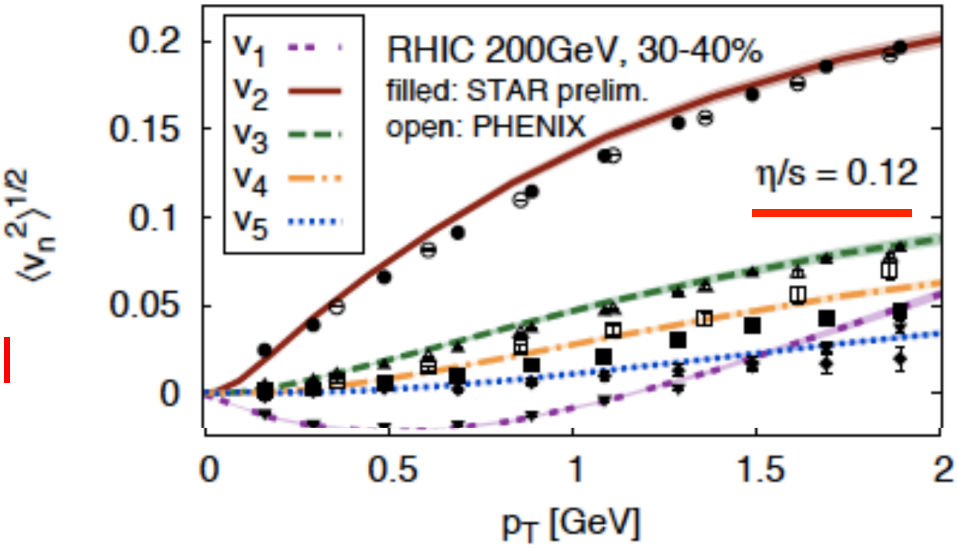
Event-averaged quantities



Initial state geometry



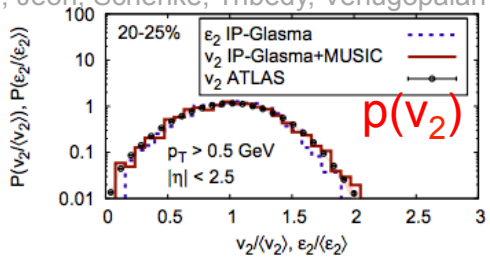
Matter properties

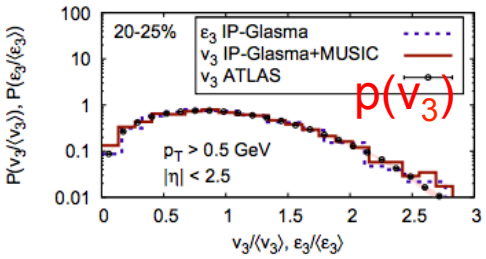


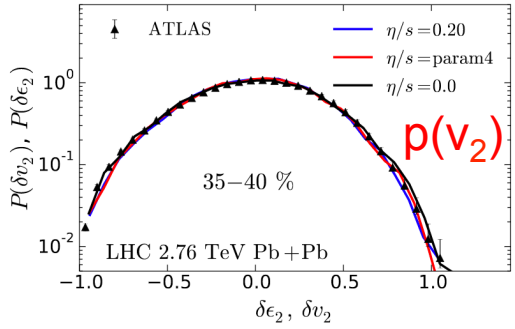
Simultaneously constrain initial geometry and shear viscosity

Data/hydro comparison: EbyE flow fluctuations

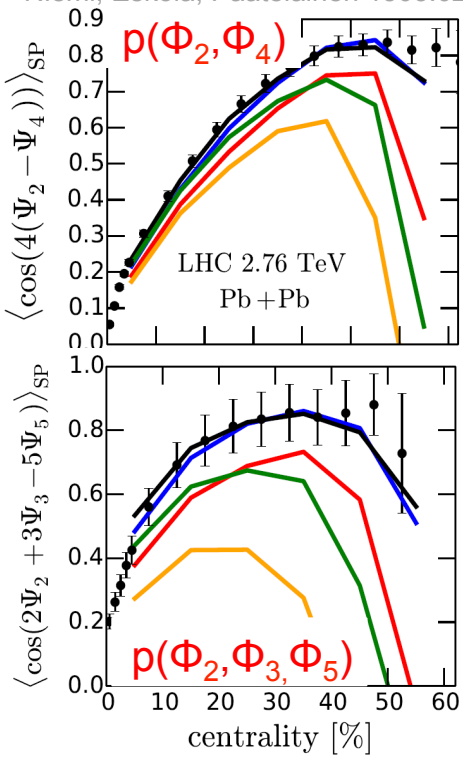
Gale, Jeon, Schenke, Tribedy, Venugopalan 1209.6330





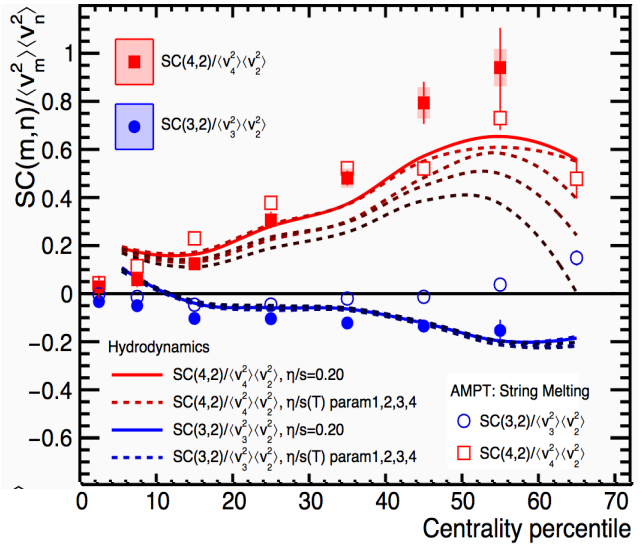


Niemi, Eskola, Paatelainen 1505.02677



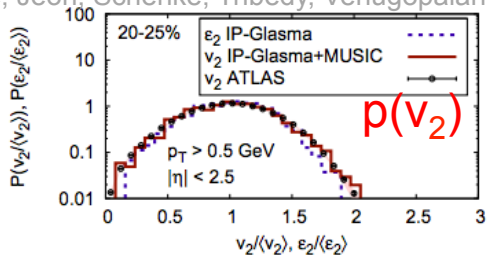
ALICE 1604.07663

$p(v_2,v_3)$, $p(v_2,v_4)$

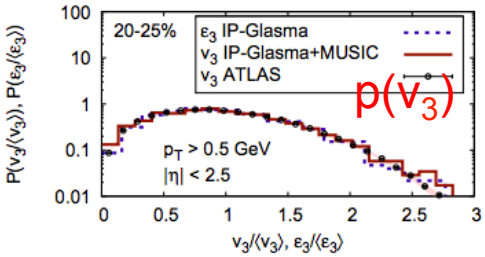


Data/hydro comparison: EbyE flow fluctuations

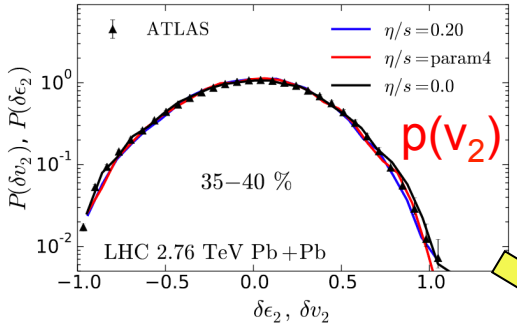
Gale, Jeon, Schenke, Tribedy, Venugopalan 1209.6330



$p(v_2)$

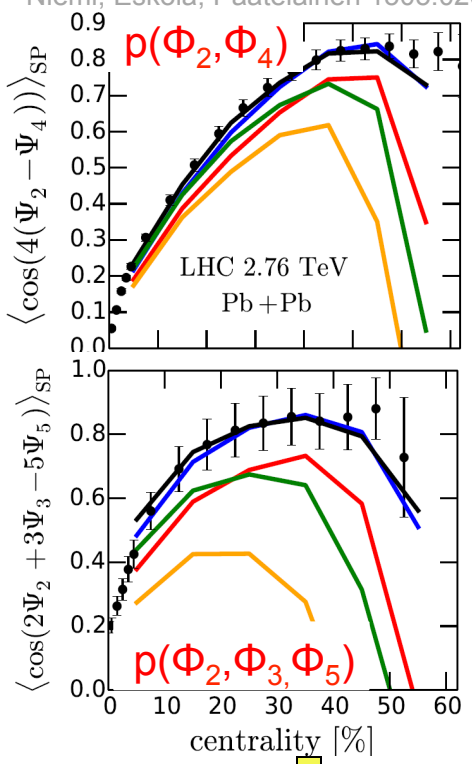


$p(v_3)$



$p(v_2)$

Niemi, Eskola, Paatelainen 1505.02677

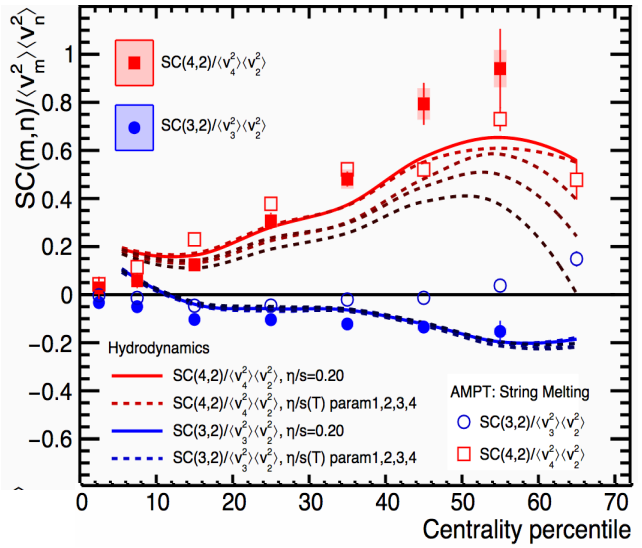


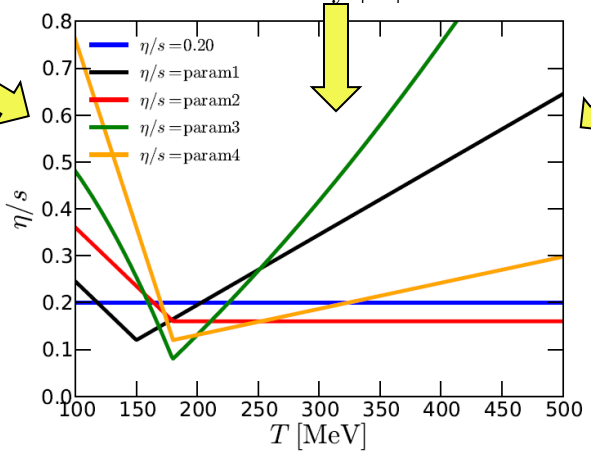
$p(\Phi_2,\Phi_4)$

$p(\Phi_2,\Phi_3,\Phi_5)$

ALICE 1604.07663

$p(v_2,v_3)$, $p(v_2,v_4)$





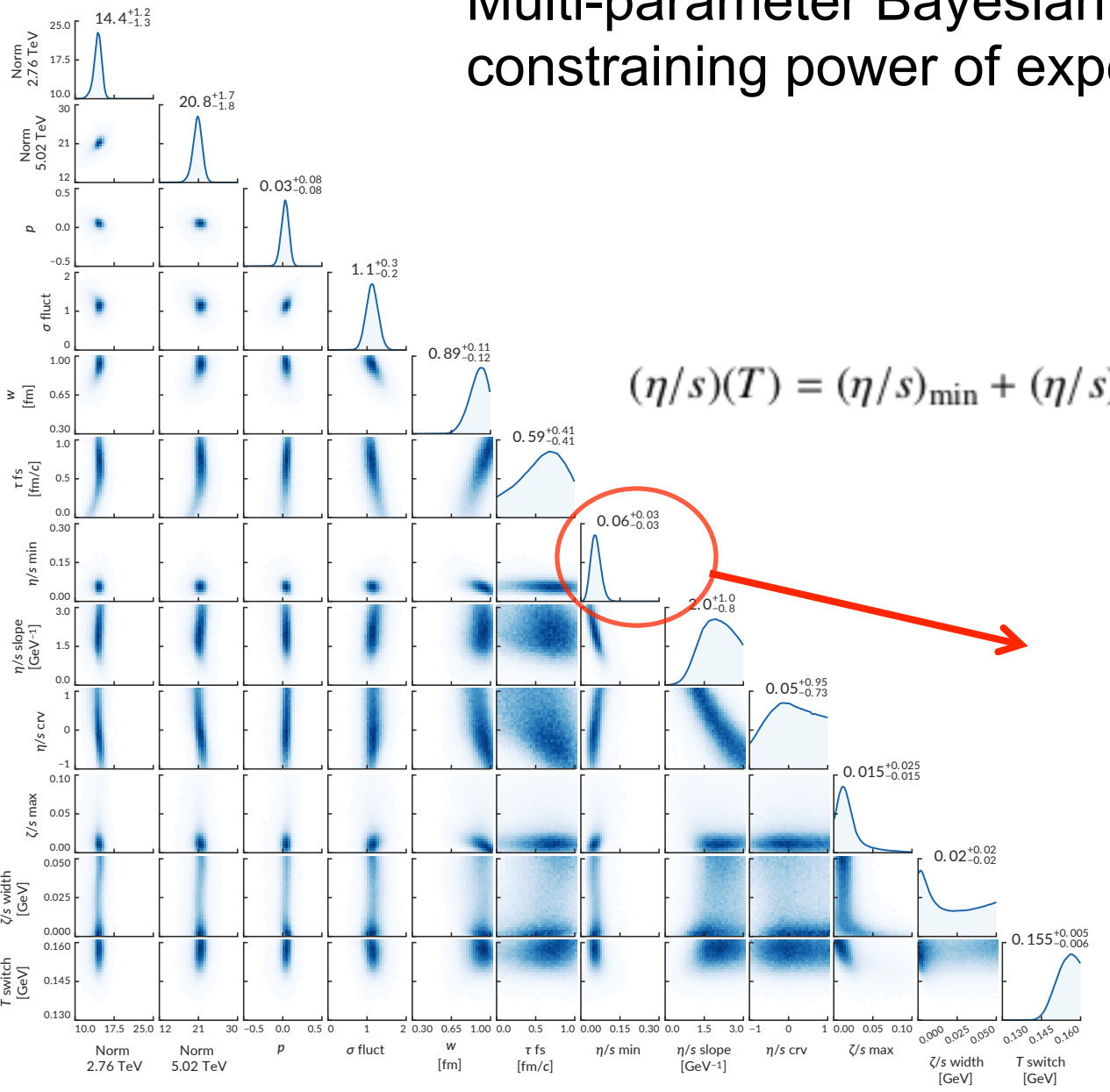
Probe the hydrodynamic response: $(\varepsilon_n,\Phi_n^*)\rightarrow(v_n,\Phi_n)$

Sensitive to detail of transport coeffis: η/s(T)

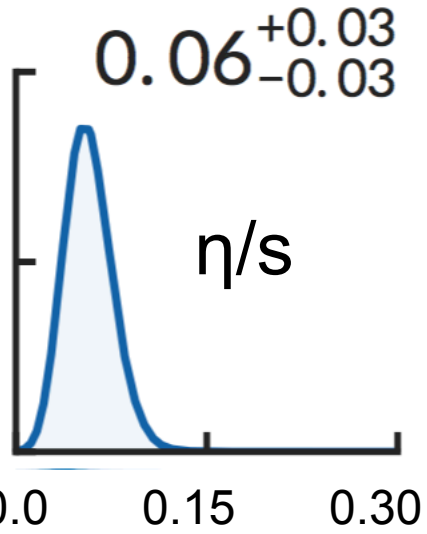
Over-constrain current hydrodynamic models

Maximizing the constraining power

Multi-parameter Bayesian analysis to maximize constraining power of experimental data.



$$(\eta/s)(T) = (\eta/s)_{\text{min}} + (\eta/s)_{\text{slope}}\,(T - T_c)\,(T - T_c)^{(\eta/s)_{\text{curvature}}}$$



S.Bass, J. Bernhard, QM2017

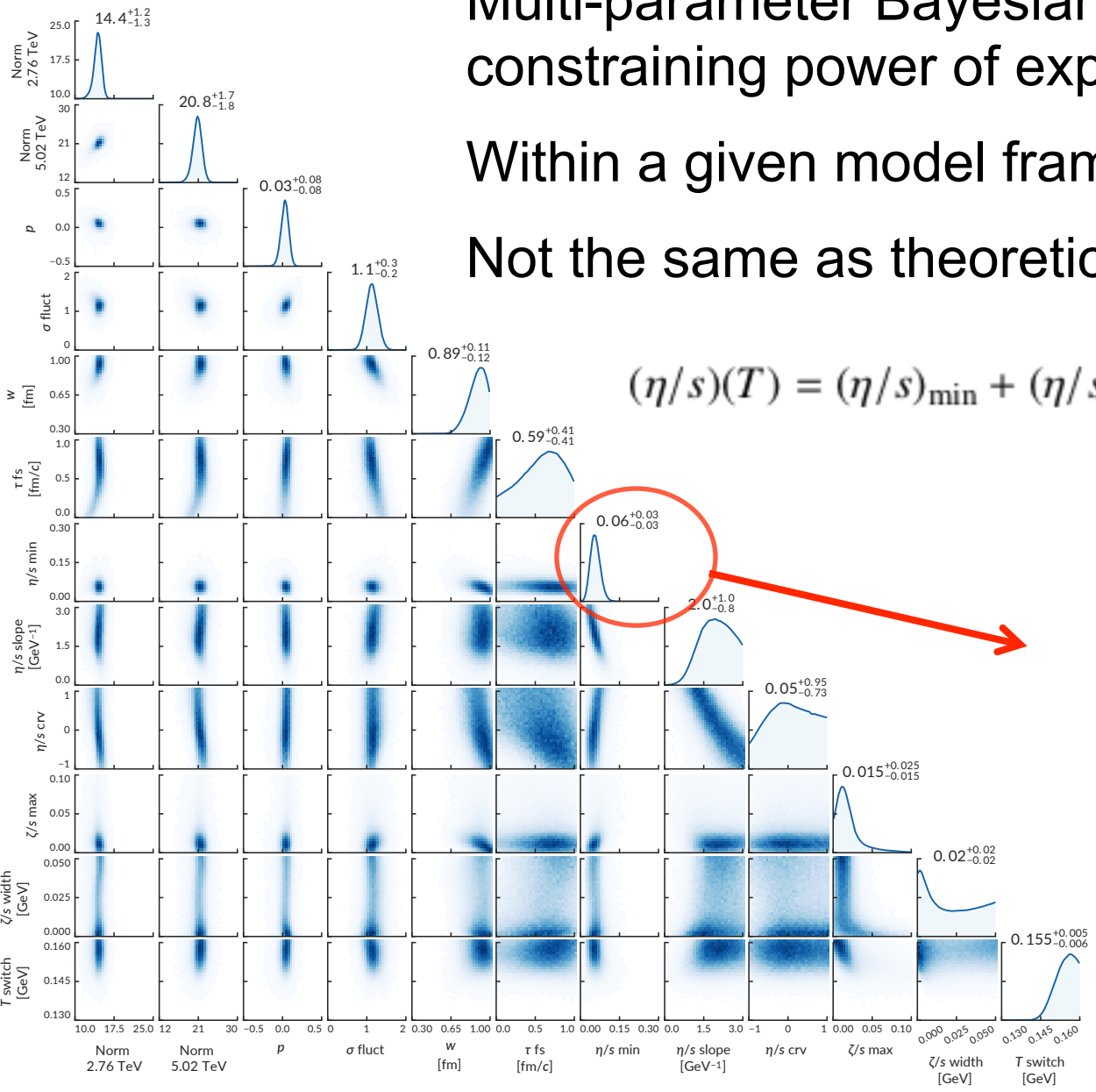
W. Zajc, NPSS2017

Maximizing the constraining power

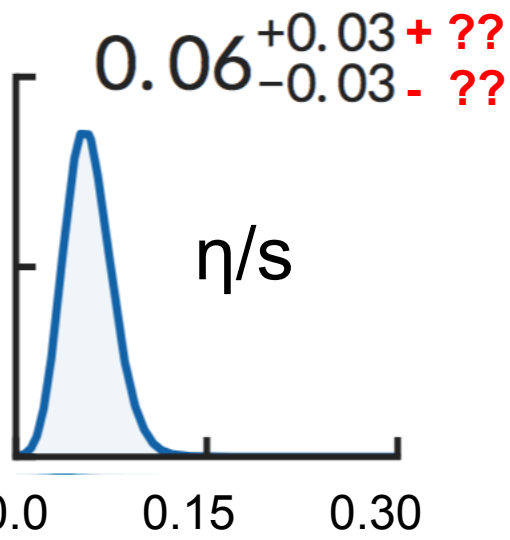
Multi-parameter Bayesian analysis to maximize constraining power of experimental data.

Within a given model framework!

Not the same as theoretical uncertainty.



$$(\eta/s)(T) = (\eta/s)_{\text{min}} + (\eta/s)_{\text{slope}}\,(T - T_c)\,(T - T_c)^{(\eta/s)_{\text{curvature}}}$$



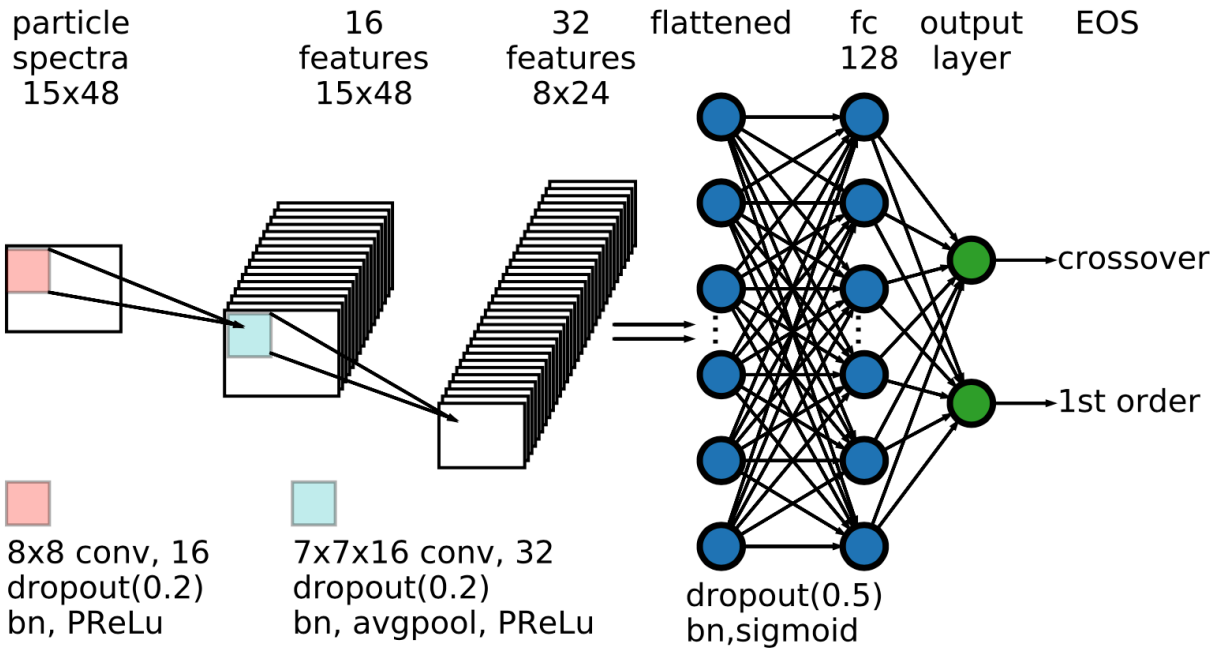
S.Bass, J. Bernhard, QM2017

W. Zajc, NPSS2017

Deep machine learning

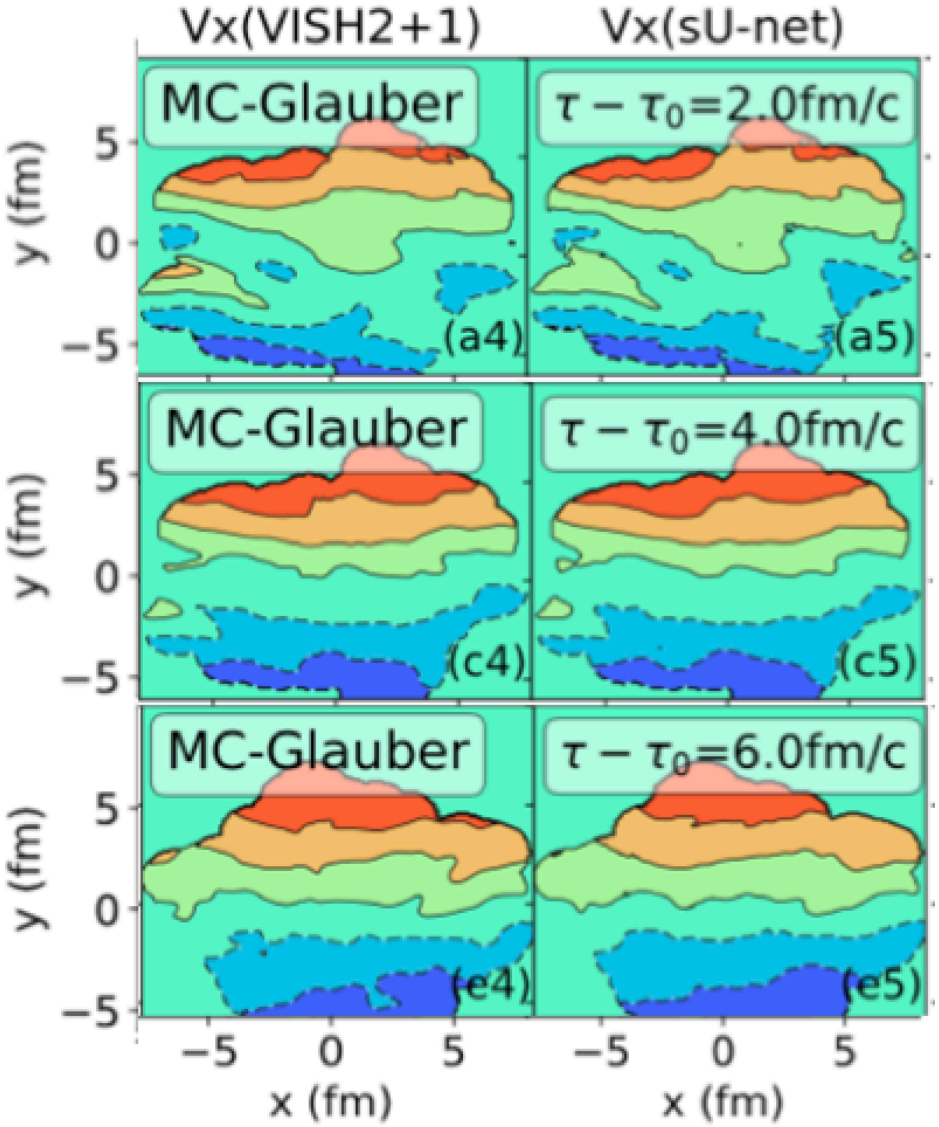
- Identify the important phase space at the final state and relate to dynamics WITHIN a model framework
- DCNN capture main features of the non-linear hydrodynamics, effective parameterization of the entire ebye space-time dynamics.

Huge saving: CPU→GPU→Machine learning



L.G.Pang, K. Zhou, N. Su, H. Petersen, H Stöcker, XN Wang Nature Physics 9, 210 (2018) 1612.04262

Full hydro Predicted



H. Huang B. Xiao H. Xiong Z. Wu Y. Mu H. Song 1801.03334

Efficient e-by-e simulations of relativistic hydrodynamics

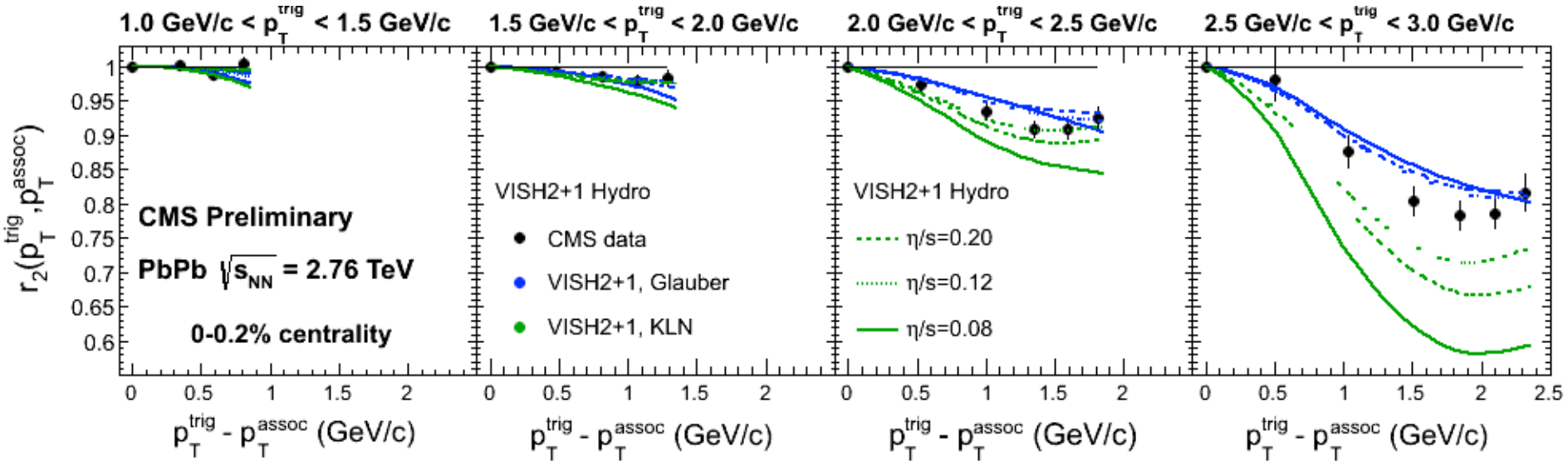
Flow fluctuation in transverse (p_T) direction

- Ollitrault saw v_n angle and amplitude fluctuates in p_T in EbyE hydro

$$\tilde{r}_n(p_{T1}, p_{T2}) := \frac{\langle v_n(p_{T1}) v_n(p_{T2}) \cos[n(\Psi_n(p_{T1}) - \Psi_n(p_{T2}))] \rangle}{\langle v_n(p_{T1}) v_n(p_{T2}) \rangle}$$

QM2012

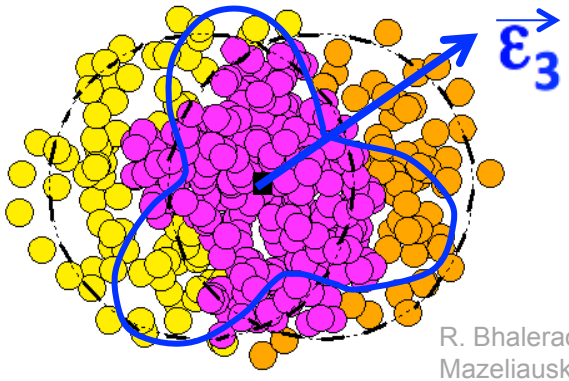
- Breaking is largest for v_2 in ultra-central Pb+Pb collisions
  - Also depends strongly on PID



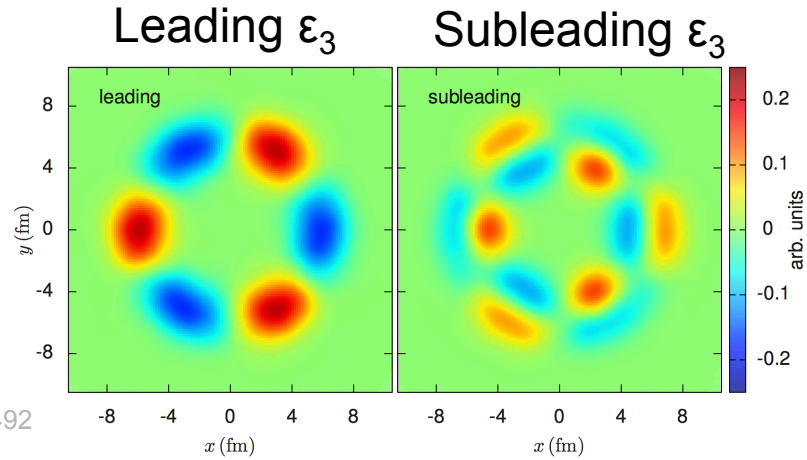
U.Heinz, Z.Qiu, C.Shen 1302.3535

Flow fluctuation in transverse (p_T) direction

Transverse density profile contains multiple eccentricity ε_n mode with different radial length scales. Each ε_n drives its own flow component with its own phase:



R. Bhalerao, J.Ollitrault, S. Pal, D. Teaney, A. Mazeliauskas 1410.7739,1501.03138,1509.07492



$$\varepsilon_3(r) \propto \varepsilon_{3;1}(r) e^{i3\Phi_3^1} + \varepsilon_{3;2}(r) e^{i3\Phi_3^2}$$

Flow fluctuation in transverse (p_T) direction

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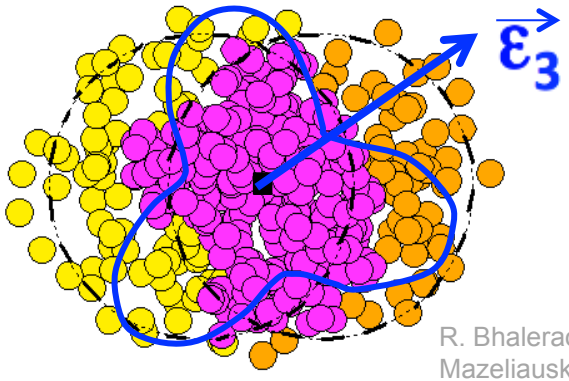
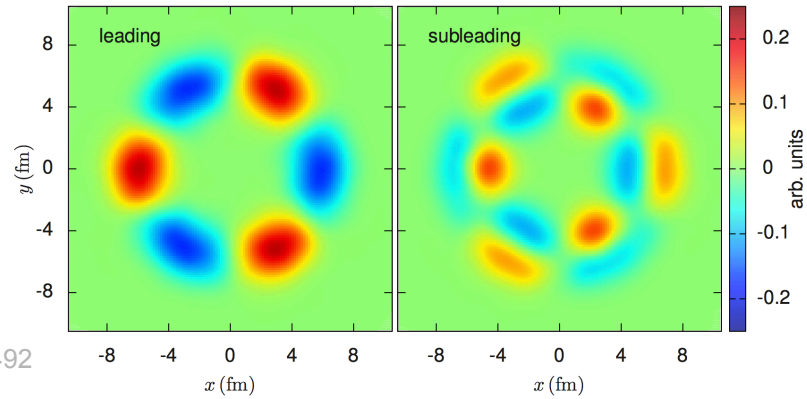


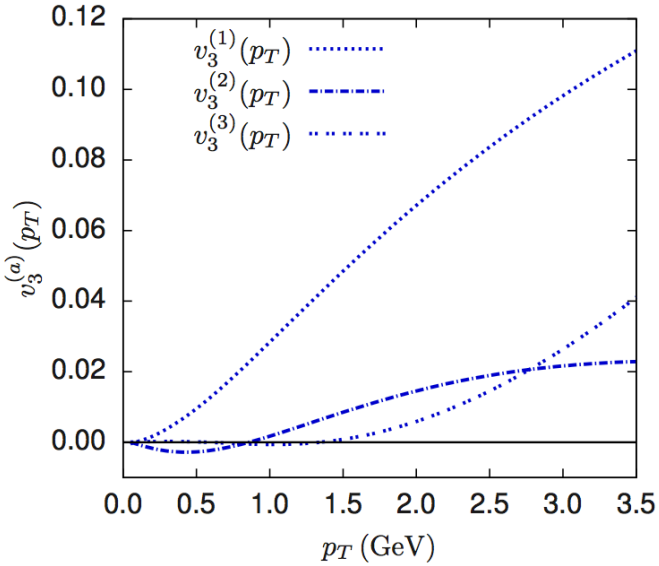
R. Bhalerao, J.Ollitrault, S. Pal, D. Teaney, A. Mazeliauskas 1410.7739,1501.03138,1509.07492

Leading ε_3 Subleading ε_3



$$\varepsilon_3(r) \propto \varepsilon_{3;1}(r) e^{i3\Phi_3^1} + \varepsilon_{3;2}(r) e^{i3\Phi_3^2}$$

$$v_3(p_T) \propto c_1(p_T) e^{i3\Phi_3^1(p_T)} + c_2(p_T) e^{i3\Phi_3^2(p_T)}$$



Flow fluctuation in transverse (p_T) direction

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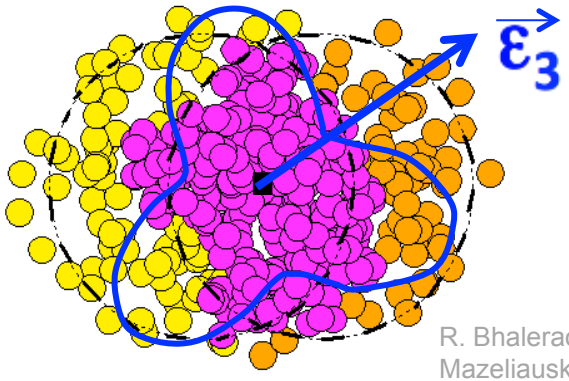
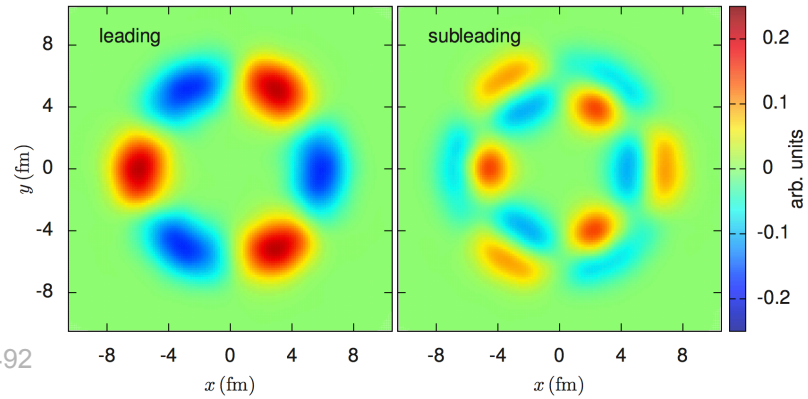


R. Bhalerao, J.Ollitrault, S. Pal, D. Teaney, A. Mazeliauskas 1410.7739,1501.03138,1509.07492

Leading ε_3 Subleading ε_3



$$\varepsilon_3(r) \propto \varepsilon_{3;1}(r) e^{i3\Phi_3^1} + \varepsilon_{3;2}(r) e^{i3\Phi_3^2}$$

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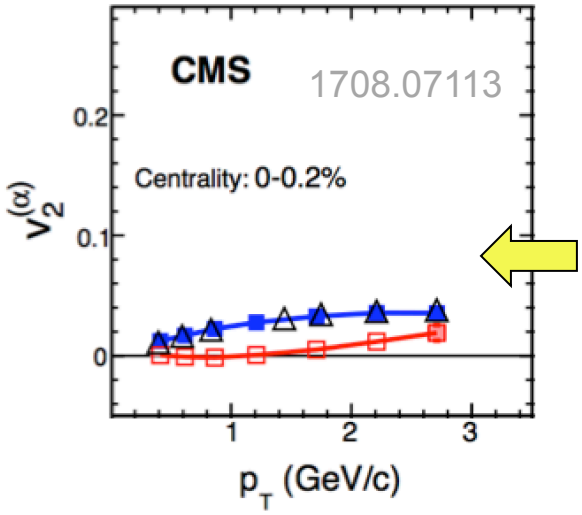
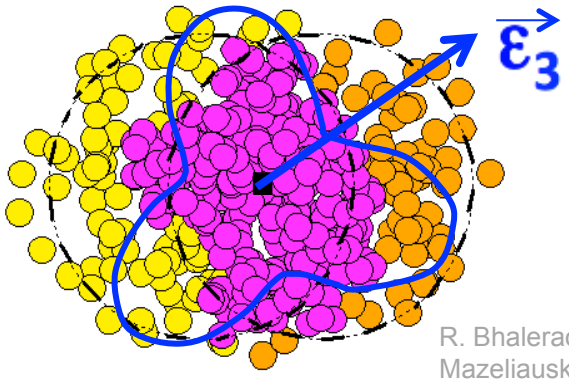
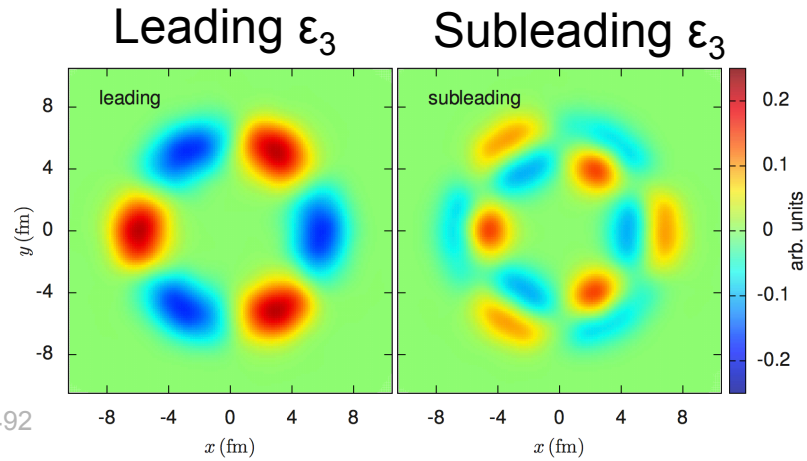


Flow fluctuation in transverse (p_T) direction

Transverse density profile contains multiple eccentricity ε_n mode with different radial length scales. Each ε_n drives its own flow component with its own phase:



R. Bhalerao, J.Ollitrault, S. Pal, D. Teaney, A. Mazeliauskas 1410.7739,1501.03138,1509.07492

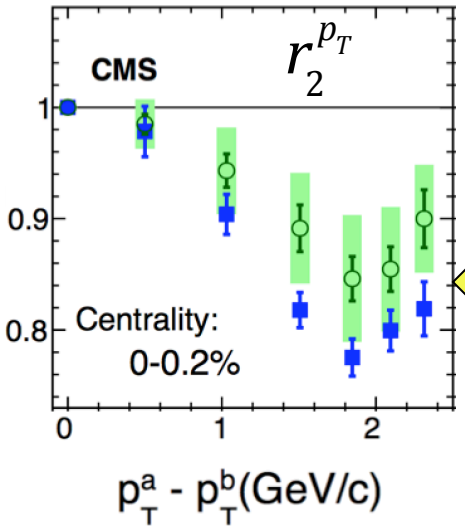


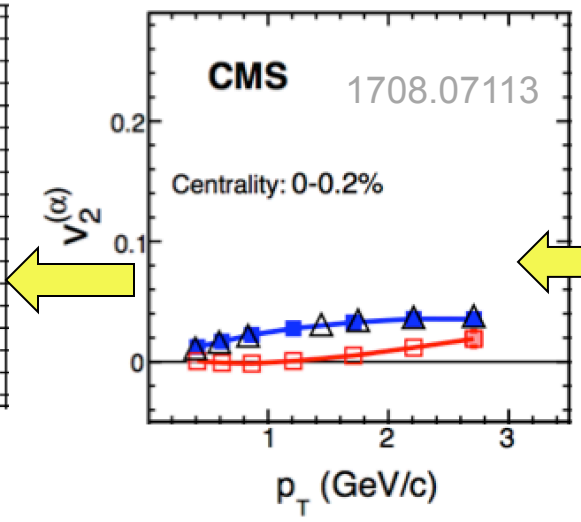
$$\varepsilon_3(r) \propto \varepsilon_{3;1}(r)e^{i3\Phi_3^1} + \varepsilon_{3;2}(r)e^{i3\Phi_3^2}$$

$$v_3(p_T) \propto c_1(p_T)e^{i3\Phi_3^1(p_T)} + c_2(p_T)e^{i3\Phi_3^2(p_T)}$$

Flow angle and amplitude fluctuates in p_T

$$V_{n\Delta}(p_T^a, p_T^b) \neq v_n(p_T^a)v_n(p_T^b)$$



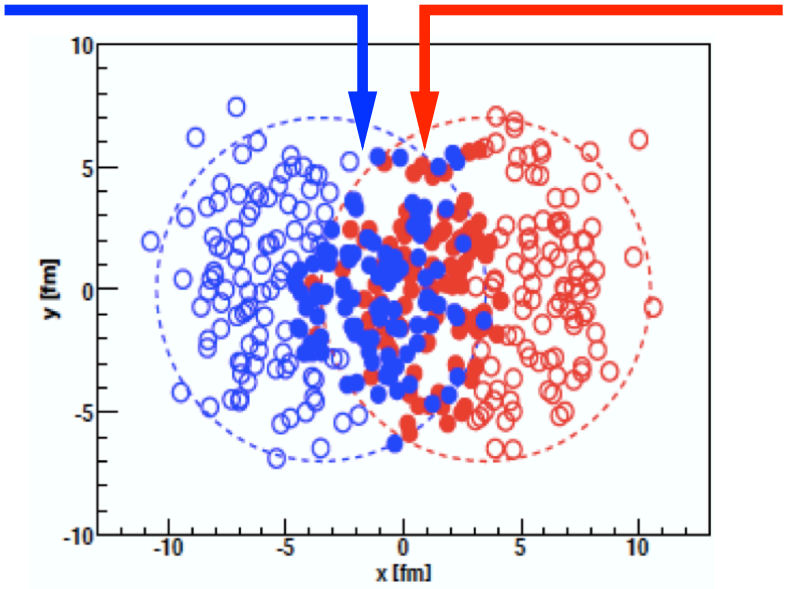


Subleading flow responsible for most factorization breaking effect

Flow fluctuation in longitudinal direction

Fluctuation of sources in two nuclei → fluctuation of transverse-shape

$v_n^{\mathrm{F}}, \Psi_n^{F}$



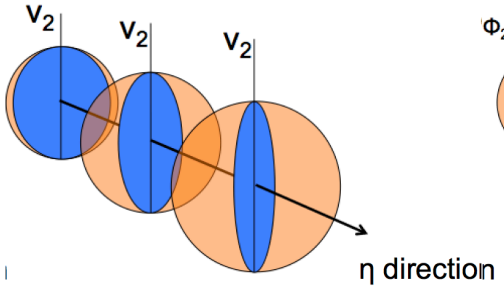
$v_n^{\mathrm{B}}, \Psi_n^{B}$

Bozek et.al., arXiv:1011.3354

$$\boldsymbol{v}_n = v_n e^{in\Psi_n}$$

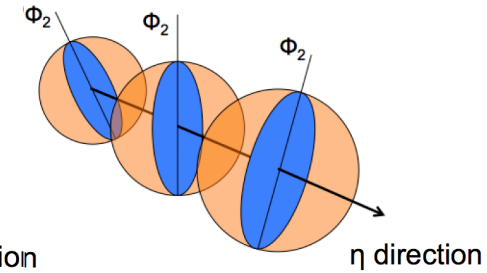
Consequences:

Asymmetry of a flow magnitude



$$v_n(\eta_1) \neq v_n(\eta_2)$$

Torque/twist of an event plane

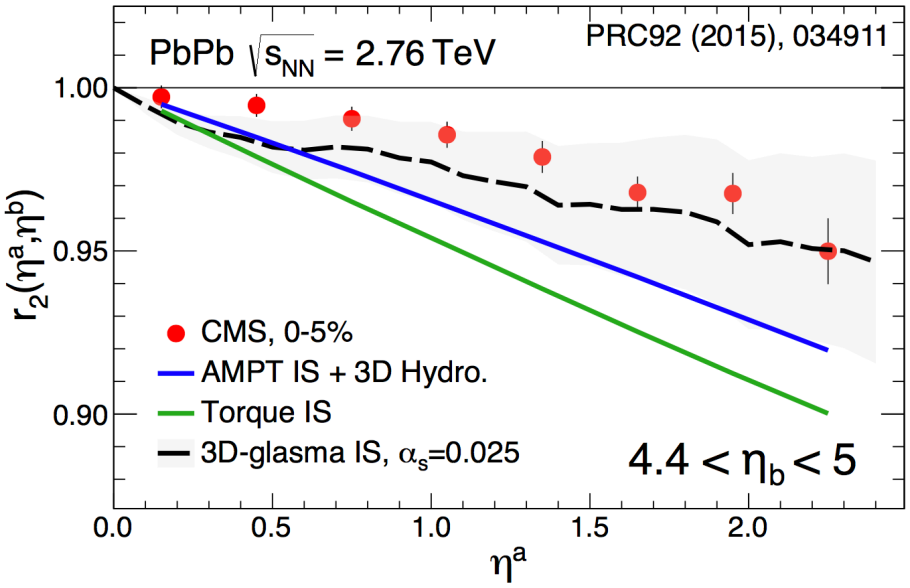


$$\Psi_n(\eta_1) \neq \Psi_n(\eta_2)$$

Flow fluctuation in longitudinal direction

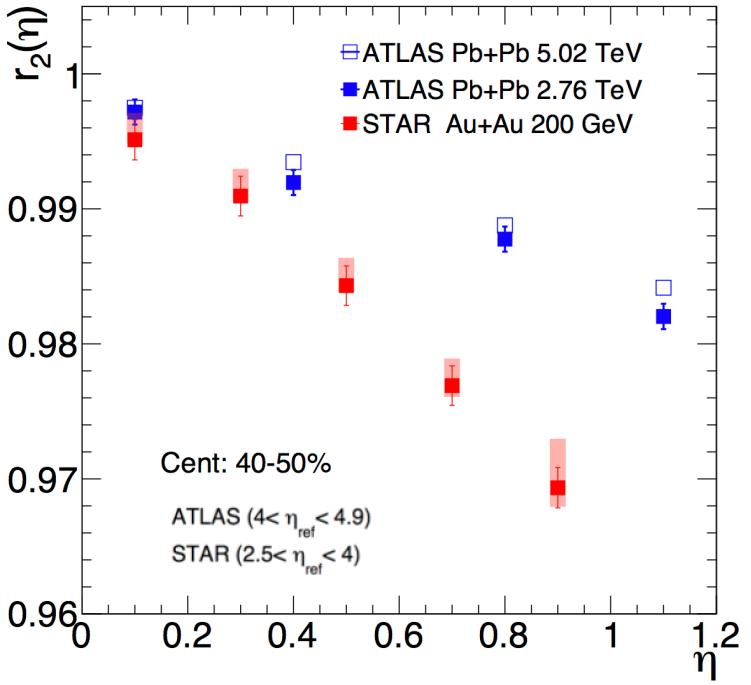
Observables: $$r_n^{\eta} = \frac{V_n(-\eta)V_n^*(\eta_{\rm ref})}{V_n(\eta)V_n^*(\eta_{\rm ref})} \sim \langle \cos n\left[\Phi_n(\eta) - \Phi_n(-\eta)\right]\rangle$$

Significant decorrelation,
not described by any models



Wei Li QM2017

Much stronger at lower √s



M. Nie IS2017

Flow fluctuation in longitudinal direction

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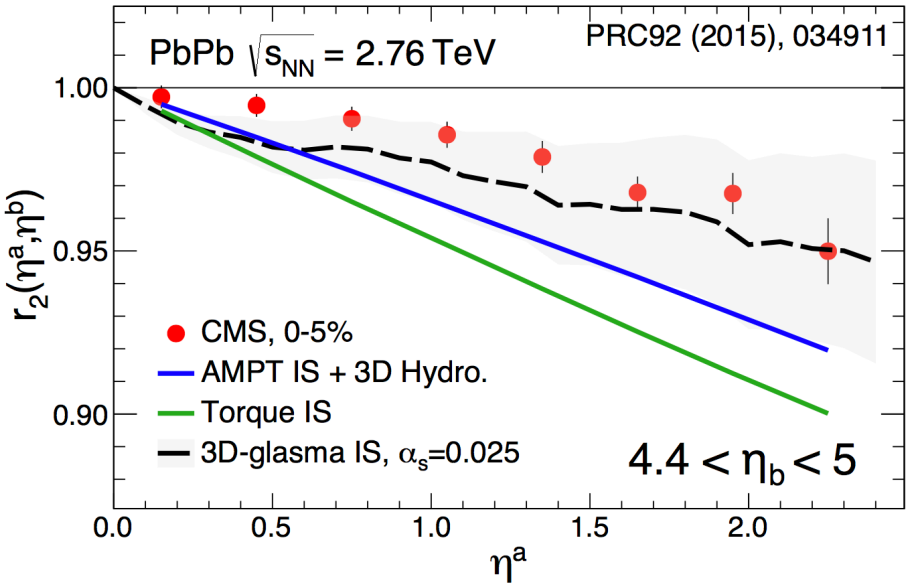
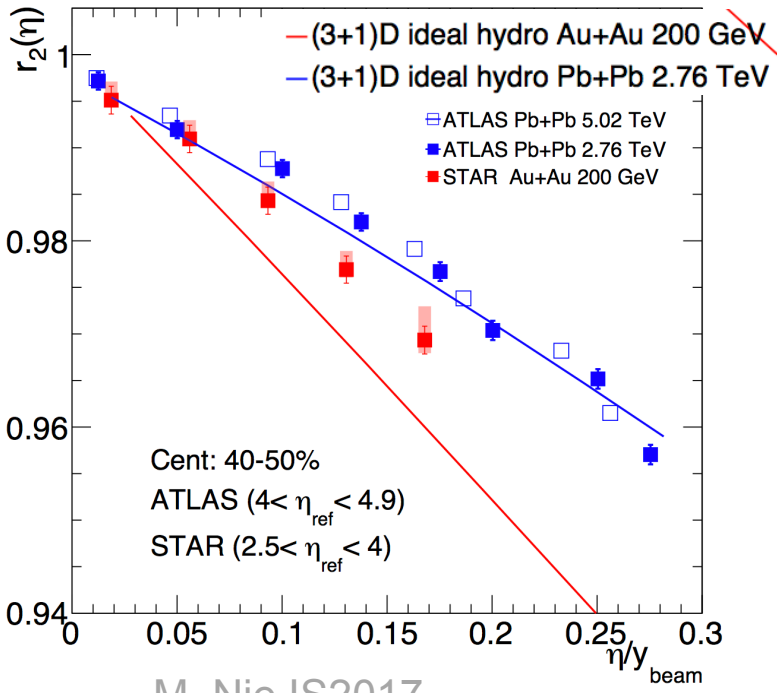
Significant decorrelation, not described by any models



Wei Li QM2017

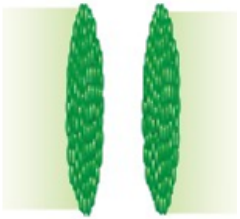
Can't be explained by beam-rapidity scaling, not described by hydro model

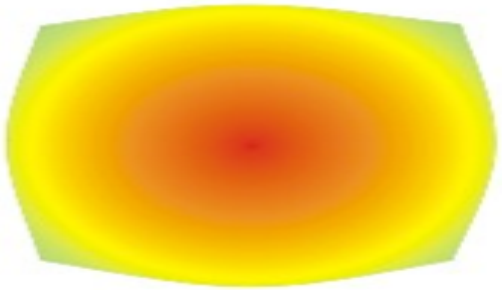
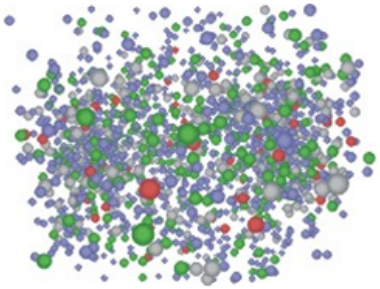


M. Nie IS2017

Challenge for precision

$\tau<0$ | $0<\tau<0.5$fm/c | $0.5<\tau<6$ fm/c | $6<\tau<10$ fm/c



initial state | **pre-equilibrium** | **QGP & expansion** | **Phase transition&freeze-out**

Glauber, CGC | Pre-flow dynamics | Hydrodynamics | Hadronization

Sub-nucl. dof
Longi. structure

Challenge for precision

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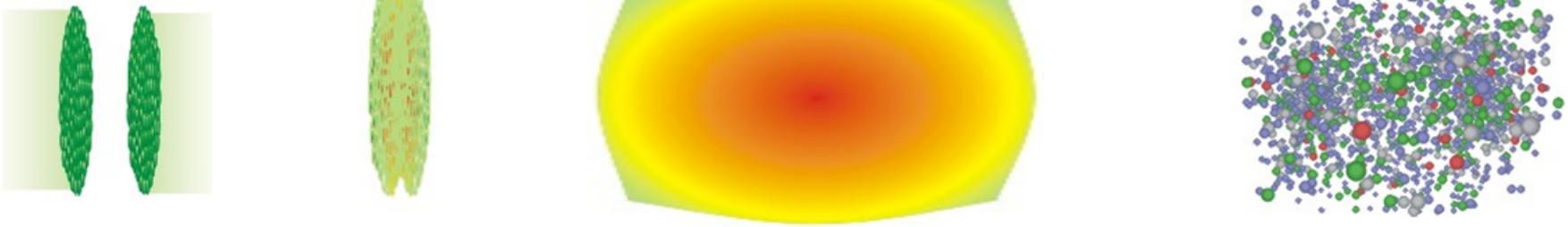


initial state | **pre-equilibrium** | **QGP & expansion** | **Phase transition&freeze-out**

Glauber, CGC

Sub-nucl. dof
Longi. structure

External input

Schenke, Shen, Schlingting, Venogopalan, Tribedy,Bzdak et.al 1804.10557, 1710.00881 1605.07158, 1304.3403, 1209.6330

Pre-flow dynamics

Non-hydro modes
Kinetic approach

C. Shen, U. Heinz 1504.02160

. Vredevoogd, S. Pratt 0810.4325z

Weller, Romatschke 1701.07145,1704.08699,1712.05815

A.Kurkela, A.Mazeliauskas,J.Paquet,S.Schlichting, D.Teaney 1805.00961, 1605.04287,

Hydrodynamics

Slow modes
Hydro-fluctuations
Critial-flucuations

M. Stephanov, Y.Yin K.Rajagopal,. C.Young, T. Hirano,A.Mazeliauskas, D.Teaney et.al, 1712.10305, 1606.07742, 1407.1077,1304.3243. hep-ph/9806219,

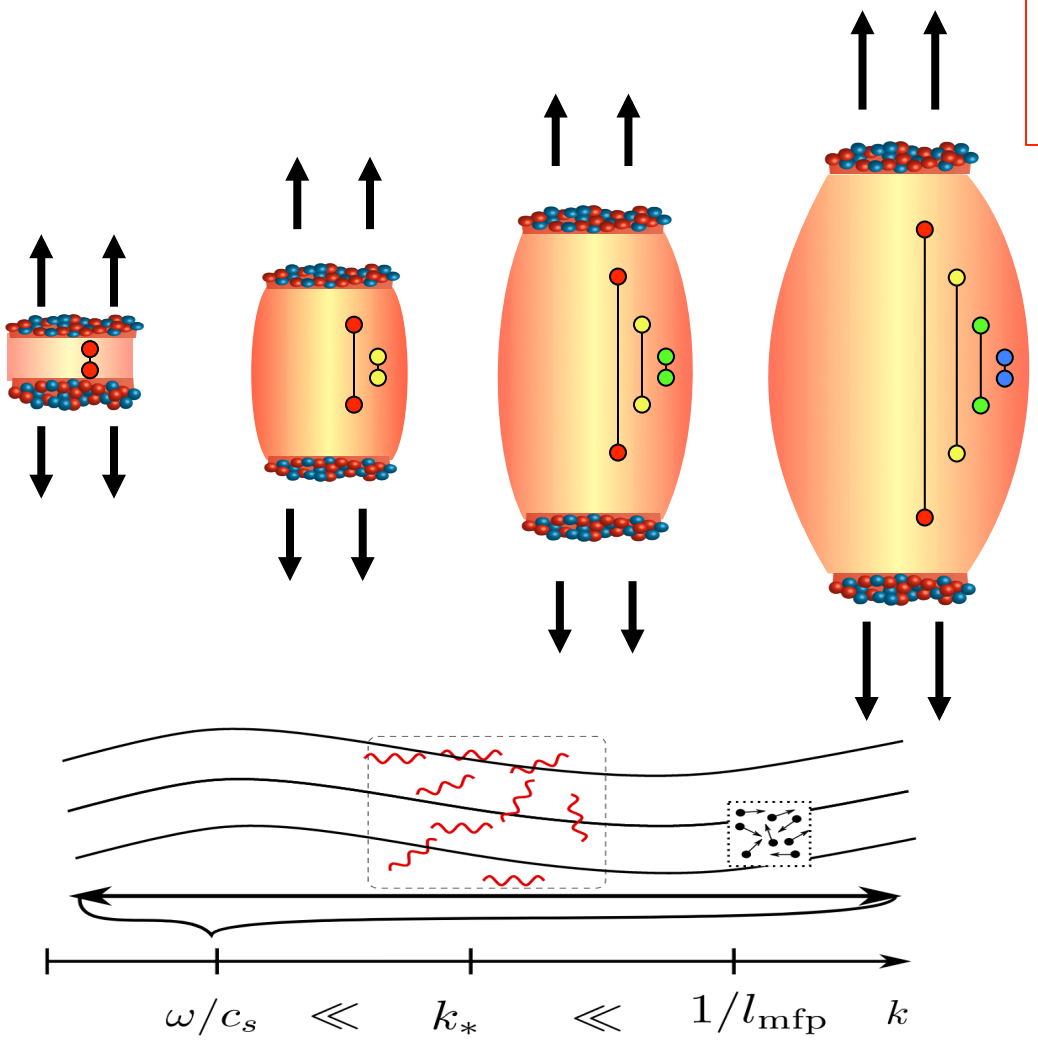
Hadronization

Bulk viscosity

S. Ryu, G.Denicol,B. Schenke,S. Jeon,C. Gale, H. Song, K. Dusling, 1704.04216, 1704.04216, 1502.01675, 0903.3595, 0909.1549, 1109.5181, 1305.1981

Fluctuations are everywhere

Initial fluctuation	non-hydro modes
Thermal fluctuation	Jet quenching,
Critical fluctuation	HBT
…	Resonance decays,
	…



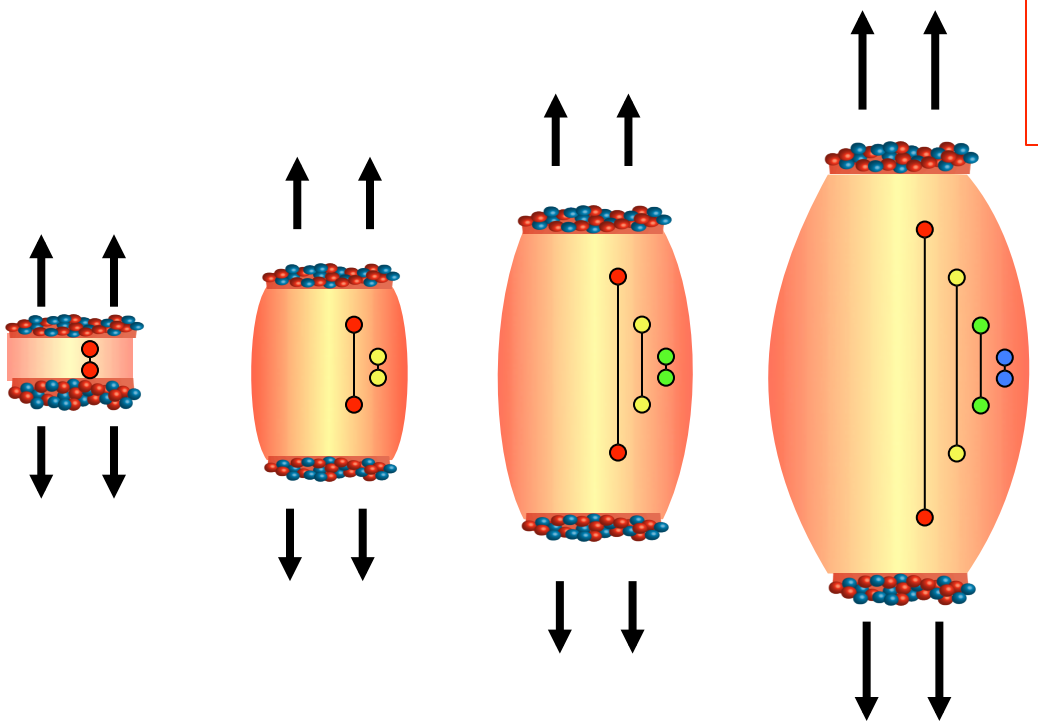
Y. Akamatsu, A. Mazeliauskas, D.Teaney 1606.07742

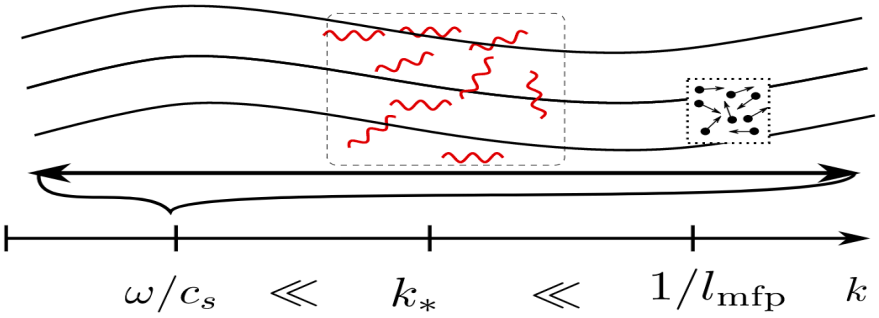
With varying length scale

Fluctuations are everywhere

Initial fluctuation
Thermal fluctuation
Critical fluctuation
…

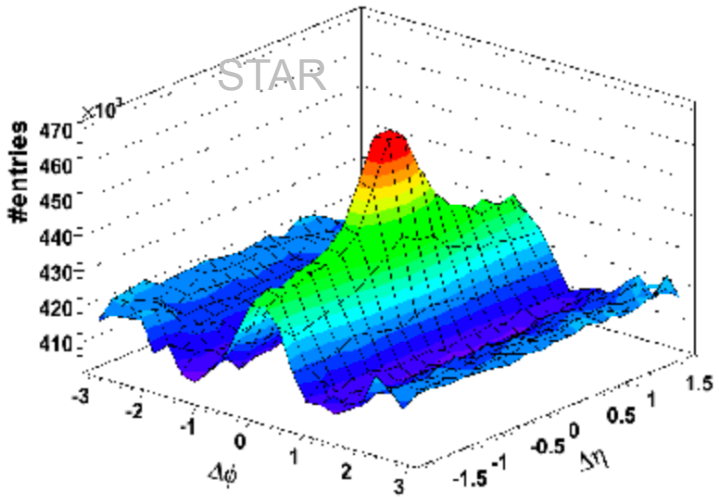
non-hydro modes
Jet quenching,
HBT
Resonance decays,
…

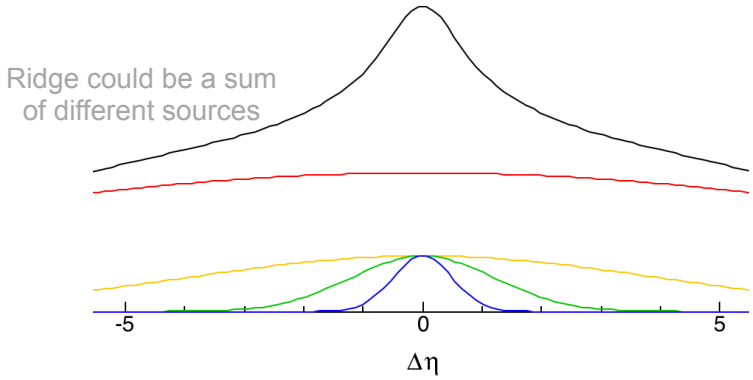




Y. Akamatsu, A. Mazeliauskas, D.Teaney 1606.07742

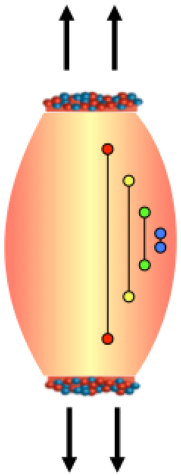
With varying length scale

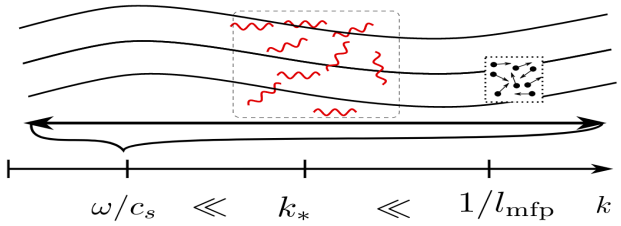




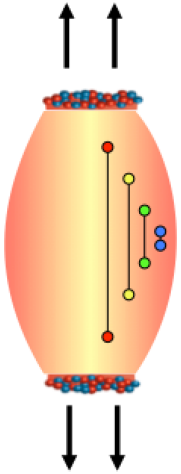
Disentangle various time-scales
via Δη correlation, but how?

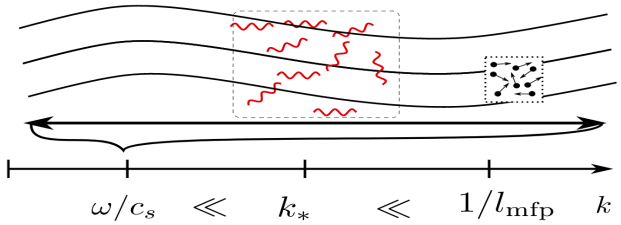
Resolve fluctuations at different time and length scale



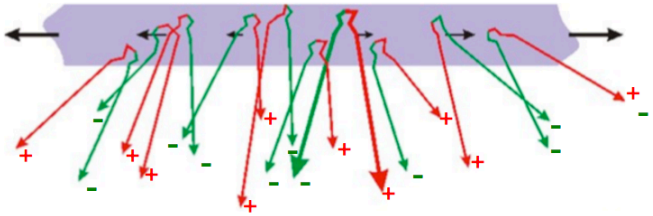


Resolve fluctuations at different time and length scale



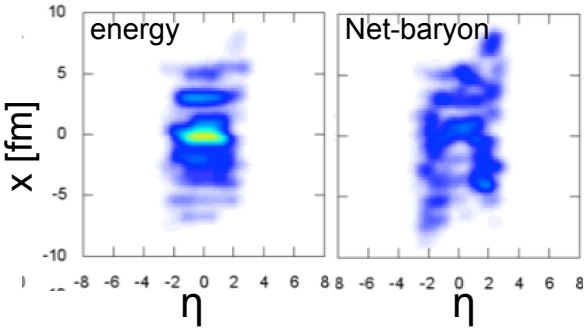


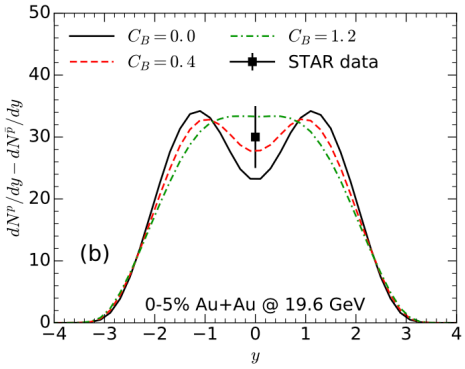
Charge transport



background for CME

Net-baryon transport



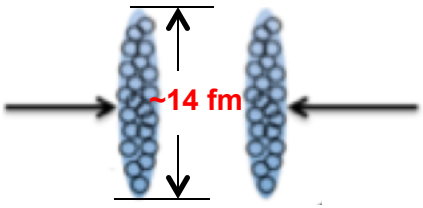


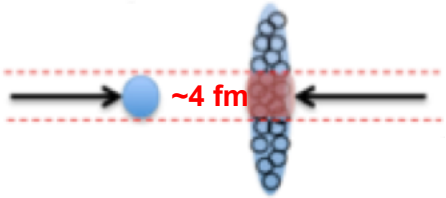
background for critical fluctuation

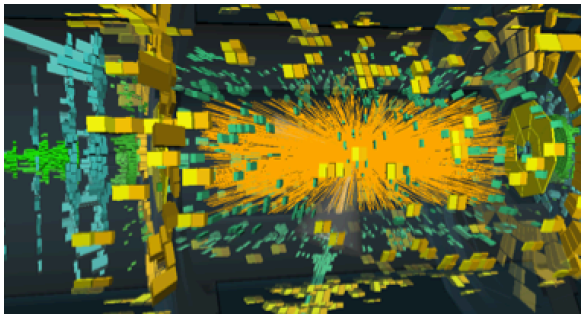
G. Denicol, C.Gale, S.Jeon, A.monnai, B.Schenke C.Shen 1804.10557

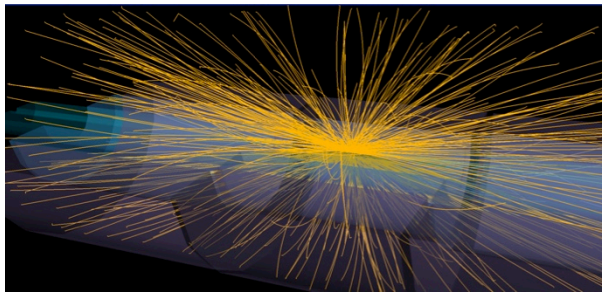
Large η coverage from forward upgrade is important

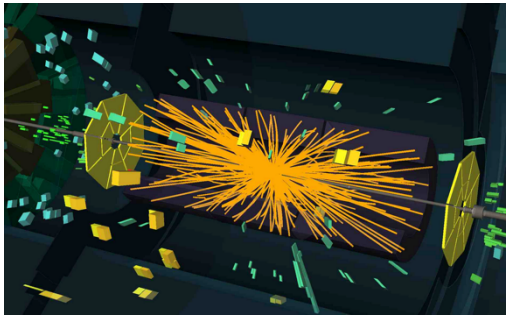
Small systems and early time dynamics











~30000 particles* ~2000 particles* ~ 600 particles*

What is the smallest droplet of QGP created in these collisions?

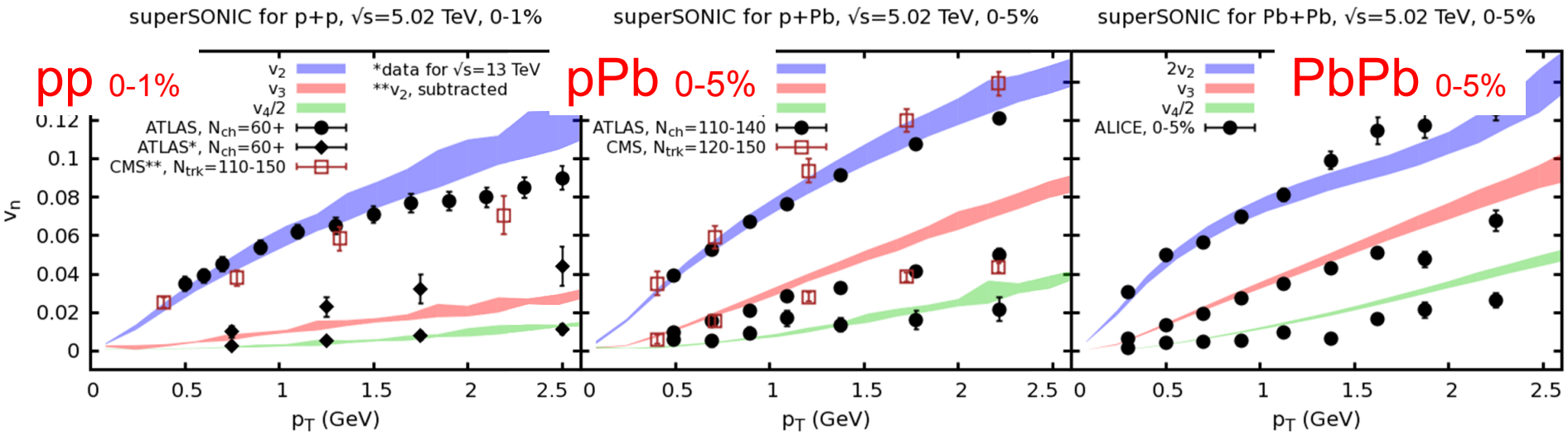
→ Change matter size, life-time and space-time dynamics @ RHIC and LHC

* Rough number in very high-multiplicity events, integrated over full phase space at LHC

Unreasonable success of hydro?

P. Romatschke, R. Weller 1701.07145

Same 2nd-order viscous hydro equations describe all three systems



Actual space-time dynamics & properties should still be different!

- This does not mean $T^{uv}(\mathbf{x},t)_{pp}=T^{uv}(\mathbf{x},t)_{pPb}=T^{uv}(\mathbf{x},t)_{PbPb}$
- The pre-equilibrium effects are not the same

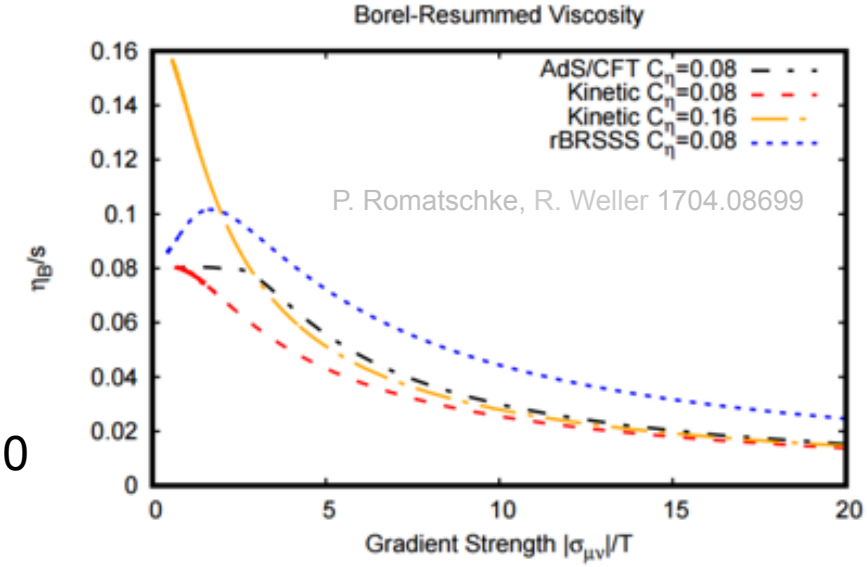
Unreasonable success of hydro?

- In far from equilibrium region, hydro still fit the data, but gives wrong viscosity

$$T^{\mu\nu}_{\rm hydro} = (\epsilon + P_B)u^\mu u^\nu + P_B g^{\mu\nu} - \eta_B \sigma^{\mu\nu}$$

Small gradients $\eta_B \sim \eta$ Large gradients $\eta_B \to 0$

Also A.Kurkela, U.Wiedemann, B. Wu 1805.04081



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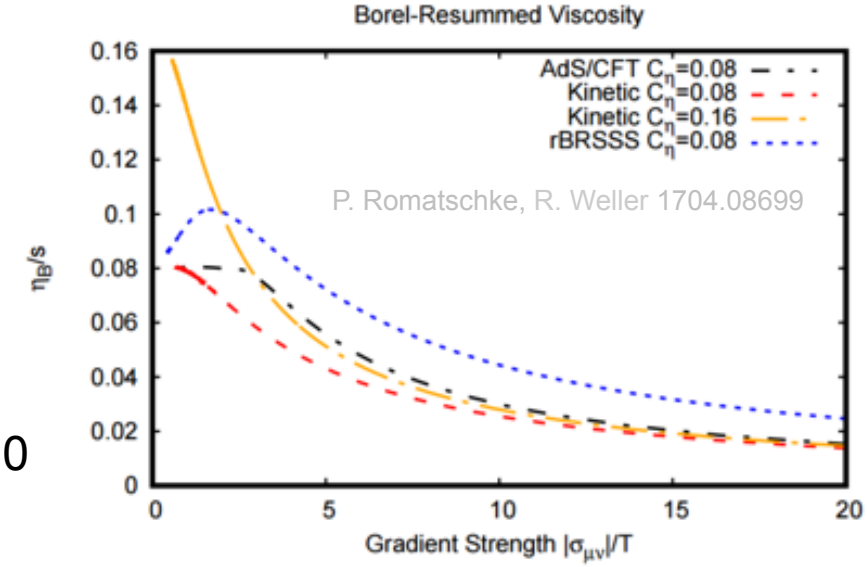
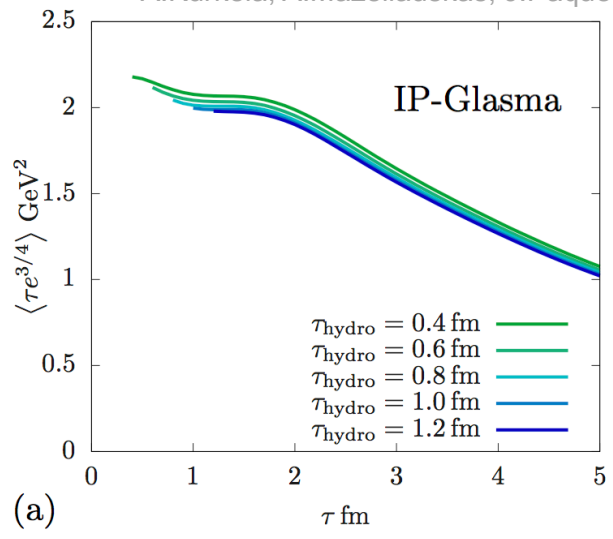
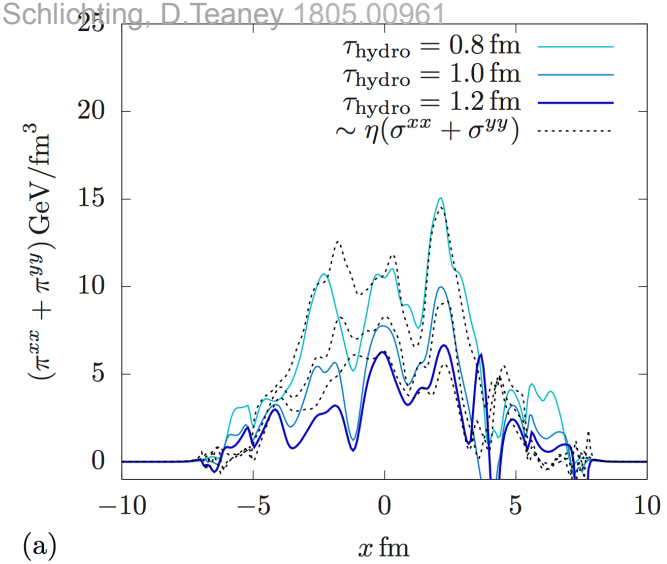
Also A.Kurkela, U.Wiedemann, B. Wu 1805.04081



- Different models for early-time dynamics have similar average hydro-field, but different differential distri., e.g shear tensor $\pi^{\mu\nu}(\mathbf{x})$.

A.Kurkela, A.Mazeliauskas, J.Paquet, S.Schlichting, D.Teaney 1805.00961



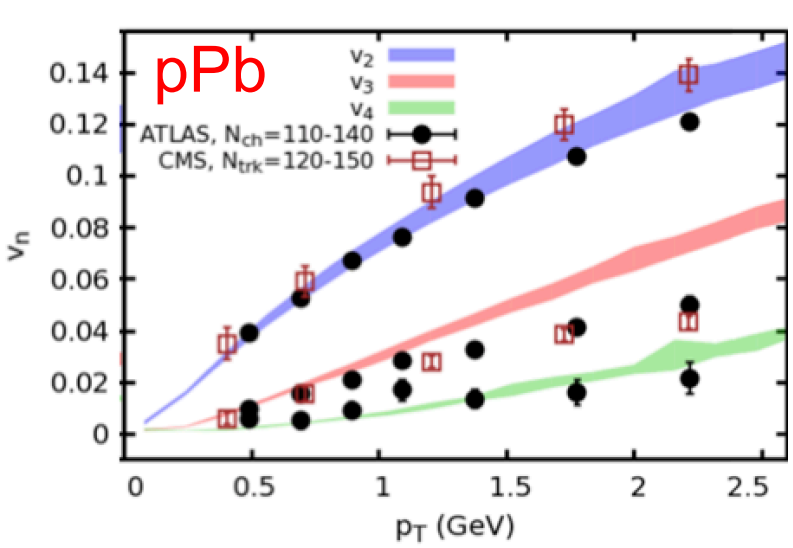


Unreasonable success of hydro?

- Non-hydro approaches can also describe the same data

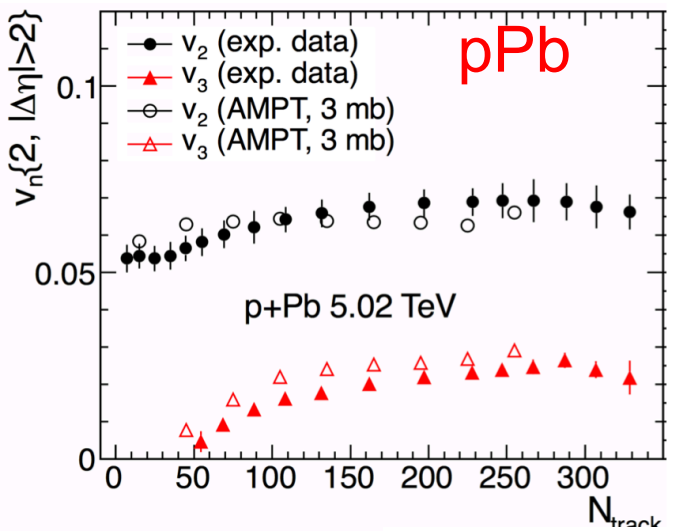
P. Romatschke, R. Weller 1701.07145

Hydrodynamics

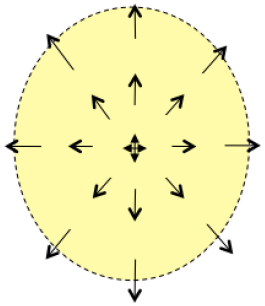


L.He, T.Edmonds, Z.Lin, F.Liu, D. Molnar and F. Wang 1502.05572, G. Ma and A Bzdak 1406.2804

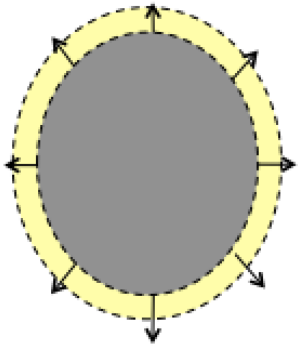
AMPT transport



Pressure driven



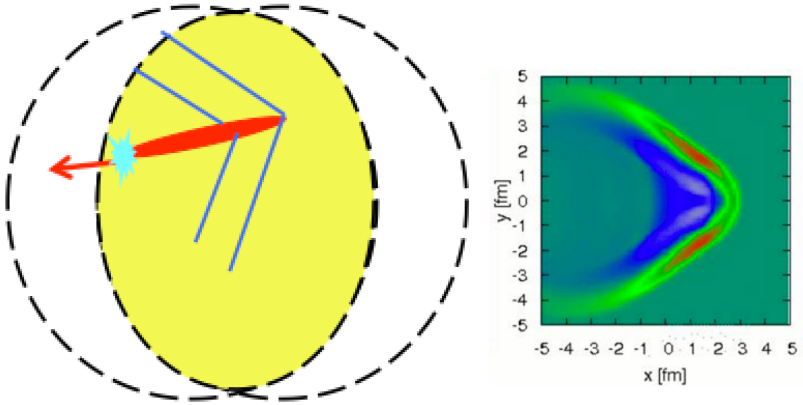
Anistropic escape
Anistropic absorption,
surface emission



Actual space-time dynamics are very different
How to distinguish the two experimentally?

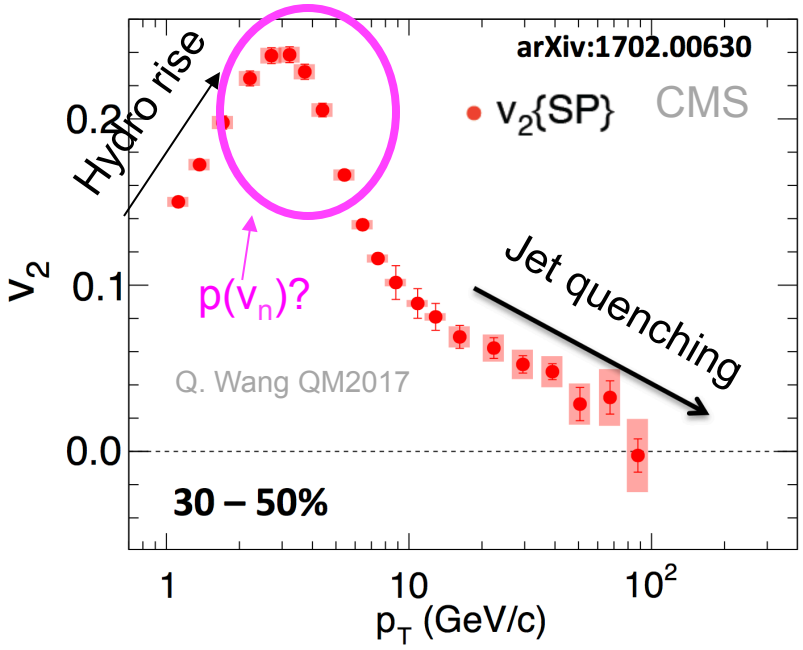
Direct search for non-equilibrium features in v_n?

Response of the medium to quenched jet, Mach cone?



hard-soft correlations to understand relaxation of non-hydrodynamic perturbations

Nature of flow fluctuations at intermediate p_T?



Look for systematic failure of gradient expansion of hydrodynamics

Summary

- Flow observables efficiently characterize particle correlations at $\tau=\infty$.
- Hydrodynamic model is a good tool to unfold flow data to extract space-time dynamics of QGP and its properties for $\tau < 10$fm/c.
- Precision knowledge of HI collisions require more theoretical progress to disentangle contributions from different stages of evolution.
- Some possible directions.
  - Differential measurements of flow fluctuations, mixed correlations
  - Observables to probe fluctuations at different time scales, transport mechanisms for charge and baryon number.
  - Small systems to constrain the pre-equilibrium dynamics.
  - Direct experimental search for effects of non-hydro dynamics.

HOT QUARKS 2018

Scientific Program

- QCD at high temperature/density and lattice QCD
- Initial state effects and Color Glass Condensate
- Relativistic hydrodynamics and collective phenomena
- Correlations and fluctuations
- Jets in the vacuum and in the medium
- Baryons and strangeness
- Heavy-flavour, dileptons and photons
- Application of String Theory and AdS/CFT
- Experimental techniques and future programs

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A workshop for young scientists on the physics of ultrarelativistic nucleus-nucleus collisions

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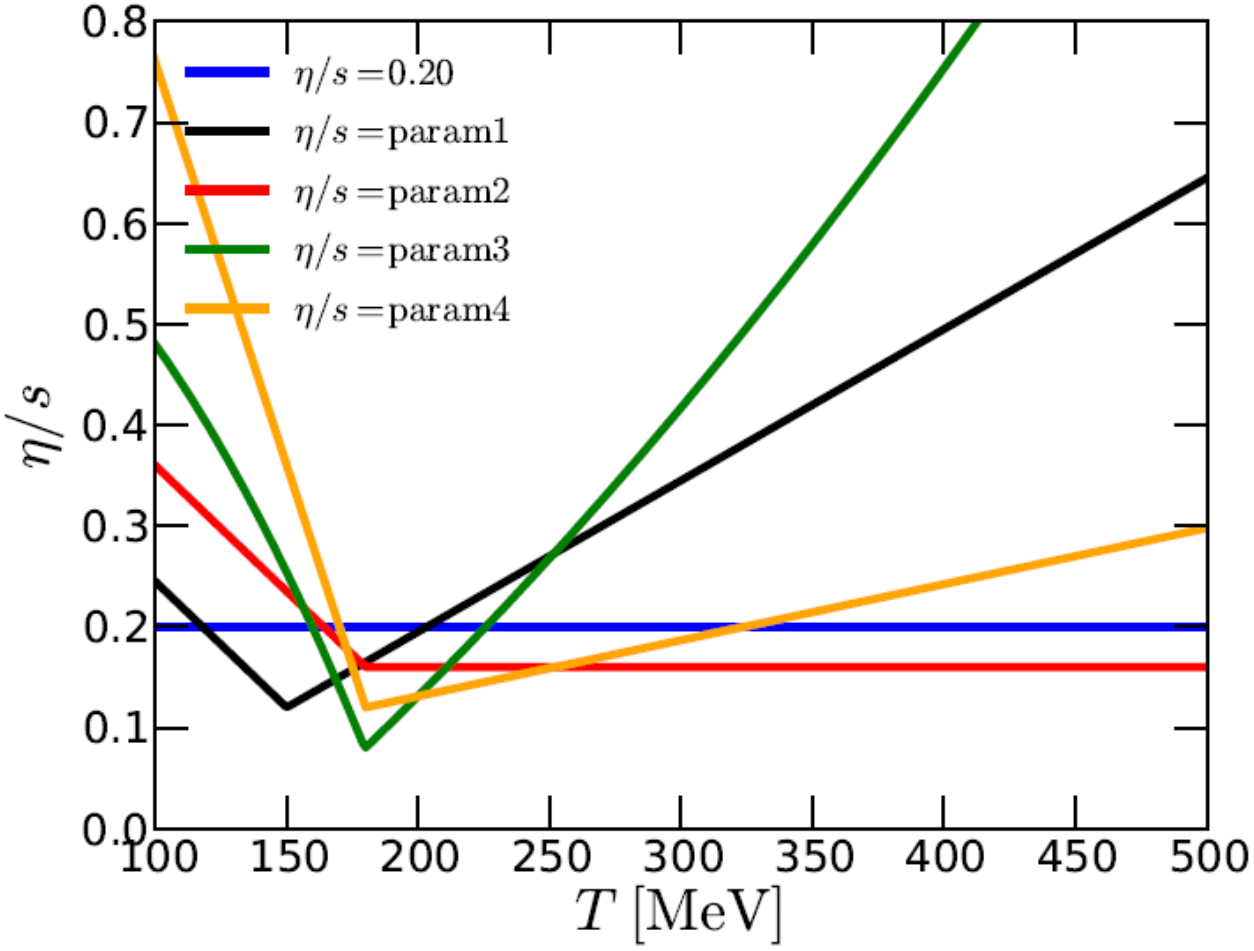
Temperature dependence of η/s

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