The Modern View of Quark-Gluon Plasma

presented at

Quark Matter 2018 Student Day

Venice, Italy
May 13th, 2018

W.A. Zajc
Columbia University

Outline

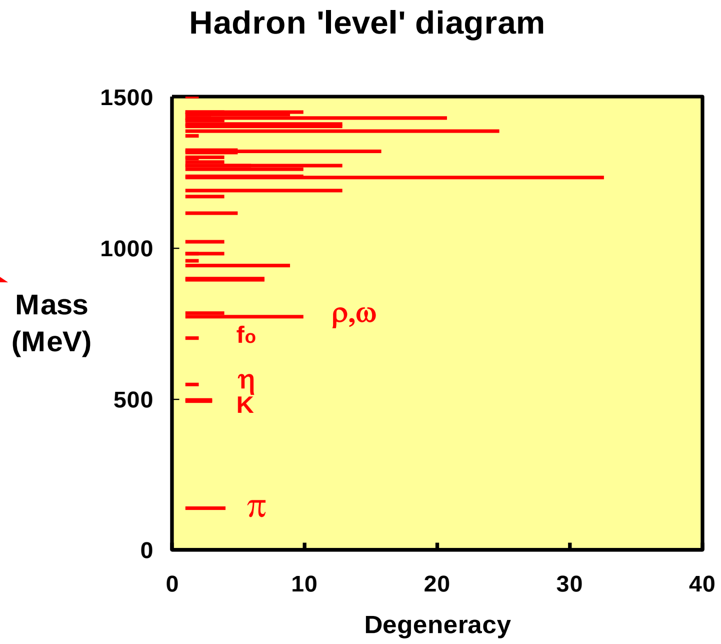
- Original expectations
- Exact solutions
- Why “almost” doesn’t work
- Measures of coupling
- The emerging modern view

Thanks to (JN)^3 :

Jamie Nagle, Jorge Noronha, Jaki Noronha-Hostler
for many useful discussions

A “Strong” Hint (prior to QCD)

- Hagedorn ~1968: *Is there an ultimate temperature?*
- The very rapid increase of hadron levels with mass



R. Hagedorn, Nuovo Cim. **A**56, 1027 (1968) ; S. Fraustchi, Phys. Rev. **D**3:2821-2834,1971

A "Strong" Hint (prior to QCD)

- Hagedorn ~1968: *Is there an ultimate temperature?*
- The very rapid increase of hadron levels with mass
- ~ equivalent to an *exponential* level density

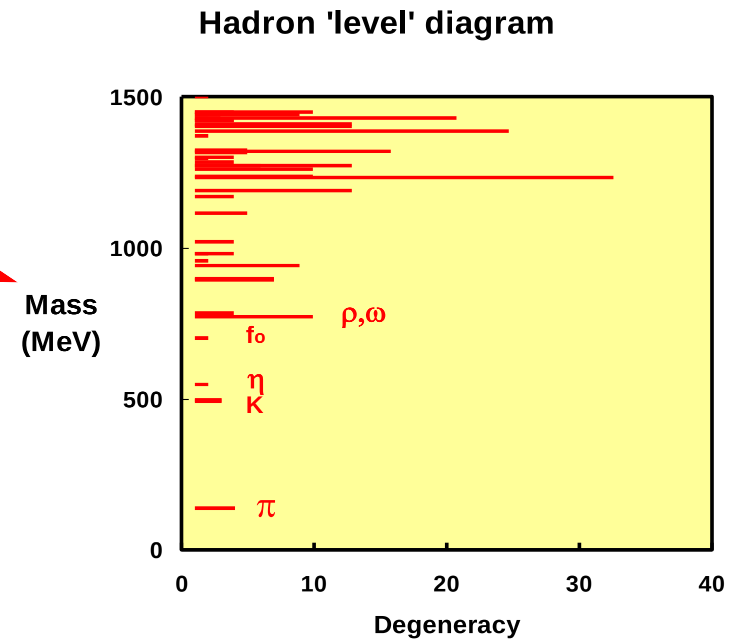
$$\rho(m) \equiv \frac{dn}{dm} \sim m^a \; e^{m/T_H}$$

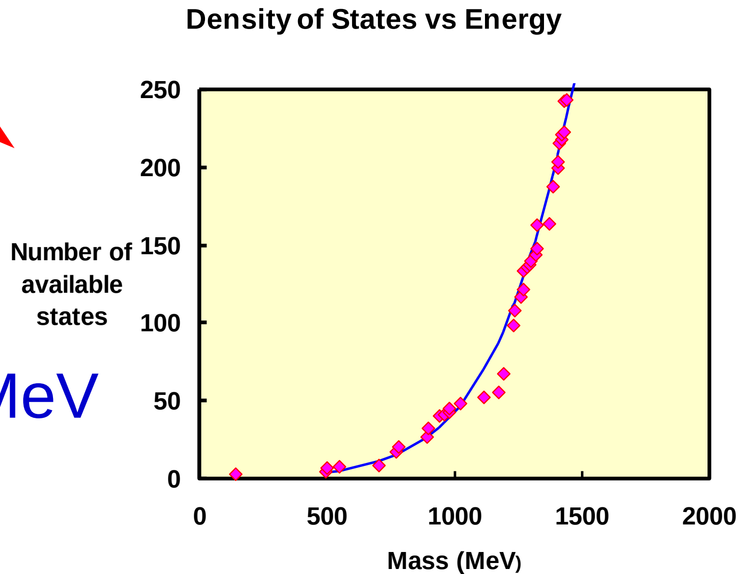
$$\Rightarrow \qquad Z = \int \rho(m)\; e^{-m/T}\; dm$$

$$\sim \int m^a \; e^{m/T_H}\; e^{-m/T}\; dm$$

$$\to \; \infty \quad \text{when} \quad T > T_H \;\; (!)$$

- Which implies an "Ultimate Temperature" (!) $T_H \sim 170$ MeV

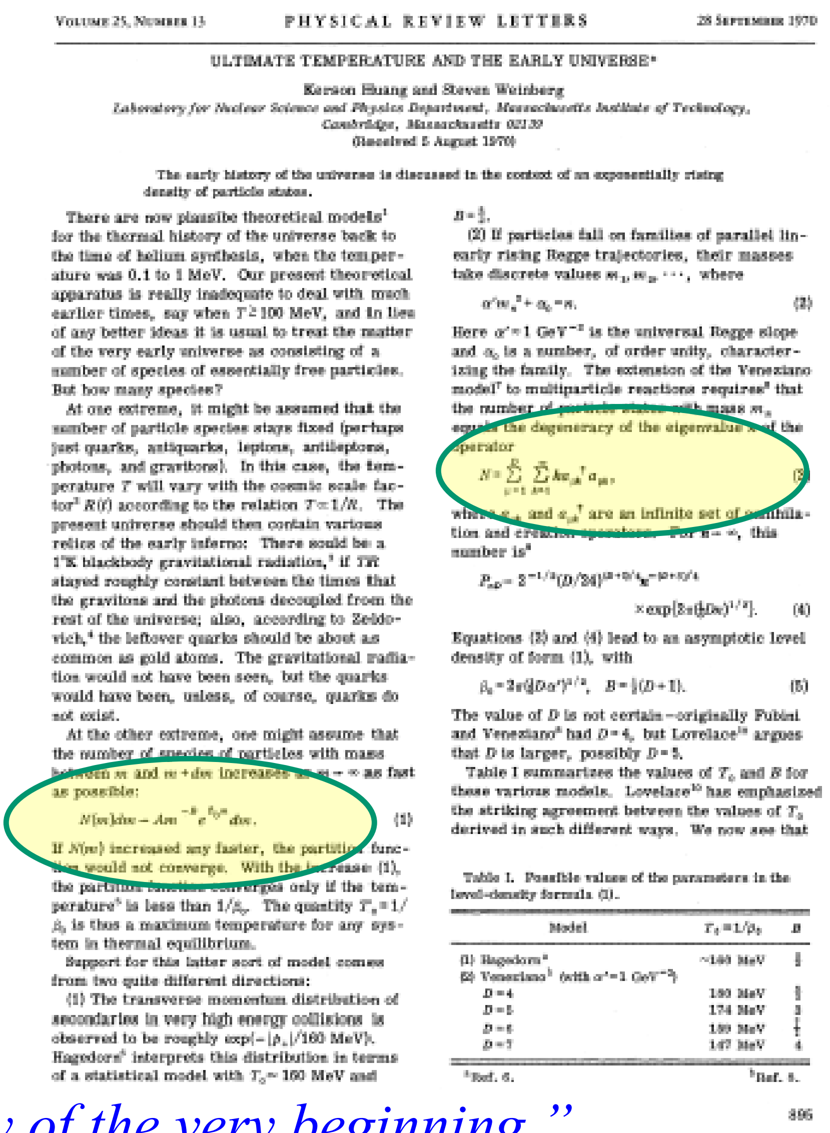




R. Hagedorn, Nuovo Cim. **A**56, 1027 (1968) ; S. Fraustchi, Phys. Rev. **D**3:2821-2834,1971

Puzzles from pre-History

- Huang and Weinberg (1970):
  - *Ultimate Temperature* *and the Early Universe*, Phys. Rev. Lett. **25**, 896 (1970)
  - Difficulties in constructing a consistent theory of the early universe with a limiting temperature
  - Its own fine-tuning problem(s)
    - ☐ “A curious tentative view of cosmic history emerges from these considerations… at earlier times ($T \sim T_0$), ρ was, once again, dominated by non-relativistic baryons!”



VOLUME 25, NUMBER 13 PHYSICAL REVIEW LETTERS 28 SEPTEMBER 1970

ULTIMATE TEMPERATURE AND THE EARLY UNIVERSE*

Kerson Huang and Steven Weinberg
Laboratory for Nuclear Science and Physics Department, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 5 August 1970)

The early history of the universe is discussed in the context of an exponentially rising density of particle states.

There are now plausible theoretical models[1] for the thermal history of the universe back to the time of helium synthesis, when the temperature was 0.1 to 1 MeV. Our present theoretical apparatus is really inadequate to deal with much earlier times, say when $T \gtrsim 100$ MeV, and in lieu of any better ideas it is usual to treat the matter of the very early universe as consisting of a number of species of essentially free particles. But how many species?

At one extreme, it might be assumed that the number of particle species stays fixed (perhaps just quarks, antiquarks, leptons, antileptons, photons, and gravitons). In this case, the temperature T will vary with the cosmic scale factor[2] $R(t)$ according to the relation $T \propto 1/R$. The present universe should then contain various relics of the early inferno: There would be a 1°K blackbody gravitational radiation,[3] if TR stayed roughly constant between the times that the gravitons and the photons decoupled from the rest of the universe; also, according to Zeldovich,[4] the leftover quarks should be about as common as gold atoms. The gravitational radiation would not have been seen, but the quarks would have been, unless, of course, quarks do not exist.

At the other extreme, one might assume that the number of species of particles with mass between m and $m+dm$ increases as $m \to \infty$ as fast as possible:

$$N(m)dm \sim Am^{-B}e^{\beta_0 m}dm. \tag{1}$$

If $N(m)$ increased any faster, the partition function would not converge. With the increase (1), the partition function converges only if the temperature[5] is less than $1/\beta_0$. The quantity $T_0 = 1/\beta_0$ is thus a maximum temperature for any system in thermal equilibrium.

Support for this latter sort of model comes from two quite different directions:

(1) The transverse momentum distribution of secondaries in very high energy collisions is observed to be roughly $\exp(-|p_\perp|/160 \text{ MeV})$. Hagedorn[6] interprets this distribution in terms of a statistical model with $T_0 \approx 160$ MeV and $B = \frac{5}{2}$.

(2) If particles fall on families of parallel linearly rising Regge trajectories, their masses take discrete values $m_1, m_2, \cdots$, where

$$\alpha' m_n^2 + \alpha_0 = n. \tag{2}$$

Here $\alpha' \approx 1 \text{ GeV}^{-2}$ is the universal Regge slope and α_0 is a number, of order unity, characterizing the family. The extension of the Veneziano model[7] to multiparticle reactions requires[8] that the number of particle states with mass m_n equals the degeneracy of the eigenvalue n of the operator

$$N = \sum_{\mu=1}^{D} \sum_{k=1}^{\infty} k a_{k\mu}^\dagger a_{k\mu}, \tag{3}$$

where $a_{k\mu}$ and $a_{k\mu}^\dagger$ are an infinite set of annihilation and creation operators. For $n \to \infty$, this number is[8]

$$P_{nD} \approx 2^{-1/2}(D/24)^{(D+1)/4} n^{-(D+3)/4} \times \exp[2\pi(\tfrac{1}{6}Dn)^{1/2}]. \tag{4}$$

Equations (2) and (4) lead to an asymptotic level density of form (1), with

$$\beta_0 = 2\pi(\tfrac{1}{6}D\alpha')^{1/2}, \quad B = \tfrac{1}{2}(D+1). \tag{5}$$

The value of D is not certain—originally Fubini and Veneziano[9] had $D=4$, but Lovelace[10] argues that D is larger, possibly $D=5$.

Table I summarizes the values of T_0 and B for these various models. Lovelace[10] has emphasized the striking agreement between the values of T_0 derived in such different ways. We now see that

Table I. Possible values of the parameters in the level-density formula (1).

Model	$T_0 = 1/\beta_0$	B
(1) Hagedorn[a]	~160 MeV	$\frac{5}{2}$
(2) Veneziano[b] (with $\alpha' = 1 \text{ GeV}^{-2}$)		
$D=4$	195 MeV	$\frac{5}{2}$
$D=5$	174 MeV	3
$D=6$	159 MeV	$\frac{7}{2}$
$D=7$	147 MeV	4

[a] Ref. 6.
[b] Ref. 7.

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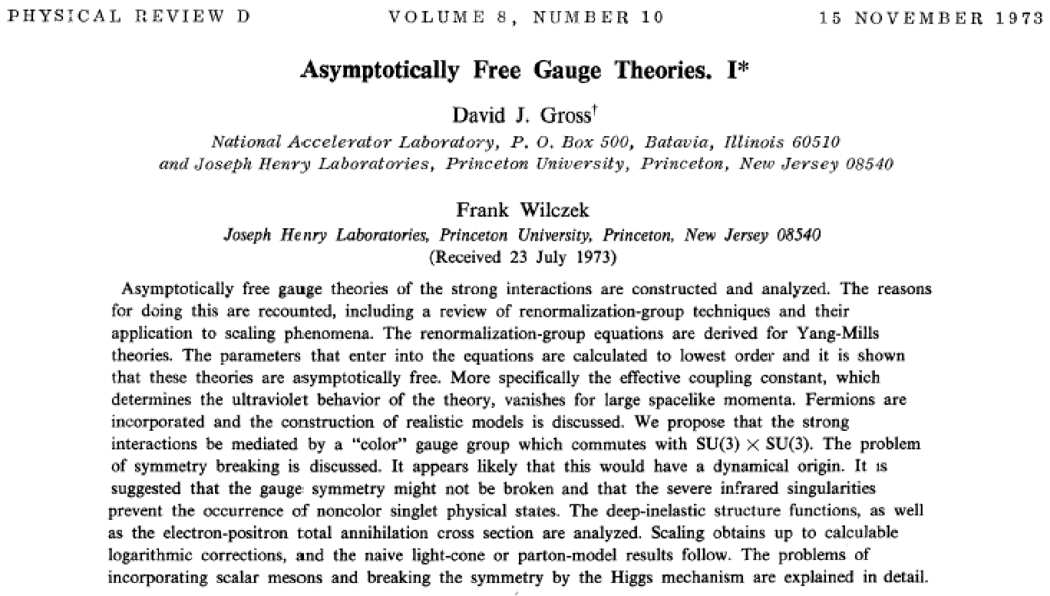
- *“…a veil, obscuring our view of the very beginning.”*

Steven Weinberg, The First Three Minutes (1977)

1973

- 1973 = Birth of QCD
- Gross, Politzer, Wilczek



PHYSICAL REVIEW D VOLUME 8, NUMBER 10 15 NOVEMBER 1973

Asymptotically Free Gauge Theories. I*

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and Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540

Frank Wilczek
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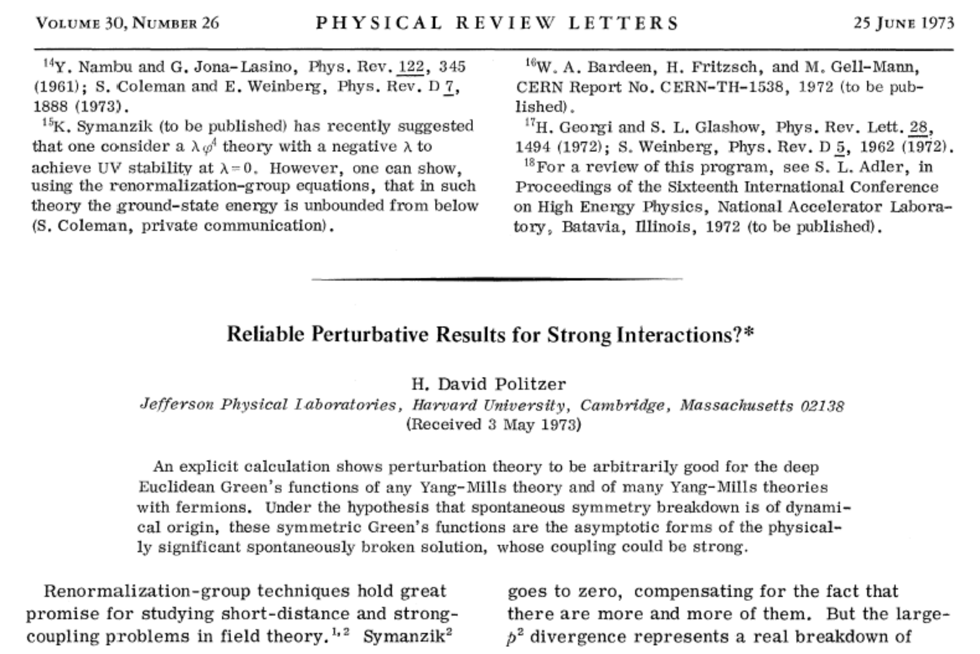
(Received 23 July 1973)

Asymptotically free gauge theories of the strong interactions are constructed and analyzed. The reasons for doing this are recounted, including a review of renormalization-group techniques and their application to scaling phenomena. The renormalization-group equations are derived for Yang-Mills theories. The parameters that enter into the equations are calculated to lowest order and it is shown that these theories are asymptotically free. More specifically the effective coupling constant, which determines the ultraviolet behavior of the theory, vanishes for large spacelike momenta. Fermions are incorporated and the construction of realistic models is discussed. We propose that the strong interactions be mediated by a "color" gauge group which commutes with SU(3) × SU(3). The problem of symmetry breaking is discussed. It appears likely that this would have a dynamical origin. It is suggested that the gauge symmetry might not be broken and that the severe infrared singularities prevent the occurrence of noncolor singlet physical states. The deep-inelastic structure functions, as well as the electron-positron total annihilation cross section are analyzed. Scaling obtains up to calculable logarithmic corrections, and the naive light-cone or parton-model results follow. The problems of incorporating scalar mesons and breaking the symmetry by the Higgs mechanism are explained in detail.



VOLUME 30, NUMBER 26 PHYSICAL REVIEW LETTERS 25 JUNE 1973

[14] Y. Nambu and G. Jona-Lasino, Phys. Rev. 122, 345 (1961); S. Coleman and E. Weinberg, Phys. Rev. D 7, 1888 (1973).

[15] K. Symanzik (to be published) has recently suggested that one consider a $\lambda\varphi^4$ theory with a negative λ to achieve UV stability at $\lambda=0$. However, one can show, using the renormalization-group equations, that in such theory the ground-state energy is unbounded from below (S. Coleman, private communication).

[16] W. A. Bardeen, H. Fritzsch, and M. Gell-Mann, CERN Report No. CERN-TH-1538, 1972 (to be published).

[17] H. Georgi and S. L. Glashow, Phys. Rev. Lett. 28, 1494 (1972); S. Weinberg, Phys. Rev. D 5, 1962 (1972).

[18] For a review of this program, see S. L. Adler, in Proceedings of the Sixteenth International Conference on High Energy Physics, National Accelerator Laboratory, Batavia, Illinois, 1972 (to be published).

Reliable Perturbative Results for Strong Interactions?*

H. David Politzer
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(Received 3 May 1973)

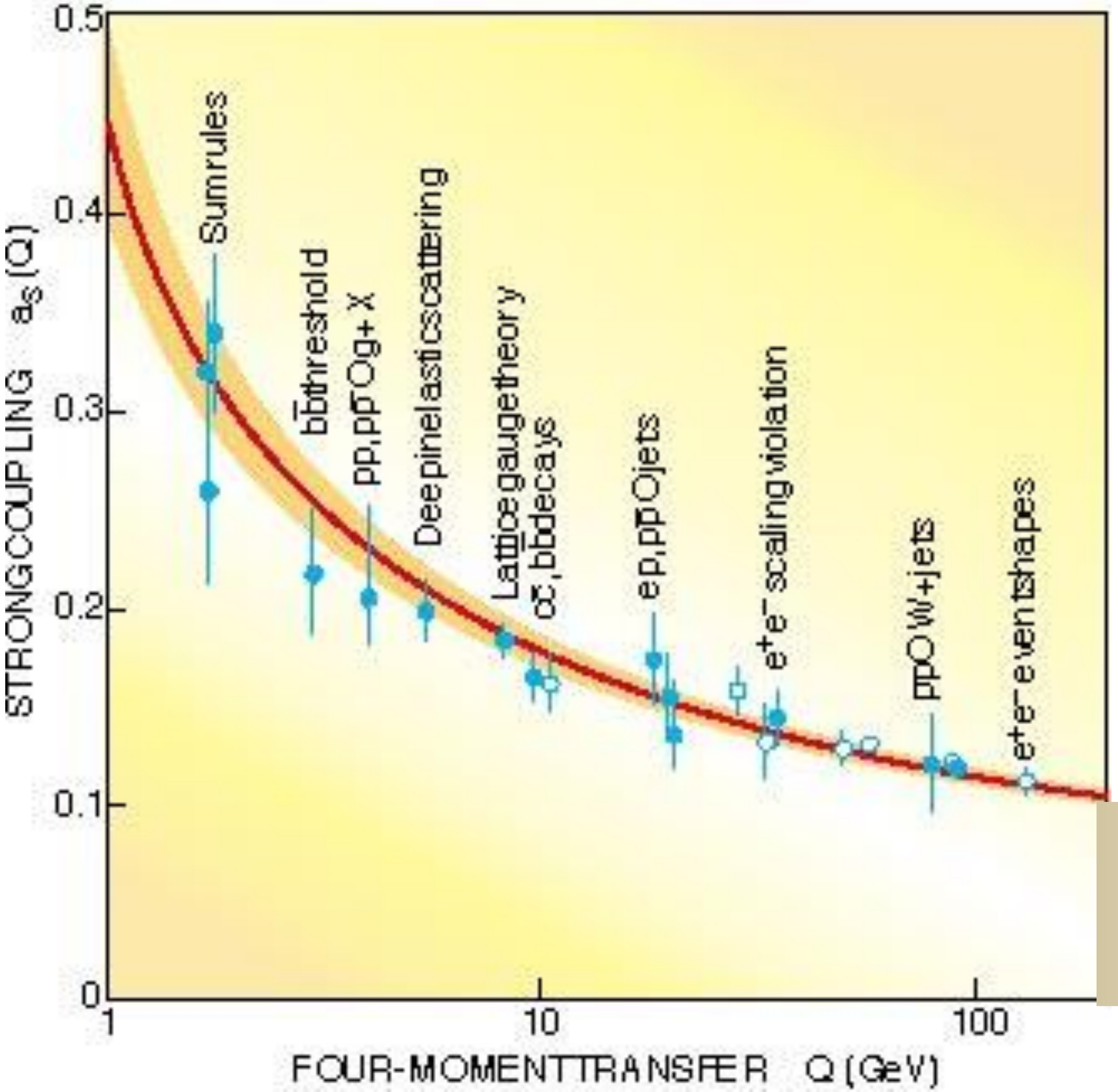
An explicit calculation shows perturbation theory to be arbitrarily good for the deep Euclidean Green's functions of any Yang-Mills theory and of many Yang-Mills theories with fermions. Under the hypothesis that spontaneous symmetry breakdown is of dynamical origin, these symmetric Green's functions are the asymptotic forms of the physically significant spontaneously broken solution, whose coupling could be strong.

Renormalization-group techniques hold great promise for studying short-distance and strong-coupling problems in field theory.[1,2] Symanzik[2]

goes to zero, compensating for the fact that there are more and more of them. But the large-p^2 divergence represents a real breakdown of

QCD’s Running Coupling “Constant”

Strong coupling ➔
non-perturbative
confinement
“infrared slavery”



🡐 Weak coupling
perturbative
“asymptotic freedom”

QCD’s Running Coupling “Constant”

Strong coupling ➔
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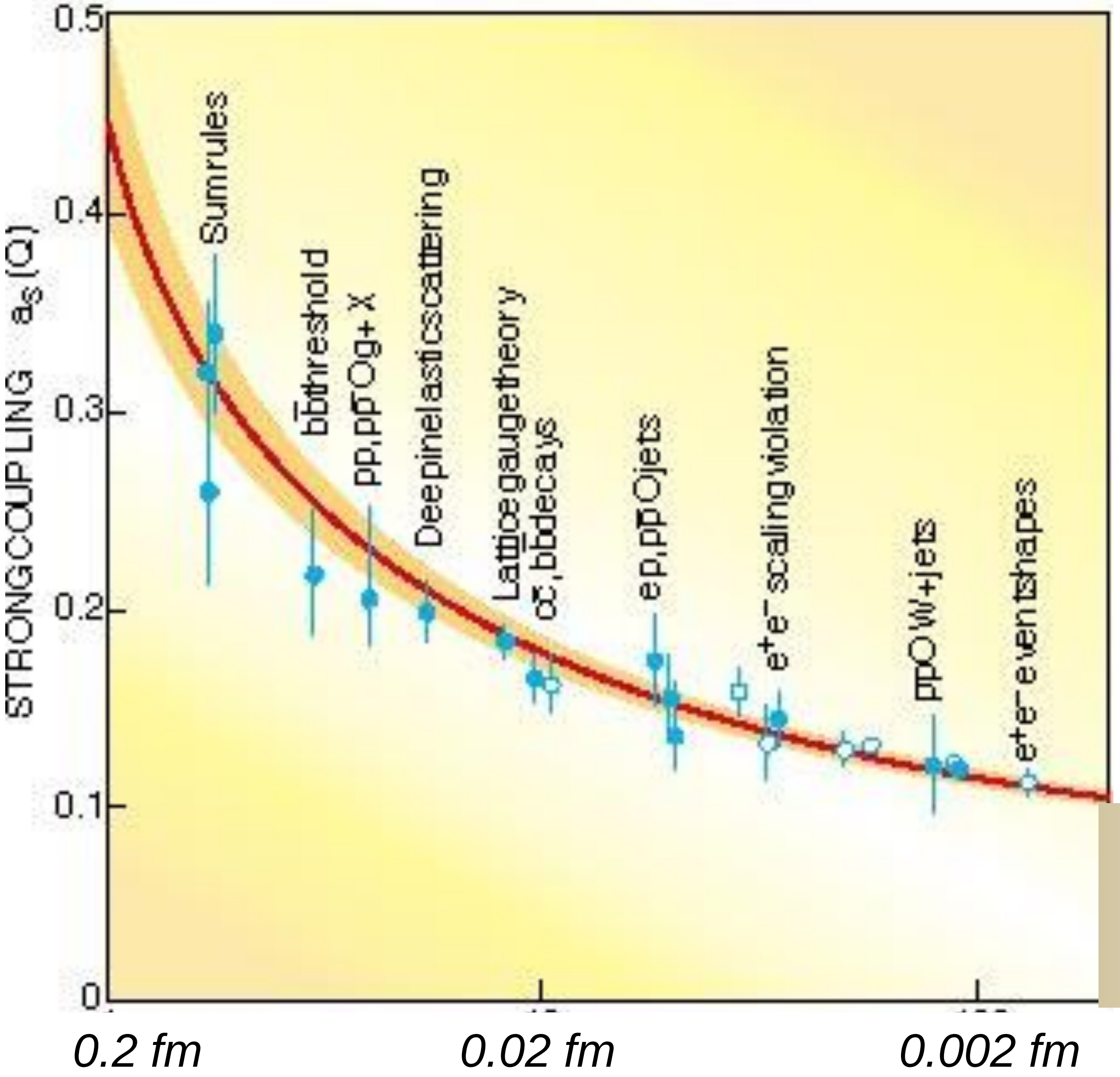


🡐 Weak coupling
perturbative
“asymptotic freedom”

Q. What Sets the Scale ?

- Simple answer – Quantum Mechanics:
  - $\hbar$ c = 200 Mev-fm = 0.2 GeV-fm
  - So $\hbar$ /(1 fm) = 200 MeV ~ light hadron masses

- Deeper answer – vacuum condensate that establishes confinement scale Λ ~ 200 MeV

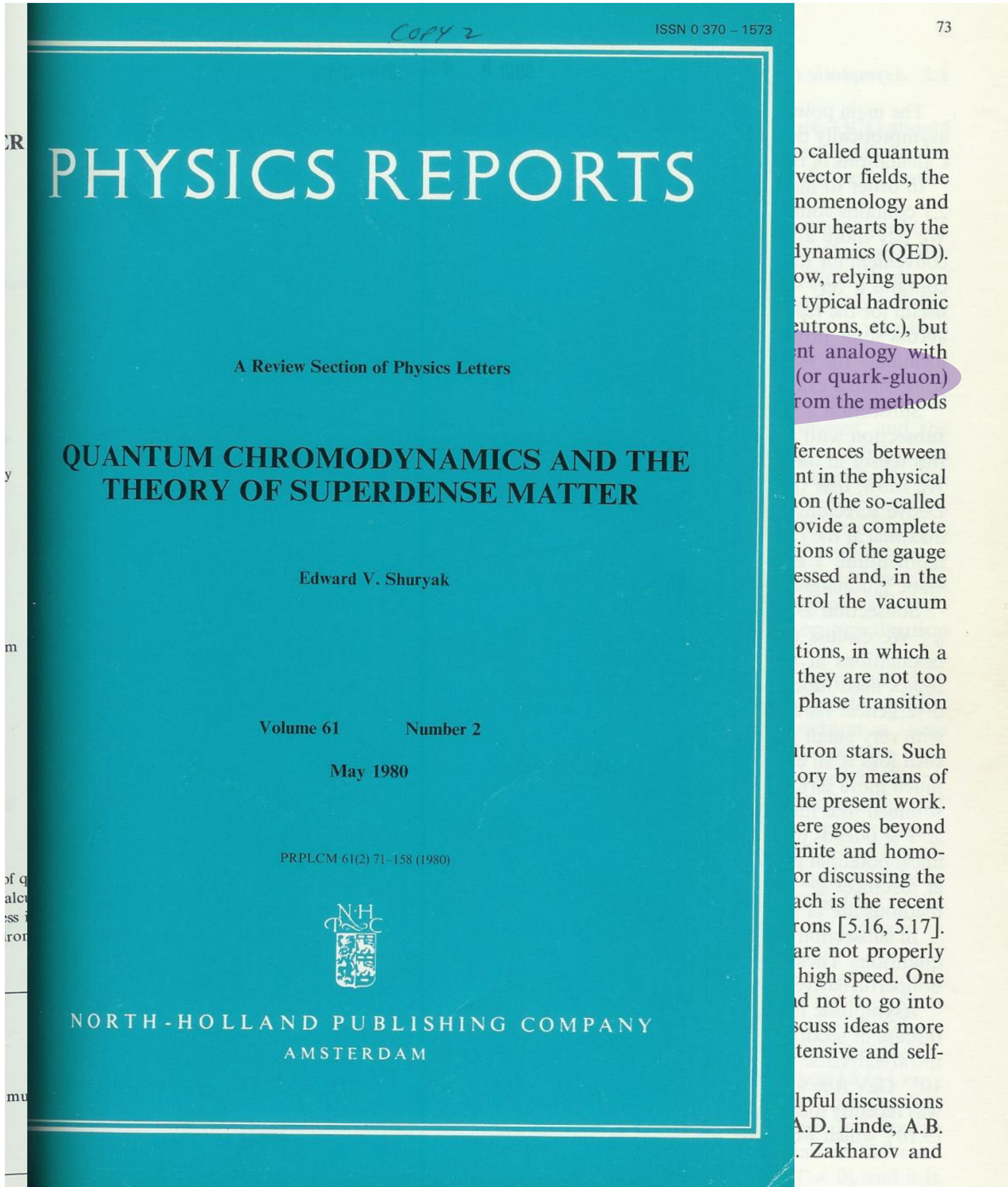
$$\alpha_S(Q^2) = \frac{12\pi}{(33-2N_F)\log(\frac{Q^2}{\Lambda^2})} \sim \frac{1}{\log(\frac{Q^2}{\Lambda^2})} \qquad \Lambda \approx \frac{\hbar c}{r_o} \approx 0.2\,GeV$$

Naming It

- It was quickly realized that asymptotic freedom would lead to deconfined quarks and gluons at high densities and/or pressures:
  - *Superdense Matter: Neutrons or Asymptotically Free Quarks?,*
    J.C. Collins and M.J. Perry, Phys. Rev. Lett. 34, 1353 (1975).
    - “We suggest that matter at such high densities is a quark soup.”
  - *Hot Quark Soup,*
    L. Susskind, submitted to Phys. Rev. D (1978…)
    available as http://slac.stanford.edu/pubs/slacpubs/2000/slac-pub-2070.pdf .
    - “At high temperatures a transition to a plasma-like phase occurs.”

Naming It

- Shuryak publishes first “review” of thermal QCD- and coins a phrase:



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PHYSICS REPORTS

A Review Section of Physics Letters

QUANTUM CHROMODYNAMICS AND THE THEORY OF SUPERDENSE MATTER

Edward V. Shuryak

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AMSTERDAM

73

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vector fields, the
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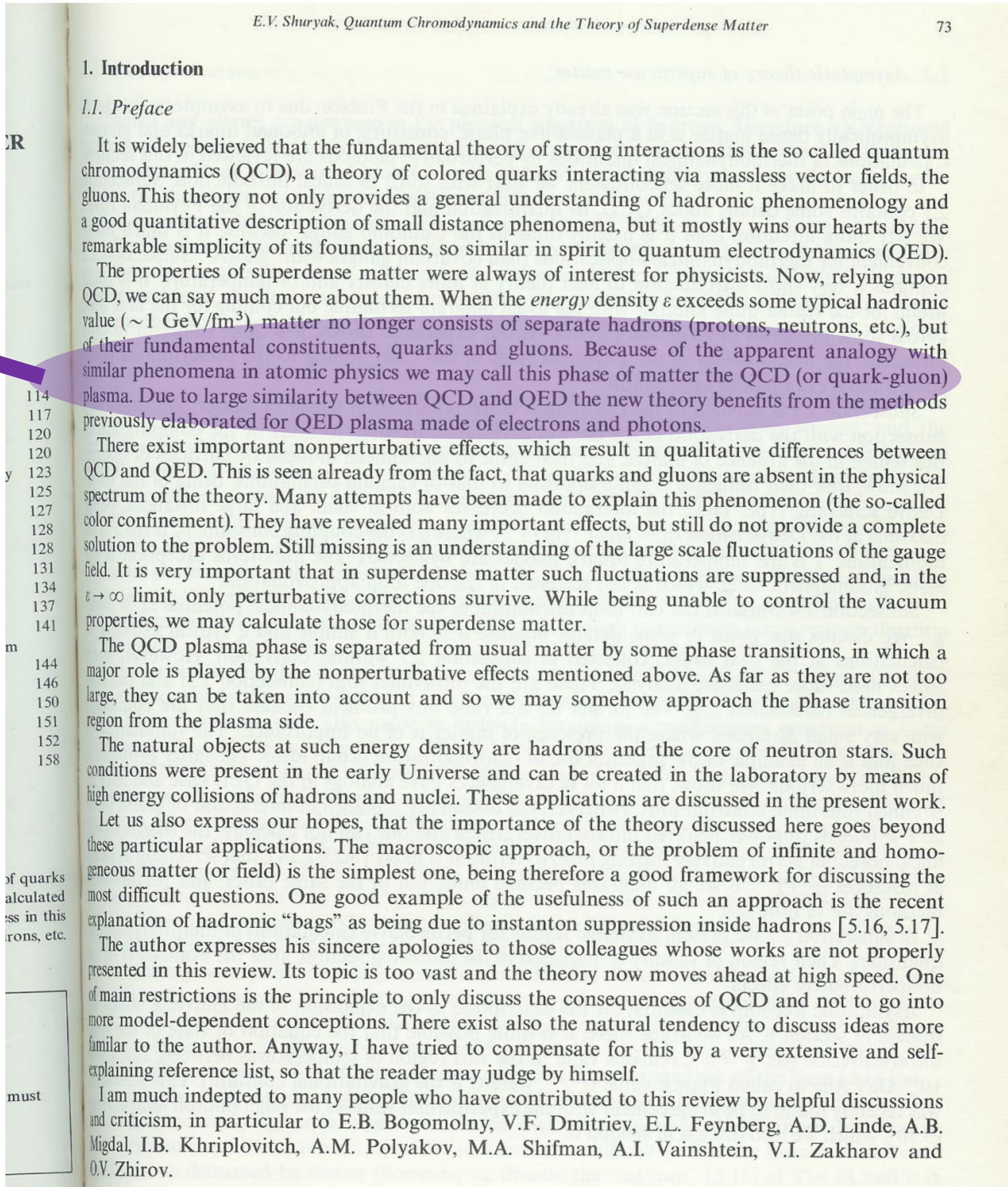
Naming It

- Shuryak publishes first “review” of thermal QCD- and coins a phrase:

  > “Because of the apparent analogy with similar phenomena in atomic physics, we may call this phase of matter the QCD (or quark-gluon) plasma.”

QGP



E.V. Shuryak, *Quantum Chromodynamics and the Theory of Superdense Matter* 73

1. Introduction

1.1. Preface

It is widely believed that the fundamental theory of strong interactions is the so called quantum chromodynamics (QCD), a theory of colored quarks interacting via massless vector fields, the gluons. This theory not only provides a general understanding of hadronic phenomenology and a good quantitative description of small distance phenomena, but it mostly wins our hearts by the remarkable simplicity of its foundations, so similar in spirit to quantum electrodynamics (QED).

The properties of superdense matter were always of interest for physicists. Now, relying upon QCD, we can say much more about them. When the *energy* density ε exceeds some typical hadronic value (~ 1 GeV/fm^3), matter no longer consists of separate hadrons (protons, neutrons, etc.), but of their fundamental constituents, quarks and gluons. Because of the apparent analogy with similar phenomena in atomic physics we may call this phase of matter the QCD (or quark-gluon) plasma. Due to large similarity between QCD and QED the new theory benefits from the methods previously elaborated for QED plasma made of electrons and photons.

There exist important nonperturbative effects, which result in qualitative differences between QCD and QED. This is seen already from the fact, that quarks and gluons are absent in the physical spectrum of the theory. Many attempts have been made to explain this phenomenon (the so-called color confinement). They have revealed many important effects, but still do not provide a complete solution to the problem. Still missing is an understanding of the large scale fluctuations of the gauge field. It is very important that in superdense matter such fluctuations are suppressed and, in the $\varepsilon \to \infty$ limit, only perturbative corrections survive. While being unable to control the vacuum properties, we may calculate those for superdense matter.

The QCD plasma phase is separated from usual matter by some phase transitions, in which a major role is played by the nonperturbative effects mentioned above. As far as they are not too large, they can be taken into account and so we may somehow approach the phase transition region from the plasma side.

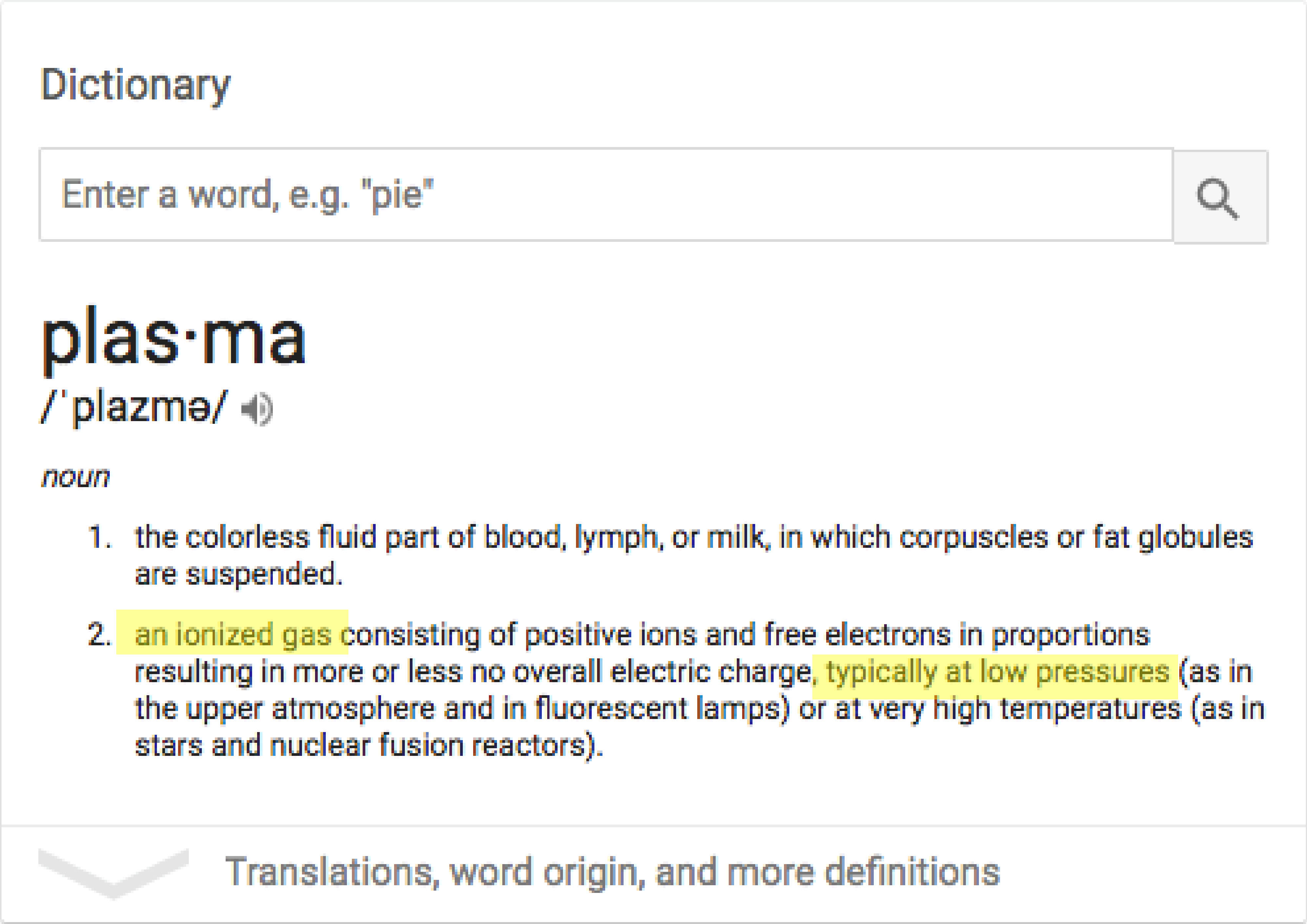
The natural objects at such energy density are hadrons and the core of neutron stars. Such conditions were present in the early Universe and can be created in the laboratory by means of high energy collisions of hadrons and nuclei. These applications are discussed in the present work.

Let us also express our hopes, that the importance of the theory discussed here goes beyond these particular applications. The macroscopic approach, or the problem of infinite and homogeneous matter (or field) is the simplest one, being therefore a good framework for discussing the most difficult questions. One good example of the usefulness of such an approach is the recent explanation of hadronic “bags” as being due to instanton suppression inside hadrons [5.16, 5.17].

The author expresses his sincere apologies to those colleagues whose works are not properly presented in this review. Its topic is too vast and the theory now moves ahead at high speed. One of main restrictions is the principle to only discuss the consequences of QCD and not to go into more model-dependent conceptions. There exist also the natural tendency to discuss ideas more familar to the author. Anyway, I have tried to compensate for this by a very extensive and self-explaining reference list, so that the reader may judge by himself.

I am much indepted to many people who have contributed to this review by helpful discussions and criticism, in particular to E.B. Bogomolny, V.F. Dmitriev, E.L. Feynberg, A.D. Linde, A.B. Migdal, I.B. Khriplovitch, A.M. Polyakov, M.A. Shifman, A.I. Vainshtein, V.I. Zakharov and O.V. Zhirov.

“Plasma” Leads To Prejudice



Dictionary

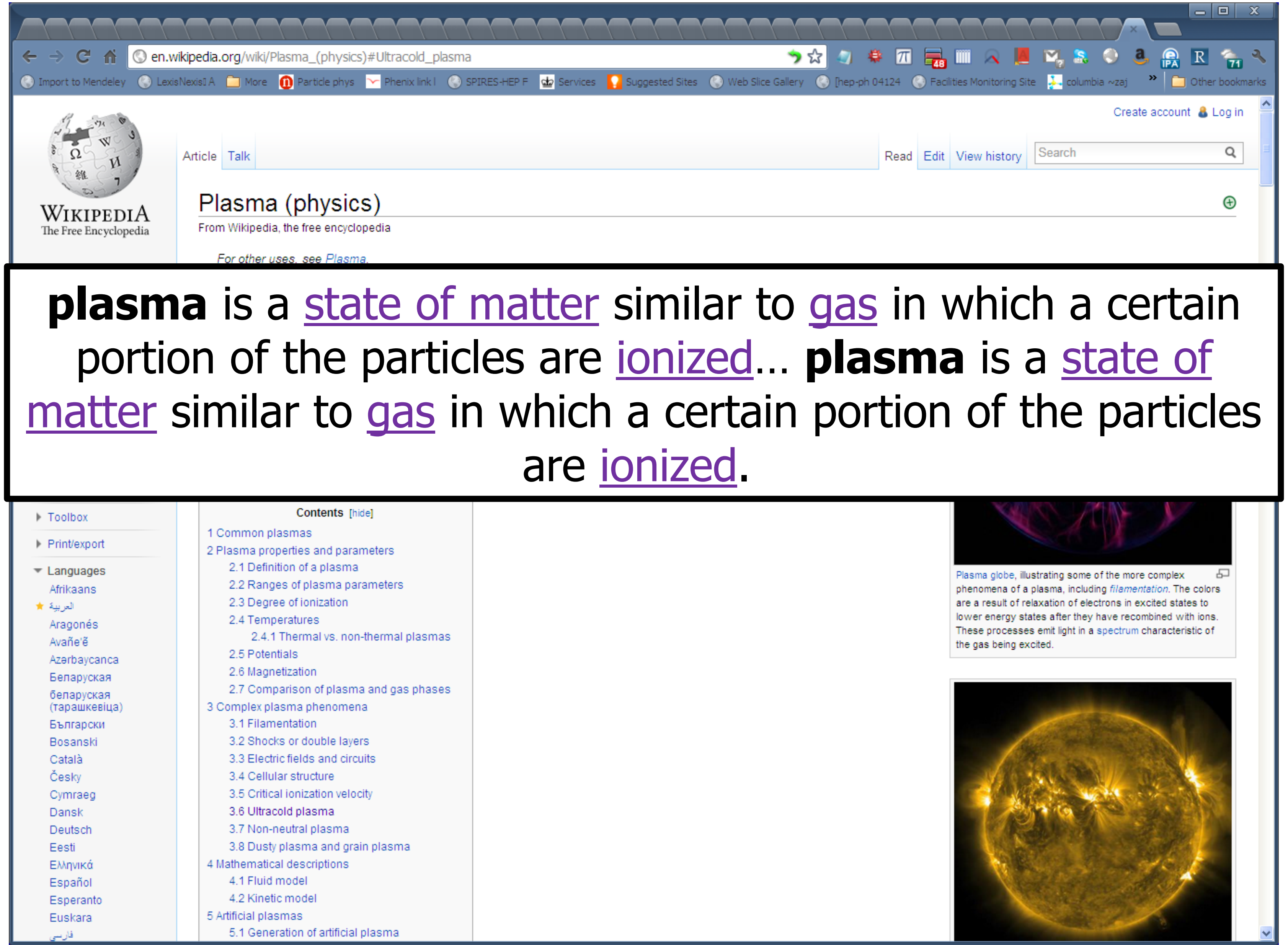
plas·ma

/ˈplazmə/

noun

1. the colorless fluid part of blood, lymph, or milk, in which corpuscles or fat globules are suspended.
2. an ionized gas consisting of positive ions and free electrons in proportions resulting in more or less no overall electric charge, typically at low pressures (as in the upper atmosphere and in fluorescent lamps) or at very high temperatures (as in stars and nuclear fusion reactors).

“Plasma” Leads To Prejudice



en.wikipedia.org/wiki/Plasma_(physics)#Ultracold_plasma

Plasma (physics)

From Wikipedia, the free encyclopedia

For other uses, see Plasma.

plasma is a state of matter similar to gas in which a certain portion of the particles are ionized… **plasma** is a state of matter similar to gas in which a certain portion of the particles are ionized.

Contents [hide]

Plasma globe, illustrating some of the more complex phenomena of a plasma, including *filamentation*. The colors are a result of relaxation of electrons in excited states to lower energy states after they have recombined with ions. These processes emit light in a spectrum characteristic of the gas being excited.

Q. What Temperature Is Required for Deconfinement?

I.e., How to compute location of transition from

A gas of hadrons at temperature T

to

A gas of deconfined quarks and gluons at T ?

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A1: The only scale in QCD is $\Lambda_{QCD} \rightarrow T_c \sim \Lambda_{QCD}$

Q. What Temperature Is Required for Deconfinement?

I.e., How to compute location of transition from

A gas of hadrons at temperature T

to

A gas of deconfined quarks and gluons at T ?

A1: The only scale in QCD is $\Lambda_{QCD} \rightarrow$ Tc ~ Λ_{QCD}

A2: Thermodynamics of phase equilibria

- Compute the pressure P in each phase
- The phase with the higher pressure wins

- (See back-up for refresher on phase equilibria):

Needed Statistical Mechanics

- For massless non-interacting *bosons* :
  - Number density $$n(T)=\frac{1.202}{\pi^2}T^3\approx\left(\frac{T}{2}\right)^3$$
  - Energy density $$\varepsilon(T)=\frac{\pi^2}{30}T^4$$
  - Pressure $$P(T)=\frac{1}{3}\varepsilon(T)=\frac{\pi^2}{90}T^4$$

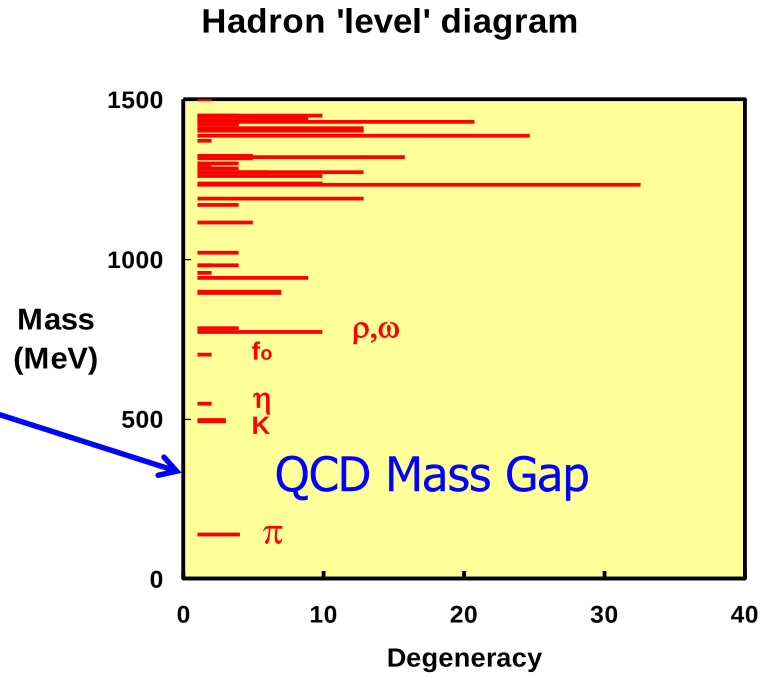
Exercise: Convince yourself this handy pocket formal is correct (in natural units). Use it to compute the density of microwave background photons (T=2.7 K)

☞ *Per degree of freedom*

⇒Next steps: Counting degrees of freedom

Counting Degrees of Freedom

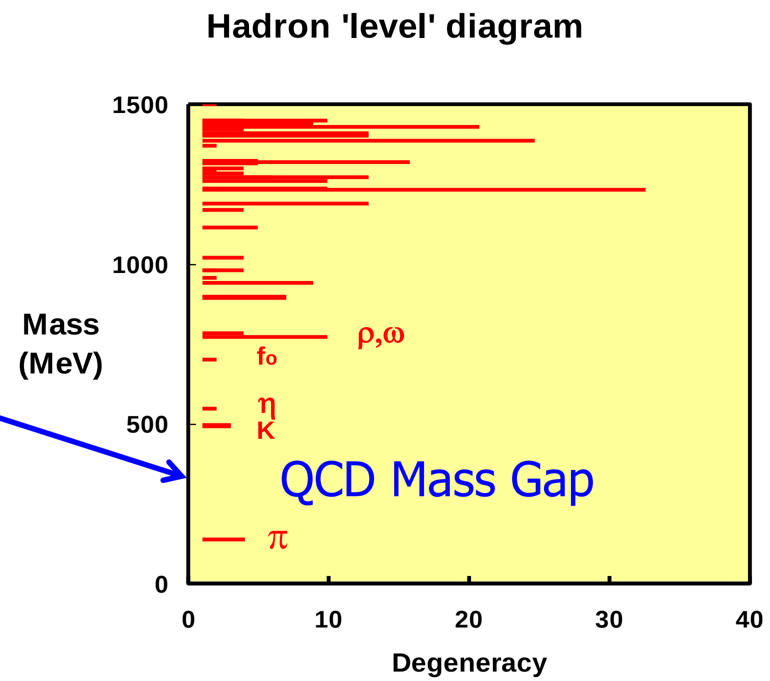
- Hadronic phase
  - Assume relevant $T < 500$ MeV
  - ndf = 3 (π^-, π^0, π^+)

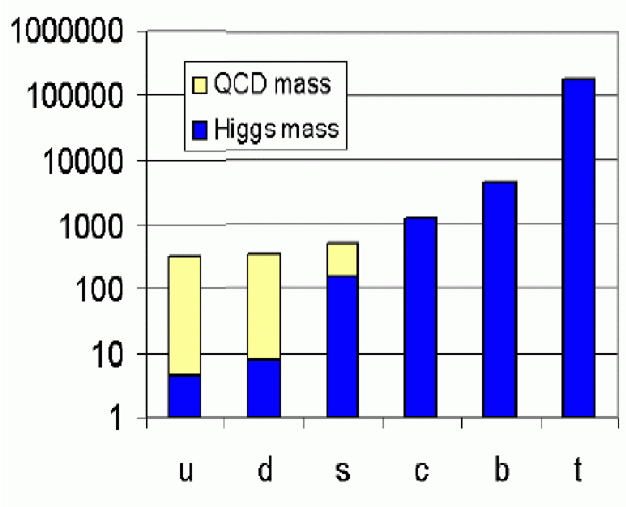


Counting Degrees of Freedom

- Hadronic phase
  - Assume relevant T < 500 MeV
  - ndf = 3 (π^-, π^0, π^+)
- Quark-gluon phase
  - Gluons: ndf = $2_s \times 8_c$ = 16
  - Quarks: ndf = $(7/8) \times 2_s \times 2_f \times 2_a \times 3_c$ = $\underline{21}$
  - Total ndf = 37

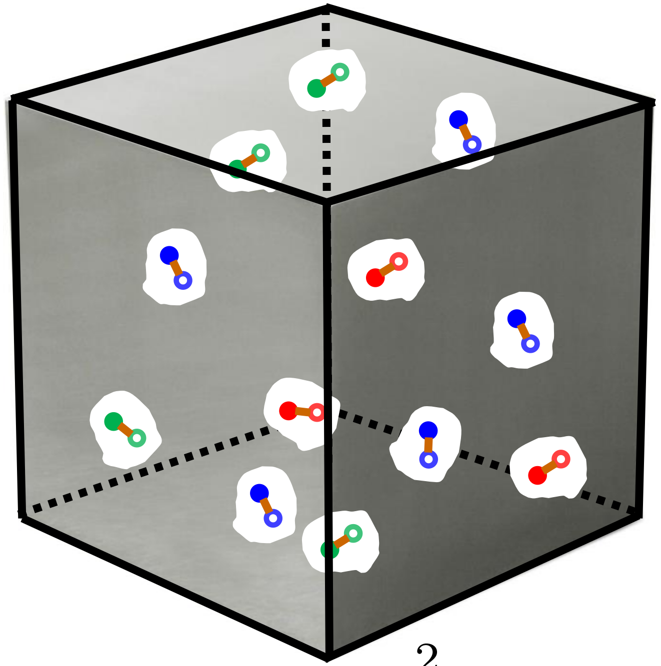
    47.5 (3_f)
- Bottom line: $ndf_{QGP} \sim 10 \times ndf_{Hadrons}$



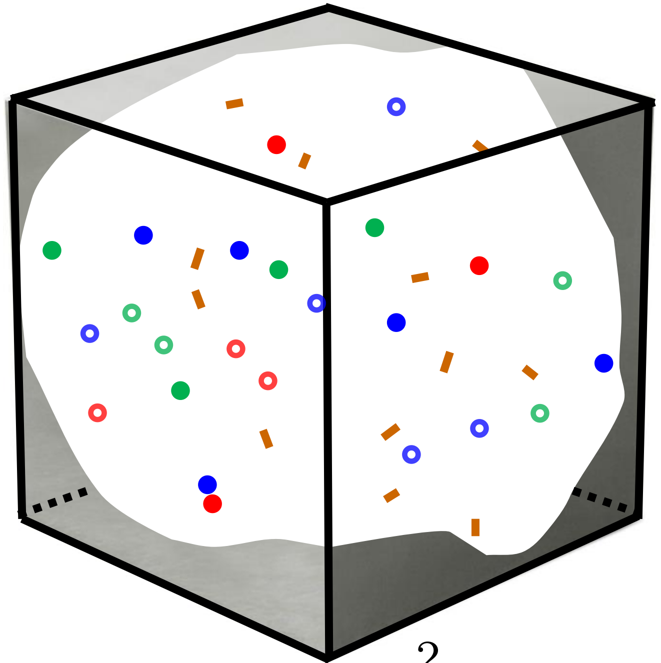


It's a High Pressure Game

- Compare pressure of pion gas to that of a bag of q's and g's



$$P_\pi(T) = 3\,\frac{\pi^2}{90}\,T^4$$

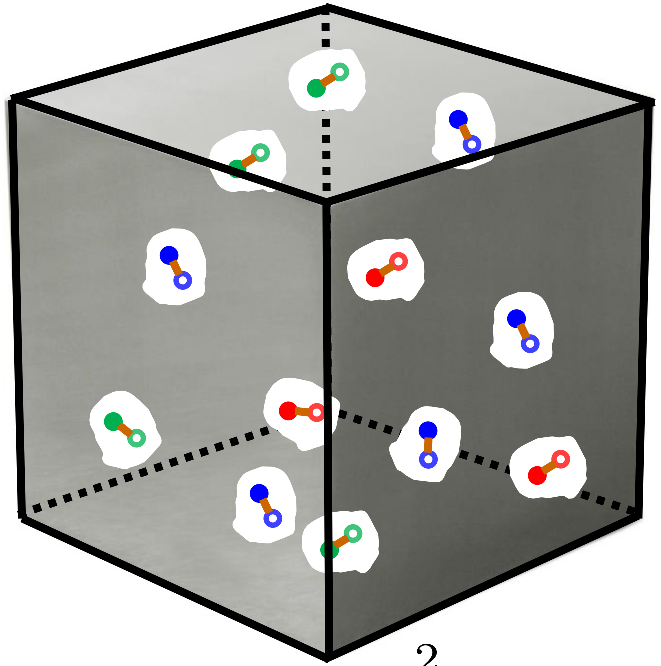


$$P_{QGP}(T) = g\,\frac{\pi^2}{90}\,T^4 - B$$

$$g \sim 37\text{--}47.5$$

It’s a High Pressure Game

- Compare pressure of pion gas to that of a bag of q’s and g’s



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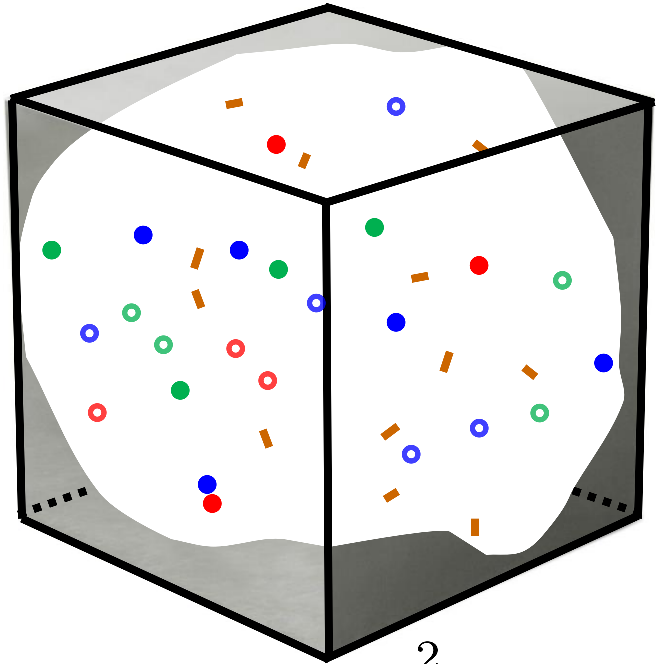


$$P_{QGP}(T) = g\,\frac{\pi^2}{90}\,T^4 - B$$

$$g \sim 37\text{–}47.5$$

- Nature selects system with higher pressure…

Simple Model Result

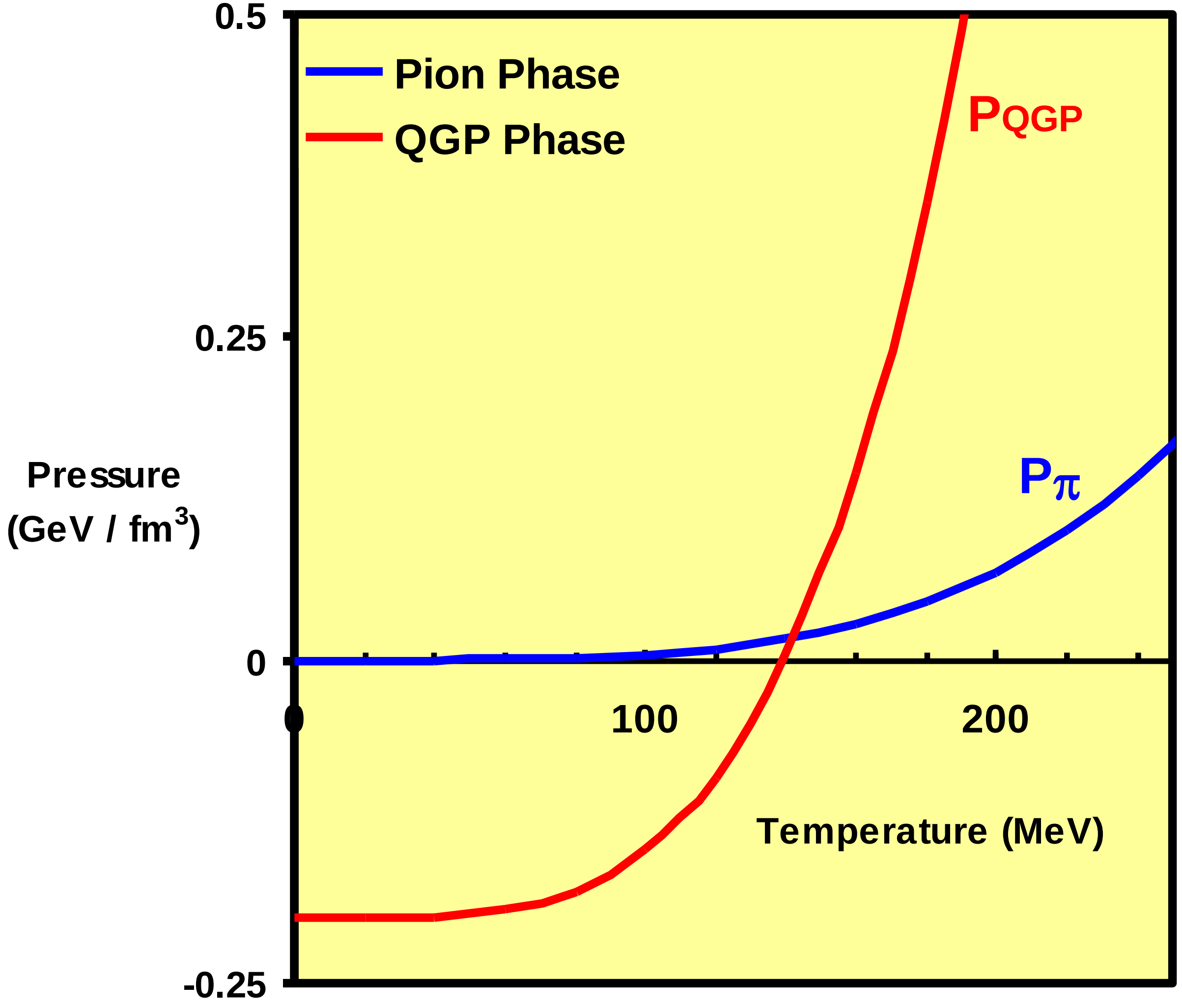
- For $T > (0.68\text{-}0.72)\ B^{1/4}$,

$$P_{QGP} > P_{\pi}$$

➔ a phase transition
to deconfined state

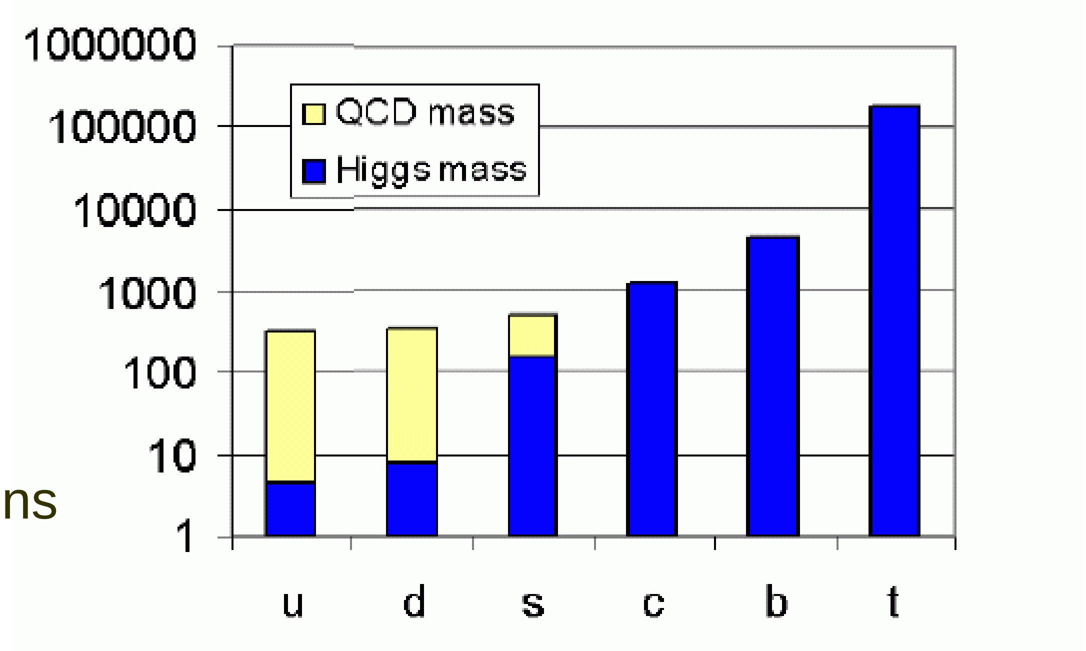
- For $B^{1/4} \sim 200$ MeV ,
$T_C \sim 140$ MeV ,

with modest latent heat = 4B



Damage Control

- In reality, this is not such a great estimate:
  - Hadron side
    - ☐ Pions hardly massless relative to 140 MeV ☺
    - ☐ Ignores exponential growth at higher T
    - ☐ Ignores (strong!) interactions
  - QGP side
    - ☐ Strange quark neither massless nor massive
    - ☐ Bag constant stand-in for QCD vacuum fluctuations
    - ☐ Ignores (strong!) interactions



- To do better
  - Program some of this in *Mathematica*

Damage Control

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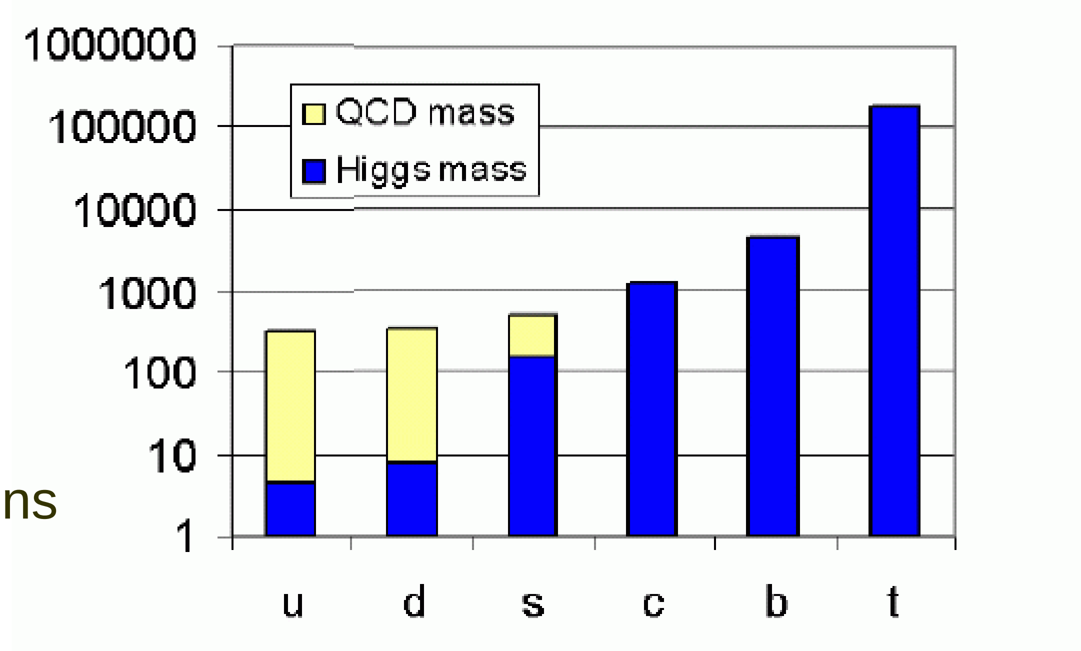
    - ☐ Bag constant stand-in for QCD vacuum fluctuations
    - ☐ Ignores (strong!) interactions



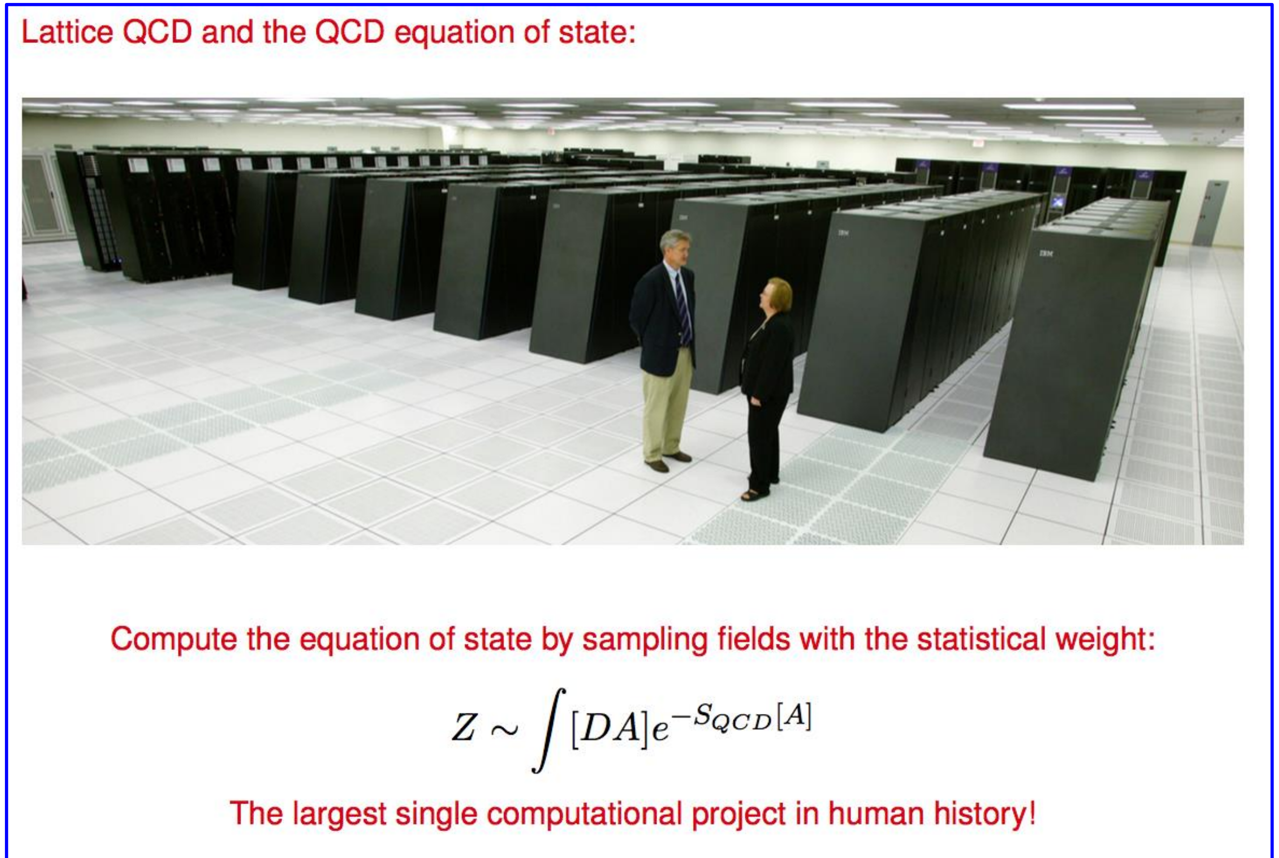
- To do better
  - Program some of this in *Mathematica*

    or
  - Calculate ~ 1 TeraFlops x 100 days ~ 10^{19} Flops

Lattice QCD

- "Solve" the theory on a discrete space-time lattice
- Requires massive (parallel) computing

Below slide from Derek Teaney

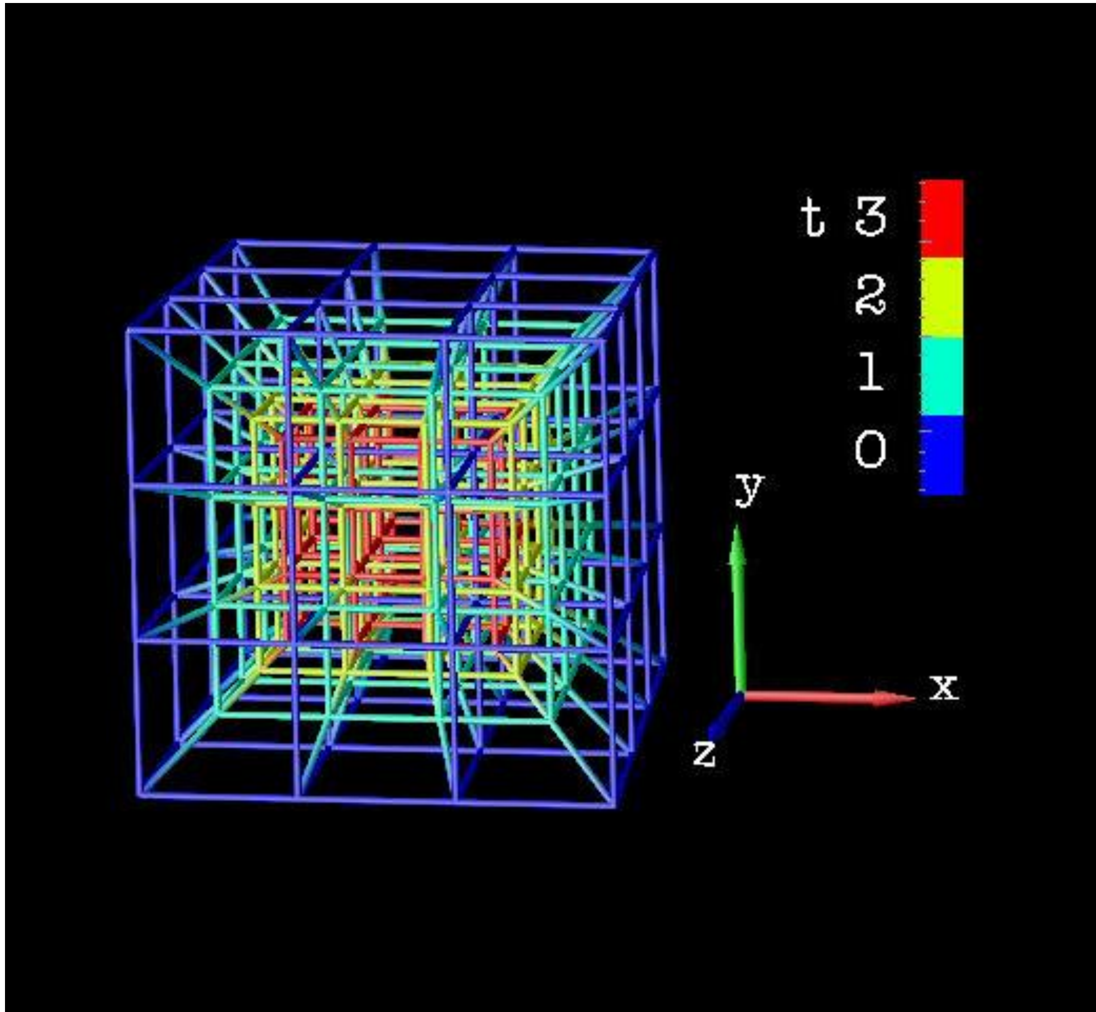


Lattice QCD and the QCD equation of state:

Compute the equation of state by sampling fields with the statistical weight:

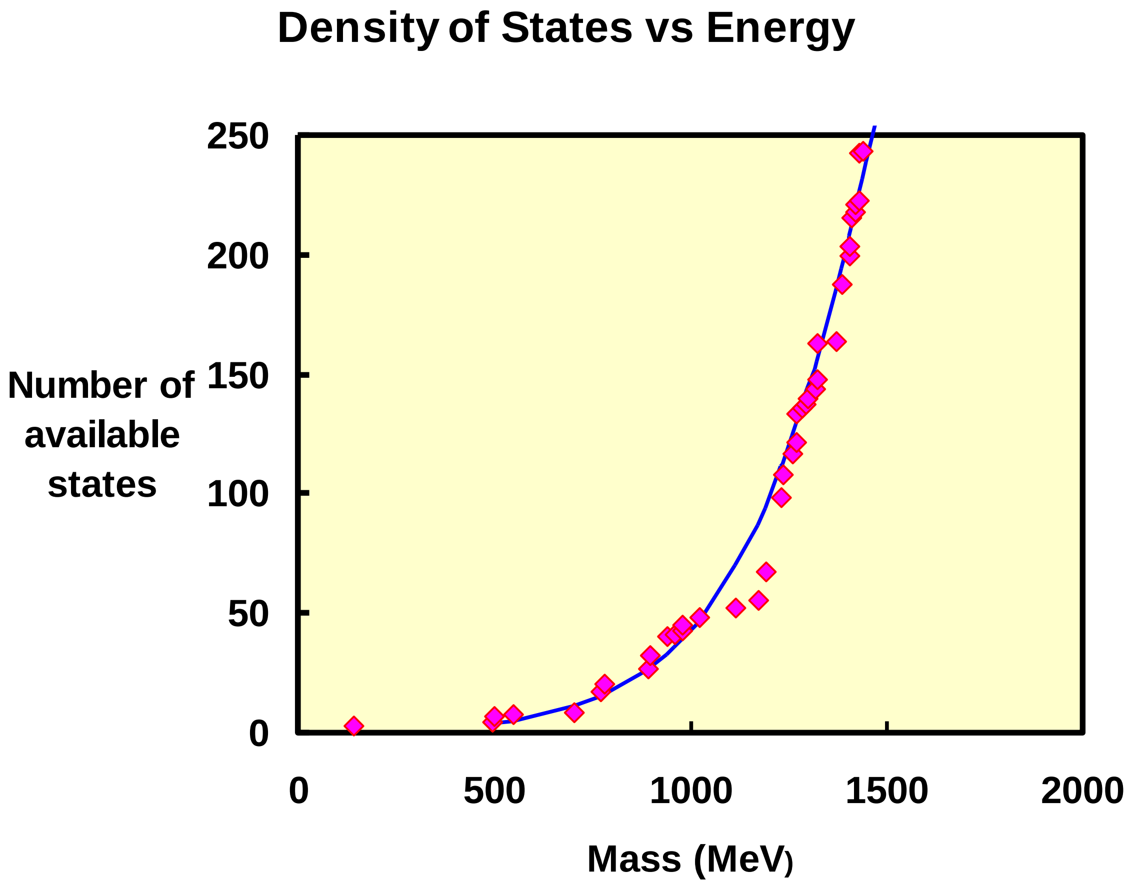
$$Z \sim \int [DA] e^{-S_{QCD}[A]}$$

The largest single computational project in human history!



Selected Lattice QCD Results (I)

- The density of states really is exponential:



(hand-compilation by WAZ c. 1995)

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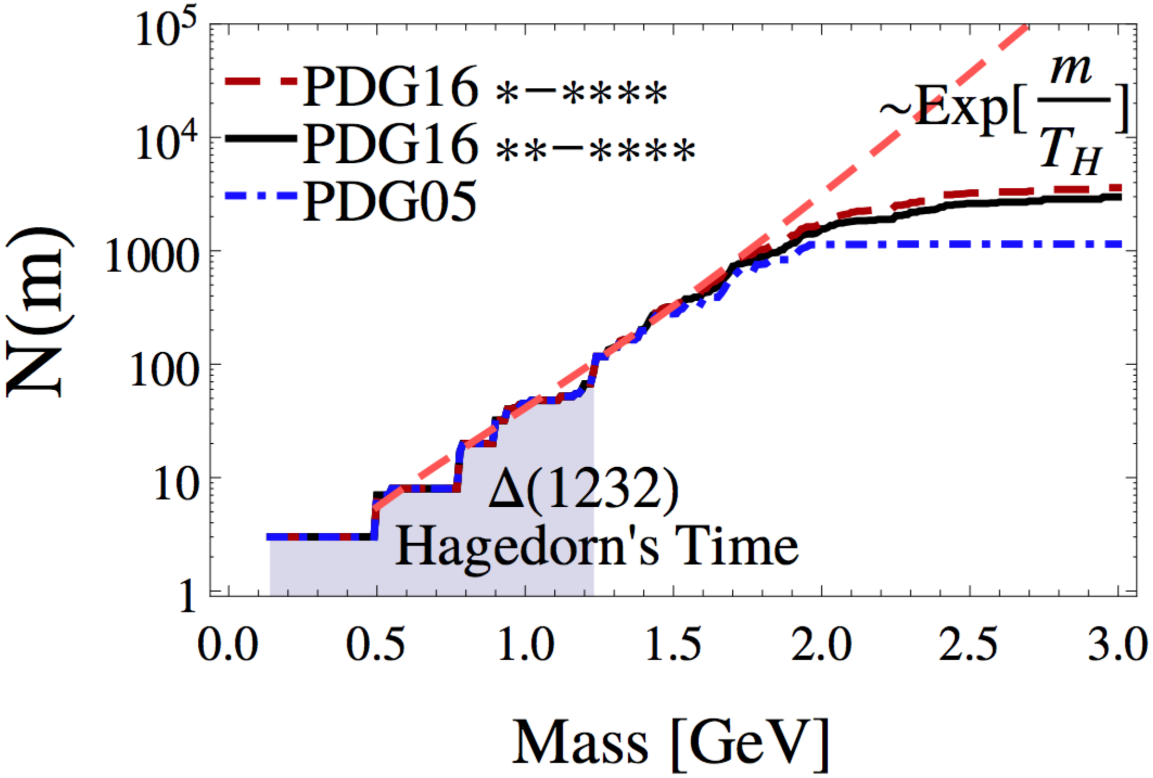
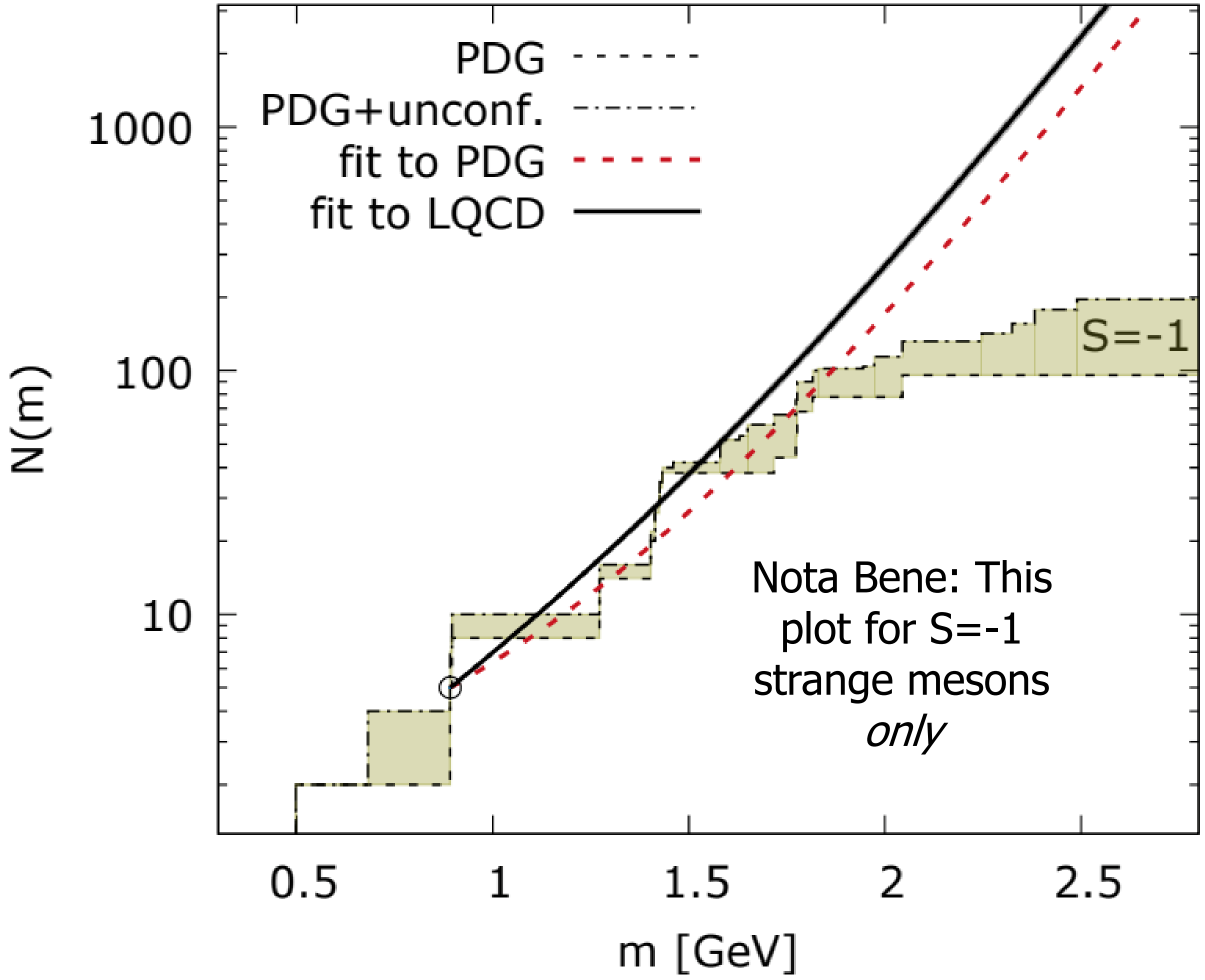


J. Noronha-Hostler, arXiv:1612.07765

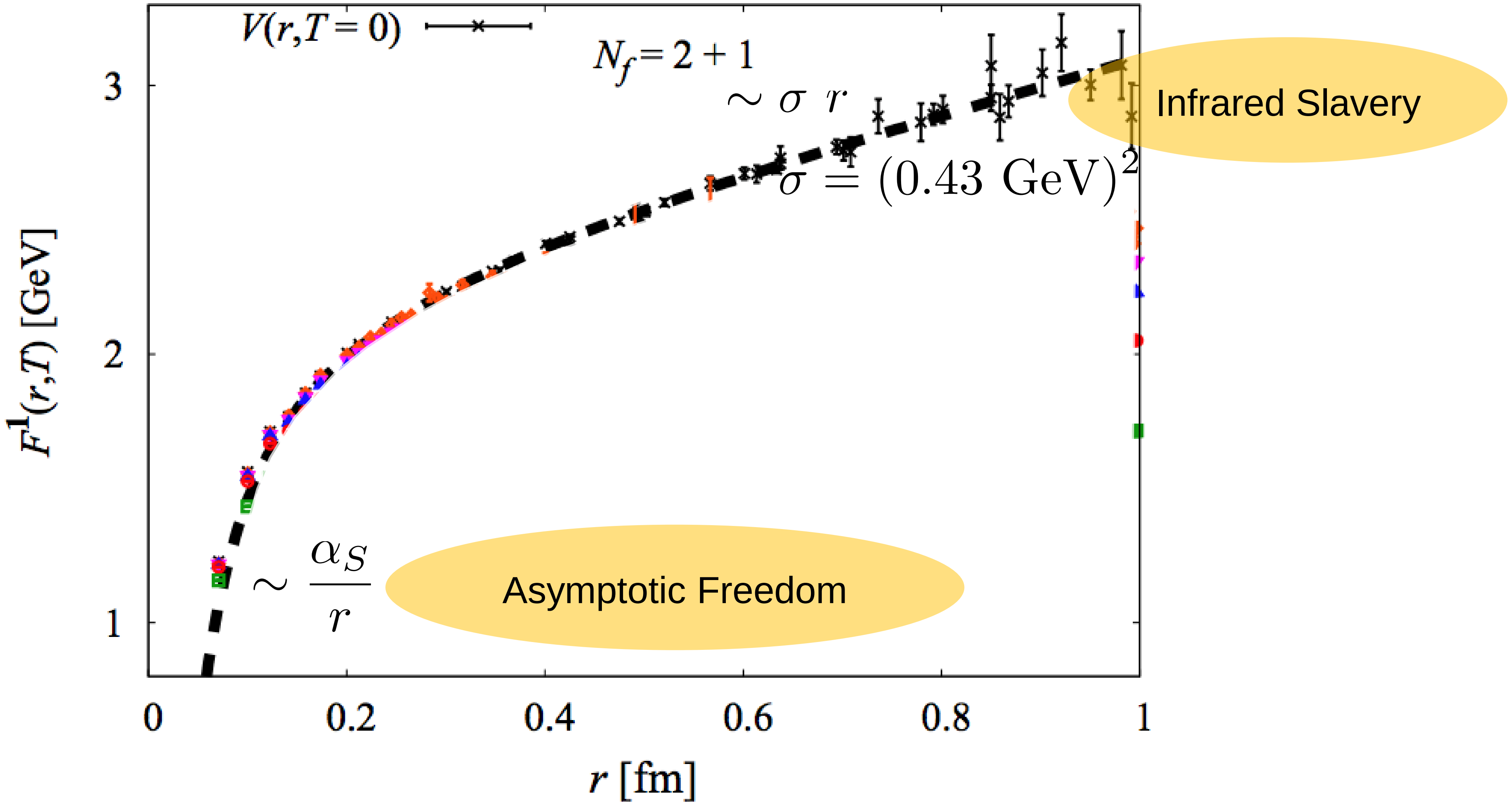


Nota Bene: This plot for S=-1 strange mesons *only*

P.M. Lo *et al.*, arXiv: 1507.06398

Selected Lattice QCD Results (II)

- The quark-antiquark potential is strongly modified at $T > T_H$.



Y. Maezawa et al. arXiv:1112.2756

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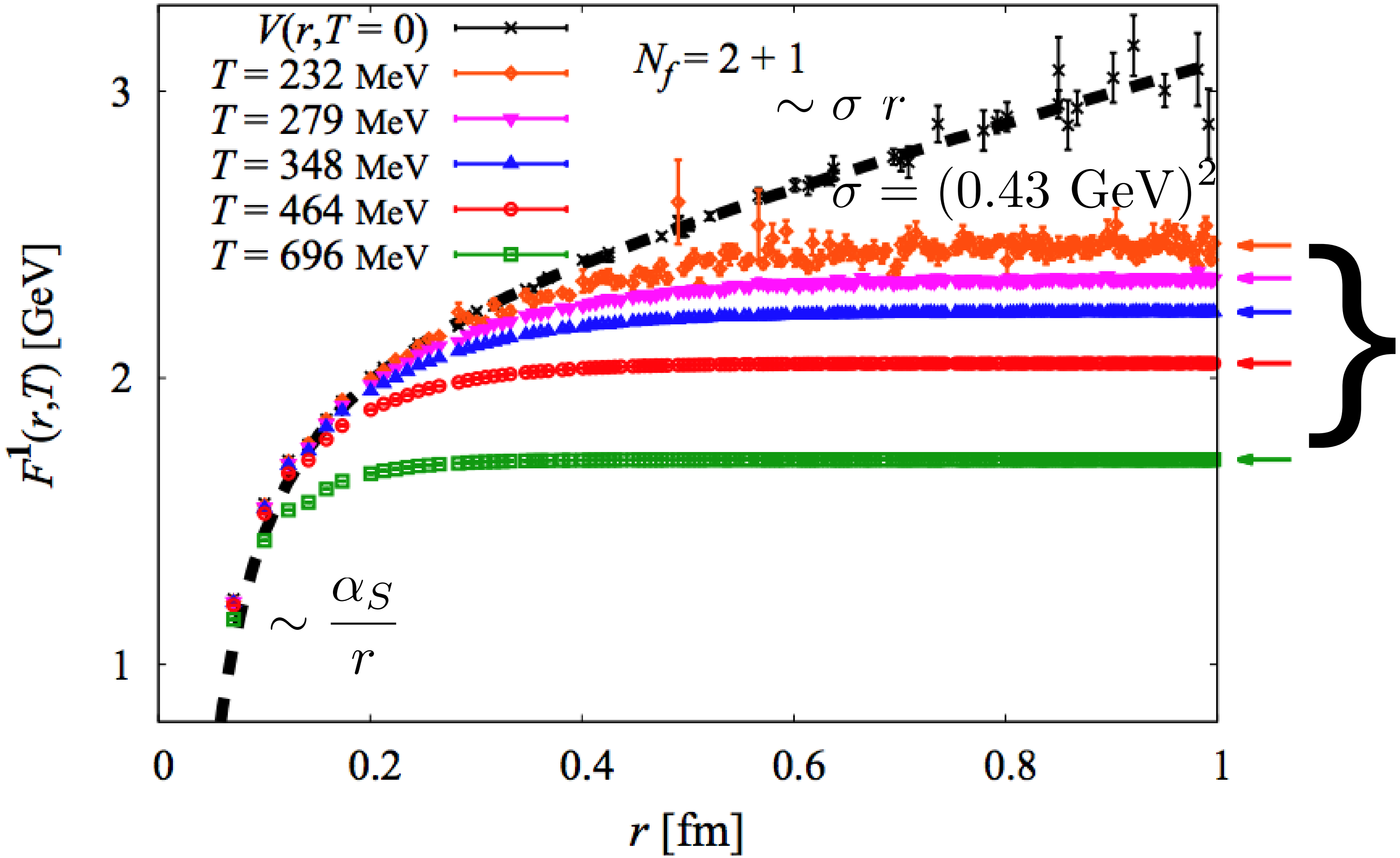
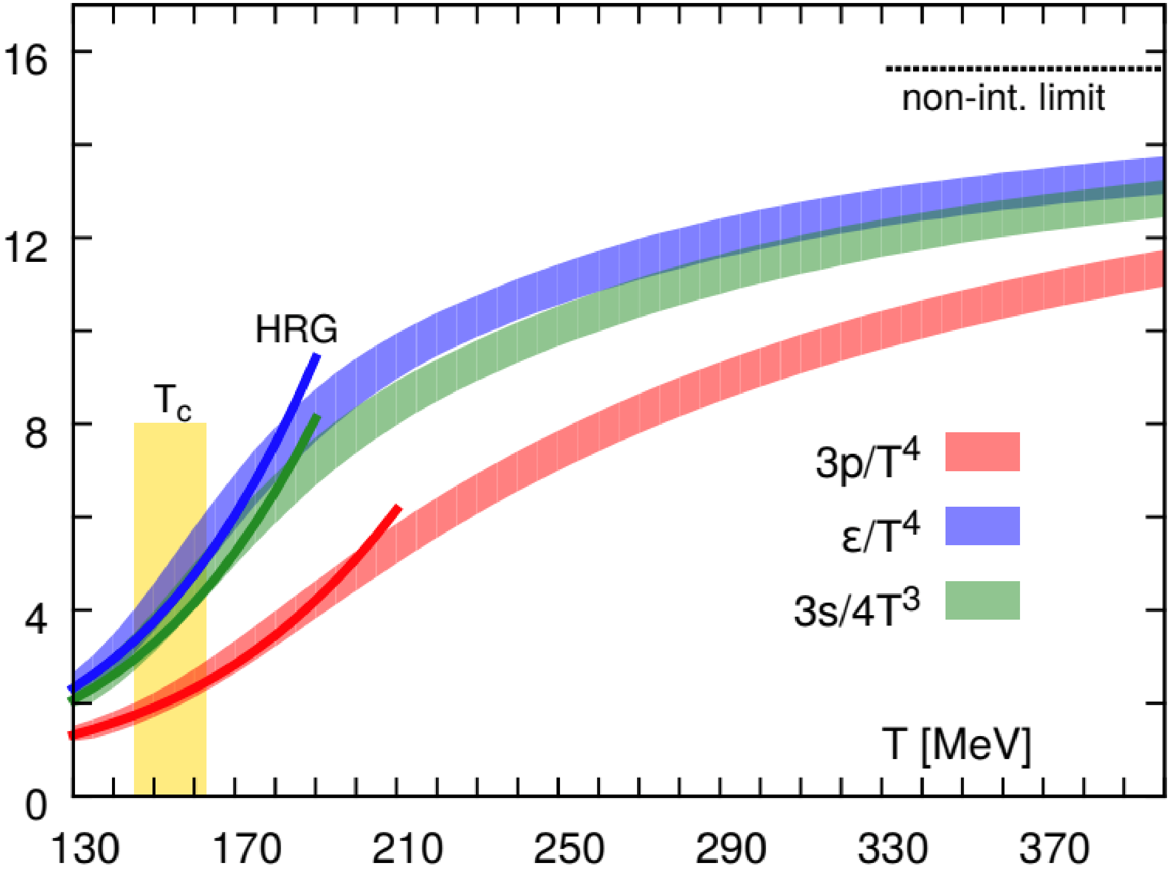


Arrows indicate twice the free energy of isolated quarks

Y. Maezawa et al. arXiv:1112.2756

Selected Lattice QCD Results (III)

- The pressure *p*, energy density ε and entropy density *s* all increase dramatically for T > ~ 140 MeV
- Smooth crossover (like ionization in forming regular plasma)
- At low temperature, LQCD results nicely consistent with a Hadron Resonance Gas (HRG) calculation containing exponential increase of states
- At high temperature, ε/T^4 approaches estimates from counting free quarks and gluons



HotQCD Collaboration, arXiv:1407.6387

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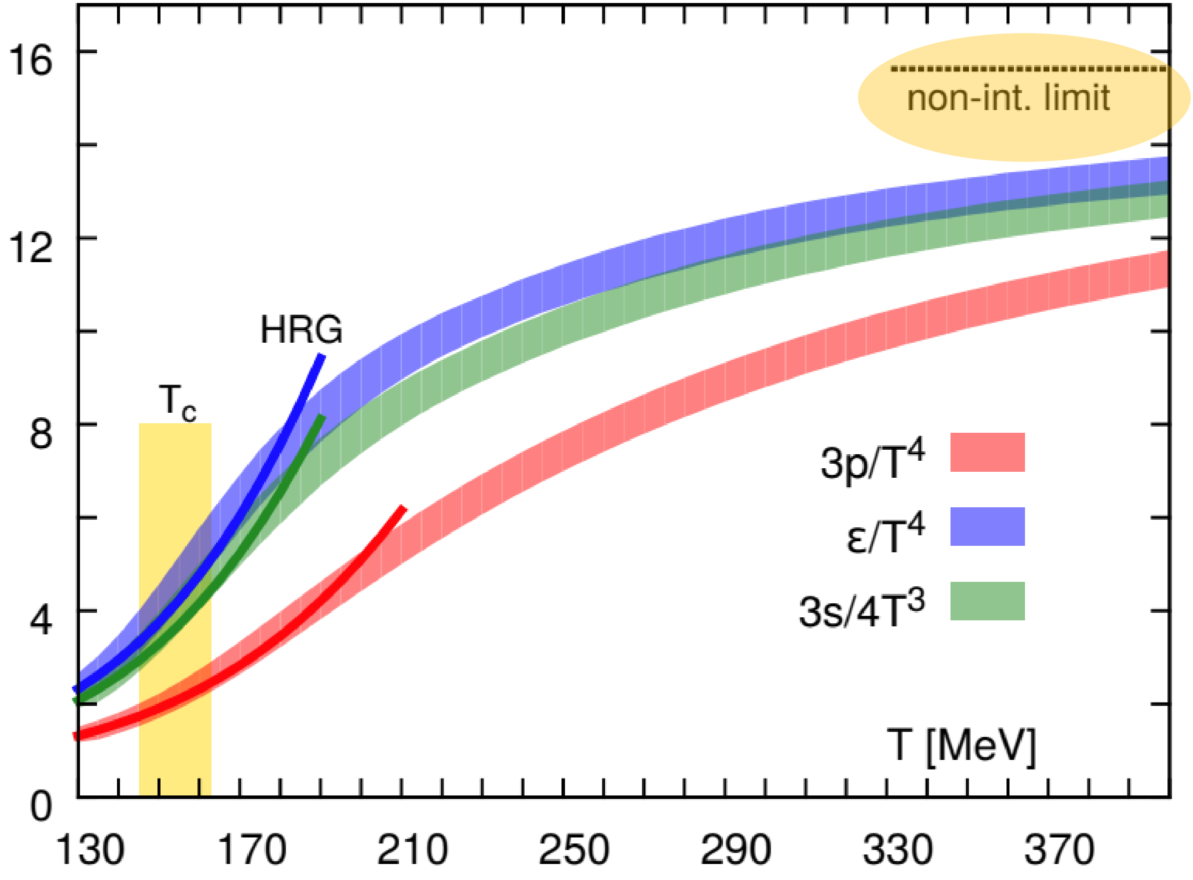
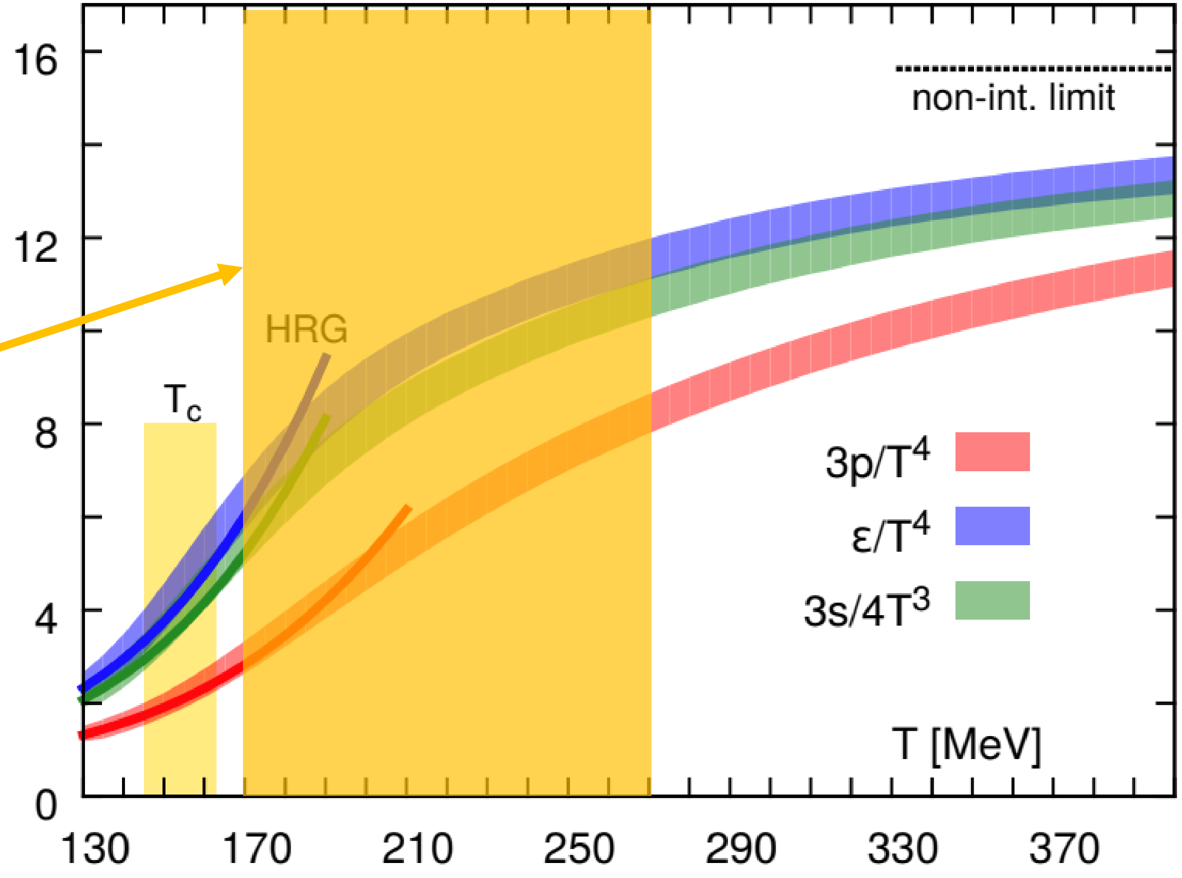


HotQCD Collaboration, arXiv:1407.6387

Resisting Temptation

- First results from RHIC in 2000-2002 especially
  - thermal spectra indicative of temperatures ~140 MeV (or higher)
  - Strong suppression of high p_T particles
- Compelling evidence for matter in this regime
- Very tempting to identify this with free quarks and gluons!



Theoretical Guidance

- 1983: “an extended ***quark-gluon plasma*** within which the quarks are deconfined and move independently”

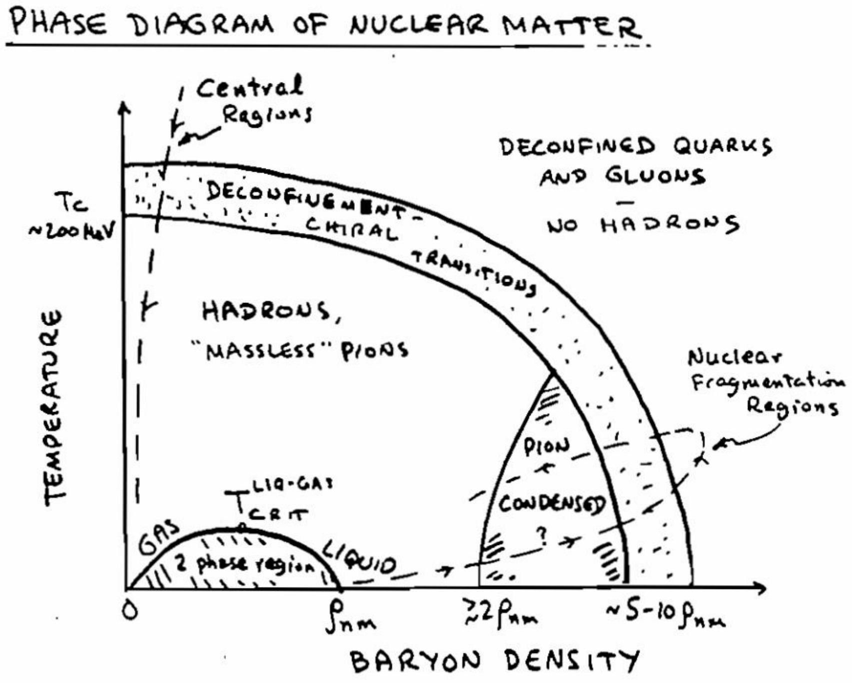


Fig. II.9-A. Expected phases of nuclear matter at various temperatures and baryon (or nucleon) densities, showing the “hadronic phase” including a gas-liquid phase transition region, and *the transition region to deconfined quarks and gluons.* The dashed lines illustrate trajectories in this phase diagram that can be explored in ultra-relativistic heavy ion collisions.

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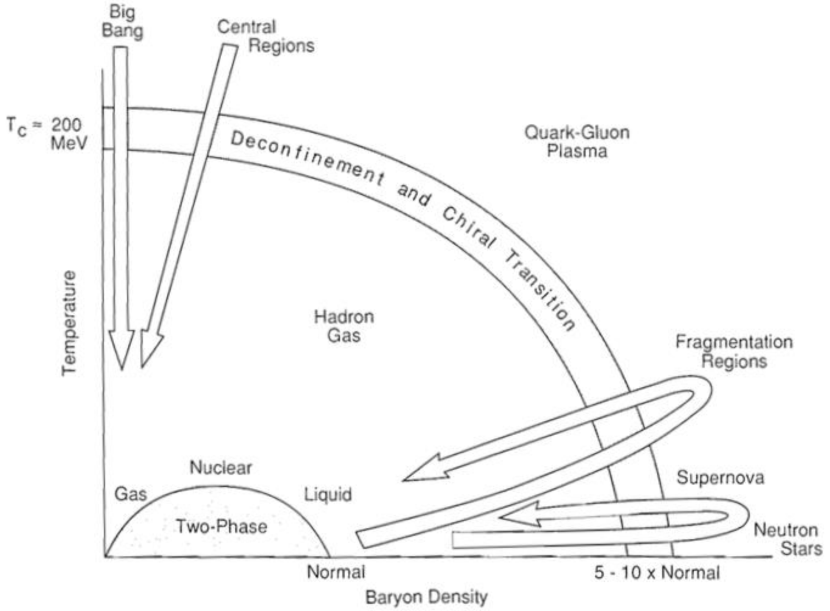

Figure 24: Expected phases of nuclear matter at various temperatures and baryon (or nucleon) densities, showing the "hadronic phase," including a gas-liquid phase-transition region, and the transition region to deconfined quarks and gluons. The dashed lines illustrate trajectories in this phase diagram that can be explored in ultrarelativistic heavy-ion collisions.

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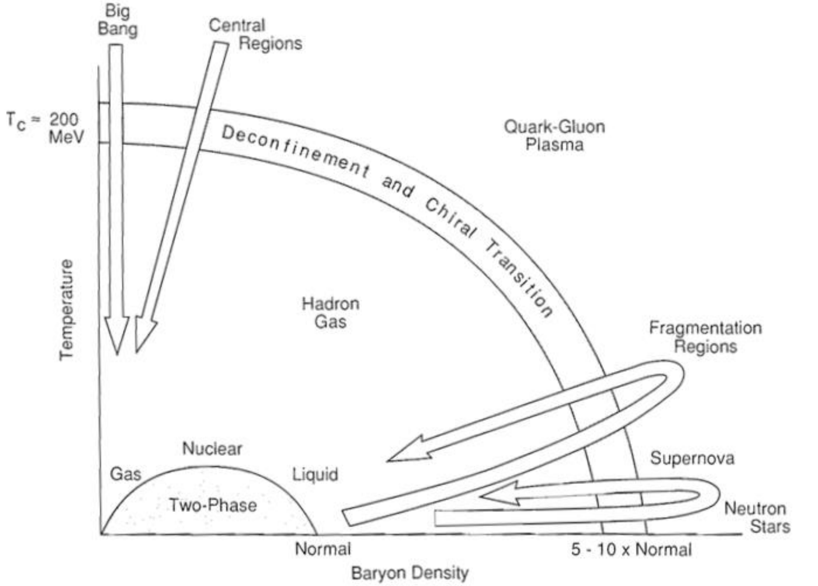


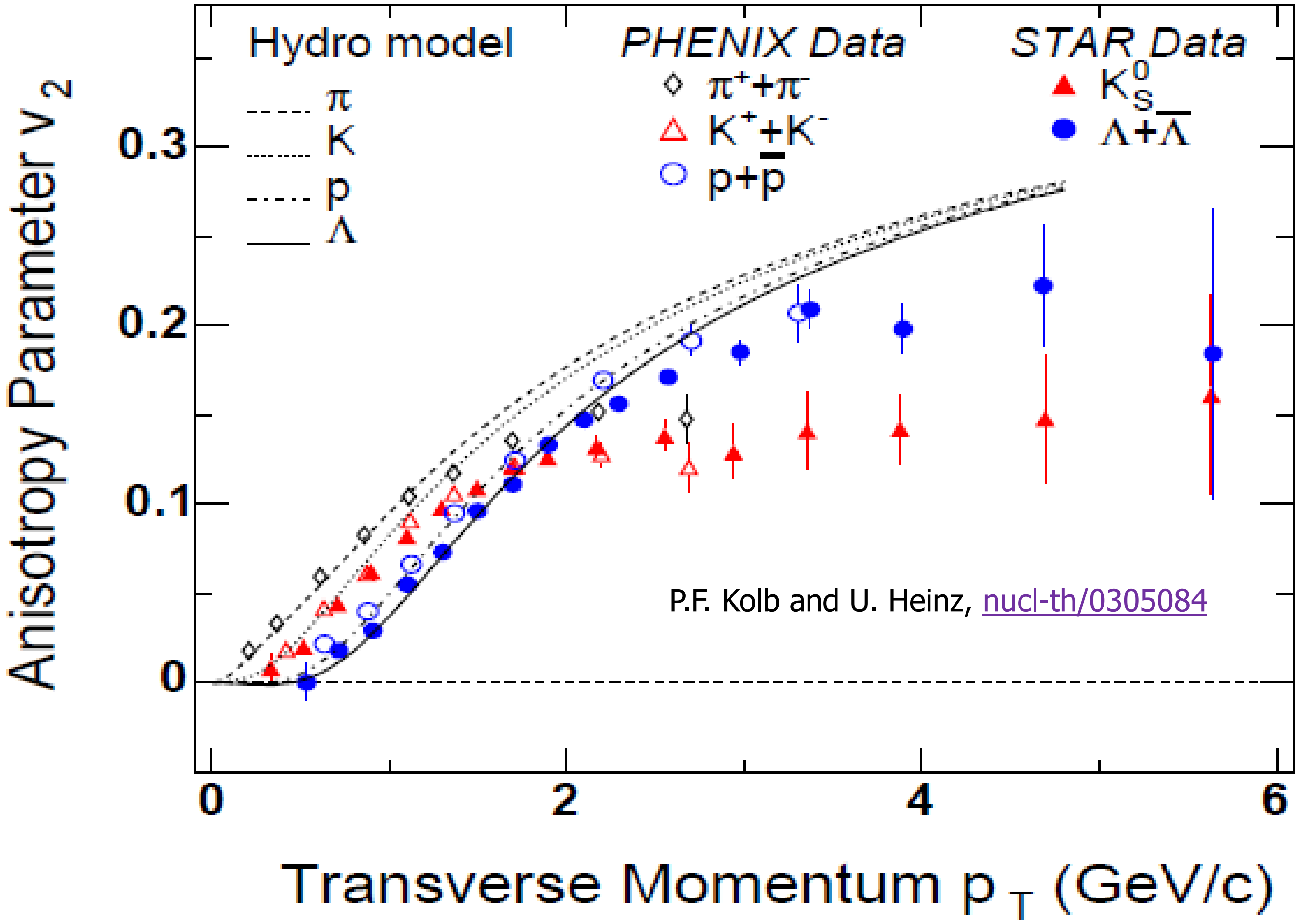
Figure 24: Expected phases of nuclear matter at various temperatures and baryon (or nucleon) densities, showing the “hadronic phase,” including a gas-liquid phase-transition region, and the transition region to deconfined quarks and gluons. The dashed lines illustrate trajectories in this phase diagram that can be explored in ultrarelativistic heavy-ion collisions.

- 1989: “quark-gluon plasma, in which hadrons dissolve into a plasma of quarks and gluons, which are then free to move over a large volume.”

- 2000: “Quarks and gluons would then freely roam within the volume of the fireball created by the collision.”

Lead Us Not Into Temptation…

- Those putatively “free” quarks and gluons exhibited flow patterns consistent with *ideal* hydrodynamics



P.F. Kolb and U. Heinz, nucl-th/0305084

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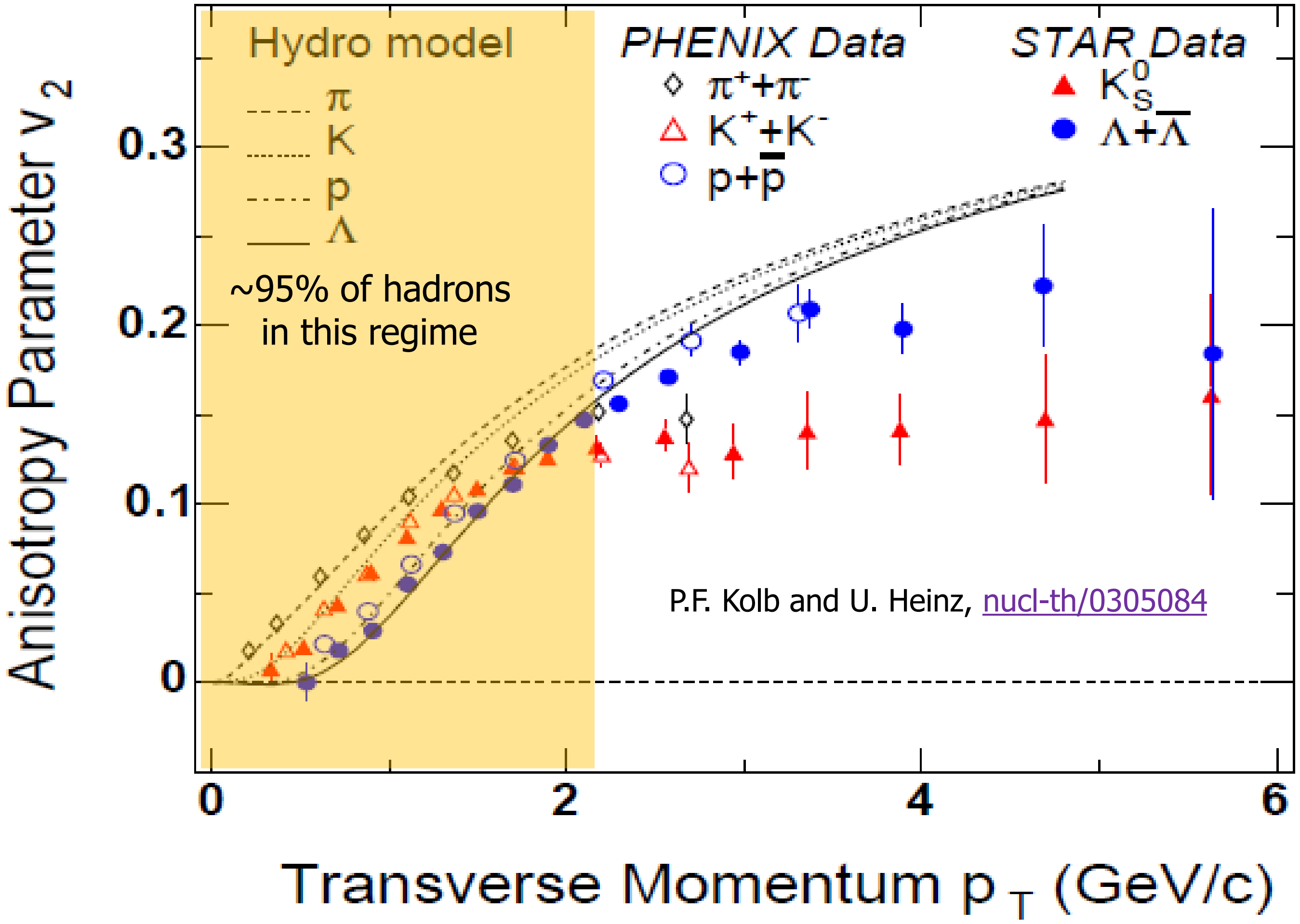
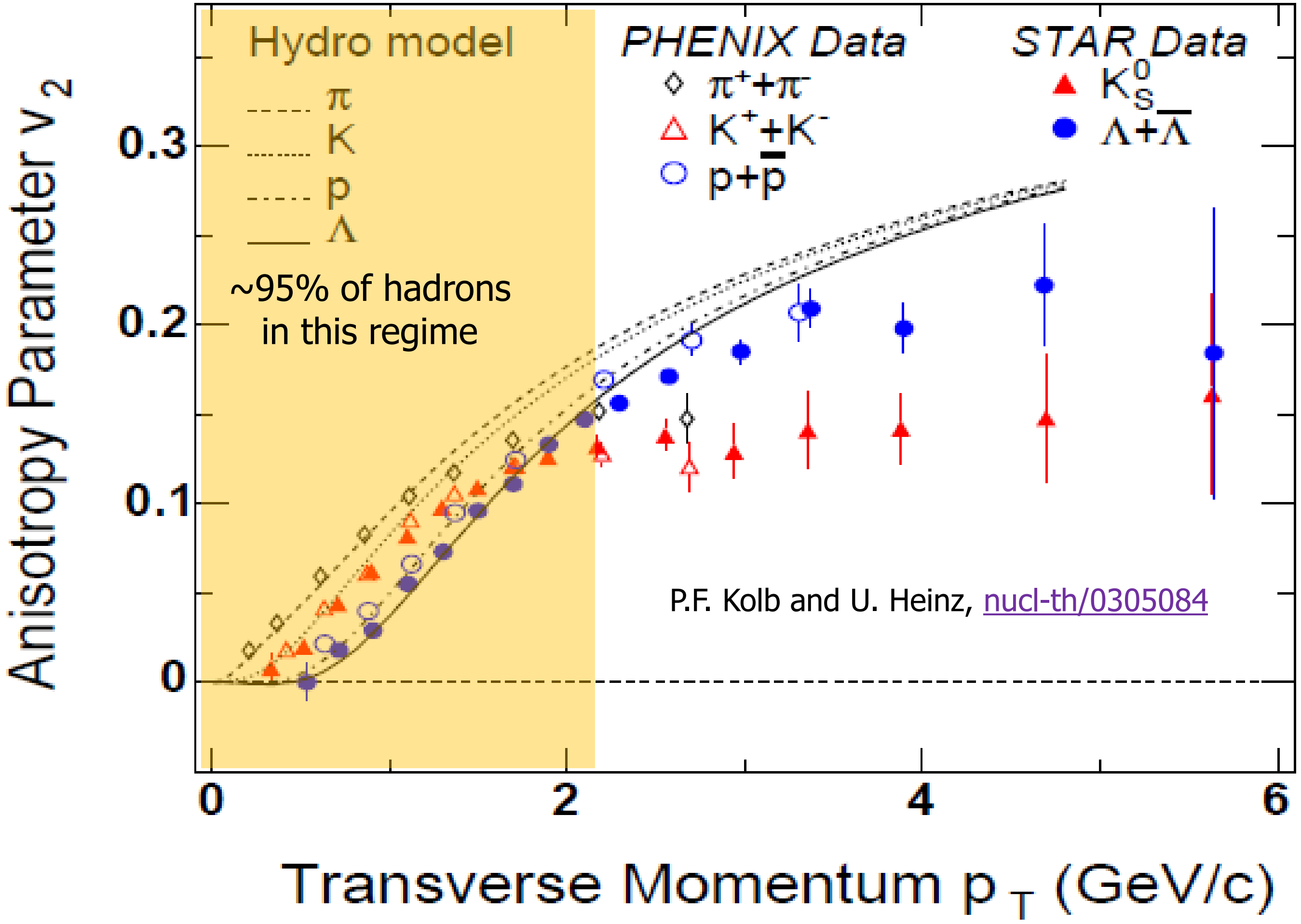


~95% of hadrons in this regime

P.F. Kolb and U. Heinz, nucl-th/0305084

Lead Us Not Into Temptation…

- Those putatively “free” quarks and gluons exhibited flow patterns consistent with *ideal* hydrodynamics
- Ideal hydro → viscosity set to *zero*
- Zero viscosity limit → *infinite* cross sections between our “free” quarks and gluons (!)



P.F. Kolb and U. Heinz, nucl-th/0305084

A Funny Thing About Viscosity

- *Strong* interactions imply *small* viscosity:
  - Viscosity η ~ Transverse momentum diffusion

    ~ $n \langle p \rangle \lambda$, n = number density

    , $\langle p \rangle$ = mean momentum

    , λ = mean free path

    $= 1 / n\sigma$,

    σ = cross section

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    , $\langle p \rangle$ = mean momentum

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    = $1 / n\sigma$,

    σ = cross section
  - So

$$\eta \sim n \langle p \rangle \lambda \sim \langle p \rangle / \sigma$$

  - *Large* inter-particle cross section ➔ *small* viscosity

A Perturbative Calculation for Viscosity

- Lattice suggests deconfined quarks and gluons above T_c .
- A harbinger of asymptotic freedom?
- → Try perturbation in *g*, where $\alpha_S = \frac{g^2}{4\pi}$
  - ☐ P. Arnold, G. Moore, L. Yaffe, hep-ph/0010177

- Leading-order result:

$$\eta \sim \#_1 \frac{T^3}{g^4 \ \log(\frac{\#_2}{g})} \quad \text{where } \#_1 \sim 90 \ \ ; \ \ \#_2 \ \sim 2.95$$

- Small perturbative cross-sections (< mb) produce a *large* viscosity

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  - Small perturbative cross-sections (< mb) produce a *large* viscosity

Aside: This result is often quoted as

"parametrically, $\eta \sim \dfrac{T^3}{g^4 \;\log(\frac{1}{g})}$ "

This is confusing,
well, more accurately, *wrong*.

Thinking Like a Physicist

- Viscosity is a dimensionful quantity
  → we must ask “*Small or large compared to what?* ”
- Answer: *Entropy density s*
- Motivation:
  - Relativistic F = ma results in terms containing $\frac{\eta}{\epsilon + P} \sim \frac{\text{drag force}}{\text{inertial mass}}$
  - Thermodynamics (at zero chemical potential) → $\epsilon + P = T\ s$
  - So $\frac{\eta}{\epsilon + P} = \left(\frac{\eta}{s}\right) \frac{1}{T} \sim (\text{dimensionless number}) \times (\text{natural thermal time})$

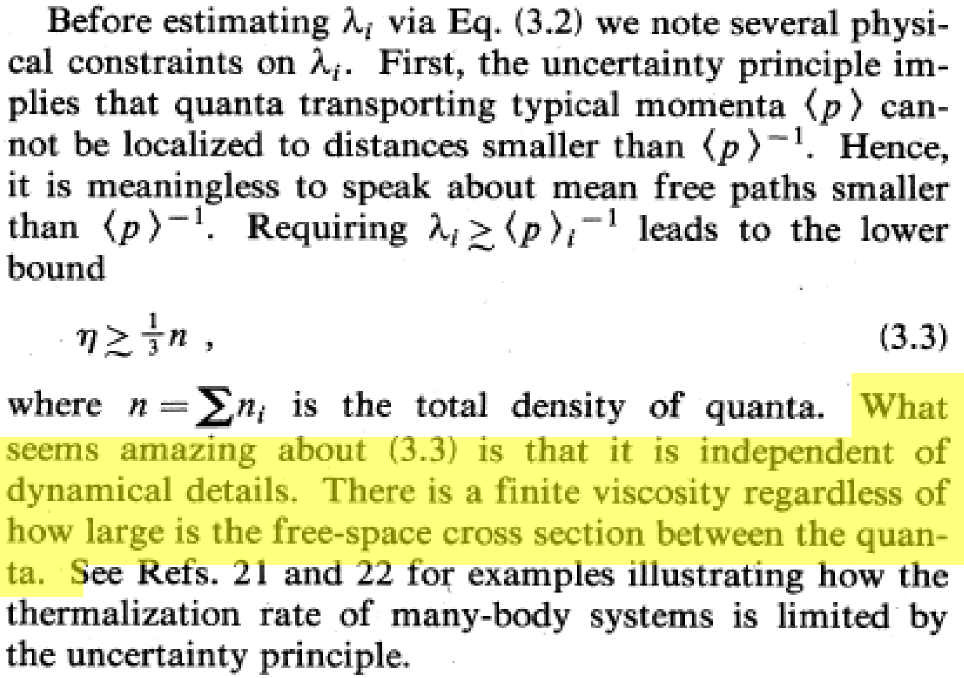
How Small Can We Make η/s ?

- Recall
  - Viscosity η ~ Transverse momentum diffusion ~ n <p> λ
  - Entropy density s ~ 4 n (massless non-interacting quanta)
- Then $\dfrac{\eta}{s} \sim \dfrac{n\langle p\rangle\lambda}{4n} \sim \dfrac{1}{4}\langle p\rangle\lambda$
- But uncertainty principle ➔ $\langle p\rangle\ \lambda\ \gtrsim\ \hbar$
- Which in turn implies $\dfrac{\eta}{s}\ \gtrsim\ \dfrac{1}{4}\hbar$

Lost Knowledge (1985)

- Miklos Gyulassy and Pawel Danielewicz:
  - *Dissipative Phenomena*
    In Quark-Gluon Plasmas
    P. Danielewicz, M. Gyulassy
    Phys.Rev. D31, 53,1985.

noted restrictions on smallest allowed



Before estimating λ_i via Eq. (3.2) we note several physical constraints on λ_i. First, the uncertainty principle implies that quanta transporting typical momenta $\langle p \rangle$ cannot be localized to distances smaller than $\langle p \rangle^{-1}$. Hence, it is meaningless to speak about mean free paths smaller than $\langle p \rangle^{-1}$. Requiring $\lambda_i \gtrsim \langle p \rangle_i^{-1}$ leads to the lower bound

$$\eta \gtrsim \tfrac{1}{3} n \, , \qquad (3.3)$$

where $n = \sum n_i$ is the total density of quanta. What seems amazing about (3.3) is that it is independent of dynamical details. There is a finite viscosity regardless of how large is the free-space cross section between the quanta. See Refs. 21 and 22 for examples illustrating how the thermalization rate of many-body systems is limited by the uncertainty principle.

- Most restrictive:
- $\lambda > \hbar / \langle p \rangle \quad \Rightarrow \quad \eta > \sim n / 3$
- But recall s = 4 n for
  the quanta they were considering
- $\Rightarrow \eta/s > \hbar / (4 \times 3) \sim \hbar / (4\pi)$

$$\frac{\eta}{s} \gtrsim \frac{1}{4\pi}\hbar$$

Simultaneously in 2003-4

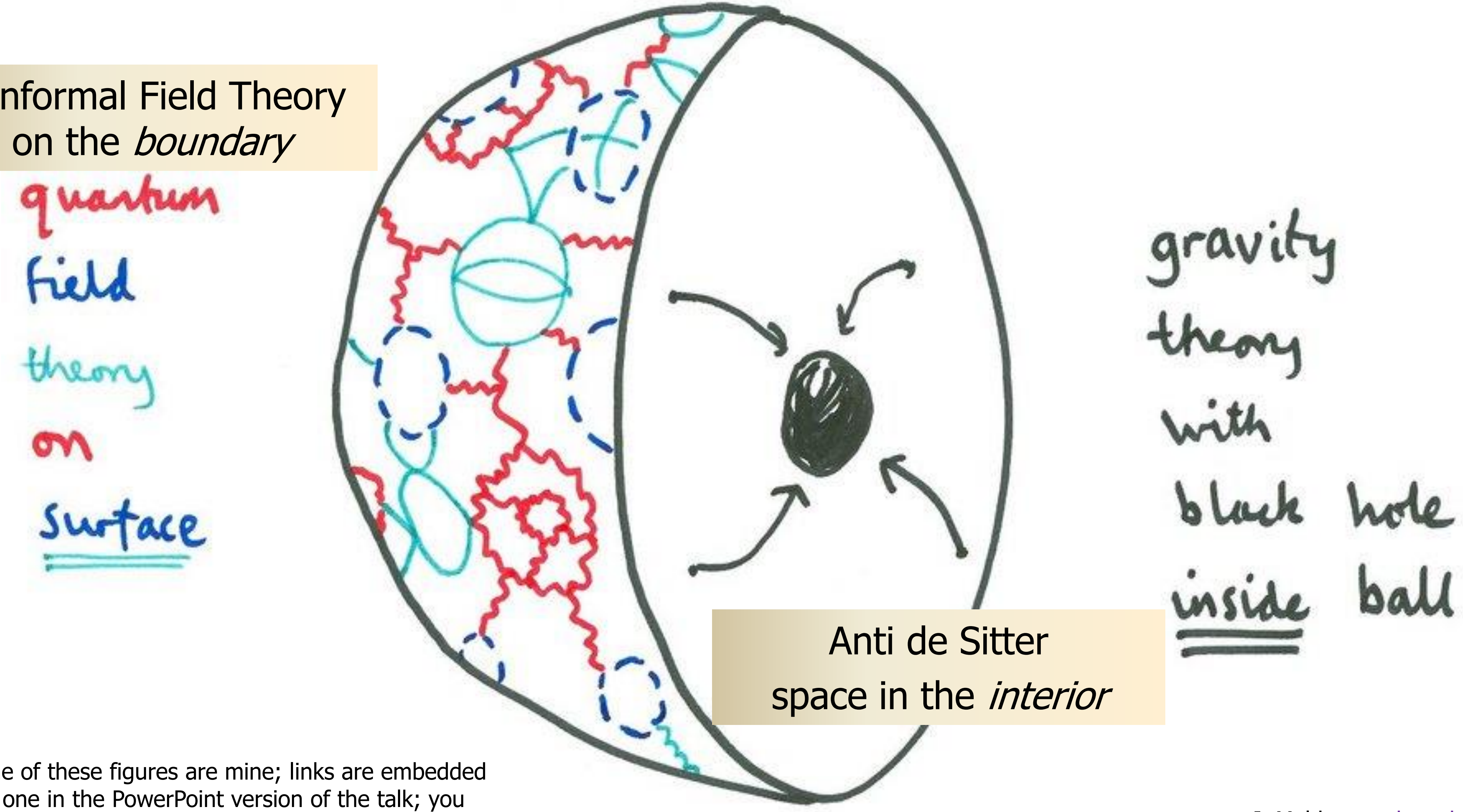
- An estimate (~~bound~~?) on viscosity appeared from string theory's AdS/CFT correspondence:
  - *A Viscosity Bound Conjecture*, P. Kovtun, D.T. Son, A.O. Starinets, hep-th/0405231

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi}$$

⇒ Very important to quantify *departures* from ideal hydrodynamics

AdS / CFT in a Picture

Conformal Field Theory
on the *boundary*



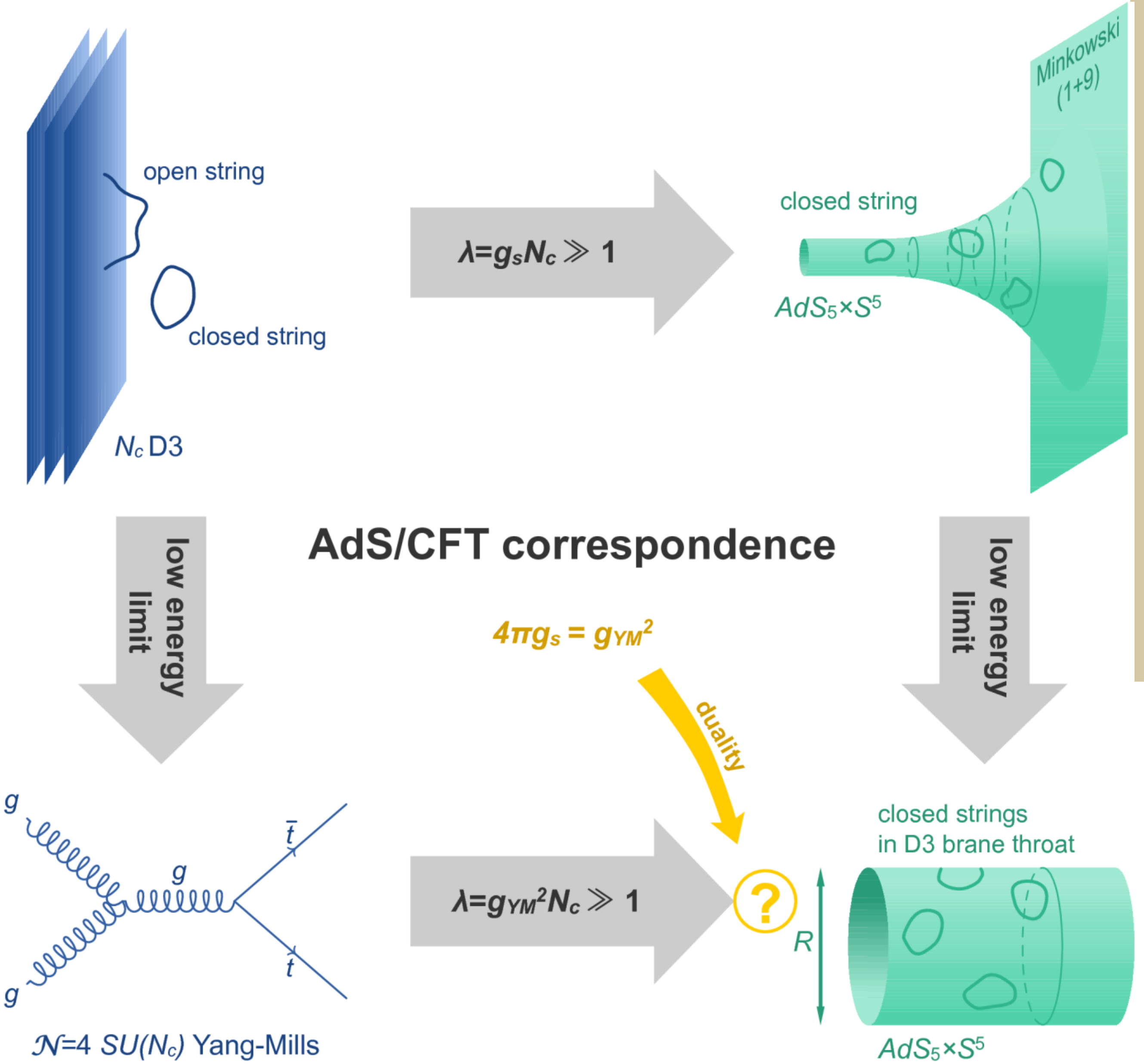
Anti de Sitter
space in the *interior*

☞ *A hard (strongly-coupled) gauge theory calculation is* ***dual*** *to an easy semi-classical gravitational calculation.*

Note: None of these figures are mine; links are embedded with each one in the PowerPoint version of the talk; you can find all of them simply by searching on "AdS/CFT"

J. Maldacena, hep-th/9711200

AdS / CFT in a Picture



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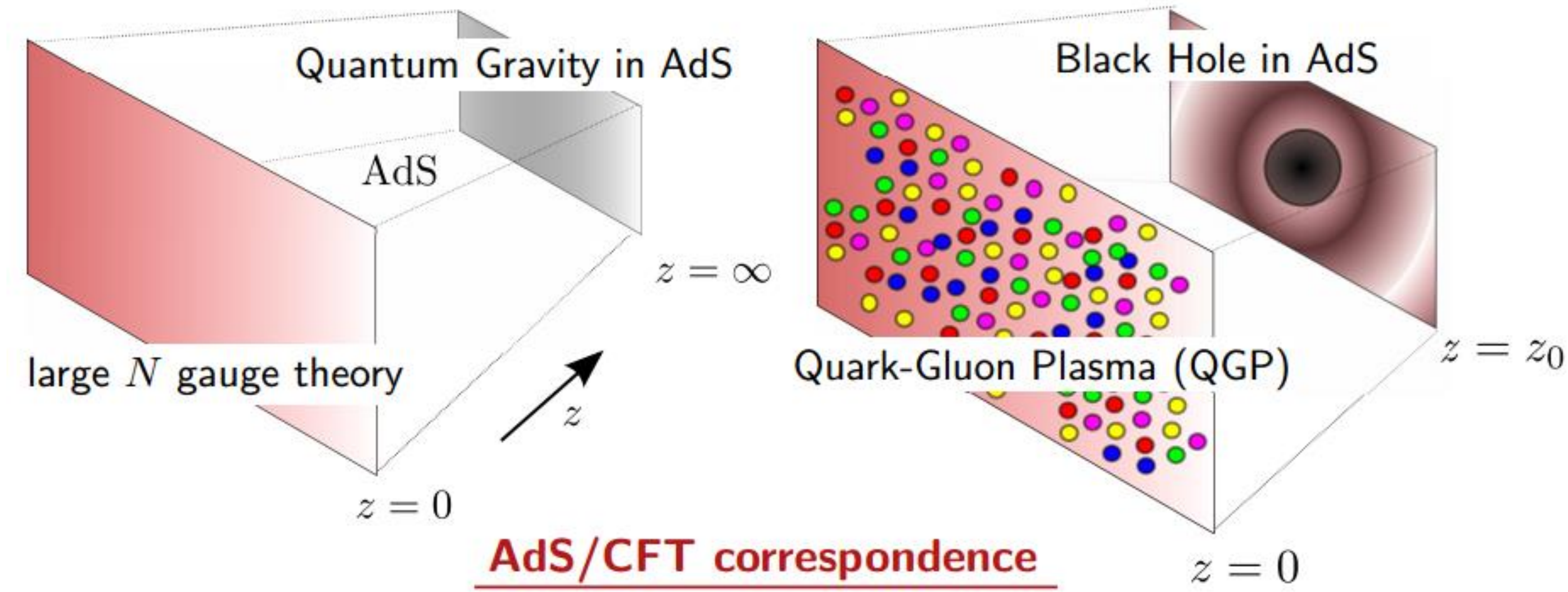

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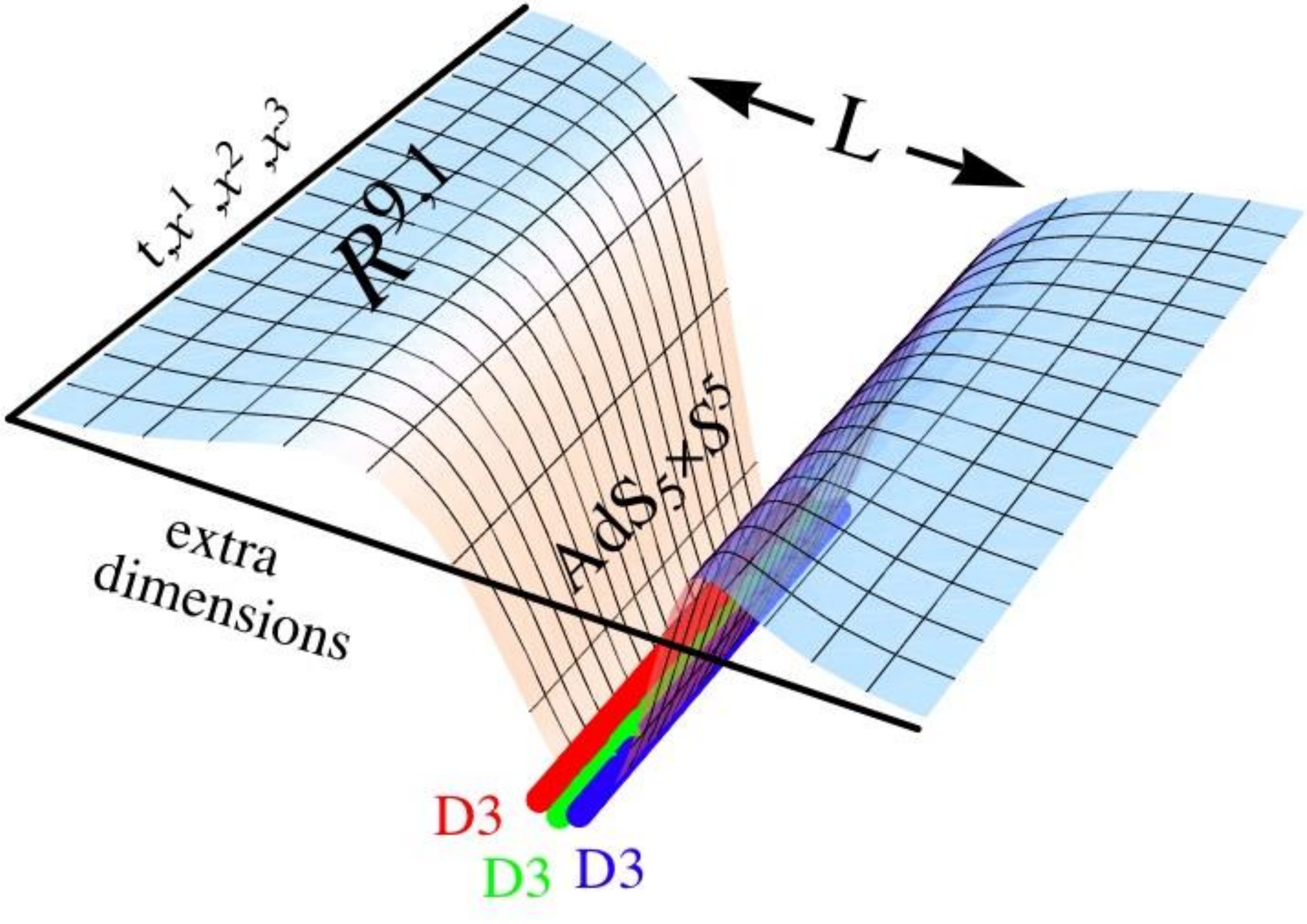

strong coupling

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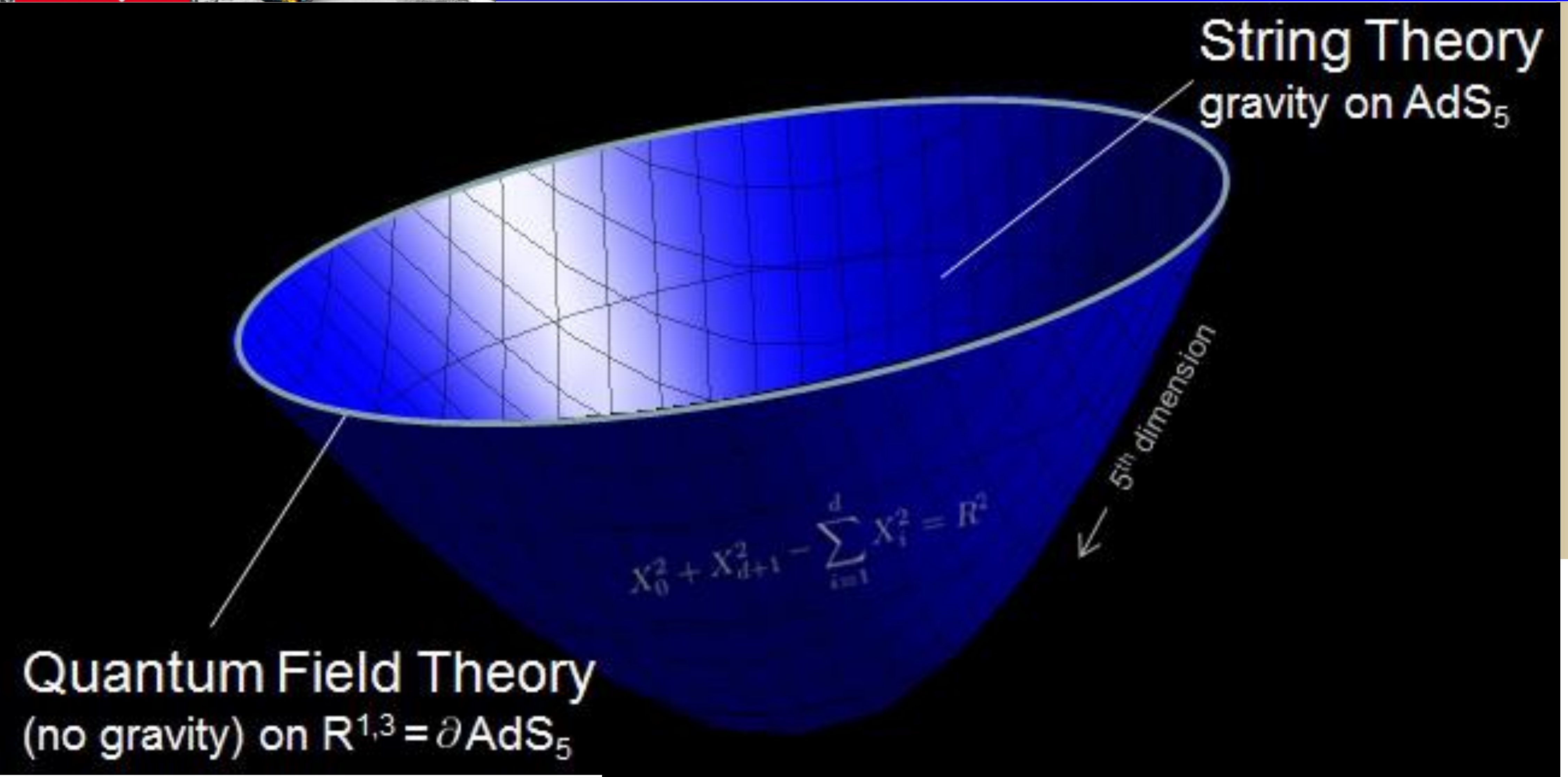

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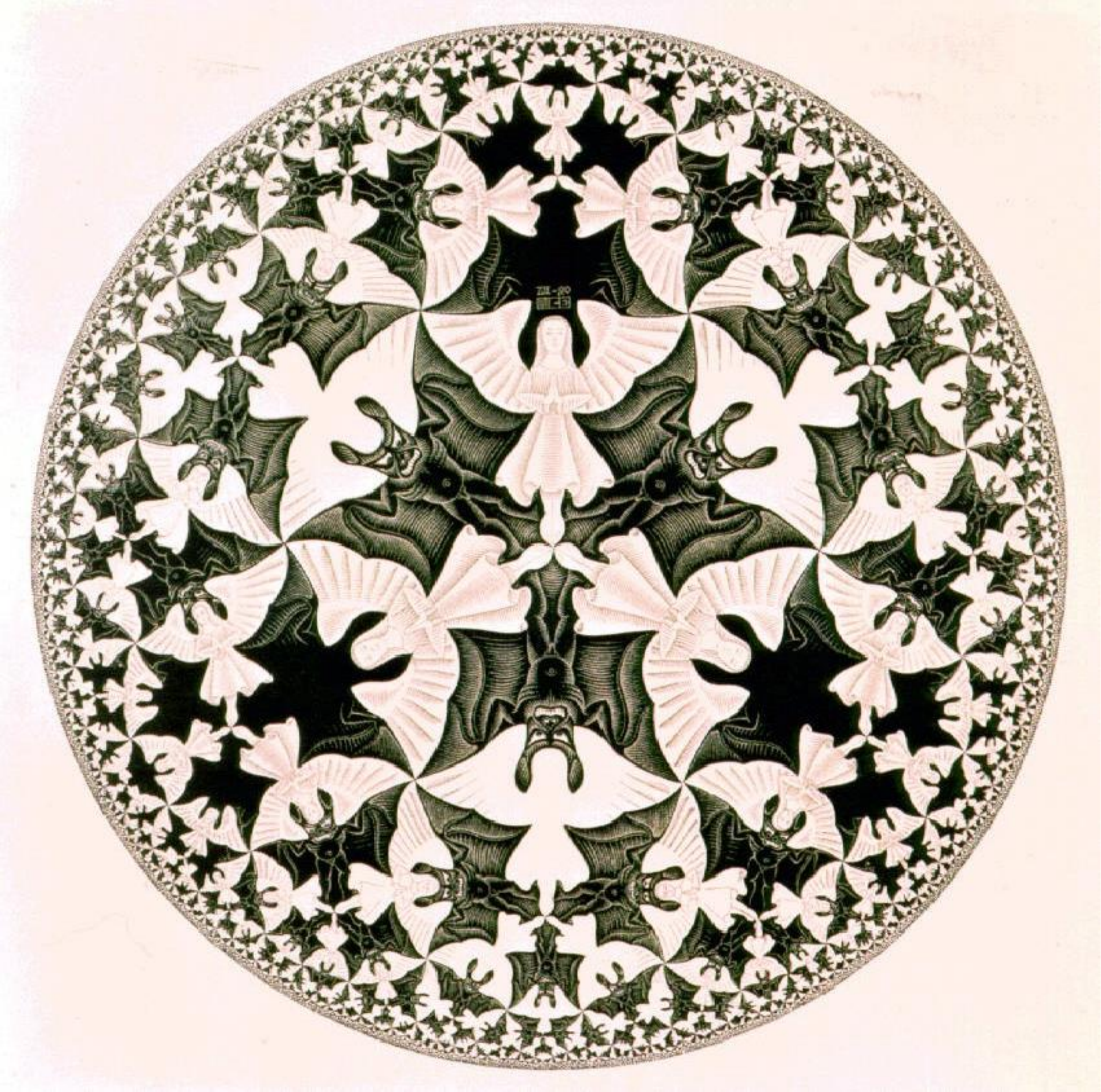

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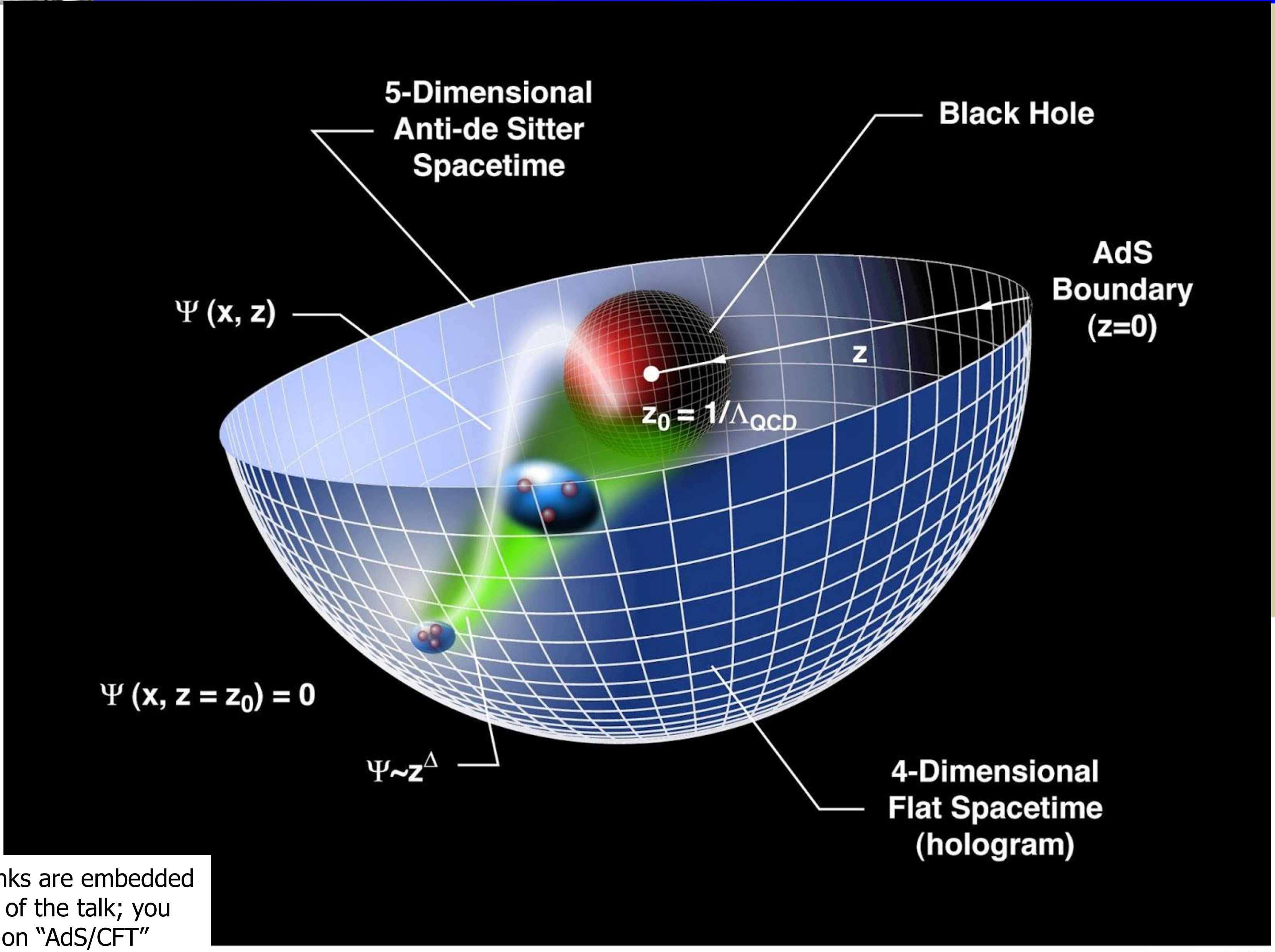

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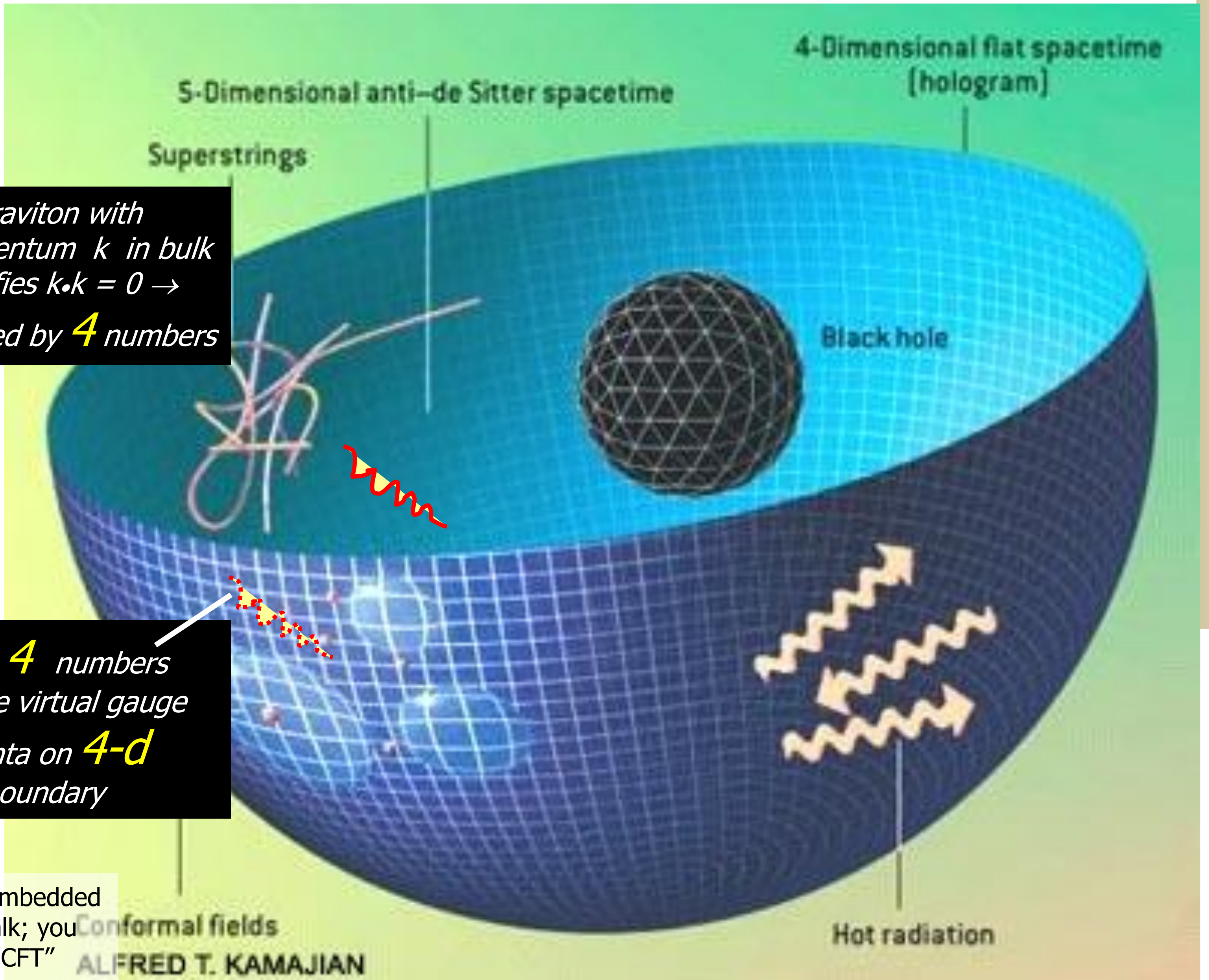
AdS / CFT in a Picture

Graviton with 5-momentum k in bulk satisfies k•k = 0 → described by 4 numbers

Those 4 numbers describe virtual gauge quanta on 4-d boundary



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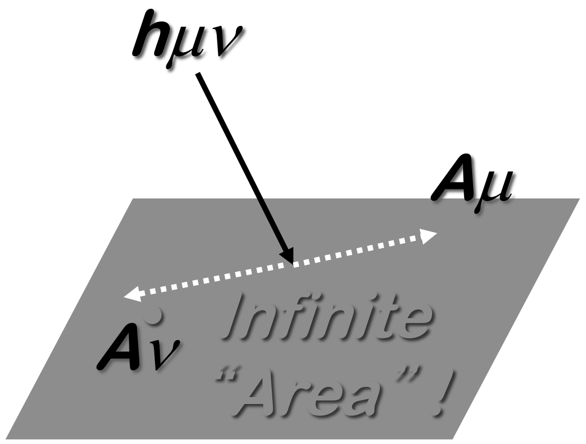
The (Assumed) Connection

- *Exploit* Maldacena's "D-dimensional strongly coupled gauge theory ⇔ (D+1)-dimensional stringy gravity"
- *Thermalize* with massive black brane



- *Calculate* viscosity η = "Area"/16πG
- *Normalize* by entropy (density) s = "Area" / 4G
- *Dividing out* the infinite "areas" :

$$\frac{\eta}{s} = \left(\frac{\hbar}{k}\right)\frac{1}{4\pi}$$

- *Conjectured* to be a lower bound "*for all relativistic quantum field theories at finite temperature and zero chemical potential*".
- *See* "*Viscosity in strongly interacting quantum field theories from black hole physics*", P. Kovtun, D.T. Son, A.O. Starinets, Phys.Rev.Lett.94:111601, 2005, hep-th/0405231

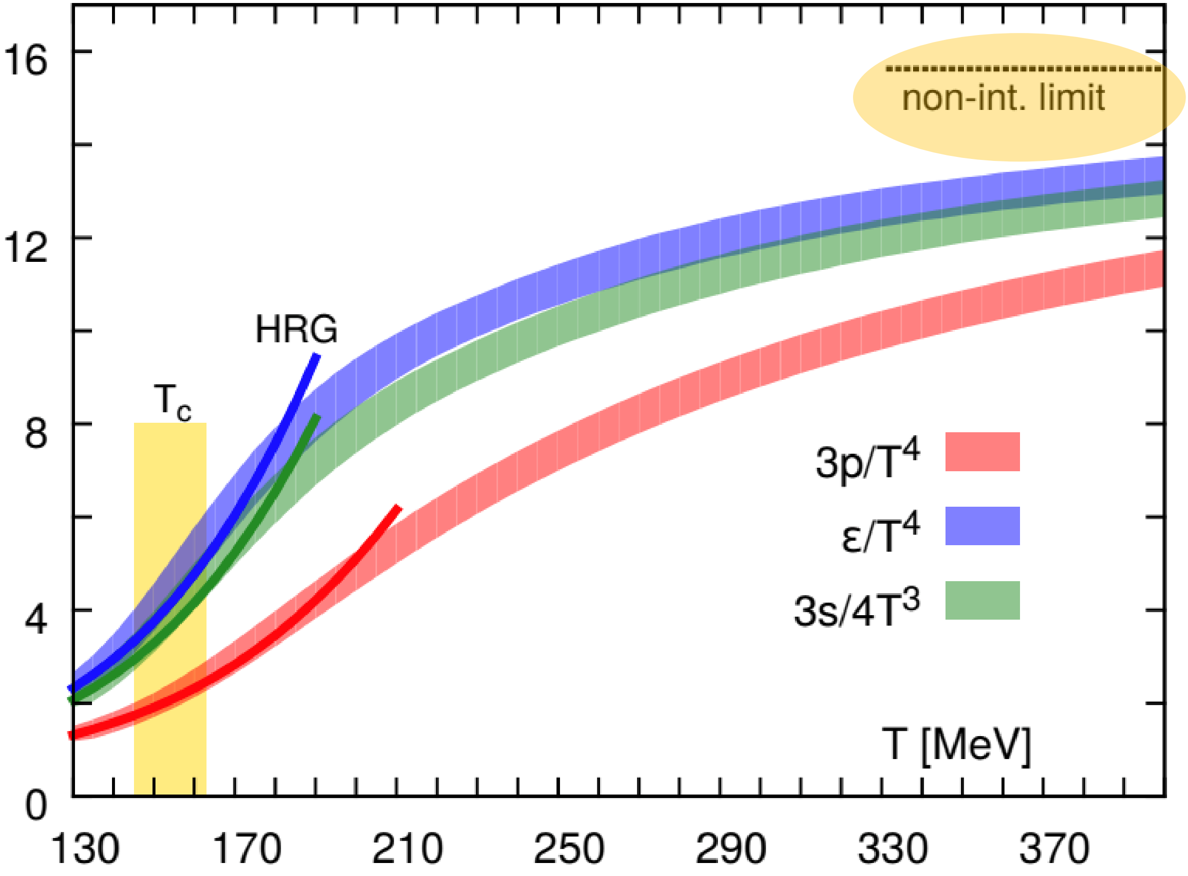
J. Maldacena, hep-th/9711200

Implications

- The bulk behavior of “strongly-coupled QGP” (sQGP) depends only on $\alpha_s(T)$ and $\hbar$.
  - A true “quantum liquid”.
- The observed very low value of $\eta/s \sim 0.1\text{-}0.2 \Rightarrow$ no quasiparticles!
  - Urs Wiedemann, Quark Matter 2012:
    “This plasma is unique in that it does not carry quasi-particle excitations”
  - National Academy Decadal Survey:
    “Does the QGP have a particulate description
    at any length scale?”

Another Lesson From AdS/CFT

- The ”approach” observed to the non-interacting limit may be misleading.
- The Super Yang-Mills plasma on the boundary in AdS/CFT in some sense has infinitely strong coupling
- But...



HotQCD Collaboration, arXiv:1407.6387

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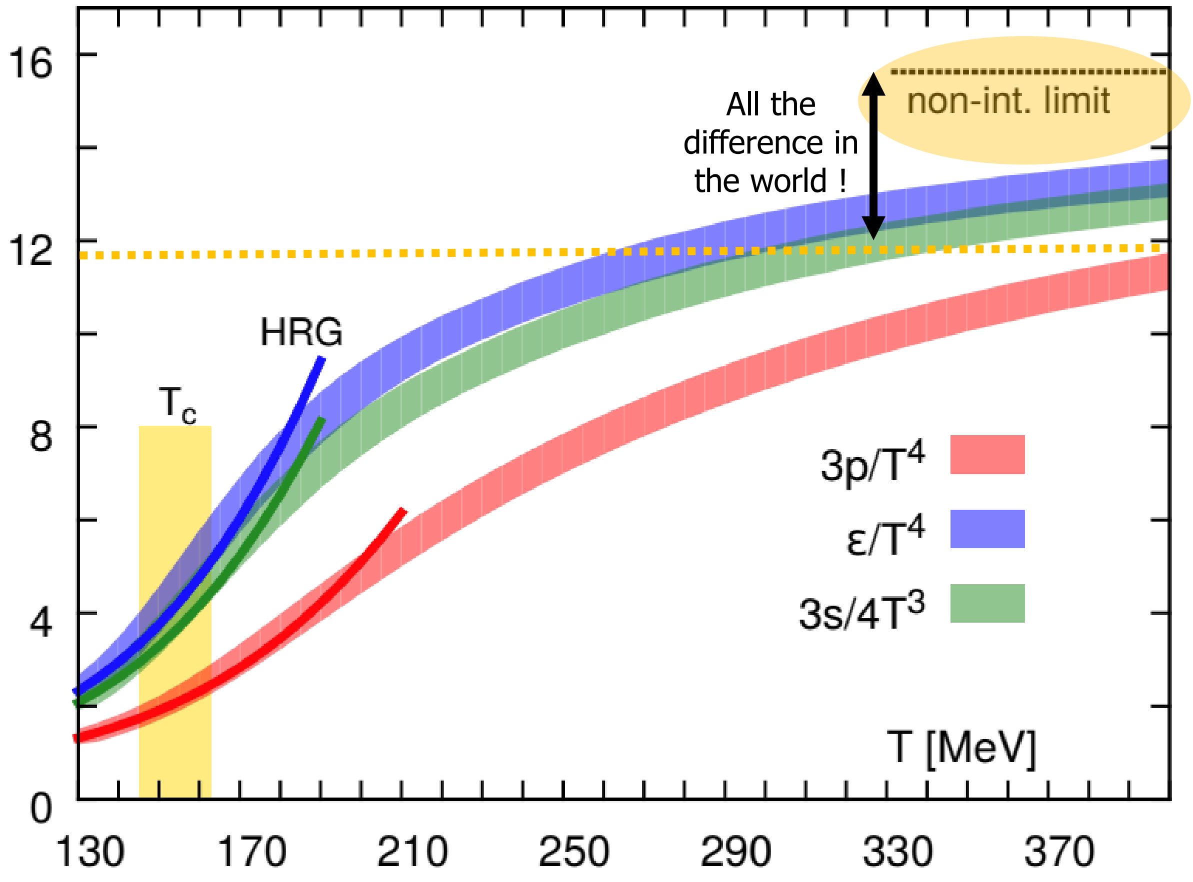
- But

$$s(g=\infty) \;=\; \frac{3}{4}\, s(g=0) \quad (!)$$



HotQCD Collaboration, arXiv:1407.6387

Asymptotic Freedom Redux

- The colloquial is not the physical:
- QCD potential $V(r) \sim \frac{\alpha_s(r)}{r} \sim \frac{\alpha_0}{\log(1/r\Lambda)} \frac{1}{r}$
- But: QCD *number density* $n \sim T^3 \sim \frac{1}{\langle r \rangle^3} \Rightarrow \frac{1}{\langle r \rangle} \sim T$
- So – in a *thermal* system, $\Rightarrow \langle V(r) \rangle \sim \frac{\alpha_0}{\log(1/\langle r \rangle \Lambda)} \frac{1}{\langle r \rangle} \sim \frac{\alpha_0}{\log(T/\Lambda)} T$
  the slow *log(T/Λ)* decrease in α_S
  is overwhelmed by the fast increase in 1/r ~ T

➔ asymptotic freedom is asymptotic indeed !

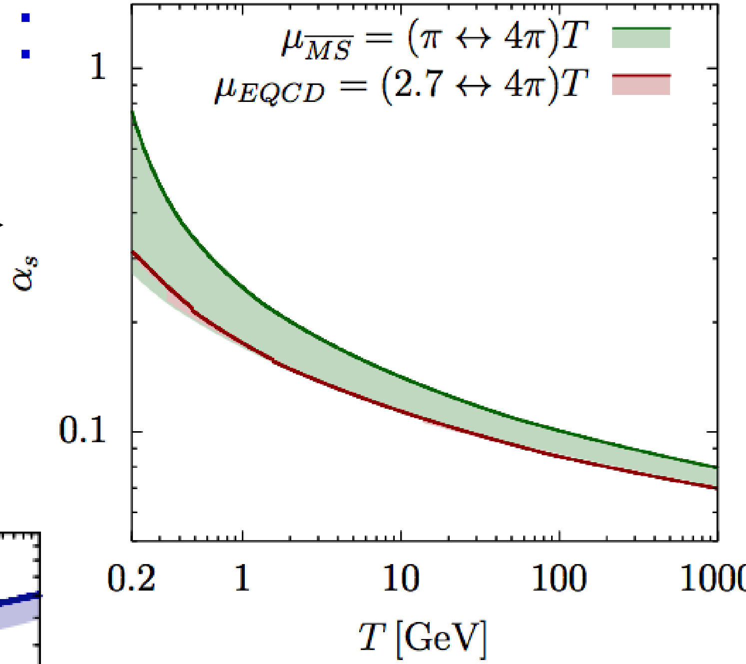
Strongly Coupled Plasmas

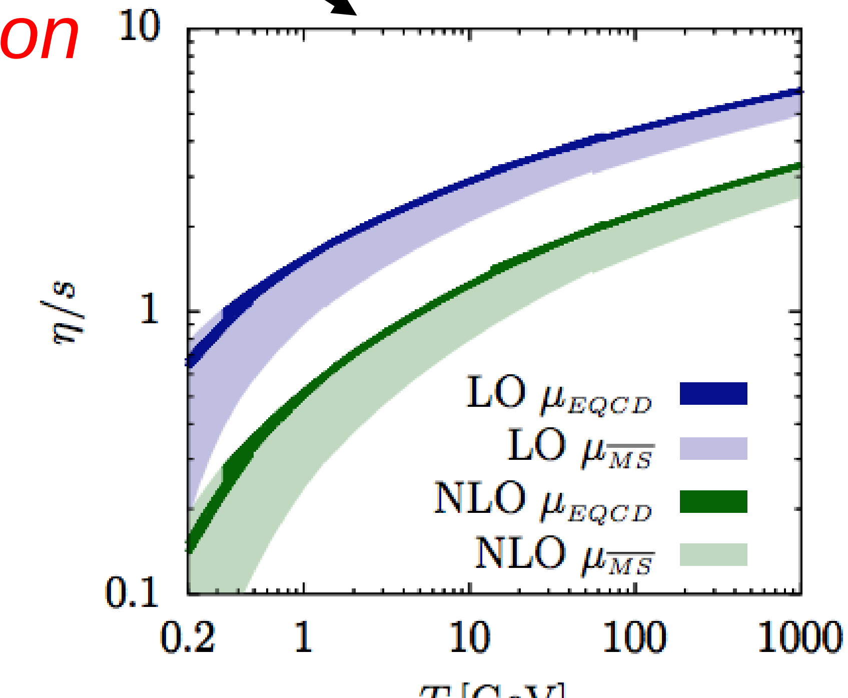
- The absence of asymptotic freedom known long enough to be forgotten several times:
  - 1982: Gordon Baym, proceedings of Quark Matter '82:
    - ☐ A hint of trouble can be seem from the first order result for the entropy density ($N_f = 3$)

      which turns negative for $\alpha_s > 1.1$ $$s(T) = \frac{19\pi^2}{9}\{\ 1 - \frac{54}{19\pi}\alpha_S(T)\ + \ldots\ \}\, T^4$$
  - 1992: Berndt Mueller, Proc. of NATO Advanced Study Institute
    - ☐ For plasma conditions realistically obtainable in the nuclear collisions (T ~250 MeV, $g = \sqrt{(4\pi\alpha_s)} = 2$) the effective gluon mass $m_g^* \sim 300$ MeV. We must conclude, therefore, that the notion of almost free gluons (and quarks) in the high temperature phase of QCD is quite far from the truth. Certainly one has $m_g^* << T$ when $g << 1$, but this condition is never really satisfied in QCD, because g ~ 1/2 even at the Planck scale (10^{19} GeV), and g<1 only at energies above 100 GeV.
  - 2002: Ulrich Heinz, Proceedings of PANIC conference:
    - ☐ Perturbative mechanisms seem unable to explain the phenomenologically required very short thermalization time scale, pointing to strong non-perturbative dynamics in the QGP even at or above 2Tc.... The quark-hadron phase transition is arguably the most strongly coupled regime of QCD.

Recent Developments

- New results on perturbative calculation of η/s :
  - Investigate sensitivity to subtraction scheme, renormalization scale
  - Find NLO corrections are *large*
  - *"The perturbative expansion is problematic even for T ~ 100 GeV…"*

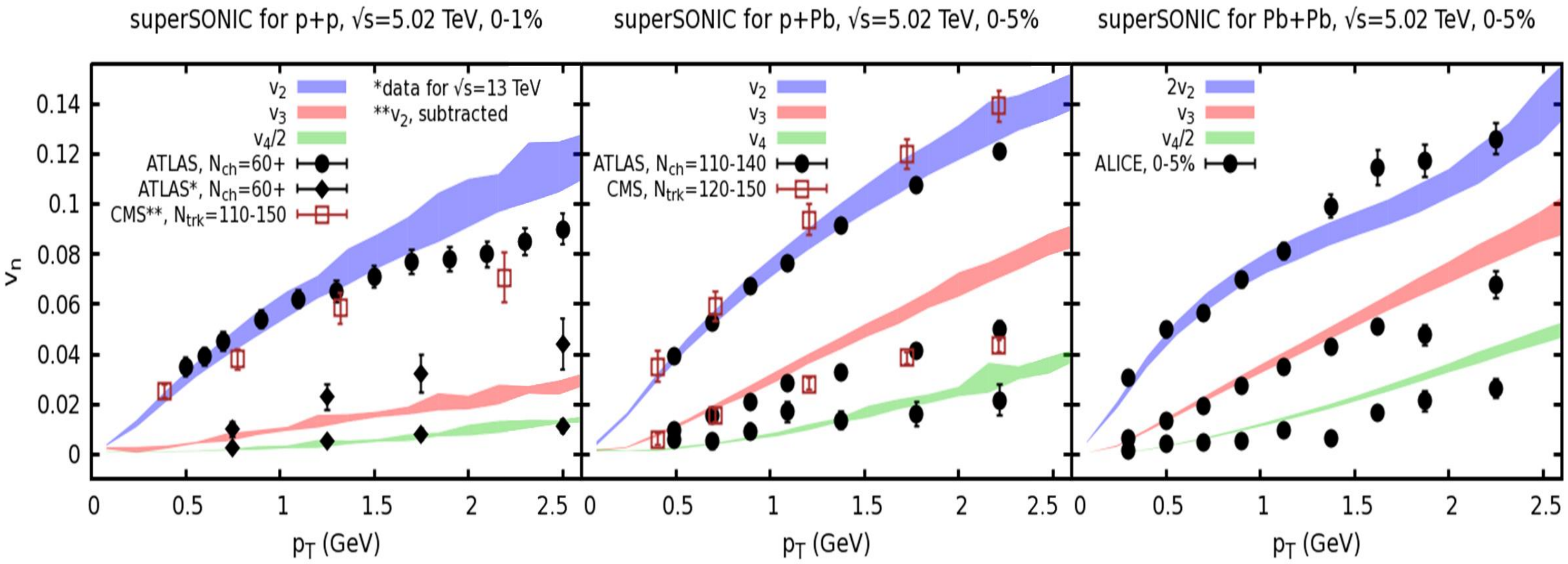




QCD Shear Viscosity at (Almost) NLO,
J. Ghiglieri, G. Moore and D. Teaney,
arXiv:1802:09535

Hydrodynamics in Small Systems

- *Single* hydro description provides “reasonable” *simultaneous* description of $v_2(p_T)$, $v_3(p_T)$ and $v_4(p_T)$ in p+p, p+Pb, and Pb+Pb:



superSONIC for p+p, √s=5.02 TeV, 0-1%

superSONIC for p+Pb, √s=5.02 TeV, 0-5%

superSONIC for Pb+Pb, √s=5.02 TeV, 0-5%

R.D. Weller and P. Romatschke, arXiv:1701:07145

- “One fluid to rule them all…”
- N.B.: *Geometry initialized at constituent quark level*

Recent review of collective behavior in small systems: J.L. Nagle and WAZ, arXiv:1801.03477

Recent Developments

- Hydrodynamic behavior $\not\Rightarrow$ thermalization
  - That is, applicability of hydrodynamics does not *require* thermalization

    and therefore

    P. Romatschke, arXiv:1609.02820
  - Should not be assumed to be *evidence* for thermalization

Reviews:

W. Florkowski, M.P. Heller, and M. Spalinski, arXiv:1707.02282

P. Romatschke and U. Romatschke, arXiv:1712.05815

Recent Developments

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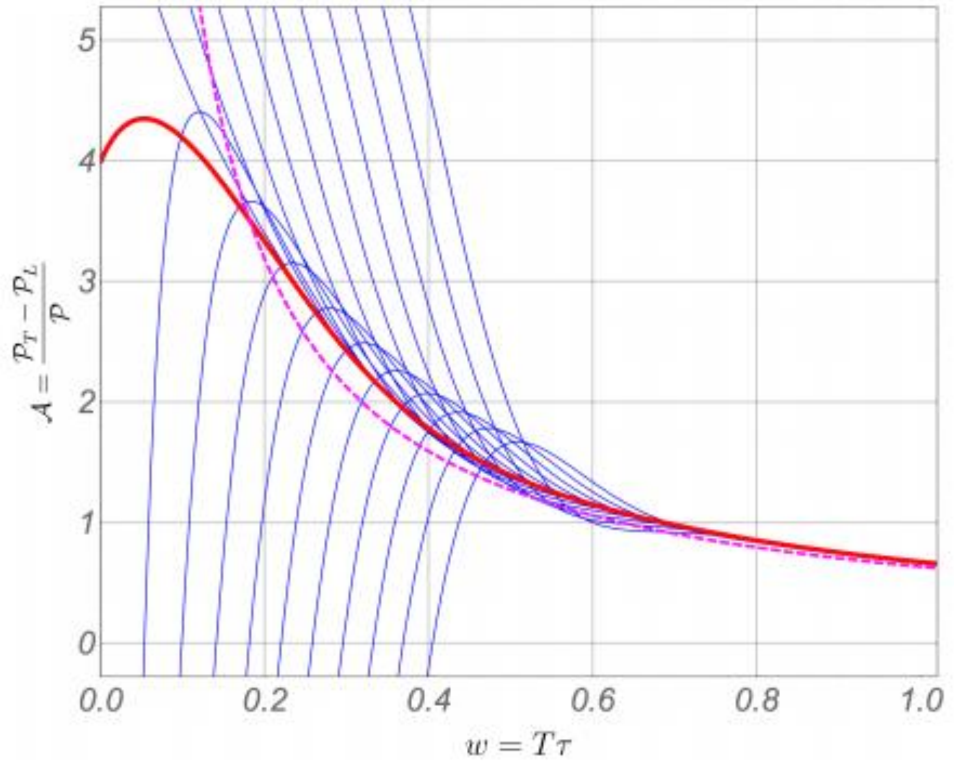
    P. Romatschke, arXiv:1609.02820
  - Should not be assumed to be *evidence* for thermalization
- Instead – strongly-coupled systems exhibit *relaxation* to *universal attractor* for $\tau > 0.7 / T$



Reviews:

W. Florkowski, M.P. Heller, and M. Spalinski, arXiv:1707.02282
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Open Questions

- How does non-zero baryon chemical potential affect the fluid properties of QGP?
- Are there experimentally accessible regimes where the plasma become weakly coupled?
- Can we use jets to probe the microscopic structure of QGP?
- How can we understand the success of parton transport models that seemingly violate quantum mechanical limits yet reproduce flow-like features in small systems?
- Are there experimental observables sensitive to the non-hydro modes? Can they be used to determine the associated relaxation parameters?
- What is the smallest drop of QGP describable by relativistic viscous hydrodynamics?

- Good News…

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- Good News: *There is much work to be done!*

Thank You!

Back-up

A Question

- Why does the system select the *higher* pressure ?
- After all, systems tend to ‘select’ the lowest energy…
- Possible answers:
  - To get the right answer
  - Higher pressure pushes harder on lower pressure…
  - It’s more chaotic
  - 2nd Law of Thermodynamics
    - Basic result in phase equilibria at fixed volume and temperature
    - $dS = - d\Phi/T$, Φ = Grand Potential = $- P V$
    - Details provided in back-up slides

Statistical Mechanics I

- Density of states: $$dN = \frac{d^3r\,d^3p}{h^3}$$
    - (Incredibly ubiquitous and useful)
- Boson occupation factor: $$f_B(p) = \frac{1}{e^{(E(p)-\mu)/T} - 1}$$
- Fermion occupation factor: $$f_F(p) = \frac{1}{e^{(E(p)-\mu)/T} + 1}$$
- Then

$$N = \int \frac{1}{e^{(E(p)-\mu)/T} \pm 1} \frac{d^3r\,d^3p}{h^3} = \frac{V}{h^3} \int \frac{1}{e^{(E(p)-\mu)/T} \pm 1} \frac{d^3p}{h^3}$$

$$U = \int \frac{E(p)}{e^{(E(p)-\mu)/T} \pm 1} \frac{d^3r\,d^3p}{h^3} = \frac{V}{h^3} \int \frac{E(p)}{e^{(E(p)-\mu)/T} \pm 1} \frac{d^3p}{h^3}$$

Statistical Mechanics II

- Huge simplification for (non-interacting) massless quanta at zero chemical potential μ .
- Mathematics: $\int \frac{s^a\,ds}{e^s-1} = \Gamma(a+1)\zeta(a+1)$

  $\int \frac{s^a\,ds}{e^s+1} = (1-\frac{1}{2^a})\Gamma(a+1)\zeta(a+1)$

Exercise: "Prove" this

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- Physics: $n_B(T) \equiv \frac{N_B}{V} = \frac{\xi(3)}{\pi^2}T^3 \quad , \quad n_F(T) = \frac{3}{4}n_B(T)$

  $\varepsilon_B(T) \equiv \frac{U_B}{V} = \frac{3\xi(4)}{\pi^2}T^4 \quad , \quad \varepsilon_F(T) = \frac{7}{8}\varepsilon_B(T)$

Exercise: Derive these expressions using relations above.

- Mathematics: $\xi(2) = \frac{\pi^2}{6} \quad , \quad \xi(4) = \frac{\pi^4}{90}$

Exercise: "Prove" this. (Either you know the tricks or this will be very challenging…

Thermodynamics I

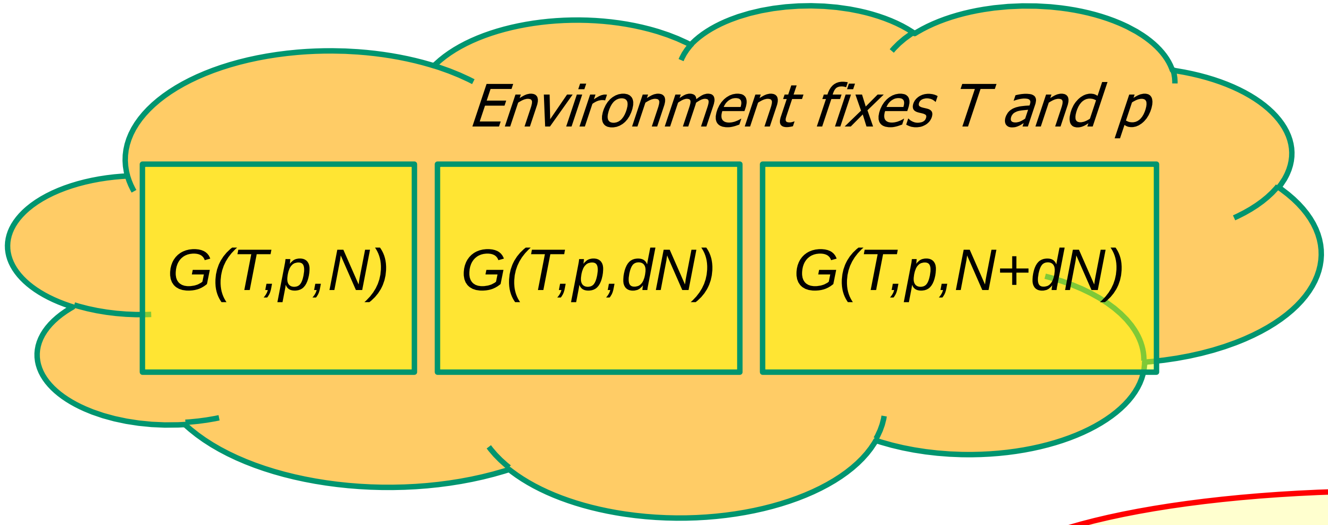
- $U \equiv$ Internal energy of a system
  - $dU = dQ + dW$ (1st Law, energy conservation)
  - $dU = T\,dS - p\,dV + \mu\,dN \Rightarrow U(S,V,N)$

Always remember this form, derive the rest

- Enthalpy $H \equiv U + pV$
  - $dH = T\,dS + V\,dp + \mu\,dN \Rightarrow H(S,p,N)$
- Free Energy $F \equiv U - TS$
  - $dF = -S\,dT - p\,dV + \mu\,dN \Rightarrow F(S,T,N)$
- Gibbs Free Energy $G \equiv F + pV$
  - $dG = -S\,dT + V\,dp + \mu\,dN \Rightarrow G(T,p,N)$
- Grand Potential $\Phi \equiv F - \mu N$
  - $d\Phi = -S\,dT + V\,dp - N\,d\mu \Rightarrow \Phi(T,p,\mu)$

Thermodynamics II

- Hiding in the Legendre Transformation formalism is some very useful physics:
- Gibbs Free Energy $G \equiv F + pV = U - TS + pV$
  - $dG = -S\,dT + V\,dp + \mu\,dN \Rightarrow G(T,p,N)$
  - So $\mu = (\partial G/\partial N)_{T,p}$, which implies $G \sim N$



Environment fixes T and p

G(T,p,N)

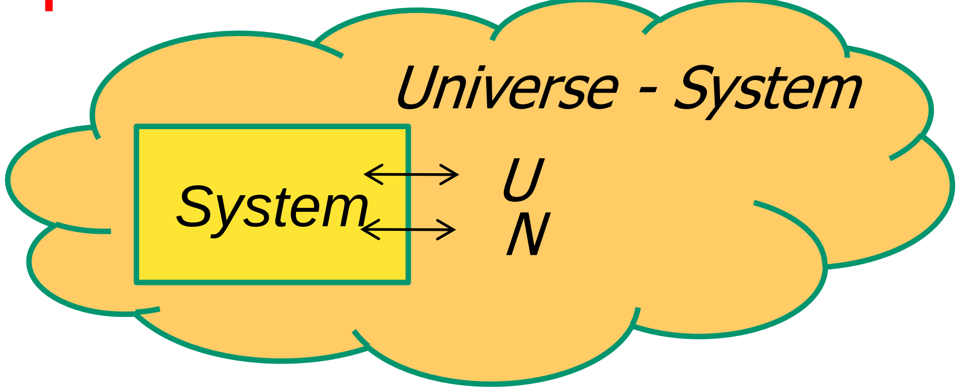
G(T,p,dN)

G(T,p,N+dN)

- So $G = \mu N = U - TS - pV \Rightarrow \boxed{U = TS - pV + \mu N}$

Thermodynamics III

- Now use $U = TS - pV + \mu N$ together with definition of grand potential $\Phi \equiv F - \mu N$:
- $\Phi = \Phi(T,p,\mu) \equiv F - \mu N = U - TS - \mu N = -pV$
- Consider system at fixed (T,p,μ) in equilibrium with rest of universe:
- $dS_{TOT} = dS_S + dS_{U\text{-}S}$



- $T\,dS_{U\text{-}S} = dU_{U\text{-}S} + p\,dV_{U\text{-}S} - \mu\,dN_{U\text{-}S}$
  $= dU_{U\text{-}S} + \qquad - \mu\,dN_{U\text{-}S}$ (Since $V_{U\text{-}S}$ fixed)
- Then $dS_{TOT} = \quad dS_S + (dU_{U\text{-}S} - \mu\,dN_{U\text{-}S})/T$
  $= (T\,dS_S - dU_S \quad + \mu\,dN_S \quad)/T = \boxed{- d\Phi/T}$

Exercise: Work through this argument

Alternative History

- So the “perfect fluid” observed at RHIC with

$$\left(\frac{\eta}{s}\right)_{RHIC} \sim 0.1\,\hbar$$

was immediately recognized as confirming the 1985 uncertainty principle estimate of Danielewicz and Gyulassy

- Except that’s not what happened…

Instead …

- In 2003-4 a new ~~estimate~~ (bound?) appeared from the AdS/CFT correspondence
  in string theory (!):
  - *A Viscosity Bound Conjecture*,
    P. Kovtun, D.T. Son, A.O. Starinets,
    hep-th/0405231

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi} \sim 0.08\hbar$$

in a rigorous calculation with no (apparent)
appeal to the uncertainty principle.

Theoretical Discovery 2003-4

- An ~~estimate~~ (bound?) on viscosity appeared from string theory's AdS/CFT correspondence:
  - *A Viscosity Bound Conjecture*,
    P. Kovtun, D.T. Son, A.O. Starinets,
    hep-th/0405231 (1300+ citations!)

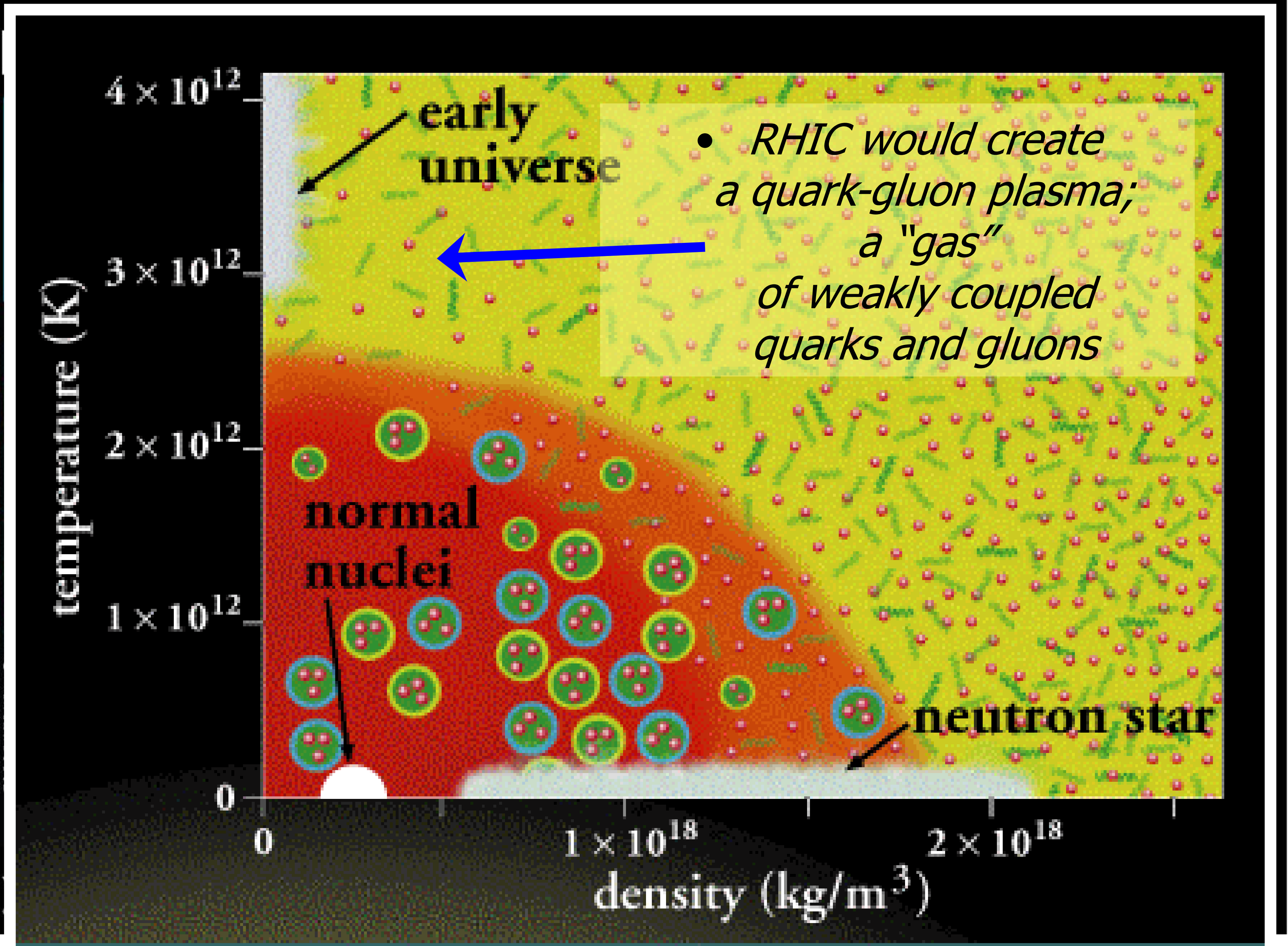
$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi} \sim 0.08\hbar$$

$\Rightarrow$ *Fundamental* measure of strong coupling

$\Rightarrow$ Cleanest result from gauge/gravity duality

$\Rightarrow$ A measure of "quantum liquidity"

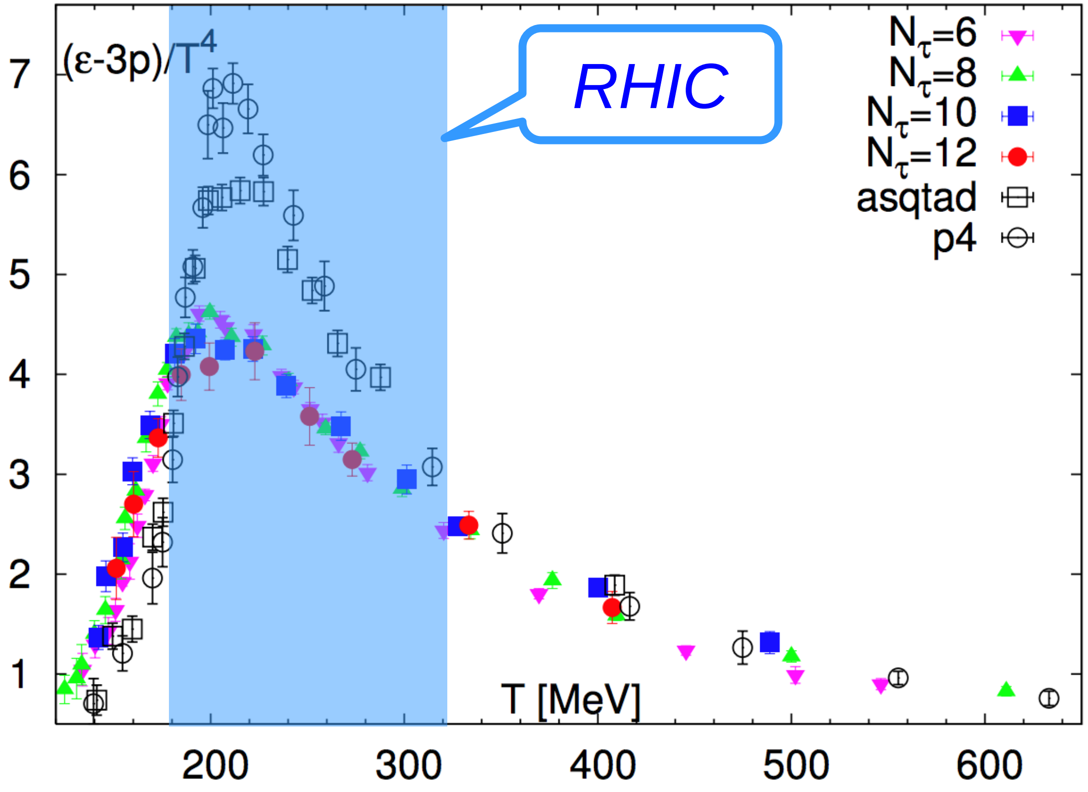
Expectations circa 2000



- *RHIC would create a quark-gluon plasma; a "gas" of weakly coupled quarks and gluons*

The Key Technique

- Working at the *strongest* possible coupling
- Tune temperature to produce ~"most liquid" QGP



- Cross sections as large as allowed by unitarity (?)

- *The Equation of State in 2+1 QCD*, A. Bazavov *et al.*, Phys. Rev. **D90** 094503 (2014), arXiv:1407.6387