

Mode coupling instability of the colliding beam

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@ Pierre and Marie Curie University & CERN



Supervised by Dr. Xavier Buffat

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D. Amorim and Dr. E. Metral

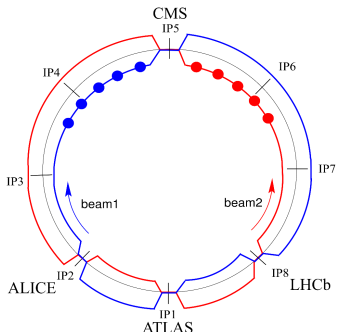
...and thanks to EPFL for providing parallel computing resources



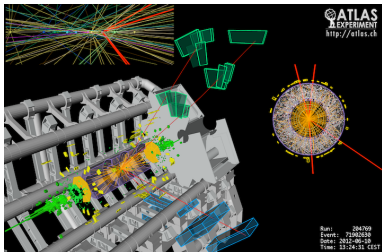
Monday 11th September, 2017 - HSC Meeting - CERN, Geneva

Introduction - the Large Hadron Collider (LHC)

- The purpose of this machine is to guide, **accelerate** hadron close to the speed of light and **collide** them.



$$\frac{dR}{dt}_p = \mathcal{L}\sigma_p$$



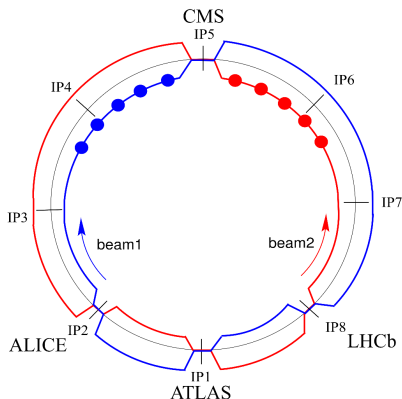
$H^0 \rightarrow ZZ \rightarrow 4\mu^-$ event

- Hard Collision event rate p , noted $\frac{dR}{dt}_p$, is link to the luminosity \mathcal{L} and the cross section of the event :

$$\mathcal{L} = \frac{n_b \omega_0 N^2}{4\pi\sigma^2}$$

Introduction - High Luminosity upgrade

- The goal of the High Luminosity Large Hardron Collider is to **increase the Luminosity by a factor 10.**

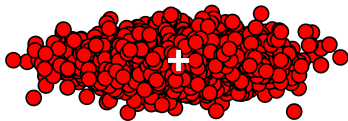
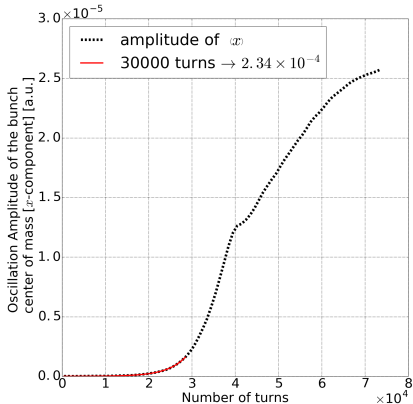


$$\mathcal{L} = \frac{n_b \omega_0 N^2}{4\pi\sigma^2}$$

- This increasing will be realised by **reduce the size** of the beam at the IP and **increase the intensity.**
- This reducing of the size will be reached by **upgrading the final focusing magnets.**
- This increasing of the intensity will be reached by **upgrading the injector.**

Introduction - Instability

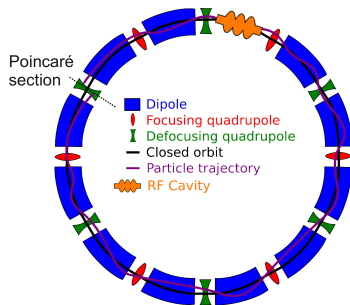
- The **electromagnetic interaction** of the beam with the **surroundings** and with the **opposite beam** could be made the beam **unstable**.



- This coherent instability can **increase the beam size and reduce the Luminosity**.
- The incoherent motion of the particle of the beam can, on particular condition, **stabilize the beam and prevent this instability**.

The effect of magnetic lattice and the electromagnetic cavity (RF cavity) could be describe :

$$\underline{x} = m_{dip} \circ \dots \circ m_{RF} \circ \dots \circ m_F \circ m_D \underline{x}_0 = M_{oneturn} \underline{x}_0$$

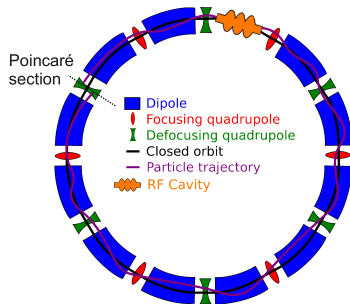


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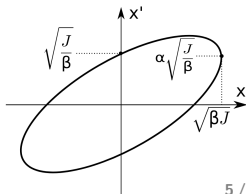
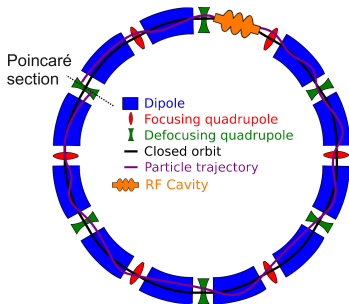
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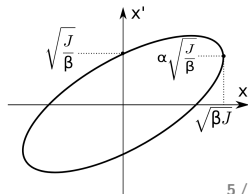
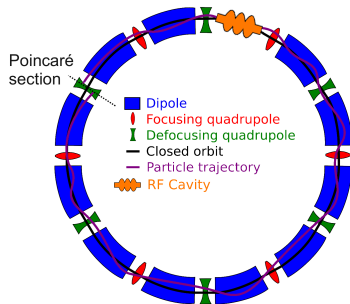
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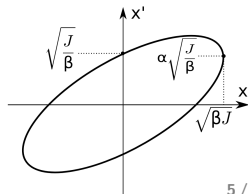
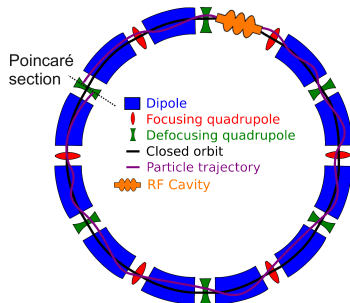
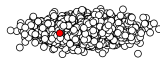
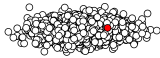
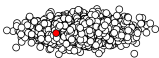
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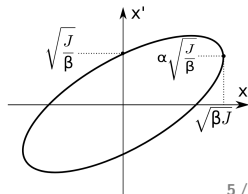
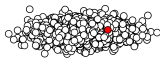
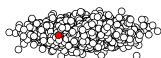
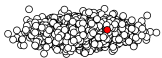
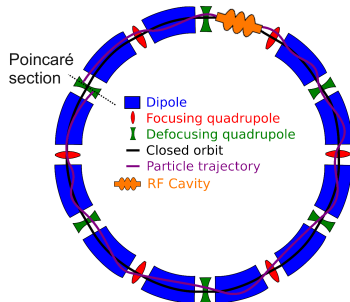
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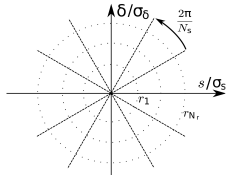
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The **Circulant Matrix Model** is the discretisation of the **longitudinal phase space** of the bunch



$$\begin{pmatrix} x_1^1(s, \delta) \\ x_1^1(s, \delta) \\ \dots \\ x_{N_s}^{N_s}(s, \delta) \\ x_{N_r}^{N_s}(s, \delta) \\ y_1^1(s, \delta) \\ y_1^1(s, \delta) \\ \dots \\ y_{N_s}^{N_s}(s, \delta) \\ y_{N_r}^{N_s}(s, \delta) \\ y_{N_r}^{N_s}(s, \delta) \end{pmatrix} = M \begin{pmatrix} x_1^0(s, \delta) \\ x_1^0(s, \delta) \\ \dots \\ x_{N_s}^0(s, \delta) \\ x_{N_r}^0(s, \delta) \\ y_1^0(s, \delta) \\ y_1^0(s, \delta) \\ \dots \\ y_{N_s}^0(s, \delta) \\ y_{N_r}^0(s, \delta) \\ y_{N_r}^0(s, \delta) \end{pmatrix}$$

$$M = W^{-1}DW$$

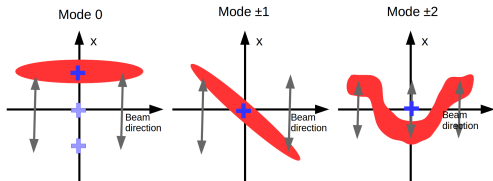
$$\lambda_n = e^{-2\pi i Q_{coh, n_a}}$$

$Re(Q_{coh, n_a})$: the **frequency** of the mode

$Im(Q_{coh, n_a})$: the **growth rate** of the mode

If we consider $M = M_{oneturn}$, the degenerate spectrum have this form :

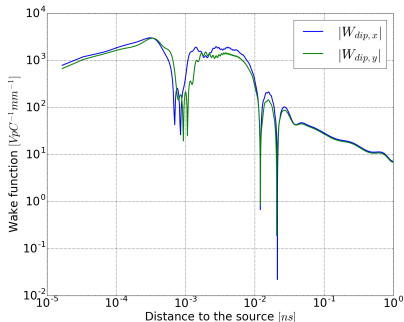
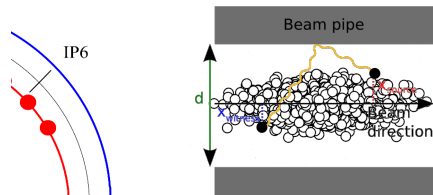
$$Q = \left\{ Re(Q_{coh, n}) = \pm(Q_\beta + n_a Q_s) : n_\alpha \in \mathbb{Z}, n_a < \left\lfloor \frac{N_s - 1}{2} \right\rfloor \right\}$$



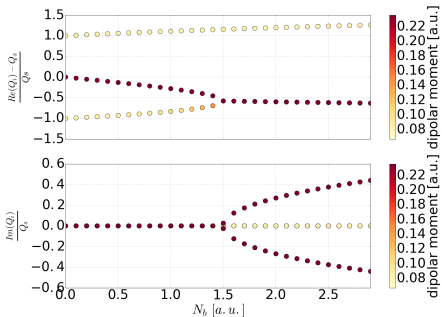
This is the so called **head-tail mode basis**

The dipolar moment : $\langle x \rangle_\tau = \frac{1}{\tau N} \sum_\tau \sum_{bunch} x$

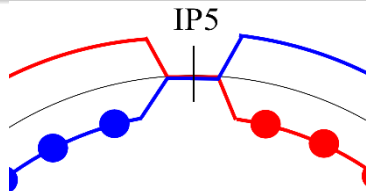
- The particle of the **head** of the bunch could become **coupled** with the particle of the **tail** due to the **electromagnetic interaction** with the surroundings



- This wake **deform** the head-tail basis of normal oscillation of the bunch



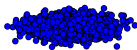
- The **TMCI** is the **coupling** between the **0** and **-1** head-tail mode





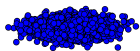


$$\Delta x'_{coh}(x, y) \underset{\text{linearized}}{=} \Delta x'_{coh}(x_0, y_0) + \frac{\partial \Delta x'_{coh}}{\partial x}(x_0, y_0) \Delta x$$



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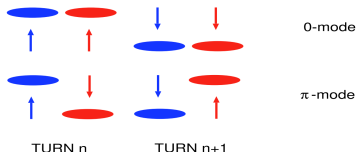
- The two bunch colliding are coupled by this coherent force and the system has two mode of oscillation

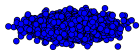


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$$\begin{cases} Q_\sigma = Q_\beta \\ Q_\pi = Q_\beta + \frac{\beta^* \frac{\partial \Delta x'_{coh}}{\partial x}(x_0, y_0)}{2\pi} = Q_\beta + \xi \end{cases}$$

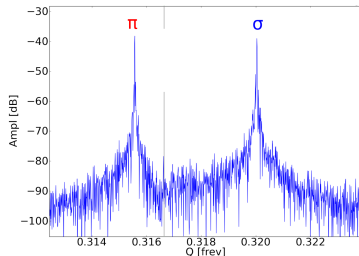
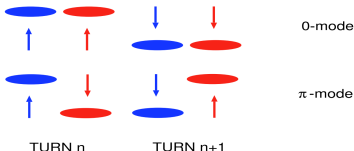


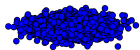


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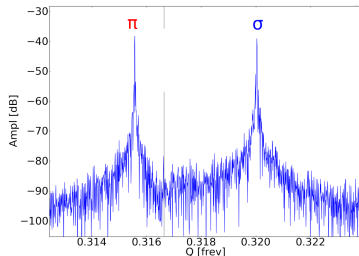
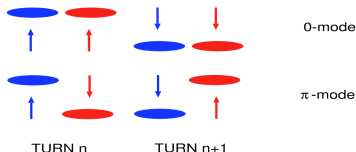




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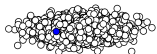


- This two mode could be **coupled** with oscillation mode driven by the wake field to create the so-called mode coupling instability of the colliding beam.

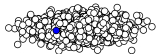
Incoherent tune spread



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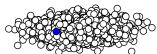


Incoherent tune spread



$$\Delta r'(r) \underset{\text{Head-on}}{=} \frac{2Nr_0}{\gamma r} \left[1 - e\left(-\frac{r^2}{2\sigma^2}\right) \right] \underset{r \rightarrow 0}{=} -\frac{N_b r_0}{\gamma \sigma^2} r$$

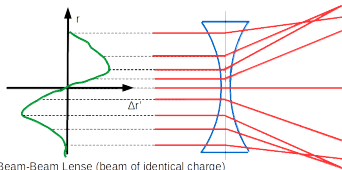
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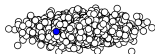
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$$\xi = \Delta Q_{bb} = \frac{1}{4\pi} \frac{\beta^*}{f} = \frac{N_b r_0 \beta^*}{4\pi \gamma \sigma^2}$$



Beam-Beam Lens (beam of identical charge)

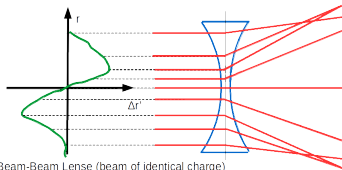
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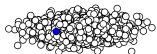
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- Due to the **non linearity** of the beam-beam force the particle experienced a different kick **depending of their amplitude** generates a **tune spread** of frequency of the **incoherent motion** of the particle.

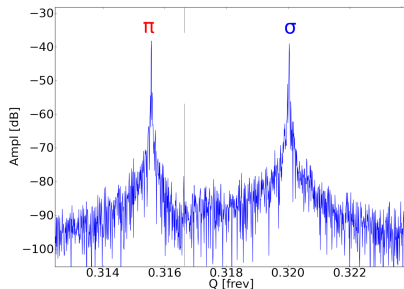
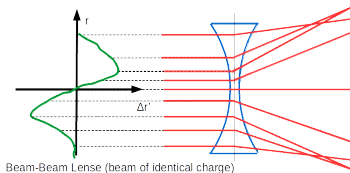
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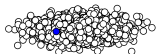
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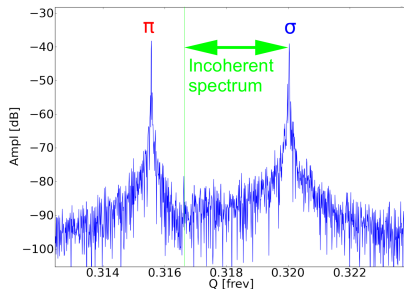
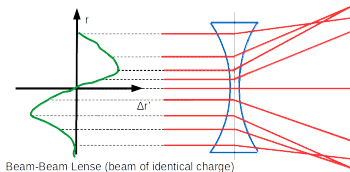


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$$Q_\pi = Q_x + \Lambda_{yokoya} \xi$$

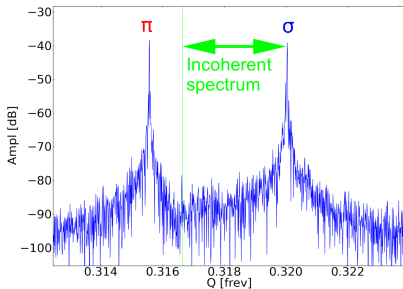
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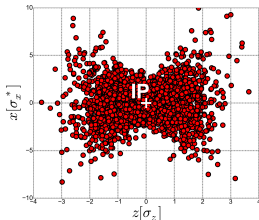
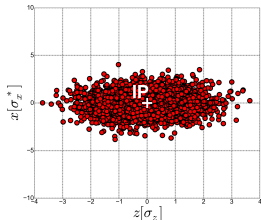
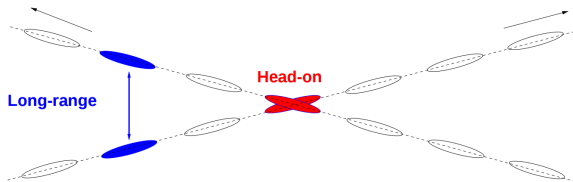
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Landau Damping



- This incoherent motion can stabilize and prevent instability thanks to the **Landau Damping mechanism**.
- This mechanism is similar to Landau Damping in Plasma physics.
- The Landau Damping is strongly dependant to the distribution of the incoherent motion.

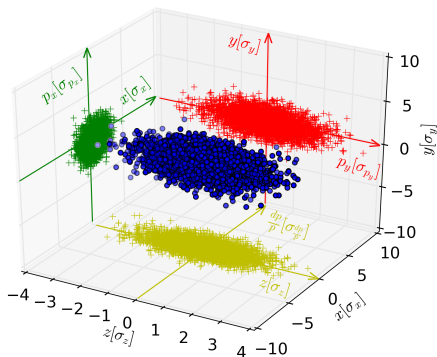
Interaction Point Configuration



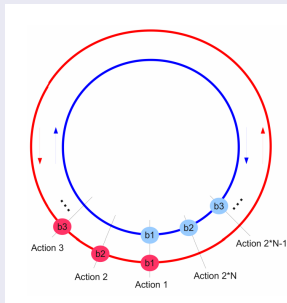
- Due to the long range effect and to have only **one-one bunches colliding**, a crossing angle are induce at the IP.
- The focusing of the bunch at the IP **deform longitudinally** the bunch, this phenomenon is called **hourglass**

- The large crossing angle and hourglass effect allow **synchrotron coupling** : **modulation** of the incoherent tune spread (transverse) by the synchrotron tune (longitudinal)

COMBI, a 6D macro-particle code

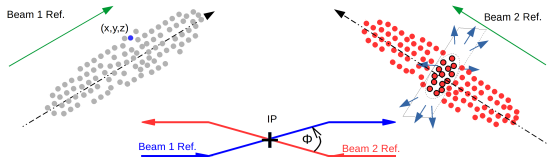


Actions



- Lattice Action already implemented
- Wake field Action already implemented
- 4D beam-beam kick already implemented

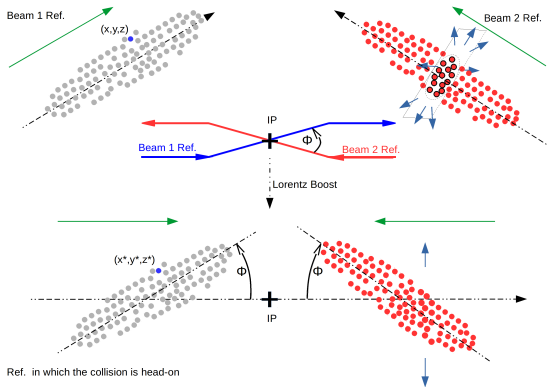
Boost - 6D kick - Anti boost



Steps of symplectic 6D soft gaussian action

$$(x, p_x, y, p_y, z, \frac{dp}{p})$$

Boost - 6D kick - Anti boost

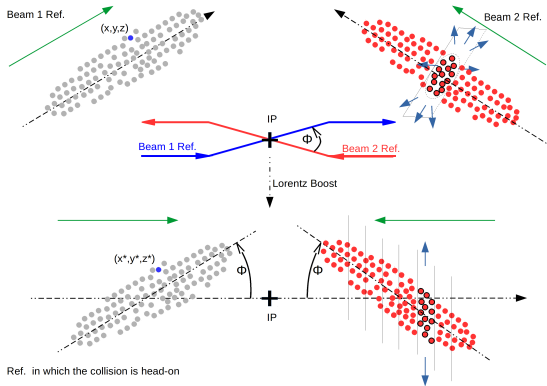
Steps of symplectic
6D soft gaussian action

$$(x, p_x, y, p_y, z, \frac{dp}{p})$$

Lorentz Boost

$$(x^*, p_x^*, y^*, p_y^*, z^*, \frac{dp}{p}^*)$$

Boost - 6D kick - Anti boost

Steps of symplectic
6D soft gaussian action

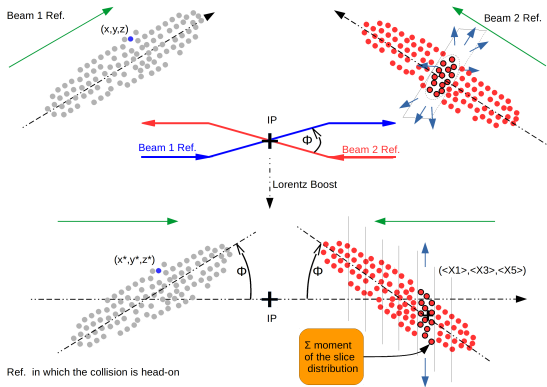
$$(x, p_x, y, p_y, z, \frac{dp}{p})$$

Lorentz Boost

$$(x^*, p_x^*, y^*, p_y^*, z^*, \frac{dp^*}{p^*})$$

Longitudinal Slicing

Boost - 6D kick - Anti boost

Steps of symplectic
6D soft gaussian action

$$(x, p_x, y, p_y, z, \frac{dp}{p})$$

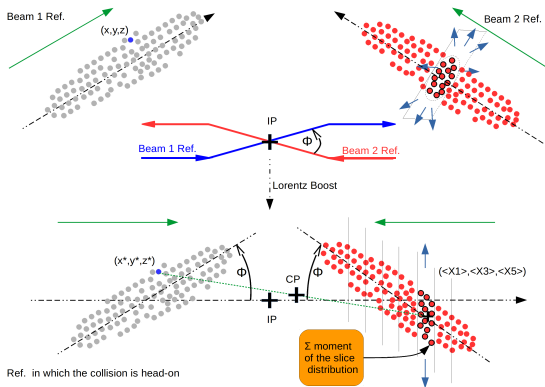
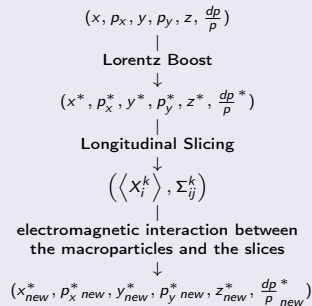
Lorentz Boost

$$(x^*, p_x^*, y^*, p_y^*, z^*, \frac{dp}{p}^*)$$

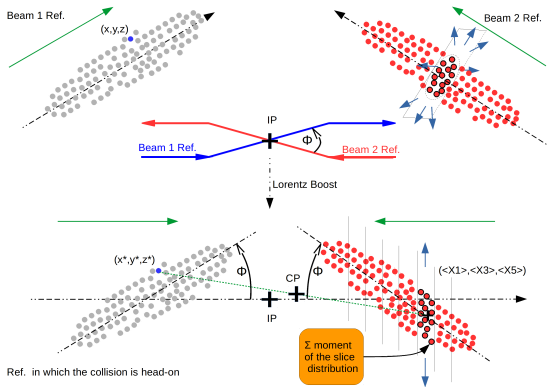
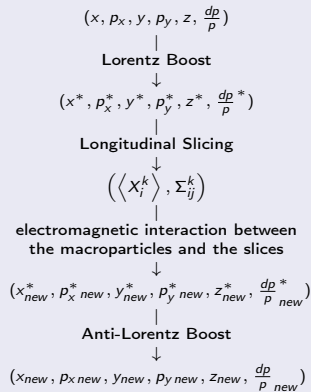
Longitudinal Slicing

$$(\langle X_i^k \rangle, \Sigma_{ij}^k)$$

Boost - 6D kick - Anti boost

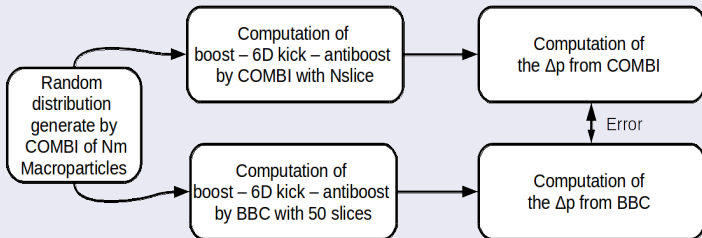
Steps of symplectic
6D soft gaussian action

Boost - 6D kick - Anti boost

Steps of symplectic
6D soft gaussian action

Convergence method

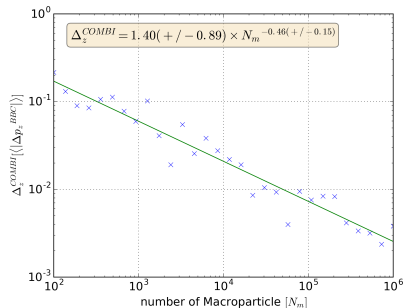
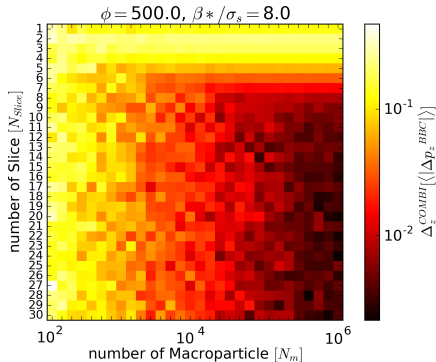
Benchmark method



Error Definition

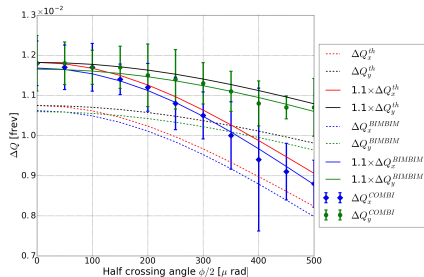
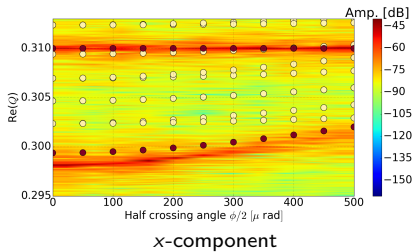
$$\Delta_i^{COMBI} = \frac{\langle |\Delta p_i^{COMBI} - \Delta p_i^{BBC}| \rangle}{\langle |\Delta p_i^{BBC}| \rangle}$$

Global view of the convergence



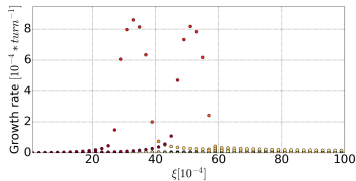
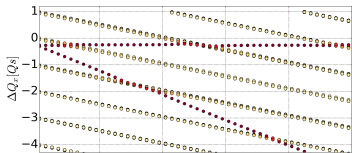
- Statistical convergence in term of Macro-particles (in $\frac{1}{\sqrt{n}}$)
- For more than 10 longitudinal slices, we reach the statistical convergence

Spectrogram COMBI and Modal decomposition from BIMBIM



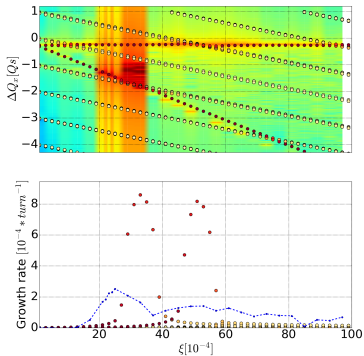
- $\Lambda_{Yokoya} \approx 1.1$ in agreement with the theory for soft gaussian kick approach
- For more than $\frac{\phi}{2} = 300$, we observe a decrease of this Yokoya factor, meaning the π -mode become closer of the incoherent spectrum
- The possible crossing of the π -mode and the incoherent spectrum enable the Landau Damping of this mode

Mode coupling instability of the colliding beam in 4D approach



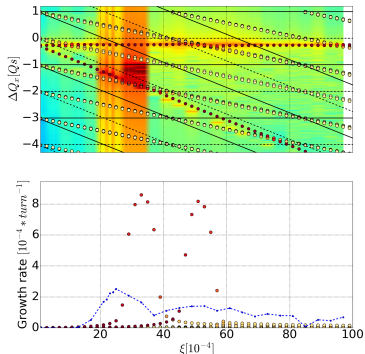
- Mode coupling between the beam-beam mode and the mode of oscillation driven by the wake field
- Strong coupling instability between the -1 mode with the π -mode and the 1 mode with the σ -mode

Mode coupling instability of the colliding beam in 4D approach



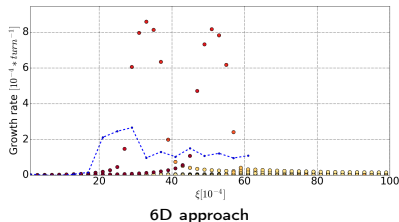
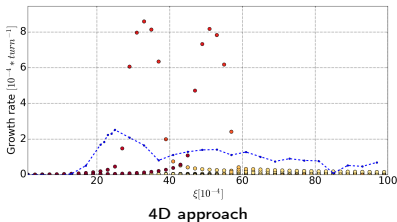
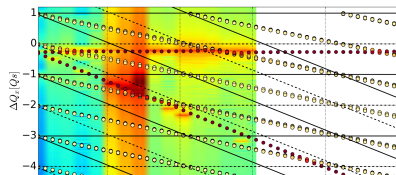
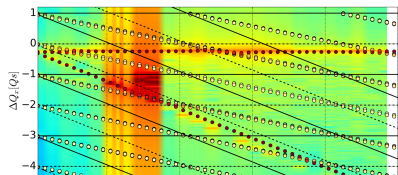
- Mode coupling between the beam-beam mode and the mode of oscillation driven by the wake field
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- The shifting between the modal and macro-particle approach could be explain by the non linearity and taking to account of the Yokoya factor

Mode coupling instability of the colliding beam in 4D approach



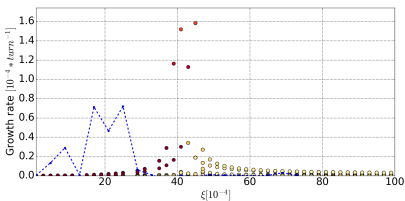
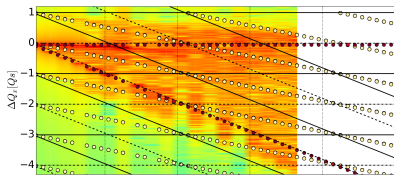
- Mode coupling between the beam-beam mode and the mode of oscillation driven by the wake field
- Strong coupling instability between the -1 mode with the π -mode and the 1 mode with the σ -mode
- The shifting between the modal and macro-particle approach could be explain by the non linearity and taking to account of the Yokoya factor
- Damping by the first sideband incoherent spectrum is not enough strength to completely dump the instability
- The high order coupling instability (for $\xi > 0.006$) observed only in the macro-particle approach

Comparison 4D and 6D soft gaussian kick

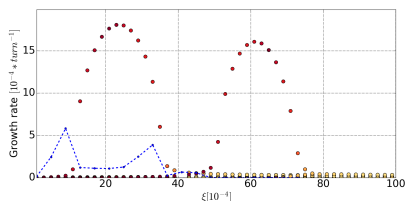
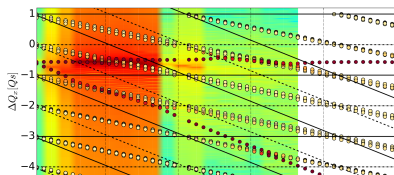


- In the case of low hourglass effect the 6D approach must follow the 4D approach

Influence of the strength of the Wake field

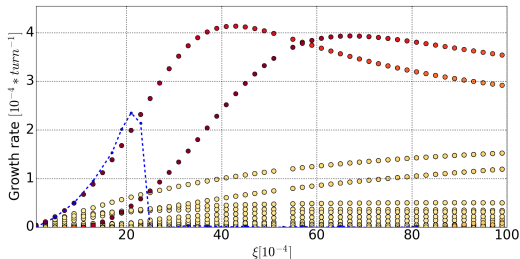
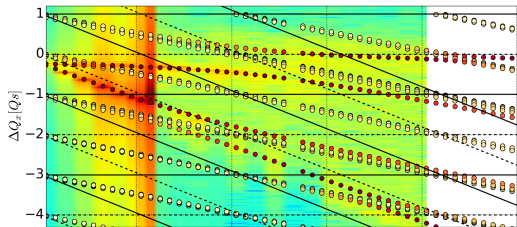


weak wake - 0.5×10^{11} p/b



strong wake - 4.5×10^{11} p/b

- For low intensity, the damping due to the incoherent spectrum of the main side-band is not enough efficient because the σ -mode is at the border of the side-band
- For high intensity, the damping of this side-band is efficient of the σ -mode due to the large tune shift due to the strong intensity of the wake.

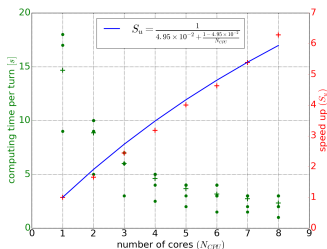
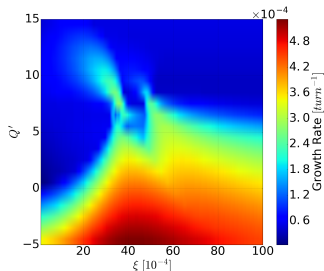
Impact of the hourglass, $\beta^* = 0.2m$ 

- In case of hourglass effect, the mode coupling instability of colliding beam start for a low ξ
- The behaviour of the growth match for the two approach, in case of low ξ
- When the σ and π mode enter on the respective incoherent spectrum of the first side band the Landau Damping is efficient to damp completely the instability for $\xi > 0.0021 \approx Q_s$

Conclusion

- The 6D soft-Gaussian kick was implemented and benchmarked (statistical convergence against BBC, Yokoya factor and similarity with 4D) in COMBI
- A damping mechanism for intense wake have been observed due to a large tune shift of the σ mode in the main side band
- A efficient damping mechanism by the first sideband was observed in case of strong hourglass

Outlook



- For high chromaticity, the mode coupling stay unstable
- For $G > 2 \times 10^{-3}$, the main mode coupling are efficiently damped
- For large crossing angle, the strong betatroncoupling deform the head tail basis and allow high order coupling.
- The modeling of the very low instability (like with high gain or chromaticity) will be a computational problem to model with the macroparticle approach
- The convergence issue of the high order coupling in the modal approach need to be investigated

Single beam behaviour

Beam-beam effect and Landau Damping

6D beam-beam soft-Gaussian kick

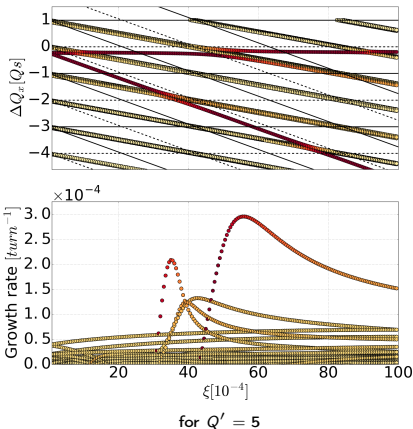
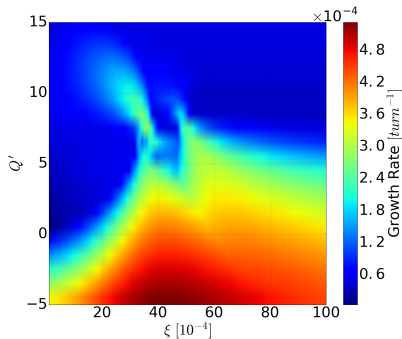
Head-Tail and beam-beam mode coupling instability

Influence of the strength of the Wake field
Impact of the strong hourglass effect

Any Questions?

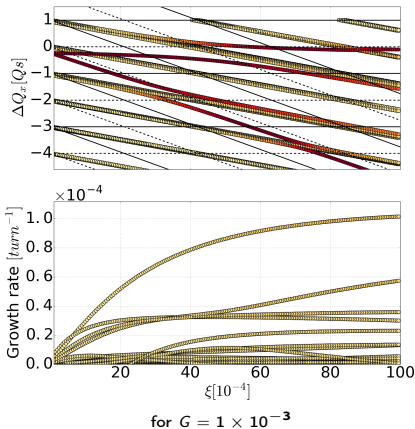
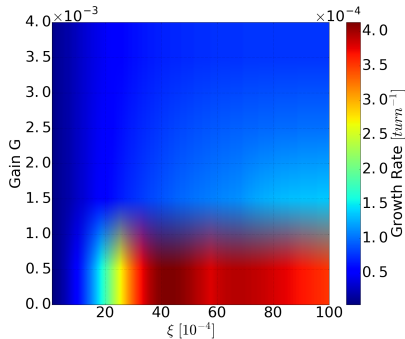
Thank you for your attention

Influence of the chromaticity



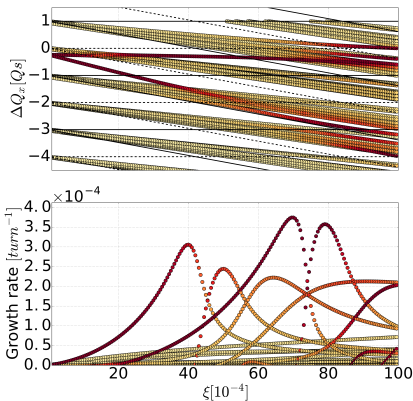
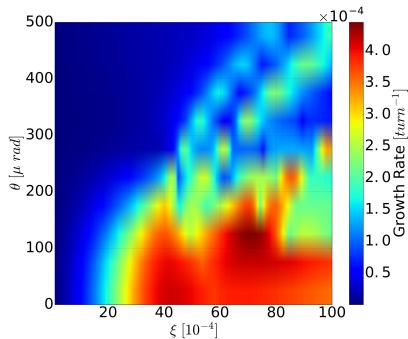
- For negative chromaticity, the head-tail mode are unstable (single beam theory)
- For large positive chromaticity, we reach a area of stability
- Nevertheless, the two mode coupling are shifted for larger ξ and stay unstable even for high chromaticity

Influence of the gain



- For $G > 2 \times 10^{-3}$, all unstable instability seem efficiency damped
- For $G = 1 \times 10^{-3}$, the two mode coupling with dipolar component are damped.

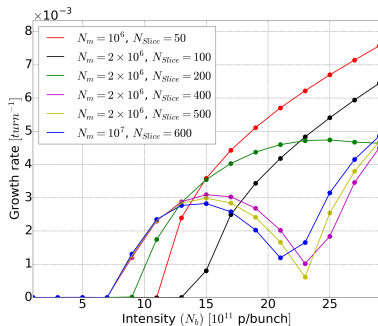
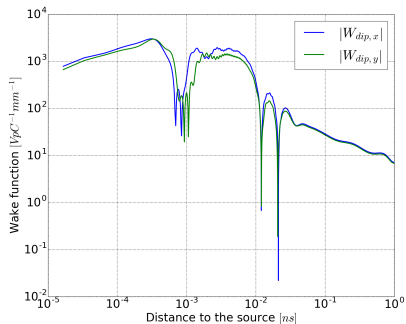
Influence of the crossing angle



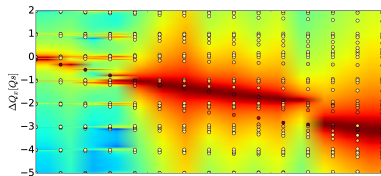
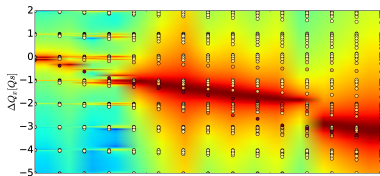
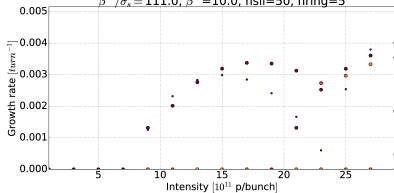
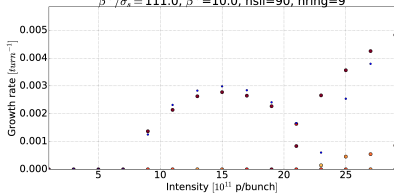
for $\phi = 150 \mu\text{rad}$

- For large crossing angle, the mode coupling instability are shifted for granter ξ
- A strong synchrotron coupling deform the head tail basis and the radial mode become spited
- The convergence of the modal approach need to cheked for high order coupling

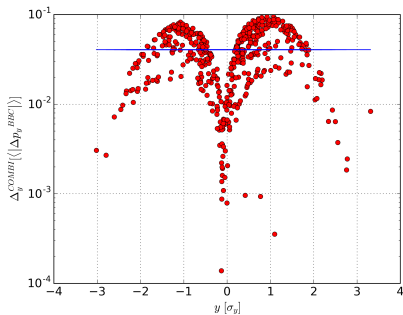
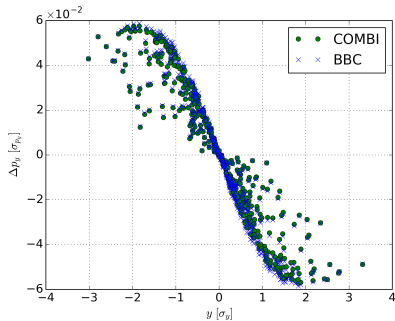
TMCI convergence



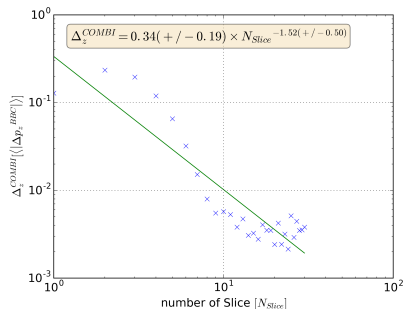
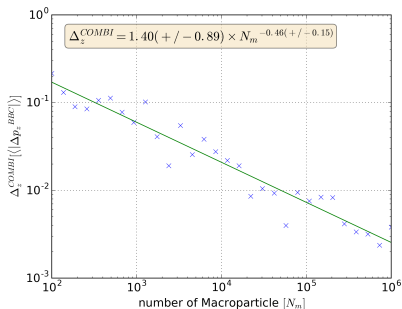
TMCI convergence


 $\beta^*/\sigma_x = 111.0, \beta^* = 10.0, \text{nsli} = 50, \text{nring} = 5$

 $\beta^*/\sigma_x = 111.0, \beta^* = 10.0, \text{nsli} = 90, \text{nring} = 9$


6D Kick convergence

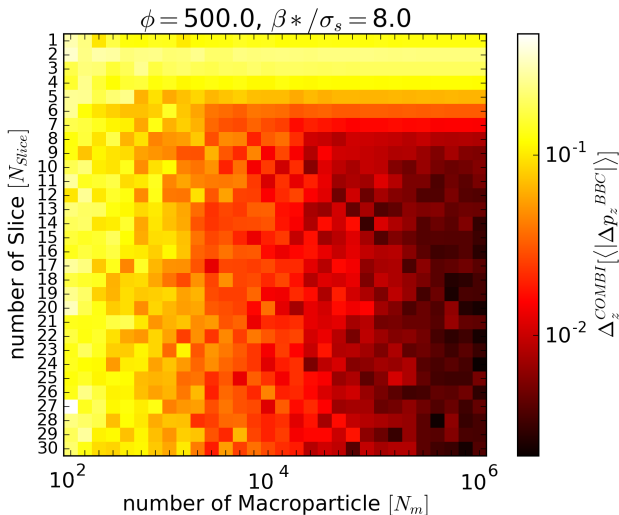


Result of convergence

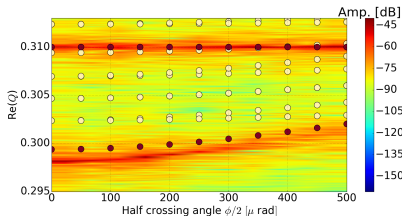


- Statistical convergence in term of Macro-particles (in $\frac{1}{\sqrt{n}}$)
- For more than 10 longitudinal slices, we reach the statistical convergence

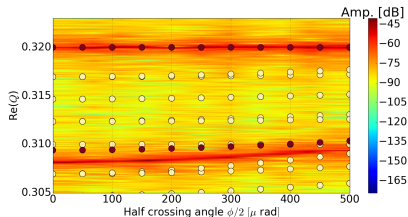
Global view of the convergence



Spectrogram COMBI and Modal decomposition from BIMBIM



x-component

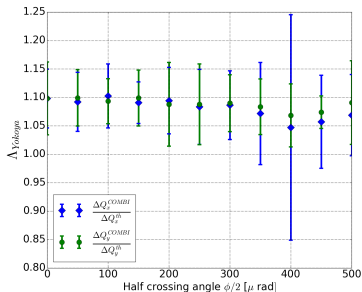
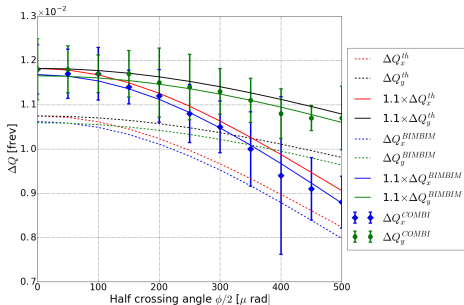


y-component

Comparison with theoretical tune spread

$$\Delta Q_x^{th} = \frac{N_{br} r_0 \beta_x^*}{2\pi \gamma \sigma_x^0 \sqrt{1 + P_x^2} \left(\sigma_x^0 \sqrt{1 + P_x^2} + \sigma_y \right)}$$

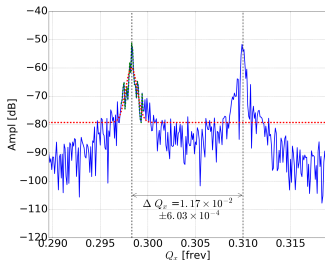
Yokoya Factor Observation



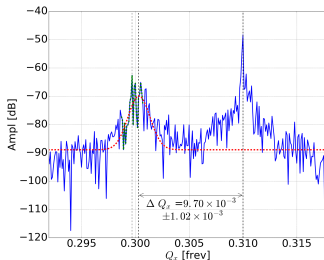
Conclusion of Yokoya Factor observation

- $\Lambda_{Yokoya} \approx 1.1$ in agreement with the theory for soft gaussian kick approach
- For more than $\frac{\phi}{2} = 300$, the error bar increase

This increasing of the error bar could be explain by ...



100 μ rad



300 μ rad

Hypothesis of the widening of the π -mode

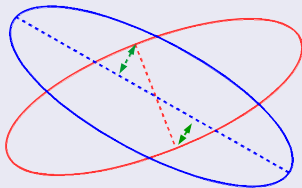
- π -mode seem outside the tune spread according the computation of Q^{th}
- Computation of the ΔQ^{th} could be not correct for the large crossing angle
- Possible Landau Damping due to the tune spread

Landau Damping in Accelerator Physics

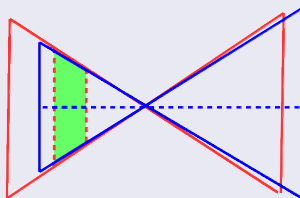
$$\langle x \rangle (t) = \frac{A}{2\omega_{\text{unp}}} \left(\cos(\omega_c t) P.V. \int_{-\infty}^{+\infty} \frac{\rho(\omega_i)}{\omega_i - \omega_c} d\omega_i + \pi \rho(\omega_c) \sin(\omega_c t) \right)$$

- The Landau damping depend of the distribution of the incoherent spectrum of the particle.
- More the density of incoherent spectrum of the particle is high near to the coherent frequency, more efficient is the damping.

Crossing Angle



Hourglass



Synchrotronbetatron coupling

- Oscillation of one bunch could influencing the longitudinal motion of the opposed bunch
- possible modulation of the spread (transverse) by the synchrotron tune (longitudinal)