Mode coupling instability of the colliding beam

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Supervised by Dr. Xavier Buffat

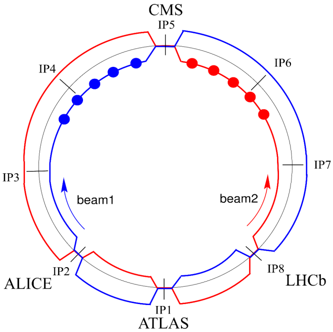
Many thanks to Dr. J. Barranco, Dr. S. Antipov, Dr. T. Pieloni, D. Amorim and Dr. E. Metral

...and thanks to EPFL for providing parallel computing resources

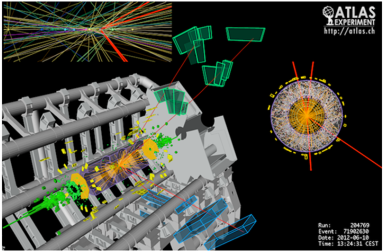
Tuesday 12th September, 2017 - UPMC, Paris

Introduction - the Large Hadron Collider (LHC)

- The purpose of this machine is to guide, **accelerate** hadron close to the speed of light and **collide** them.



$$\frac{dR}{dt}_p = \mathcal{L}\sigma_p$$



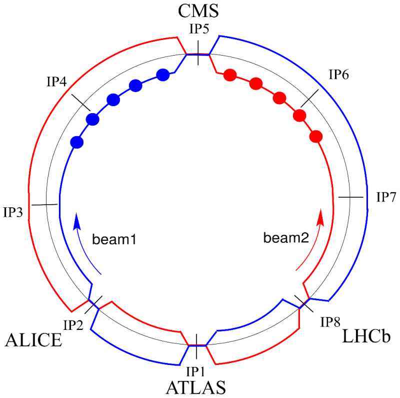
$H^0 \rightarrow ZZ \rightarrow 4\mu^-$ event

- Hard Collision event rate p, noted $\frac{dR}{dt}_p$, is link to the luminosity $\mathcal{L}$ and the cross section of the event :

$$\mathcal{L} = \frac{n_b f_0 N^2}{4\pi\sigma^2}$$

Introduction - High Luminosity upgrade

- The goal of the High Luminosity Large Hardron Collider is to **increase the Luminosity by a factor 10**.

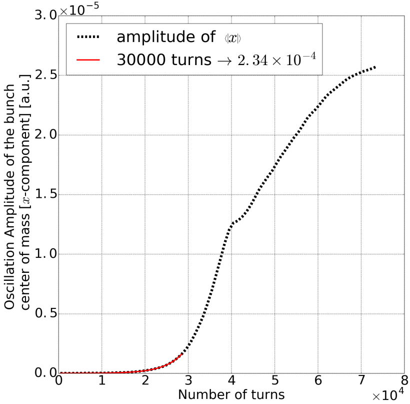


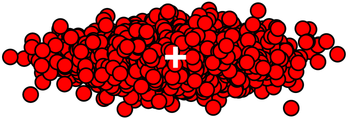
$$\mathcal{L} = \frac{n_b f_0 N^2}{4\pi\sigma^2}$$

- This increase will be realised by **reduicing the size** of the beam at the IP and **increasing its intensity**.
- This size reduction will be reached by **upgrading the final focusing magnets**.
- Increase of the intensity will be reached by **upgrading the injectors**.

Introduction - Instability

- The **electromagnetic interaction** of the beam with its **surroundings** and with the **opposite beam** can render the beam **instable**.

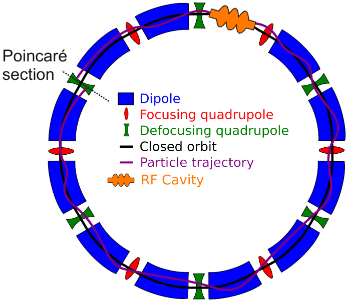




- This coherent instability can **increase the beam size and reduce the Luminosity**.
- The incoherent motion of the particle of the beam can, on particular conditions, **stabilize the beam and prevent instabilities**.

The effect of magnetic lattice and the electromagnetic cavity (RF cavity) could be described :

$$\underline{x} = m_{dip} \circ \ldots \circ m_{RF} \circ \ldots \circ m_F \circ m_D \underline{x_0} = M_{oneturn} \underline{x_0}$$



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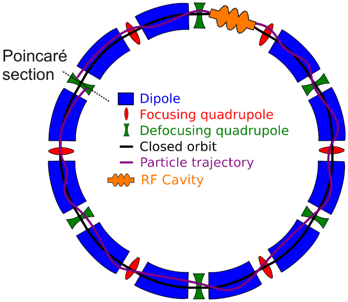

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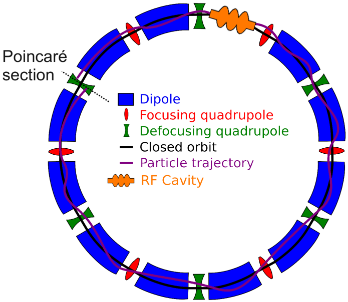
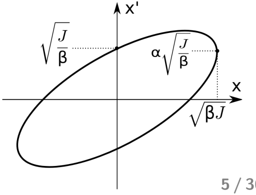
- The particle **oscillate transversely** around this closed orbit at the normalize frequency, called **betatron tune** :

$$Q_\beta = \frac{\omega_\beta}{\omega_0}$$





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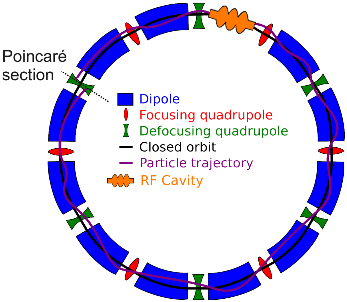

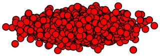

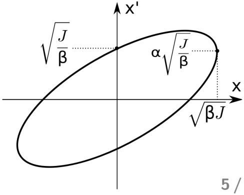

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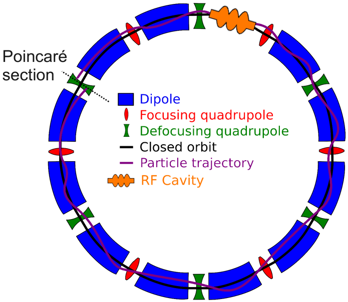

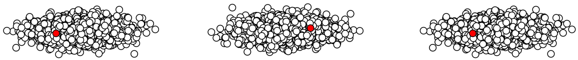

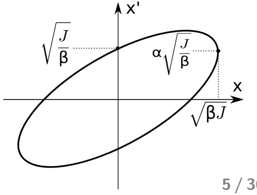

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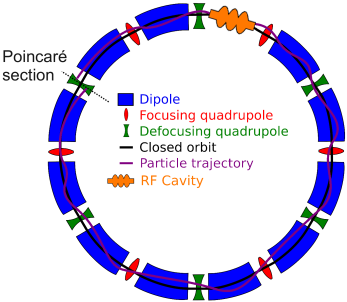
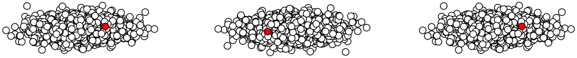
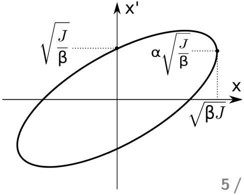
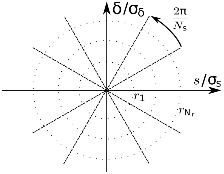






The **Circulant Matrix Model** is the discretisation of the **longitudinal phase space** of the bunch



$$M = W^{-1} D W$$

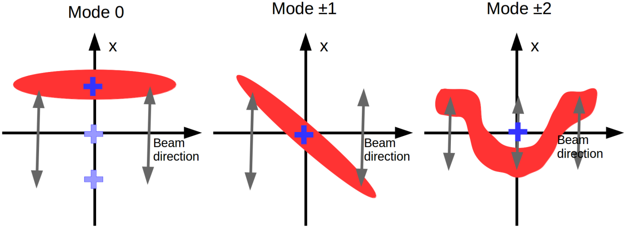
$$\lambda_n = e^{-2\pi i Q_{\mathbf{coh},n_a}}$$

$Re(Q_{\mathrm{coh},n_a})$: the **frequency** of the mode
$Im(Q_{\mathrm{coh},n_a})$: the **growth rate** of the mode

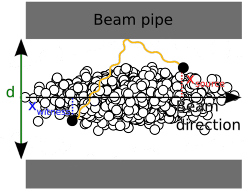
If we consider $M = M_{oneturn}$, the degenerate spectrum have this form :

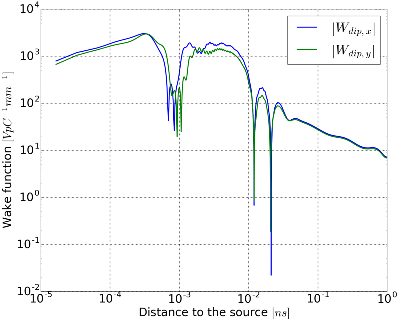
$$\mathbf{Q} = \left\{ Re(Q_{coh,n}) = \pm(Q_\beta + n_a Q_s) : n_\alpha \in \mathbb{Z}, n_a < \left| \frac{N_s - 1}{2} \right| \right\}$$

This is the so called **head-tail mode basis**
The dipolar moment : $\langle x \rangle_\tau = \frac{1}{\tau N} \sum_\tau \sum_{bunch} x$

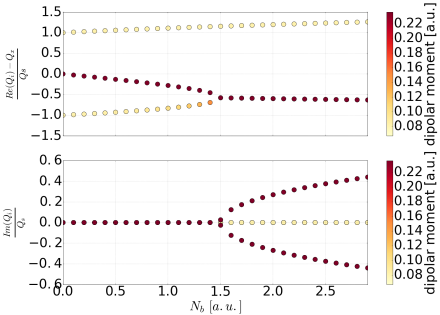


- The particle at the **head** of the bunch could become **coupled** with the particles at the **tail** due to the **electromagnetic interaction** with the surroundings

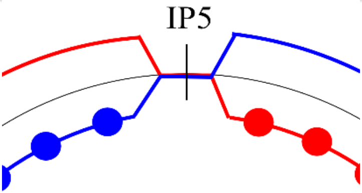


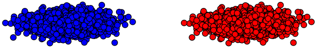


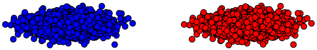
- This wake **deforms** the head-tail basis of normal oscillation of the bunch



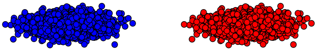
- The **TMCI** is the **coupling** between the **0 and -1** head-tail mode


IP5





$$\Delta x'_{coh}(x,y) \underset{\text{linearized}}{=} \Delta x'_{coh}(x_0,y_0) + \frac{\partial \Delta x'_{coh}}{\partial x}(x_0,y_0)\Delta x$$



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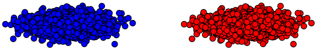
- The two bunch colliding are coupled by this coherent force and the system has two mode of oscillation



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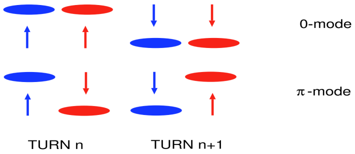
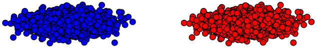
$$\begin{cases} Q_\sigma = Q_\beta \\ Q_\pi = Q_\beta + \dfrac{\beta^* \frac{\partial \Delta x'_{coh}}{\partial x}(x_0,y_0)}{2\pi} = Q_\beta + \xi \end{cases}$$





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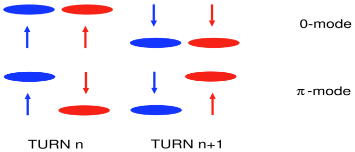

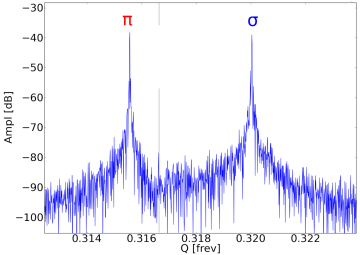

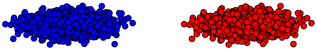

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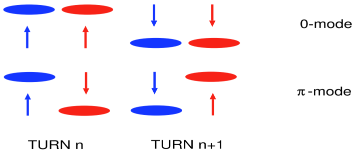
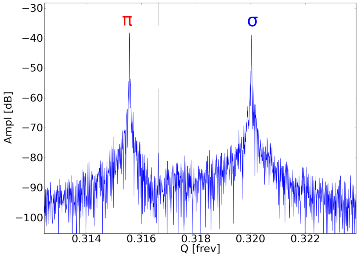
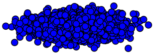
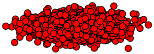




- This two mode could be **coupled** with oscillation mode driven by the wake field to create the so-called mode coupling instability of the colliding beam.

Incoherent tune spread





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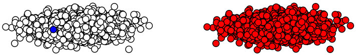

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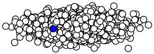

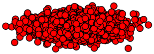

$$\Delta r'(r) \underset{Head-on}{=} \frac{2Nr_0}{\gamma r}\left[1 - e^{\left(-\frac{r^2}{2\sigma^2}\right)}\right] \underset{r\to 0}{=} -\frac{N_b r_0}{\gamma\sigma^2} r$$

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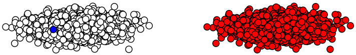

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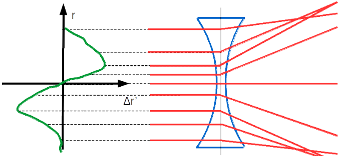
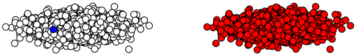
The **beam-beam parameter** is defined as :

$$\xi = \Delta Q_{bb} = \frac{1}{4\pi}\frac{\beta^*}{f} = \frac{N_b r_0 \beta^*}{4\pi\gamma\sigma^2}$$



Beam-Beam Lense (beam of identical charge)

Incoherent tune spread



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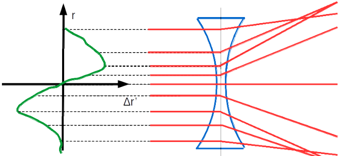
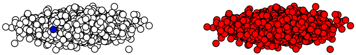


Beam-Beam Lense (beam of identical charge)

- Due to the **non linearity** of the beam-beam force, particles experience a different kick **depending their amplitude**, which generates a **tune spread** of the the **incoherent motion** of the particle.

Incoherent tune spread



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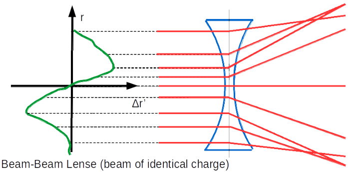

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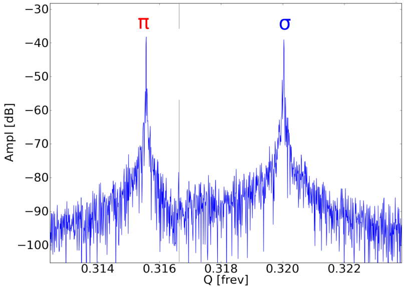

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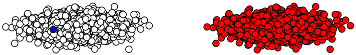

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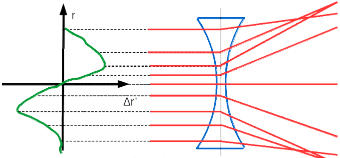
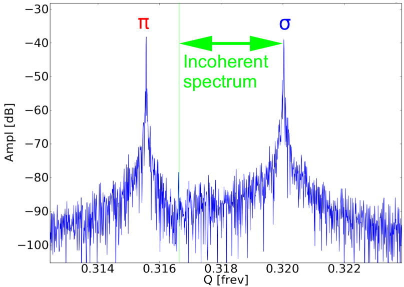
$$Q_\pi = Q_x + \Lambda_{\text{yokoya}}\xi$$

The **beam-beam parameter** is defined as :

$$\xi = \Delta Q_{bb} = \frac{1}{4\pi}\frac{\beta^*}{f} = \frac{N_b r_0 \beta^*}{4\pi\gamma\sigma^2}$$

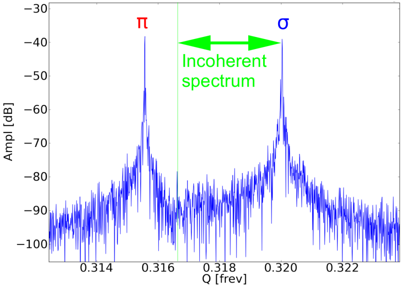


Beam-Beam Lense (beam of identical charge)



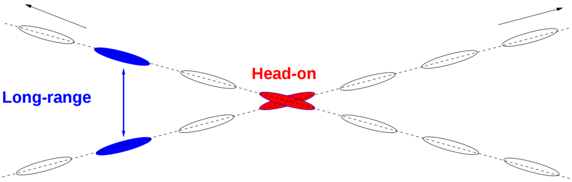
- Due to the **non linearity** of the beam-beam force, particles experience a different kick **depending their amplitude**, which generates a **tune spread** of the **incoherent motion** of the particle.

Landau Damping



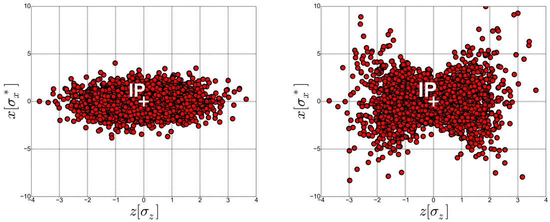
- This incoherent motion can stabilize and prevent instability thanks to the **Landau Damping mechanism.**
- This mechanism is similar to Landau Damping in Plasma physics.
- The Landau Damping is strongly dependant to the distribution of the incoherent motion.

Interaction Point Configuration



weak hourglass effect

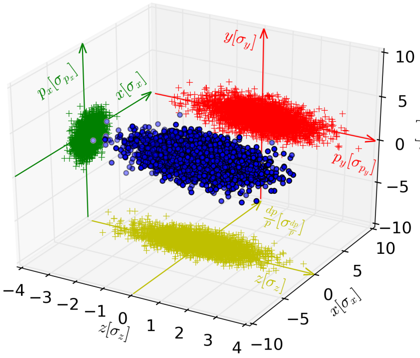
strong hourglass effect



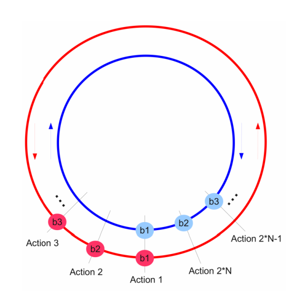
- Due to the long range effect and to have only **one-one bunches colliding**, crossing angles are induced at the IP.
- The focusing of the bunch at the IP **deforms it longitudinally**, this phenomenon is called **hourglass**

- The large crossing angle and hourglass effect allow **synchrobetatron coupling** : **modulation** of the incoherent tune spread (transverse) by the synchrotron tune (longitudinal)

COMBI, a 6D macro-particle code

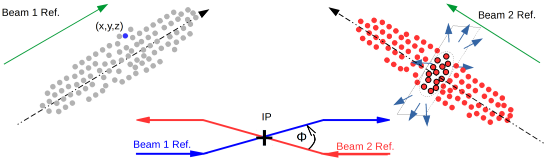


Actions



- Lattice Action already implemented
- Wake field Action already implemented
- 4D beam-beam kick already implemented

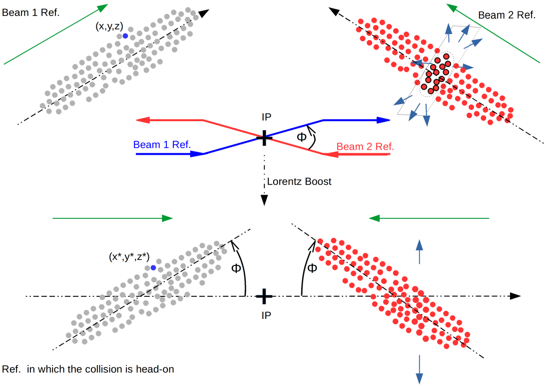
Boost - 6D kick - Anti boost



Steps of simpletic 6D soft gaussian action

$$(x, p_x, y, p_y, z, \frac{dp}{p})$$

Boost - 6D kick - Anti boost



Steps of simpletic 6D soft gaussian action

$$(x, p_x, y, p_y, z, \frac{dp}{p})$$

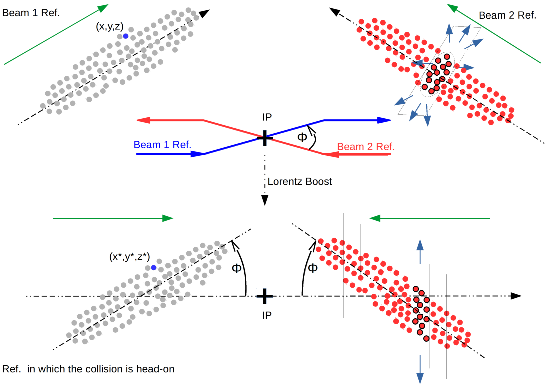
|

Lorentz Boost

↓

$$(x^*, p_x^*, y^*, p_y^*, z^*, \frac{dp}{p}^*)$$

Boost - 6D kick - Anti boost



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|

Lorentz Boost

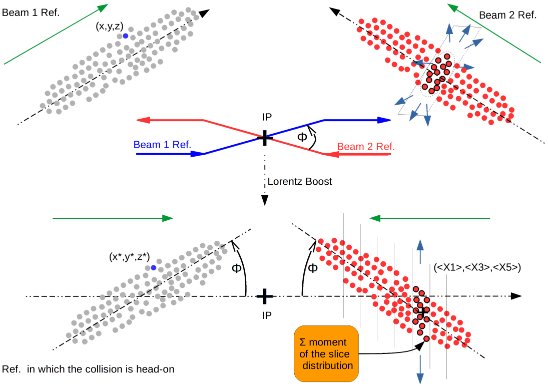
↓

$$(x^*, p_x^*, y^*, p_y^*, z^*, \frac{dp}{p}^*)$$

|

Longitudinal Slicing

Boost - 6D kick - Anti boost



Steps of simpletic 6D soft gaussian action

$$\left(x, p_x, y, p_y, z, \frac{dp}{p}\right)$$

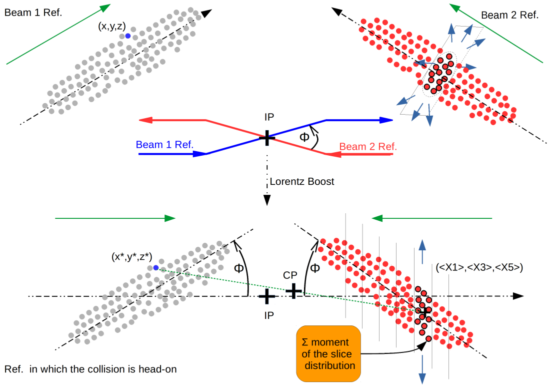
Lorentz Boost

$$\left(x^*, p_x^*, y^*, p_y^*, z^*, \frac{dp}{p}^*\right)$$

Longitudinal Slicing

$$\left(\left\langle X_i^k \right\rangle, \Sigma_{ij}^k\right)$$

Boost - 6D kick - Anti boost



Steps of simpletic 6D soft gaussian action

$(x, p_x, y, p_y, z, \frac{dp}{p})$

Lorentz Boost

$(x^*, p_x^*, y^*, p_y^*, z^*, \frac{dp}{p}^*)$

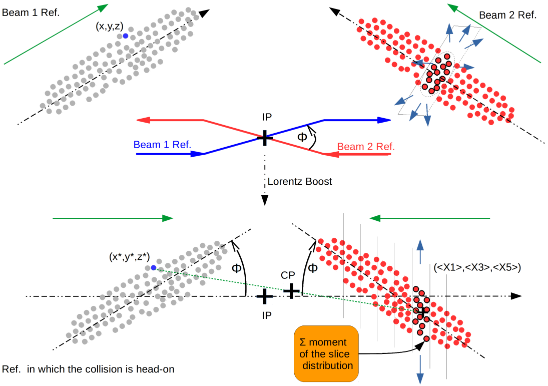
Longitudinal Slicing

$\left(\left\langle X_i^k \right\rangle, \Sigma_{ij}^k\right)$

electromagnetic interaction between the macroparticles and the slices

$(x_{new}^*, p_x^*{}_{new}, y_{new}^*, p_y^*{}_{new}, z_{new}^*, \frac{dp}{p}^*_{new})$

Boost - 6D kick - Anti boost



Steps of simpletic 6D soft gaussian action

$(x, p_x, y, p_y, z, \frac{dp}{p})$

Lorentz Boost

$(x^*, p_x^*, y^*, p_y^*, z^*, \frac{dp}{p}^*)$

Longitudinal Slicing

$\left(\left\langle X_i^k \right\rangle, \Sigma_{ij}^k\right)$

electromagnetic interaction between the macroparticles and the slices

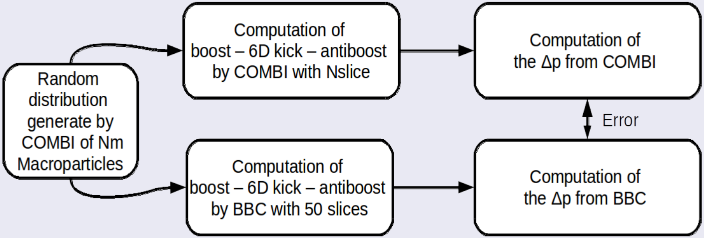
$(x^*_{new}, p_x^*{}_{new}, y^*_{new}, p_y^*{}_{new}, z^*_{new}, \frac{dp}{p}^*_{new})$

Anti-Lorentz Boost

$(x_{new}, p_{x\,new}, y_{new}, p_{y\,new}, z_{new}, \frac{dp}{p}_{new})$

Convergence method

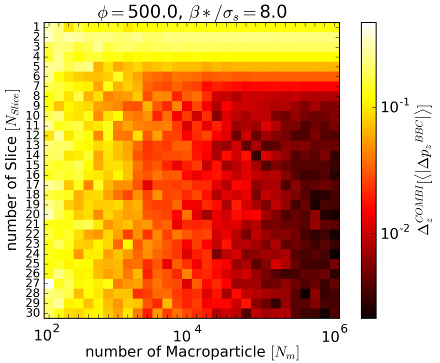
Benchmark method

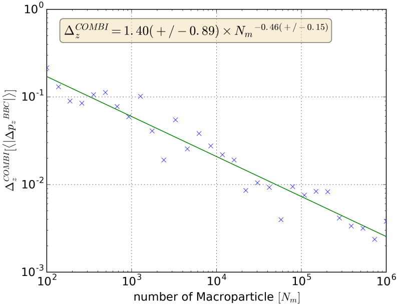


Error Definition

$$\Delta_i^{COMBI} = \frac{\left\langle \left| \Delta p_i{}^{\mathrm{COMBI}} - \Delta p_i{}^{\mathrm{BBC}} \right| \right\rangle}{\left\langle \left| \Delta p_i{}^{\mathrm{BBC}} \right| \right\rangle}$$

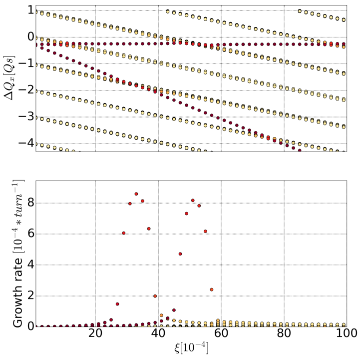
Global view of the convergence





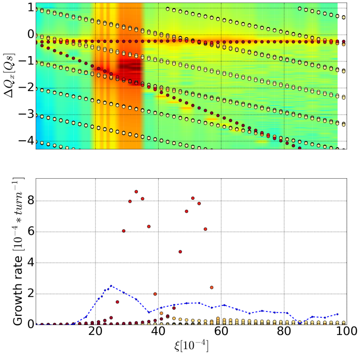
- Statistical convergence in term of Macro-particles (in $\frac{1}{\sqrt{n}}$)
- For more than 10 longitudinal slices, we reach the statistical convergence

Mode coupling instability of the colliding beam in 4D approach



- The wake effect stay constant during all the scan
- Mode coupling between the beam-beam mode and the mode of oscillation driven by the wake field
- Strong coupling instability between the -1 mode with the π-mode and the 1 mode with the σ-mode

Mode coupling instability of the colliding beam in 4D approach



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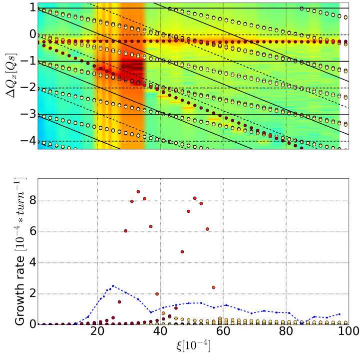
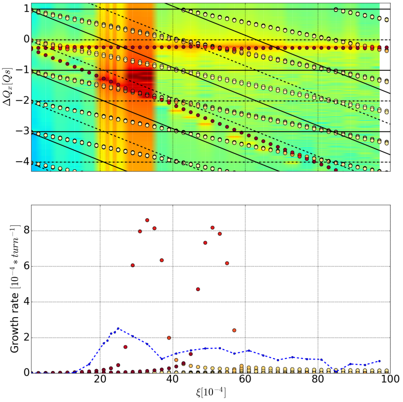
- The shift between the modal and macro-particle approach could be explain by the non linearity and taking to account of the Yokoya factor

Mode coupling instability of the colliding beam in 4D approach

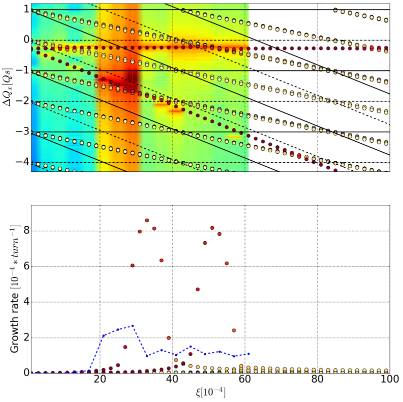


- The wake effect stay constant during all the scan
- Mode coupling between the beam-beam mode and the mode of oscillation driven by the wake field
- Strong coupling instability between the -1 mode with the π-mode and the 1 mode with the σ-mode
- The shift between the modal and macro-particle approach could be explain by the non linearity and taking to account of the Yokoya factor
- Damping by the first sideband incoherent spectrum is not strong enough to completely damp the instability
- The high order coupling instability (for $\xi > 0.006$) is only observed in the macro-particle approach

Comparaison 4D and 6D soft gaussian kick



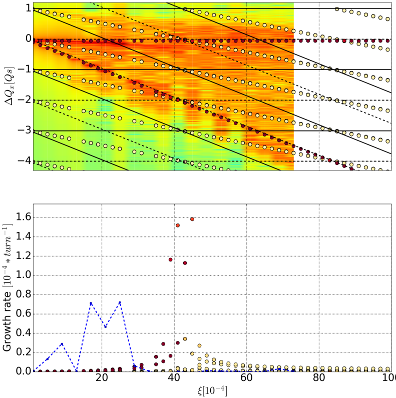
4D approach



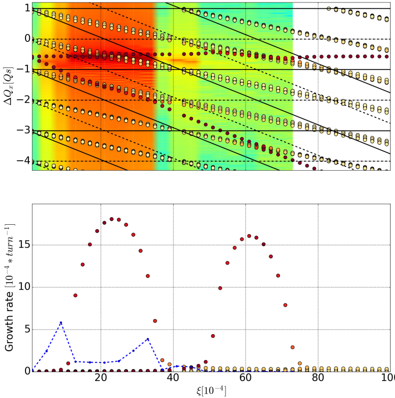
6D approach

- In the case of low hourglass effect the 6D approach must follow the 4D approach

Influence of the strength of the Wake field



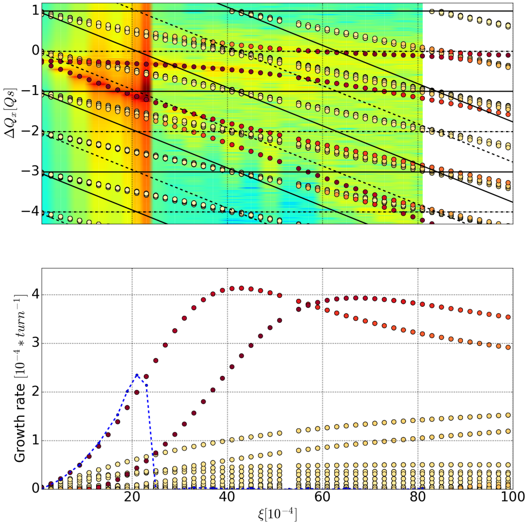
weak wake - 0.5 $\times$ 10^{11} p/b



strong wake - 4.5 $\times$ 10^{11} p/b

- For low intensity, the damping due to the incoherent spectrum of the main side-band is not enough efficient because the σ-mode is at the border of the side-band
- For high intensity, the damping of this side-band is efficient of the σ-mode due to the large tune shift due to the strong intensity of the wake.

Impact of the hourglass, $\beta^* = 0.2m$

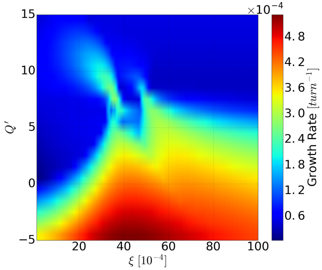


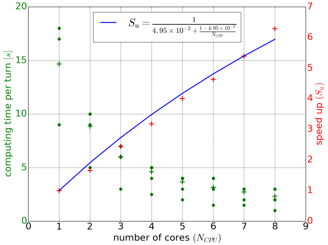
- In case of hourglass effect, the mode coupling instability of colliding beam start for a low ξ
- The growth behavior matches for the two approach, in case of low ξ
- When the σ and π modes enter on the respective incoherent spectrum of the first side band the Landau Damping is efficient to damp completely the instability for $\xi > 0.0021 \approx Q_s$

Conclusion

- The 6D soft-Gaussian kick was implemented and bencharked (statistical convergence against BBC, Yokoya factor and similarity with 4D) in COMBI
- A damping mechanism for intense wake have been observed due to a large tune shift of the σ mode in the main side band
- A efficient damping mechanism by the first sideband was observed in case of strong hourglass
- Potential experimental studies

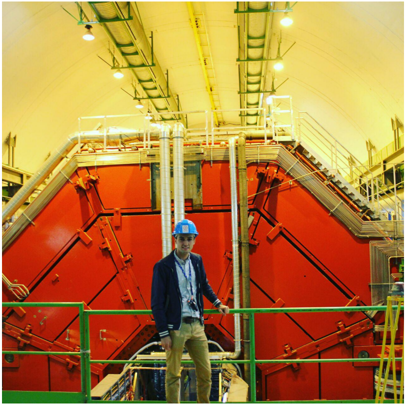
Outlook





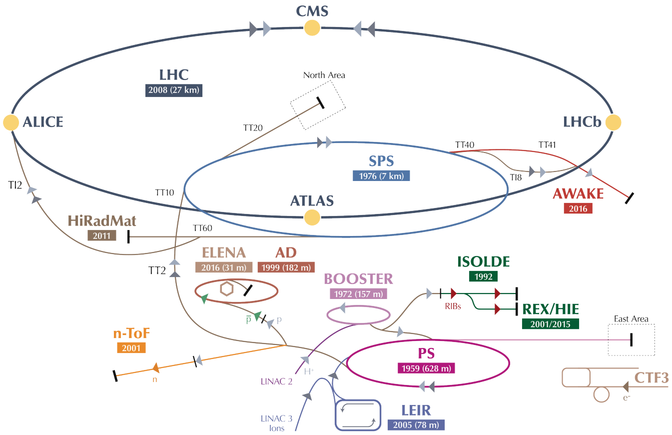
- For high chromaticity, the mode coupling stay unstable
- For $G > 2 \times 10^{-3}$, the main mode coupling are efficiently damped
- For large crossing angle, the strong betatroncoupling deform the head tail basis and allow high order coupling.
- The modeling of the very low instability (like with high gain or chromaticity) will be a computational problem to model with the macroparticle approach
- The convergence issue of the high order coupling in the modal approach need to be investigated

Any Questions ?

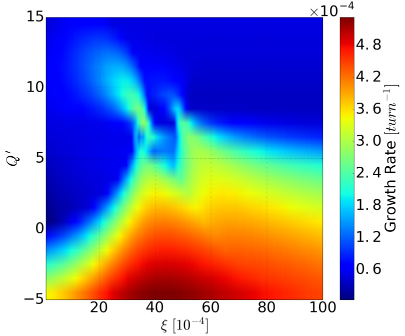


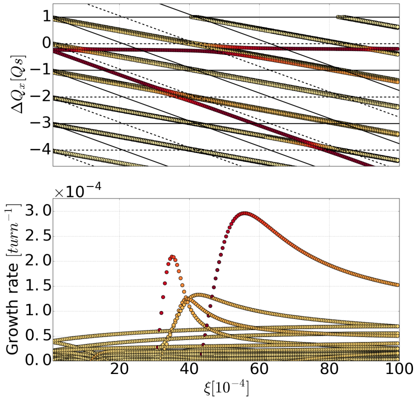
Thank you for your attention

CERN Accelerator complex



Influence of the chromaticity

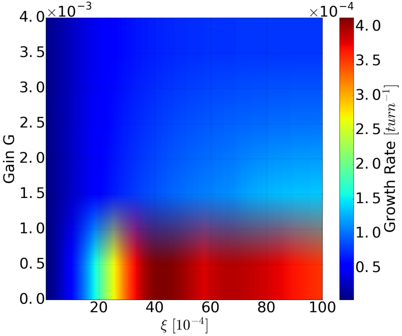


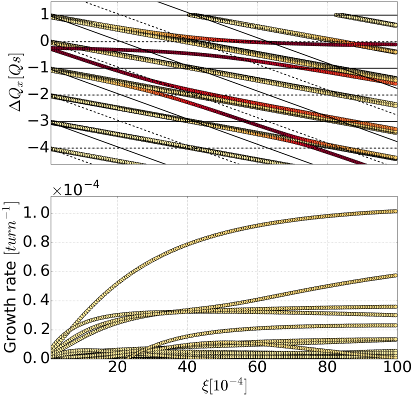


for $Q' = 5$

- For negative chromaticity, the head-tail mode are unstable (single beam theory)
- For large positive chromaticity, we reach a area of stability
- Nevertheless, the two mode coupling are shifted for larger ξ and stay unstable even for high chromaticity

Influence of the gain

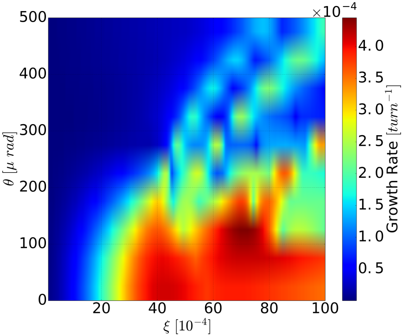


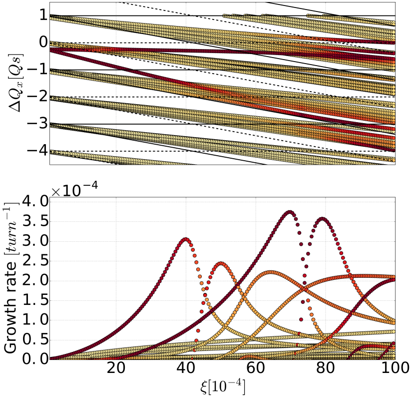


for $G = 1 \times 10^{-3}$

- For $G > 2 \times 10^{-3}$, all unstable instability seem efficiency damped
- For $G = 1 \times 10^{-3}$, the two mode coupling with dipolar componante are damped.

Influence of the crossing angle

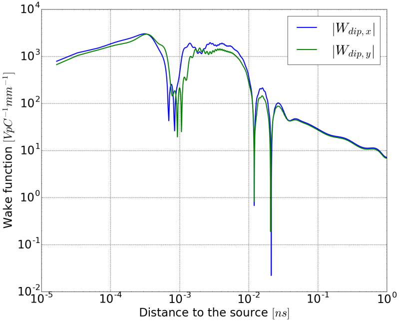


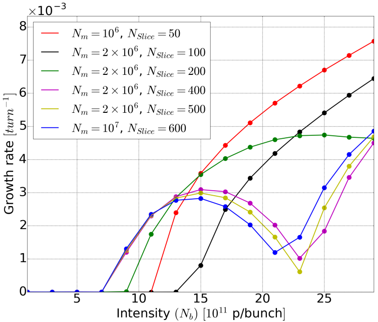


for $\phi = 150\mu$rad

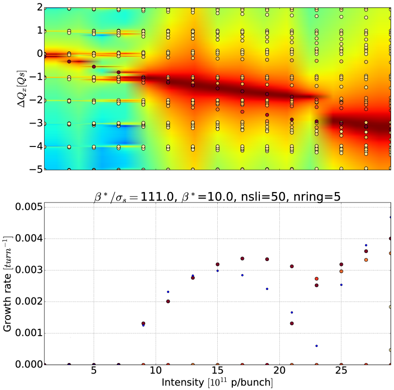
- For large crossing angle, the mode coupling instability are shifted for granter ξ
- A strong synchrobetatron coupling deform the head tail basis and the radial mode become spited
- The convergence of the modal approach need to cheked for high order coupling

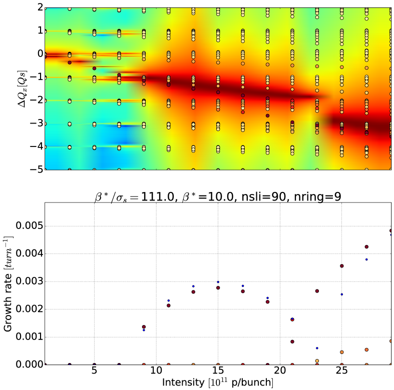
TMCI convergence



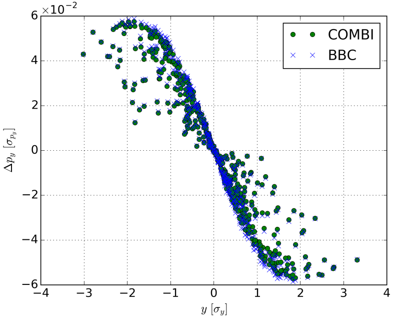


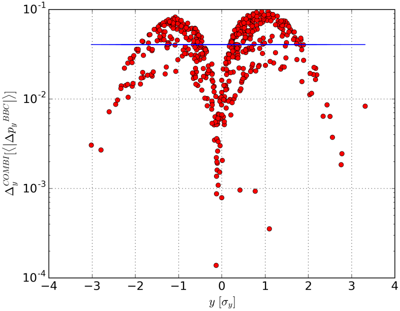
TMCI convergence



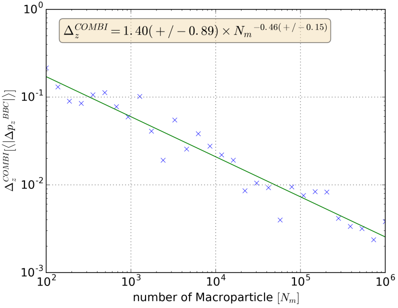


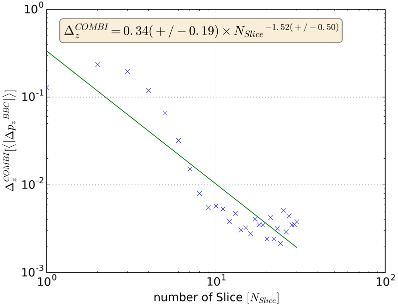
6D Kick convergence





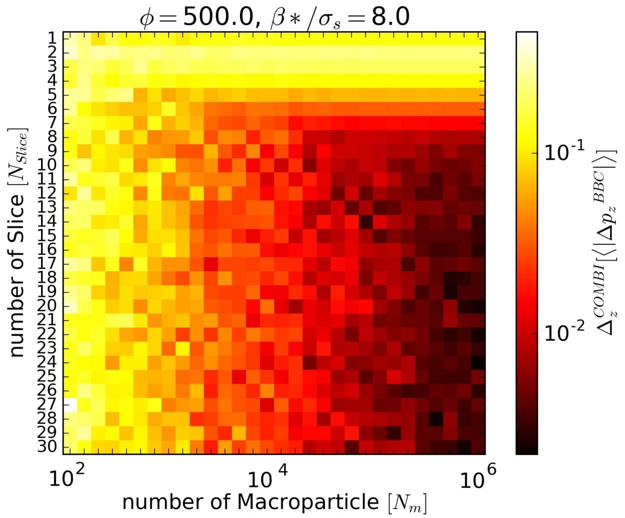
Result of convergence



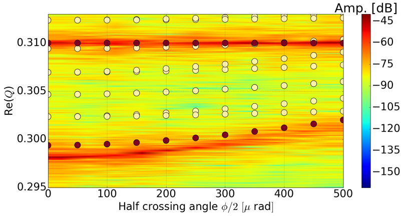


- Statistical convergence in term of Macro-particles (in $\frac{1}{\sqrt{n}}$)
- For more than 10 longitudinal slices, we reach the statistical convergence

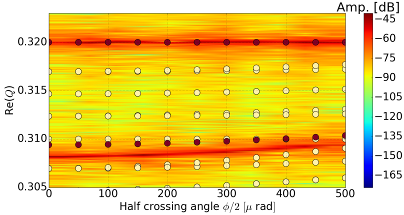
Global view of the convergence



Spectogram COMBI and Modal decomposition from BIMBIM



x-component

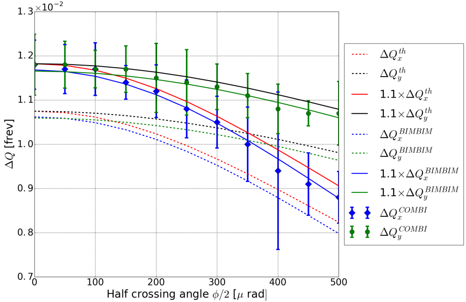


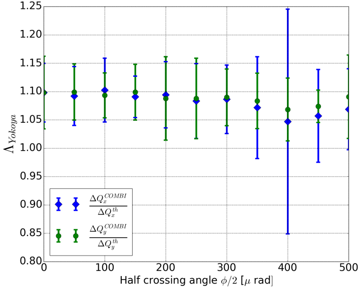
y-component

Comparaison with theoretical tune spread

$$\Delta Q_x^{th} = \frac{N_b r_0 \beta_x^*}{2\pi\gamma\sigma_x^0\sqrt{1+{P_x}^2}\left(\sigma_x^0\sqrt{1+{P_x}^2}+\sigma_y\right)}$$

Yokaya Factor Observation

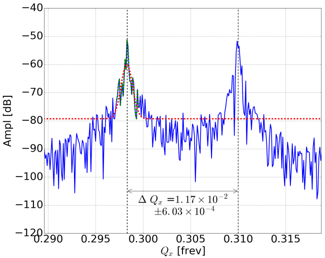




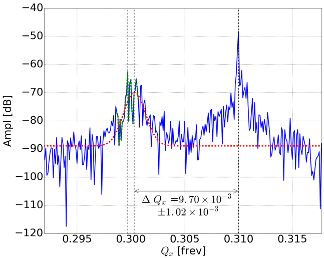
Conclusion of Yokoya Factor observation

- $\Lambda_{Yokaya} \approx 1.1$ in agreement with the theory for soft gaussian kick approach
- For more than $\frac{\phi}{2} = 300$, the error bar increase

This increasing of the error bar could be explain by ...



100 μrad



300 μrad

Hypothesis of the widening of the π-mode

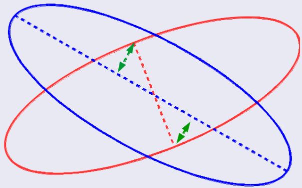
- π-mode seem outside the tune spread according the computation of Q^{th}
- Computation of the ΔQ^{th} could be not correct for the large crossing angle
- Possible Landau Damping due to the tune spread

Landau Damping in Accelerator Physics

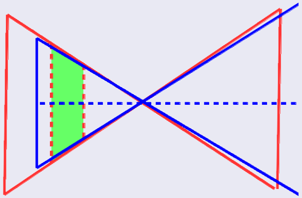
$$\langle x\rangle(t) = \frac{A}{2\omega_{\text{unp}}}\left(cos(\omega_c t)P.V.\int_{-\infty}^{+\infty}\frac{\rho(\omega_i)}{\omega_i-\omega_c}d\omega_i + \pi\rho(\omega_c)sin(\omega_c t)\right)$$

- The Landau damping depend of the distribution of the incoherent spectrum of the particle.
- More the density of incoherent spectrum of the particle is high near to the coherent frequency, more efficient is the damping.

Crossing Angle



Hourglass



Synchrotronbetatron coupling

- Oscillation of one bunch could influencing the longitudinal motion of the opposed bunch
- possible modulation of the spread (transverse) by the synchrotron tune (longitudinal)