Lattice Holographic Cosmology (HC)

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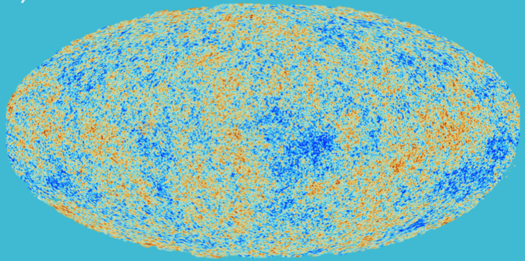

Image credit: ESA and the Planck Collaboration

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Introduction to Lattice HC – Motivations

Recent observations of the cosmic microwave background (CMB) provide a window into Planck scale physics.

This provides an excellent opportunity for testing quantum theories of gravity.

Additionally, holography has also advanced in recent decades. It states that any quantum gravity theory should have a QFT dual in one dimension fewer without gravity.

Introduction to Lattice HC – Motivations

Afshordi et al[1] showed that a holographic model provides an excellent fit to data.

For low multipoles, the QFT dual is expected to be non-perturbative.

Lattice methods are necessary to model the low multipole region.

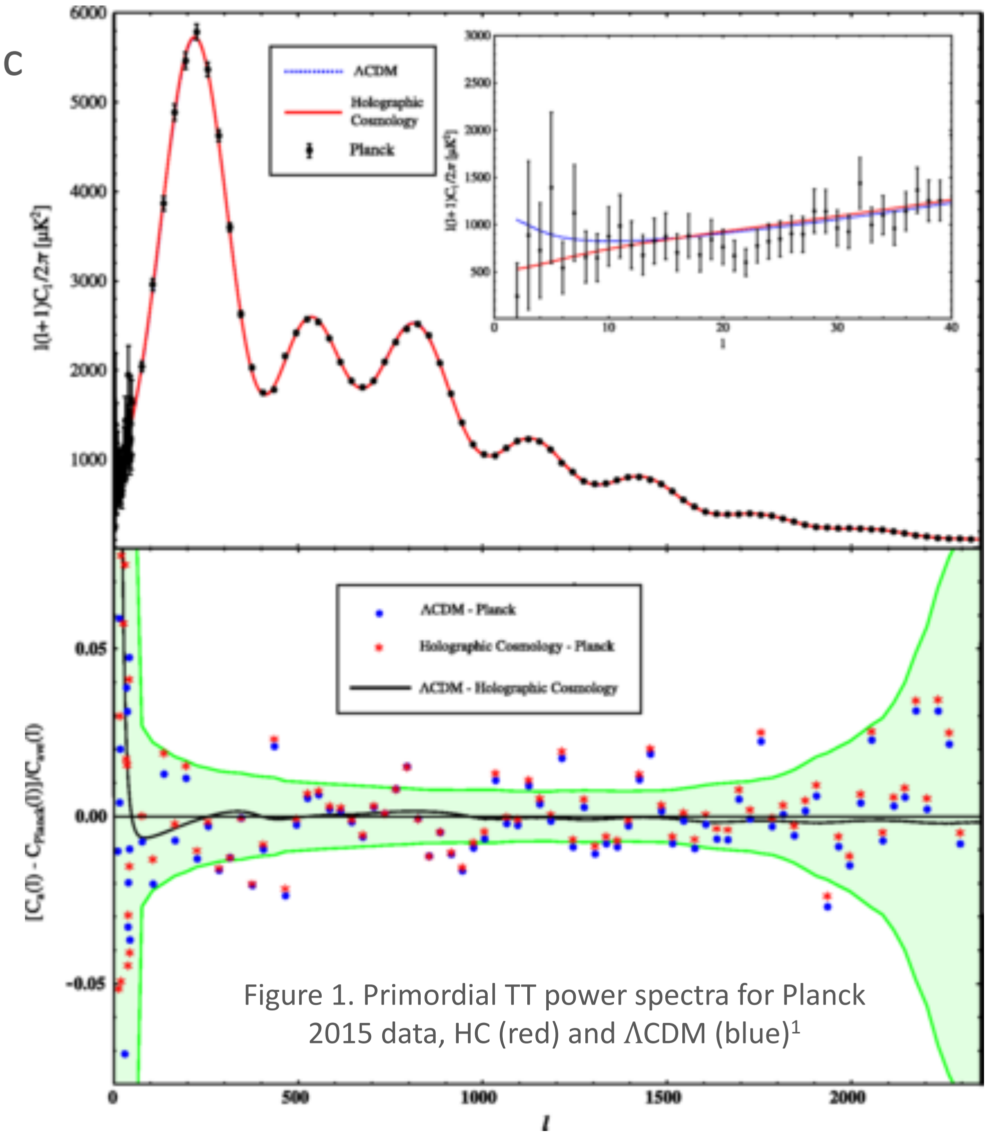


Figure 1. Primordial TT power spectra for Planck 2015 data, HC (red) and ΛCDM (blue)[1]

1. N. Afshordi, C. Coriano, L. Delle Rose, E. Gould, K. Skenderis, Phys. Rev. Lett. **118** (2017), 041301

Introduction to Lattice HC – First Aims

The results of Afshordi et al suggest that the dual QFT is a Yang-Mills theory coupled to massless scalars with a ϕ^4 interaction.

Lattice field theories are the method of choice for non-perturbative calculations…

- Start from a simpler model: scalar ϕ^4 theory without gauge fields (on lattice)
- Locate the massless point
- Find the conserved energy-momentum tensor (EMT)
- Calculate non-perturbative 2-point EMT on the lattice
- Compare with CMB data

Current Work – Scalar ϕ^4 heory

We want to study a d=3 massless Euclidean QFT on the lattice. Starting from the massive theory:

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}\Phi \mathcal{O}(\Phi) e^{-S[\Phi]}$$

$$S = \int d^3x \mathrm{Tr} \left\{ \frac{1}{2} [\partial \Phi(x)]^2 + \frac{1}{2} m^2 \Phi(x)^2 + \frac{\lambda}{4!} \Phi(x)^4 \right\}$$

$\phi(x)$ is in the adjoint of SU(N)

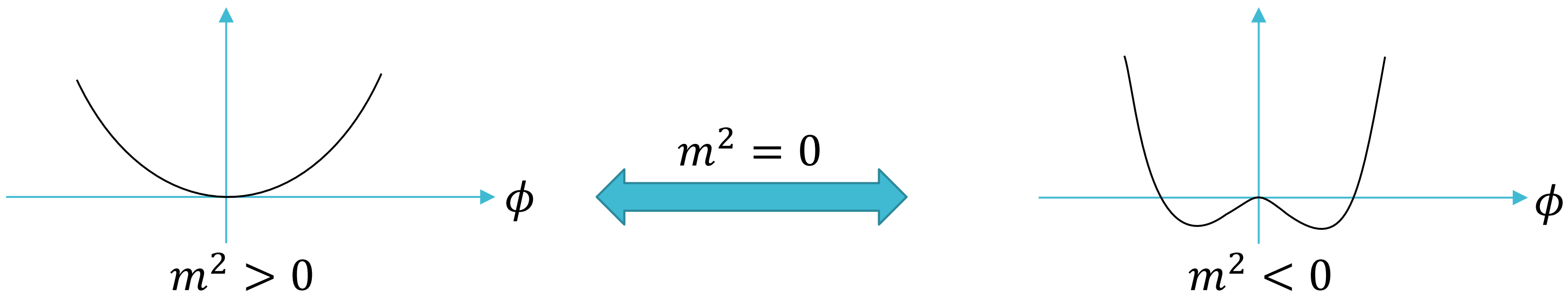
Then we apply this on a lattice with spacing, a:

$$S = \frac{a^3 N}{g} \sum_{x \in \Lambda^3} \mathrm{Tr} \left\{ \left[\partial_\mu \phi(x) \right]^2 + m^2 \phi + \phi(x)^4 \right\}$$

$$\phi(x) = \sqrt{\frac{g}{N}} \Phi(x) \qquad g = N\lambda$$

Current Work – Near-critical simulations

Next we wish to determine the bare mass at which our theory becomes massless.



Symmetric

Spontaneously broken symmetry

In order to do this, we take guidance from lattice perturbation theory where we determine the mass correction term at 2-loop order:

$$\delta_m^2 = -g\left(\frac{Z_0}{a} - \frac{m}{4\pi}\right)\left(2 - \frac{3}{N^2}\right) + g^2 D_0(0)\left(1 - \frac{6}{N^2} + \frac{18}{N^4}\right)$$

where Z_0 and $D_0(0)$ are the massless 1- and 2-loop integrals respectively.

Current Work – Near-critical simulations

By using this correction term, we can predict the bare mass value for a massless theory:

$$a^2\delta_m^2 = -agZ_0\left(2-\frac{3}{N^2}\right)+a^2g^2D_0(0)\left(1-\frac{6}{N^2}+\frac{18}{N^4}\right)=(am_c)^2$$

Current Work – Binder cumulants

Since the perturbation theory result is not applicable, we are using it as guidance.

Need to scan across bare masses near this guide point to locate the critical mass.

Binder cumulants[2], U, have previously been used to study the critical phenomena and phase transitions in nearest neighbour (nn) Ising models.

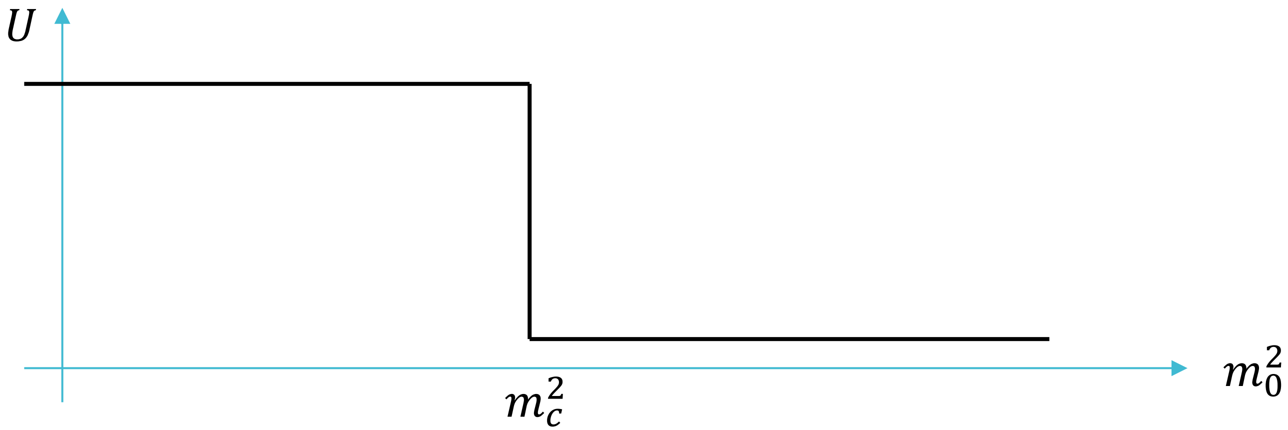
$$U = \frac{\langle M^2 \rangle^2}{\langle M^4 \rangle}$$

They are extremely useful as they cancel the critical exponents we see in observables on the lattice.

2. K. Binder, Z. Phys. B. **43** (1981)

Current Work – Binder cumulants

We also take the U of the total field value, or "magnetisation" (M), on our lattice. This allows us to predict a step function in the vicinity of the critical mass.



Current Work – Binder cumulants

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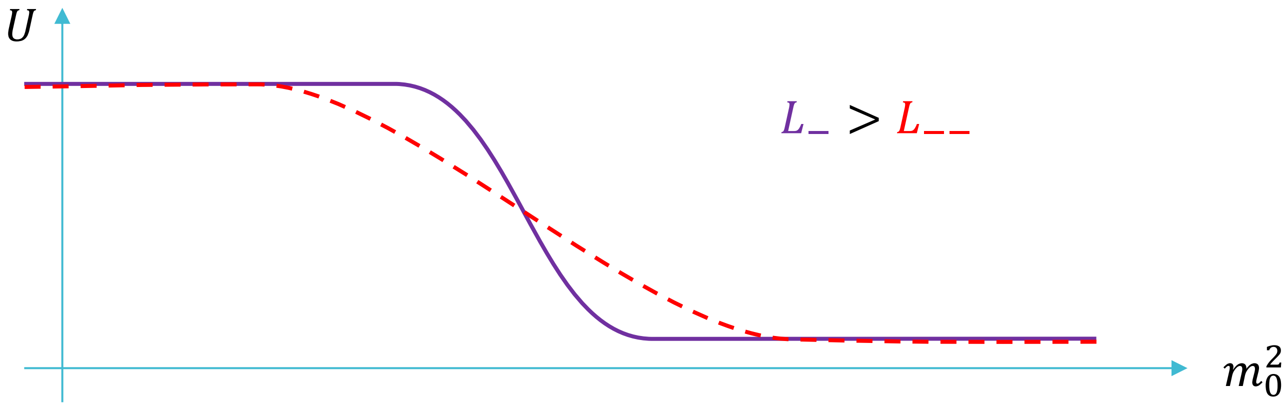

However finite volume effects actually smooth this out. As $L > \infty$, the curve retrieves its Heaviside form.

We aim to calculate the crossing points for pairs of subsequent volumes.

So to determine the critical mass, we have to take the infinite volume limit…

Preliminary results for Binder cumulants

We simulated series of bare mass QFTs near critical region, for different values of L:

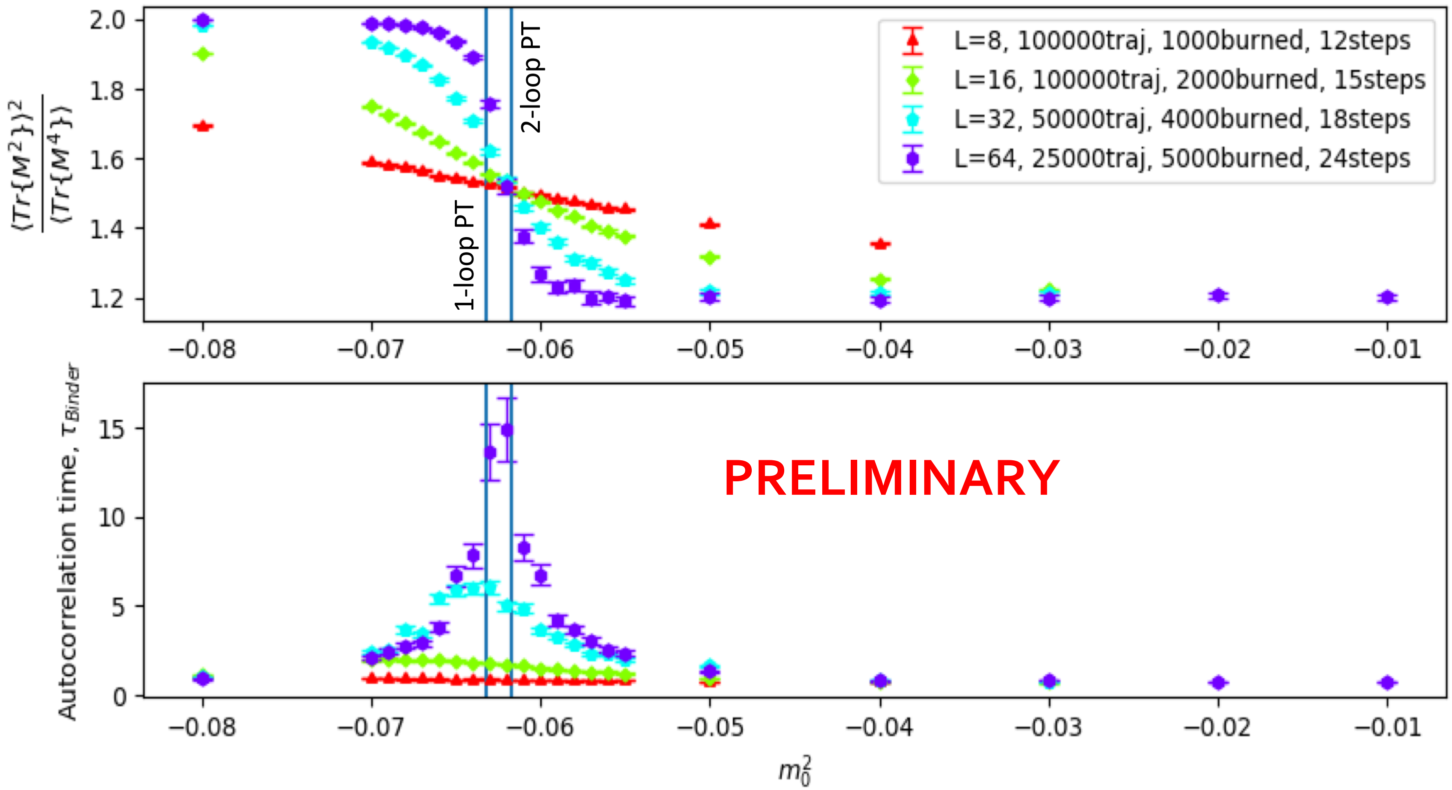

Figure 2. Results of Binder cumulant analysis for SU(2) and lattice coupling ag=0.2. Top plot shows the values of $U(M)$ where a phase transition is apparent. Bottom plot shows the autocorrelation time of the MCMC simulation which shows a spike close to the critical mass predictions by perturbation theory (PT).

Preliminary results for Binder cumulants

One can use a technique called multi-histogram reweighting to create a continuous plot of the Binder cumulant:

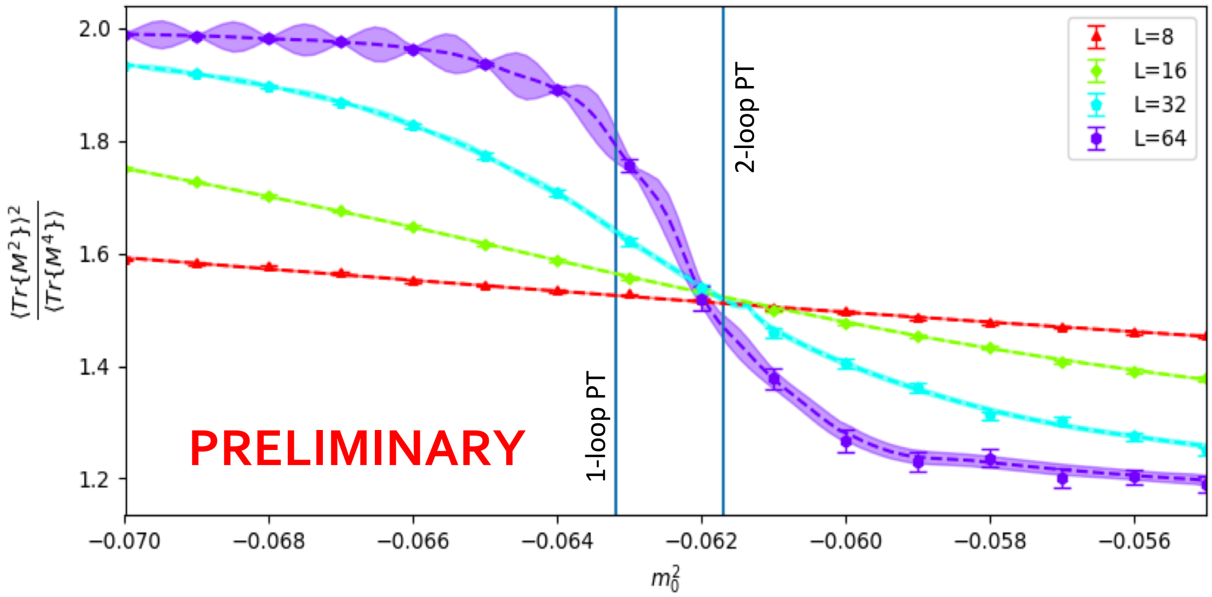

Figure 3. Zoomed in view of the critical region with MHRW providing the continuous results. The bands represent the standard deviation of contributions from the mean at each reweighted point. SU(2), ag=0.2

Preliminary results for m_c^2

We can actually use this method with a solver algorithm to find the bare masses where pairs of subsequent volumes cross over and then extrapolate to the infinite volume limit:

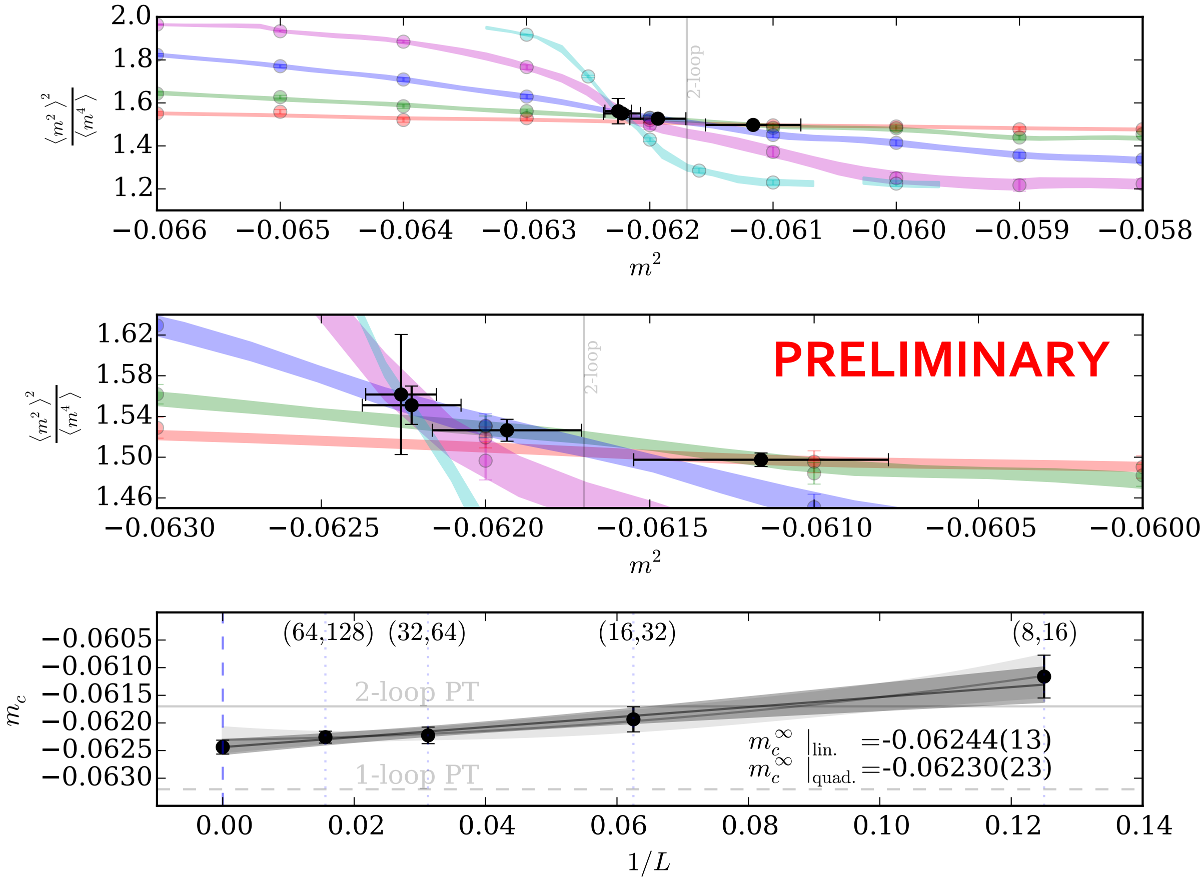

Figure 4. Top plot: Binder cumulants for SU(2), ag=0.2 now including L=128 (cyan) also. Middle plot: Zoomed in version, black points are cross-over points between subsequent volumes with errors. Bottom plot: Convergence of the critical mass as we use larger volumes. Estimates provided are for extrapolation to infinite volume limit.

Next steps…

- Start from a simpler model: scalar ϕ^4 theory without gauge fields (on lattice) ✓
- Locate the massless point ✓
- Find the conserved energy-momentum tensor (EMT)
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Conclusions

We used holography to model Planck scale physics

Currently have class of holographic cosmology models that agree with Planck data

Use non-perturbative lattice methods to properly model the low multipole part of the CMB data

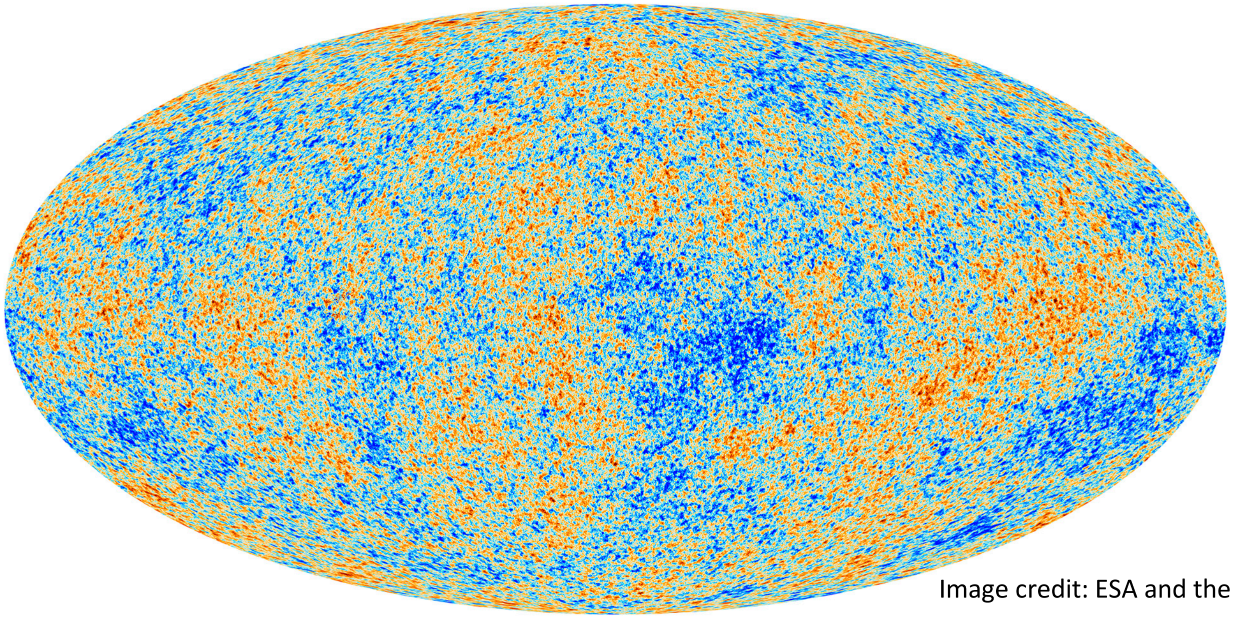

Image credit: ESA and the Planck Collaboration

Multi-Histogram Reweighting for QFT

We can use the technique of multi-histogram reweighting (MHRW) in such a way that it allows us to probe physics at a bare mass, $m^{2\prime}$, away from the simulated points:

$$\langle U \rangle_{m^{2\prime}} = \frac{{\langle M^2 \rangle_{m'^2}}^2}{\langle M^4 \rangle_{m'^2}}$$

After performing some maths on our lattice action, we obtain an expression that allows us to compute these reweighted observables:

$$\langle U \rangle_{m^{2\prime}} = \frac{\left\langle M_{m^2}^2 W \right\rangle^2}{\langle W \rangle \langle M_{m^2}^4 W \rangle}$$

Where the weight, W is:

$$W = \exp\left[-(m'^2 - m^2) N L^3 tr\{\phi_{m^2}^2\}/g\right]$$