UV conformal window for asymptotic safety

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Content

- Motivation
- Fixed points in gauge theories
- Fixed point at next-to-next-to-leading order
- Constraints of the conformal window
  - Constraints from fixed points
  - Constraints from beta functions
- Conclusions

Motivation

- Asymptotic safety: interactive fixed point; residual interactions in the UV
- Critical phenomena: Wilson-Fisher fixed point (ϕ^4 theory in $d = 4 - \epsilon$ dimensions) (Wilson & Fisher, 1972)
- A quantum theory of gravity may be non-perturbatively renormalizable provided there is an interactive UV fixed point. (Weinberg, 1979)
- More recently, necessary conditions and no-go theorems have been derived for the existence of UV fixed points in general gauge theories. (Bond & Litim, 2016)
- The search for asymptotic safety has also extended to supersymmetric and phenomenological motivated models (Martin & Wells, 2000) (Bond & Litim, 2017) (Bond, Hiller, et al., 2017)
- An important question relates to the range of the conformal window of such theories.
- IR conformal windows in QCD-like theories have been extensively studied, and are known to extend past the domain of perturbation theory (Banks & Zaks, 1982) (Appelquist et al., 2008) (Del Debbio., 2011)
- We want to understand what is the extent of the UV conformal window in asymptotically safe gauge theories

Fixed points in gauge theories

- Fixed points are obtained from the renormalization group flow of couplings (beta functions), e.g.:

$$\beta^{(2)} = -B\alpha^2 + C\alpha^3$$

- With a non-trivial fixed point:

$$\alpha^* = \frac{B}{C}$$

- If $|B| \ll |C|$, then the fixed point is weakly interacting and can be expressed as a series expansion in a small parameter ϵ in perturbation theory, i.e.:

$$\alpha^* = \lambda_1\epsilon + \lambda_2\epsilon^2 + \lambda_3\epsilon^3 + O\left(\epsilon^n\right)$$

Fixed points in gauge theories

- A weakly coupled interacting UV fixed point requires: $\alpha^* = \frac{B}{C}$

$$B < 0; \quad C < 0$$
$$|B| \ll |C|$$

- The first condition can only be realized with the help of Yukawa interactions (Bond & Litim, 2016)
- The second one is model dependent, e.g.: for N_F fermions transforming on the fundamental of $SU(N_C)$, we can take the Veneziano limit: (Litim & Sannino, 2014)

$$N_C \to \infty; \quad N_F \to \infty$$
$$B \propto \epsilon$$
$$\epsilon = \frac{N_F}{N_C} - \frac{11}{2}$$

The model – Field content

$$L = L_{\rm YM} + L_{\rm kin.} + L_{\rm Yuk.} + L_{\rm pot.}$$

$$L_{\rm YM} = -\frac{1}{2} {\rm Tr}\, F^{\mu\nu} F_{\mu\nu}$$

$$L_{\rm kin.} = {\rm Tr}\left(\overline{Q}\, i\not{D}\, Q\right) + {\rm Tr}\left(\partial_\mu H^\dagger\, \partial^\mu H\right)$$

$$L_{\rm Yuk.} = -y\, {\rm Tr}\left(\overline{Q}_L\, H\, Q_R\right) + {\rm h.c.}$$

$$L_{\rm pot.} = -u\, {\rm Tr}\left(H^\dagger H\, H^\dagger H\right) - v\left({\rm Tr}\, H^\dagger H\right)^2$$

$$\alpha_g = \frac{g^2 N_C}{(4\pi)^2}; \quad \alpha_y = \frac{y^2 N_C}{(4\pi)^2}$$

$$\alpha_u = \frac{u N_F}{(4\pi)^2}; \quad \alpha_v = \frac{v N_F^2}{(4\pi)^2}$$

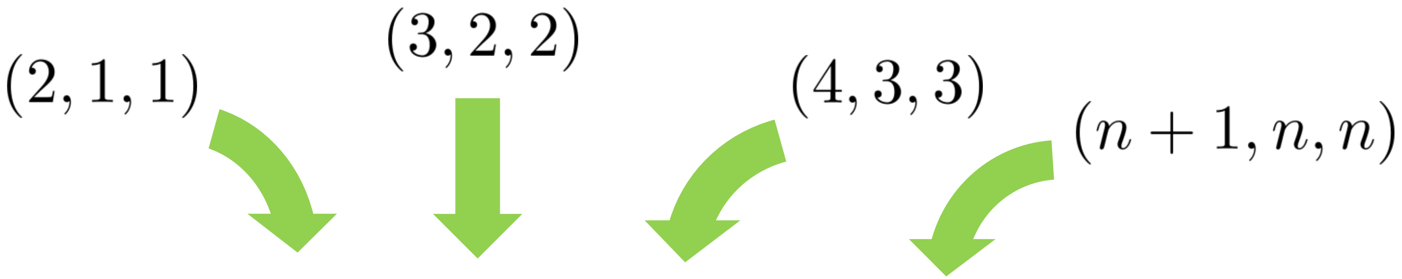
- 4d gauge theory
- Fermions charged under a gauge group $\rm SU(N_C)$ in the fundamental representation
- Scalar singlets with self-interactions and Yukawa interactions
- $\rm N_F$ flavours of fermions and scalars

Fixed point – Sub-leading corrections

- The approximation is denoted by the number of loop orders retained:

$$\left(\beta_g^{(n)},\ \beta_y^{(m)},\ \beta_{u,v}^{(l)}\right) \equiv (n, m, l)$$

- We find that the approximation $(n+1, n, n)$ completely determines the coefficient of order ϵ^n in the series expansion



$$\alpha_i^* = \lambda_{1i}\epsilon + \lambda_{2i}\epsilon^2 + \lambda_{3i}\epsilon^3 + O\left(\epsilon^n\right)$$

Fixed point – Sub-leading corrections

- Following this ordering, we can compute the ϵ^2 coefficients analytically from the beta functions. Numerically, we find:

$$\alpha_g^* = 0.4561\epsilon + 0.7808\epsilon^2 + O\left(\epsilon^3\right)$$
$$\alpha_y^* = 0.2105\epsilon + 0.5082\epsilon^2 + O\left(\epsilon^3\right)$$
$$\alpha_u^* = 0.1998\epsilon + 0.4403\epsilon^2 + O\left(\epsilon^3\right)$$
$$\alpha_v^* = -0.1373\epsilon - 0.6318\epsilon^2 + O\left(\epsilon^3\right)$$

(Bond et al., 2017)

- Sub-leading corrections show no change in sign. This argues in favor of the stability of the fixed point, but may be a hint of a slow rate of convergence.
- We should check vacuum stability, which is dictated by the scalar potential.

Vacuum stability

- In order to have a stable vacuum state, we require for the scalar potential to be bounded from below. In the present setting, this means:

$$\alpha_u^* > 0, \quad \alpha_u^* + \alpha_v^* > 0$$

(Litim, Mojaza & Sannino, 2016)

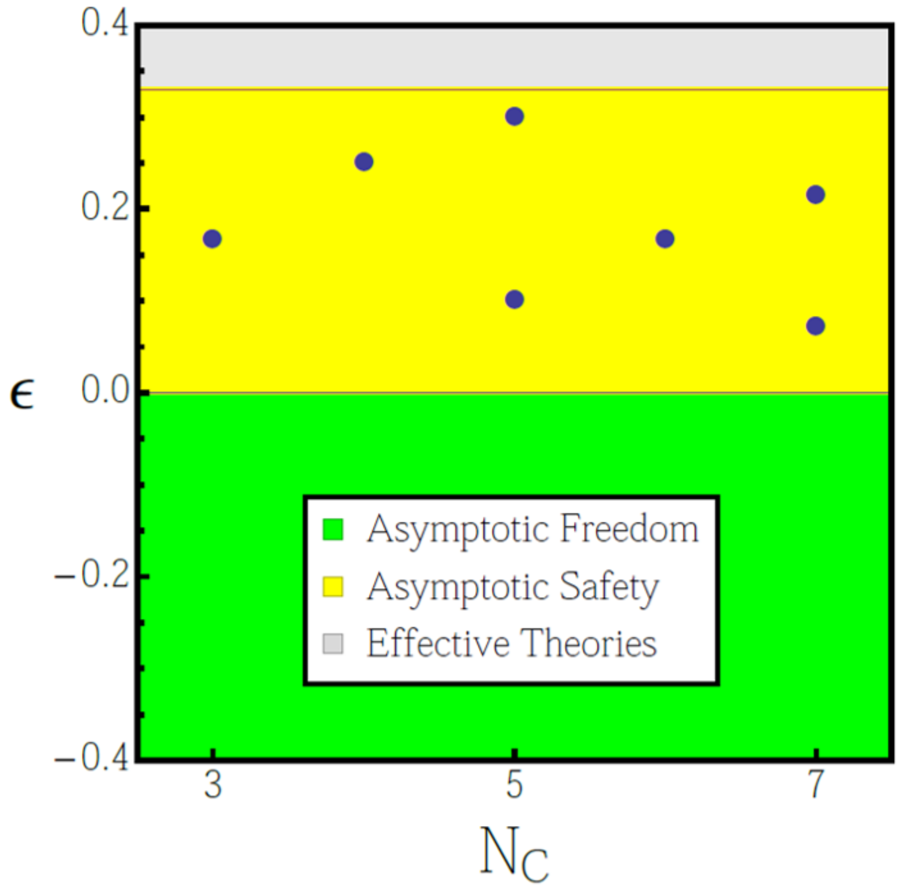
- As we improve our approximation, we notice a change in sign in the quadratic term of the fixed point expansion:

$$\alpha_u^* + \alpha_v^*|_{(211)} = 0.0625\epsilon + O\left(\epsilon^3\right)$$
$$\alpha_u^* + \alpha_v^*|_{(321)} = 0.0625\epsilon + 0.1535\epsilon^2 + O\left(\epsilon^3\right)$$
$$\alpha_u^* + \alpha_v^*|_{(322)} = 0.0625\epsilon - 0.1915\epsilon^2 + O\left(\epsilon^3\right)$$

Constraints from fixed points

- This remarkable discovery sets the boundary of the conformal window in the current approximation (3,2,2)
- In the plot, the blue dots indicate the value of ϵ for the first integer values of N_F
- The conformal window is delimited from below by asymptotic freedom and from above by vacuum instability:

$$0 < \epsilon < 0.326$$



(Bond et al., 2017)

Constraints from beta functions

- Instead of using a series expansion, we could also compute constraints from the beta functions.
- Suppose the beta functions are exact at the chosen approximation. This approach is sensitive to sub-leading corrections in epsilon, e.g.:
  At 2 loop in the gauge beta function we should expect at most terms of order ϵ^3

$$\beta_g^{(2)} = \left(25 + \frac{26}{3}\epsilon\right)\alpha_g^3 - 2\left(\frac{11}{2} + \epsilon\right)^2 \alpha_y \alpha_g^2 \qquad \beta_g^{(n)} \sim \epsilon^{n+1}, \quad n \geq 2$$

- Terms with higher powers of epsilon will compete with higher loop order terms, and can be regarded as incomplete corrections.

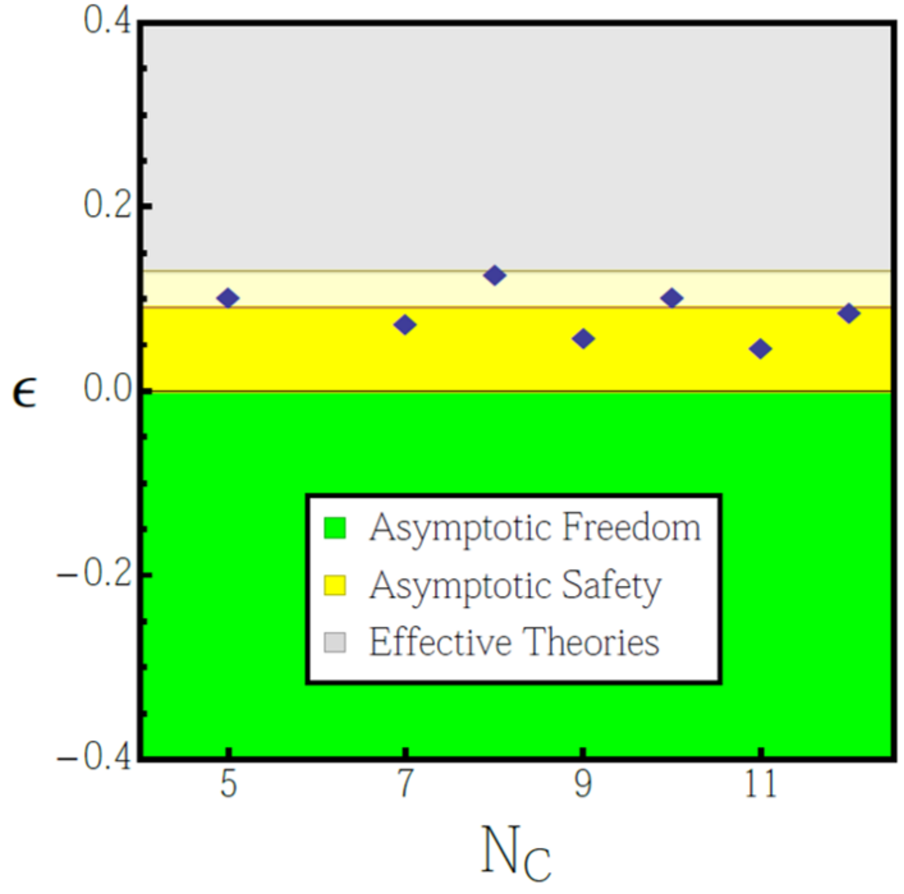
$$\epsilon\,\alpha_g^3,\ \epsilon\,\alpha_y\alpha_g^2 \ \sim O\left(\epsilon^4\right) \sim \beta_g^{(3)}$$

$$\epsilon^2\alpha_y\alpha_g^2 \sim O\left(\epsilon^5\right) \sim \beta_g^{(4)}$$

Constraints from beta functions

- The lower dark yellow region is the asymptotically safe region in the (3,2,2) approximation (constraint from vacuum stability)
- The upper light yellow region represents the (3,2,1) approximation (constraint from fixed point merger)
- In the plot, the blue dots indicate the value of ϵ for the first integer value of N_F
- This approach imposes a tighter bound on the conformal window:

$$\epsilon_{max} \approx 0.09 \ldots 0.13$$



(Bond et al., 2017)

Conclusions

- With the inclusion of the 2-loop scalar beta functions, the UV fixed point has been consistently determined to order ϵ^2.
- Estimates for the conformal window have been computed in two approximations. Constraints from fixed point convergence leads to a wider conformal window, while those derived from beta functions tend to reduce it.
- Vacuum stability provides the strongest constraint at the complete next-to-next-to-leading order (3,2,2).
- The full conformal window spans a region entirely within the domain of perturbation theory. Therefore the perturbative description of the UV fixed point is completely reliable up to the bounds provided.

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