Renormalisation group fixed points and physics at highest energies

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Goals

- Want QFTs that are predictive up to arbitrarily large energies
- Have non-SUSY examples of theory with weakly coupled UV fixed point
- Are perturbative fixed points compatible with supersymmetry?
- What conditions must such theories satisfy?

Renormalisation group

- Couplings λ_i in QFT run with energy scale — described by renormalisation group equations (RGEs)

$$\frac{\partial \lambda_i}{\partial \log \mu} = \beta_i(\{\lambda\})$$

- Beta functions β_i determined by field content and symmetries
- Various approaches available to compute the β_i in some approximation

Fixed points

- Fixed points λ_i^* are points in coupling space that satisfy

$$\beta_i(\{\lambda^*\}) = 0$$

- Infrared means have solutions to RGEs which satisfy $\lim_{\mu \to 0^+} \lambda(\mu) = \lambda^*$
- Ultraviolet means have solutions to RGEs which satisfy $\lim_{\mu \to \infty} \lambda(\mu) = \lambda^*$
- Ultraviolet fixed points allow us to define QFTs up to arbitrarily large energies

UV critical surface

Can solve RGEs in vicinity of a fixed point — approximately linear.

$$\alpha_i(t) \approx \alpha_i^* + \sum_n c_n V_i^{(n)} e^{\theta_n t} \,.$$

θ_n are the critical exponents.

$$\begin{aligned} \mathrm{Re}(\theta_n) < 0 : &\quad V^{(n)} \text{ is a relevant direction.} \\ \mathrm{Re}(\theta_n) > 0 : &\quad V^{(n)} \text{ is an irrelevant direction.} \end{aligned}$$

Eigensystem of stability matrix

$$M_{ij} \equiv \left.\frac{\partial \beta_i}{\partial \alpha_j}\right|_{\alpha=\alpha^*}$$

UV critical surface

- Space of trajectories coming from relevant directions = UV critical surface.
- For a fixed choice of RG scale, all points on critical surface are valid UV theories — need to make n measurements to determine our theory from an n-dimensional critical surface.
- Important that we have only a finite number of relevant directions — don't know beforehand in general!

Perturbation theory

- Small couplings $\implies$ compute β-functions perturbatively as series expansion

$$\beta(\lambda) = c_1\lambda^2 + c_2\lambda^3 + \dots$$

- β-functions for 4d theories known to low orders — can exploit structure to constrain possible fixed points
- Fixed points can be brought under strict control with an adjustable small parameter
- Useful starting point to understand non-perturbatively

Ultraviolet fixed points in perturbation theory

- Two possible fixed point scenarios:
  - Gaussian fixed point $\lambda^* = 0$ — asymptotic freedom
  - Interacting fixed point $\lambda^* \neq 0$ — asymptotic safety
- Perturbation theory $\implies$ need couplings to be small
  - For asymptotic safety need $0 < |\lambda^*| \ll 1$
  - Small corrections to anomalous dimensions — classical mass dimension still governs relevance

The general story

[AB, D Litim, 1608.00519]

- Can write simple gauge beta function as

$$\beta = \alpha(-B + C\alpha + \dots)$$

- Interacting fixed points are of the form

$$\alpha^* = B/C$$

- Only gauge interactions mean $C > 0$ when $B \leq 0 \implies$ fixed points are IR
- Encode effect of Yukawas as shift in effective two-loop $C \to C' \leq C$, can have $C' < 0 \implies$ UV
- Generalises to semisimple groups

Partially interacting fixed points

- For semisimple gauge groups have multiple independent gauge couplings α_a
- Partially interacting fixed points (some $\alpha_a^* = 0$) give marginal directions
- Relevancy determined by effective one-loop coefficient

$$\begin{aligned}\beta_a &= \alpha_a^2(-B_a + C_{ab}\alpha_b^* - D_a\alpha_y^*) \\ &\equiv -B'_a\alpha_a^2\end{aligned}$$

- Sign of B' can be different from $B \to$ couplings can change between being IR or UV free

Features of SUSY

- Gauge and Yukawa only marginal couplings
- Have all-orders expressions for beta functions — door to nonperturbative physics
- More constraining — not obvious general mechanism will still work
- $\mathcal{N} \geq 2$ have interactions fixed — at most can choose gauge reps
- RG dynamics of extended supersymmetry highly constrained — focus on $\mathcal{N} = 1$

General $\mathcal{N} = 1$ theory

- Gauge group G, with some vector superfields
- Matter consists of some chiral superfields Φ_i
- Allowed non-gauge interactions via superpotential $W = \frac{1}{6} Y^{ijk} \Phi_i \Phi_j \Phi_k$

Beta functions

- In appropriate scheme have all-orders beta functions in terms of chiral superfield anomalous dimensions γ

NSVZ:

$$\beta_g^{NSVZ} = -\alpha^2 \frac{B + \frac{4}{d_G}\ \mathrm{Tr}(\gamma\, C_2^R)}{1 - 2\, C_2^G\, \alpha}$$

Yukawa non-renormalisation:

$$\beta_Y^{ijk} = Y^{ij\ell}\,\gamma_\ell^k + (i \leftrightarrow k) + (j \leftrightarrow k)$$

- Can determine perturbatively $\gamma = \frac{1}{2}Y^2 - 2\alpha C_2^R + \dots$

Yukawa flow

Can look at flow of sum of squared Yukawas

$$\frac{1}{12}\partial_t(Y_{ijk}\,Y^{ijk}) = [\gamma^{(1)\,k}{}_{\ell} + 2\,\alpha_a\,C_2^{R_a}(k)\,\delta^k_\ell]\;\gamma^\ell_k$$
$$\approx d(R)|\gamma(R)|^2 - \frac{1}{2}\,B_a\,\alpha_a\,d(G_a)\left(1 + 2\,C_2^{G_a}\,\alpha_a\right)$$

- To get a zero need some $B_a > 0$
- For UV fixed point need semisimple gauge group!
- $C' \geq -2\,B\,C_2^G$

SUSY model

Chiral superfields	ψ_L	ψ_R	Ψ_L	Ψ_R	χ_L	χ_R	Q_L	Q_R
$SU(N_1)$	$\overline{\Box}$	$\Box$	$\Box$	$\overline{\Box}$	1	1	1	1
$SU(N_2)$	1	1	$\Box$	$\overline{\Box}$	$\overline{\Box}$	$\Box$	$\overline{\Box}$	$\Box$

- Superpotential $W = y\,\mathrm{Tr}\left[\psi_L\,\Psi_L\,\chi_L + \psi_R\,\Psi_R\,\chi_R\right]$,
- Parameters $R = \frac{N_2}{N_1}$, $P = \frac{N_1}{N_2}\frac{N_Q+N_1+N_F-3N_2}{N_F+N_2-3N_1} \propto \frac{B_2}{B_1}$, $\epsilon \propto B_1$
- Veneziano limit: all $N \to \infty$, ratios fixed
- Perturbativity for $|\epsilon| \ll 1$

Beta functions and fixed points

- Have a range of potential perturbative $O(\epsilon)$ fixed points

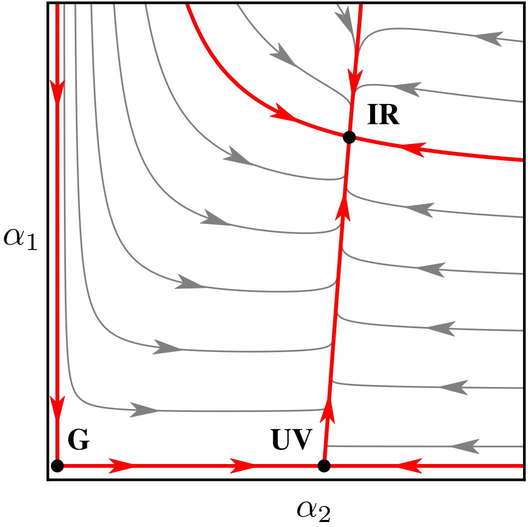
Fixed point	G	BZ_1	BZ_2	GY_1	GY_2	BZ_{12}	GY_{12}
α_1^*	0	$-\frac{\epsilon}{6}$	0	$\frac{-\epsilon}{2(3-3R+R^2)}$	0	$\frac{PR-3}{16}\epsilon$	$\frac{3-4R-2PR^2+PR^3}{(R-1)(9-8R+3R^2)}\frac{\epsilon}{2}$
α_2^*	0	0	$-\frac{P\epsilon}{6}$	0	$\frac{-PR}{4R-3}\frac{\epsilon}{2}$	$\frac{1-3PR}{16R}\epsilon$	$\frac{R-2-3PR+3PR^2-PR^3}{(R-1)(9-8R+3R^2)}\frac{\epsilon}{2}$
α_y^*	0	0	0	$\frac{1}{2}\alpha_1^*$	$\frac{1}{2}\alpha_2^*$	0	$\frac{1}{2}(\alpha_1^*+\alpha_2^*)$

- In vicinity of e.g. GY_2, have beta function

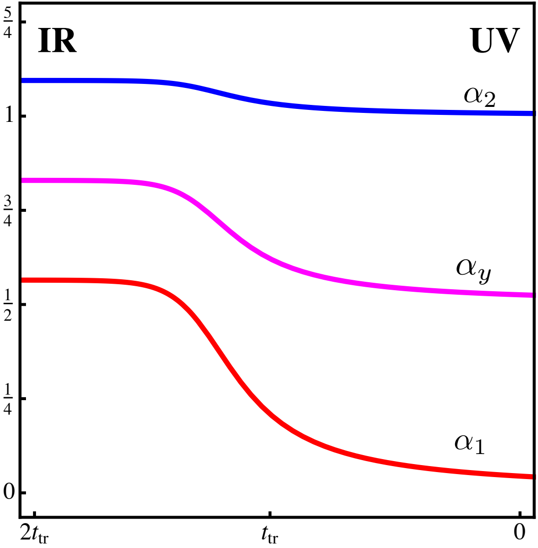
$$\beta_1 = -B_1'\alpha_1^2\,, \qquad B_1' = B_1 - C_{12}\alpha_2^* + D_1\alpha_y^*$$

- In fact $B_1' > 0 > B_1$ — UV relevancy generated by fixed point!

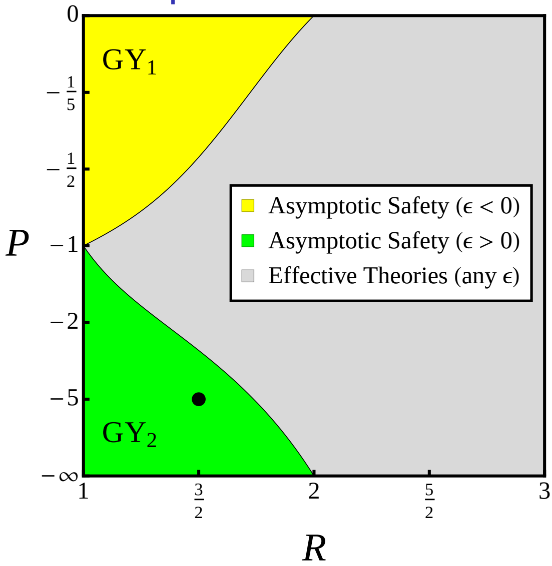
SUSY phase diagram



SUSY separatrix



Parameter space



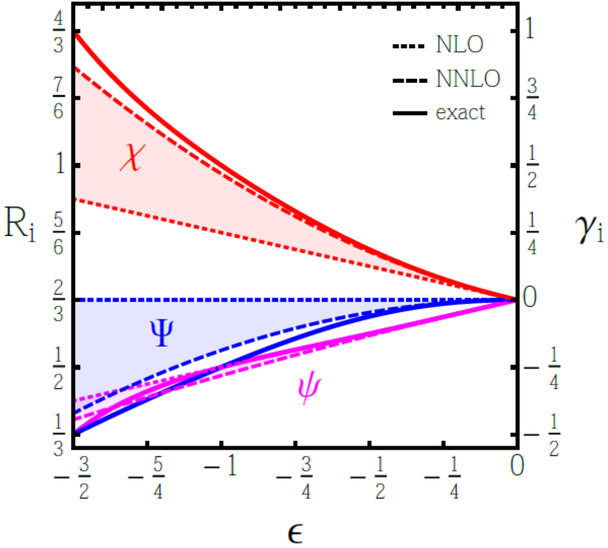
R-charges

- Superconformal algebra contains $U(1)_R$ factor — obeyed at fixed points
- Related to anomalous dimension $R_i = \frac{2}{3}(1 + \gamma_i)$
- Can be determined exactly via a-maximisation [Intriligator, Wecht, hep-th/0304128]

e.g. at FP$_{1y}$

$$R_\psi = \frac{3N_F(N_F - (N_2 + N_1))^2 - N_1\sqrt{P_4(N_1, N_2, N_F)}}{3(N_1 - N_F)(N_2(N_1 + N_F) + 2N_1N_F - N_1^2 - N_F^2)}$$

R-charges FP$_{1u}$



a-theorem

- a-theorem states that the central charge a decreases along RG trajectories
- $a_{UV} - a_{IR} > 0$
- Offers consistency check — a is determined by R-charges

$$a = \frac{3}{32}\left[3\,\mathrm{Tr}\,R^3 - \mathrm{Tr}\,R\right]$$

- Window to nonperturbative relevancy

Summary

- Interacting UV fixed points exist in supersymmetric theories
- Perturbative fixed points require semisimple gauge groups and superpotentials
- Supersymmtery allows us access towards stronger coupling