Conformally Coupled General Relativity

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Motivation

- New symmetry due to Ogievetsky theorem
- Conformal coupling due to the large number hypothesis
- ADM formalism due to necessity of time parametrization

Ogievetsky theorem

General coordinate transformations

$$x_\mu = x_\mu(x_0', x_1', x_2', x_3')$$

$$L_k^{n_1,n_2,n_3,n_4} = -i\ x_1^{n_1} x_2^{n_2} x_3^{n_3} x_4^{n_4} \partial_k$$

Ogievetsky, V.I. Lett. Nuovo Cim. 1973, 8, 988–990.

"Any generator of the general covariant group $L_k^{n_1,n_2,n_3,n_4}$ is representable as some linear combination of the commutators of generators of the special linear and conformal groups"

$$\overbrace{R_{\mu\nu}, \underbrace{L_{\mu\nu}}^{SL(4,\mathbb{R})}, P_{\mu}, K_{\mu}, D}_{C(1,3)} \rightarrow L_k^{n_1,n_2,n_3,n_4}$$

Nonlinear symmetry realization

- Group G with algebra $\mathcal{G}$
- Subgroup H with algebra $\mathcal{H} = \langle\{V_k\}\rangle$
- $\mathcal{G} = \mathcal{H} \oplus \langle\{A_k\}\rangle$

Group G is covered by the exponential map:

$$g = e^{\zeta_k A_k} e^{\phi_l V_l}$$

It spawn the nonlinear group realization:

$$g e^{\zeta_k A_k} = e^{\zeta'_k A_k} e^{u_l V_l}$$

Coleman, Wess, Zumino, Phys. Rev. 1969, 177, 2239–2250.

Nonlinear realization of $SL(4,\mathbb{R}) \rtimes \langle P_\mu \rangle$ on L:

$$G(x,h) = \exp[i x_{(\mu)} P_{(\mu)}] \exp\left[\frac{i}{2} h_{(\mu)(\nu)} R_{(\mu)(\nu)}\right]$$

$$G^{-1}dG = i\left(\omega_{(\mu)} P_{(\mu)} + \omega^{L}_{(\mu)(\nu)} L_{(\mu)(\nu)} + \omega^{R}_{(\mu)(\nu)} R_{(\mu)(\nu)}\right)$$

$$\omega_{(\mu)} = \omega_{(\mu)\nu} dx^{\nu}$$

$$\omega^{R/L} = 1/2\left(\omega^{\sigma}_{(\mu)} d\omega_{(\nu)\sigma} \pm \omega^{\sigma}_{(\nu)} d\omega_{(\mu)\sigma} p\right)$$

Nonlinear realization of $SL(4,\mathbb{R})$ and $C(1,3)$

$$\nabla_{(\mu)}\Psi = \omega^{\sigma}_{(\mu)}\partial_\sigma\Psi + \frac{i}{2}V_{(\mu),(\alpha)(\beta)}L^{\Psi}_{(\alpha)(\beta)}\Psi$$

$$[\nabla_{(\mu)},\nabla_{(\nu)}] = \frac{i}{2}R_{(\mu)(\nu)(\alpha)(\beta)}L^{\Psi}_{(\mu)(\nu)}\Psi$$

Large Number Hypothesis

"All dimensionless numbers of this order that turn up in nature are connected. One of these large numbers is the epoch t, the present time reckoned from the time of creation as zero, and this increases with the passage of time"

Dirac, Naturwiss. 1973, 60, 529–531.
Dirac, Proc. Roy. Soc. Lond. 1973, A333, 403–418.

- g is governed by Einstein equation; describes gravitational field self-dynamic
- $\tilde{g}$ is coupled to matter; defines particles' motion

† Metrics g and $\tilde{g}$ are related by a conformal factor

Conformal metric and the conformal factor are related to the physical metric

$$g_{\mu\nu} = e^{-2D}\tilde{g}_{\mu\nu}$$

The action reads

$$S_{\mathrm{GR}} \to S_{\mathrm{CCGR}} = \int d^4\chi\sqrt{-\tilde{g}}\left[\frac{\tilde{M}_P^2}{16\pi}\tilde{R} + \frac{3\tilde{M}_P^2}{8\pi}\tilde{g}^{\mu\nu}\partial_\mu D\partial_\nu D\right]$$

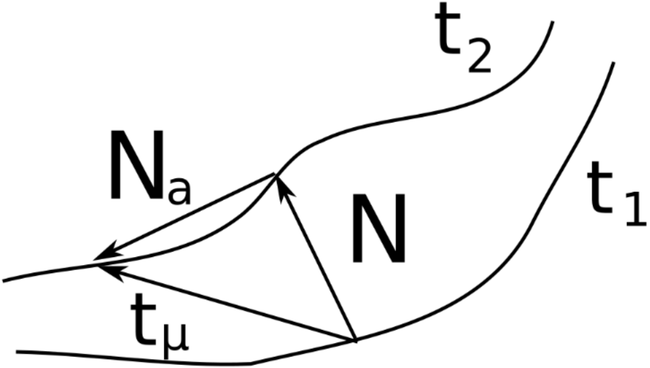
$$+ \text{gauge}$$

$3 + 1$ Decomposition

Four-dimensional spacetime is foliated in a series of three-layers

$$g = -(Ndt) \otimes (Ndt) + \gamma_{ab}(dx^a + N^a dt) \otimes (dx^b + N^b dt)$$

Lapse function and shift vector



Three-layer mean value

$$\langle \cdot \rangle = \frac{1}{V} \int d^3V \ \cdot$$

The conformal factor admits the following decomposition:

$$D(x^0, x^1, x^2, x^3) = \langle D \rangle (x^0) + \bar{D}(x^0, x^1, x^2, x^3).$$

Conformally Coupled General Relativity

$$S_{\text{CCGR}} = S_{\text{Global}} + S_{\text{Local}} + S_{\text{Conformal}}$$

$$S_{\text{Global}} = -V\frac{3\tilde{M}_P^2}{8\pi}\int dt\left(\frac{d}{dt}\langle D\rangle\right)^2$$

$$S_{\text{Local}} = -\frac{3\tilde{M}_P^2}{8\pi}\int dtd^3\chi\ \bar{D}\Delta\bar{D}$$

$$S_{\text{Conformal}} = \frac{\tilde{M}_P^2}{16\pi}\int dtd^3\chi\ \tilde{R}$$

Nonlinear Pane Wave

The model admits a nonlinear plane wave solution

$$g = -\, d\chi^0 \otimes d\chi^0 + d\chi^3 \otimes d\chi^3$$
$$e^{\Sigma}\left[e^{\sigma} d\chi^1 \otimes d\chi^1 + e^{-\sigma} d\chi^2 \otimes d\chi^2\right]$$

$$S_{\text{Conformal}} \to \int d\chi^0 d^3\chi \left\{ \frac{1}{2}\left[\left(\frac{d\sigma}{d\chi^0}\right)^2 - \left(\frac{d\sigma}{d\chi^3}\right)^2\right]\right.$$
$$\left. -e^{-\Sigma}\left(e^{-\sigma}\frac{\partial^2\Sigma}{(\partial\chi^1)^2}\right) + e^{\sigma}\frac{\partial^2\Sigma}{(\partial\chi^2)^2}\right\}$$

Nonlinear plane wave solution in terms of ω-forms

$$\omega_{(0)0} = 1 \quad \omega_{(3)3} = 1 \quad \omega_{(1)1} = e^{1/2\sigma} \quad \omega_{(2)2} = e^{-1/2\sigma}$$
$$\omega^{0}_{(0)} = -1 \quad \omega^{3}_{(3)} = 1 \quad \omega^{1}_{(1)} = e^{-1/2\sigma} \quad \omega^{2}_{(2)} = e^{1/2\sigma}$$

$$\omega^{R}_{(\mu)(\nu)}(\partial_\alpha) = \frac{1}{2}\partial_\alpha \sigma \left(\delta_{(\mu)(1)}\delta_{(\nu)(1)} + \delta_{(\mu)(2)}\delta_{(\nu)(2)}\right)$$

The model admits the following nonlinear plane wave expansion

$$\omega^{R}_{(a)(b)}(\partial_{(c)}) = \int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2\omega_k}} ik_c \left[\epsilon^{R}_{(a)(b)}(k) g^{+}_{k} e^{ik\cdot x} + \text{h.c}\right]$$

$$S_{\text{Conformal}} \to \int \frac{d^3k}{(2\pi)^3}\frac{1}{2}\varepsilon(k)\left[g^{+}(k)g^{-}(k) + g^{-}(k)g^{+}(k)\right]$$

Conclusions

$$S_{\text{CCGR}} = S_{\text{Global}} + S_{\text{Local}} + S_{\text{Conformal}}$$

$$S_{\text{Conformal}} = \frac{\tilde{M}_P^2}{16\pi}\int dt d^3\chi \tilde{R} \rightarrow \int \frac{d^3k}{(2\pi)^3}\ \ \varepsilon g^+ g^-$$

$$S_{\text{Global}} = -V\frac{3\tilde{M}_P^2}{8\pi}\int dt \left(\frac{\partial\langle D\rangle}{\partial t}\right)^2$$

$$S_{\text{Local}} = -\frac{\tilde{M}_P^2}{16\pi}\int dt d^3\chi \bar{D}\Delta\bar{D}$$

Conclusions

- † New symmetry decouples global and local dynamics
- † New symmetry points on quantisation of nonlinear plane waves
- Quantum features of the model requires further study
- Renormalization behaviour is still unclear

Literature

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