Infinite Derivative Gravity

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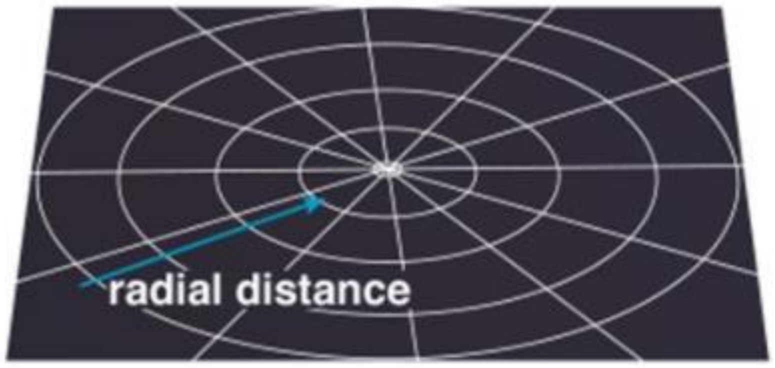
arXiv:1604.01989, arXiv:1611.05062, arXiv:1705.02382, arXiv:1710.01366, arXiv:1801.00834

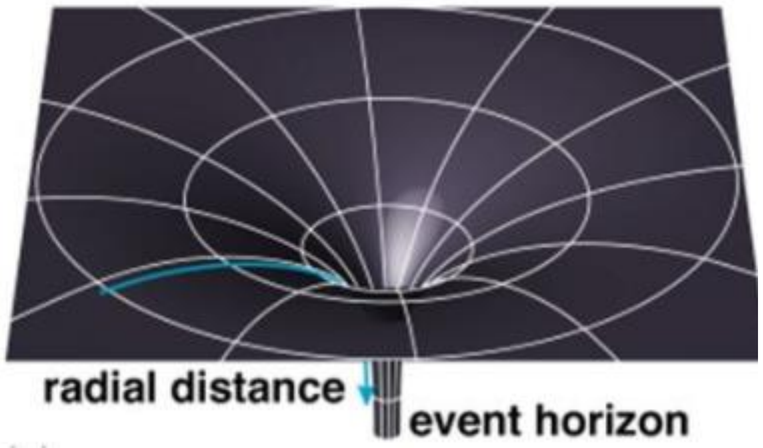
SPACETIME METRIC

Flat space: $ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$

Or in spherical coordinates:

$ds^2 = -dt^2 + dr^2 + r^2 d\Omega^2$ where $d\Omega^2$ is the angular part





GENERAL RELATIVITY

Lagrangian: $$\mathcal{L}_{\rm GR} = \frac{1}{16\pi G}\sqrt{-g}R$$

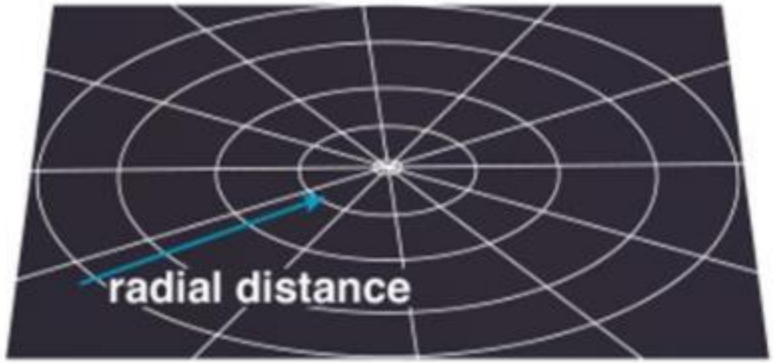
Equations of motion: $$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

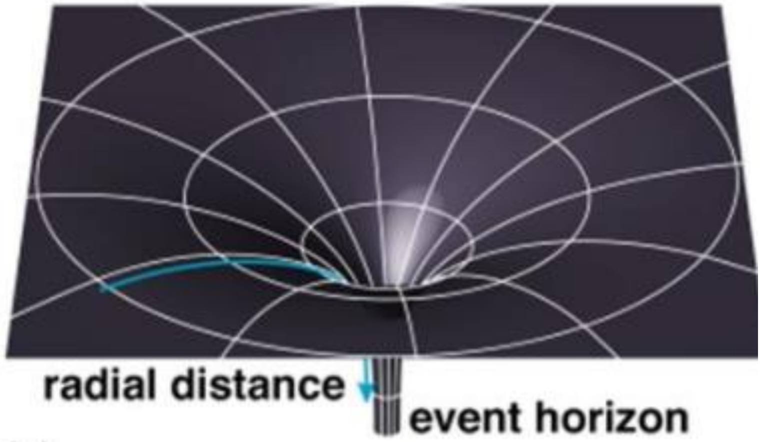
SCHWARZSCHILD METRIC

- Flat space: $ds^2 = -dt^2 + dr^2 + r^2 d\Omega^2$
- Schwarzschild metric for a black hole of mass μ:

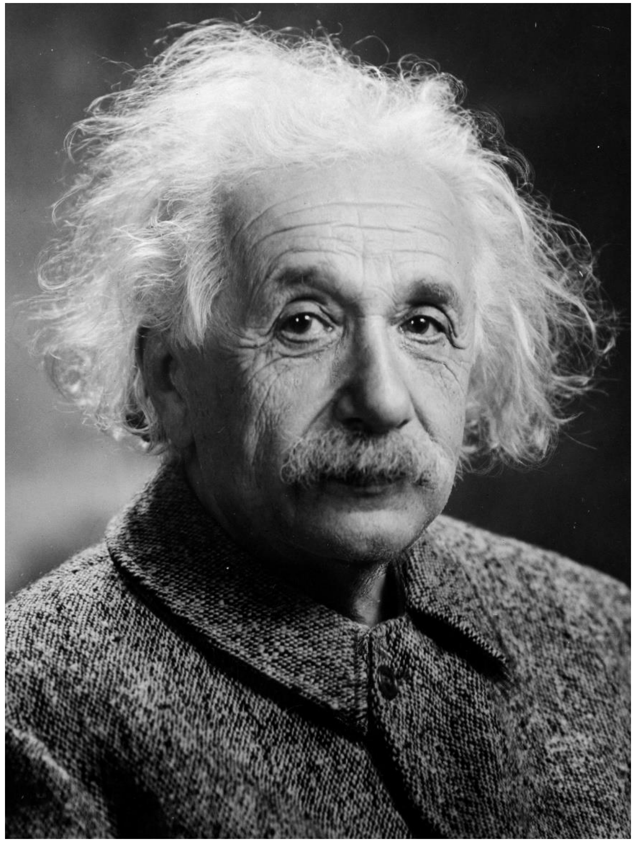
$$ds^2 = -\left(1 - \boxed{\frac{2G\mu}{r}}\right) dt^2 + \boxed{\frac{1}{1-\frac{2G\mu}{r}}} dr^2 + r^2 d\Omega^2$$

$$K \equiv R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} \propto \frac{1}{r^6}$$



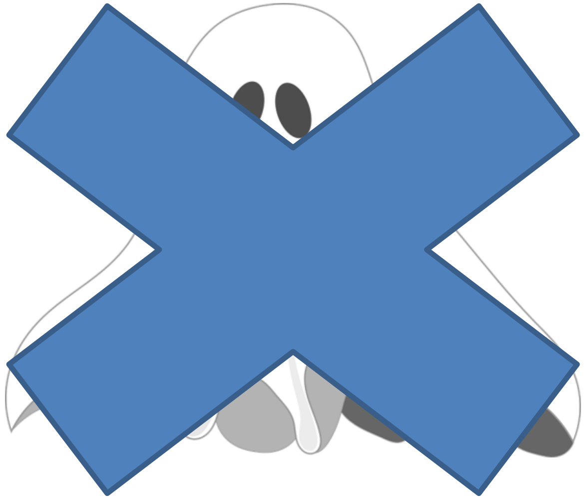


GENERAL RELATIVITY



- Works well at large distances
- Breaks down at short distances – produces singularities

INFINITE DERIVATIVE GRAVITY (IDG)



- Ghosts are physical excitations with negative kinetic energy

$$\mathcal{L}_{\mathrm{IDG}} = \frac{1}{8\pi G}\sqrt{-g}\left[R + \alpha\left(R F_1(\Box) R + R^{\mu\nu} F_2(\Box) R_{\mu\nu} + R^{\mu\nu\rho\sigma} F_3(\Box) R_{\mu\nu\rho\sigma}\right)\right]$$

where $F_i(\Box) = \sum_{n=0}^{\infty} f_{i_n}\left(\frac{\Box}{M^2}\right)^n$ $\qquad \Box = g^{\mu\nu}\nabla_\mu\nabla_\nu$

PROPAGATOR

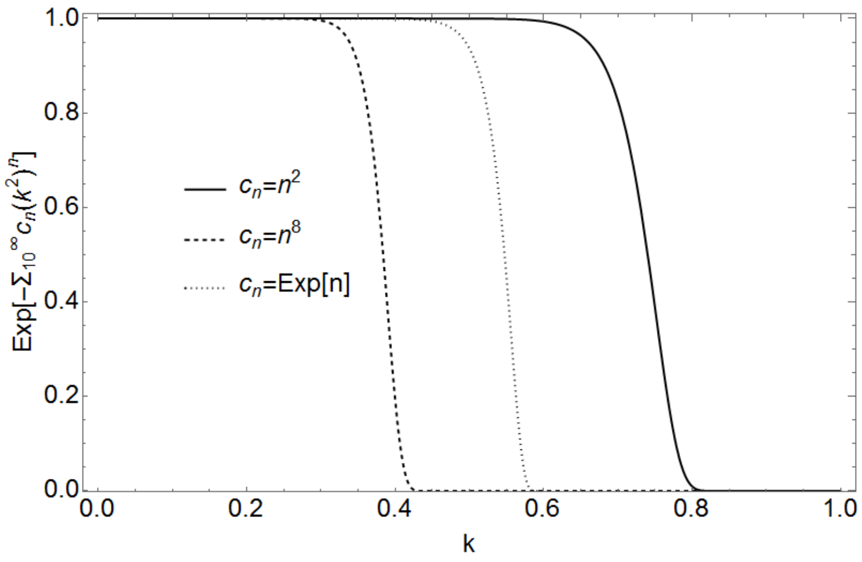
- IDG propagator around a flat background is

$$\Pi_{\mathrm{IDG}} = \frac{1}{a(k^2/M^2)}\left(\frac{P^2}{k^2} - \frac{P_s^0}{2k^2}\right) = \frac{1}{a(k^2/M^2)}\Pi_{\mathrm{GR}}$$

- where $\Box \to -k^2$ in momentum space
- $a(k^2/M^2)$ is a combination of the $F_i(\Box)$ s from earlier
- To avoid ghosts, set $a(k^2/M^2) = e^{\gamma(k^2/M^2)}$
- This has $\boxed{\text{no zeroes}}$
- Therefore no poles in the propagator so no ghosts!

FUNCTIONAL FREEDOM

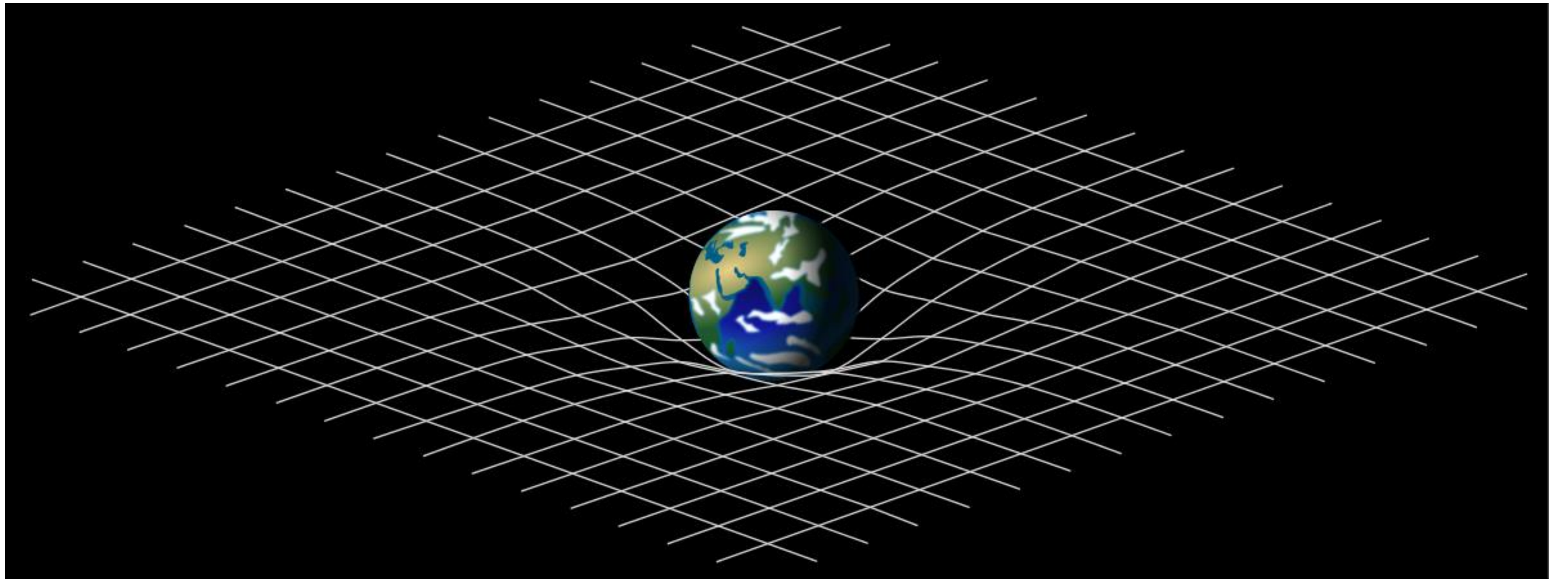
- An entire function can be written as a polynomial $\gamma(k^2/M^2) = \sum_{n=1}^{\infty} c_n \frac{k^{2n}}{M^{2n}}$
- A priori, we have infinite freedom to set coefficients
- Turns out that exponential of larger coefficients can be described by a rectangle function of a single parameter
- Reduced DOF down from infinity to ~10



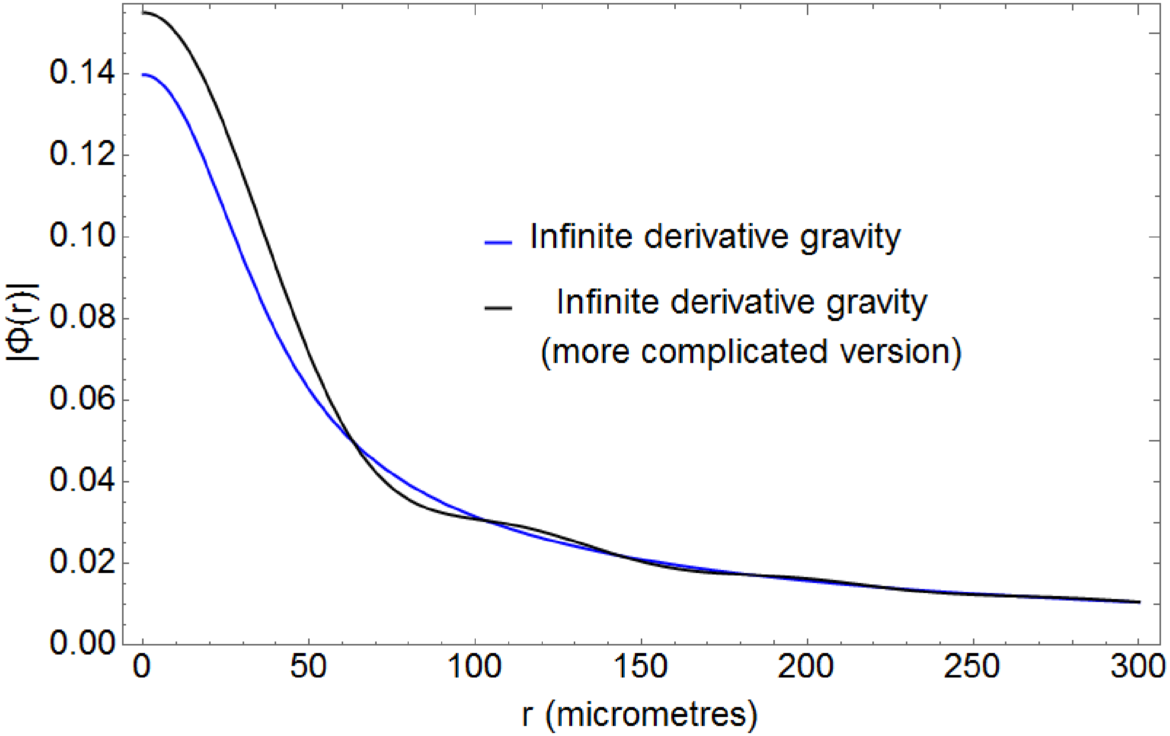
LINEARISED GRAVITY

- Static point mass added to flat background

$$ds^2 = -(1 - 2\Phi(r))dt^2 + (1 + 2\Psi(r))\left(dr^2 + r^2 d\Omega^2\right)$$



NEWTONIAN POTENTIAL

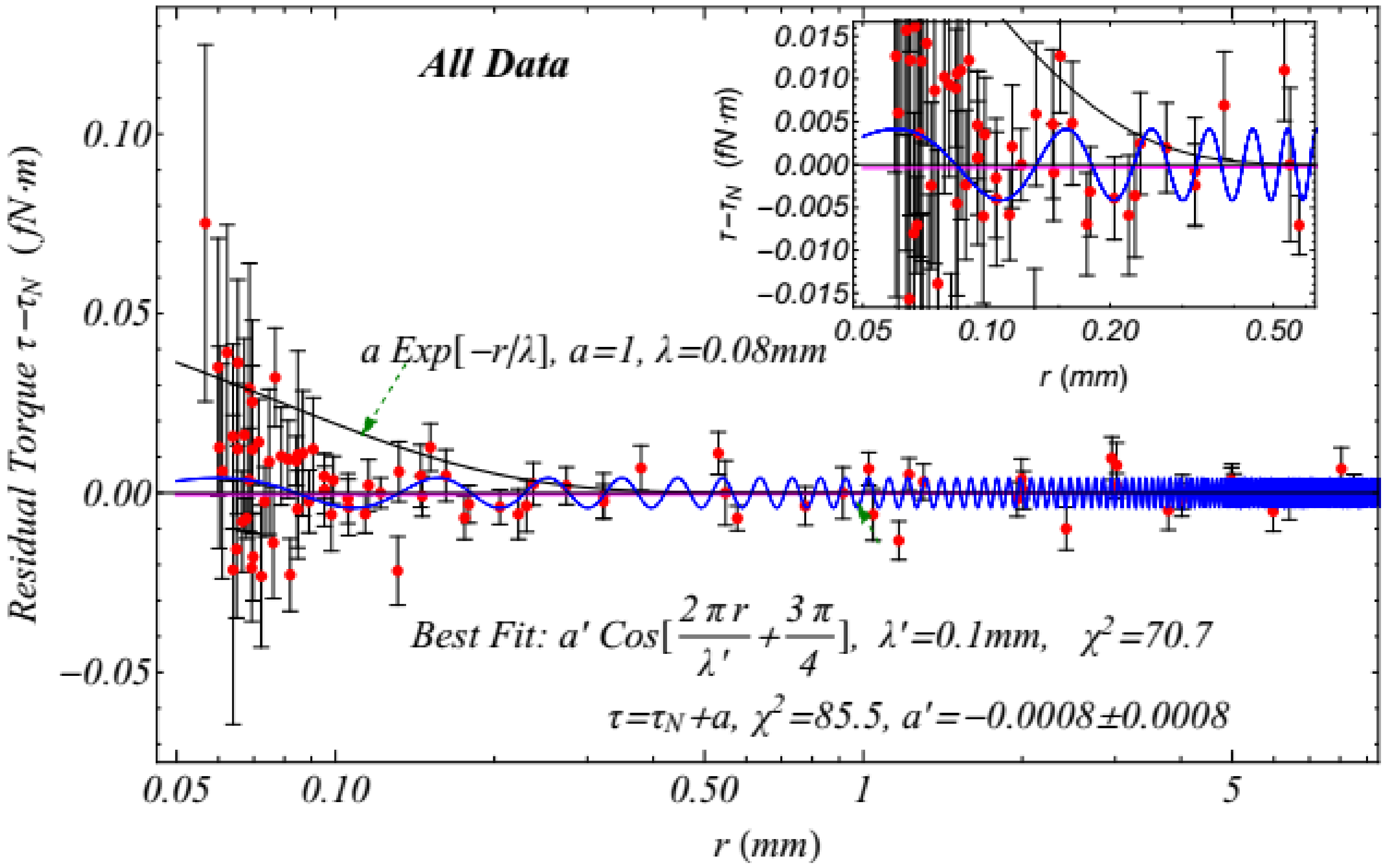


$$\Phi_{\mathrm{GR}}(r) \propto -\frac{1}{r}$$

$$\Phi_{\mathrm{IDG}}(r) \propto -\frac{\mathrm{Erf}(\frac{Mr}{2})}{r} \propto \begin{array}{l} -\frac{1}{r} \text{ for large } r \\ -1 \text{ for small } r \end{array}$$

arXiv:1604.01989

COMPARISON TO DATA



Credit: *Leandros Perivolaropoulos* "Submillimeter spatial oscillations of Newton's constant: Theoretical models and laboratory tests" arXiv:1611.07293

POTENTIAL AROUND A CURVED BACKGROUND

- Used approximation $H^2 r^2 \ll 1$
- Found full equation for potential
- At any distance where IDG makes a difference, curved background has negligible effect and vice-versa

arXiv:1801.00834

HAWKING-PENROSE SINGULARITIES

- Hawking-Penrose singularities are produced in GR
- IDG can avoid these singularities under certain conditions
- Avoiding singularities for perturbations around flat background requires extra degree of freedom in propagator
- Even with extra DOF, still avoid singularity in the potential from earlier
- Extra DOF is not necessary around a curved background

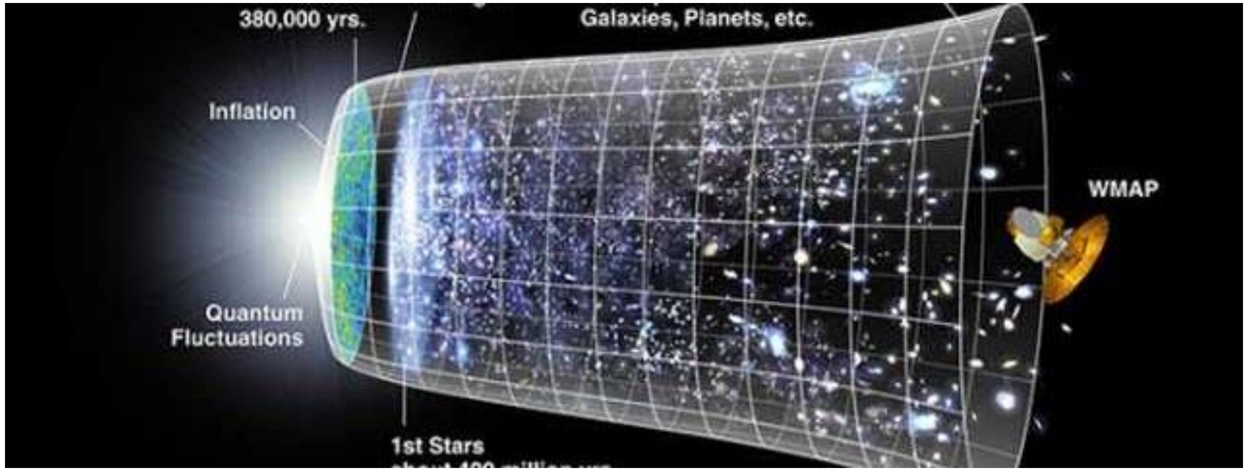
arXiv:1705.02382, arXiv:1710.01366

INFLATION

- IDG contains R+R^2 (Starobinsky gravity)+ extra terms

$$\mathcal{L}_{\rm IDG} = \frac{1}{8\pi G}\sqrt{-g}\left[R + \alpha\left(R\cancelto{1}{F_1(\Box)}R + R^{\mu\nu}\cancelto{0}{F_2(\Box)}R_{\mu\nu} + R^{\mu\nu\rho\sigma}\cancelto{0}{F_3(\Box)}R_{\mu\nu\rho\sigma}\right)\right]$$

$$\mathcal{L}_{\rm Staro} = \frac{1}{8\pi G}\sqrt{-g}\left(R + \alpha R^2\right)$$



EFFECT ON INFLATIONARY VARIABLES

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- IDG does not affect scalar perturbations $P_s^{\mathrm{IDG}} = P_s^{R+R^2}$
- Spectral tilt of scalar perturbations is unaffected $n_s^{\mathrm{IDG}} = n_s^{R+R^2}$
- It does affect tensor perturbations

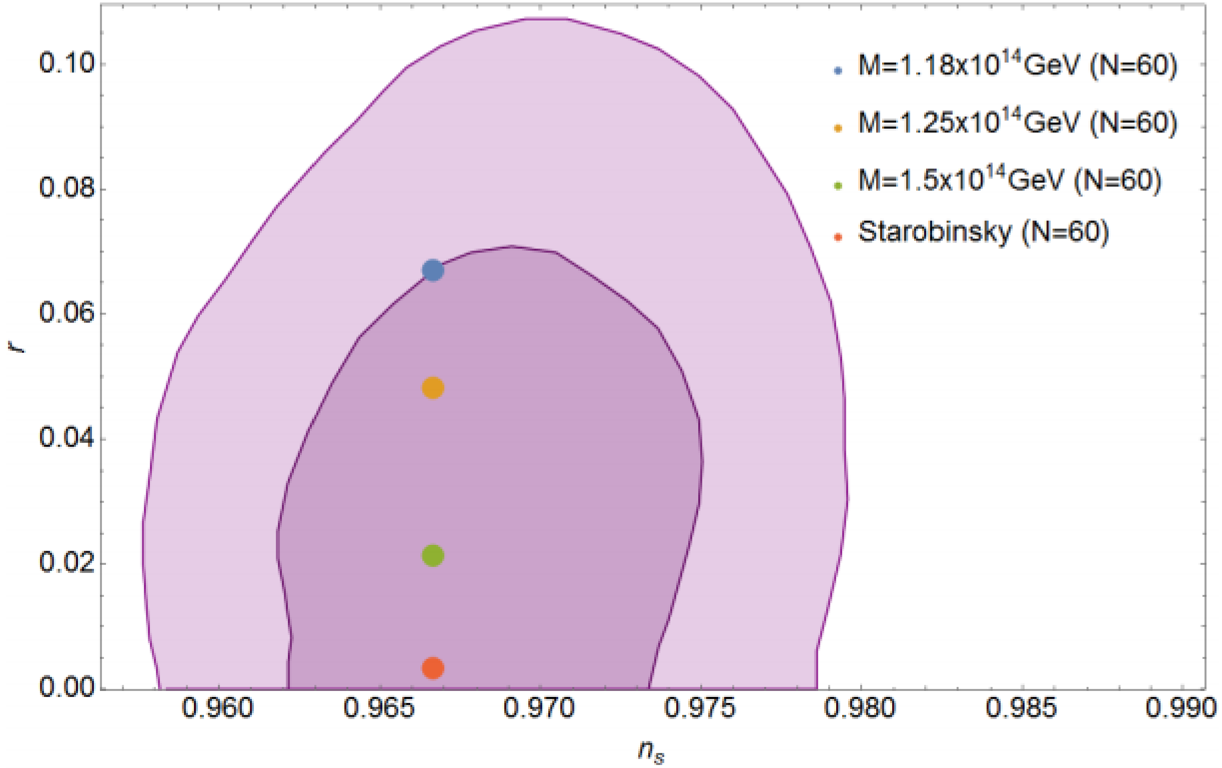
$$P_h^{\mathrm{IDG}} = P_h^{R+R^2} \exp[\omega(\Box/M^2)]\Big|_{\Box=\bar{R}/6}$$

- So the ratio between them is

$$r_{\mathrm{IDG}} = \frac{P_h^{\mathrm{IDG}}}{P_S^{\mathrm{IDG}}} = r_{R+R^2} \exp[\omega(\Box/M^2)]|_{\Box=\bar{R}/6}$$

COMPARISON TO DATA

- Tensor-scalar ratio against spectral tilt of scalar perturbations (from Planck satellite)



$$r_{\mathrm{IDG}} = r_{R+R^2} \exp\left[\omega\left(\Box/M^2\right)\right]\Big|_{\Box=\bar{R}/6}$$

$$n_s^{\mathrm{IDG}} = n_s^{R+R^2}$$

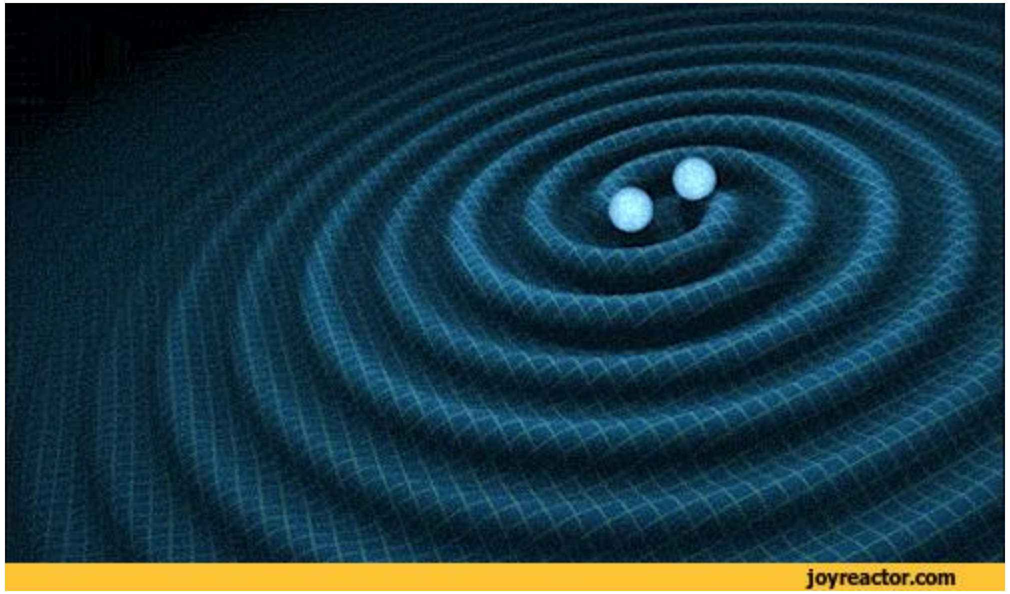
arXiv:1611.05062

CONCLUSION

- IDG can help resolve singularities
- Conforms well to experimental results
- Can be constrained using inflation data

Thanks for listening!

- Any questions?



Collabor8 conference

Lancaster University

- 14th-15th May 2018 in Lancaster
- www.collabor8research.com
- Cosmology, strings, gravity and related topics
- Aimed at PhDs and postdocs
- Keynote speaker: Ruth Gregory
- Abstract submission deadline: 5 April