Quantum Gravity in 2-dim and the Sachdev - Ye - Kitaev Model

Spenta R. Wadia
ICTS - TIFR

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G. Mandal, P. Nayak & SRW

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A bit of history:

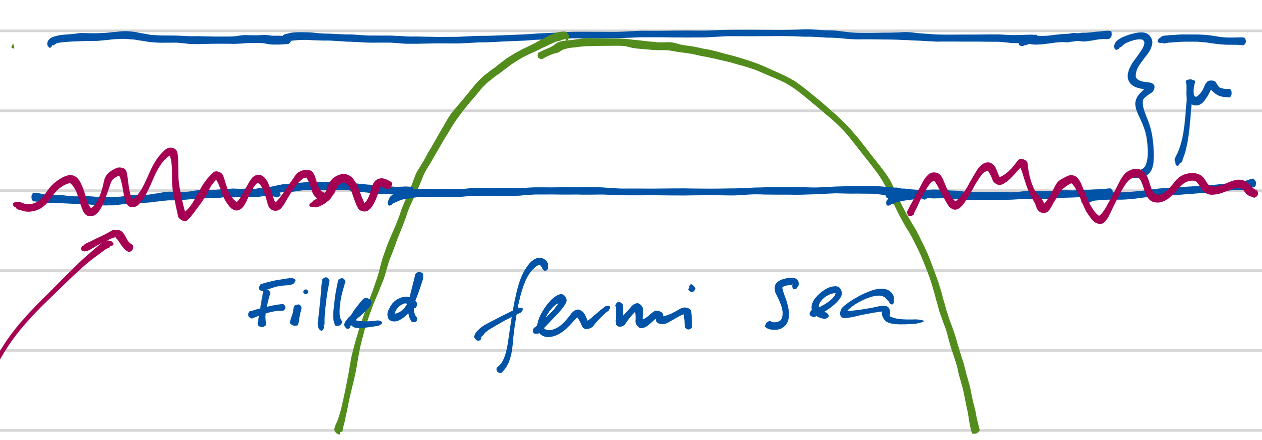
The search for soluble models to resolve problems concerning unitarity & information loss have a long history.

$C=1$ matrix quantum mechanics (~ 1990)

$$L = \frac{1}{2}\,\mathrm{tr}\,\dot{M}^2 - \left(-\frac{1}{2}\,\mathrm{tr}\,M^2 + \cdots\right)$$

$M(t)$ is a $N\times N$ matrix $M^\dagger = M$

Singlet sector: N non-relativistic fermions



A.M. Sengupta
SRW
Gross, Klebanov

massless excitation around fermi level $\tilde{T}(t,x)$
(C=1 hydrodynamics)

corresponds to massless scalar in 2-dim. string theory

Boundary to Bulk map

$\tilde{T}(t,x) \rightarrow T(t,\eta)$

$$\left(-\partial_t^2 + \partial_\eta^2\right) T(t,\eta) + e^{-\sqrt{2}\eta}\, T^2(t,\eta) + \dots = 0$$

η = Liouville mode

(Das, Naik, SRW)

Dilaton-Gravity limit of 2-dim string theory

$$S = \int dt\, d\eta\; e^{-2\Phi} \sqrt{G} \left(R - 4(\nabla\Phi)^2 + (\nabla T)^2 + V(T)\right)$$

(eqn for $T(t,\eta)$: $\Phi = \sqrt{2}\,\eta$, $G_{\mu\nu} = \eta_{\mu\nu}$)

There is more : 2-dim. BH

Mandal, Sengupta, SRW

E. Witten

Elitzur, Forge, Rabinovici

$$e^{-2\Phi} = 2(t^2 - \eta^2) + a$$

$$ds^2 = \frac{dt\, d\eta}{2(t^2-\eta^2)+a}\, , \quad a = \text{mass of BH}$$

However there is no signature of the BH in the singlet sector of the $c=1$ matrix model or its deformations!

[Integrable with ∞ conserved charges]

Finite temperature matrix model

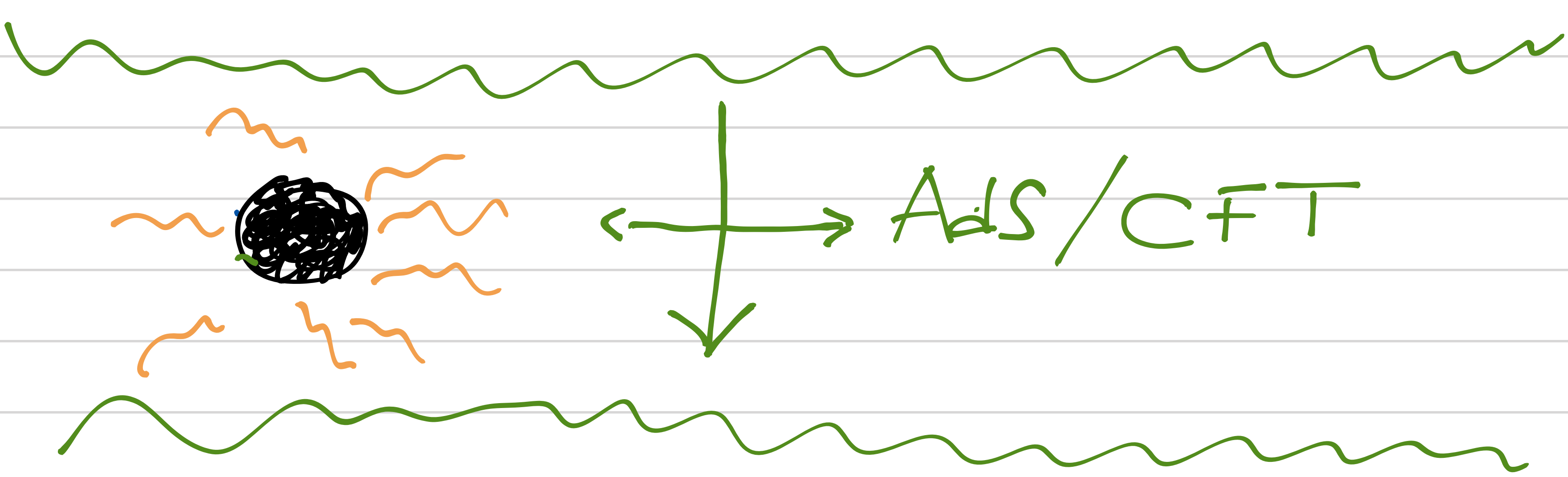
Gauged $c=1$ matrix model at finite temp.

$\Rightarrow$ inclusion of non-singlets $M(\beta) = U\, M(0)\, U^{+}$

where U is a unitary matrix.

$$Z = \int [dU]\, e^{-S_{\text{eff}}(U)}$$, calculable.

may exhibit a signature of a BH ....



A new diagnostic of BH properties:

maximal chaos, i.e. saturation of the

Quantum Liapunov exponent :

$$\lambda_L = \frac{2\pi}{\beta\hbar}$$

Shenker, Stanford, Maldacena
Susskind group

Kitaev proposed a slightly modified version of Sachdev-Ye model that is solvable at large N!

Kitaev, ..., Maldacena
Stanford

Sachdev - Ye - Kitaev Model

Simplest version :

QM of N Majorana fermions :

$\psi_i(t)$, $i = 1, 2, \ldots, N$

$$H = \sum_{1 \leq i < j < k \leq N} J_{ijkl} \psi_i \psi_j \psi_k \psi_l$$

J_{ijkl} is a random coupling

$\langle J_{ijkl} \rangle = 0$, $\langle J_{ijkl}^2 \rangle = 6 \dfrac{J^2}{N^3}$

Some important properties :

1. Well defined at finite N; soluble for large N and strong coupling $\beta J \gg 1$.

2. Emergent reparametrization symmetry as $\beta J \to \infty$

Chaos bound
density of states

3. $$\frac{\langle \psi_i(0)\psi_j(t)\psi_i(0)\psi_j(t)\rangle}{\langle \psi_i(0)\psi_i(0)\rangle\langle \psi_j(t)\psi_j(t)\rangle} \sim \left(\frac{\beta J}{N}\right) e^{\lambda_L t}$$

$$\lambda_L = \frac{2\pi}{\beta\hbar}$$ chaos bound

$$\Rightarrow \quad t_s \sim \frac{\hbar}{kT} \ln N$$ scrambling time

4. Thermodynamics : (SYK partition fn exists at finite β)

$$\log Z = -\beta E_0 + S_0 + \frac{c}{2\beta} + \cdots$$

$$\frac{c}{2} = \left(\frac{N}{J}\right) \#$$

$$\rho(E) = \frac{1}{2\pi i}\int_{\gamma + i\mathbb{R}} d\beta \, Z(\beta) e^{\beta E} \sim \sqrt{\frac{2\pi}{c J^3}}\; e^{S_0 + \sqrt{2cE}}$$

Solving SYK at large N

Averaging over J_{ijkl}, the large N SD equations are expressed in terms of bilocal fields: $G(\tau_1,\tau_2)$, $\Sigma(\tau_1,\tau_2)$

(time is euclidean)

$$\left.\begin{aligned} \frac{1}{G(\omega)} &= -i\omega - \Sigma(\omega) \\ \Sigma(\tau_1,\tau_2) &= J^2 G(\tau_1,\tau_2)^3 \end{aligned}\right\} \quad A$$

They are large N classical solutions of

$$Z = \int \mathcal{D}G\,\mathcal{D}\Sigma\; e^{-N S(\Sigma,G)}$$

$$S(\Sigma,G) = -\frac{1}{2}\log \mathrm{Pf}(\partial_\tau - \Sigma)$$

$$+ \int d\tau_1 d\tau_2 \left[\Sigma G - \frac{J^2}{4} G^4\right]$$

$$\frac{\delta S}{\delta G(\tau_1,\tau_2)} = \frac{\delta S}{\delta \Sigma(\tau_1,\tau_2)} = 0 \Rightarrow A$$

Emergent Symmetry

$\omega \ll J$ (or $\beta J \gg 1$ at finite temp)

Action S + SD equations are invariant under:

$$G(\tau_1,\tau_2) = [f'(\tau_1) f'(\tau_2)]^{1/4}\, G(f(\tau_1), f(\tau_2))$$

$$\Sigma(\tau_1,\tau_2) = [f'(\tau_1) f'(\tau_2)]^{3/4}\, \Sigma(f(\tau_1), f(\tau_2))$$

$f(\tau) \in \mathrm{Diff}(\mathbb{1})$, $\mathbb{1} \equiv \mathbb{R}^1$ or S^1

$\Rightarrow$

$$G_c(\tau) \sim \frac{\mathrm{Sgn}(\tau)}{|J\tau|^{1/4}}, \quad \Sigma_c(\tau) \sim J^2 G_c^3(\tau)$$

finite temperature : $\tau \to \tan\frac{\pi\tau}{\beta}$

$$G_c(\tau) \sim \mathrm{sgn}(\tau)\left(\frac{\pi}{\beta J \sin\frac{\pi\tau}{\beta}}\right)^{1/2}$$

Diff(1) spontaneously broken to $SL(2,\mathbb{R})$

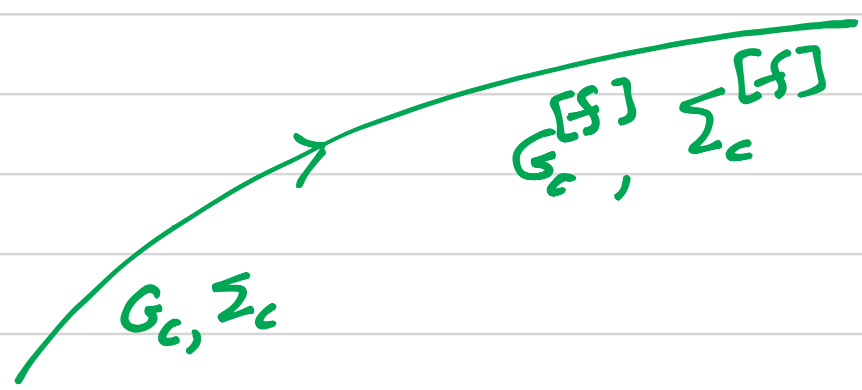
$SL(2,\mathbb{R})$: $\tau \to \frac{a\tau+b}{c\tau+d}$, $ad-bc=1$, $a,b,c,d \in \mathbb{R}^1$

Configuration space of low lying states:

$$\mathrm{Diff}(1) \longrightarrow SL(2,\mathbb{R})$$

$\mathbb{R}^1$ $\quad$ S^1

$$G_c^{[f]}(\tau),\ \Sigma_c^{[f]} \qquad f(\tau) \in \frac{\mathrm{Diff}(1)}{SL(2,\mathbb{R})}$$



$$\beta J \to \infty \ \left(\text{or } \frac{\omega}{J} \to 0\right), \quad S(G_c, \Sigma_c) = S(G_c^{[f]}, \Sigma_c^{[f]})$$

$$Z \sim \int_{\frac{\mathrm{Diff}(1)}{SL(2,\mathbb{R})}} d\mu(f)\, e^{-N S(G_c^{[f]}, \Sigma_c^{[f]})}$$

$$= \infty$$

Breaking SL(2,R)

Diff(1) / SL(2,R) dynamics

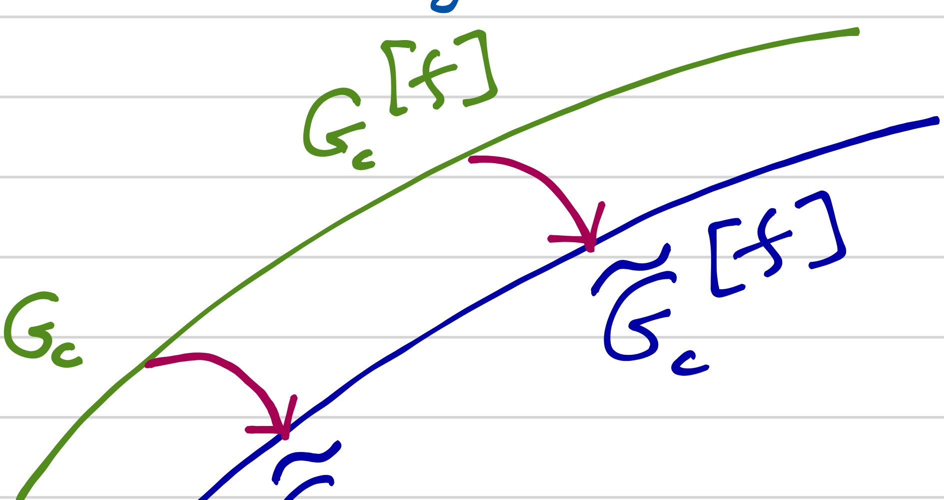
Solve SD eqn. for large $J\tau$ / βJ :

$$\tilde G_c(\tau) = G_c(\tau)\left[1 - \frac{1}{2|J\tau|} + \cdots\right]$$

$$\Big\downarrow \ \tau \to \tan\frac{\pi\tau}{\beta}$$

$$\tilde G_c(\tau,\beta) = G_c(\tau,\beta)\left[1 - \frac{1}{2\beta J}\left(\frac{2+\pi - 2\pi|\tau|/\beta}{\tan\frac{\pi|\tau|}{\beta}}\right)+\cdots\right]$$

Correct solution breaks SL(2,R) symmetry explicitly.



Perturbation theory around $\tilde G_c[f]$ well defined!

One can calculate the effective action of the low lying spectrum.

(There is a J independent spectrum of $O(1)$ spaced levels corresponding to $\mathcal{O}_m \sim \sum_i \psi_i \partial_\tau^{2m+1}\psi_i$)

$= 1, \quad , \ldots$

Effective action is the Schwarzian

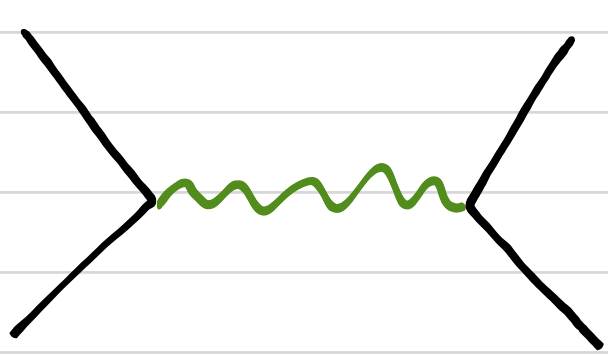
$$S_{Sch} = \alpha \frac{N}{J} \int_0^{\beta} d\tau \left[\left(\frac{f''}{f'}\right)^2 - \left(\frac{2\pi}{\beta}\right)^2 (f')^2 \right]$$

α is a number that depends on the UV properties.

$$f(\tau) = \tau + \epsilon(\tau)$$

$$\langle \epsilon(\tau)\, \epsilon(0) \rangle \sim \frac{\beta J}{N} \left(- \frac{(|\tau| - \pi)^2}{2} + (|\tau| - \pi) \sin|\tau| + a + b \cos\tau \right)$$

after appropriate analytic continuation



$$\sim \frac{\beta J}{N} e^{\frac{2\pi}{\beta \hbar} t}$$

(dominant exchange in OTO 4-pt function)

real time

(Similar conclusions for tensor models)

Dual representation in terms of 2-dim gravity

Can one derive or strongly motivate the dual representation?

Another candidate: Jackiw-Teitelboim

SYK model has emergent Diff(1) symmetry broken to $SL(2,R)$

The low lying states lie on

$\mathrm{Diff}(1)/SL(2,R)$

Hence one can consider the quantization of the co-adjoint orbit of

$\mathrm{Diff}(1)/SL(2,R)$

We will assume that Diff(1) in SYK has a central extension

$\widetilde{\mathrm{Diff}}(1)$, with $c \sim N$

<u>Proposal</u>: Dual rep. emerges from <u>quantization</u> of co-adjoint orbit of $M = \widetilde{\mathrm{Diff}}(1)/SL(2,R)$

The 'action' on the co-adjoint orbit of $\widetilde{\text{Diff}}(1) / SL(2,R)$

$\{ f(\tau,\sigma) \}$ is a set of reparametrizations labeled by 'σ'.

Witten

Alexeev, Shatashville, Rai, Rodgers

$$S_{\text{Kirilov}} = \int d\sigma d\tau \left[-b_0 f' \dot{f} + \frac{c}{48\pi} \frac{f'}{\dot{f}} \left(\frac{\dddot{f}}{\dot{f}} - 2\frac{\ddot{f}^2}{\dot{f}^2} \right) \right]$$

$f' = \partial_\sigma f$, $\dot{f} = \partial_\tau f$, c = central charge

For $SL(2,R)$ $\quad b_0 = -\frac{c n^2}{48\pi}$ (Witten)

$n \in \mathbb{Z}$

Alexeev, Shatashville

For the choice $b_0 = 0$

$$\boxed{S_{\text{Kirilov}} = \frac{c}{24\pi} \int \sqrt{g}\, R \frac{1}{\Box} R \qquad ds^2 = \partial_\sigma f \, d\sigma d\tau}$$

Polyakov Action

Quantization of $\widetilde{\text{Diff}} / SL(2,R)$ naturally leads to Polyakov gravity in 2-dim.

Dual for SYK (A proposal) (MNW)

$$S_{cov}[g] = \frac{1}{16\pi b^2} \int_{\Gamma} \sqrt{g} \left[R \frac{1}{\Box} R - 16\pi\mu \right]$$

$$+ \frac{1}{4\pi b^2} \int_{\partial\Gamma} \sqrt{\gamma}\, K \frac{1}{\Box} R$$

$$+ \frac{1}{4\pi b^2} \int_{\partial\Gamma} \sqrt{\gamma}\, K \frac{1}{\Box} K$$

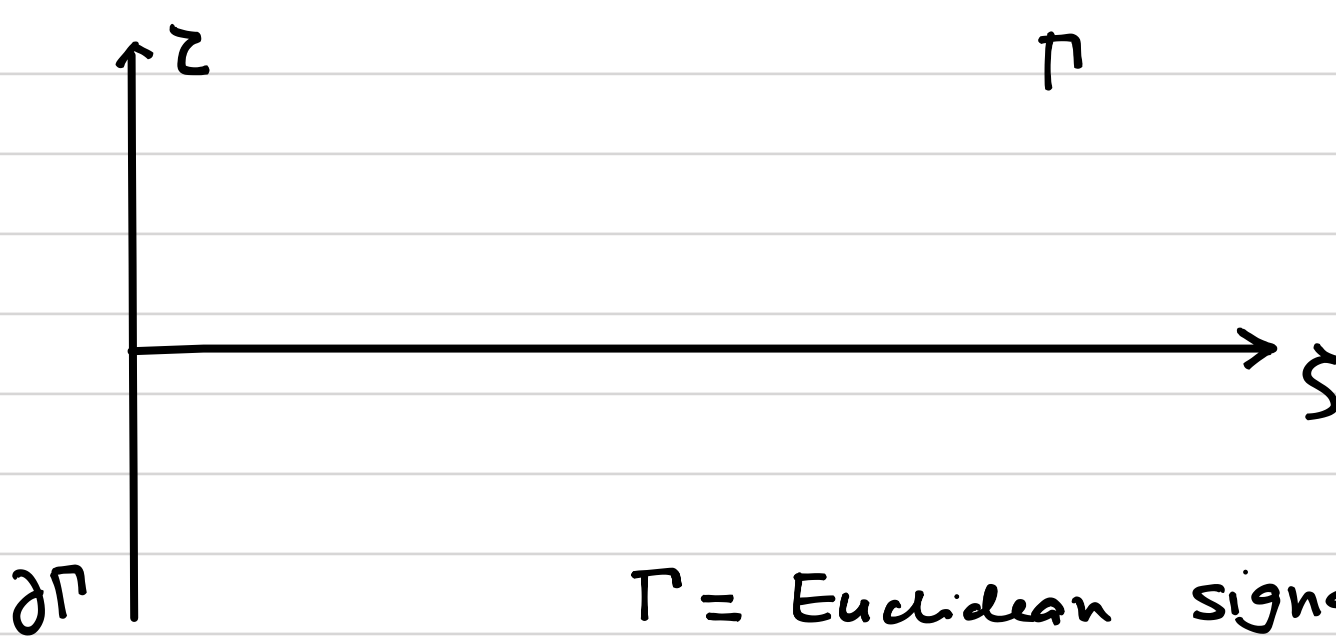
- $-\mu < 0 \Rightarrow$ cosmological constant, $b^2 = \frac{3}{2c}$
- $\Box$ = Laplacian, R = Ricci scalar
- $\partial\Gamma$ is the boundary of Γ a domain in
- 2-dim., γ is the induced metric on $\partial\Gamma$
- K is the extrinsic curvature on $\partial\Gamma$.

1. Boundary terms are determined by the usual requirement that eqns of motion are implied by a variational principle.

2. Requirement of asymptotic AdS_2 geometry $\Rightarrow$ $\mu \int d^2x \sqrt{g}$

Also $R\left(\frac{1}{\Box}R\right)^n$, $n>1$ do not admit asymptotic AdS_2 geometries.

3.



Γ = Euclidean signature Poincare right half plane

Equations of motion

$$R(x) = -8\pi\mu \qquad (\mathrm{I})$$

$$\left.\begin{aligned} &-2\left(\nabla_\mu^{(\omega)}\nabla_\nu^{(\omega)} - \tfrac{1}{2} g_{\mu\nu}(\omega)\,\Box^{(\omega)}\right)\chi(\omega) \\ &+ \partial_\mu^{(\omega)}\chi\,\partial_\nu^{(\omega)}\chi - \tfrac{1}{2} g_{\mu\nu}(\omega)\,\partial_\alpha\chi\,\partial^\alpha\chi = 0 \end{aligned}\right\}(\mathrm{II})$$

$$\chi(\omega) = \int G(\omega,x)\,R(x) \quad , \quad G(\omega,x) = \langle \omega | \tfrac{1}{\Box} | x\rangle$$

Solutions :

Step 1 :

Look for solutions of the form :

$$g_{\alpha\beta} = e^{2\phi}\,\hat{g}_{\alpha\beta}$$

AdS$_2$

$$\hat{ds}^2 = \hat{g}_{\alpha\beta}\,dx^\alpha dx^\beta = \frac{dz\,d\bar{z}}{\pi\mu\,(z+\bar{z})^2} = \frac{1}{4\pi\mu}\,\frac{d\zeta^2 + dz^2}{\zeta^2}$$

(I): $R = -8\pi\mu \;\Rightarrow\; 2\hat{\Box}\phi = \hat{R} + 8\pi\mu e^{2\phi}$

$$\Rightarrow \quad \phi = \frac{1}{2}\log\left[\frac{(z+\bar{z})^2\,\partial g(z)\,\bar{\partial}\bar{g}(\bar{z})}{(g(z)+\bar{g}(\bar{z}))^2}\right]$$

$\partial_{\bar{z}}\, g = 0 = \partial_z \bar{g}$

$$ds^2 = \frac{1}{\pi\mu}\frac{e^{2\phi}}{(z+\bar z)^2}\,dz\,d\bar z = \frac{1}{\pi\mu}\frac{(dz\,\partial g)(d\bar z\,\bar\partial \bar g)}{(g(z)+\bar g(\bar z))^2}$$

This defines a new class of metrics except when $g(z) = \frac{az+b}{cz+d} \in SL(2,R)$

(Schwarz-Pick Lemma)

(II): $$\partial^2\phi - (\partial\phi)^2 + 2\frac{\partial\phi}{(z+\bar z)} = 0$$

$$\bar\partial^2\phi - (\bar\partial\phi)^2 + 2\frac{\bar\partial\phi}{(z+\bar z)} = 0$$

$$\Rightarrow \quad \{g(z), z\} = 0 = \{\bar g(\bar z), \bar z\}$$

$$\Rightarrow \quad g(z) = \frac{az+b}{cz+d}, \quad a,b,c,d \in \mathbb{C}$$

$$ad - bc = 1$$

If $g(z) \in SL(2,R)$ then

$$g(z) + \bar g(\bar z)\Big|_{z+\bar z} = 0$$ i.e. the boundary is left invariant by the isometry.

If $g(z) \notin SL(2,\mathbb{R})$ then

$$g(z) + \bar{g}(\bar{z})\Big|_{z+\bar{z}} \neq 0$$

$\Rightarrow$ a 3-parameter deformation that changes the boundary

$\Rightarrow$ metrics which are deformations of AdS_2 (call them $NAdS_2$)

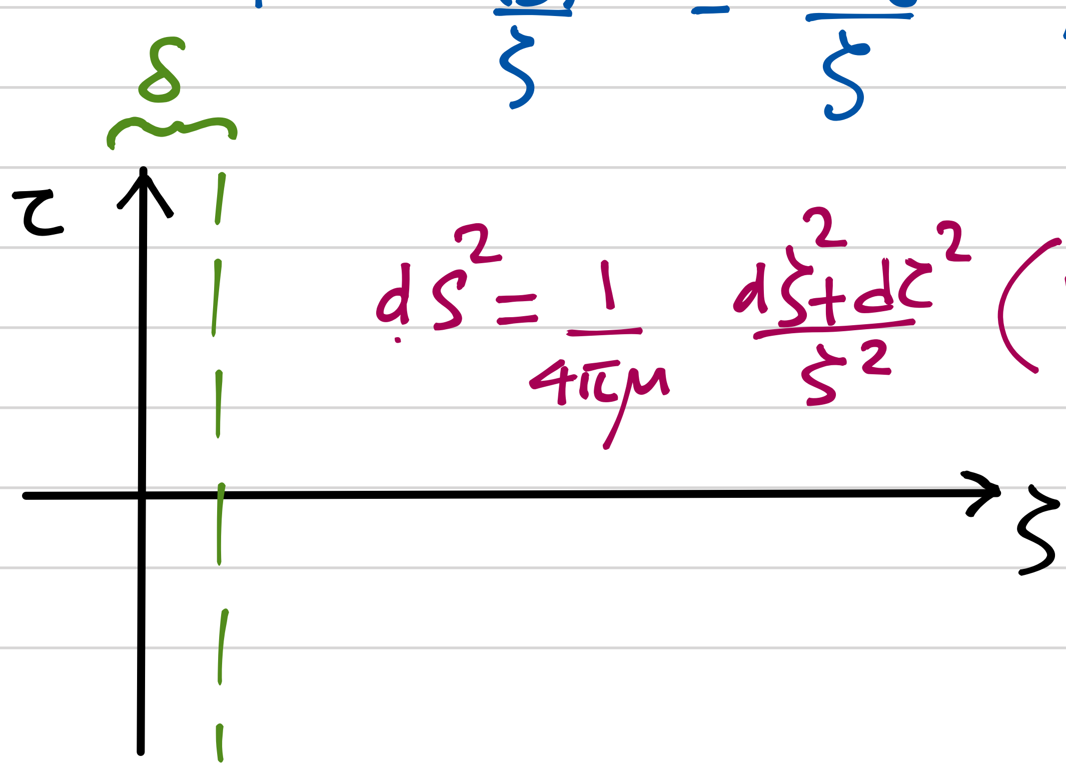
Consider the simplest one parameter deformation:

$$g(z) = z + b \qquad , \qquad |b| \ll 1$$

$\Rightarrow$

$$\Phi \sim \frac{\mathrm{Re}(b)}{\zeta} \equiv \frac{\delta g}{\zeta} \quad , \quad |\delta g| \lesssim \delta$$

AdS_2 cutoff



$$ds^2 = \frac{1}{4\pi\mu}\,\frac{d\zeta^2 + d\tau^2}{\zeta^2}\left(1 + 2\,\frac{\delta g}{\zeta} + \dots\right)$$

$NAdS_2$

Asymptotically AdS_2 geometries ($AAdS_2$)

1. Fall off conditions:

$$g_{\zeta\zeta} = \frac{1}{4\pi\mu\,\zeta^2}\,, \quad g_{\zeta z} = o(\zeta^0)\,, \quad g_{zz} = \frac{1}{4\pi\mu}\frac{1}{\zeta^2} \;\ldots$$

2. Diffs of AdS_2 that preserve these conditions

$$\delta g_{\alpha\beta} = \nabla_\alpha \epsilon_\beta + \nabla_\beta \epsilon_\alpha$$

3. Fix gauge: $\delta g_{\zeta\zeta} = 0 = \delta g_{\zeta\tau}$

Solution:

$$\epsilon^\zeta = \zeta\, \delta f(\tau)\,, \quad \epsilon^\tau = \delta f(\tau) - \frac{1}{2}\zeta^2\, \delta f''(\tau)$$

$\delta f(\tau)$ is an arbitrary function, $|\delta f| \ll 1$

Tangential to the boundary of AdS_2 $(\zeta = 0)$.

Finite 'large' diffs

$$\tilde{z} = f(z) - \frac{2\zeta^2 f''(z) f'(z)^2}{4f'(z)^2 + \zeta^2 f''(z)^2}$$

$$\tilde{\zeta} = \frac{4\zeta f'(z)^3}{4f'(z)^2 + \zeta^2 f''(z)^2}$$

restricting large diffs in AdS_3 to the $(\zeta, z, x=0)$ plane

$$d\hat{s}^2 = \frac{1}{4\pi\mu} \frac{d\tilde{\zeta}^2 + d\tilde{z}^2}{\tilde{\zeta}^2}$$

$$\Big\downarrow f(z)$$

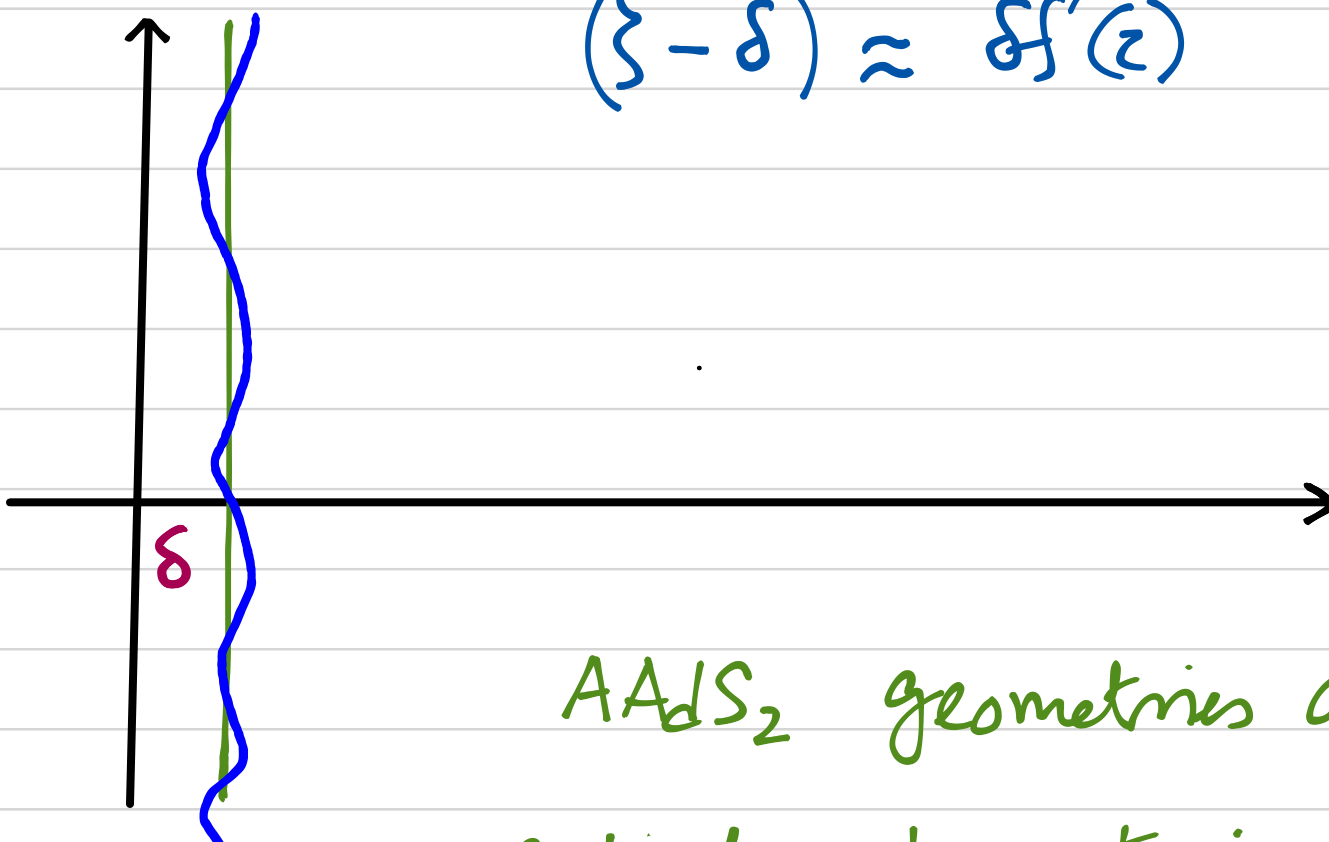
$$\hat{g} \xrightarrow{f} \hat{g}[f]$$

$$\boxed{\begin{aligned} d\hat{s}^2 &= \frac{1}{4\pi\mu}\frac{1}{\zeta^2}\left(d\zeta^2 + dz^2\left[1 - \zeta^2 \frac{\{f(z), z\}}{2}\right]^2\right) \\ \{f, z\} &= \frac{f'''}{f'} - \frac{3}{2}\left(\frac{f''}{f'}\right)^2, \quad f' \neq 0 \end{aligned}}$$

Transformation of Cutoff:

$$\tilde{\zeta} = \delta \xrightarrow{f(z)} \zeta \approx \delta\left(1 + \delta f'(z)\right)$$

$$(\zeta - \delta) \approx \delta f'(z)$$



$AAdS_2$ geometries are entirely characterized by $\{f(z)\}$.

Summary of full solution

$$ds^2 = e^{2\phi}\, d\hat{s}^2$$

$$d\hat{s}_f^2 = \frac{1}{4\pi\mu\zeta^2}\left(d\zeta^2 + dz^2\left[1 - \zeta^2 \frac{\{f(z), z\}}{2}\right]^2\right)$$

$$\phi = \frac{\delta g}{\zeta} + \dots$$

Classical Action

Results :

$$\delta S_{bulk} = S_{bulk}\left(\hat{g}^{[f]}, \phi\right) - S_{bulk}\left(\hat{g}^{[f=\tau]}, \phi\right)$$

$$= \frac{1}{4\pi b^2} \frac{1}{\delta}\left[\epsilon(+\infty) - \epsilon(-\infty)\right] = 0 .$$

$f(\tau) = \tau + \epsilon(\tau)$, $\epsilon(\infty) = \epsilon(-\infty)$

2. $$\delta S_{bd} = S_{bd}^{AAdS_2} - S_{bd}^{AdS_2}$$

$$= \frac{1}{2\pi b^2} \delta g \int d\tilde{\tau} \left\{ \tilde{f}(\tilde{\tau}), \tilde{\tau} \right\}$$

Path Integral of 2-dim. gravity

$$Z \sim \int \left[\frac{\mathcal{D} f(\tau)}{f'(\tau)}\right]' e^{-\frac{\delta g}{2\pi b^2}\int d\tau \{f(\tau), \tau\}}$$

At finite temp: $S_\beta = \frac{\delta g}{2\pi b^2} \int d\theta \left\{ \frac{\beta}{2} \tan\left(\frac{\pi f(\theta)}{\beta}\right), \theta \right\}$

Correspondence with SYK

$\delta g \sim \frac{1}{J} \ll 1 \, , \quad b^2 \sim \frac{1}{N} \ll 1$

$$\ln Z = -\beta F = \underbrace{\frac{\delta g}{2b^2}\left(\frac{1}{\beta}\right)}_{\text{matches with SYK}} + (\text{constant})$$

Incorporation of the higher spectrum of SYK

SYK : $\delta G(\tau_1,\tau_2) = G(\tau_1,\tau_2) - G_{cl}(\tau_1-\tau_2)$

$\Rightarrow$ tower $\sim$ evenly spaced states

with dim. h_m corresponding to

$$O_m = \sum_i \psi_i \, \partial_z^{2m+1} \psi_i \, , \quad m = 1, 2, \ldots \infty$$

Correspond to massive scalar fields $\eta_r(\zeta,\tau)$ with mass m_r given in terms of h_n.

Quantum Chaos in 2-dim quantum gravity

Bulk scalar (probe) action

$$S = \int dz\, d\zeta \sqrt{g} \left((\partial\phi)^2 + m^2\phi^2 \right)$$

metrics : $ds^2 = \frac{d\zeta^2}{\zeta^2} + \frac{1}{\zeta^2}\left(1 - \zeta^2 \frac{\{f,z\}}{2}\right)^2 dz^2$

$\{f, z\} = 0$ corresponds to AdS_2.

Solution in AdS_2 regular as $\zeta \to \infty$

$$e^{ikz}\,\tilde{\Phi}(k, \tilde{\zeta}) \sim \sqrt{\zeta}\, B(k)\, K_\nu(k\tilde{\zeta})\, e^{ikz}$$

$$\nu = \frac{1}{2}\left(1 + 4m^2\right)^{1/2}$$

Boundary coupling: $S_{int} = \int dz\, \tilde{J}(z)\, \tilde{O}_\Delta(z)$

"AdS/CFT"

$$\tilde{J}(\tilde{z}) = \lim_{\tilde{\zeta}\to 0} \zeta^{\Delta-1}\tilde{\Phi}(\tilde{z}, \zeta), \quad \Delta - 1 = \nu - \frac{1}{2}$$

Now consider the large diffs. :

$$(\tilde{\tau}, \tilde{\zeta}) \to (\tau, \zeta)$$

$$\tilde{\tau} = f(\tau) - \frac{2\zeta^2 f''(\tau) f'(\tau)^2}{4f'(\tau)^2 + \zeta^2 f''(\tau)^2}$$

$$\tilde{\zeta} = \frac{4\zeta f'(\tau)^3}{4f'(\tau)^2 + \zeta^2 f''(\tau)^2}$$

Since ϕ is a scaler,

$$\tilde{\Phi}(\tilde{\zeta}, \tilde{\tau}) = \phi(\zeta, \tau)$$

$$\Rightarrow \tilde{J}(\tilde{\tau})\, \tilde{\zeta}^{1-\Delta} = J(\tau)\, \zeta^{1-\Delta},$$

$$\frac{\tilde{\zeta}}{\zeta} = f'(\tau), \qquad \zeta \to 0$$

$$\Rightarrow \quad \tilde{J}(\tilde{\tau}) = J(\tau) \left[f'(\tau)\right]^{\Delta - 1}$$

Bulk on-shell action :

$$S^{int}_{Bulk} = \int_{\partial M} d\tilde{\tau} \sqrt{\tilde{\gamma}}\ \tilde{\Phi}(\tilde{\tau},\tilde{\xi}) \frac{\partial}{\partial \tilde{\xi}} \tilde{\Phi}(\tilde{\tau},\tilde{\xi})$$

$\partial M = \partial AdS_2 \qquad \xi \to 0\ ,\ \tau \in (-\infty, +\infty)$

$$= \int d\tilde{\tau} d\tilde{\tau}' \frac{\tilde{J}(\tilde{\tau})\tilde{J}(\tilde{\tau}')}{|\tilde{\tau} - \tilde{\tau}'|^{2\Delta}}$$

$$\Rightarrow \boxed{S^{int}_{Bulk}(f) = \int_{\tau,\tau'} J(\tau) \frac{[f'(\tau) f'(\tau')]^{\Delta}}{|f(\tau) - f(\tau')|^{2\Delta}} J(\tau')}$$

This is the main formula

Quantum Chaos

2-dim. quantum gravity path integral

$$Z[J] = \int [\mathcal{D}f(\tau)]\, e^{-S}$$

$$S = S^{\text{Int}}_{\text{BULK}}(f) + S_{\text{Sch}}(f)$$

$$S^{\text{Int}}_{\text{Bulk}}(f) = \int d\tau d\tau' J(\tau) \frac{f'(\tau)^{\Delta} f'(\tau')^{\Delta}}{|f(\tau)-f(\tau')|^{2\Delta}} J(\tau')$$

$$S_{\text{Sch}}(f) = \frac{N}{J}\int d\tau \, \{f(\tau), \tau\}$$

$$\langle O(\tau_1) O(\tau_2) O(\tau_3) O(\tau_4)\rangle =$$

$$\left. \frac{1}{Z[J]} \frac{\delta}{\delta J(\tau_1)} \frac{\delta}{\delta J(\tau_2)} \frac{\delta}{\delta J(\tau_3)} \frac{\delta}{\delta J(\tau_4)} Z[J] \right|_{J=0}$$

From here on calculation is same as Kitaev + Maldacena, Stanford ($\tau \to it$, OTO...)

$$\lambda_L = \left(\frac{2\pi}{\beta\hbar}\right)$$

Conclusions

SYK / tensor models:

- Soluble in large N limit
- Strong coupling / infrared $\rightarrow$

  Emergent infinite dim. symmetry Diff[1]

$$\downarrow$$

$$\mathrm{Diff}[1] \,/\, SL(2,R)$$

$$\downarrow$$

Hydrodynamics description $\longrightarrow$ Schwarzian action

$$\downarrow$$

Saturation of chaos bound.

- SYK can be used to study black hole problems in a concrete setting.

- $\boxed{\text{Solve SYK for finite N + learn to take large N limit.}}$

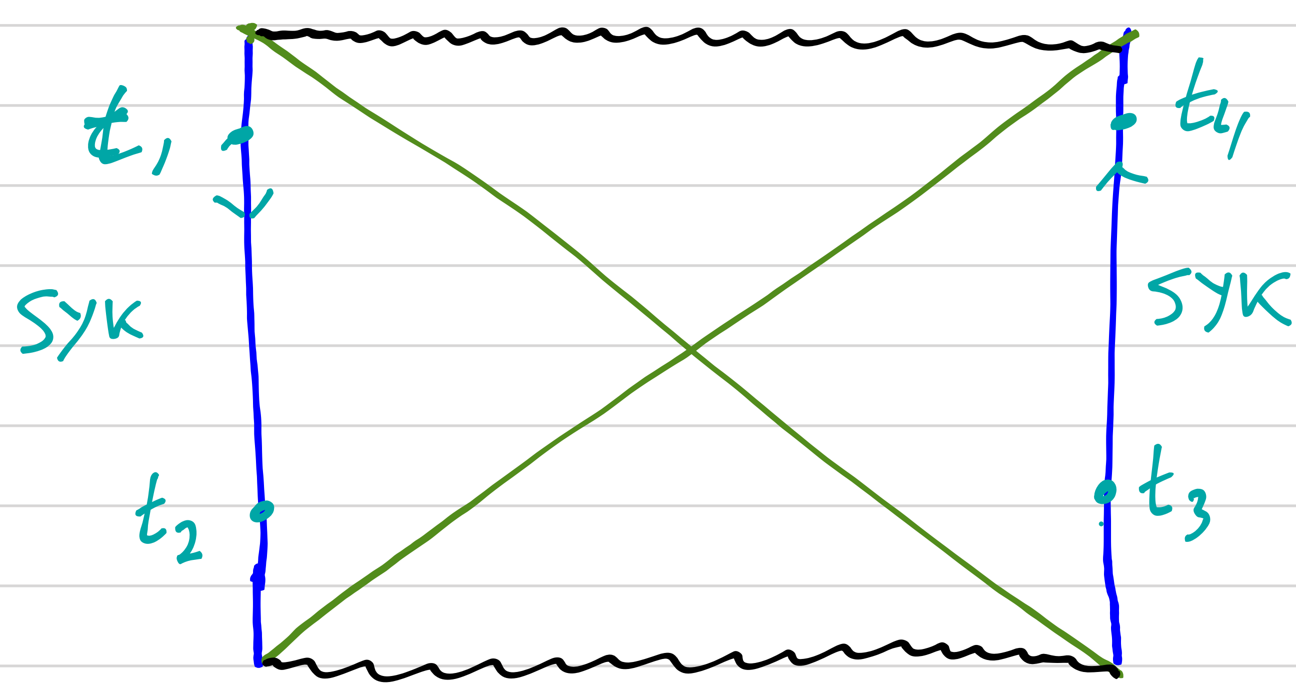
- Emergent symmetry at strong coupling / infrared

- Quantization of co-adjoint orbits
$\Rightarrow$ (Polyakov) 2-dim. quantum gravity.

$\Downarrow$

Schwarzian, chaos bound

- Exact solution of this theory at finite $G_N \sim \frac{1}{N}$ can throw light on black hole conundrums



Thank you