14th HXSWG Workshop 26 March 2018

Higgs Boson Pair Production: Introduction NLO & NLO+PS

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(Werner-Heisenberg-Institut)

Stephen Jones

Borowka, Greiner, Heinrich, Kerner, Kuttimalai, Luisoni, Schlenk, Schubert, Vryonidou, Zirke

JHEP 02 (2018) 176 [1711.03319]
JHEP 08 (2017) 088 [1703.09252]
JHEP 10 (2016) 107 [1608.04798]
PRL 117 (2016) 012001, Erratum 079901 [1604.06447]

MAX-PLANCK-GESELLSCHAFT

Outline

HH Production Channels

Gluon Fusion

- Currently available results
- Recap/appraisal of approximations at NLO

Fixed Order NLO Results

NLO + Parton Shower Results

Motivation

Standard Model Higgs Lagrangian:

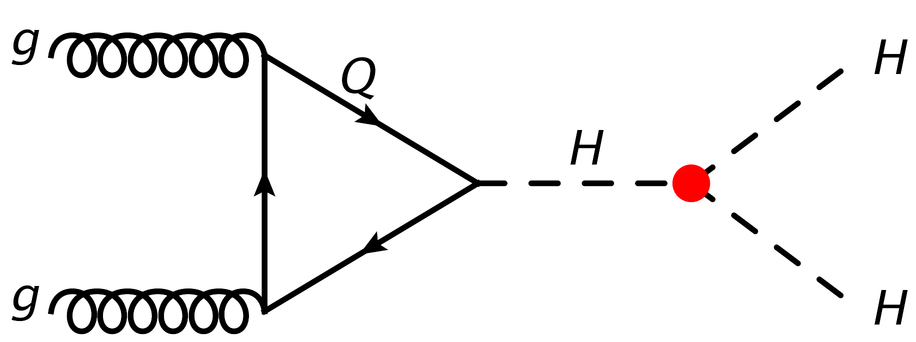
$$\mathcal{L} \supset -V(\phi), \quad V(\Phi) = -\mu^2(\Phi^\dagger\Phi) + \lambda(\Phi^\dagger\Phi)^2$$

EW symmetry breaking

$$V(H) = \frac{1}{2}m_H^2 H^2 + \boxed{\lambda v H^3} + \frac{\lambda}{4}H^4, \qquad \mu^2 = \lambda v^2, \quad m_H^2 = 2\lambda v^2$$

SM: self-couplings determined by m_H, v

Higgs pair production probes triple-Higgs coupling



Higgs pair production extremely challenging to measure at LHC due to $\mathcal{O}(\mathrm{fb})$ cross section and difficult backgrounds

Production Channels

$\sigma(pp \to HH + X)$ @ 14 TeV

Gluon Fusion

Vector Boson Fusion (VBF)

NLO [1,2] NNLO [3]

+ non-negligible contribution from $gg \to HHjj$ LO [6]

Top-Quark Associated

NLO [2]

Higgs-strahlung

NLO [1,2] NNLO [1,4,5]

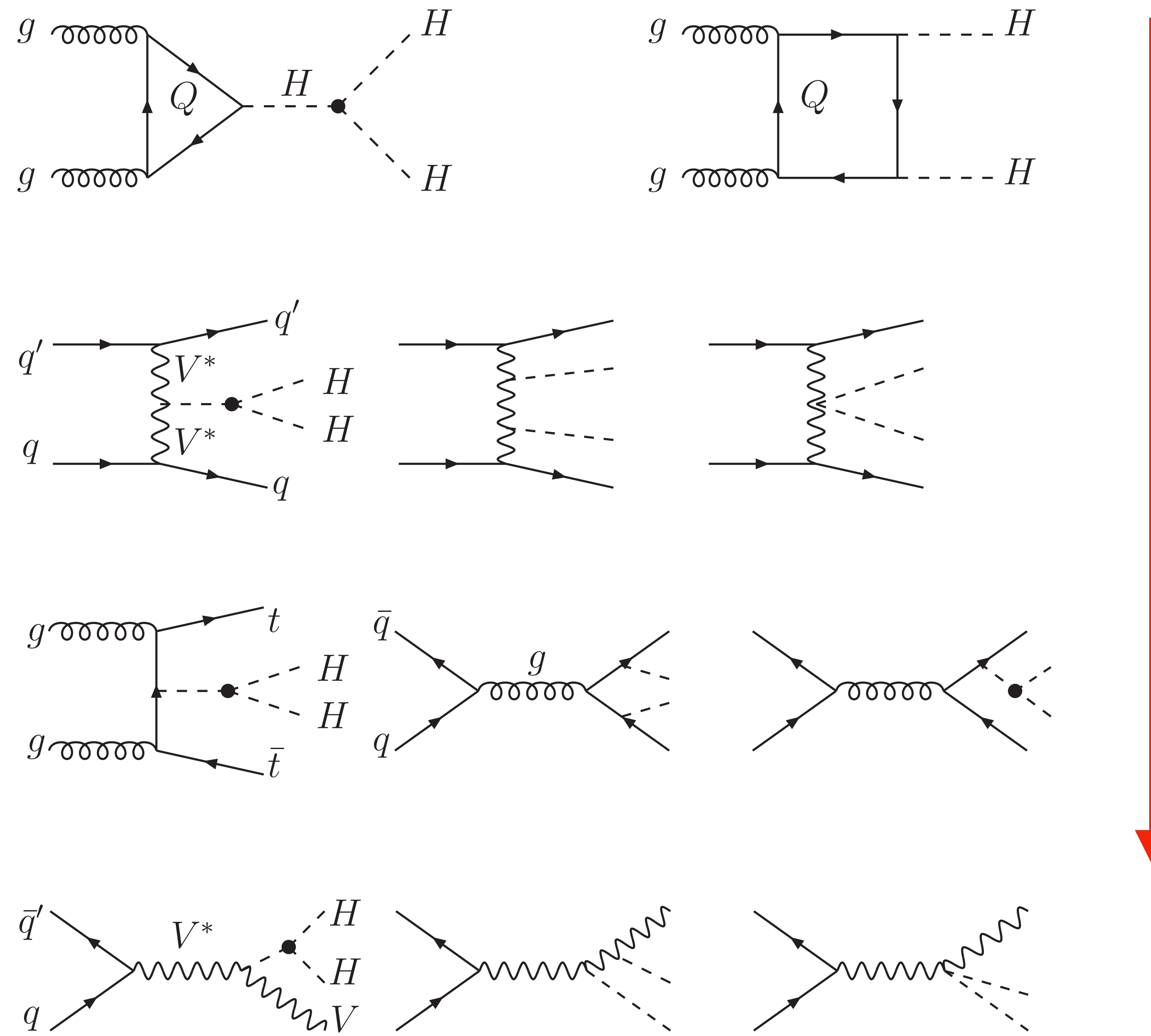


Figure from [1]

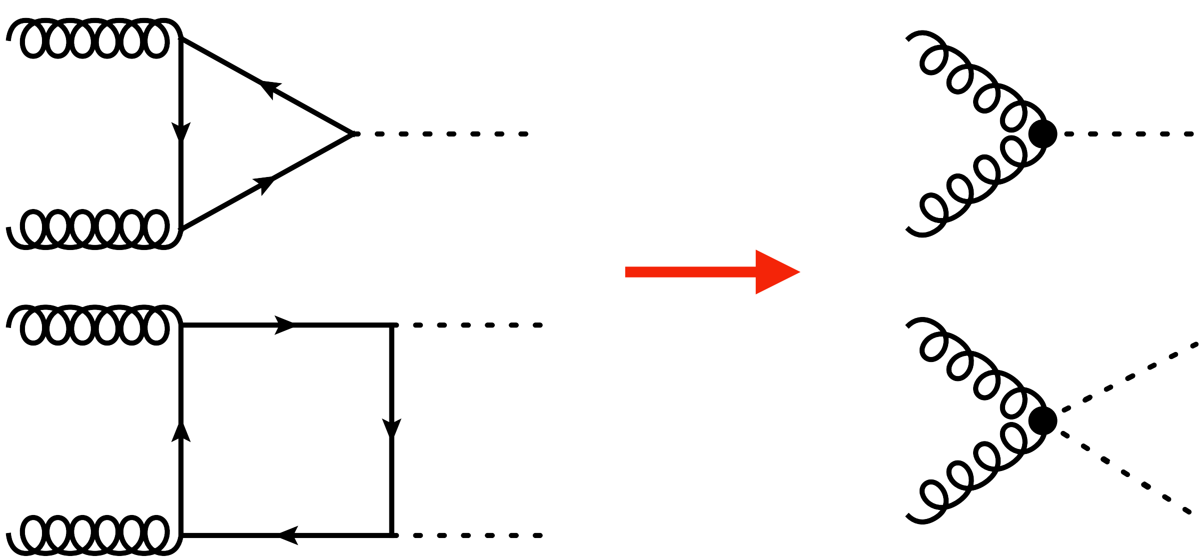
[1] Baglio, Djouadi, Gröber, Mühlleitner, Quevillon, Spira 12;
[2] Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Torrielli, Vryonidou, Zaro 14;
[3] Ling, Zhang, Ma, Guo, Li, Li 14; [4] Li, Wang 16; [5] Li, Li, Wang 17;
[6] Dolan, Englert, Greiner, Nordstrom, Spannowsky 15;

Higgs EFT

H(iggs)EFT: $m_T \to \infty$

Effective tree-level couplings between gluons and Higgs

Lowers number of loops by 1



HEFT valid for

$$\sqrt{\hat{s}} \ll 2m_T$$

HH production for

$$2m_H < \sqrt{\hat{s}}$$

Small energy range in which HEFT is technically justified

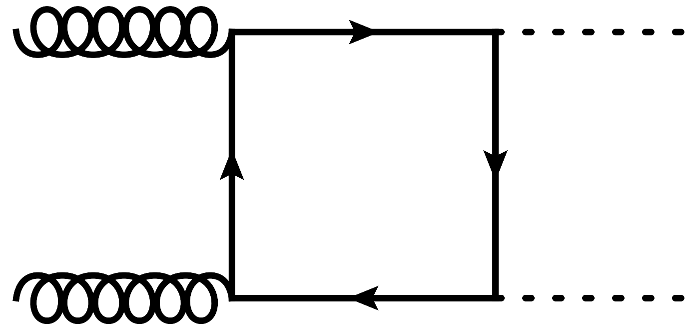
Born improved NLO HEFT:

$$\mathrm{d}\sigma_{\mathrm{NLO}}(m_T) \approx \mathrm{d}\bar{\sigma}_{\mathrm{NLO}}(m_T) \equiv \frac{\mathrm{d}\sigma_{\mathrm{NLO}}(m_T \to \infty)}{\mathrm{d}\sigma_{\mathrm{LO}}(m_T \to \infty)} \mathrm{d}\sigma_{\mathrm{LO}}(m_T)$$

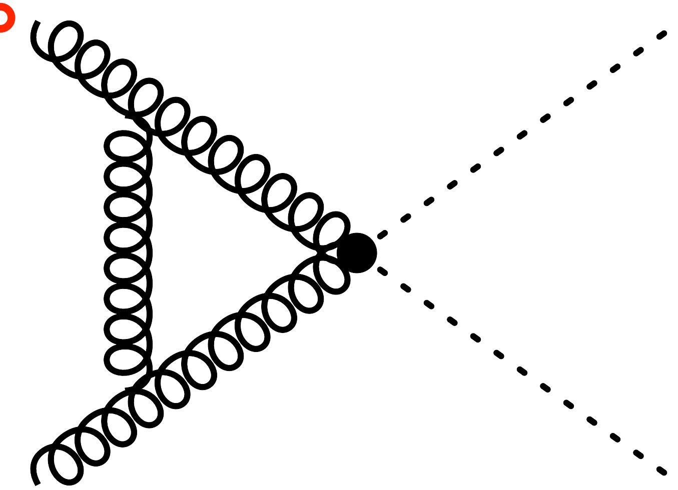
Spira et al. (HPAIR)

HH Production (Gluon Fusion)

1. LO (1-loop), Dominated by Top (Bottom <1%)
   Glover, van der Bij 88



2. Born Improved NLO H(iggs)EFT $m_T \to \infty$ **+90%**
   Dawson, Dittmaier, Spira 98



A. Including m_T in Real Radiation "FTapprox" **-10%**
   Maltoni, Vryonidou, Zaro 14

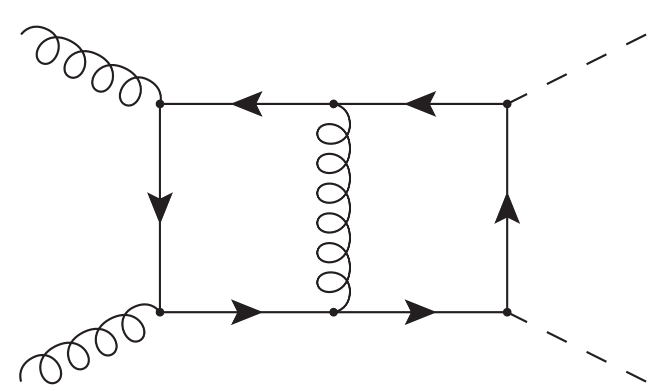
B. Including $\mathcal{O}(1/m_T^{12})$ Terms in Virtual MEs **±10%**
   Grigo, Hoff, Melnikov, Steinhauser 13; Grigo, Hoff 14; Grigo, Hoff, Steinhauser 15

3. NLO QCD (2-loop) with Full Top Mass
   Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Schubert, Zirke 16;
   Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Zirke 16

   (Transverse momentum) NLL + NLO
   Ferrera, Pires 16

   Parton Shower (POWHEG/MG5_aMC@NLO)
   Heinrich, SPJ, Kerner, Luisoni, Vryonidou 17

   Parton Shower (Sherpa) SPJ, Kuttimalai 17

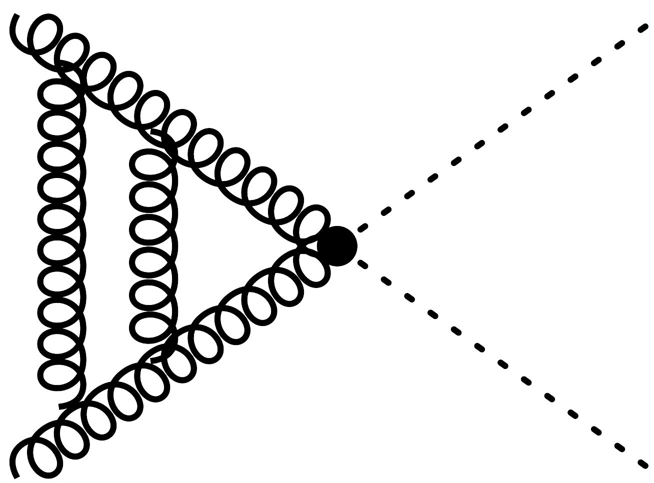


Focus of this talk

HH Production (Gluon Fusion) (II)

4. Born Improved NNLO HEFT
   De Florian, Mazzitelli 13
   Including Matching Coefficients
   Grigo, Melnikov, Steinhauser 14
   Including Terms $\mathcal{O}(1/m_T^4)$ in Virtual MEs
   Grigo, Hoff, Steinhauser 15
   (Threshold) NNLL + NNLO Matching
   (SCET) Shao, Li, Li, Wang 13; de Florian, Mazzitelli 15

+20%



+1%

With $\mu_R = \mu_F = \frac{m_{HH}}{2}$

5. NNLO HEFT (Differential)
   de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16

6. NLO Improved NNLO HEFT
   Grazzini, Heinrich, SPJ, Kallweit, Kerner, Lindert, Mazzitelli 18

Focus of Javier's Talk

Total Cross Section @ 14 TeV

End of 2017:

	$\sigma_{\rm LO}$ (fb)	$\sigma_{\rm NLO}$ (fb)	$\sigma_{\rm NNLO}$ (fb)
Basic HEFT	$17.07^{+30.9\%}_{-22.2\%}$	$31.93^{+17.6\%}_{-15.2\%}$	$37.52^{+5.2\%}_{-7.6\%}$
B-i. HEFT	$19.85^{+27.6\%}_{-20.5\%}$	$38.32^{+18.1\%}_{-14.9\%}$	$43.63^{+5.3\%}_{-7.7\%}$*
FTapprox	$19.85^{+27.6\%}_{-20.5\%}$	$34.25^{+14.7\%}_{-13.2\%}$	–
Full Theory	$19.85^{+27.6\%}_{-20.5\%}$	$32.88^{+13.5\%}_{-12.5\%}$	–
NLO-i. HEFT	–	$32.88^{+13.5\%}_{-12.5\%}$	$38.66^{+5.3\%}_{-7.7\%}$*

PDF4LHC15_nlo/nnlo
$m_H = 125$ GeV
$m_T = 173$ GeV
Uncertainty:
$\mu_R = \mu_F = \frac{m_{HH}}{2}$
$\mu \in \left[\frac{\mu_0}{2}, 2\mu_0\right]$ (7 – point)

* re-weighted on total cross-section level

de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16;
Maltoni, Vryonidou, Zaro 14 (recalculated);
Borowka, Greiner, Heinrich, Kerner, Schlenk, Schubert, Zirke 16;
Dawson, Dittmaier, Spira 98 (recalculated); Glover, van der Bij 88 (recalculated)

But: we can do better than this!
(see Javier's talk)

Comparison to Full Theory

	$\Delta\sigma^{\rm Full}_{\rm LO}$	$\Delta\sigma^{\rm Full}_{\rm NLO}$
Basic HEFT	−14%	−2.9%
B.I. HEFT	0%	+17%
FTapprox	0%	+4.2%

Can do a similar exercise @ 100 TeV, differences typically larger

Obtaining NLO Results

Showered Results:

`MadGraph5_aMC@NLO:`

Contact eleni.vryonidou@cern.ch

Heinrich, SPJ, Kerner, Luisoni, Vryonidou 17

`POWHEG-BOX:` **20/03/17: 380 min→30 min (10k Events, Stage 1/2)**

```
svn co --username anonymous --password anonymous svn://
powhegbox.mib.infn.it/trunk/User-Processes-V2/ggHH
```

Heinrich, SPJ, Kerner, Luisoni, Vryonidou 17

`Sherpa:`

Contact silvan@slac.stanford.edu

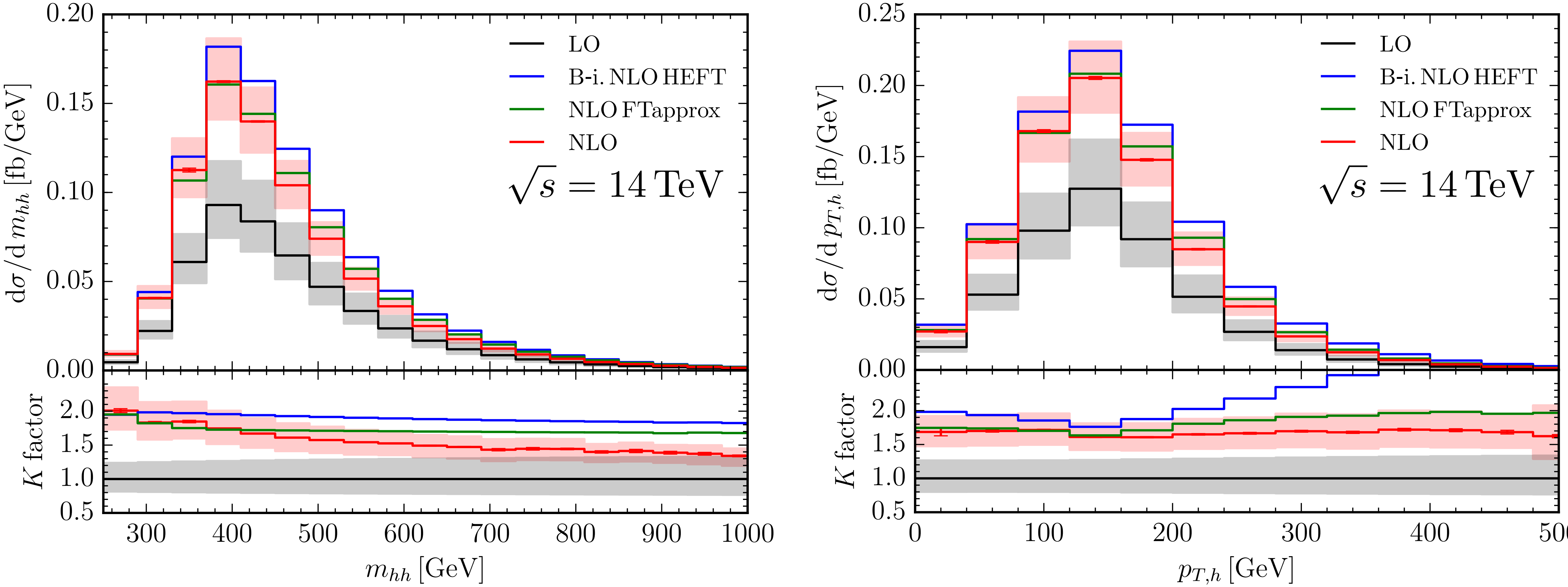
SPJ, Kuttimalai 17

NLO Fixed Order Grid:

https://github.com/mppmu/hhgrid

Borowka, Greiner, Heinrich, SPJ, Kerner, Luisoni, Schlenk, Schubert, Zirke 16; Heinrich, SPJ, Kerner, Luisoni, Vryonidou 17

NLO Differential Results



Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Zirke 16

B-i. HEFT: Outside scale var.

$$m_{hh} > 420\,\text{GeV}$$

FTapp: Outside scale var.

$$m_{hh} > 620\,\text{GeV}$$

Note: Ambiguous how to rescale HEFT real radiation by full LO born at event level

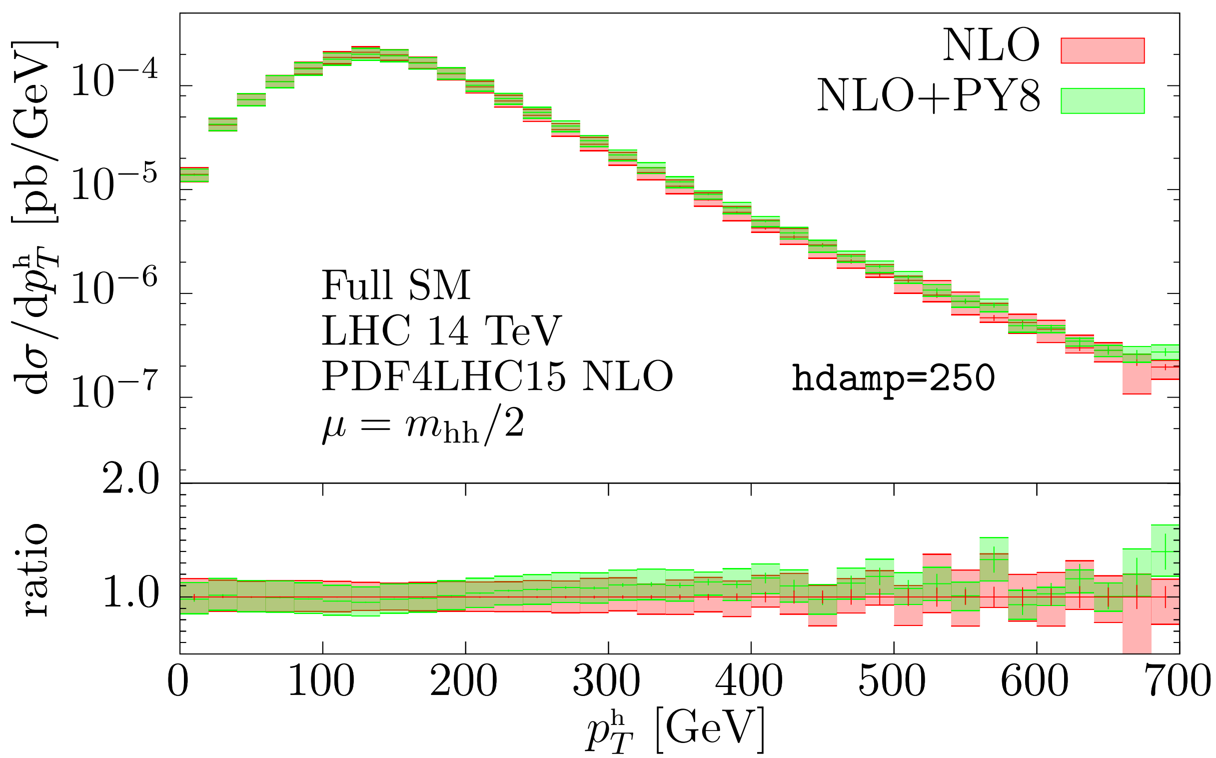
Including m_T in real radiation does improve over HEFT in tails

NLO Showered Results

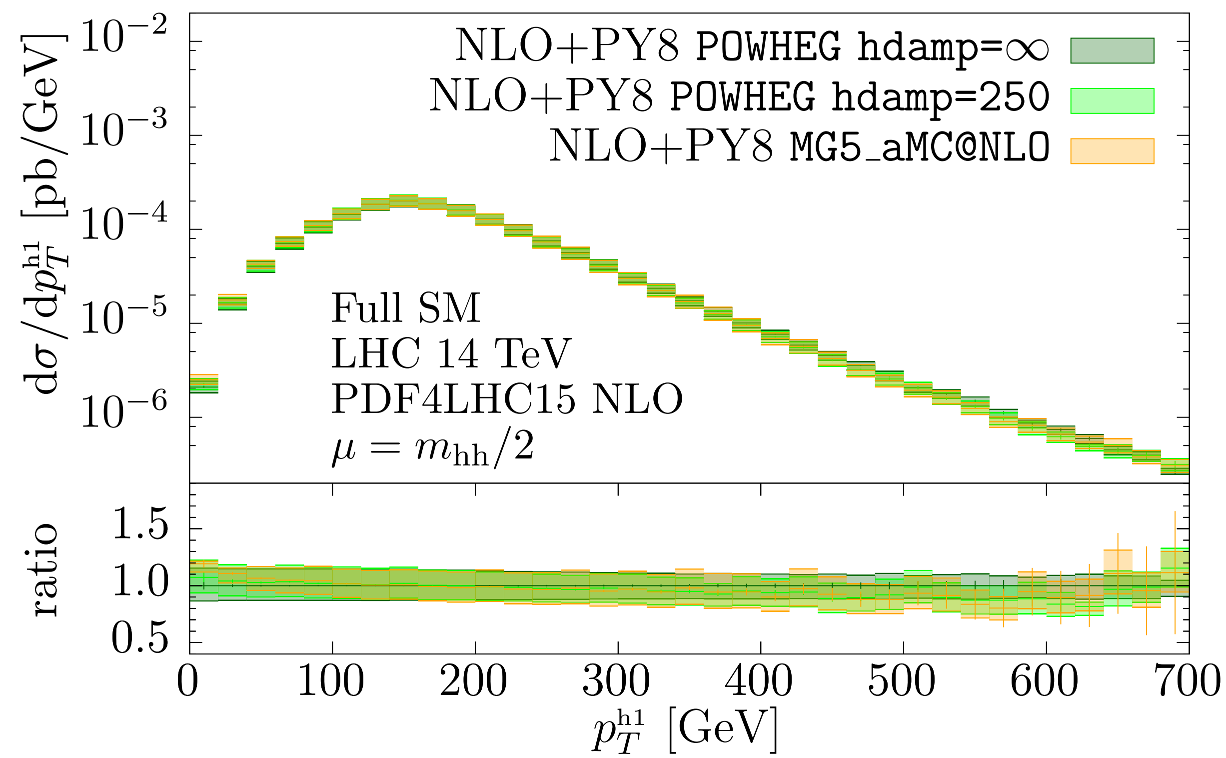
No Higgs decay or hadronization included

Assume $\Gamma_h = 0$ (decay can be attached e.g. in narrow width approx.)

POWHEG-BOX



POWHEG-BOX/MG5_aMC@NLO

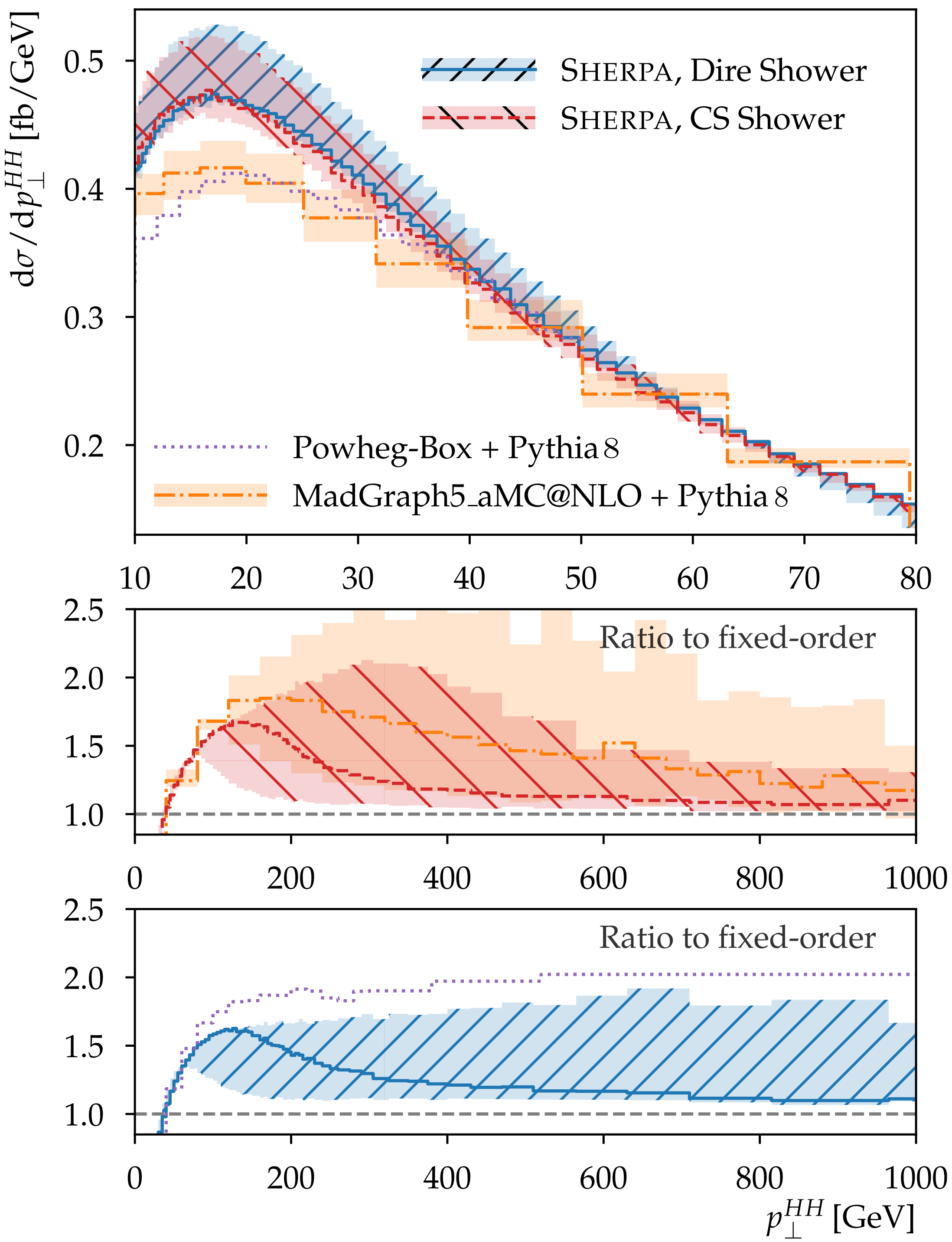


Shower has moderate impact on NLO accurate observables

NLO accurate observables ($m_{hh}, p_T^h, p_T^{h_1}, p_T^{h_2}$) only moderately sensitive to matching procedure

Heinrich, SPJ, Kerner, Luisoni, Vryonidou 17

NLO Showered Results (II)



$\mu_{\rm PS}$ variation

SPJ, Kuttimalai 17

Matching/shower has significant impact on LO accurate observables

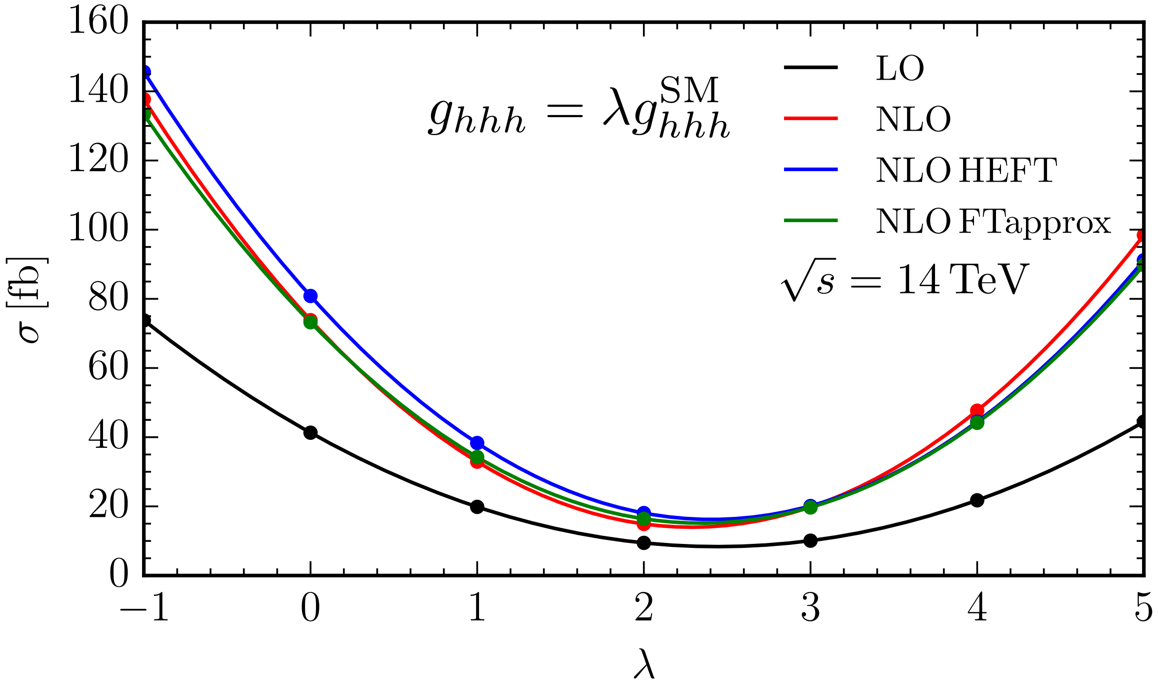
Can have large matching uncertainties

$$\langle \mathcal{O} \rangle = \int \left[\bar{B}(\phi_B) - B(\phi_B)\right] \frac{D(\phi_B,\phi_1)}{B(\phi_B)} \Theta(\mu_{\rm PS}^2 - t) \mathcal{O}(\phi_R)\, d\phi_B\, d\phi_1 + \int R(\phi_R)\mathcal{O}(\phi_R)\, d\phi_R.$$

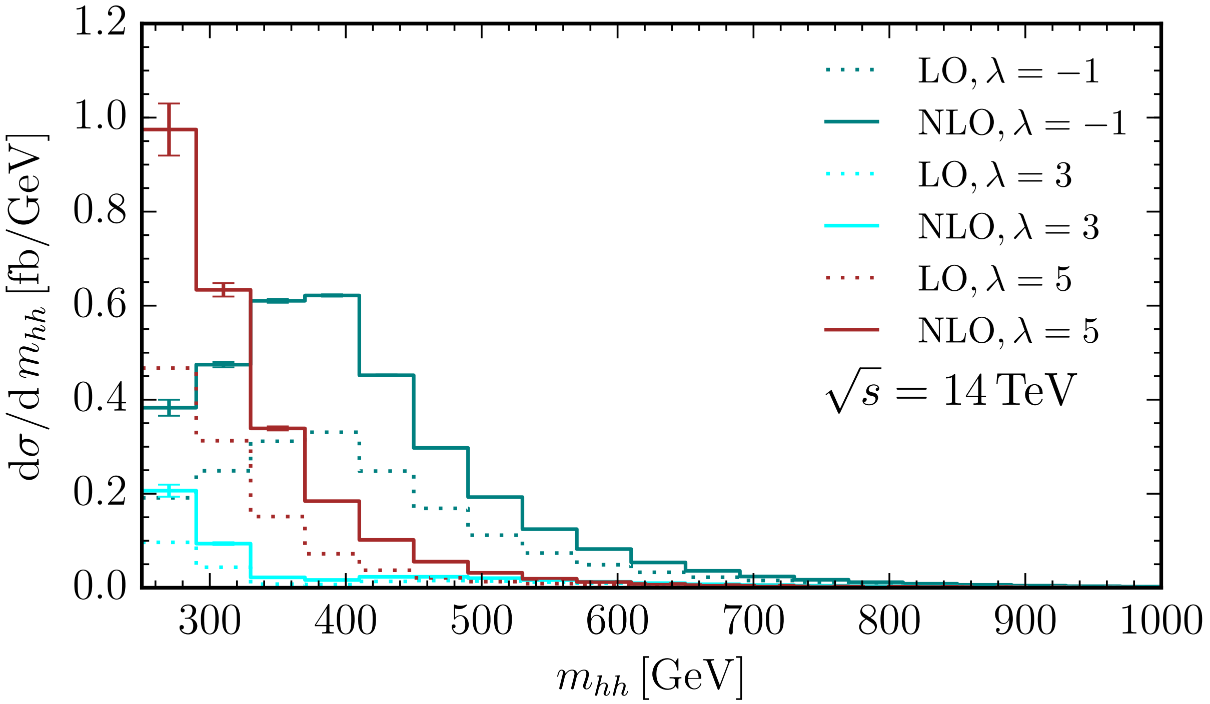
Cancellation spoiled if:

- Large NLO corrections ($\bar{B} - B$)
- Splitting kernels over Born (D/B) numerically large compared to real radiation
- Phase space accessible to the PS (depends on scale $\mu_{\rm PS}$ and PS evolution variable t)

Triple-Higgs Coupling Sensitivity



SM: Destructive interference between g_{hhh} and y_T^2 contributions



Distributions: Can help to distinguish between λ values

NLO results for $\lambda \neq 1$ published (but not yet available in public codes)

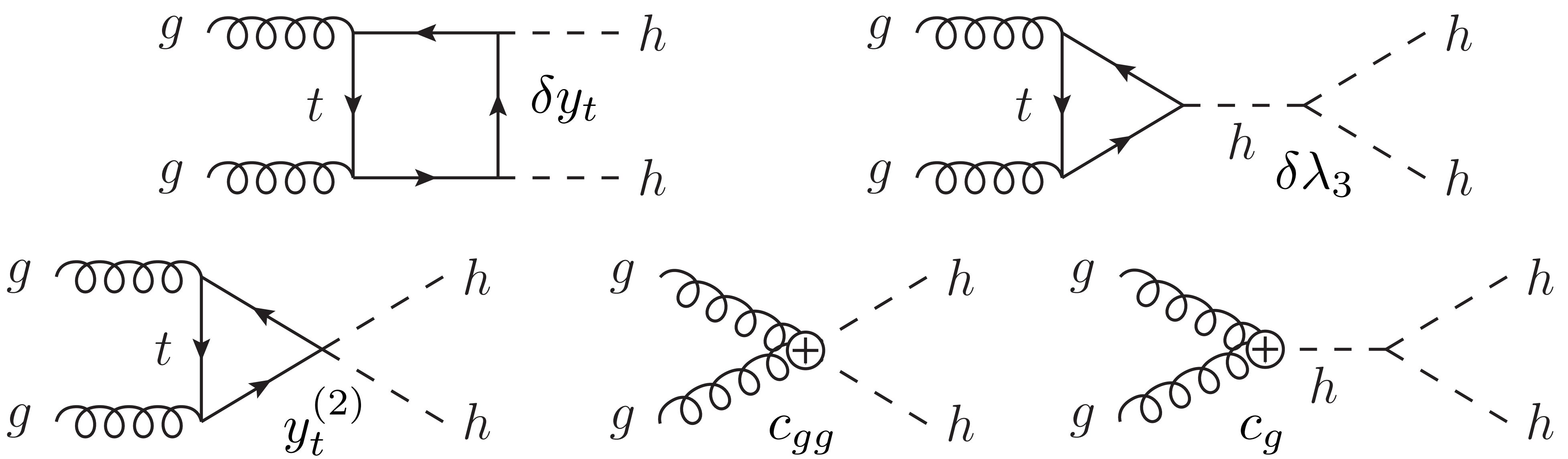
Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Zirke 16

Also: VBF more sensitive (but smaller cross section)

BSM EFT

But: Just varying λ: one ``direction'' in EFT parameter space

Parametrise **non-resonant** new physics with EFT (5 parameters):



Azatov, Contino, Panico, Son 15;
Buchalla, Cata, Celis, Krause 15;
(B.I. NLO HEFT) Gröber, Mühlleitner, Spira, Streicher 15;
(B.I. NNLO HEFT) de Florian, Fabre, Mazzitelli 17;
(Cluster analysis) Dall'Osso, Dorigo, Gottardo, Oliveira, Tosi, Goertz 15;
+ Carvalho, Manzano, Dorigo, Gouzevich 16;
Kim, Sakaki, Son 18;

← 12 representative ``clusters''

Work currently in progress for NLO analysis including top-quark mass

Buchalla, Capozi, Celis, Heinrich, Scyboz (To appear)

Conclusion

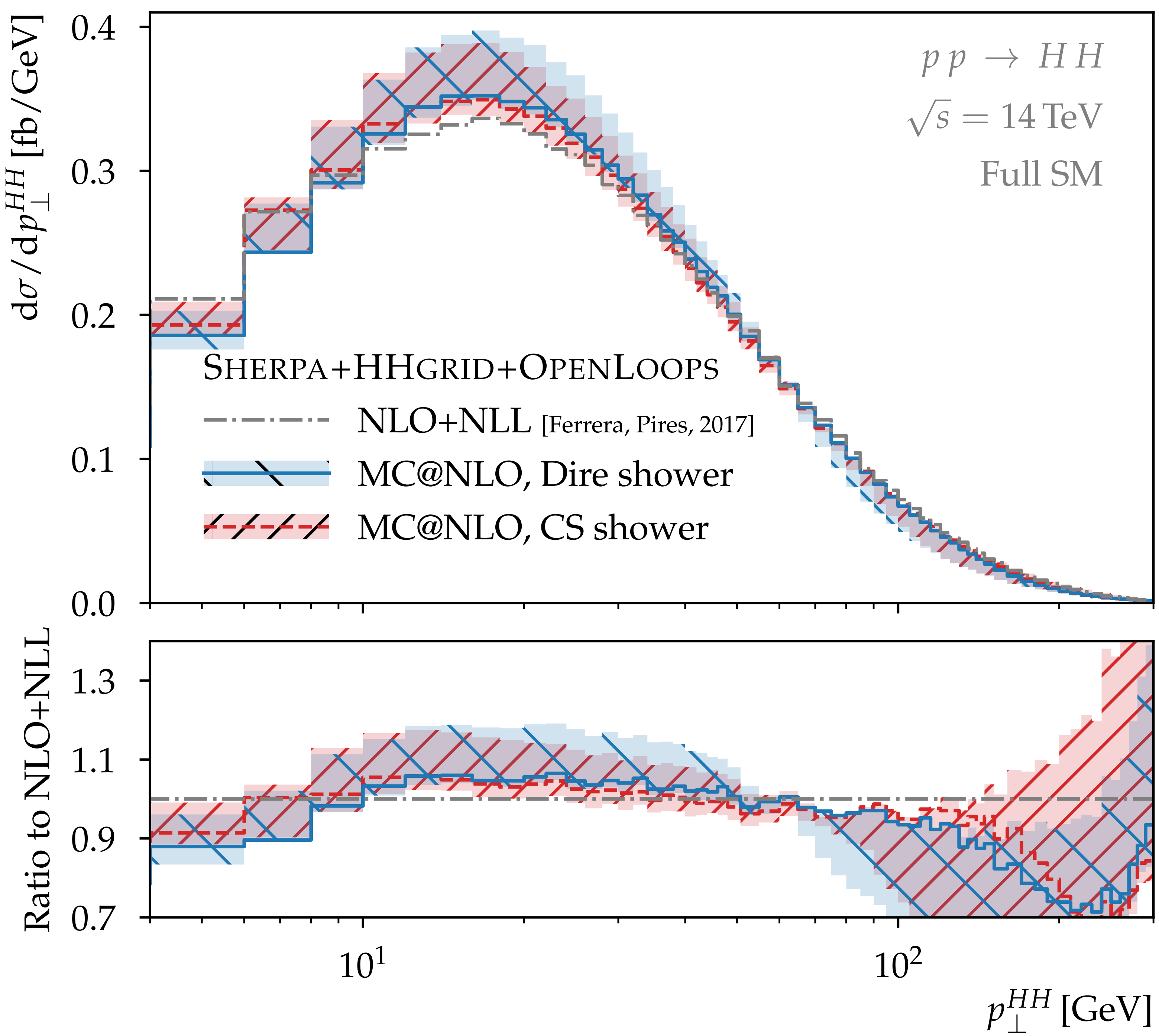
Higgs Boson Pair Production via Gluon Fusion

- Important measurement for probing the self coupling (HL-LHC era)
- Large (K≈2) NLO correction, deviates from Born Improved HEFT
  -14% @ 14 TeV, -24% @ 100 TeV
- Distributions altered significantly
- NLO result interfaced to `MG5_aMC@NLO`, `POWHEG`, and `SHERPA` for parton shower

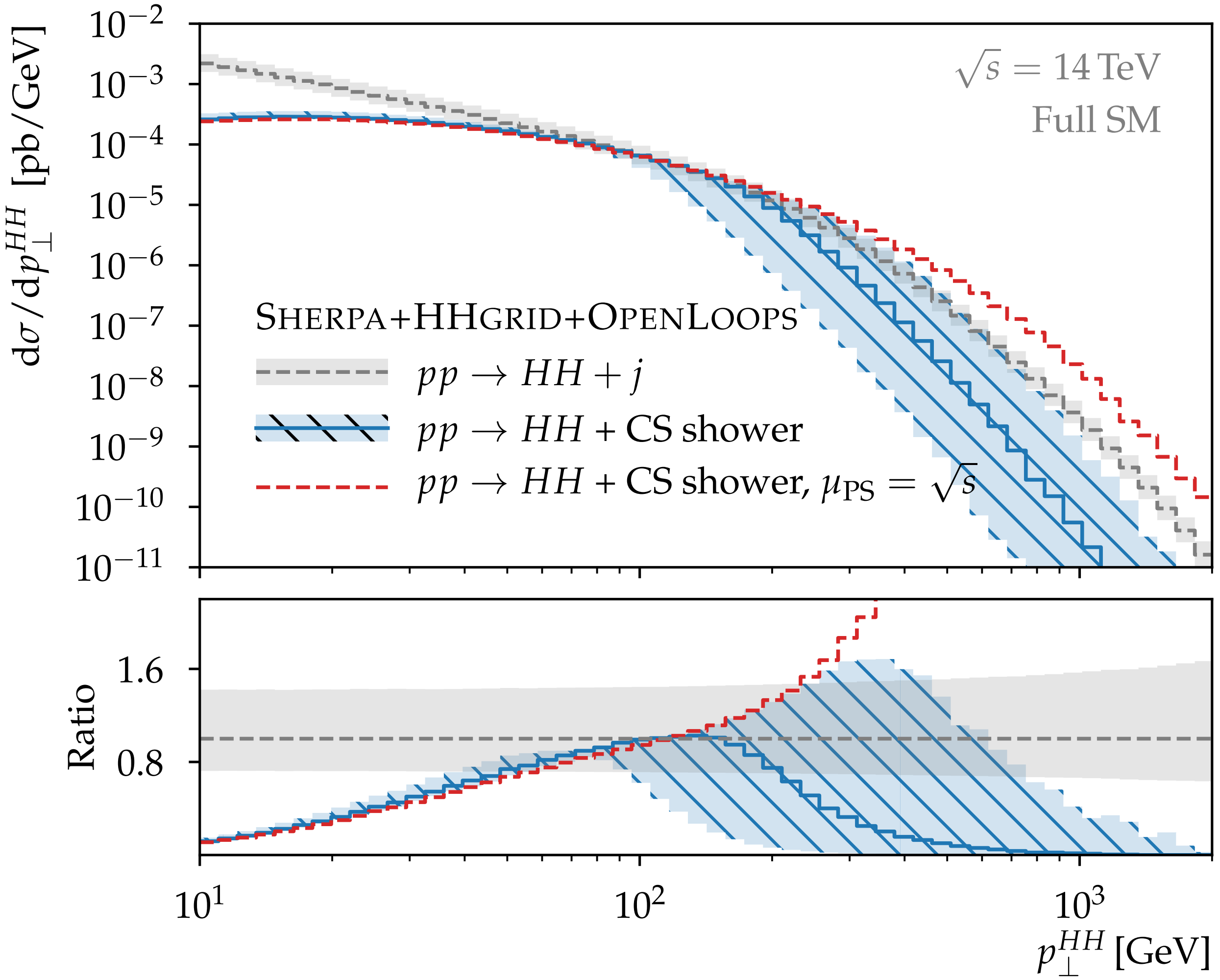
Thank you for listening!

Backup

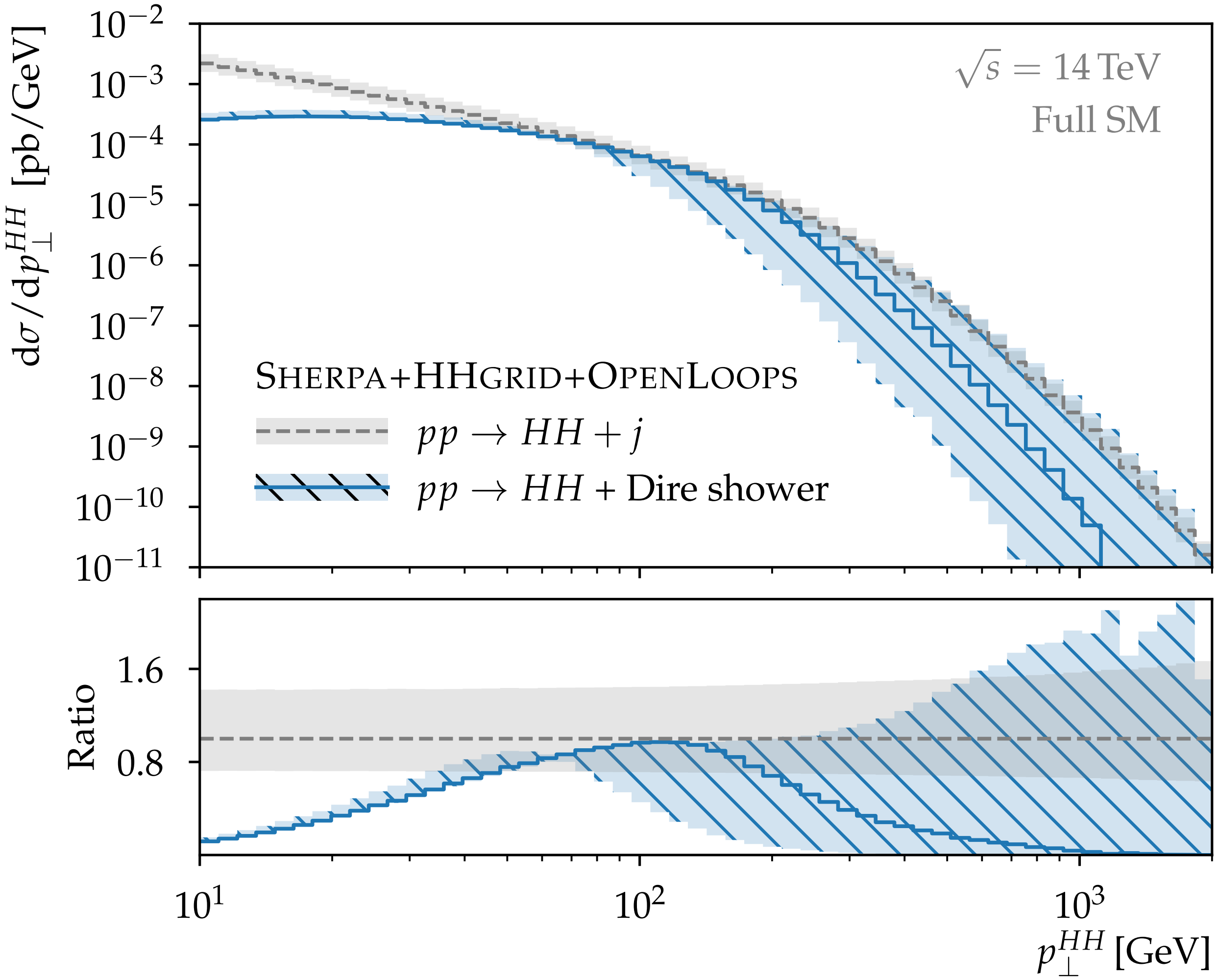
NLO+PS vs Analytic Resummation



LO+PS vs HHj

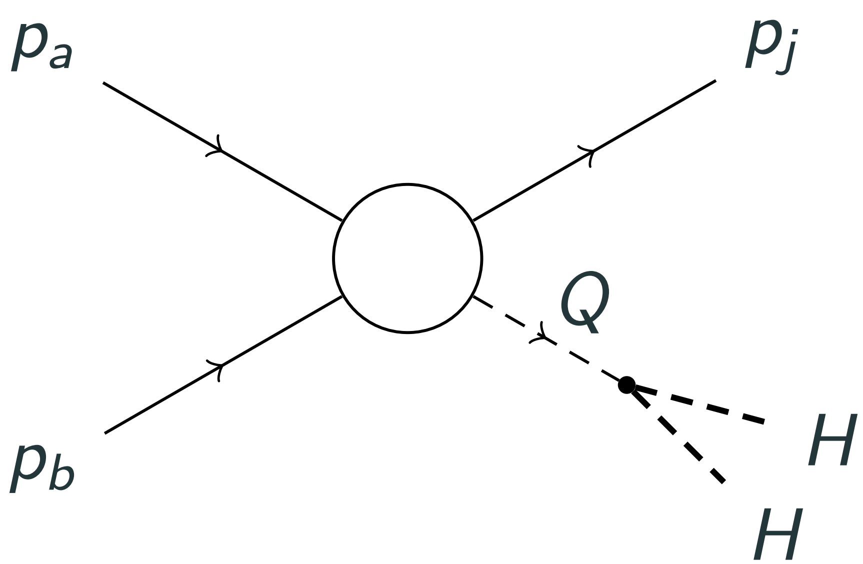


$\sqrt{s} = 14$ TeV
Full SM
SHERPA+HHGRID+OPENLOOPS
$pp \to HH + j$
$pp \to HH$ + CS shower
$pp \to HH$ + CS shower, $\mu_{PS} = \sqrt{s}$
$d\sigma/dp_\perp^{HH}$ [pb/GeV]
Ratio
$p_\perp^{HH}$ [GeV]



$\sqrt{s} = 14$ TeV
Full SM
SHERPA+HHGRID+OPENLOOPS
$pp \to HH + j$
$pp \to HH$ + Dire shower
$d\sigma/dp_\perp^{HH}$ [pb/GeV]
Ratio
$p_\perp^{HH}$ [GeV]

Parton shower kinematics



$$\hat{t} = (p_a - p_j)^2$$
$$\hat{u} = (p_b - p_j)^2$$
$$\hat{s} = (p_a + p_b)^2$$

$$v = \frac{p_a p_j}{p_a p_b} = \frac{-\hat{t}}{\hat{s}} \geq 0 \qquad w = \frac{p_b p_j}{p_a p_b} = \frac{-\hat{u}}{\hat{s}} \geq 0$$

$$\hat{s} + \hat{t} + \hat{u} = Q^2 \quad \Rightarrow \quad v + w = (1 - \frac{Q^2}{\hat{s}}) < 1 \quad \Rightarrow \quad vw < \frac{1}{4}$$

$$\frac{t^{\mathrm{Dire}}}{Q^2} = \frac{(p_a p_j)(p_b p_j)}{(p_a p_b)^2} = vw \qquad t^{\mathrm{Dire}} < \frac{Q^2}{4}$$

$$\frac{t^{\mathrm{CSS}}}{Q^2} = \frac{vw}{1 - (v + w)}.$$

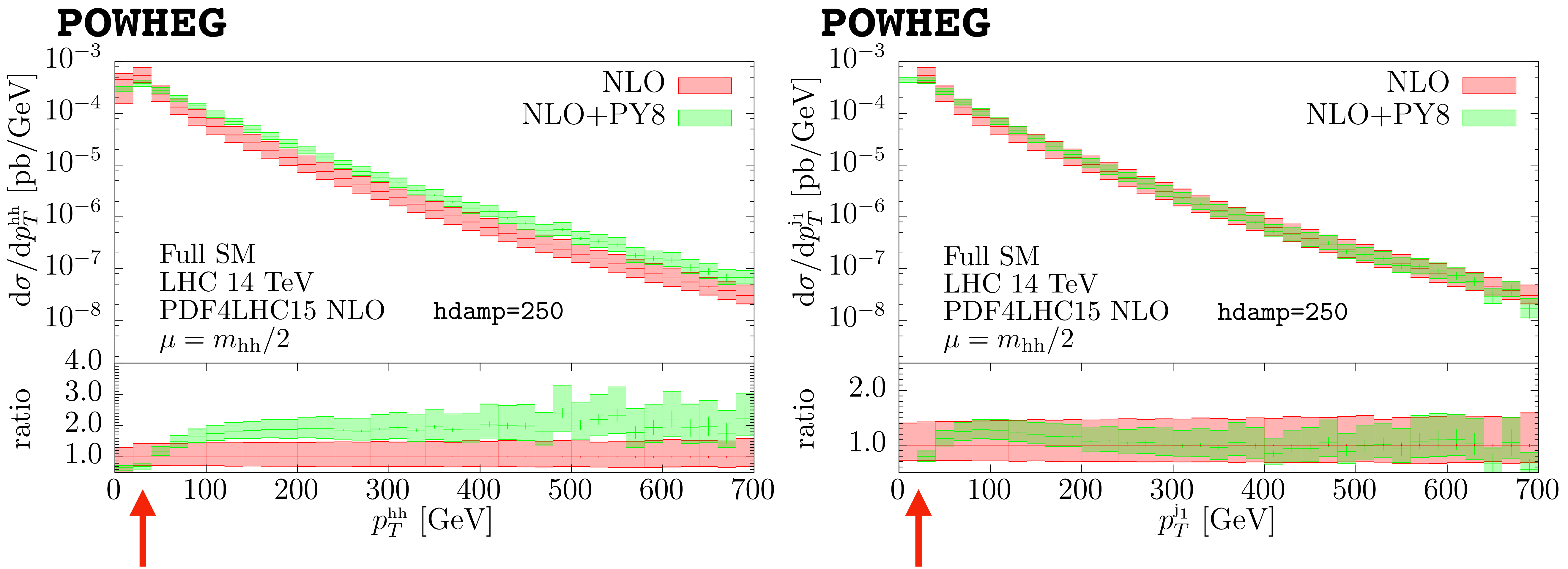
Slide:
Silvan Kuttimalai
(HC2017)

Showered Results

Shower has larger impact on LO accurate observables

Fastjet anti$-k_T$ $R = 0.4$

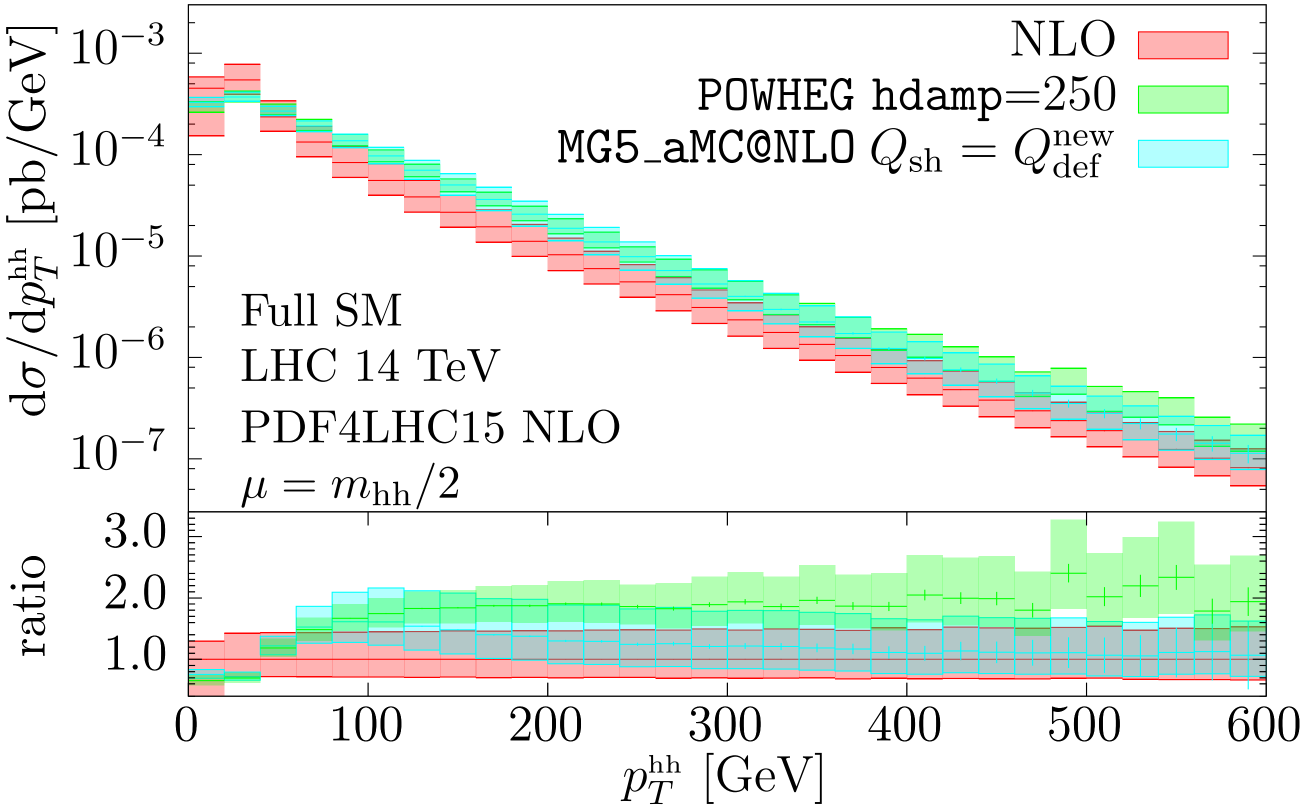
$p^j_{T,\mathrm{min}} = 20$ GeV



Parton shower needed to provide reliable predictions at low p_T^{hh}, $p_T^{j_1}$

Showered Results

POWHEG/MG5_aMC@NLO



p_T^{hh} is sensitive to matching

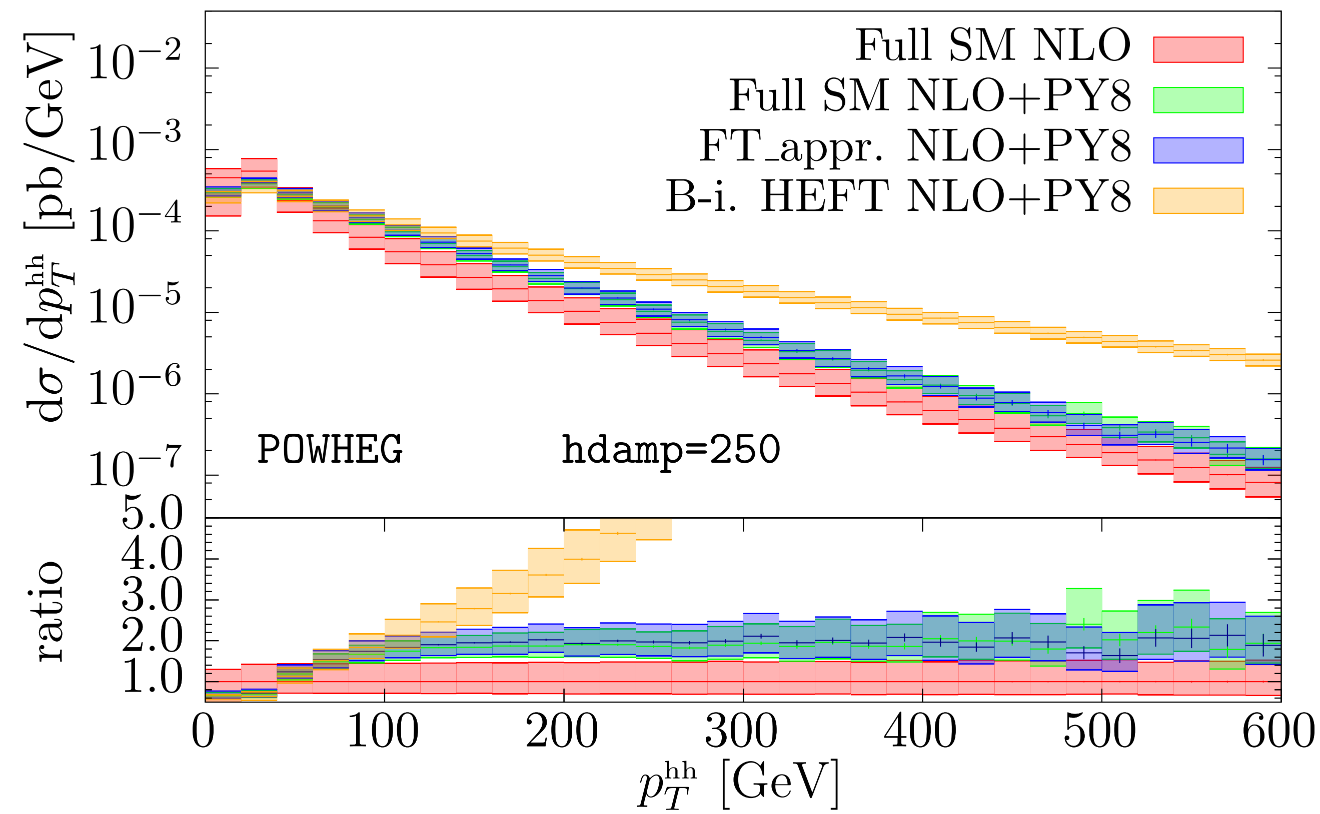
MC@NLO (2.5.3) shower scale:

$\texttt{shower\_scale\_factor} \times [0.1 H_T/2, H_T/2]$

Reducing shower scale gives softer distributions

B.I HEFT: much harder than exact computation

FTapp: gives good description at high p_T^{hh} (contains exact real ME)



POWHEG: hdamp & matching

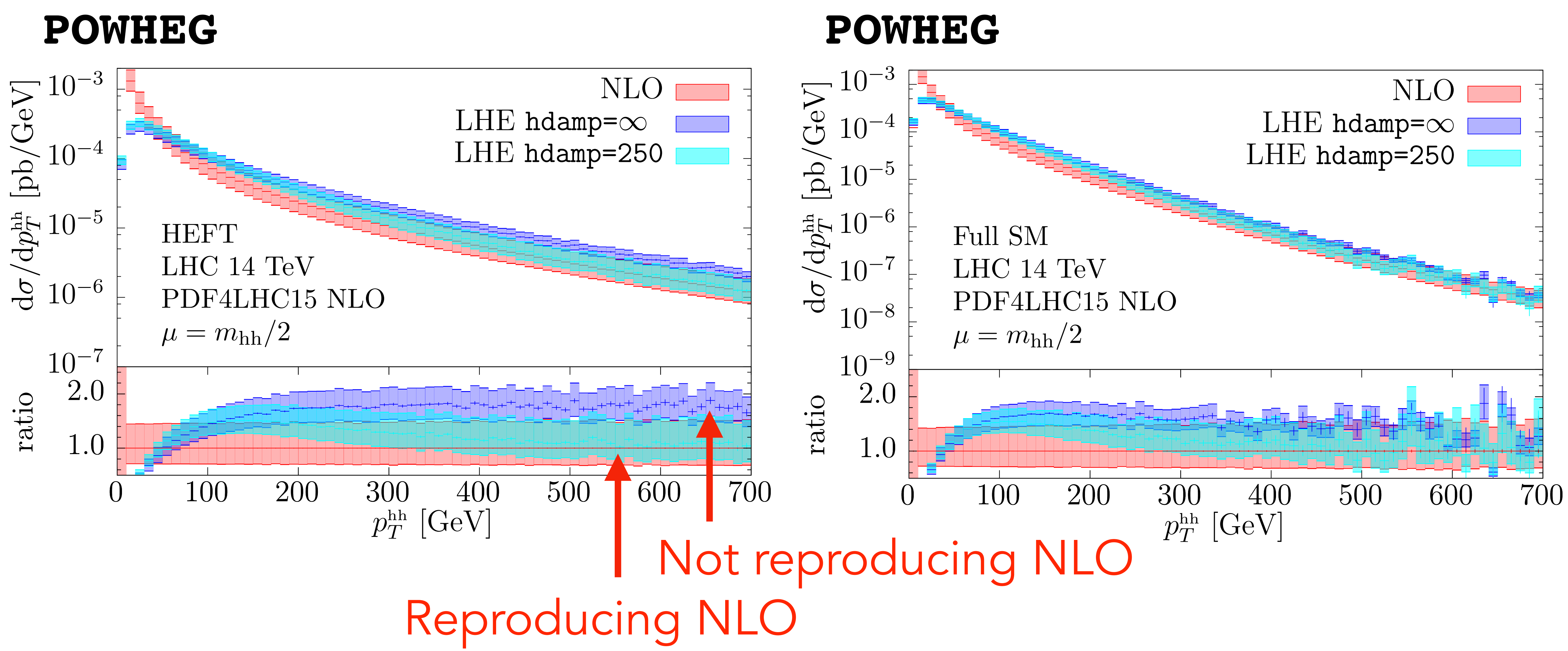
LHE (before shower): at large p_T^{hh} we do not reproduce NLO

Can introduce $h_{\rm damp}$ to limit amount of hard radiation exponentiated:

Exponentiated → $R_{\rm sing} = R \times F$

$$R_{\rm reg} = R \times (1 - F)$$

$$F = \frac{h_{\rm damp}^2}{(p_T^{hh})^2 + h_{\rm damp}^2}$$

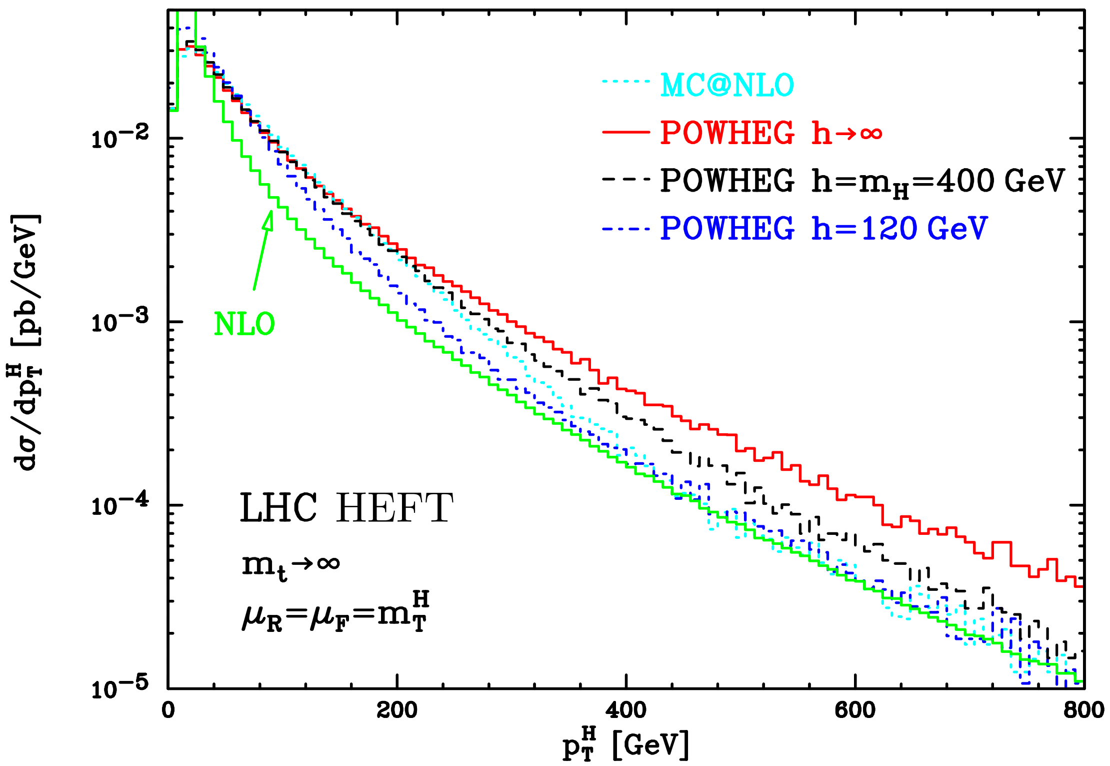


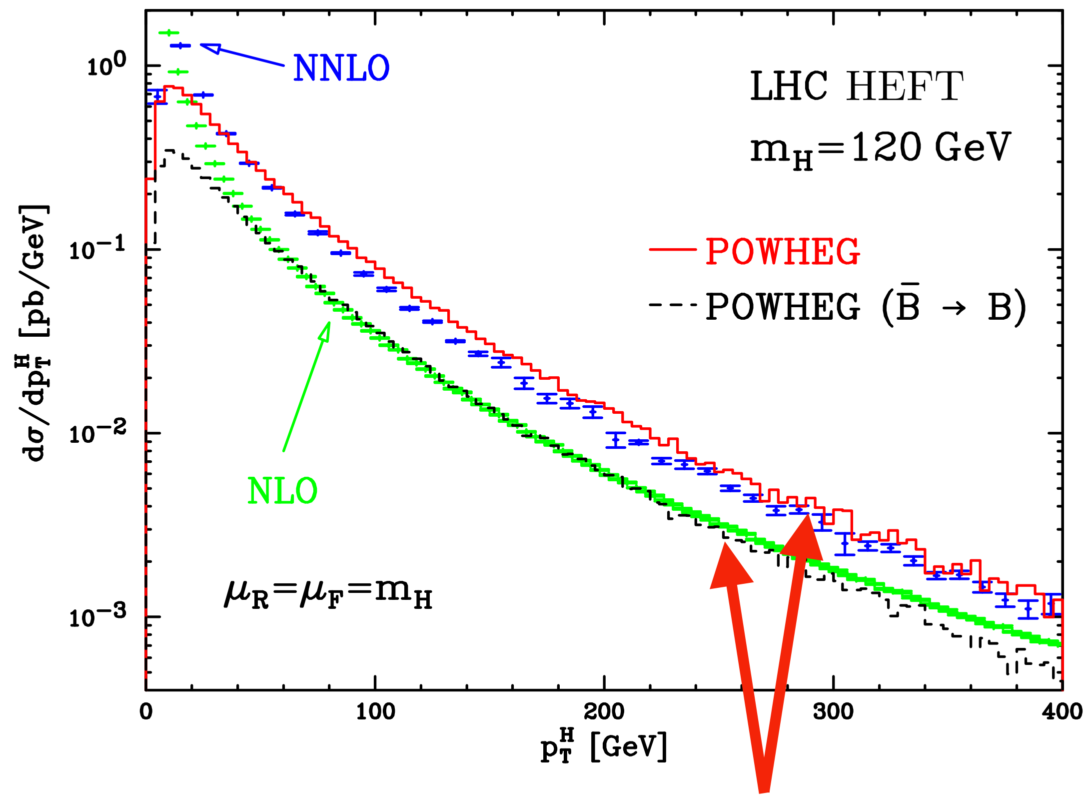
POWHEG: hdamp & matching (II)

Similar situation for Higgs p_T

Without $h_{\rm damp}$ **POWHEG** events look similar to NNLO

Difference between NLO/**POWHEG** due to terms of $\mathcal{O}(\alpha_s)$





Alioli, Nason, Oleari, Re 09

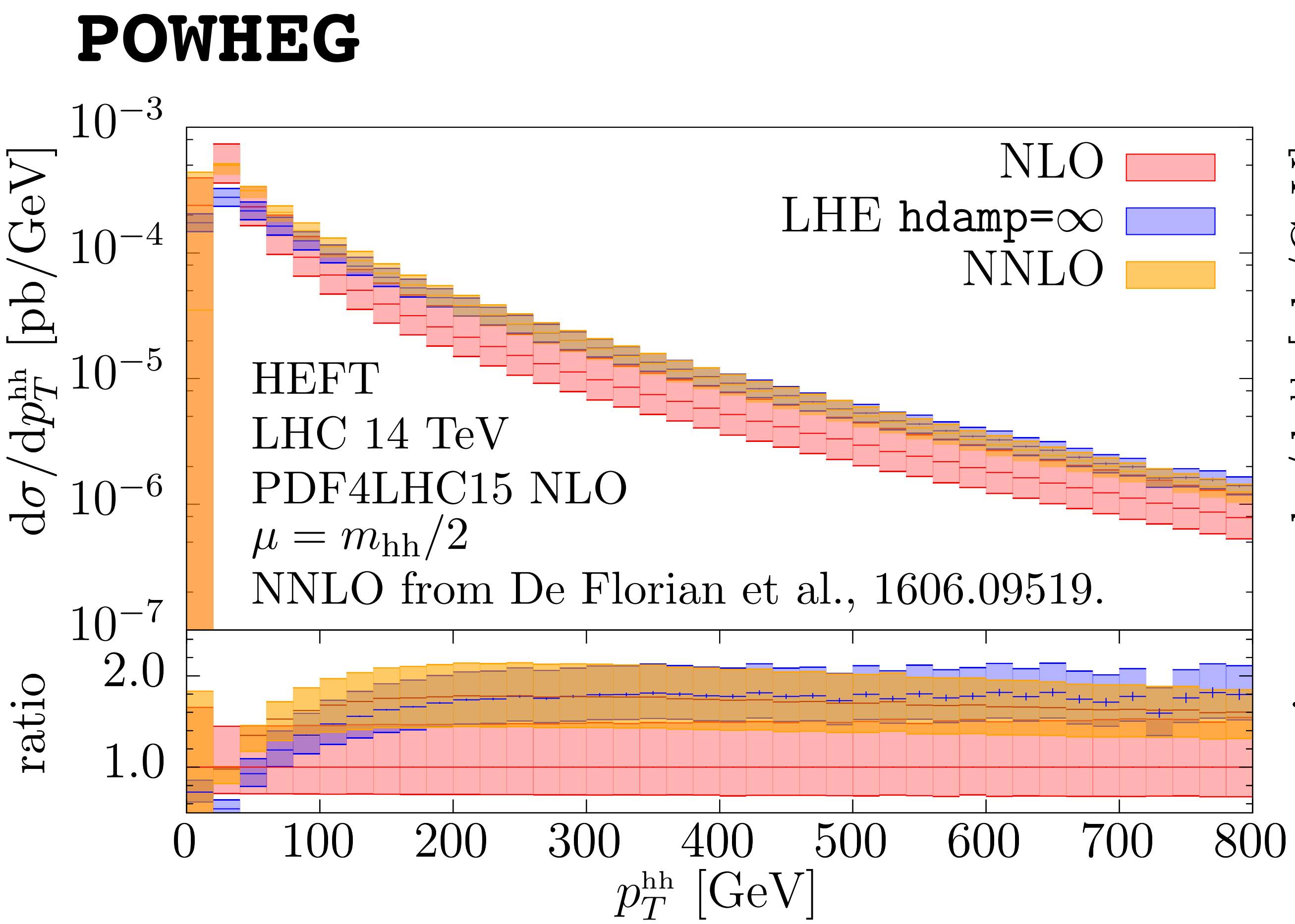
Difference $\mathcal{O}(\alpha_s)$

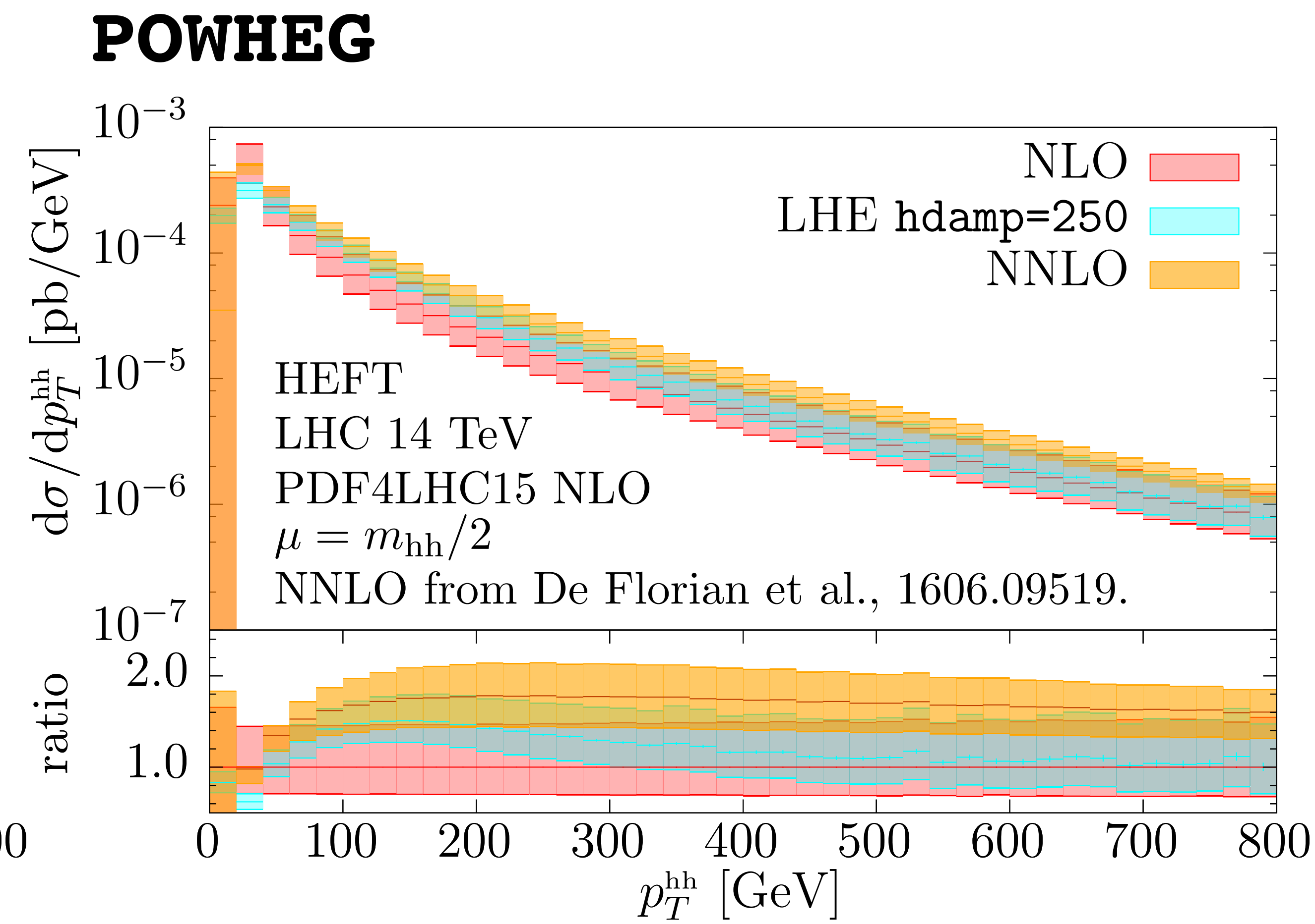
Only real part of the NLO contributes to p_T tail (variable is LO accurate)

NLO corrections are large (K≈2), can expect sensitivity to matching

POWHEG: hdamp & matching (III)

In the HEFT we know the NNLO result, can compare to LHE at NLO





$h_{\rm damp} = \infty$ leads to NLO+PS that agrees more with NNLO

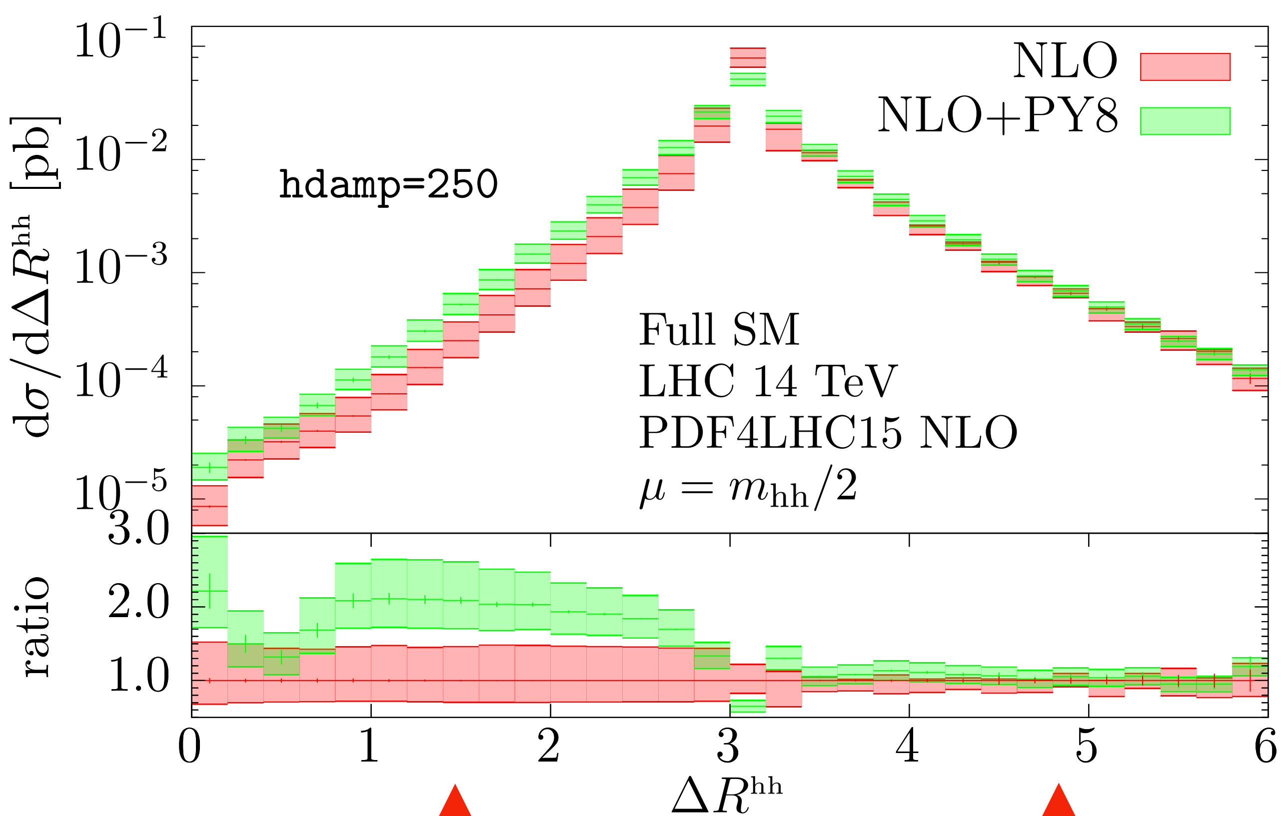
$h_{\rm damp} = 250$ GeV gives tail closer to NLO (also in full theory)

NNLO not known in the full theory

Radial Separation

Radial separation: $\Delta R^{\mathrm{hh}} = \sqrt{(\eta_1 - \eta_2)^2 + (\Phi_1 - \Phi_2)^2}$

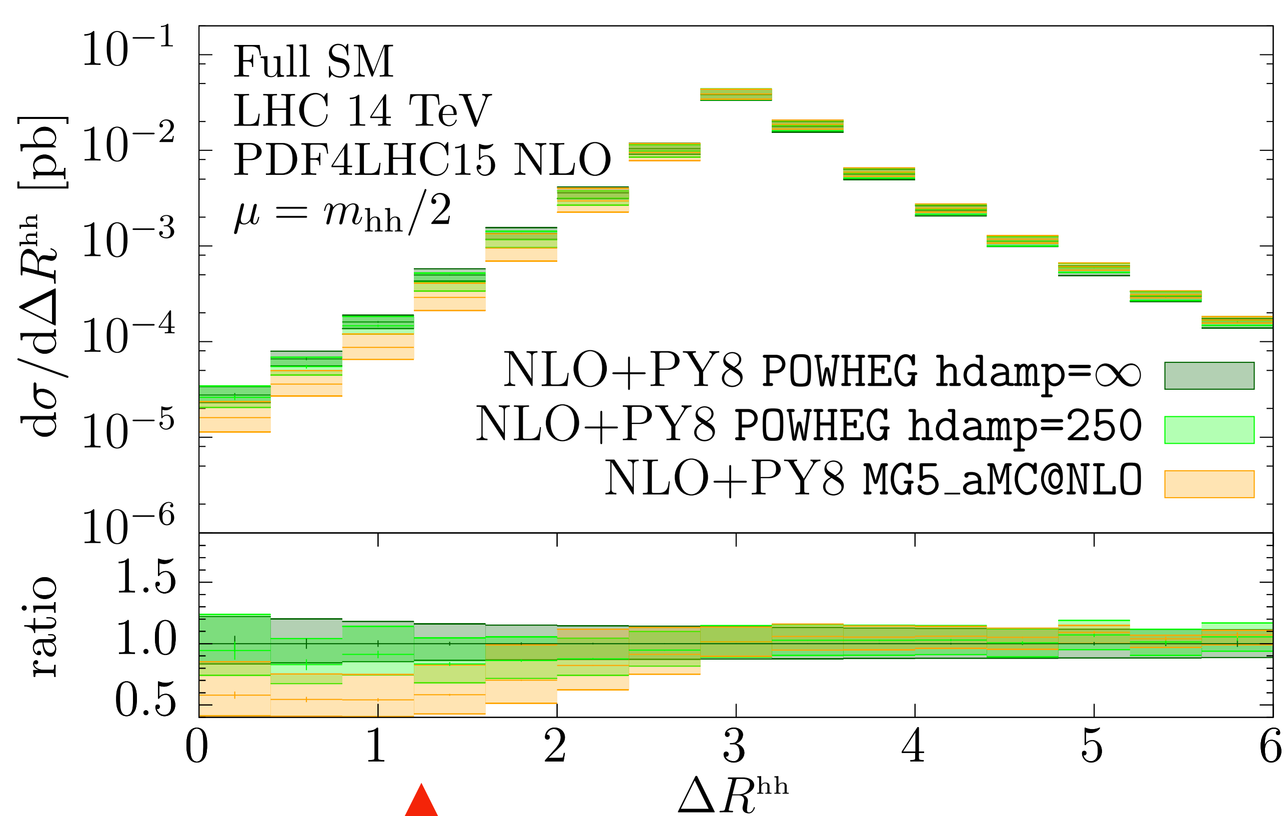
POWHEG



LO accurate

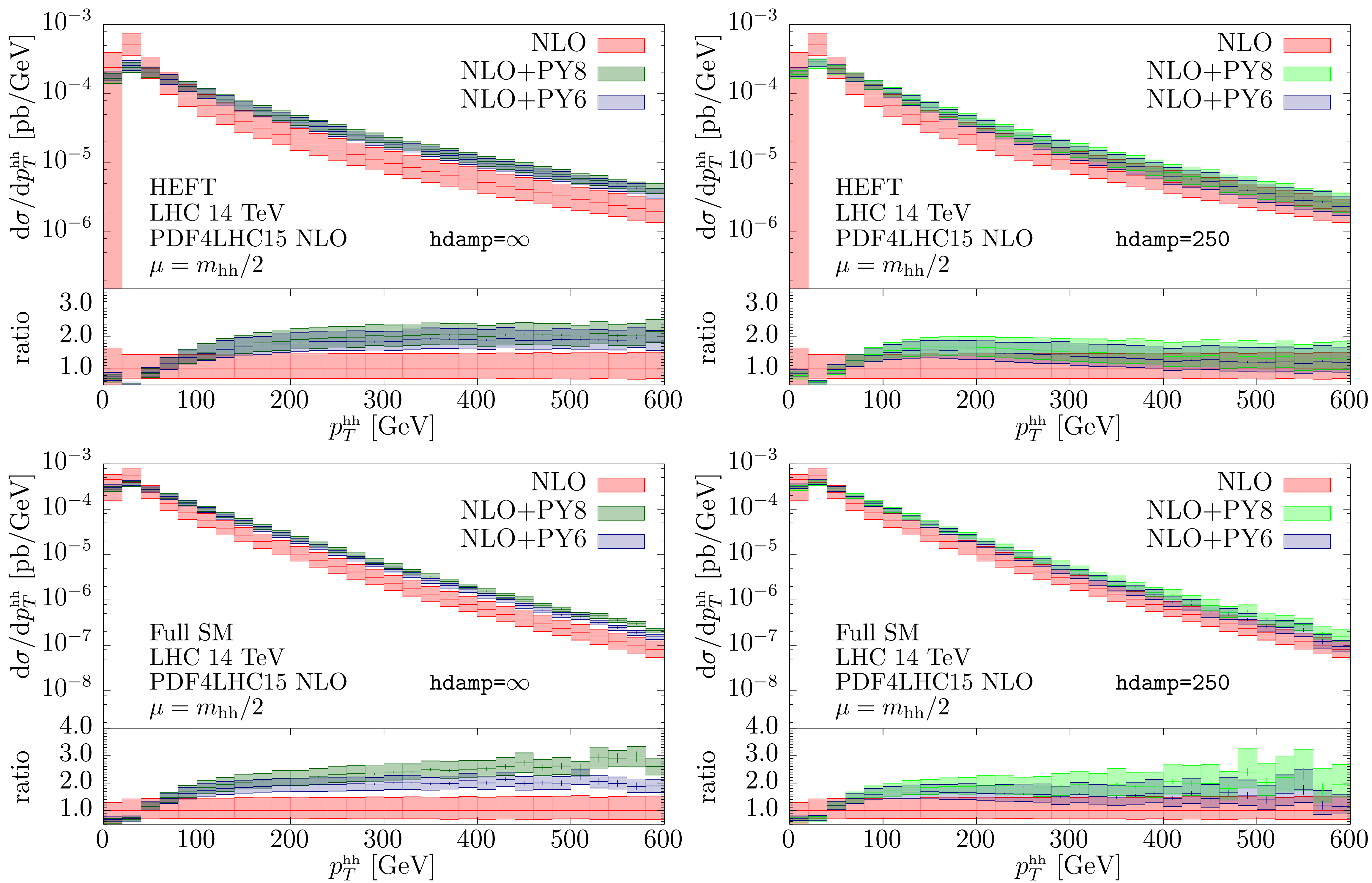
NLO accurate

POWHEG/MG5_aMC@NLO

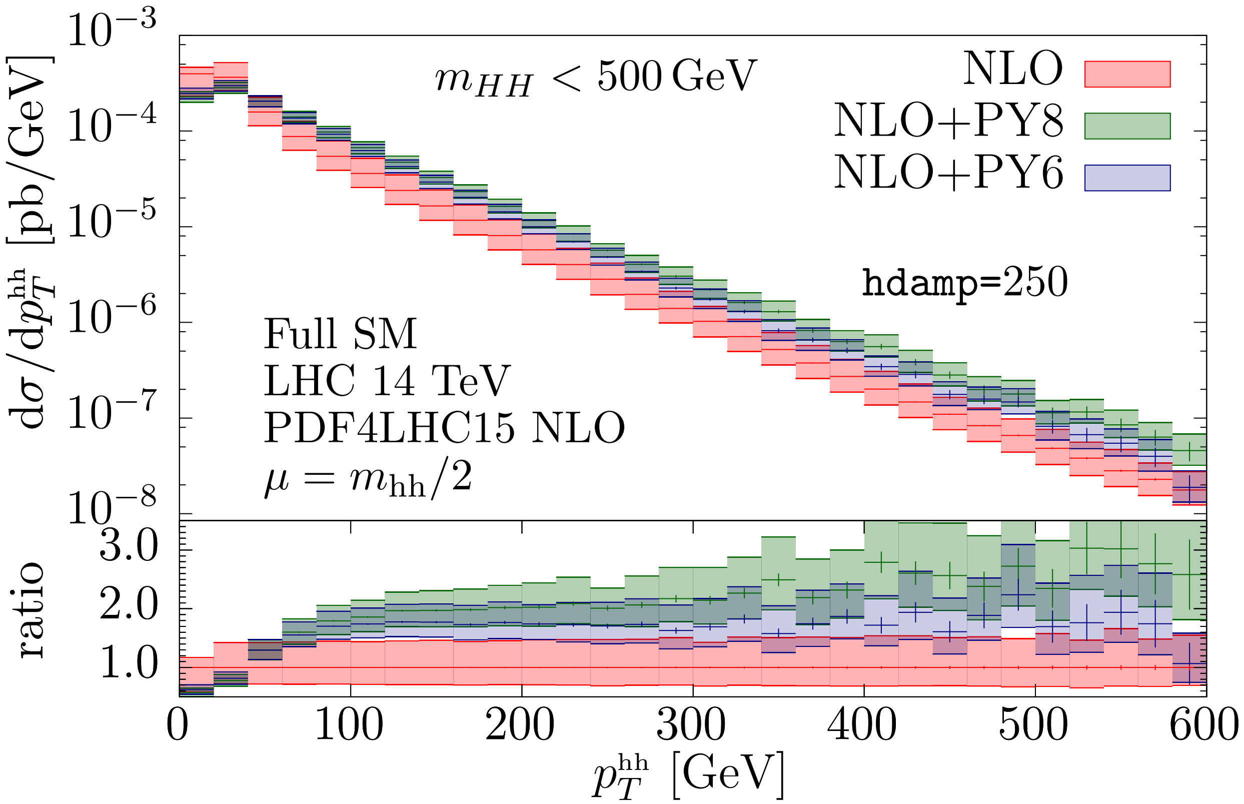


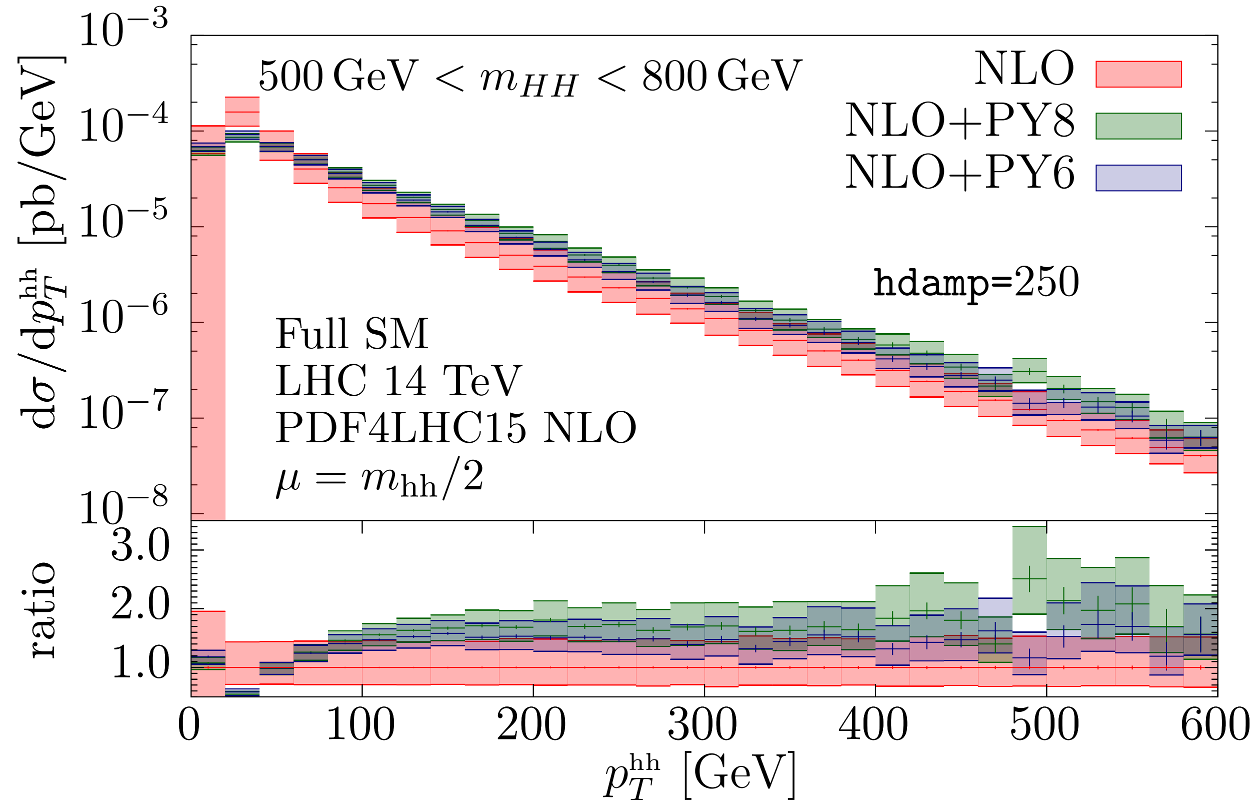
Accordingly, matching scheme uncertainties larger for small radial separation

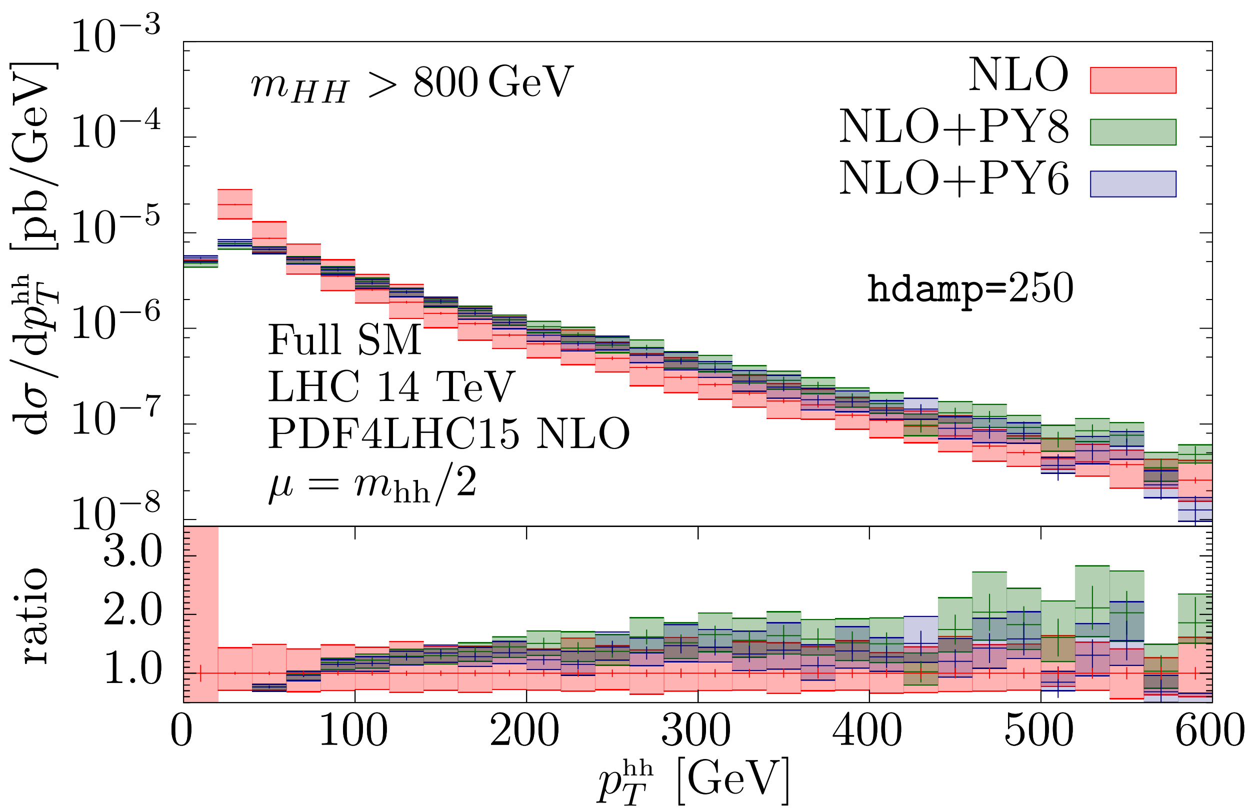
Pythia6 vs Pythia8



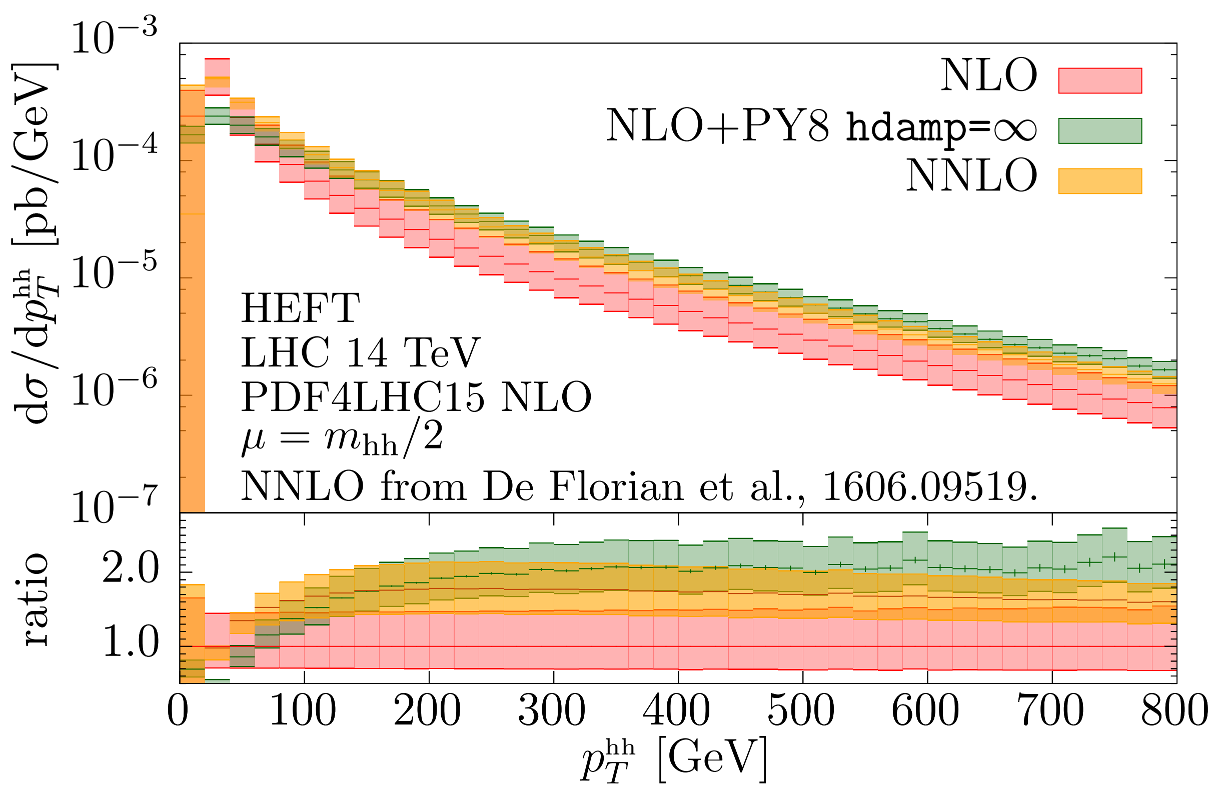
p_T^{hh} in bins of m_{hh}

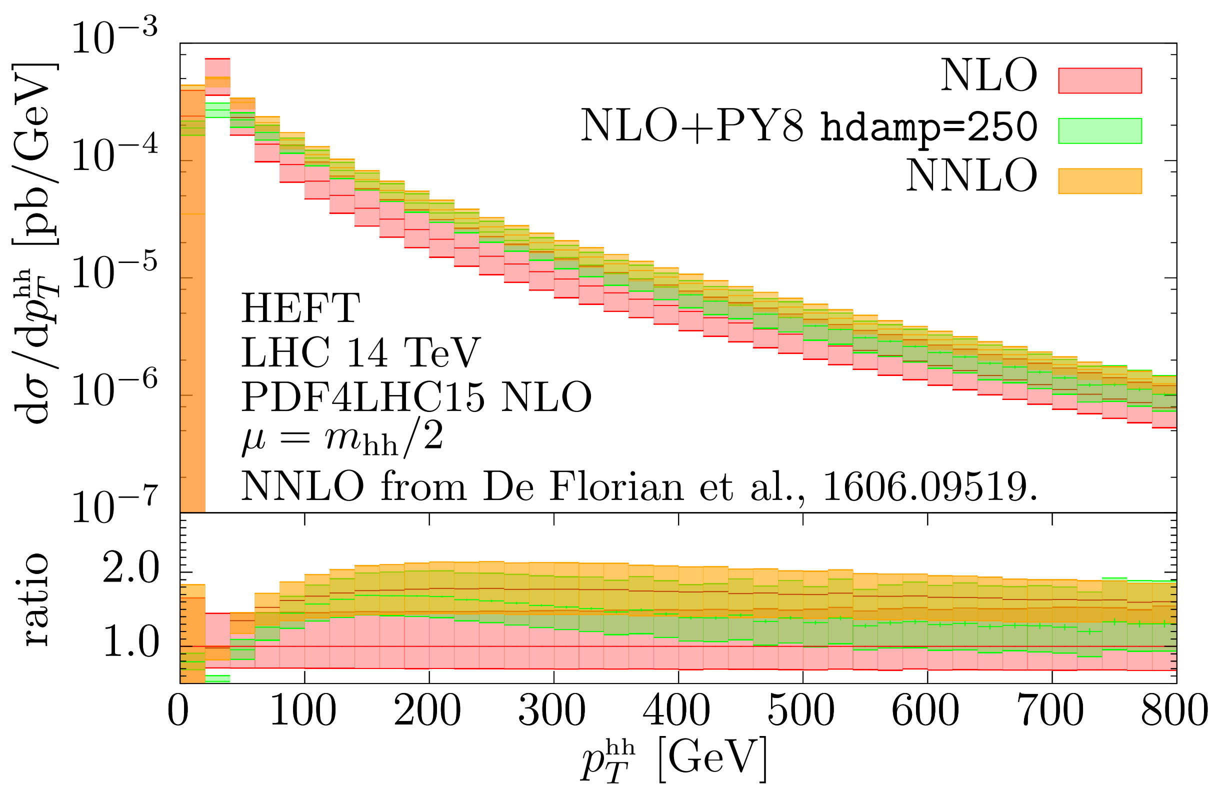




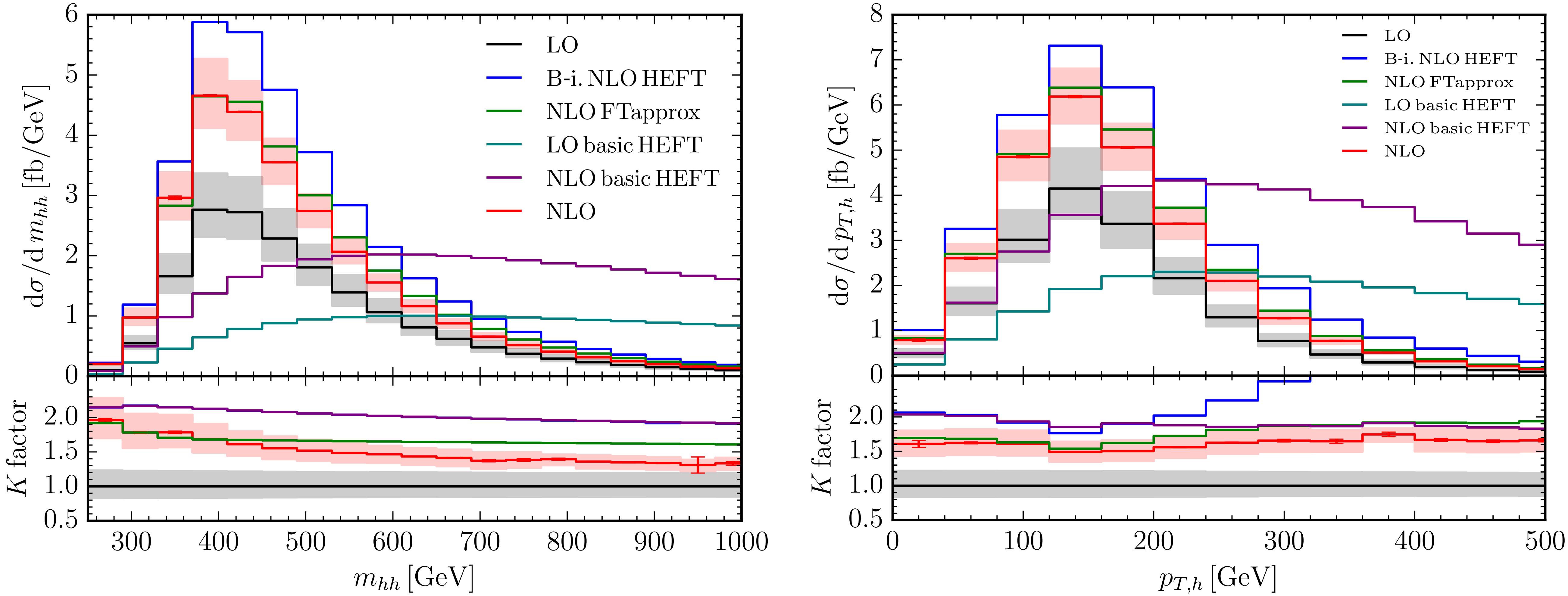


HEFT: Showered Results





Results: 100TeV



	σ_{LO} (fb)	σ_{NLO} (fb)
B.I. HEFT	–	$1511^{+16.0\%}_{-13.0\%}$
FTapprox	–	$1220^{+11.9\%}_{-10.7\%}$
Full Theory	$731.3^{+20.9\%}_{-15.9\%}$	$1149^{+10.8\%}_{-10.0\%}$

HEFT overestimates by 32%

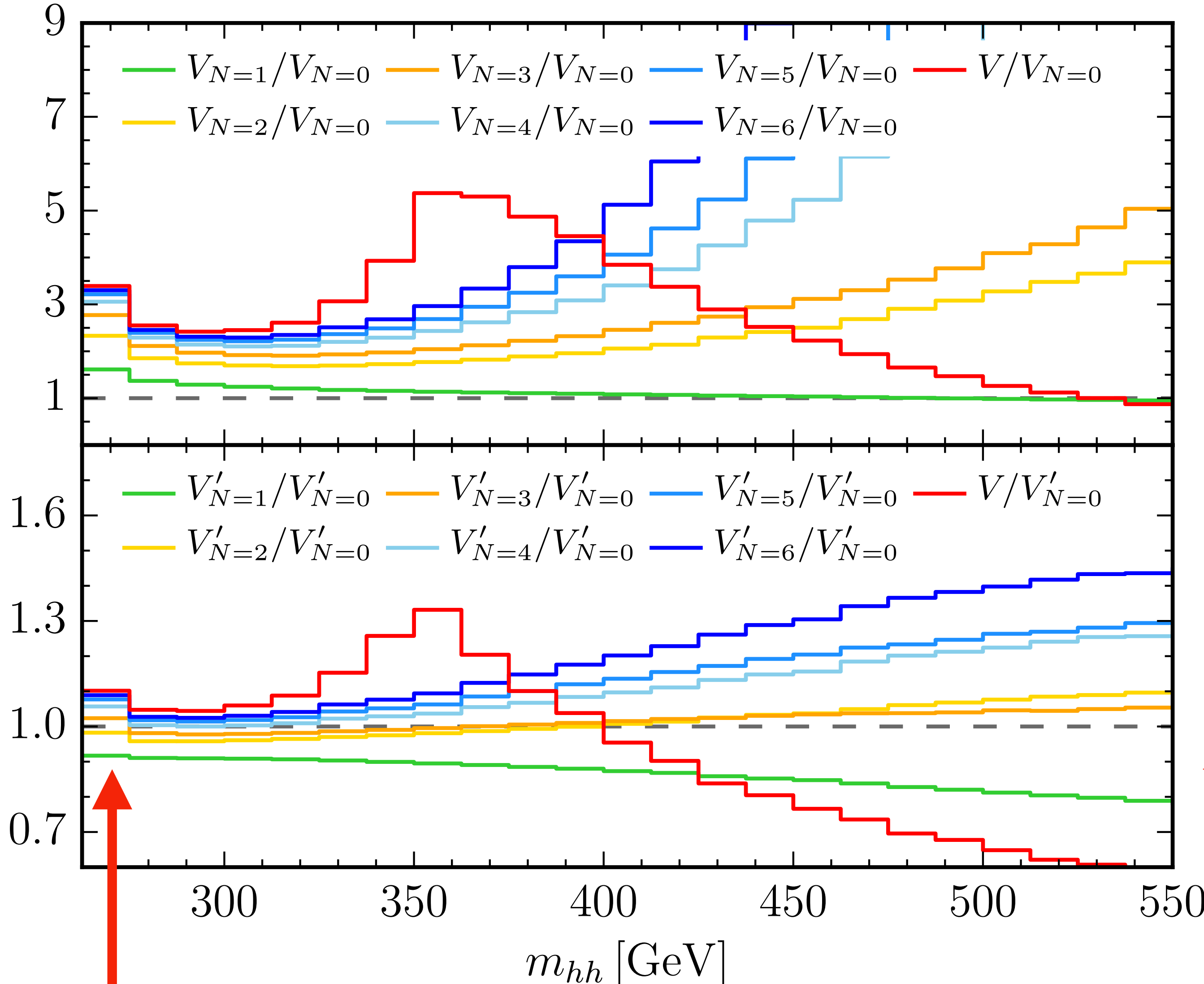
FTap. overestimates by 6%

Difference between full theory and HEFT more pronounced

Comparison to Expansion

Can compare just virtual ME to expansion:

$$d\hat{\sigma}_N = \sum_{\rho=0}^{N} d\hat{\sigma}^{(\rho)} \left(\frac{\Lambda}{m_t}\right)^{2\rho} \qquad \Lambda \in \left\{\sqrt{\hat{s}}, \sqrt{\hat{t}}, \sqrt{\hat{u}}, m_h\right\}$$



$$V_N = \left(d\hat{\sigma}_N^V + d\hat{\sigma}_N^{LO} \otimes \mathbf{I}\right)$$

$$V'_N = V_N \frac{d\hat{\sigma}^{LO}}{d\hat{\sigma}_N^{LO}}$$

Rescaled better but does not describe full above threshold

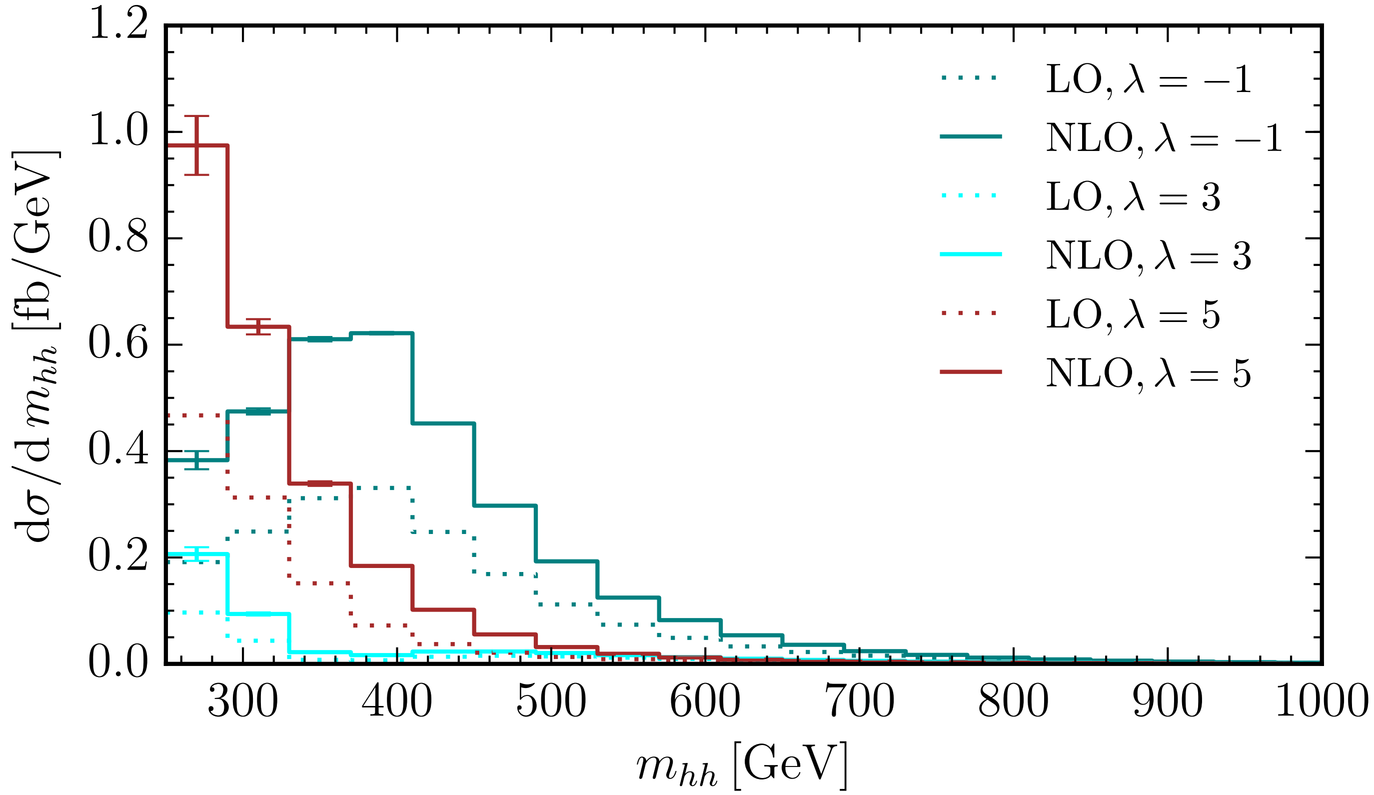
Expansion converges on full $\sqrt{\hat{s}} < 2m_T$

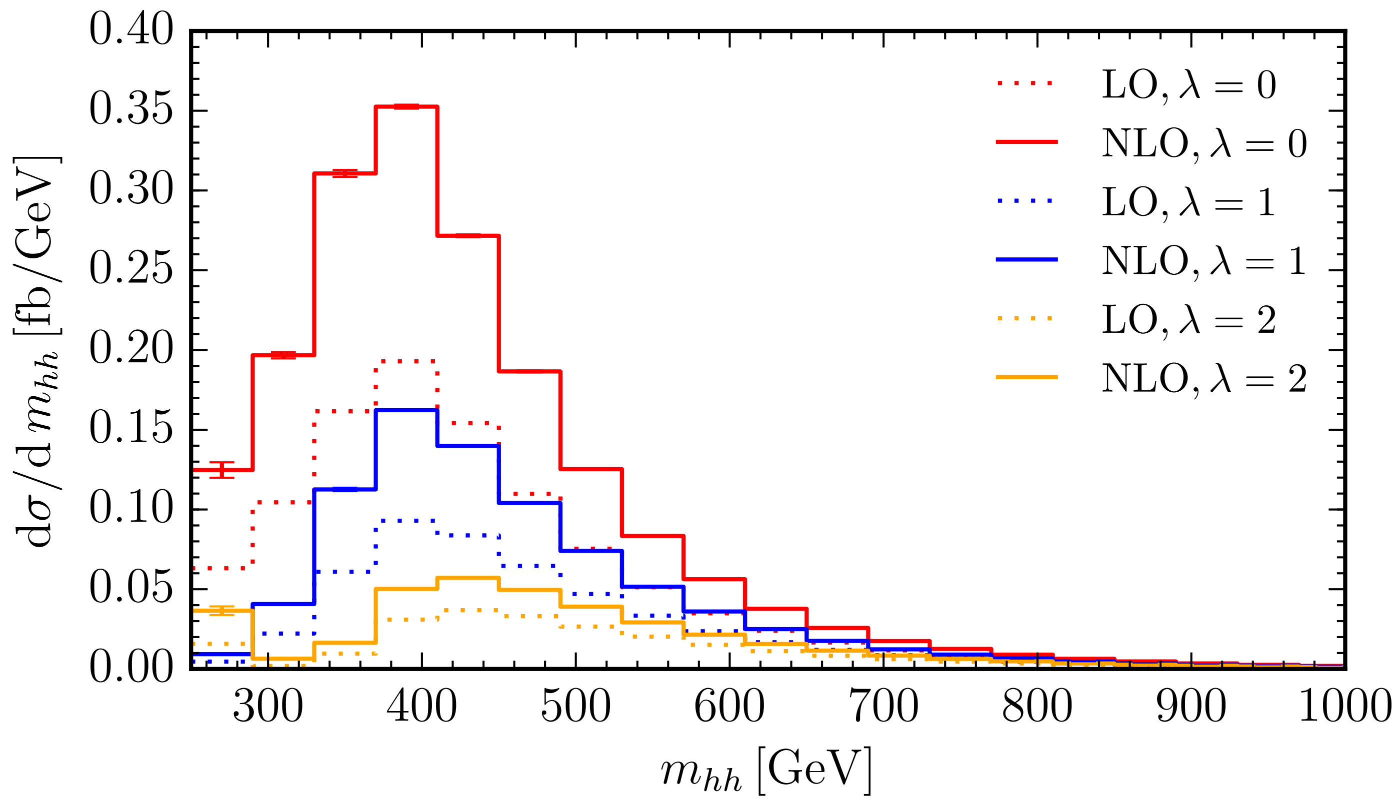
$V_{N\geq4}$ thanks to J. Hoff

Grigo, Hoff, Steinhauser 15

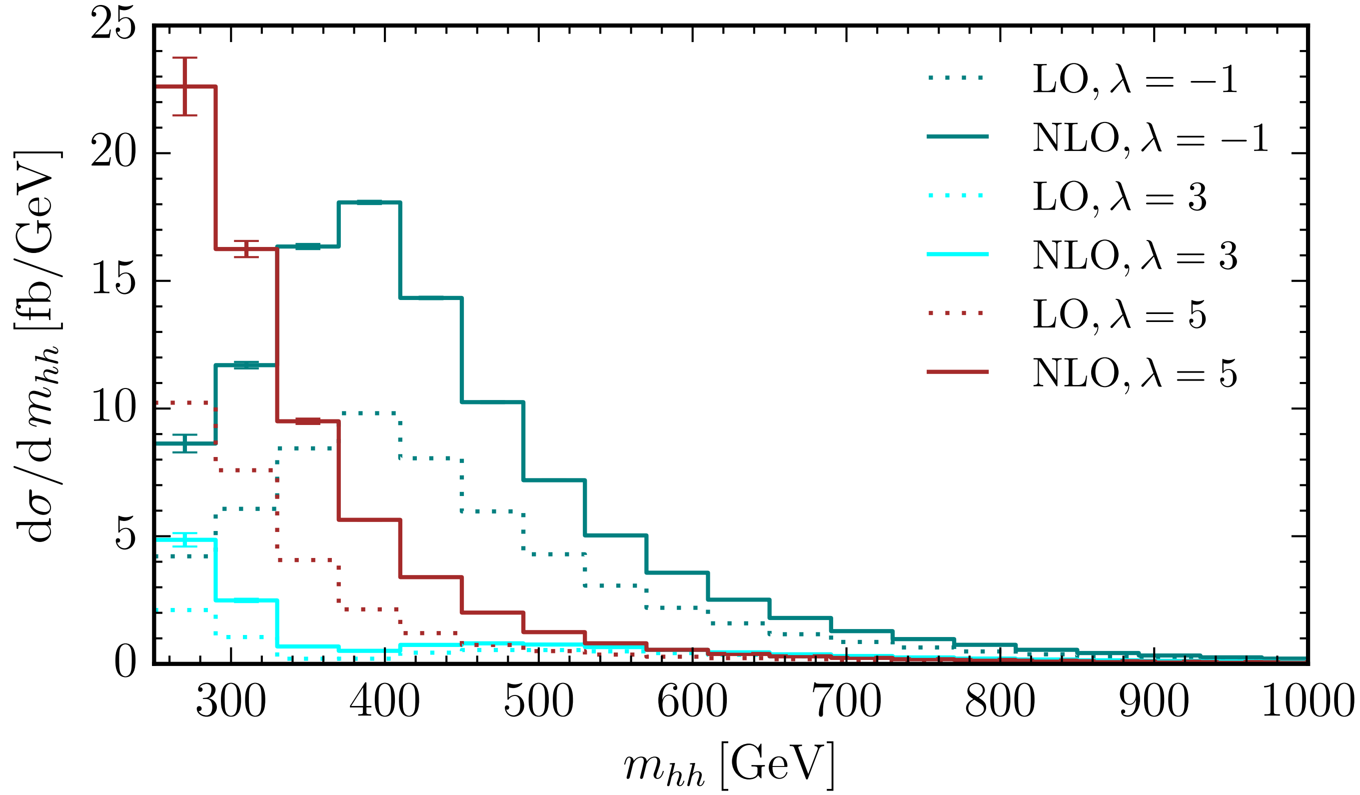
Lambda Variation

$\sqrt{s} = 14\,\text{TeV}$

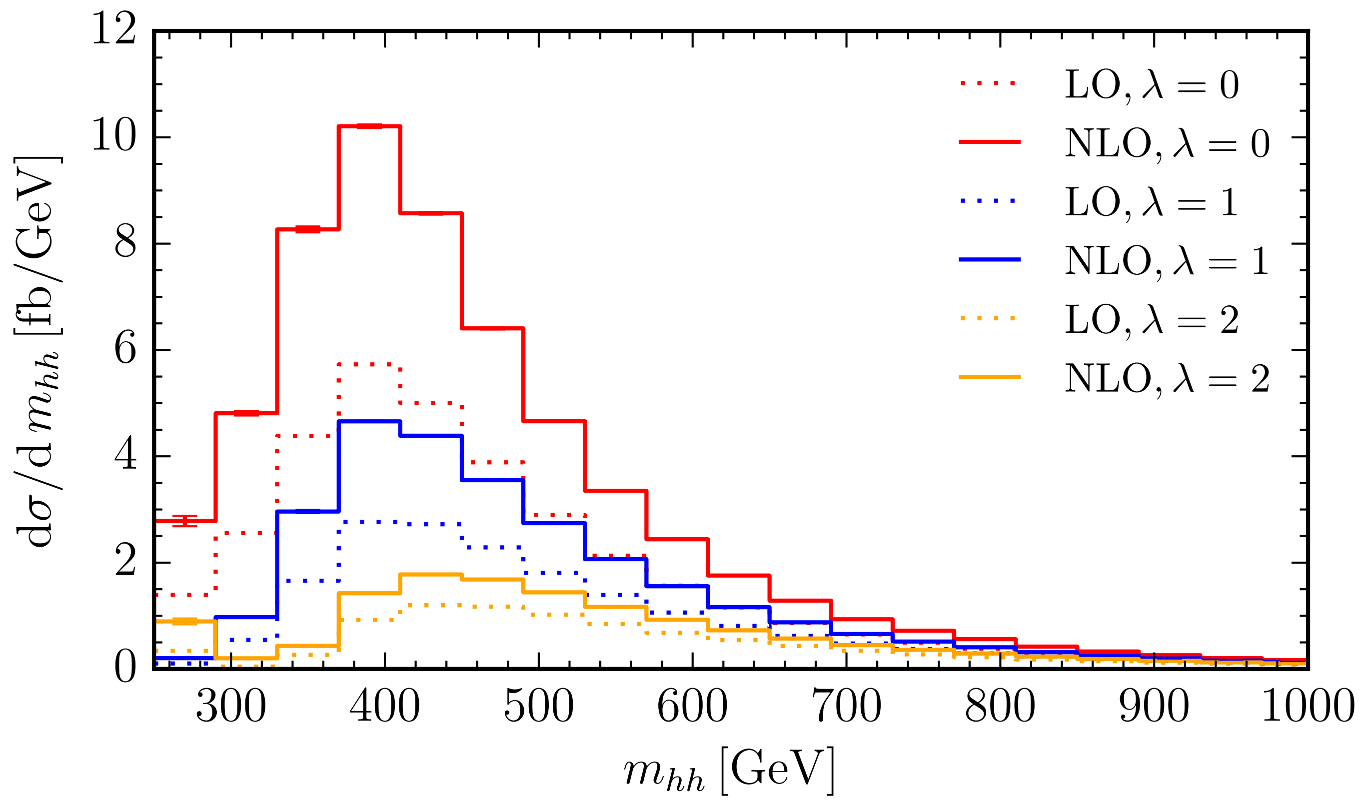




Lambda Variation

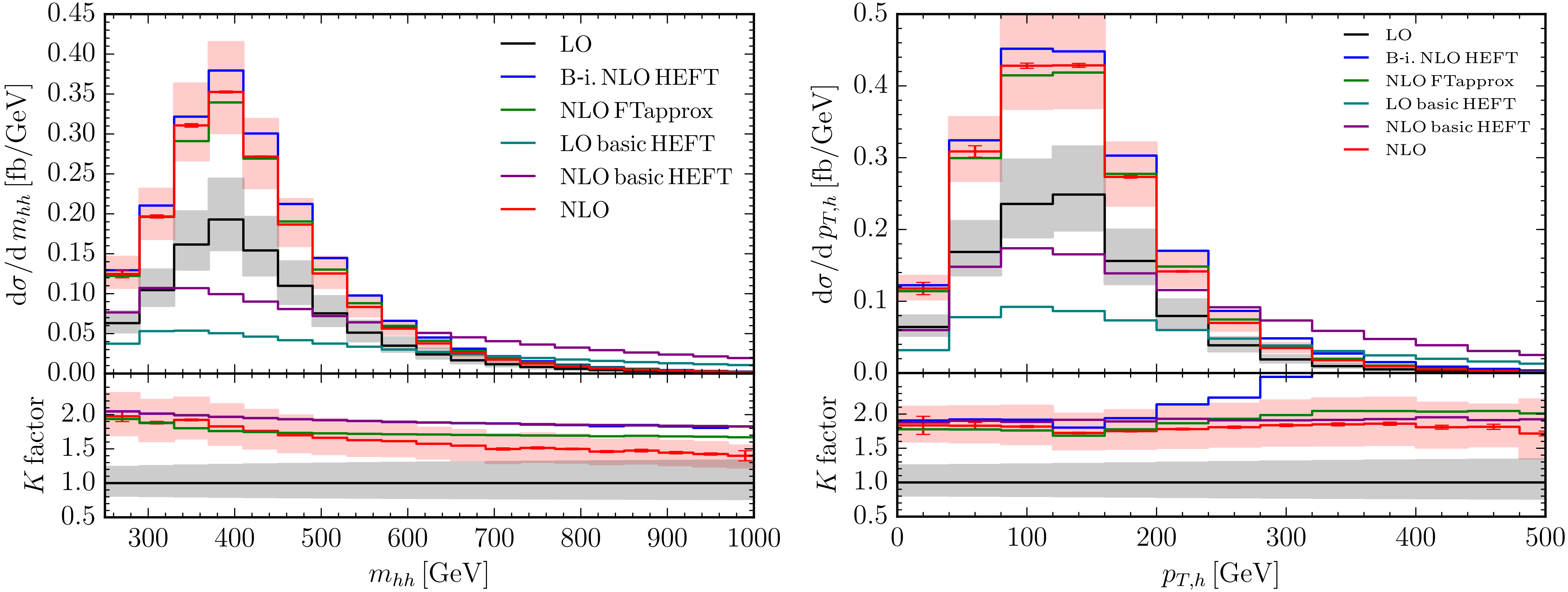


$\sqrt{s} = 100\,\mathrm{TeV}$



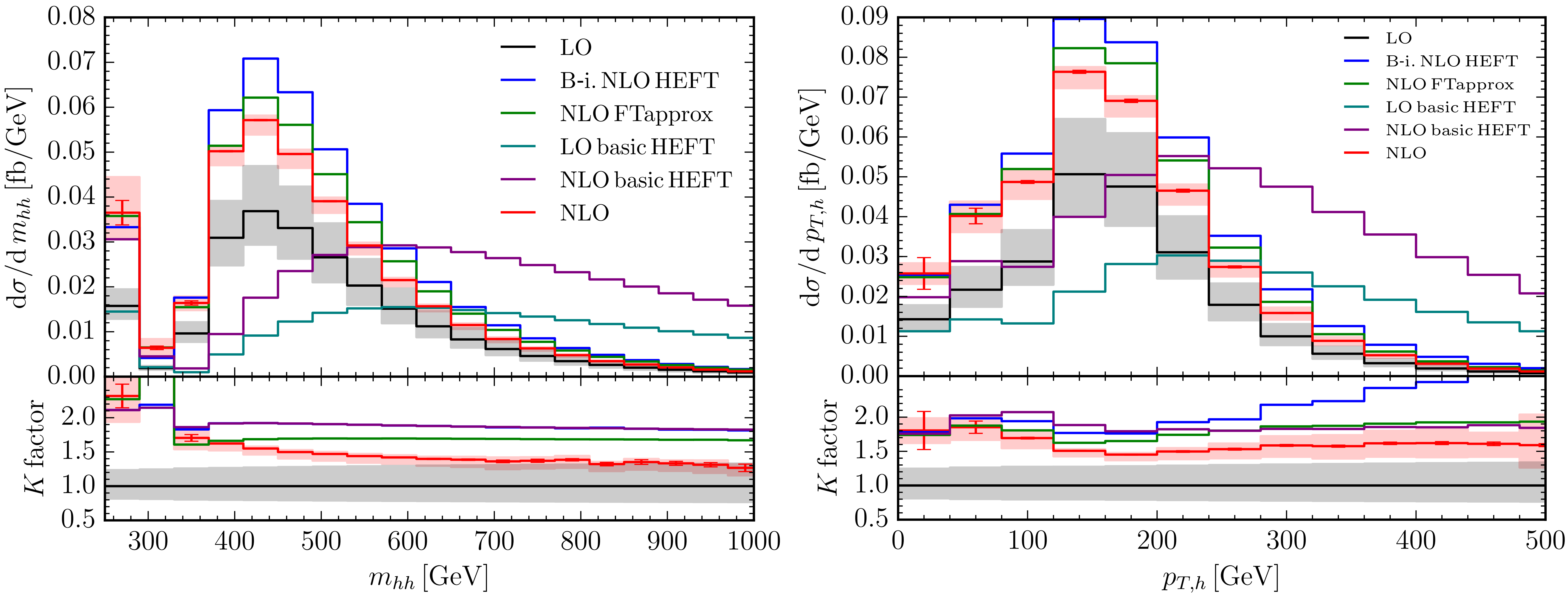
Lambda 0 x SM

$\sqrt{s} = 14\,\mathrm{TeV}$



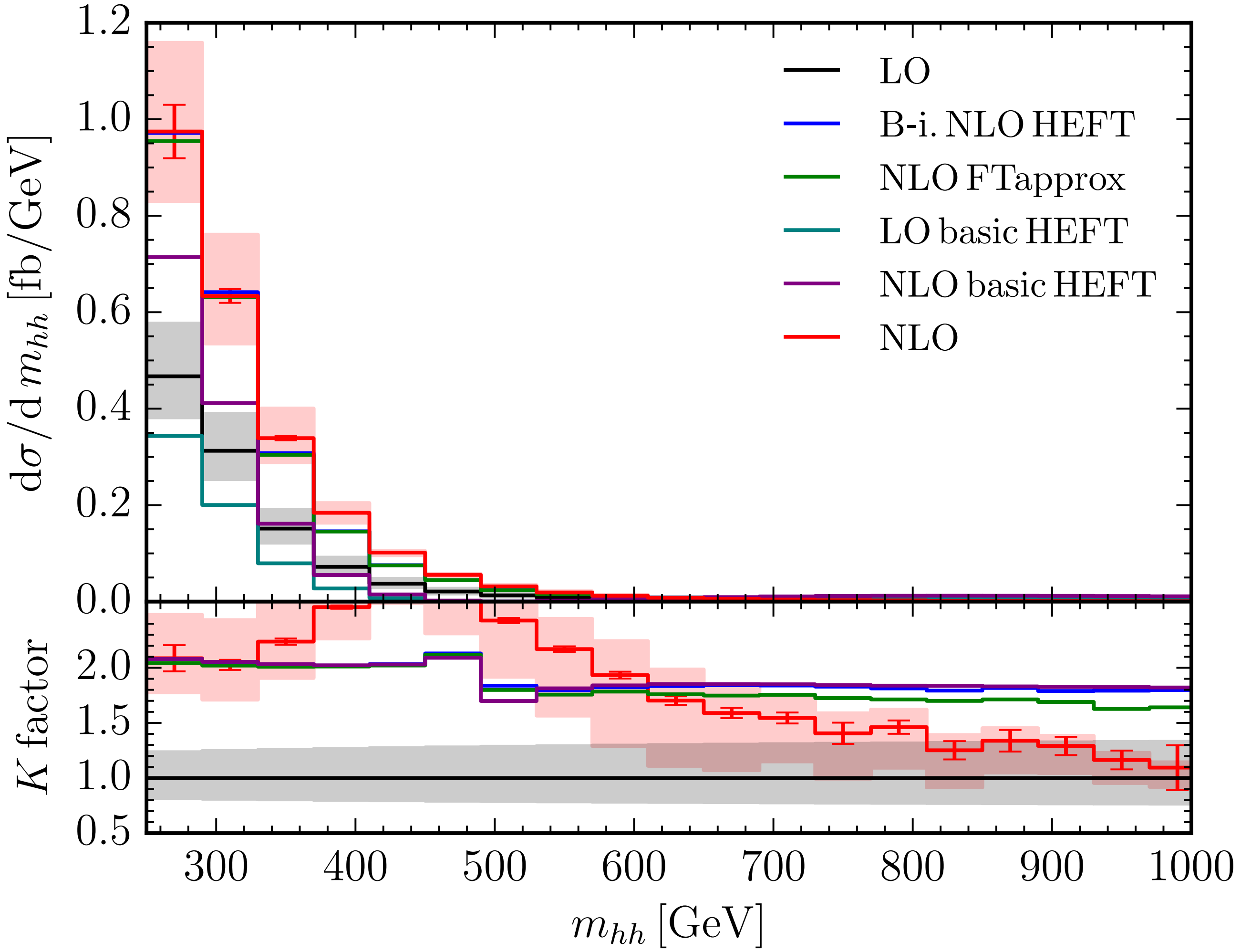
Lambda 2 x SM

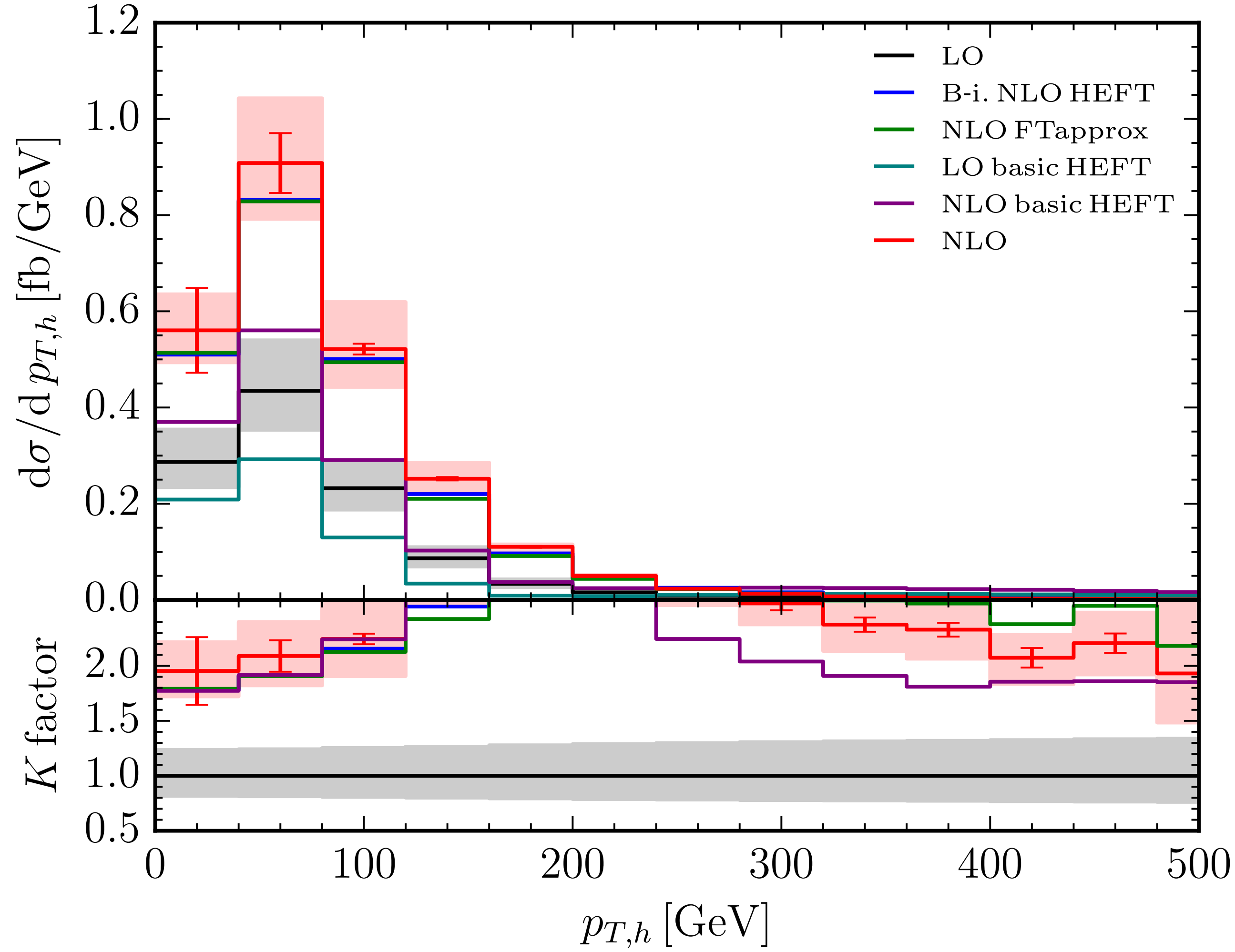
$\sqrt{s} = 14\,\text{TeV}$



Lambda 5 x SM

$\sqrt{s} = 14\,\mathrm{TeV}$





Top-quark Width Effects

Total XS @ LO: reduced by 2% by including top-quark width

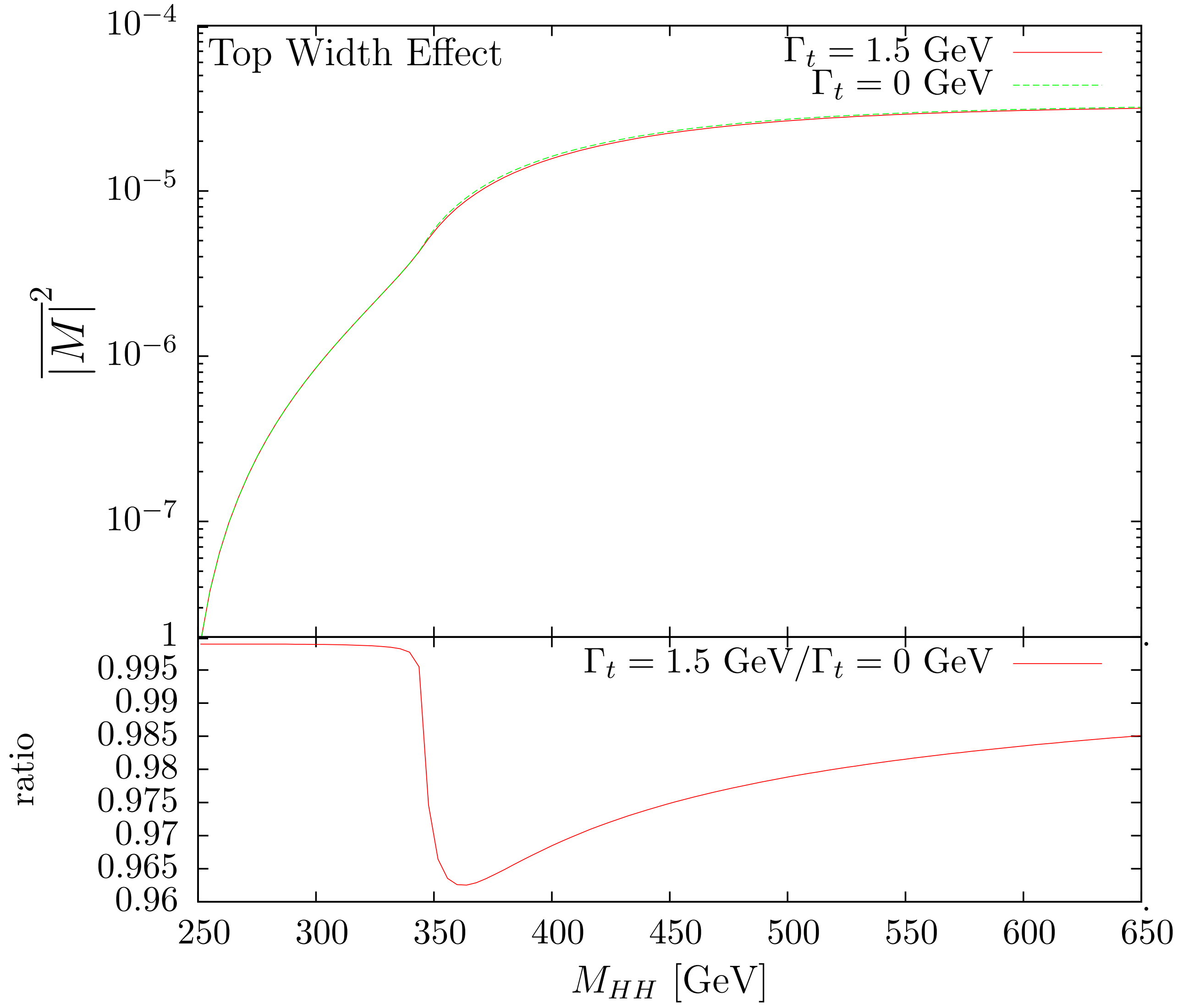


Figure 3: Top width effect on the one-loop (Born) matrix element squared for $gg \to HH$. The results for $\Gamma_t = 0$ and 1.5 GeV are shown along with the corresponding ratio.

Maltoni, Vryonidou, Zaro 14

Monte Carlo Interface

Monte Carlo Interface

Amplitude depends on 2 form factors:

$$\mathcal{M}^{\mu\nu} = F_1(\hat{s},\hat{t},m_h^2,m_t^2,D)T_1^{\mu\nu} + F_2(\hat{s},\hat{t},m_h^2,m_t^2,D)T_2^{\mu\nu}$$

Amplitude is slow to evaluate:

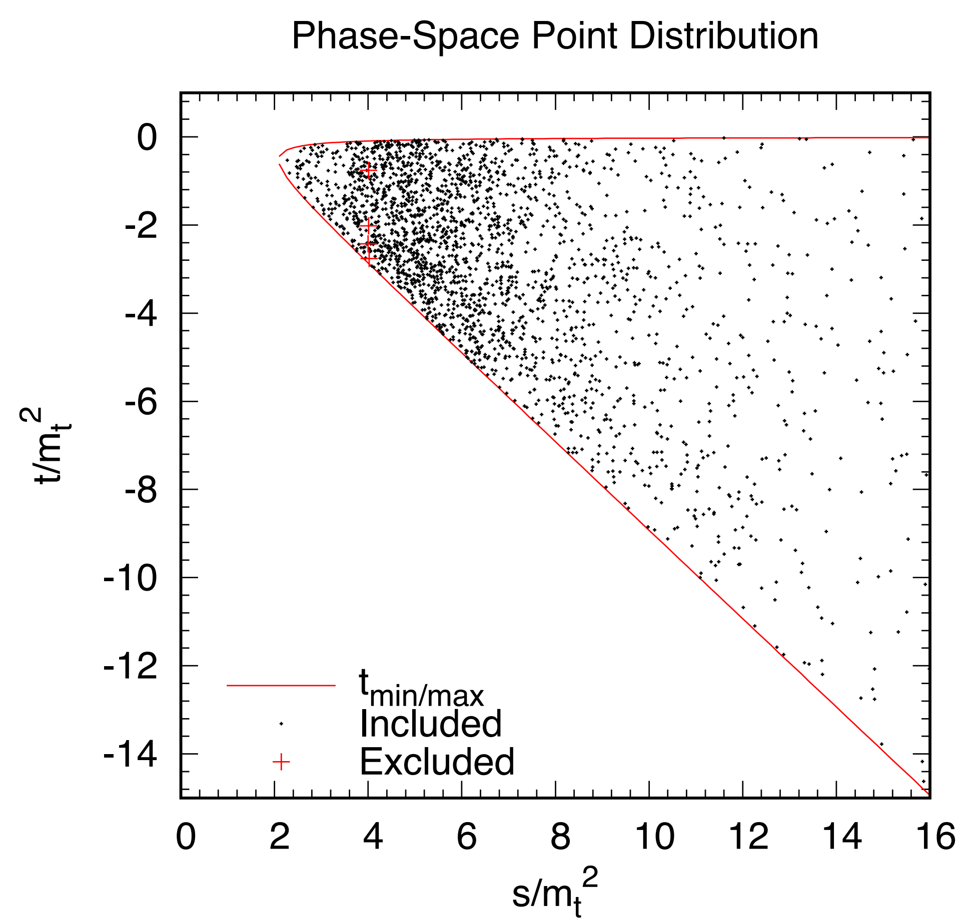
Accuracy goal: 3% for F_1, 5-20% for F_2 (depending on F_2/F_1)

GPU Time/PS point: 80 min - 2 days (median 2 hours)

Can not put directly into a Monte Carlo

But: Virtual matrix element depends only on $\hat{s},\hat{t}$ (fixed m_T, m_H)

Can build 2D grid of our phase-space points and interpolate between 3741 pre-calculated points



Monte Carlo Interface (II)

Parametrisation:

$$x = f(\beta(\hat{s})), \quad c_\theta = |\cos\theta| = \left|\frac{\hat{s} + 2\hat{t} - 2m_H^2}{\hat{s}\beta(\hat{s})}\right|, \quad \beta = \left(1 - \frac{4m_H^2}{\hat{s}}\right)^{\frac{1}{2}}$$

Choose $f(\beta)$ according to cumulative distribution function of phase space points used in the original calculation
Obtain nearly uniform distribution in (x, c_θ) unit square

Two-step interpolation procedure:

1. Choose equidistant grid points, estimate result at each grid point with linear interpolation of amplitude results in vicinity
2. Clough-Tocher interpolation (as implemented in `SciPy`) to estimate amplitude at arbitrary sampling points

Procedure reduces sensitivity to uncertainties of input data points

Monte Carlo Interface (III)

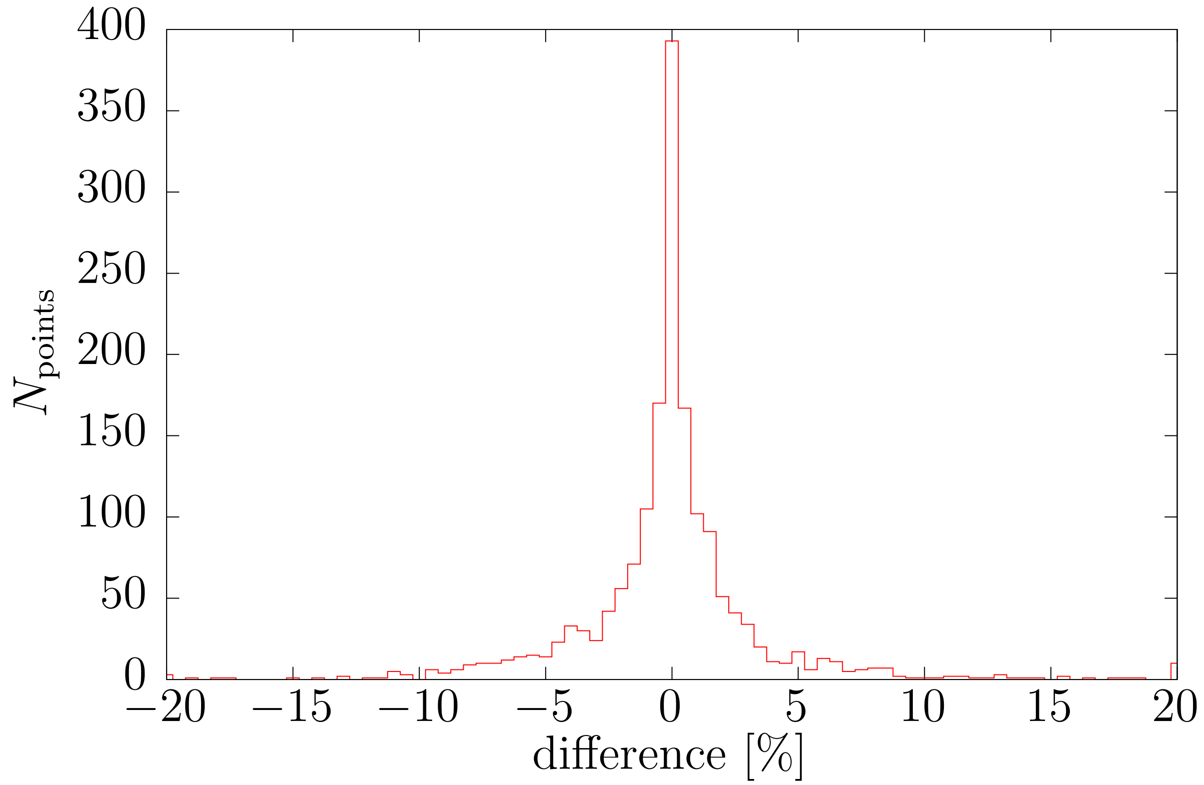
Grid of $\mathcal{V}_{\text{fin}}$ (1-loop x 2-loop interference) implemented in `Python`

Interfaced to `FORTRAN`, `C`, `C++` via `Python/C API` (examples in repo)

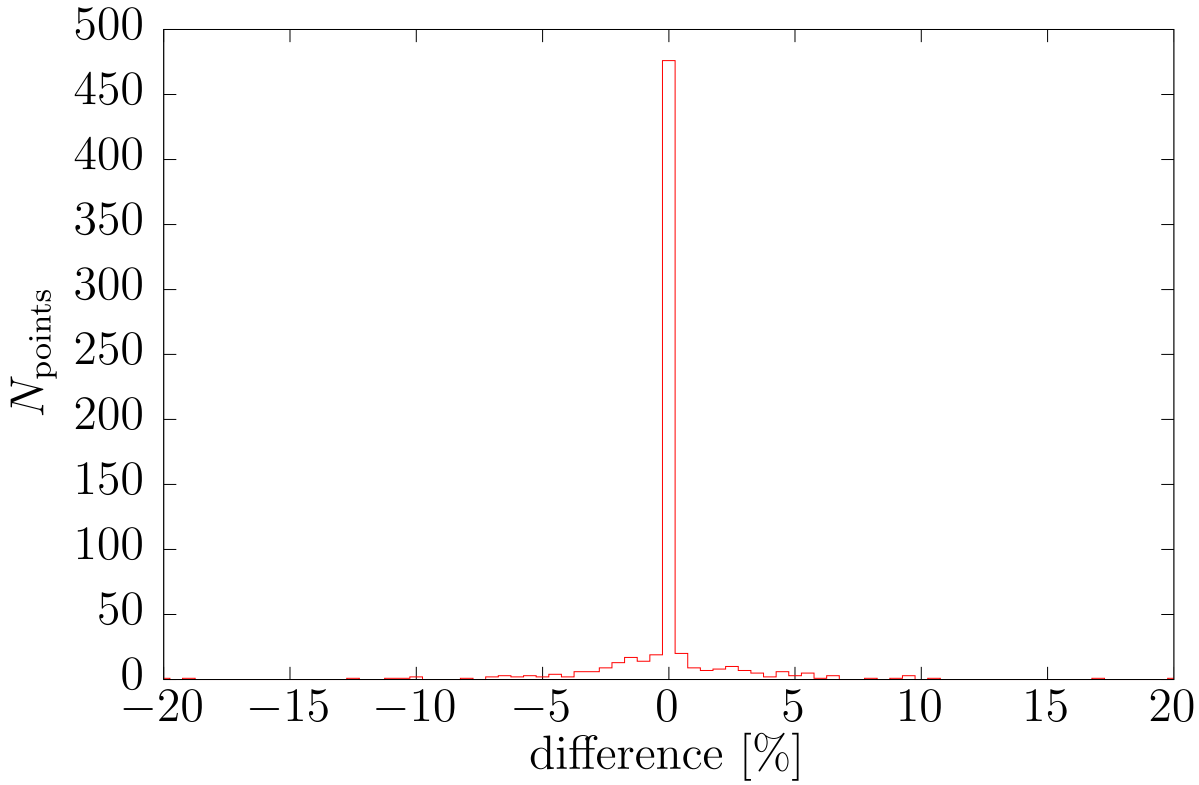
https://github.com/mppmu/hhgrid

Heinrich, SPJ, Kerner, Luisoni, Vryonidou 17

Grid Stability:



Grid vs grid with 50% of points



Grid vs grid with 80% of points

Monte Carlo Interface (IV)

Example (Calling Grid via C++):

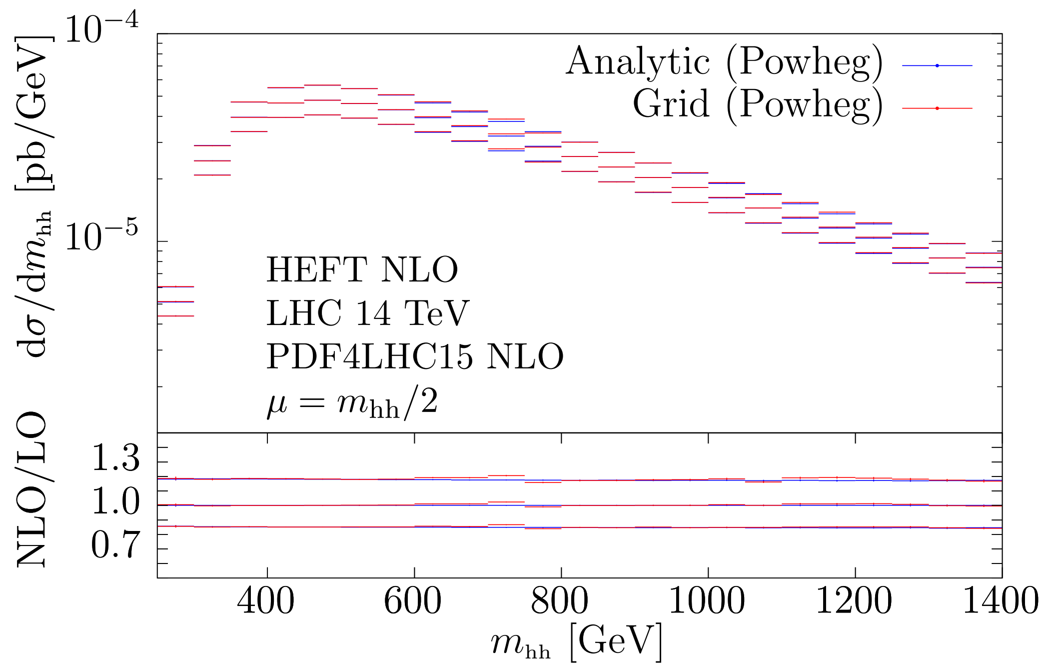
```
extern "C" {
#include "hhgrid.h"
}

#include <iostream> // std::cout
#include <string>   // std::string

int main()
{
    // Initialise python and grid
    python_initialize();
    python_printinfo();
    std::string grid_name = "Virt_EFT.grid";
    PyObject* pGrid = grid_initialize(grid_name.c_str());

    // Evaluate point
    double s = 2.56513e6;
    double t = -482321.e0;
    double result = grid_virt(pGrid,s,t);

    std::cout << "Sent: " << s << " " << t << std::endl;
    std::cout << "Received: " << result << std::endl;

    // Destruct grid, terminate python
    python_decref(pGrid);
    python_finalize();

    return 0;
};
```

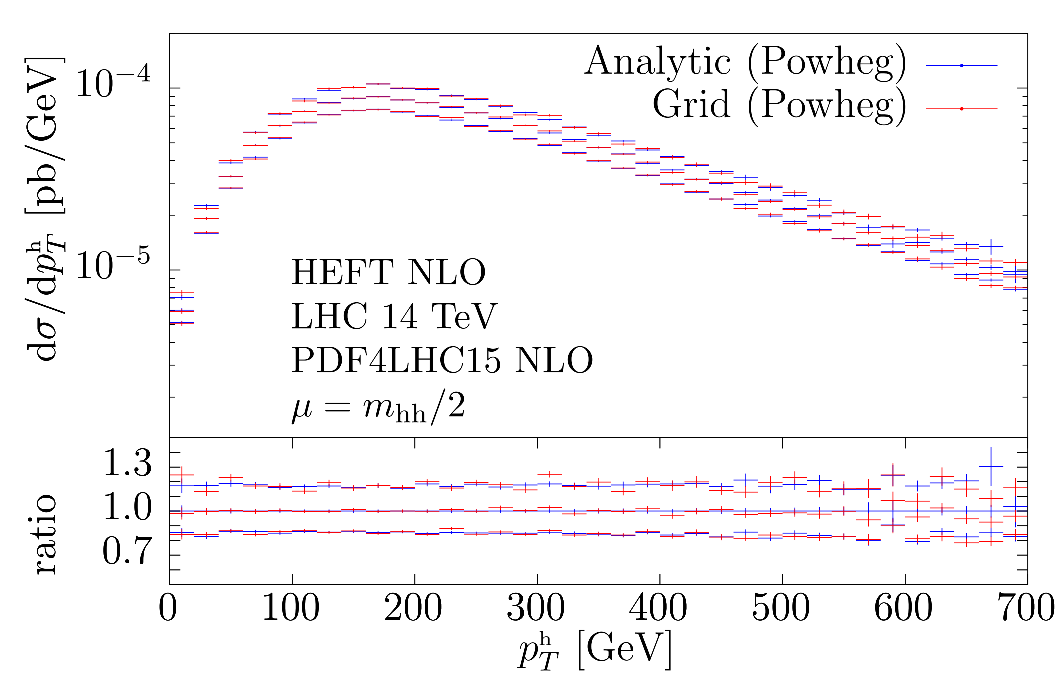
Choose:
`EFT` / `full`

Evaluate point using grid

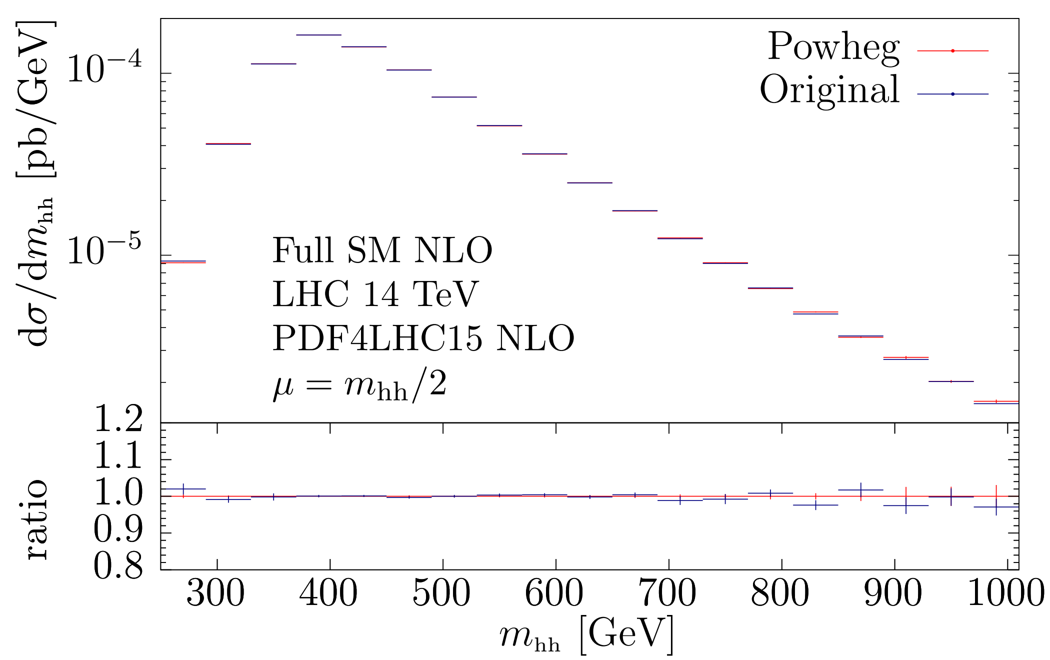
Grid Validation

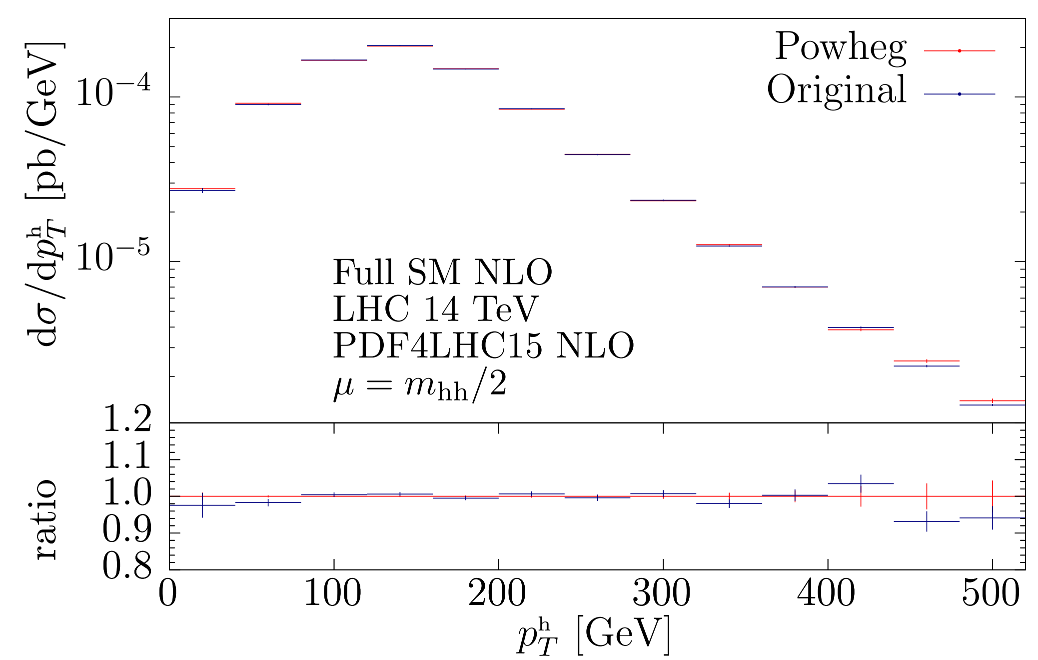
Use HEFT to study validity of grid





Full SM compare **POWHEG** (grid) with our original results

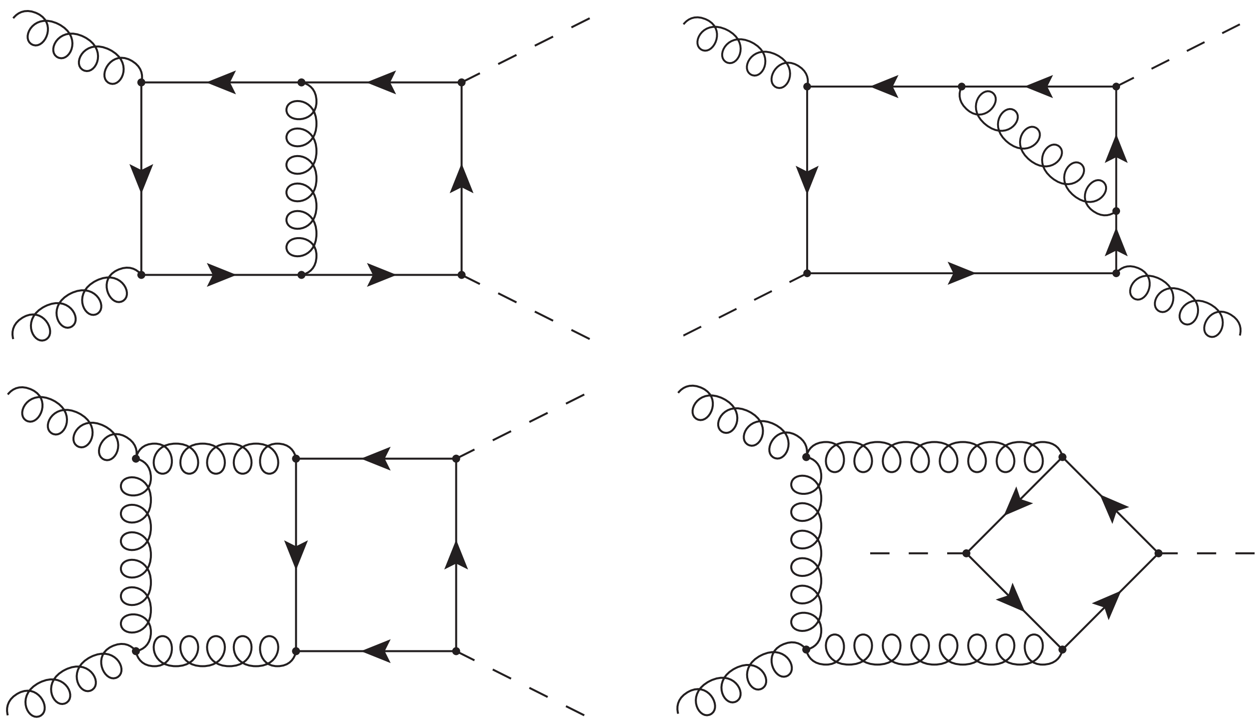


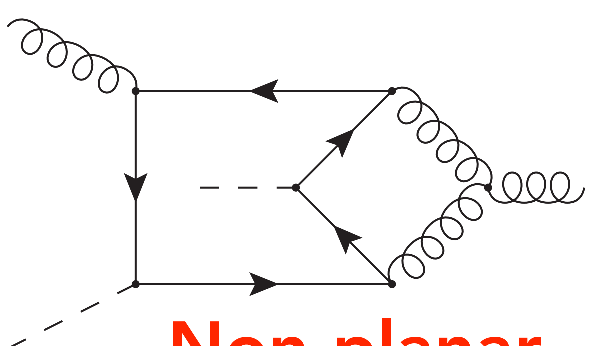


Details of Calculation

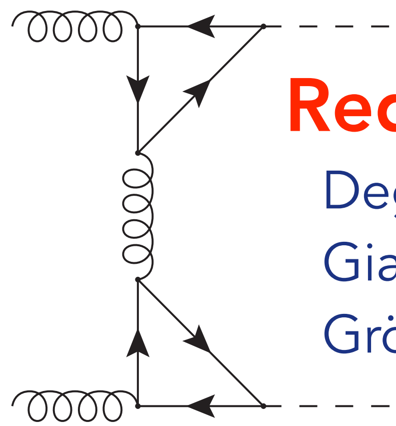
Virtual Contribution

Yukawa only (≤ 4-point)





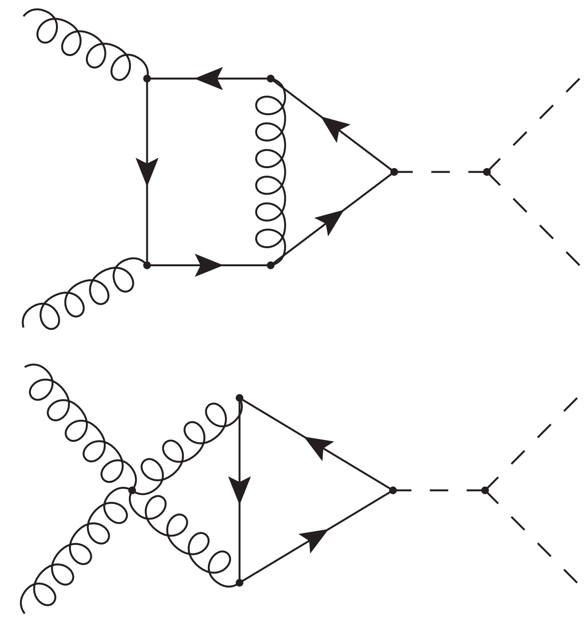
Non-planar



Reducible

Degrassi, Giardino, Gröber 16

Self-coupling (≤3-point)



Integrals Known

$gg \to H$

Spira, Djouadi et al. 93, 95;
Bonciani, P. Mastrolia 03,04;
Anastasiou, Beerli et al. 06;

Many integrals not known analytically, except:

$H \to Z\gamma$ Bonciani, Del Duca, Frellesvig et al. 15; Gehrmann, Guns, Kara 15;

Evaluating the Amplitude

Planar Integrals: Reduce to finite basis with **REDUZE 2**

von Manteuffel, Studerus 12; Panzer 14; von Manteuffel, Panzer, Schabinger 15

Non-Planar Integrals: Evaluate integrals directly

All Integrals evaluated numerically with **SecDec** (implements sector decomposition)

Borowka, Heinrich, Jahn, SPJ, Kerner, Schlenk, Zirke

Hepp 66; Denner, Roth 96; Binoth, Heinrich 00

Entire 2-loop amplitude evaluated with a single code

$$F = \sum_i \underbrace{\left(\sum_j C_{i,j} \epsilon^j \right)}_{\text{coeff.}} \underbrace{\left(\sum_k I_{i,k} \epsilon^k \right)}_{\text{integral}} = \epsilon^{-2} \left[C^{(L)}_{1,-2} I^{(L)}_{1,0} + \ldots \right] + \epsilon^{-1} \left[C^{(L)}_{1,-1} I^{(L)}_{1,0} + \ldots \right] + \ldots$$

compute once

Dynamically set target precision for each sector, minimising time

Use Quasi-Monte-Carlo (QMC) integration $\mathcal{O}(n^{-1})$ error scaling

Li, Wang, Yan, Zhao 15; (Review: Dick, Kuo, Sloan 13)

Implemented in OpenCL, evaluated on GPUs

Checks

Real Emission / Subtraction Terms

- Independence of dipole-cut α_{cut} parameter Nagy 03
- Agreement with literature Maltoni, Vryonidou, Zaro 14
- Agreement with FKS (`POWHEG/MG5_amc@NLO`)
  Frixione, Kunszt, Signer 96; Nason 04; Frixione et al 07; Alioli et al. 10; J. Alwall et al. 14

Virtual Corrections

- Two calculations of amplitude up to reduction
- Amplitude result invariant under $t \leftrightarrow u$
- Pole cancellation
- Mass renormalization using two methods:
  counter-term insertion vs. calculating $\mathrm{d}\mathcal{M}^{\mathrm{LO}}/\mathrm{d}m_T^2$ numerically
- Agreement of contributions $gg \to H \to HH$ with SusHi Harlander, Liebler, Mantler 13,16
- Convergence of $1/m_T$ expansion to full result
  where agreement is expected

Integral Families

Can rewrite tensor integrals/scalar products as inverse propagators

#scalar products

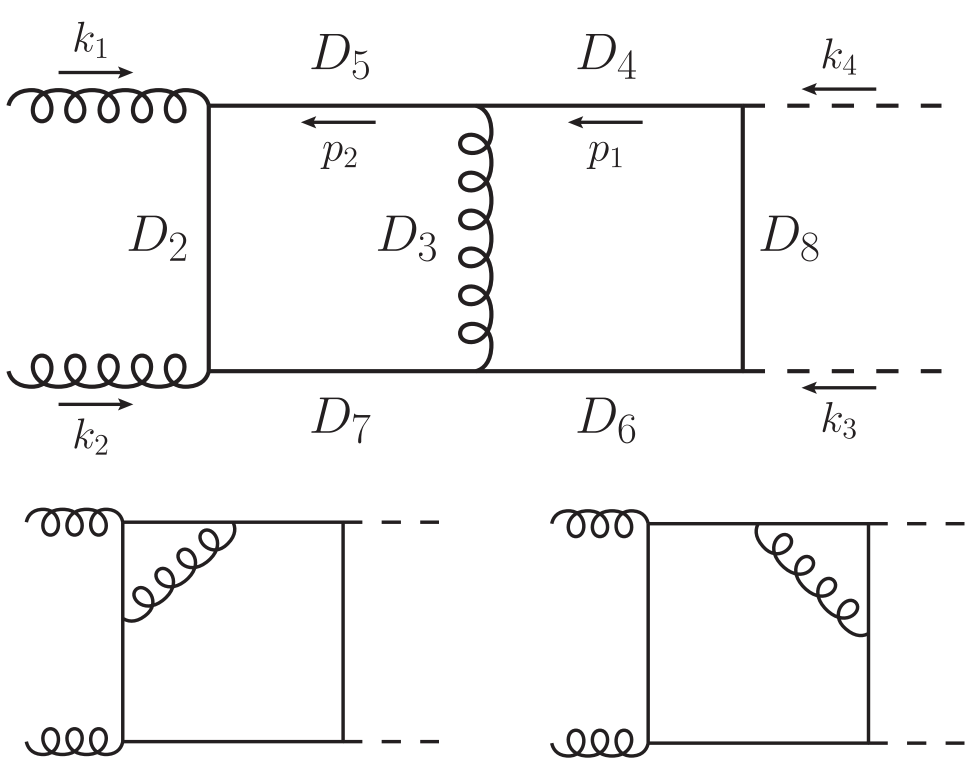
$$S = \frac{l(l+1)}{2} + lm \qquad \begin{array}{l} l = 2 \\ m = 3 \end{array} \quad \begin{array}{l} \textbf{\#loops} \\ \textbf{\#l.i. external momenta} \end{array} \quad \Rightarrow \quad S = 9$$

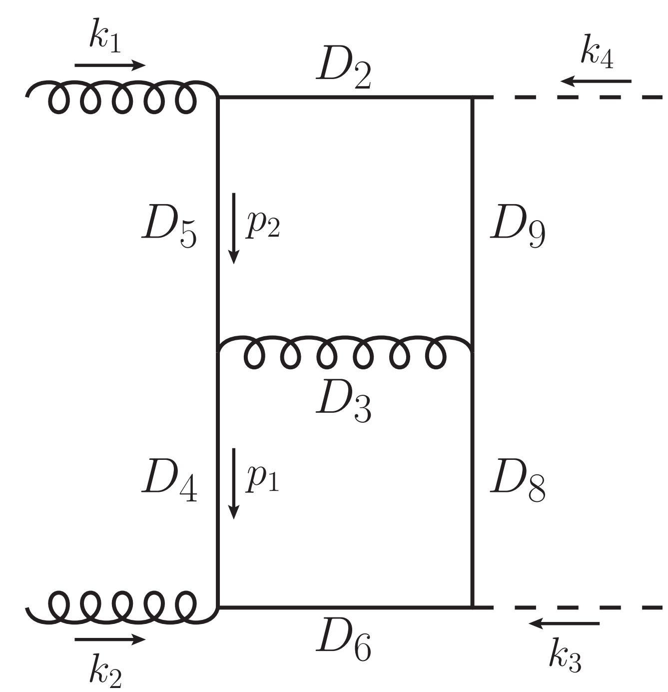
Introduce Integral Families with 9 propagators

$$I^{\text{fam}_j}_{\nu_1,\ldots,\nu_9} = \int \mathrm{d}^d p_1 \int \mathrm{d}^d p_2 \frac{1}{D_1^{\nu_1} D_2^{\nu_2} \cdots D_9^{\nu_9}} \qquad \nu_i \in \mathbb{Z}$$

Planar family 1:

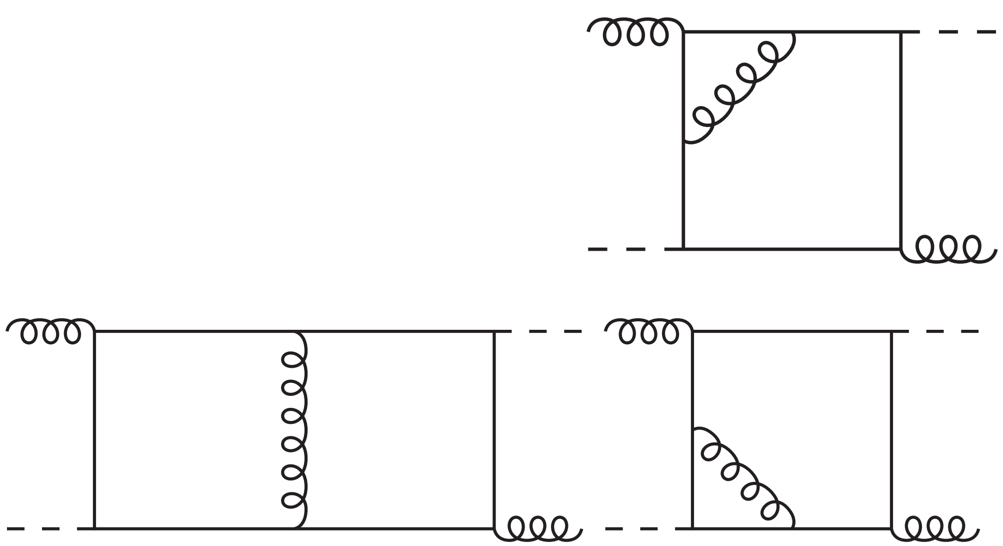
$$\begin{aligned}
D_1 &= p_1^2 - m_t^2 \\
D_2 &= p_2^2 - m_t^2 \\
D_3 &= (p_1 - p_2)^2 \\
D_4 &= (p_1 + k_1)^2 - m_t^2 \\
D_5 &= (p_2 + k_1)^2 - m_t^2 \\
D_6 &= (p_1 - k_2)^2 - m_t^2 \\
D_7 &= (p_2 - k_2)^2 - m_t^2 \\
D_8 &= (p_1 - k_2 - k_3)^2 - m_t^2 \\
D_9 &= (p_2 - k_2 - k_3)^2 - m_t^2
\end{aligned}$$



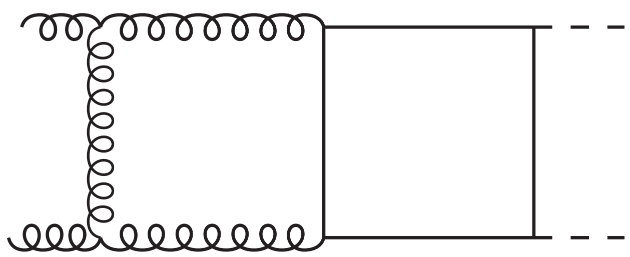


Integral Families (II)

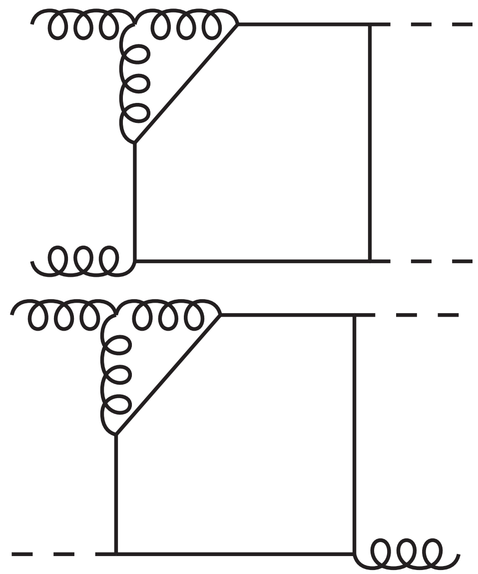
Planar Family 2



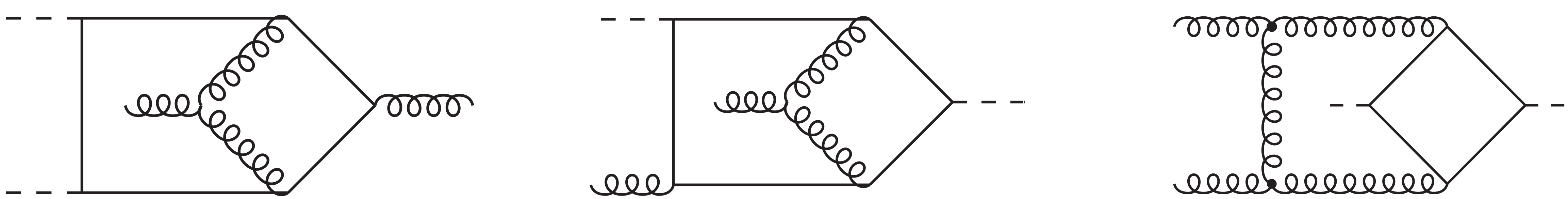
Planar Family 3



Planar Families 4/5



3 non-planar families:



Integral Reduction

Practically, 2-loop reduction with 4 scales $(\hat{s}, \hat{t}, m_T^2, m_H^2)$ and 4 inverse propagators is challenging

Simplification: Fix $m_T = 173\,\mathrm{GeV}, m_H = 125\,\mathrm{GeV}$

Price: Many arbitrary precision integers in reduction (slow)
Can not vary masses in result

Integrals	1-loop	2-loop
Direct	63	9865
+ Symmetries	21	1601
+ IBPs	8	~260-270 Currently: 327

Planar Integrals (145+83 crossed)

Reduction with `REDUZE 2`

von Manteuffel, Studerus 12

Non-Planar Integrals (70+29 crossed)

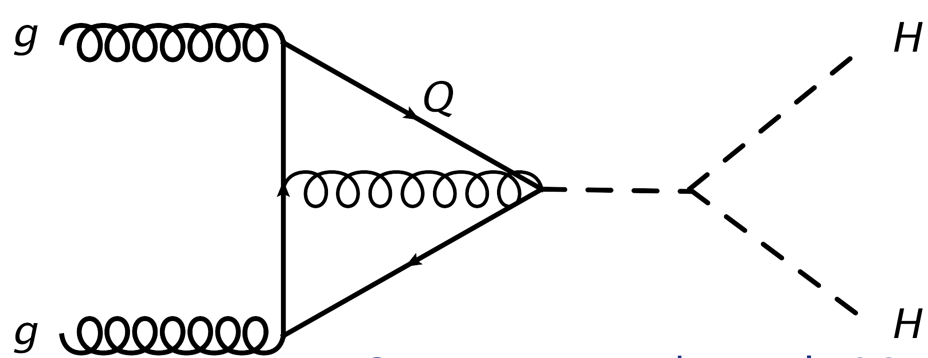
Computed mostly without reduction

For reduced integrals we choose a Finite Basis using `REDUZE`

Panzer 14; von Manteuffel, Panzer, Schabinger 15

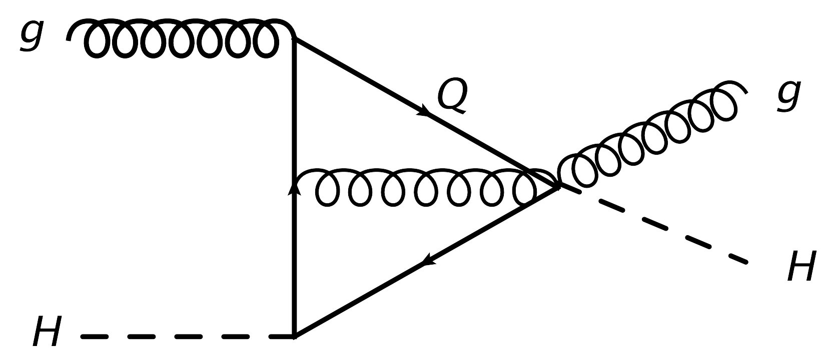
Master Integrals

Known Analytically:



Spira, Djouadi et al. 93, 95;
Bonciani, P. Mastrolia 03,04;
Anastasiou, Beerli et al. 06;

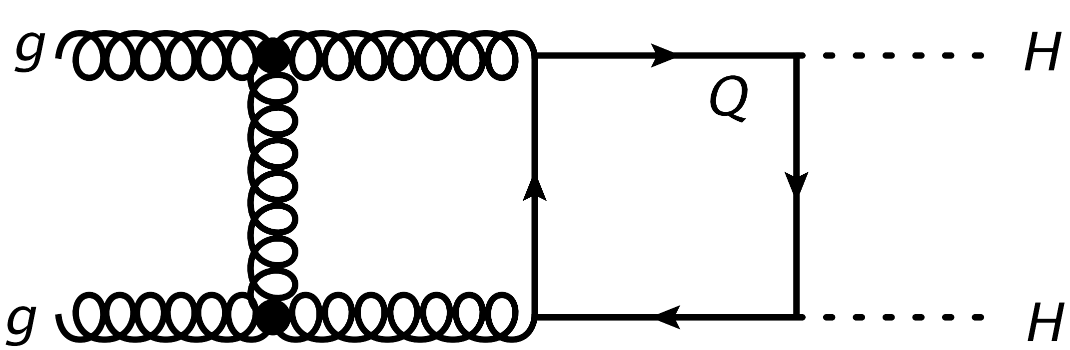
3-point, 1 off-shell leg
HPLs



Gehrmann, Guns, Kara 15

3-point, 2 off-shell legs
Generalized HPLs, 12 Letters

Numeric Evaluation:



Up to 4-point,
4 scales s, t, m_T^2, m_H^2
`SecDec`

Slide: Matthias Kerner

Finite Basis

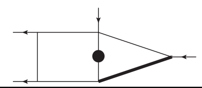
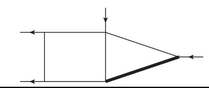
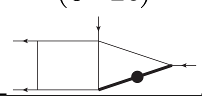
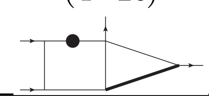
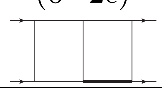
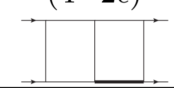
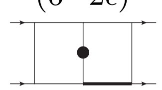
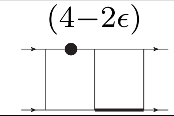
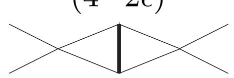
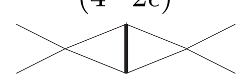
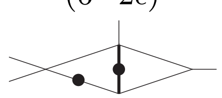
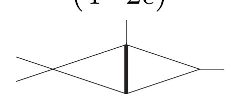
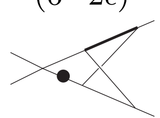
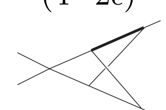
Always possible to pick finite basis of integrals, rewrite integrals using:

- Dimension Shifts Tarasov 96; Lee 10
- Dots

Panzer 14; von Manteuffel, Panzer, Schabinger 15

Two-loop EW-QCD Drell-Yan

von Manteuffel, Schabinger 17

Finite Basis...			Conventional...		
$(6-2\epsilon)$  (s,t)	201 s	2.34×10^{-4}	$(4-2\epsilon)$  (s,t)	384 s	8.12×10^{-4}
$(6-2\epsilon)$  (s,t)	150 s	4.83×10^{-4}	$(4-2\epsilon)$  (s,t)	56538 s	1.67×10^{-2}
$(6-2\epsilon)$  (s,t)	280 s	1.00×10^{-3}	$(4-2\epsilon)$  (s,t)	214135 s	8.29×10^{-3}
$(6-2\epsilon)$  (s,t)	294 s	1.21×10^{-3}	$(4-2\epsilon)$  (s,t)	3484378 s	30.9
$(4-2\epsilon)$  (s)	91 s	3.76×10^{-4}	$(4-2\epsilon)$  (s)	87 s	3.76×10^{-4}
$(6-2\epsilon)$  (s)	17 s	5.15×10^{-4}	$(4-2\epsilon)$  (s)	20 s	1.95×10^{-4}
$(6-2\epsilon)$  (s)	119 s	2.32×10^{-3}	$(4-2\epsilon)$  (s)	118 s	2.12×10^{-3}
Total/Max:	3995 s	5.84×10^{-3}	Total/Max:	5136862 s	30.9

← **Rel. Err.**

Huge decrease in time to numerically integrate and relative error

Rank 1 Shifted Lattices

$\mathcal{O}(n^{-1})$ algorithm for numerical integration:

Review: Dick, Kuo, Sloan 13

$$I_s[f] \equiv \int_{[0,1]^s} \mathrm{d}^s x f(\vec{x})$$

$$f : \mathbb{R}^s \to \mathbb{C}$$

$$I_s[f] \approx \bar{Q}_{s,n,m}[f] \equiv \frac{1}{m} \sum_{k=1}^{m} \frac{1}{n} \sum_{i=0}^{n-1} f\left(\left\{ \frac{i\vec{z}}{n} + \vec{\Delta}_k \right\}\right)$$

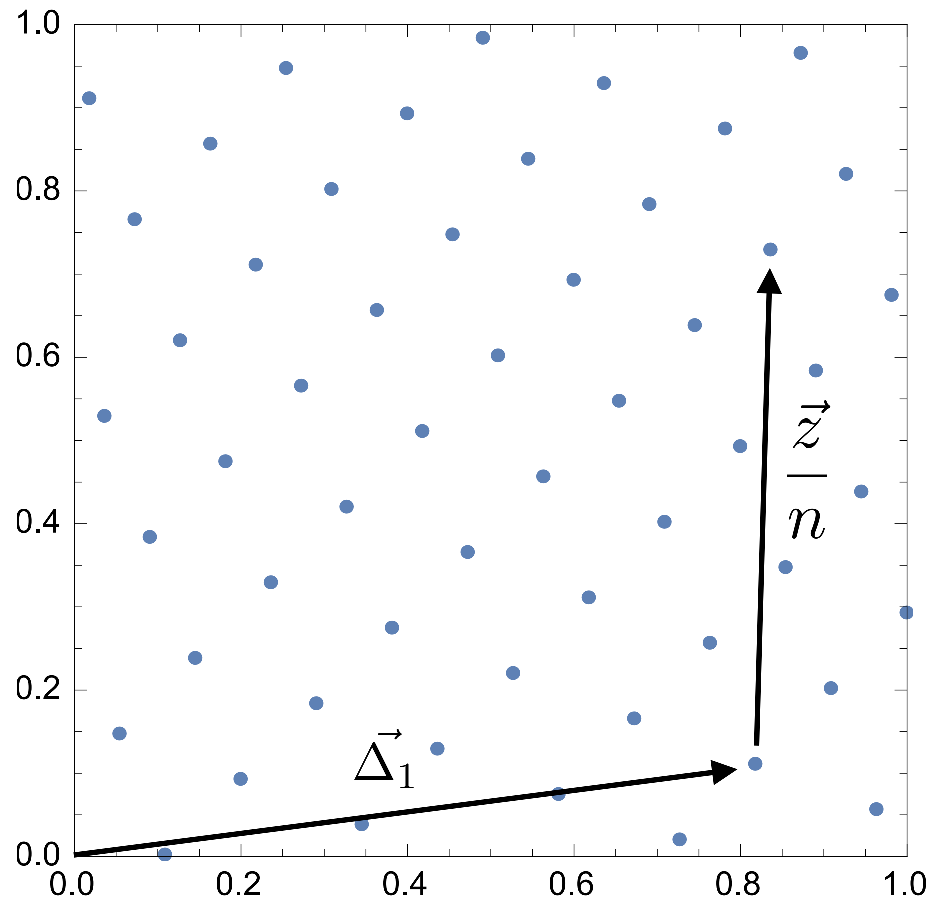
$\vec{z}$ - Generating vec.

$\vec{\Delta}_k$ - Random shift vec.

$\{\}$ - Fractional part

n - # Lattice points

m - # Random shifts

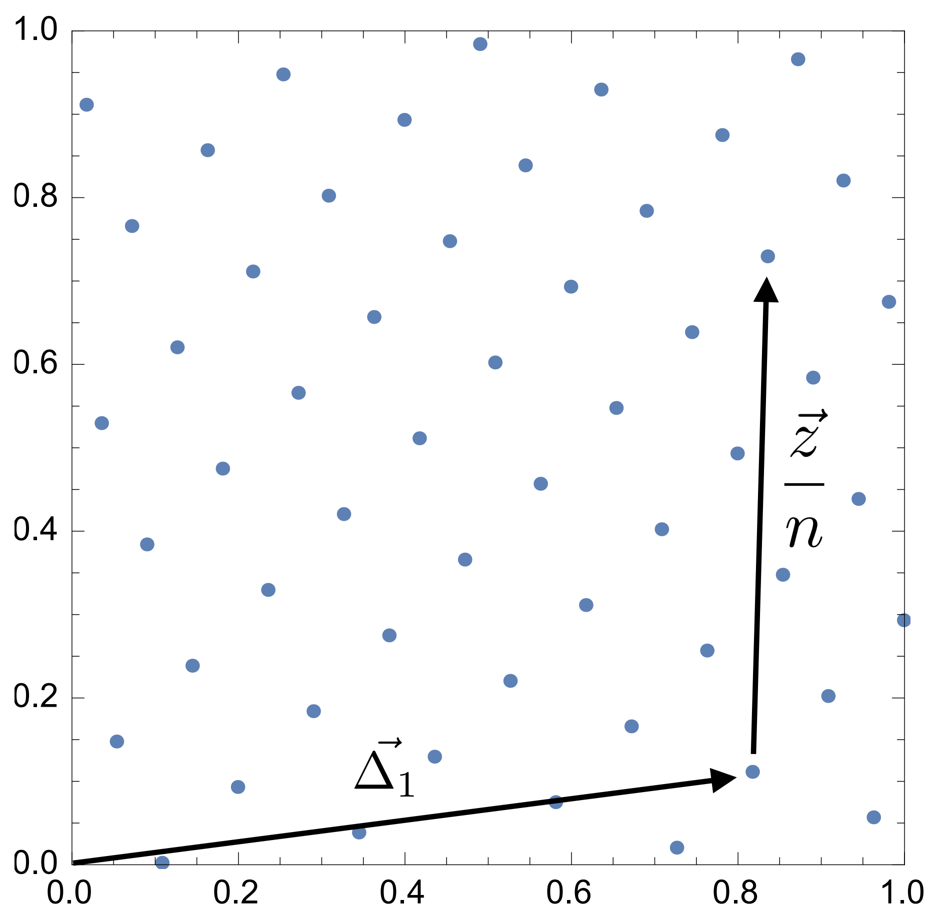


Generating vector $\vec{z}$ precomputed for a **fixed** number of lattice points, chosen to minimise worst-case error Nuyens 07

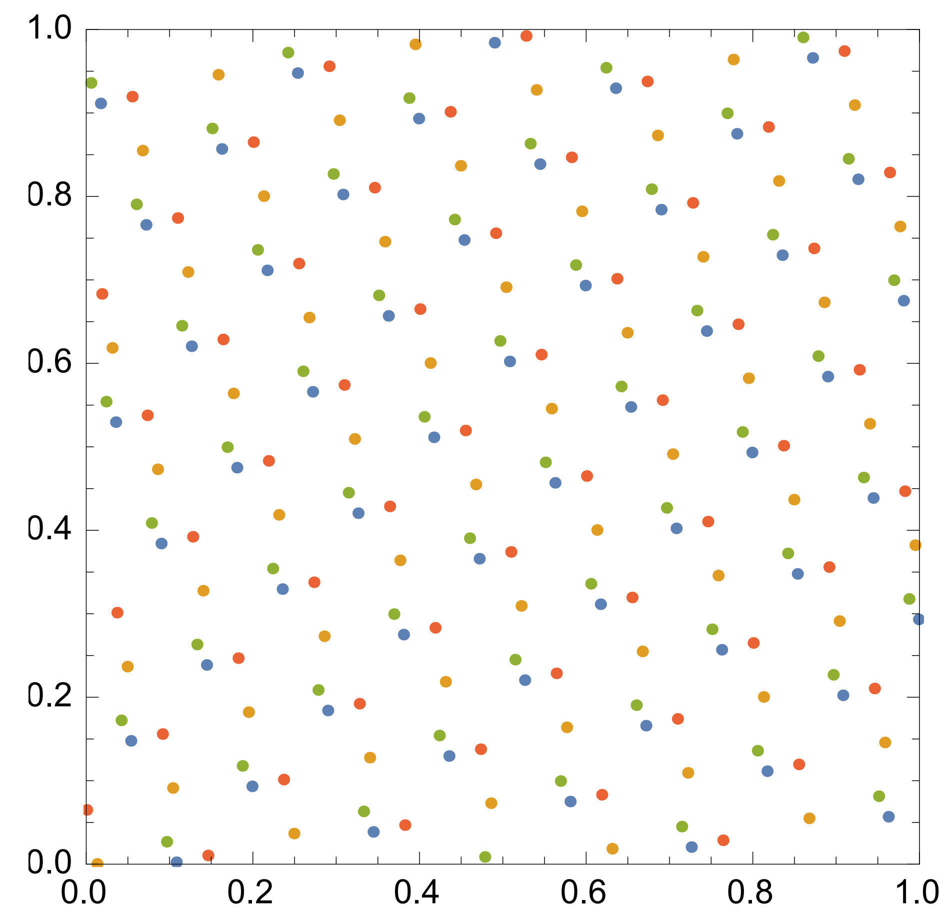
Rank 1 Shifted Lattices (II)

Unbiased error estimate computed from random shifts:

$$\mathrm{Var}[\bar{Q}_{s,n,m}[f]] \approx \frac{1}{m(m-1)} \sum_{k=1}^{m} (Q_{s,n,k} - \bar{Q}_{s,n,m})^2$$



1 Shift

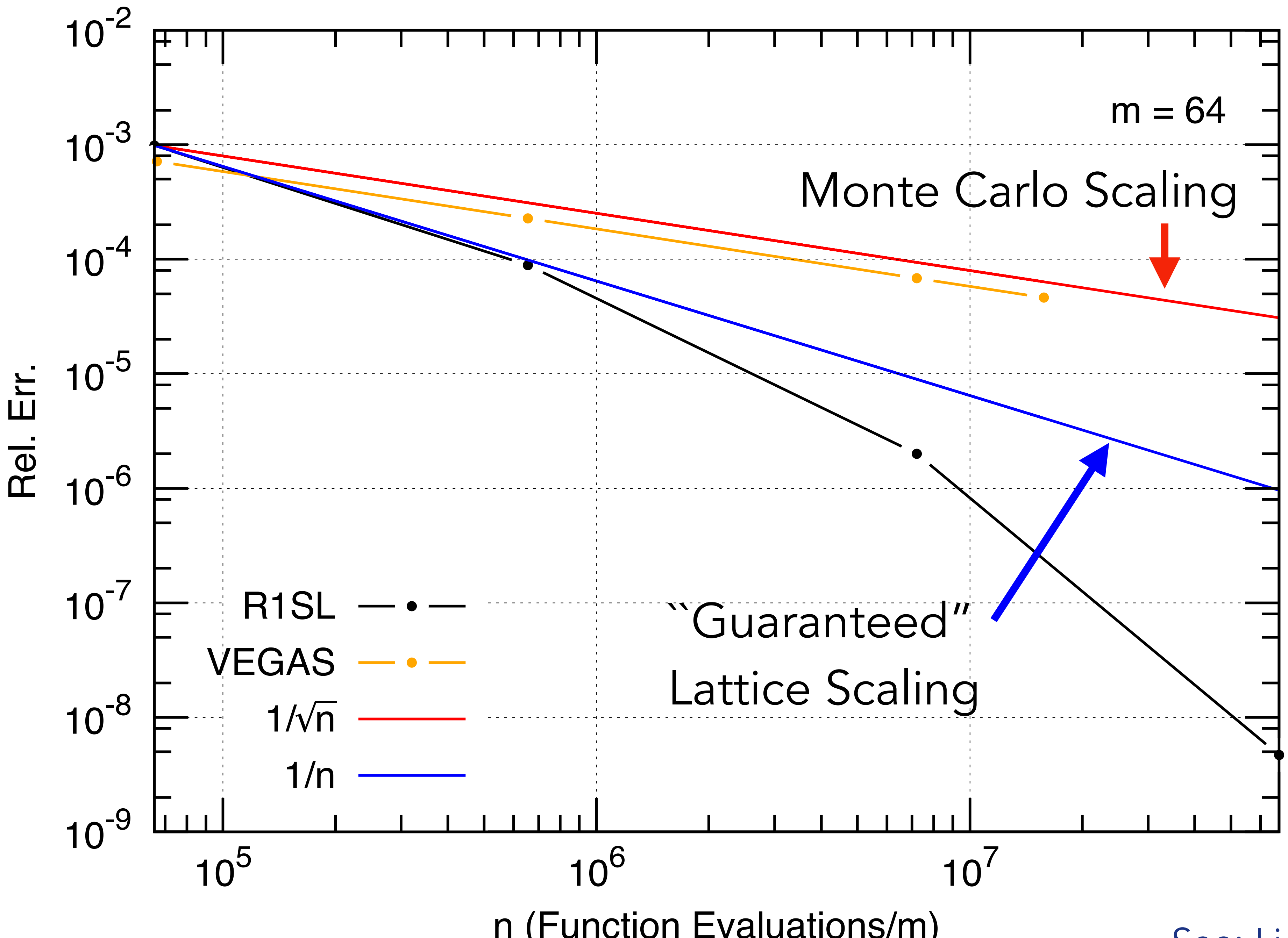


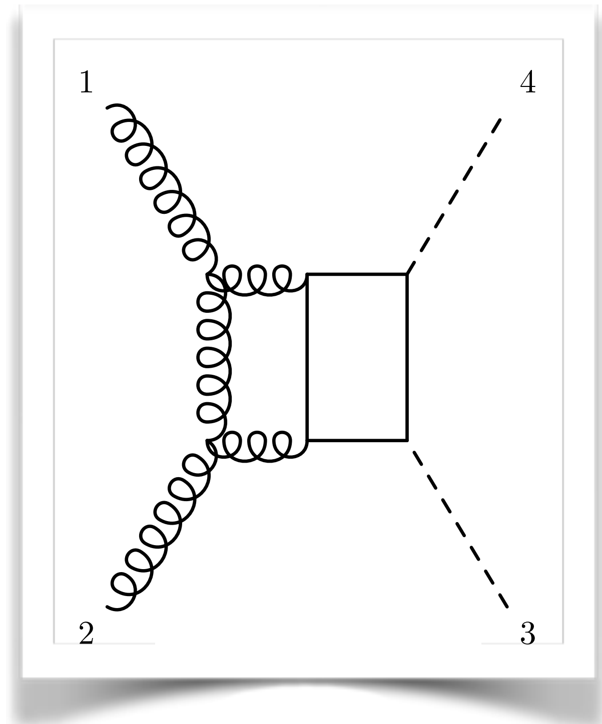
4 Shifts

Typically 10-50 shifts, production run: 20 shifts

R1SL: Algorithm Performance

Example: Rel. Err. of one sector of sector decomposed loop integral





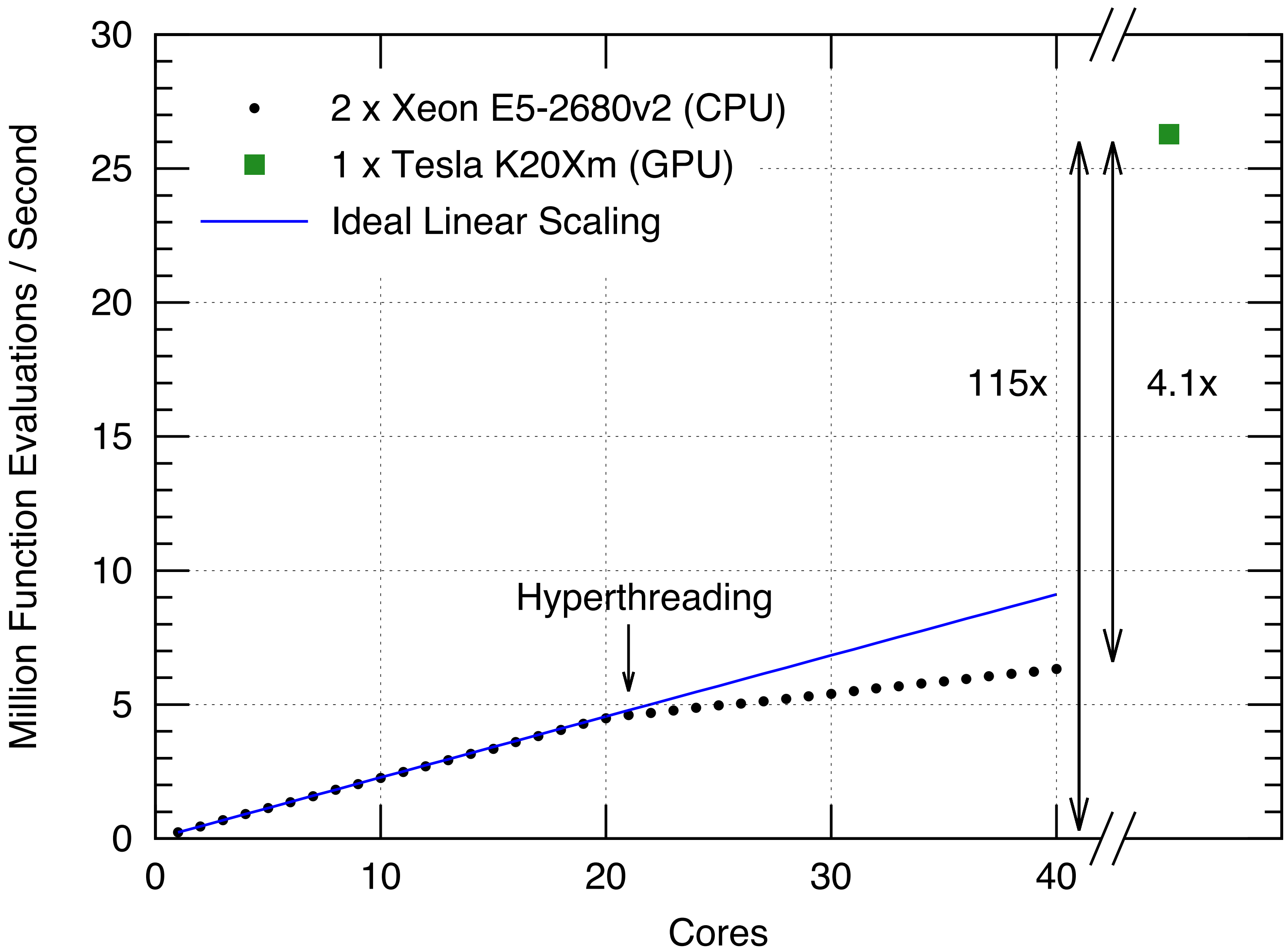
6 dimensional numerical integral

See: Li, Wang, Yan, Zhao 15

R1SL: Implementation Performance

Accuracy limited primarily by number of function evaluations

Implemented in OpenCL 1.1 for CPU & GPU, generate points on GPU/CPU core, sum blocks of points (reduce memory usage/transfers)



2 CPUs (20 Cores + HT)

1 GPU

n	CPU (s)	GPU(s)	C/G
655357	6.63	1.60	4.1
7208951	72.3	16.4	4.4
67264993	674.2	152.2	4.4

Amplitude Structure

Factor dimensionful parameter M^2 out of integrals s.t. they depend on dimensionless ratios:

#prop. in denom. **#prop. in num.**

$$I_{r,s}(\hat{s},\hat{t},m_h^2,m_t^2) = (M^2)^{-L\epsilon}(M^2)^{2L-r+s} I_{r,s}\left(\frac{\hat{s}}{M^2},\frac{\hat{t}}{M^2},\frac{m_h^2}{M^2},\frac{m_t^2}{M^2}\right)$$

$\overline{\mathrm{MS}}$ scheme strong coupling a and OS top-quark mass:

$$\mathrm{F} = a\mathrm{F}^{(1)} + a^2(\delta Z_A + \delta Z_a)\mathrm{F}^{(1)} + a^2 \delta m_t^2 \mathrm{F}^{ct,(1)} + a^2\mathrm{F}^{(2)} + O(a^3)$$

$$\mathrm{F}^{(1)} = \left(\frac{\mu_R^2}{M^2}\right)^{\epsilon}\left[b_0^{(1)} + b_1^{(1)}\epsilon + b_2^{(1)}\epsilon^2 + \mathcal{O}(\epsilon^3)\right]$$ ← 1-loop

$$\mathrm{F}^{ct,(1)} = \left(\frac{\mu_R^2}{M^2}\right)^{\epsilon}\left[c_0^{(1)} + c_1^{(1)}\epsilon + \mathcal{O}(\epsilon^2)\right]$$ ← Mass Counter-Terms

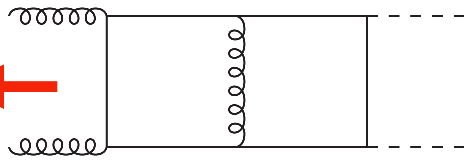
$$\mathrm{F}^{(2)} = \left(\frac{\mu_R^2}{M^2}\right)^{2\epsilon}\left[\frac{b_{-2}^{(2)}}{\epsilon^2} + \frac{b_{-1}^{(2)}}{\epsilon} + b_0^{(2)} + \mathcal{O}(\epsilon)\right]$$ ← 2-loop

Red terms contain integrals, computed numerically at each PS point, not re-evaluated for scale variations

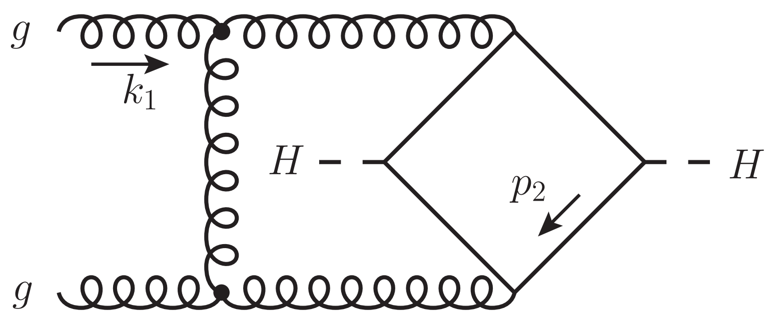
Amplitude Evaluation

Contributing integrals: $\sqrt{\hat{s}} = 327.25\,\text{GeV},\ \sqrt{-\hat{t}} = 170.05\,\text{GeV},\ M^2 = \hat{s}/4$

integral	value	error	time [s]
...			
F1_011111110_ord0	(0.484, 4.96e-05)	(4.40e-05, 4.23e-05)	11.8459
...			
N3_111111100_k1p2k2p2_ord0	(0.0929, -0.224)	(6.32e-05, 5.93e-05)	235.412
N3_111111100_1_ord0	(-0.0282, 0.179)	(8.01e-05, 9.18e-05)	265.896
N3_111111100_k1p2k1p2_ord0	(0.0245, 0.0888)	(5.06e-05, 5.31e-05)	282.794
N3_111111100_k1p2_ord0	(-0.00692, -0.108)	(3.05e-05, 3.05e-05)	433.342



$$I(s,t,m_t^2,m_h^2) = -\left(\frac{\mu^2}{M^2}\right)^{2\varepsilon}\Gamma(3+2\epsilon)M^{-4}\left(\frac{A_{-2}}{\epsilon^2}+\frac{A_{-1}}{\epsilon^1}+A_0+\mathcal{O}(\epsilon)\right)$$



Sector Decomposition

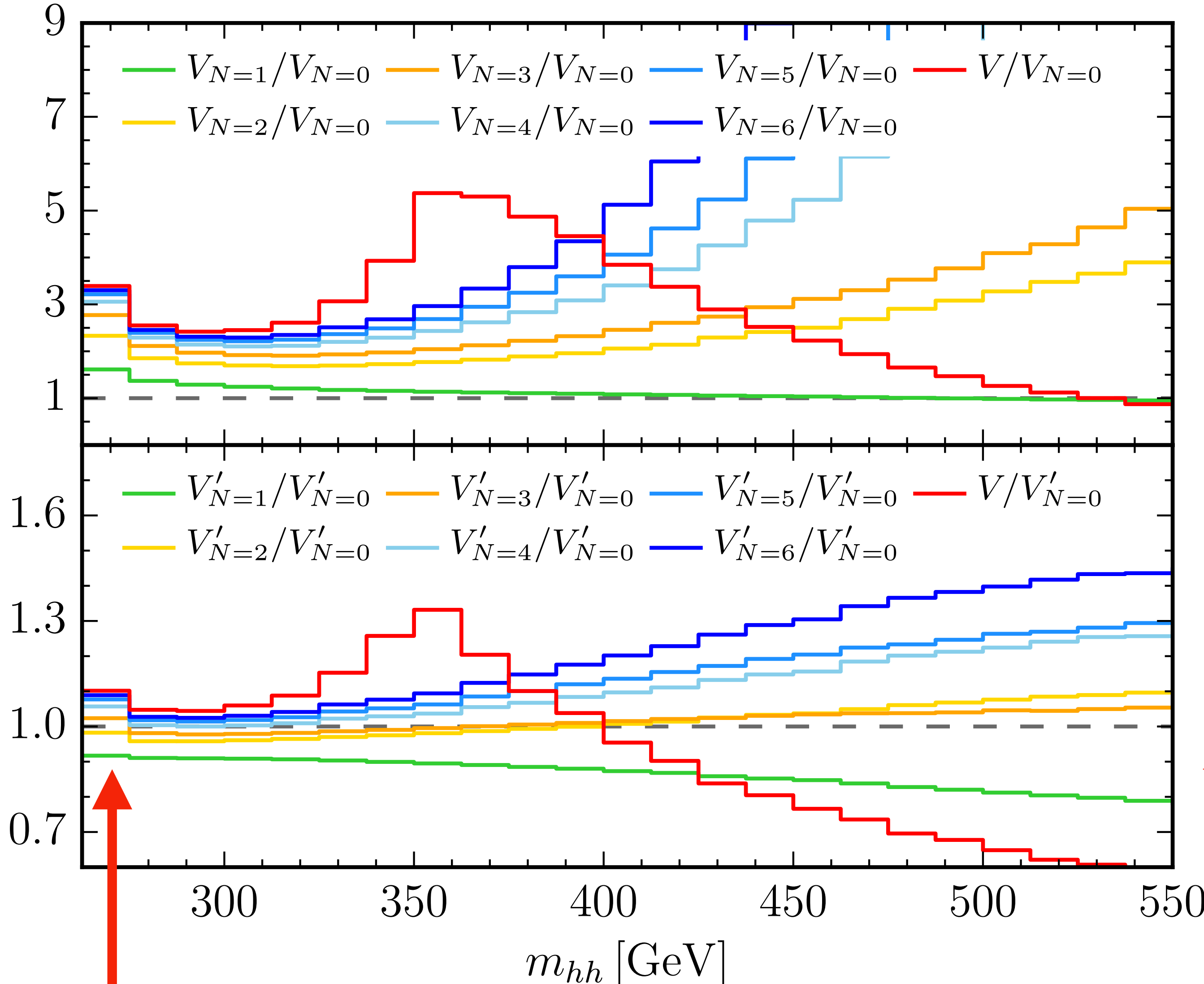
sector	integral value	error	time [s]	#points
5	(-1.34e-03, 2.00e-07)	(2.38e-07, 2.69e-07)	0.255	1310420
6	(-1.58e-03, -9.23e-05)	(7.44e-07, 5.34e-07)	0.266	1310420
...				
41	(0.179, -0.856)	(1.10e-05, 1.22e-05)	29.484	79952820
42	(0.359, -1.308)	(1.40e-06, 1.58e-06)	80.24	211436900
44	(0.0752, -1.185)	(5.44e-07, 6.76e-07)	99.301	282904860

Slide:
Matthias Kerner
(LL 2016)

Comparison to Expansion

Can compare just virtual ME to expansion:

$$d\hat{\sigma}_N = \sum_{\rho=0}^{N} d\hat{\sigma}^{(\rho)} \left(\frac{\Lambda}{m_t}\right)^{2\rho} \qquad \Lambda \in \left\{\sqrt{\hat{s}}, \sqrt{\hat{t}}, \sqrt{\hat{u}}, m_h\right\}$$



$$V_N = \left(d\hat{\sigma}_N^V + d\hat{\sigma}_N^{LO} \otimes \mathbf{I}\right)$$

$$V'_N = V_N \frac{d\hat{\sigma}^{LO}}{d\hat{\sigma}_N^{LO}}$$

Rescaled better but does not describe full above threshold

Expansion converges on full $\sqrt{\hat{s}} < 2m_T$

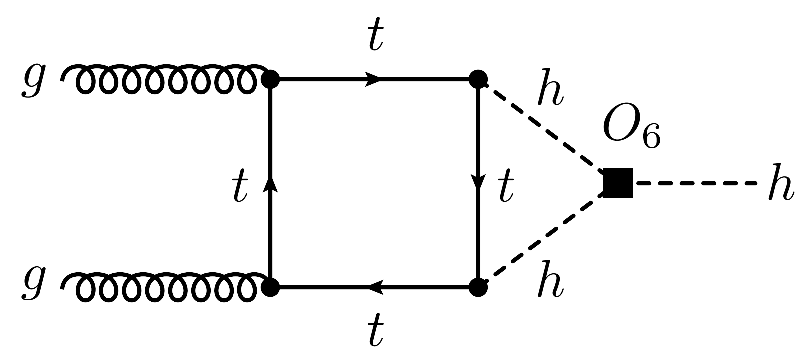
$V_{N\geq 4}$ thanks to J. Hoff

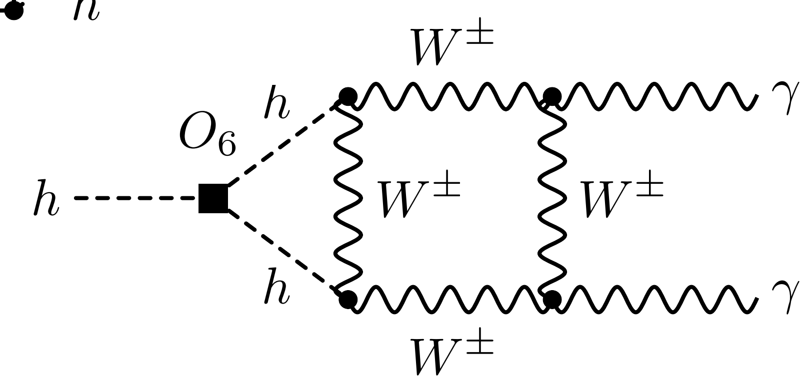
Grigo, Hoff, Steinhauser 15

Complementary Constraints

Several other promising ideas to obtain competitive/ complementary limits on deviation of self-coupling from SM:

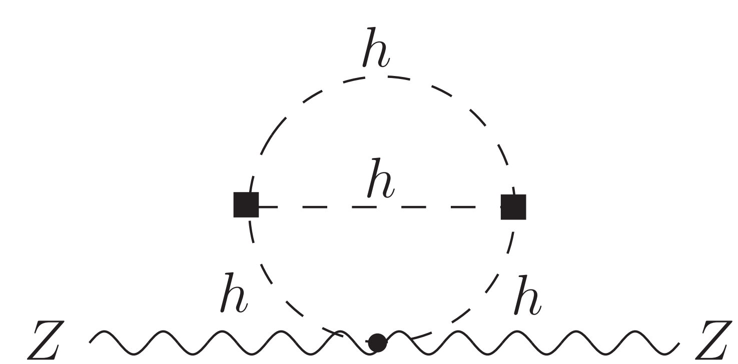
Radiative corrections to single H production (also VBF, VH, ttH, tHj)





Gorbahn, Haisch 16;
Bizoń, Gorbahn, Haisch, Zanderighi 16;
Degrassi, Giardino, Maltoni, Pagani 16;
Di Vita, Grojean, Panico, Riembau, Vantalon 17;
Maltoni, Pagani, Shivaji, Zhao 17;

Modification of precision EW observables (EW oblique corrections) S, T



Degrassi, Fedele, Giardino 17;
Kribs, Maier, Rzehak, Spannowsky, Waite 17;