Oslo School

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Multiparton interactions in QCD

[Double parton scattering (DPS)]

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MPI Workshops (2008, 2010-2016):

P. Bartalini *et al.*, **arXiv:1003.4220 [hep-ep].**

P. Bartalini *et al.*, **arXiv:1111.0469 [hep-ph].**

H. Abramowicz *et al.*, **arXiv:1306.5413 [hep-ph].**

S. Bansal *et al.*, **arXiv:1410.6664 [hep-ph].**

R. Astalos *et al.*, **arXiv:1506.05829 [hep-ph].**

mpi@lhc 2015, H. Jung *et al.*, **DESY-PROC-2016-01.**

mpi@lhc 2016 (Mexico), https://indico.nucleares.unam.mx/event/1100/

PARTON MODEL

Elastic scattering : electron — proton

——> proton (hadron) is NOT point-like

Deep inelastic scattering : electron — proton

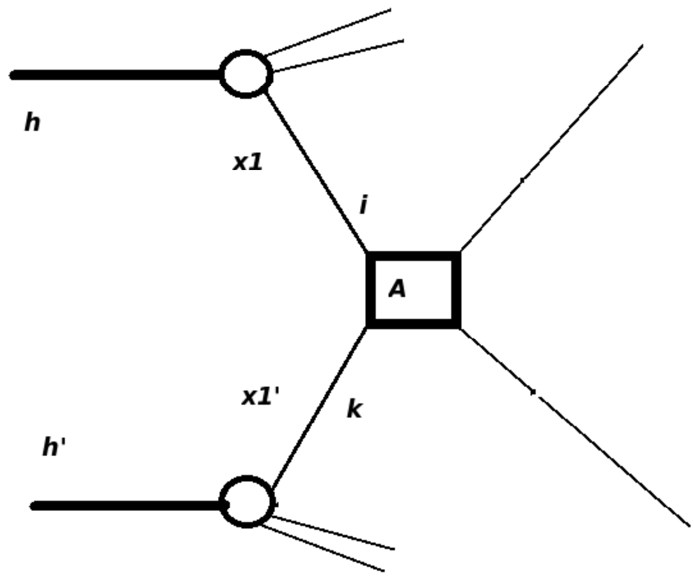
——> proton (hadron) consists of point-like particles-partons

Cross section (hadron) $=$ Σ cross section (parton) $\times$ weights

Weights — probabilities in the system of infinite momentum

(Bjorken, Feynman)

IN QCD weights depend on Q of hard processes
(SCALING VIOLATION, improved PM)



$$\sigma^A_{\rm SPS} = \sum_{i,k} \int D^i_h(x_1; Q^2_1) \hat{\sigma}^A_{ik}(x_1, x'_1) D^k_{h'}(x'_1; Q^2_1) dx_1 dx'_1$$

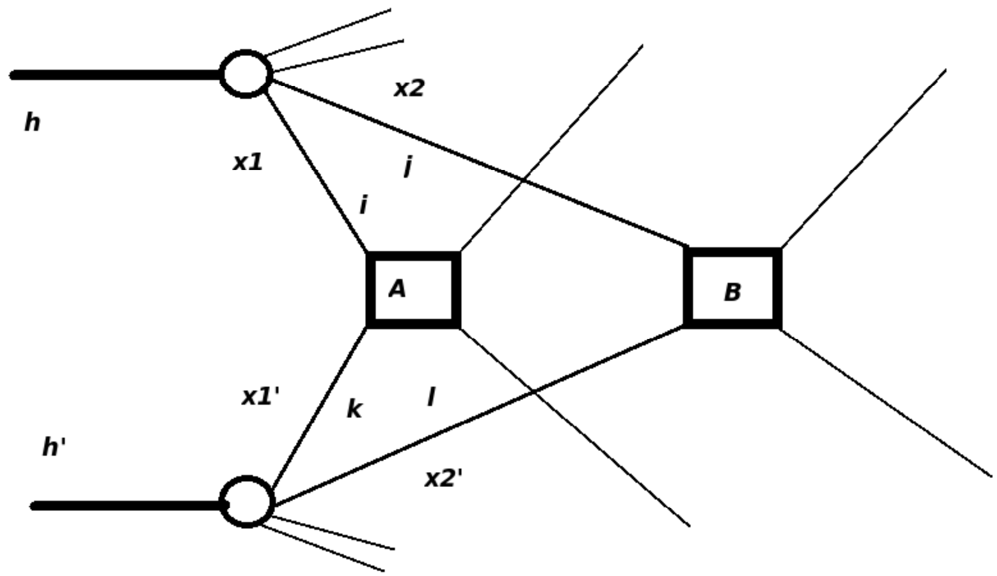
Scaling violation (dependence on Q) from
DGLAP (*Dokshitzer-Gribov-Lipatov-Altarelli-Parisi*) equations:

$$\frac{dD_i^j(x,t)}{dt} = \sum_{j'} \int_x^1 \frac{dx'}{x'} D_i^{j'}(x',t) P_{j' \to j}\left(\frac{x}{x'}\right)$$

$$t = \frac{1}{2\pi b} \ln\left[1 + \frac{g^2(\mu^2)}{4\pi} b \ln\left(\frac{Q^2}{\mu^2}\right)\right] = \frac{1}{2\pi b} \ln\left[\frac{\ln\left(\frac{Q^2}{\Lambda^2_{QCD}}\right)}{\ln\left(\frac{\mu^2}{\Lambda^2_{QCD}}\right)}\right], \quad b = \frac{33 - 2n_f}{12\pi},$$

where $g(\mu^2)$ is the running coupling constant at the reference scale μ^2,
n_f is the number of active flavours,
Λ_{QCD} is the dimensional QCD parameter.

It is possible (BUT very rarely): hard double parton scattering (subprocesses A and B)



The inclusive cross section of a double parton scattering process in a hadron collision is written in the following form (with only the assumption of factorization of the two hard parton subprocesses A and B)
(Paver, Treleani,..., Blok,...., Diehl,...).

$$\boldsymbol{\sigma^{AB}_{DPS} = \frac{m}{2} \sum_{i,j,k,l} \int \Gamma_{ij}(x_1, x_2; \mathrm{b}_1, \mathrm{b}_2; Q_1^2, Q_2^2) \hat{\sigma}^A_{ik}(x_1, x_1^{'}, Q_1^2) \hat{\sigma}^B_{jl}(x_2, x_2^{'}, Q_2^2)}$$

$$\boldsymbol{\times \Gamma_{kl}(x_1^{'}, x_2^{'}; \mathrm{b}_1 - \mathrm{b}, \mathrm{b}_2 - \mathrm{b}; Q_1^2, Q_2^2) dx_1 dx_2 dx_1^{'} dx_2^{'} d^2 b_1 d^2 b_2 d^2 b,}$$

where $\mathbf{b}$ is the impact parameter — the distance between centers of colliding (e.g., the beam and the target) hadrons in transverse plane.

$\Gamma_{ij}(x_1, x_2; \mathbf{b_1}, \mathbf{b_2}; Q_1^2, Q_2^2)$ are the double parton distribution functions, which depend on the longitudinal momentum fractions x_1 and x_2, and on the transverse position $\mathbf{b_1}$ and $\mathbf{b_2}$ of the two parton undergoing hard processes A and B at the scales Q_1 and Q_2.

$\hat{\sigma}^A_{ik}$ and $\hat{\sigma}^B_{jl}$ are the parton-level subprocess cross sections.

The factor $m/2$ appears due to the symmetry of the expression for interchanging parton species i and j. $m = 1$ if $A = B$, and $m = 2$ otherwise.

The double parton distribution functions $\Gamma_{ij}(x_1, x_2; \mathbf{b_1}, \mathbf{b_2}; Q_1^2, Q_2^2)$ are the main object of interest as concerns multiple parton interactions. In fact, these distributions contain all the information when probing the hadron in two different points simultaneously, through the hard processes A and B.

It is typically assumed that the double parton distribution functions may be decomposed in terms of longitudinal and transverse components as follows:

$$\Gamma_{ij}(x_1, x_2; \mathrm{b}_1, \mathrm{b}_2; Q_1^2, Q_2^2) = D_h^{ij}(x_1, x_2; Q_1^2, Q_2^2) f(\mathrm{b}_1) f(\mathrm{b}_2),$$

where $f(\mathbf{b_1})$ is supposed to be a universal function for all kinds of partons with the fixed normalization

$$\int f(\mathrm{b}_1) f(\mathrm{b}_1 - \mathrm{b}) d^2 b_1 d^2 b = \int T(\mathrm{b}) d^2 b = 1,$$

and

$$T(\mathbf{b}) = \int f(\mathbf{b_1}) f(\mathbf{b_1} - \mathbf{b}) d^2 b_1$$

is the overlap function (not calculated in pQCD).

If one makes the further assumption that the longitudinal components $D_h^{ij}(x_1, x_2; Q_1^2, Q_2^2)$ reduce to the product of two independent one parton distributions,

$$D_h^{ij}(x_1, x_2; Q_1^2, Q_2^2) = D_h^i(x_1; Q_1^2) D_h^j(x_2; Q_2^2),$$

the cross section of double parton scattering can be expressed in the simple form

$$\sigma_{\rm DPS}^{\rm AB} = \frac{m}{2} \frac{\sigma_{\rm SPS}^A \sigma_{\rm SPS}^B}{\sigma_{\rm eff}},$$

$$\pi R_{\rm eff}^2 = \sigma_{\rm eff} = [\int d^2b (T(\mathrm{b}))^2]^{-1}$$

is the effective interaction transverse area (effective cross section).
$R_{\rm eff}$ is an estimate of the size of the hadron.

The momentum (*instead of the mixed (momentum and coordinate)*) representation is more convenient sometimes:

$$\sigma^{AB}_{DPS} = \frac{m}{2} \sum_{i,j,k,l} \int \Gamma_{ij}(x_1, x_2; \mathrm{q}; Q_1^2, Q_2^2) \hat{\sigma}^{A}_{ik}(x_1, x_1^{'}) \hat{\sigma}^{B}_{jl}(x_2, x_2^{'})$$

$$\times \Gamma_{kl}(x_1^{'}, x_2^{'}; -\mathrm{q}; Q_1^2, Q_2^2) dx_1 dx_2 dx_1^{'} dx_2^{'} \frac{d^2 q}{(2\pi)^2}.$$

Here the transverse vector q is equal to the difference of the momenta of partons from the wave function of the colliding hadrons in the amplitude and the amplitude conjugated. Such dependence arises because the diffe - rence of parton transverse momenta within the parton pair is not conserved.

The main problems are

* to make the correct calculation of the two-parton functions $\Gamma_{ij}(x_1, x_2; \mathbf{q}; Q_1^2, Q_2^2)$ **WITHOUT simplifying factorization assumptions** (*which are not sufficiently justified and should be revised: (Blok, Dokshitzer, Frankfurt, Strikman; Diehl, Schafer; Gaunt, Stirling; Ryskin, Snigirev;...)*)

* to find (observe) longitudinal momentum parton correlations and deviation from the factorization form of DPS cross section.

These functions are available in the current literature only for $\mathbf{q} = 0$ in the collinear approximation. In this approximation the two-parton distribution functions, $\Gamma_{ij}(x_1, x_2; \mathbf{q} = 0; Q^2, Q^2) = D_h^{ij}(x_1, x_2; Q^2, Q^2)$ with the two hard scales set equal, satisfy the generalized DGLAP evolution equations *(Kirshner; Shelest, Snigirev, Zinovjev)*.

$$\frac{dD_i^{j_1 j_2}(x_1, x_2, t)}{dt} = \sum_{{j_1}'} \int_{x_1}^{1-x_2} \frac{d{x_1}'}{{x_1}'} D_i^{{j_1}' j_2}({x_1}', x_2, t) P_{{j_1}' \to j_1}\left(\frac{x_1}{{x_1}'}\right)$$

$$+ \sum_{{j_2}'} \int_{x_2}^{1-x_1} \frac{d{x_2}'}{{x_2}'} D_i^{j_1 {j_2}'}(x_1, {x_2}', t) P_{{j_2}' \to j_2}\left(\frac{x_2}{{x_2}'}\right)$$

$$+ \sum_{j'} D_i^{j'}(x_1 + x_2, t) \frac{1}{x_1 + x_2} P_{j' \to j_1 j_2}\left(\frac{x_1}{x_1 + x_2}\right)$$

The solutions of the generalized DGLAP evolution equations with the given initial conditions at the reference scales $\mu^2(t=0)$ may be written in the form:

$$D_h^{j_1 j_2}(x_1, x_2, t) \; = \; D_{h1}^{j_1 j_2}(x_1, x_2, t) \; + D_{h(QCD)}^{j_1 j_2}(x_1, x_2, t),$$

where

$$D_{h1}^{j_1 j_2}(x_1, x_2, t) =$$

$$\sum_{j_1' j_2'} \int_{x_1}^{1-x_2} \frac{dz_1}{z_1} \int_{x_2}^{1-z_1} \frac{dz_2}{z_2} \; D_h^{j_1' j_2'}(z_1, z_2, 0) D_{j_1'}^{j_1}(\frac{x_1}{z_1}, t) D_{j_2'}^{j_2}(\frac{x_2}{z_2}, t) \; ,$$

$$D_{h(QCD)}^{j_1 j_2}(x_1, x_2, t) =$$

$$\sum_{j' j_1' j_2'} \int_0^t dt' \int_{x_1}^{1-x_2} \frac{dz_1}{z_1} \int_{x_2}^{1-z_1} \frac{dz_2}{z_2} \; D_h^{j'}(z_1 + z_2, t')$$

$$\frac{1}{z_1 + z_2} P_{j' \to j_1' j_2'}(\frac{z_1}{z_1 + z_2}) D_{j_1'}^{j_1}(\frac{x_1}{z_1}, t - t') D_{j_2'}^{j_2}(\frac{x_2}{z_2}, t - t').$$

The **first term** is the solution of **homogeneous** evolution equation (**independent** evolution of two branches), where the **input two-parton** distribution is generally **NOT known** at the low scale $\mu(t=0)$. For this non-perturbative two-parton function at low z_1, z_2 one may **assume the factorization** $D_h^{j_1' j_2'}(z_1, z_2, 0) \simeq D_h^{j_1'}(z_1, 0) D_h^{j_2'}(z_2, 0)$ neglecting the constraints due to momentum conservation ($z_1 + z_2 < 1$).

This leads to

$$\boldsymbol{D_{h1}^{ij}(x_1, x_2, t) \simeq D_h^i(x_1, t) D_h^j(x_2, t)}$$

the factorization hypothesis usually used in current estimations.

This **MAIN** result shows that if the two-parton distributions are factorized at some scale μ^2, then the **evolution (second term) violates this factorization** *inevitably* **at any different scale ($Q^2 \neq \mu^2$)**, apart from the violation due to the kinematic correlations induced by the momentum conservation.

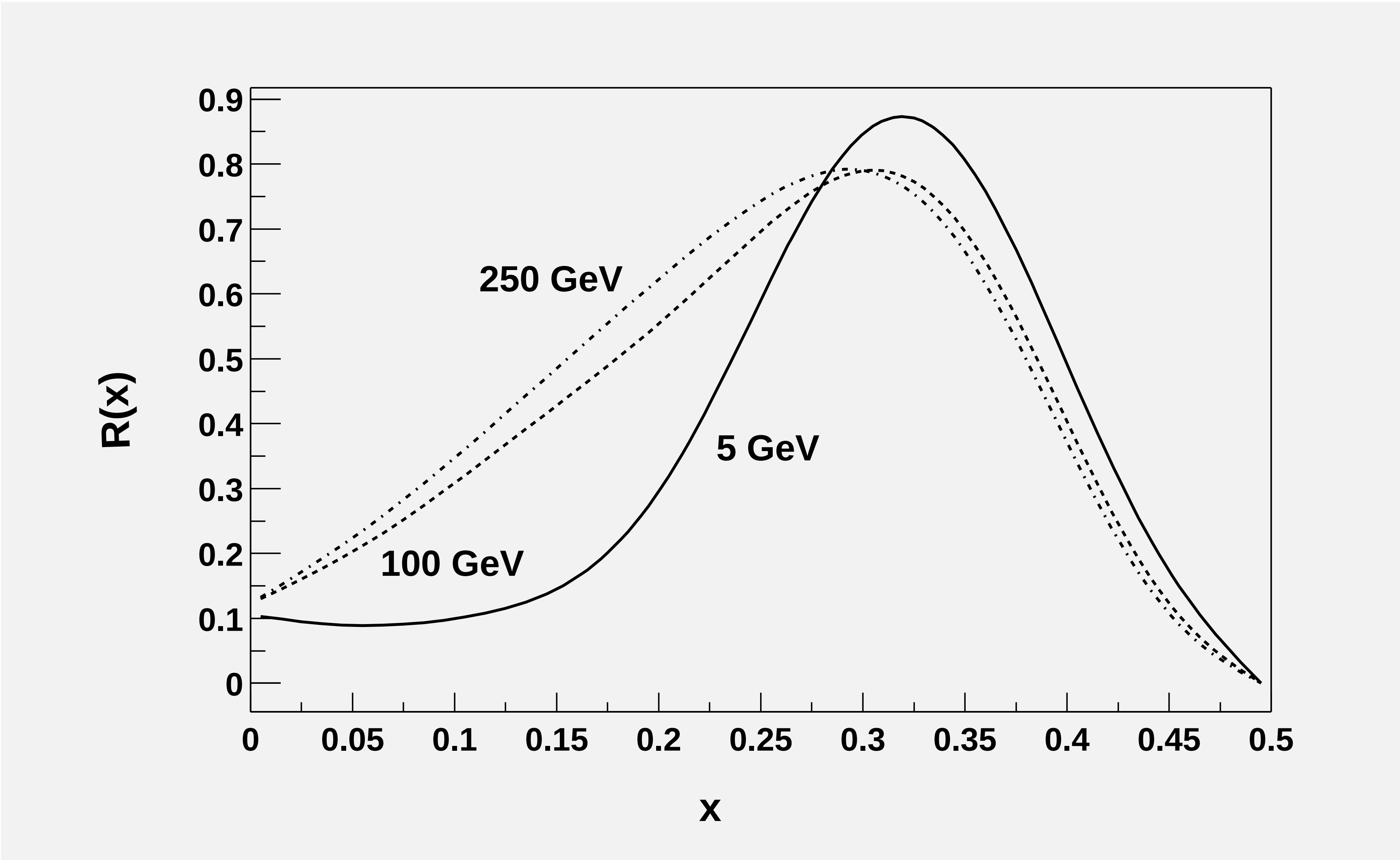
For a practical employment it is interesting to know the degree of this violation. We did *(Korotkikh, Snigirev)* it using the CTEQ fit for single distributions as an input. The nonperturbative initial conditions $D_h^j(x,0)$ are specified in a parametrized form at a fixed low-energy scale $Q_0 = \mu = 1.3$ GeV. The particular function forms and the value of Q_0 are not crucial for the CTEQ global analysis at the flexible enough parametrization, which reads

$$xD_p^j(x,0) \;=\; A_0^j x^{A_1^j}(1-x)^{A_2^j} e^{A_3^j x}(1+e^{A_4^j}x)^{A_5^j}.$$

The independent parameters $A_0^j,\ A_1^j,\ A_2^j,\ A_3^j,\ A_4^j,\ A_5^j$ for parton flavour combinations $u_v \equiv u-\bar{u}$, $d_v \equiv d-\bar{d}$, g and $\bar{u}+\bar{d}$ are given in Appendix A of work: *J.Pamplin, et al., JHEP 0207 (2002) 012.*

The results of numerical calculations are presented in Fig. for the ratio:

$$R(x,t) \;=\; (D^{gg}_{p(QCD)}(x_1,x_2,t)/D^g_p(x_1,t)D^g_p(x_2,t)(1-x_1-x_2)^2)|_{x_1=x_2=x}.$$



0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
R(x)
250 GeV
5 GeV
100 GeV
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
x

The evolution effects are getting larger with increasing hard scales. The numerical estimations by integrating directly the evolution equations (*Gaunt, Stirling; Diehl, Kasemets, Keane*) confirm also this conclusion.

The particular solutions of non-homogeneous equations contribute to the inclusive cross section of DPS with a larger weight (different effective cross section *(Cattaruzza,Del Fabbro,Treleani; Ryskin, Snigirev; Blok, Dokshitzer, Frankfurt, Strikman; Gaunt, Stirling)*) as compared to the solutions of homogeneous equations (the “traditional” factorization component).

The latter solutions are usually approximated by a factorized form if the initial nonperturbative correlations are absent. These initial correlation conditions are *a priori* unknown yet not quite arbitrary as they obey the nontrivial sum rules which are imposed upon the evolution equations. The problem of specifying the initial correlation conditions for the evolution equations, which would obey exactly these sum rules and have the correct asymptotic behavior near the kinematical boundaries, has been extensively studied *(Gaunt, Stirling; Snigirev; Ceccopieri; Chang, Manohar, Waalewijn; Rinaldi, Scopetta,Vento; Golec-Biernat, Lewandowska)*.

The experimental effective cross section, $\sigma_{\rm eff}^{\rm exp}$, which is not measured directly but is extracted by means of the normalization to the product of two single cross sections:

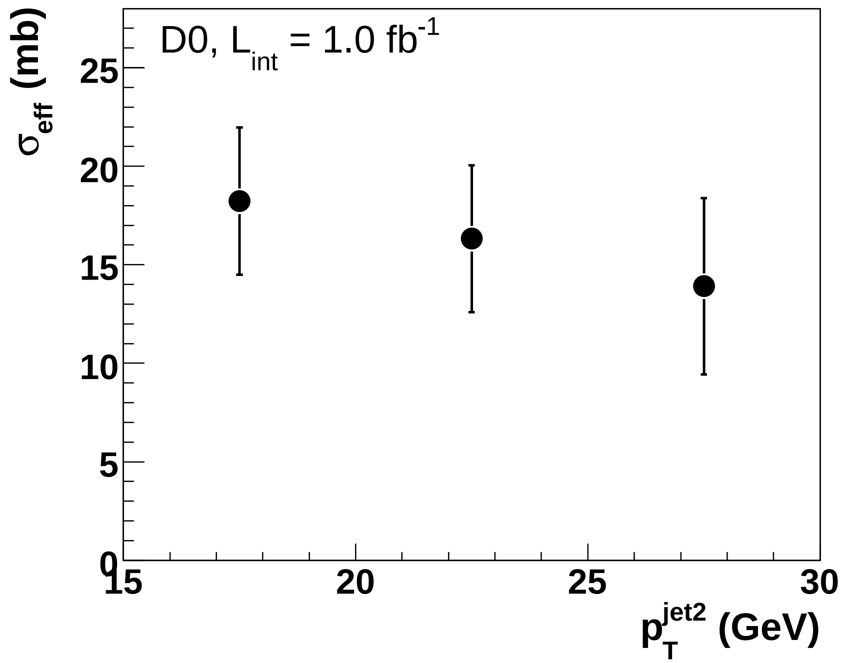
$$\frac{\boldsymbol{\sigma}_{DPS}^{\gamma+3j}}{\boldsymbol{\sigma}^{\gamma j}\boldsymbol{\sigma}^{jj}} = [\boldsymbol{\sigma}_{\rm eff}^{\rm exp}]^{-1},$$

appears to be dependent on the probing hard scale. It should DECREASE with increasing the resolution scale because all additional contributions to the cross section of double parton scattering are positive and increase.

In the above formula, $\sigma^{\gamma j}$ and σ^{jj} are the inclusive $\gamma +$ jet and dijets cross sections, $\sigma_{DPS}^{\gamma+3j}$ is the inclusive cross section of the $\gamma+3$ jets events produced in the double parton process.

It is worth noticing that the CDF and D0 Collaborations extract $\sigma_{\rm eff}^{\rm exp}$ without any theoretical predictions on the $\gamma +$ jet and dijets cross sections, by comparing the number of observed double parton $\gamma + 3$ jets events in ONE $p\bar{p}$ collision to the number of $\gamma +$ jet and dijets events occurring in TWO separate $p\bar{p}$ collisions.

The recent D0 measurements represent this effective cross section, $\sigma_{\rm eff}^{\rm exp}$, as a function of the second (ordered in the transverse momentum, p_T) jet p_T, $p_T^{\rm jet2}$, which can serve as a resolution scale. The obtained cross sections reveal a tendency to be dependent on this scale.



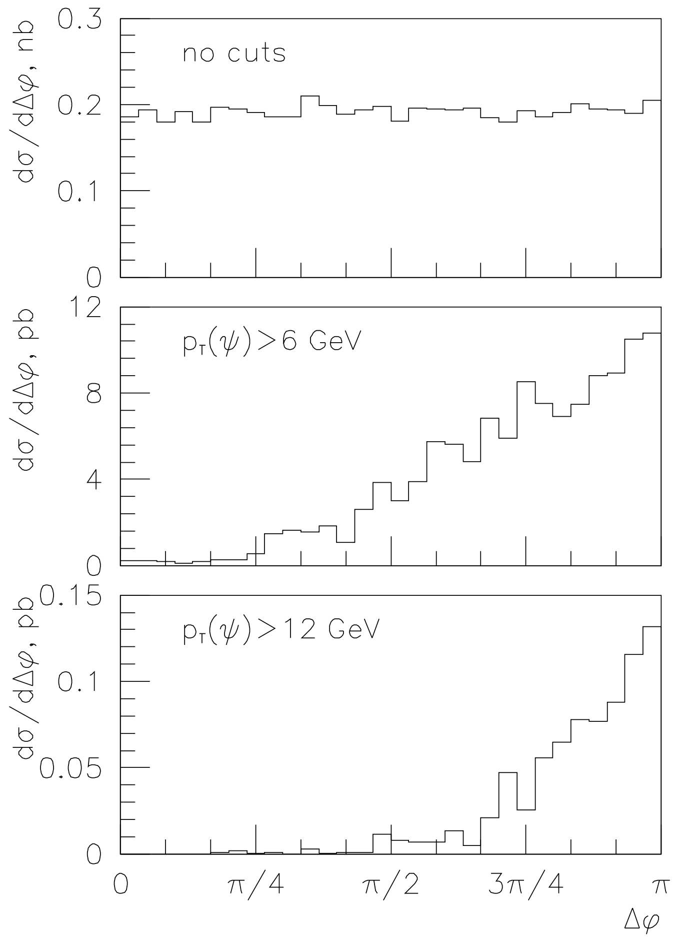
This observation can be interpreted as the first indication to the QCD evolution of double parton distributions
(*Snigirev; Flensburg, Gustafson, Lonnblad, Ster*).

Promising candidate processes to probe DPS at the LHC:

- same-sign W production (“pure”, BUT very rare)
- γ + 3 jets (Tevatron also: D0, CDF)
- $W(Z)$ + 2 jets (ATLAS — first measurement σ_{eff} at LHC)
- 4 jets (Tevatron also: CDF)
- $b\bar{b}$ pair +2 jets
- $b\bar{b}$ pair + W boson
- pairs of heavy mesons (in particular, double J/ψ production) (LHCb — first measurement of double J/ψ production)

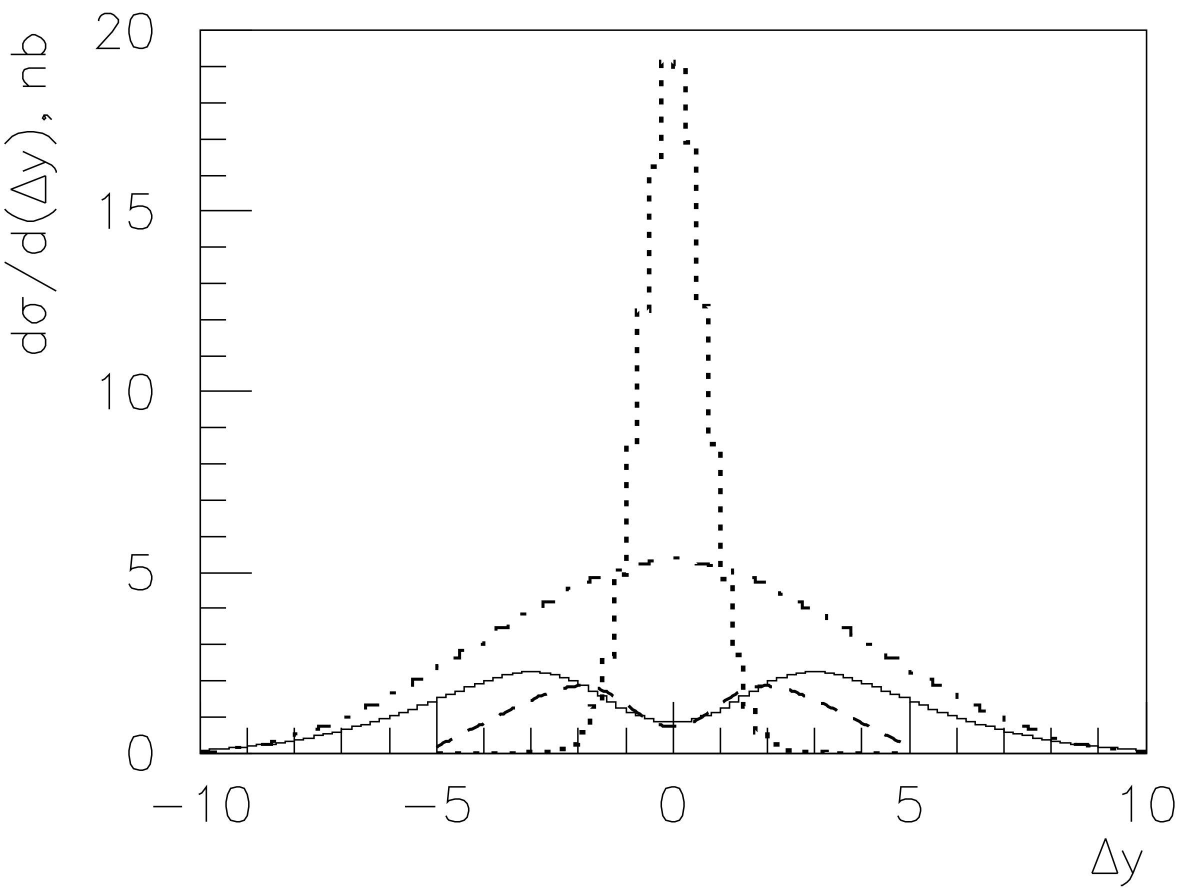
J/ψ pairs production

Azimuthal angle difference distribution after imposing cuts on the J/ψ transverse momenta for SPS



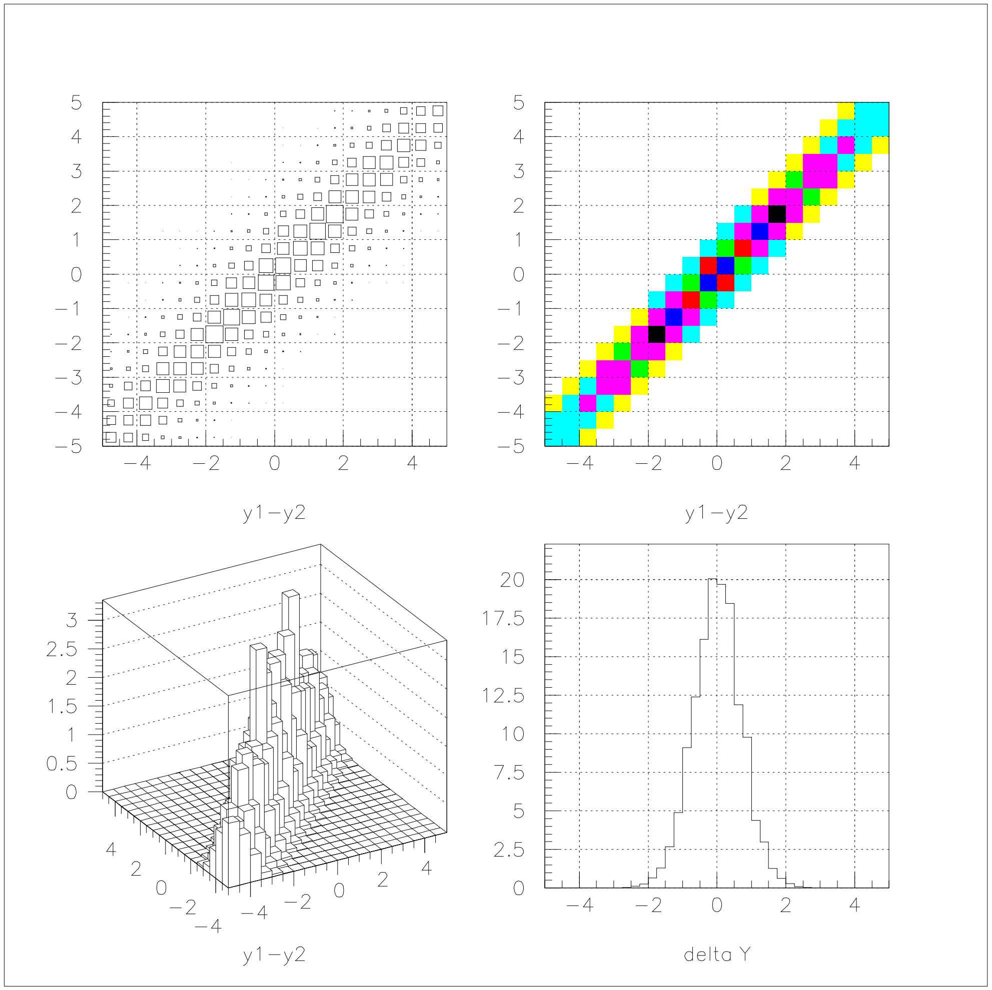
It is rather difficult to disentangle the SPS and DPS (flat) modes: the difference becomes visible only at sufficiently high cuts, where the production rates are, indeed, very small.

Distribution over the rapidity difference between J/ψ mesons. (Dotted curve: leading-order SPS, dash-dotted curve: DPS)



Selecting large rapidity difference events looks more promising to disentangle the SPS and DPS modes

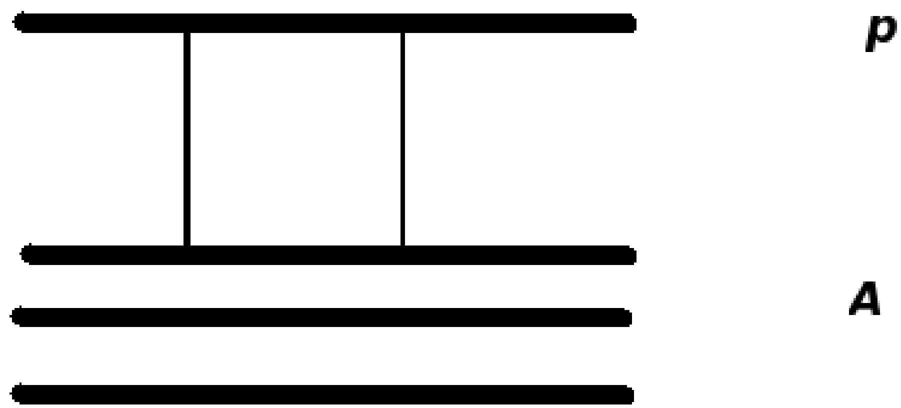
Double differential distribution for the leading-order SPS production mode



DPS in pA

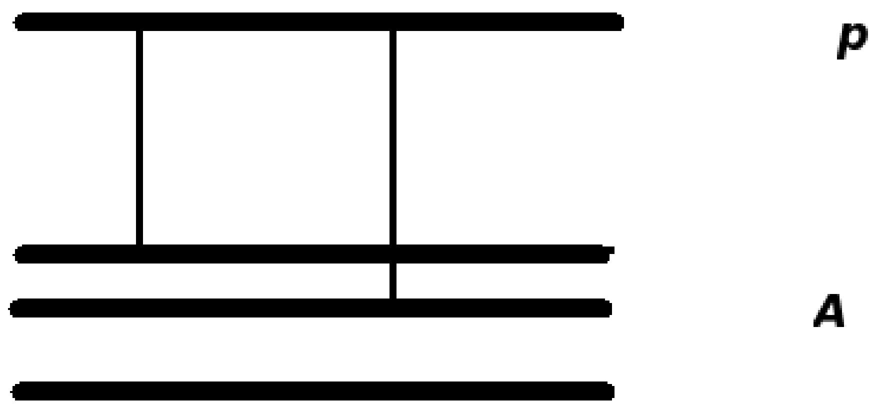
(Strikman, Treleani; Blok, Strikman, Wiedemann; d'Enterria, Snigirev,.....) **:**

1. The two partons of the nucleus belong to the same nucleon



Nuclear enhancement factor A **as for SPS**

2. The two partons of the nucleus belong to the different nucleons



Nuclear enhancement factor: $\propto A^2/A^{2/3} = A^{1+1/3}$
($A^{2/3}$ due to the difference of the transverse sizes between p and A)

The final DPS cross section "pocket formula" in pA collisions:

$$\boldsymbol{\sigma^{\rm DPS}_{(pA\to ab)} = \left(\frac{m}{2}\right)\frac{\sigma^{\rm SPS}_{(NN\to a)}\cdot\sigma^{\rm SPS}_{(NN\to b)}}{\sigma_{\rm eff,pA}}},$$

where

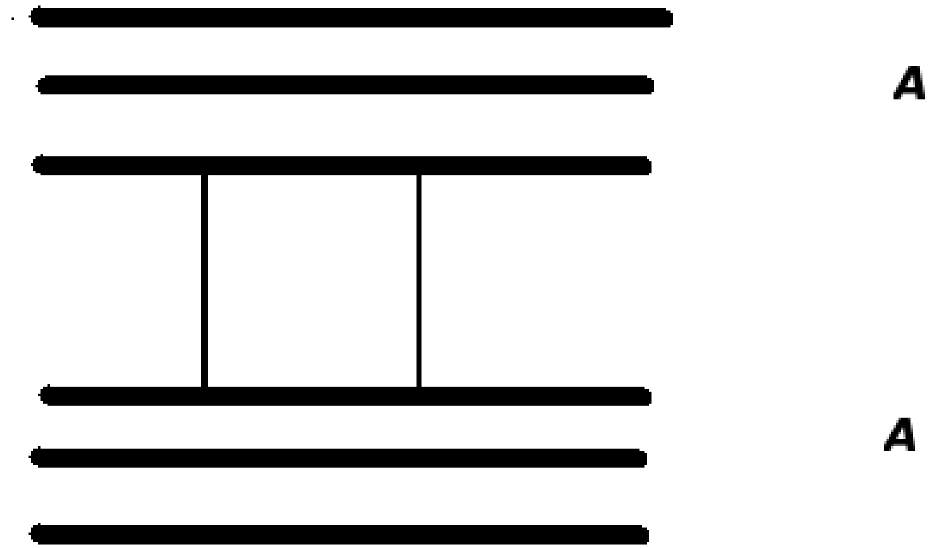
$$\boldsymbol{\sigma_{\rm eff,pA} = \frac{1}{A\left[\sigma^{-1}_{\rm eff,pp}+\frac{1}{A}\,{\rm T}_{\rm AA}(0)\right]} = 21.5\mu{\rm b}}$$

for p-Pb at $\sigma_{\rm eff,pp}$ = 14 mb and $\mathrm{T_{AA}}(0)$ = 30.4 1/mb for the standard nuclear overlap function normalized to A^2.

The relative contribution of the two terms are approximately 1 : 2

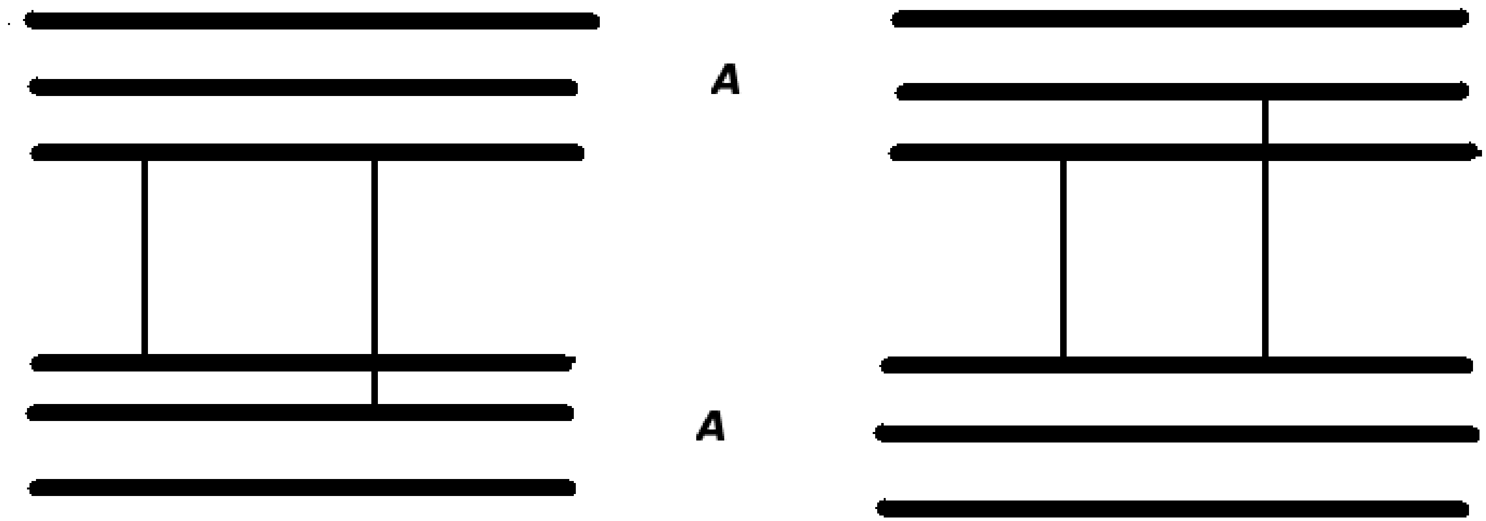
DPS in AA :

1. The two colliding partons belong to the same pair of nucleons



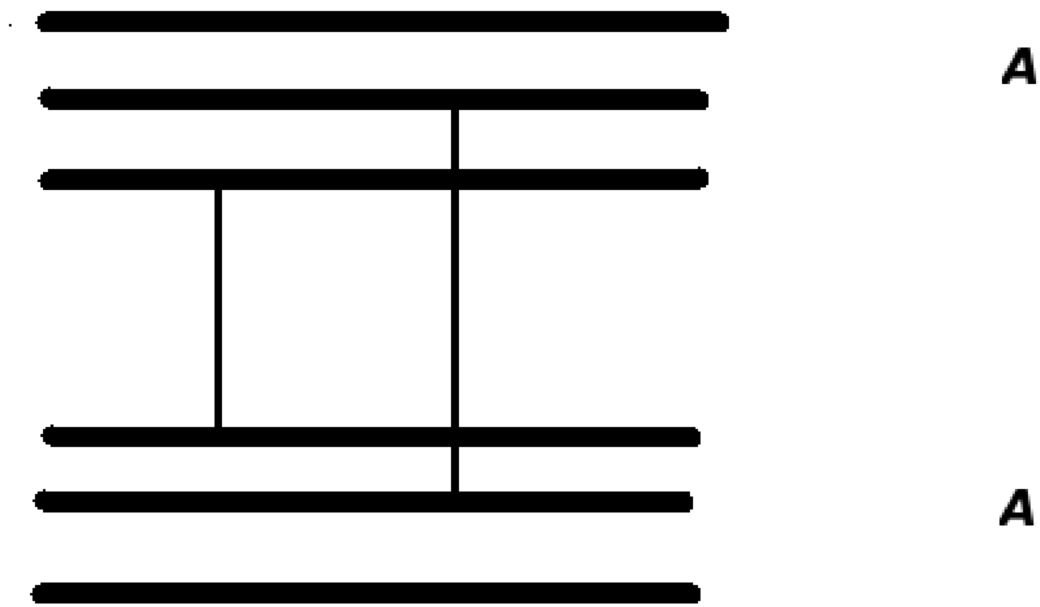
Nuclear enhancement factor A^2 as for SPS

2. Partons from one nucleon in one nucleus collide with partons from two different nucleons in the other nucleus



Nuclear enhancement factor: $\propto A^3/A^{2/3} = A^{2+1/3}$

3. The two colliding partons belong to two different nucleons from both nuclei (in fact, double nucleon scattering)



Nuclear enhancement factor: $\propto A^4/A^{2/3} = A^{2+4/3}$

The final DPS cross section "pocket formula" in AA collisions:

$$\boldsymbol{\sigma^{\mathrm{DPS}}_{(AA\to ab)} = \left(\frac{m}{2}\right) \frac{\sigma^{\mathrm{SPS}}_{(NN\to a)} \cdot \sigma^{\mathrm{SPS}}_{(NN\to b)}}{\sigma_{\mathrm{eff,AA}}}},$$

where

$$\boldsymbol{\sigma_{\mathrm{eff,AA}} = \frac{1}{A^2\left[\sigma^{-1}_{\mathrm{eff,pp}} + \frac{2}{A}\,\mathrm{T_{AA}}(0) + \frac{1}{2}\,\mathrm{T_{AA}}(0)\right]} = 1.5\ \mathrm{nb}}$$

for Pb-Pb at $\sigma_{\mathrm{eff,pp}} = 14$ mb and $\mathrm{T_{AA}}(0) = 30.4$ 1/mb for the standard nuclear overlap function normalized to A^2.

The relative contribution of the three terms are approximately 1 : 4 : 200

Centrality-dependence of the DPS

The cross section for SPS and DPS an interval of impact parameters $[b_1, b_2]$, corresponding a given centrality percentile, $f_{\%} = 0 - 100\%$, of the total A-A cross section σ_{AA}, with average overlap function $< T_{AA}[b_1, b_2] >$ are

$$\boldsymbol{\sigma^{\rm SPS}_{(AA\to ab)}[b_1, b_2] = A^2 \cdot \sigma^{\rm SPS}_{(NN\to ab)} \cdot f_1[b_1, b_2]}$$

$$\boldsymbol{= \sigma^{\rm SPS}_{(NN\to ab)} f_{\%} \sigma_{AA} < T_{AA}[b_1, b_2] >,}$$

$$\boldsymbol{\sigma^{\rm DPS}_{(AA\to ab)}[b_1, b_2] = A^2 \cdot \sigma^{\rm DPS}_{(NN\to ab)} \cdot f_1[b_1, b_2]}$$

$$\boldsymbol{\times \left[1 + \frac{2}{A} \sigma_{eff,pp} \, \mathrm{T_{AA}}(0) \, \frac{\mathrm{f_2[b_1, b_2]}}{\mathrm{f_1[b_1, b_2]}} + \sigma_{\rm eff,pp} \, \mathrm{T_{AA}}(0) \, \frac{\mathrm{f_3[b_1, b_2]}}{\mathrm{f_1[b_1, b_2]}}\right],}$$

the three dimensionless and appropriately-normalized fractions read

$$f_1[b_1, b_2] = \frac{2\pi}{A^2} \int_{b_1}^{b_2} b db \, \mathrm{T_{AA}(b)} = \frac{\mathrm{f}_{\%} \sigma_{\mathrm{AA}}}{\mathrm{A}^2} < \mathrm{T_{AA}[b_1, b_2]} >,$$

$$f_2[b_1, b_2] = \frac{2\pi}{A \, \mathrm{T_{AA}(0)}} \int_{b_1}^{b_2} b db \int d^2 b_1 \, \mathrm{T_A(b_1) T_A(b_1 - b) T_A(b_1 - b)},$$

$$f_3[b_1, b_2] = \frac{2\pi}{A^2 \, \mathrm{T_{AA}(0)}} \int_{b_1}^{b_2} b db \, \mathrm{T^2_{AA}(b)}.$$

For not very peripheral collisions ($f_{\%} < 0 - 65\%$) DPS cross section (in a thin impact-parameter range) can be approximated by third dominant term

$$\boldsymbol{\sigma^{\rm DPS}_{(AA\to ab)}[b_1,b_2] \simeq \sigma^{\rm DPS}_{(NN\to ab)} \cdot \sigma_{eff,pp} \cdot f_{\%}\sigma_{AA} \cdot < T_{AA}[b_1,b_2] >^2}$$

$$\boldsymbol{= \frac{m}{2}\sigma^{\rm SPS}_{(NN\to a)} \cdot \sigma^{\rm SPS}_{(NN\to b)} \cdot f_{\%}\sigma_{AA} \cdot < T_{AA}[b_1,b_2] >^2 \; .}$$

For ratio

$$\boldsymbol{\frac{\sigma^{\rm DPS}_{(AA\to ab)}[b_1,b_2]}{\sigma^{\rm SPS}_{(AA\to a)}[b_1,b_2]} \simeq \frac{m}{2}\sigma^{\rm SPS}_{(NN\to b)} \cdot < T_{AA}[b_1,b_2] > \; .}$$

In the centrality percentile $f_{\%} \simeq 65 - 100\%$ the second term would add about 20% more DPS cross section.

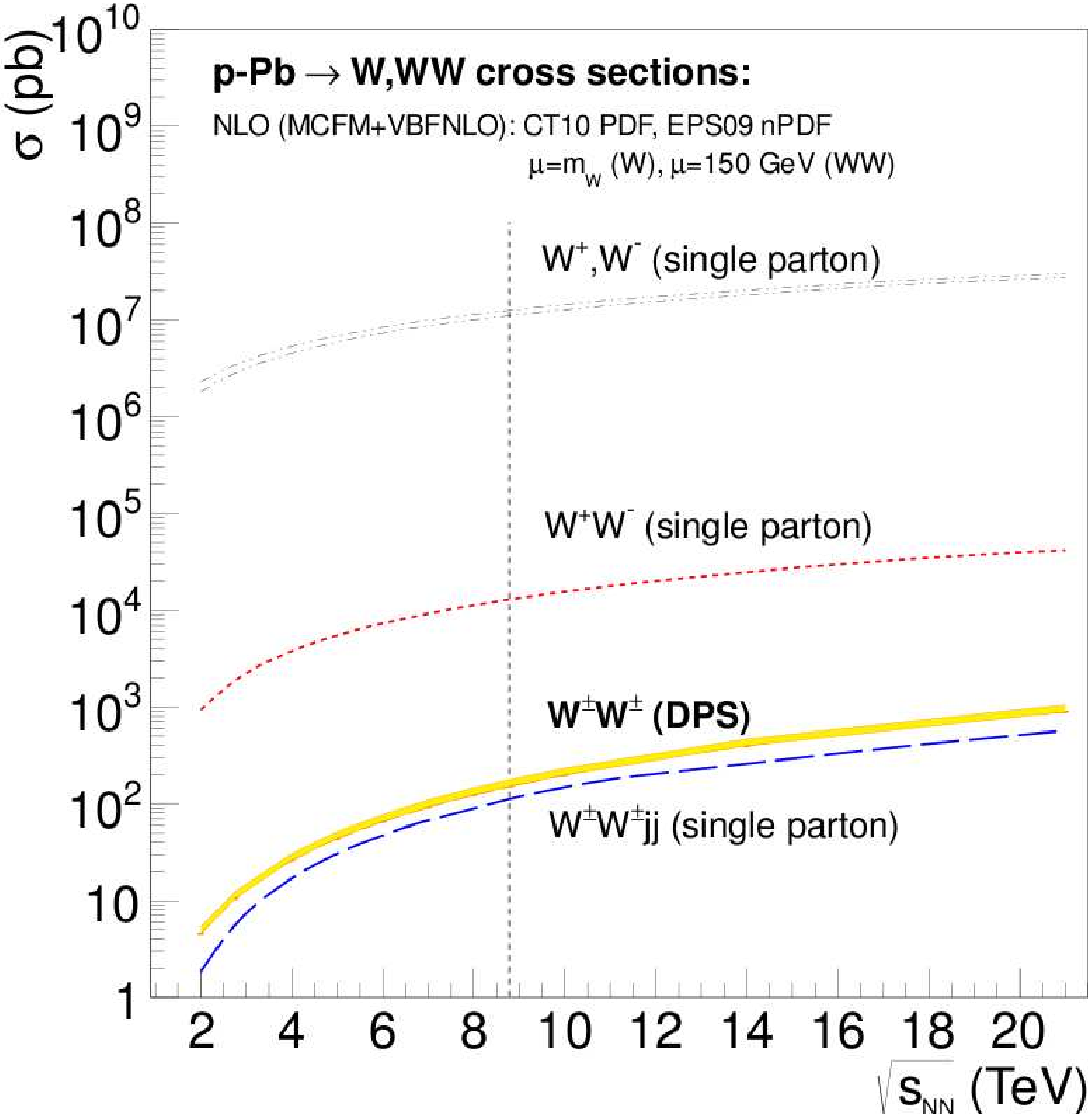
For very peripherical collisions ($f_{\%} \simeq 85 - 100\%$, where $< T_{AA}[b_1, b_2] >$ is order or less than $1/\sigma_{eff,pp}$) the contributions from the first term are also non-negligible (dominant in the limit $1/b \to 0$).

The formalism of DPS was applied to study:

same-sign W-boson pair production in pPb collisions at LHC energies

J/ψ-pair production in Pb-Pb collisions at LHC energies

Specification in calculations, results and plots
— in original papers (+ nice presentations (*d'Enterria*) on Hard Probes 2013, Quark Matter 2014)



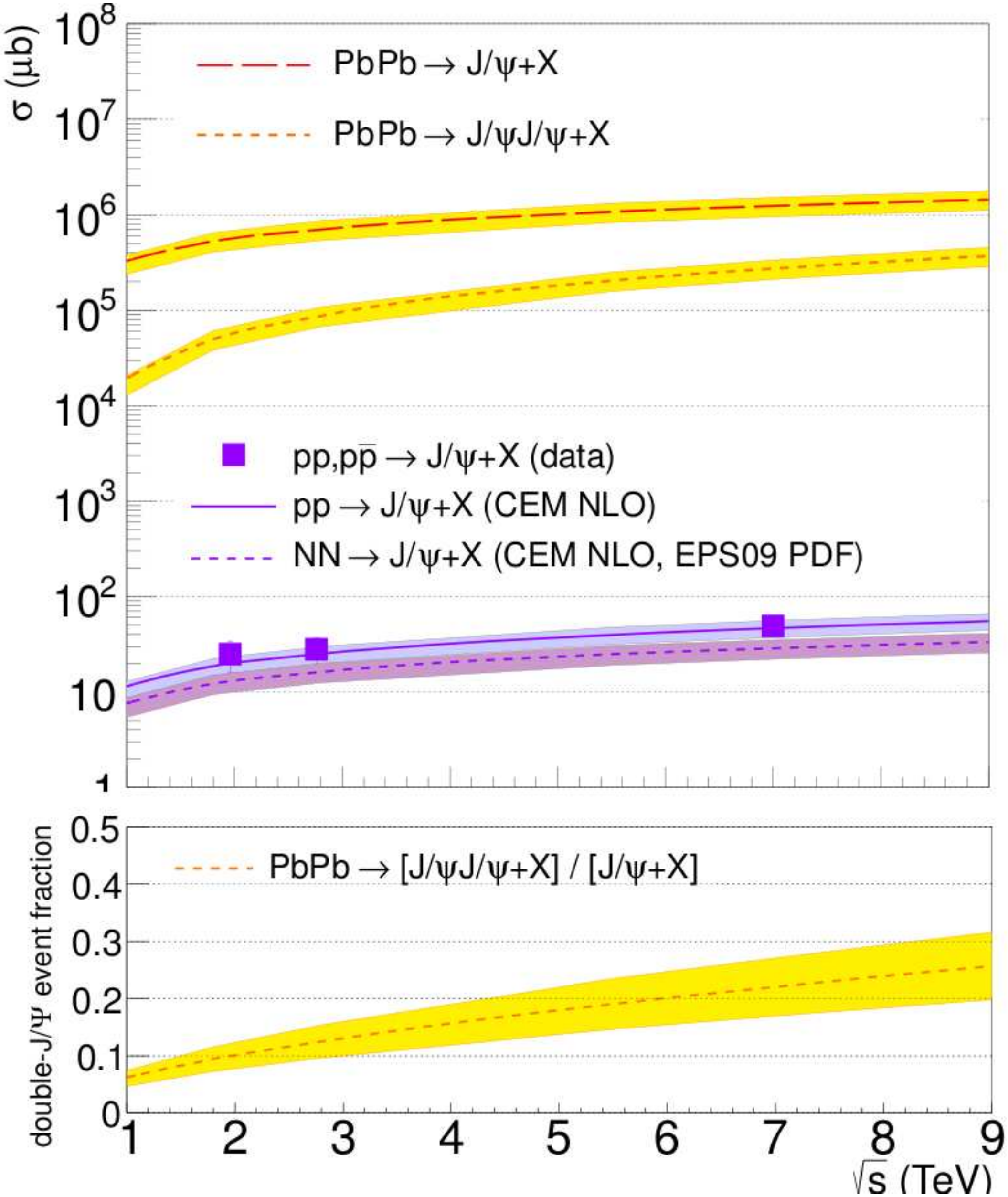
p-Pb → W,WW cross sections:
NLO (MCFM+VBFNLO): CT10 PDF, EPS09 nPDF
μ=m_W (W), μ=150 GeV (WW)
W+,W- (single parton)
W+W- (single parton)
W±W± (DPS)
W±W±jj (single parton)
σ (pb)
√s_NN (TeV)

Only main conclusions

p-Pb collisions:

* At the nominal $\sqrt{s_{NN}} = 8.8$ TeV energy, the DPS cross section for like-sign WW production is about 150 pb, i.e. 600 times larger than that in proton-proton collisions at the same c.m. energy and 1.5 times higher than the same-sign WW+2-jets background.

* The measurement of such a process, where 10 events with fully leptonic W's decays are expected after cuts in 2 pb^{-1}, would constitute an unambiguous DPS signal at the LHC, and would help determine the effective σ_{eff} parameter characterizing the area of double parton interactions in hadronic collisions.



σ (μb)
PbPb → J/ψ+X
PbPb → J/ψJ/ψ+X
pp,pp̄ → J/ψ+X (data)
pp → J/ψ+X (CEM NLO)
NN → J/ψ+X (CEM NLO, EPS09 PDF)
PbPb → [J/ψJ/ψ+X] / [J/ψ+X]
double-J/Ψ event fraction
√s (TeV)

Pb-Pb collisions:

* DPS constitute an important fraction of the total prompt-J/ψ cross sections, amounting to 20 % (35%) of the primordial production in minimum-bias (most central) Pb-Pb collisions.

* At 5.5 TeV, about 240 double-J/ψ events are expected per unit rapidity in the dilepton decay channels (in the absence of final-state suppression) for an integrated luminosity of 1 nb^{-1}, providing interesting insights on the event-by-event dynamics of J/ψ production in Pb-Pb collisions.

DPS production cross sections of

double-J/ψ, $J/\psi+\Upsilon$, J/ψ+W, J/ψ+Z,

double-Υ, Υ+W, Υ+Z, and same-sign WW

in Pb-Pb and p-Pb at the LHC:

System		$J/\psi + J/\psi$	$J/\psi + \Upsilon$	J/ψ+W	J/ψ+Z	$\Upsilon + \Upsilon$	Υ+W	Υ+Z	ss WW
Pb-Pb	σ^{DPS}	210 mb	28 mb	500 μb	330 μb	960 μb	34 μb	23 μb	630 nb
5.5 TeV	$\mathrm{N}^{\mathrm{DPS}}$ (1 nb^{-1})	∼250	∼340	∼65	∼14	∼95	∼35	∼8	∼15
p-Pb	σ^{DPS}	45 μb	5.2 μb	120 nb	70 nb	150 nb	7 nb	4 nb	150 pb
8.8 TeV	$\mathrm{N}^{\mathrm{DPS}}$ (1 pb^{-1})	∼65	∼60	∼15	∼3	∼15	∼8	∼1.5	∼4

(from arXiv:1408.5172 [hep-ph]; Nucl. Phys. A 931, 303 (2014))

The corresponding DPS yields, after (di)lepton decays and acceptance+efficiency losses, are given for 1 nb^{-1} and 1 pb^{-1} respectively.

Thus, the simultaneous production of quarkonia and/or electroweak bosons from DPS processes have large visible cross sections and are open to study in p-Pb and Pb-Pb at the LHC.

m-parton distributions:

$$\frac{dD_i^{j_1\ldots j_m}(x_1,\ldots,x_m,t)}{dt} = \sum_{l=1}^{m}\sum_{j'}\int\limits_{x_l}^{1-x_1-\ldots-x_{l-1}-x_{l+1}-\ldots-x_m}\frac{dx'}{x'}\times$$

$$\times D_i^{j_1\ldots j_{l-1}j'j_{l+1}\ldots j_m}(x_1,\ldots,x_{l-1},x',x_{l+1},\ldots,x_m,t)P_{j'\to j_l}\left(\frac{x_l}{x'}\right)$$

$$+\sum_{l=1}^{m}\sum_{p=l+1}^{m}\sum_{j'}\frac{1}{x_l+x_p}P_{j'\to j_l j_p}\left(\frac{x_l}{x_l+x_p}\right)\times$$

$$\times D_i^{j_1\ldots j_{l-1}j'j_{l+1}\ldots j_{p-1}j_{p+1}\ldots j_m}(x_1,\ldots,x_{l-1},x_l+x_p,x_{l+1},\ldots,x_{p-1},x_{p+1},\ldots,x_m,t)$$

Shelest, Snigirev, Zinovjev, Preprint ITP-83-46E, Kiev, 1983

TPS in QCD:

A.M. Snigirev, Phys. Rev. D 94, 034026 (2016)

D. d'Enterria, A.M. Snigirev, arXiv:1612.05582 [hep-ph] (2016) (PRL 118, 122001 (2017))

D. d'Enterria, A.M. Snigirev, arXiv:1612.08112 [hep-ph] (2016)

$$\sigma^{\rm TPS}_{hh'\to abc} = [m/(3!)] \cdot \sigma^{\rm SPS}_{hh'\to a} \cdot \sigma^{\rm SPS}_{hh'\to b} \cdot \sigma^{\rm SPS}_{hh'\to c} / \sigma^2_{\rm TPS,fact},$$

m is the combinatorial prefactor

$$\sigma^2_{\rm TPS,fact} = [\int d^2b (T(\mathrm{b}))^3]^{-1}.$$

$$\sigma_{\rm eff} = [\int d^2b (T(\mathrm{b}))^2]^{-1}$$

$$\sigma_{\rm TPS,fact} = k \cdot \sigma_{\rm eff}$$

with $k = 0.82 \pm 0.11$

BACK UP

EXPLICIT solution

Fortunately, the explicit form of evolution equation solutions allows us to answer the question: which correlations (perturbative or nonperturbative) are more significant at sufficiently large hard scale.

Indeed, the evolution equations are explicitly solved by introducing the Mellin transformations

$$M_h^j(n,t) = \int_0^1 dx x^n \; D_h^j(x,t),$$

$$M_h^{j_1 j_2}(n_1,n_2,t) = \int_0^1 dx_1 dx_2 \theta(1-x_1-x_2) x_1^{n_1} x_2^{n_2} D_h^{j_1 j_2}(x_1,x_2,t),$$

which lead to a system of ordinary linear differential equations of the first order. In order to obtain the distributions in x representation, an inverse Mellin transformation should be performed. In the general case this can be done only numerically. However, the asymptotic behavior can be estimated in some interesting and particularly simple limits using the same technique as above.

The exact solutions for single distributions in the moment representation can be written symbolically in a matrix form:

$$M_i^j(n,t) = [\exp P(n)t]_i^j,$$

and the solutions of the generalized DGLAP evolution equations with the given initial conditions may be written again as a convolution of single distributions; in the moment representation, they read

$$M_h^{j_1 j_2}(n_1,n_2,t) = \sum_{j_1' j_2'} M_h^{j_1' j_2'}(n_1,n_2,0) M_{j_1'}^{j_1}(n_1,t) M_{j_2'}^{j_2}(n_2,t)$$

$$+ M_{h(\mathrm{QCD})}^{j_1 j_2}(n_1,n_2,t),$$

where

$$M_{h(\mathrm{QCD})}^{j_1 j_2}(n_1,n_2,t) = \sum_i M_h^i(n_1+n_2,0) M_i^{j_1 j_2}(n_1,n_2,t)$$

are the particular solutions of the complete equations with zero initial conditions at the hadron level, and

$$M_i^{j_1 j_2}(n_1,n_2,t)$$

$$= \sum_{j j_1' j_2'} \int_0^t dt' M_i^j(n_1+n_2,t') P_{j\to j_1' j_2'}(n_1,n_2) M_{j_1'}^{j_1}(n_1,t-t') M_{j_2'}^{j_2}(n_2,t-t').$$

The kernels,

$$P_{j'\to j}(n)=\int\limits_0^1 x^n P_{j'\to j}(x)dx,$$

$$P_{j'\to j_1 j_2}(n_1,n_2)=\int\limits_0^1 x^{n_1}(1-x)^{n_2}P_{j'\to j_1 j_2}(x)dx,$$

are well-known and can be found in the explicit form.

Now we consider the initial condition effects in the asymptotic behavior ($t\to\infty$). In order to better understand the character of this dependence, at first we use a toy model with one type of partons (for instance, QCD theory with gluons only). In this case:

$$M_h^{11}(n_1,n_2,t)=M_h^{11}(n_1,n_2,0)\exp\{[P(n_1)+P(n_2)]t\}+$$

$$\frac{P(n_1,n_2)M_h^1(n_1+n_2,0)}{P(n_1+n_2)-P(n_1)-P(n_2)}\{\exp[P(n_1+n_2)t]-\exp[(P(n_1)+P(n_2))t]\}.$$

Thus, for t large enough, we have two different asymptotic regimes depending on the relation between the kernels $P(n_1+n_2)$ and $P(n_1)+P(n_2)$:

(1) If $P(n_1+n_2) < P(n_1)+P(n_2)$, then

$$\boldsymbol{M_h^{11}(n_1,n_2,t)|_{t\to\infty} = \Big[M_h^{11}(n_1,n_2,0) + \frac{P(n_1,n_2)M_h^{1}(n_1+n_2,0)}{P(n_1)+P(n_2)-P(n_1+n_2)}\Big] \times \exp\{[P(n_1)+P(n_2)]t\}.}$$

(2) If $P(n_1+n_2) > P(n_1)+P(n_2)$, then

$$\boldsymbol{M_h^{11}(n_1,n_2,t)|_{t\to\infty} = \frac{P(n_1,n_2)M_h^{1}(n_1+n_2,0)}{P(n_1+n_2)-P(n_1)-P(n_2)} \times \exp[P(n_1+n_2)t].}$$

For the second regime, the asymptotic behavior *does not dependent* on the initial correlation conditions $M_h^{11}(n_1,n_2,0)$ at all, and is specified by the correlations perturbatively calculated.

The presence of several parton types does not essentially complicate the analysis of the asymptotic behavior. Indeed, in this case one has to express single parton distributions via the eigenfunctions of corresponding DGLAP equations, put them into solutions above and take the leading contributions into consideration only.

As a result, the relation between maximum eigenvalues $\Lambda(n_1+n_2)$ and $\Lambda(n_1)+\Lambda(n_2)$ will determine the asymptotic behavior regime of the dPDFs:

(1) If $\Lambda(n_1+n_2) < \Lambda(n_1)+\Lambda(n_2)$, then the dPDFs are dependent on the initial correlation conditions $M_h^{j_1 j_2}(n_1, n_2, 0)$.

(2) If $\Lambda(n_1+n_2) > \Lambda(n_1)+\Lambda(n_2)$, then the dPDFs are independent of the initial correlation conditions $M_h^{j_1 j_2}(n_1, n_2, 0)$.

The eigenvalues and the eigenfunctions for the single distributions in QCD have been thoroughly studied. The results of these studies show that in QCD both asymptotic regimes are realized. Therefore, one needs to know the initial correlation conditions (which, generally speaking, are arbitrary and should be extracted from the experiment) to determine even the asymptotic behavior of the dPDFs. However, we come to the relation

$$\boldsymbol{\Lambda(n_1 + n_2) > \Lambda(n_1) + \Lambda(n_2)}$$

for large moments n_1 and n_2 that determines the dPDFs in the region of not parametrically small x_1 and x_2, because $\Lambda(n) \sim -\ln(n), n \gg 1.$

We conclude that the dPDFs "forget" the initial correlation conditions (unknown *a priori*) at not parametrically small longitudinal momentum fractions, and the correlations perturbatively calculated survive only in the limit of large enough hard scales.

Such a dominance is independent of the strength of the initial correlation conditions.

EVOLUTION CORRECTIONS TO DPS CROSS SECTION

The evolution equation for Γ_{ij} consists of two terms. The first term describes the independent (simultaneous) evolution of two branches of parton cascade: one branch contains the parton x_1, and another branch — the parton x_2.

The second term allows for the possibility of splitting one parton evolution (one branch k) into two different branches, i and j. It contains the usual splitting function $P_{k\to ij}(z)$. The solutions of the generalized DGLAP evolution equations with the given initial conditions at the reference scales μ^2 may be written in the form:

$$D_h^{j_1 j_2}(x_1, x_2; \mu^2, Q_1^2, Q_2^2)$$

$$= D_{h1}^{j_1 j_2}(x_1, x_2; \mu^2, Q_1^2, Q_2^2) + D_{h2}^{j_1 j_2}(x_1, x_2; \mu^2, Q_1^2, Q_2^2)$$

with

$$D_{h1}^{j_1 j_2}(x_1, x_2; \mu^2, Q_1^2, Q_2^2)$$

$$= \sum_{j_1{}' j_2{}'} \int_{x_1}^{1-x_2} \frac{dz_1}{z_1} \int_{x_2}^{1-z_1} \frac{dz_2}{z_2}\, D_h^{j_1{}' j_2{}'}(z_1, z_2; \mu^2) D_{j_1{}'}^{j_1}(\frac{x_1}{z_1}; \mu^2, Q_1^2) D_{j_2{}'}^{j_2}(\frac{x_2}{z_2}, \mu^2, Q_2^2)$$

and

$$D_{h2}^{j_1 j_2}(x_1, x_2; \mu^2, Q_1^2, Q_2^2) = \sum_{j' j_1' j_2'} \int_{\mu^2}^{\min(Q_1^2, Q_2^2)} dk^2 \frac{\alpha_s(k^2)}{2\pi k^2} \int_{x_1}^{1-x_2} \frac{dz_1}{z_1} \int_{x_2}^{1-z_1} \frac{dz_2}{z_2} \times$$

$$D_h^{j'}(z_1 + z_2; \mu^2, k^2) \frac{1}{z_1 + z_2} P_{j' \to j_1' j_2'}\left(\frac{z_1}{z_1 + z_2}\right) D_{j_1'}^{j_1}\left(\frac{x_1}{z_1}; k^2, Q_1^2\right) D_{j_2'}^{j_2}\left(\frac{x_2}{z_2}; k^2, Q_2^2\right)$$

where $\alpha_s(k^2)$ is the QCD coupling,
$D_{j_1'}^{j_1}(z; k^2, Q^2)$ are the known single distribution functions (the Green's functions) at the parton level with the specific δ-like initial conditions at $Q^2 = k^2$.

$D_h^{j_1', j_2'}(z_1, z_2, \mu^2)$ is the initial (input) two-parton distribution at the relatively low scale μ.

The one parton distribution (before splitting into the two branches at some scale k^2) is given by $D_h^{j'}(z_1 + z_2, \mu^2, k^2)$.

The first term is the solution of homogeneous evolution equation (independent evolution of two branches), where the input two-parton distribution is generally NOT known at the low scale μ. For this non-perturbative two-parton function at low z_1, z_2 one may assume the factorization $D_h^{{j_1}'{j_2}'}(z_1, z_2, \mu^2) \simeq D_h^{{j_1}'}(z_1, \mu^2) D_h^{{j_2}'}(z_2, \mu^2)$ neglecting the constraints due to momentum conservation ($z_1 + z_2 < 1$).

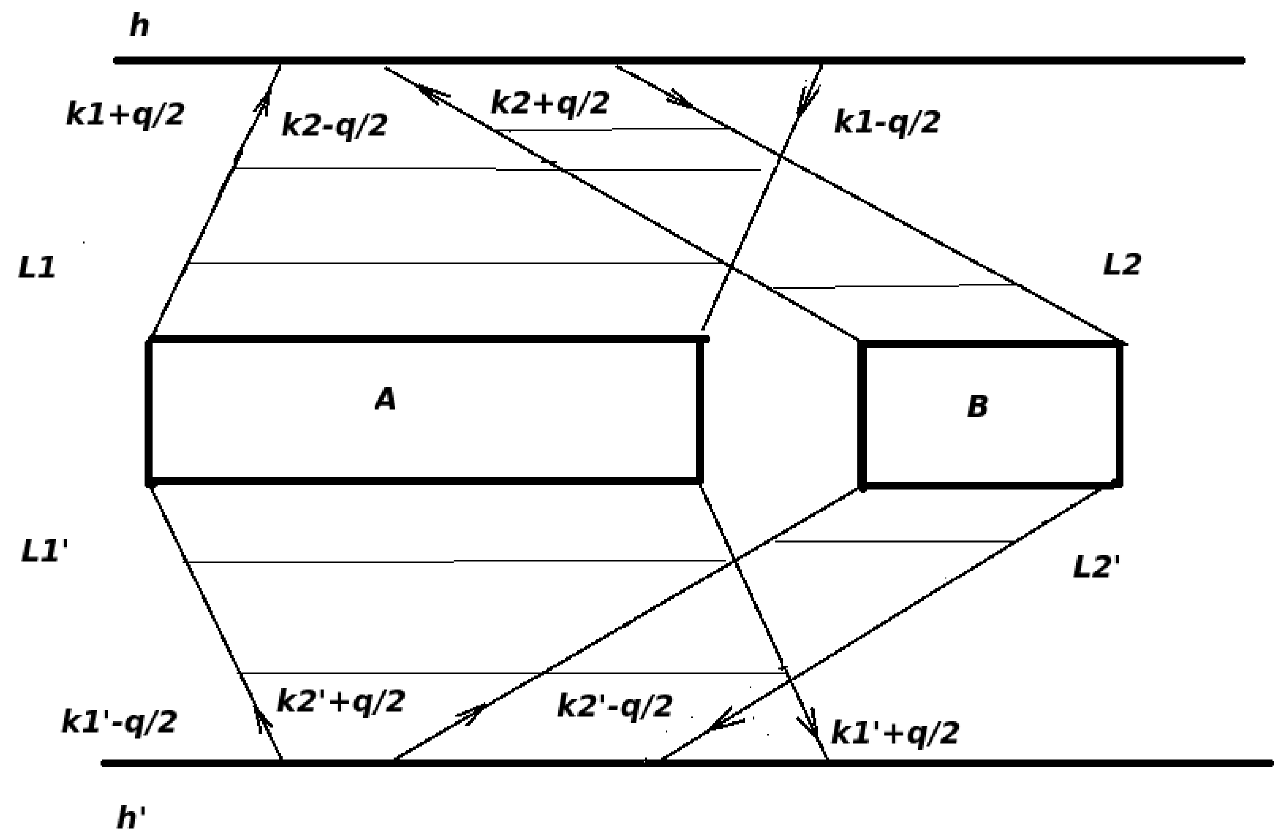
This leads to

$$\boldsymbol{D_{h1}^{ij}(x_1, x_2; \mu^2, Q_1^2, Q_2^2) \simeq D_h^i(x_1; \mu^2, Q_1^2) D_h^j(x_2; \mu^2, Q_2^2)}$$

the factorization hypothesis usually used in current estimations.

However, one should note that the input two-parton distribution $D_h^{j_1', j_2'}(z_1, z_2, \mu^2)$ may be more complicated than that given by factorization ansatz.

As a rule the multiple interactions take place at relatively low transverse momenta and low $x_{1,2}$, where the factorization hypothesis for the first term is a good approximation.



In this case the cross section for double parton scattering can be estimated, using the two-gluon form factor of the nucleon $F_{2g}(q)$ for the dominant gluon-gluon scattering mode (or something similar for other parton scattering modes)

$$\sigma^{D,1\times 1}_{(A,B)} = \frac{m}{2} \sum_{i,j,k,l} \int D^i_h(x_1;\mu^2,Q_1^2) D^j_h(x_2;\mu^2,Q_2^2) \hat{\sigma}^A_{ik}(x_1,x_1^{'}) \hat{\sigma}^B_{jl}(x_2,x_2^{'})$$

$$\times D^k_{h'}(x_1';\mu^2,Q_1^2) D^l_{h'}(x_2';\mu^2,Q_2^2) dx_1 dx_2 dx_1^{'} dx_2^{'} \int F^4_{2g}(q) \frac{d^2q}{(2\pi)^2}.$$

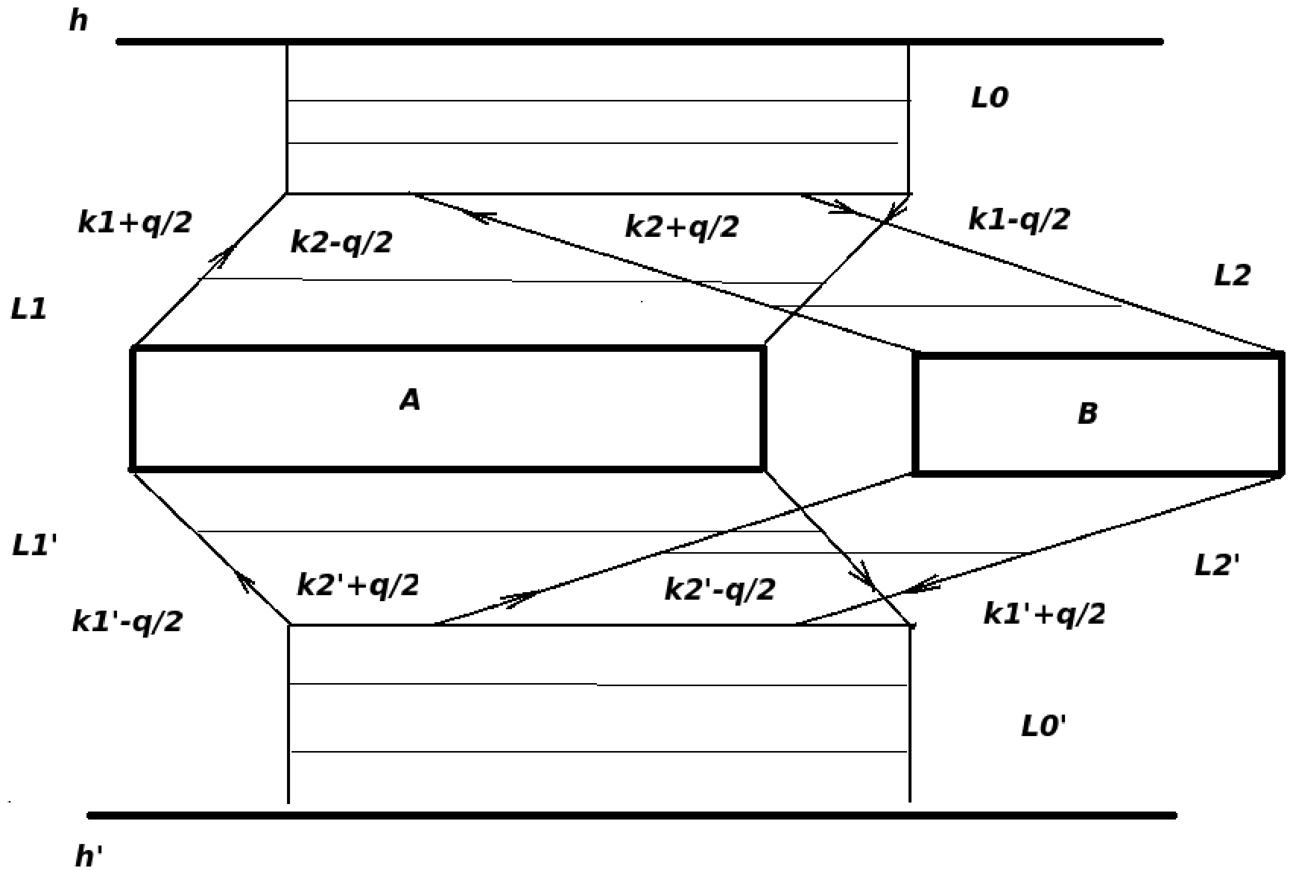
From the dipole fit $F_{2g}(q) = 1/(q^2/m_g^2+1)^2$ it follows that the characteristic value of q is of the order of "effective gluon mass" m_g. Thus the initial conditions for the single distributions can be fixed at some not large reference scale $\mu \sim m_g$, because of the weak logarithmic dependence of these distributions on the scale value.

In this approach

$$\int F^4_{2g}(q) \frac{d^2q}{(2\pi)^2}$$

gives the estimation of $[\sigma_{\text{eff}}]^{-1}$.

The second term is the solution of complete evolution equation with the evolution originating from one "nonperturbative" parton at the reference scale. Here the independent evolution of two branches starts at the scale k^2 from a point-like parton j'.



In this case, the large q_t domain is NOT suppressed by the form factor $F_{2g}(q)$ and the corresponding contribution to the cross section reads

$$\sigma^{D,2\times 2}_{(A,B)} = \frac{m}{2} \sum_{i,j,k,l} \int dx_1 dx_2 dx_1' dx_2' \int^{\min(Q_1^2,Q_2^2)} \frac{d^2q}{(2\pi)^2} \sum_{j'j_1'j_2'} \int_{q^2}^{\min(Q_1^2,Q_2^2)} dk^2 \frac{\alpha_s(k^2)}{2\pi k^2}$$

$$\times \int_{x_1}^{1-x_2} \frac{dz_1}{z_1} \int_{x_2}^{1-z_1} \frac{dz_2}{z_2} D_h^{j'}(z_1+z_2;\mu^2,k^2) \frac{1}{z_1+z_2} P_{j'\to j_1'j_2'}\left(\frac{z_1}{z_1+z_2}\right)$$

$$\times D^i_{j_1'}\left(\frac{x_1}{z_1};k^2,Q_1^2\right) D^j_{j_2'}\left(\frac{x_2}{z_2};k^2,Q_2^2\right) \hat{\sigma}^A_{ik}(x_1,x_1') \hat{\sigma}^B_{jl}(x_2,x_2')$$

$$\times \sum_{j'j_1'j_2'} \int_{q^2}^{\min(Q_1^2,Q_2^2)} dk'^2 \frac{\alpha_s(k'^2)}{2\pi k'^2} \int_{x_1'}^{1-x_2'} \frac{dz_1}{z_1} \int_{x_2'}^{1-z_1} \frac{dz_2}{z_2} D_{h'}^{j'}(z_1+z_2;\mu^2,k'^2)$$

$$\times \frac{1}{z_1+z_2} P_{j'\to j_1'j_2'}\left(\frac{z_1}{z_1+z_2}\right) D^k_{j_1'}\left(\frac{x_1'}{z_1};k'^2,Q_1^2\right) D^l_{j_2'}\left(\frac{x_2'}{z_2};k'^2,Q_2^2\right),$$

or in substantially shorter yet less transparent form:

$$\sigma^{D,2\times2}_{(A,B)}\frac{m}{2}\sum_{i,j,k,l}\int dx_1 dx_2 dx_1' dx_2' \int^{\min(Q_1^2,Q_2^2)} \frac{d^2q}{(2\pi)^2} D^{ij}_{h2}(x_1,x_2;q^2,Q_1^2,Q_2^2)$$

$$\times\hat{\sigma}^A_{ik}(x_1,x_1')\hat{\sigma}^B_{jl}(x_2,x_2')D^{kl}_{h'2}(x_1',x_2';q^2,Q_1^2,Q_2^2).$$

By analogy, the combined (“interference”) contribution may be written as

$$\sigma^{D,1\times2}_{(A,B)}=\frac{m}{2}\sum_{i,j,k,l}\int dx_1 dx_2 dx_1' dx_2' \int^{\min(Q_1^2,Q_2^2)} F^2_{2g}(q)\frac{d^2q}{(2\pi)^2}$$

$$\times[D^i_h(x_1;\mu^2,Q_1^2)D^j_h(x_2;\mu^2,Q_2^2)\hat{\sigma}^A_{ik}(x_1,x_1')\hat{\sigma}^B_{jl}(x_2,x_2')D^{kl}_{h'2}(x_1',x_2';q^2,Q_1^2,Q_2^2)$$

$$+D^{ij}_{h2}(x_1,x_2;q^2,Q_1^2,Q_2^2)\hat{\sigma}^A_{ik}(x_1,x_1')\hat{\sigma}^B_{jl}(x_2,x_2')D^k_{h'}(x_1';\mu^2,Q_1^2)D^l_{h'}(x_2';\mu^2,Q_2^2)].$$

The equations (above and below) present our solution of the problem — we obtain **the estimation of the inclusive cross section for double parton scattering, taking into account the QCD evolution and basing on the well-known collinear distributions**, extracted from deep inelastic scattering:

$$\boldsymbol{\sigma}^{D}_{(A,B)} = \boldsymbol{\sigma}^{D,1\times1}_{(A,B)} + \boldsymbol{\sigma}^{D,1\times2}_{(A,B)} + \boldsymbol{\sigma}^{D,2\times2}_{(A,B)}$$

Afterwards similar results were obtained also by Blok, Dokshitzer, Frankfurt, Strikman with an emphasis on the differential cross sections, then by Gaunt, Stirling (as concerning 1×1, 1×2 components)

2×2 component (**double splitting diagrams**) is the subject of discussion and **our disagreement** with *Blok, Dokshitzer, Frankfurt, Strikman; Gaunt, Stirling; Manohar, Waalewijn*, mainly in a terminology.

At a large final scale Q^2 the contribution of second (2×2) component should **dominate** being proportional to $q^2 \sim Q^2$, while the contributions of the 1×1 or 1×2 components $\sim m_g^2 \sim 1/\sigma_{eff}$ are limited by the nucleon (hadron) form factorF_{2g}.

In terms of impact parameters $\mathbf{b}$ this means that in the second (2×2) term two pairs of partons are **very close to each other**; $|\mathbf{b}_1 - \mathbf{b}_2| \sim 1/Q$.

We have to emphasize that the dominant contribution to the phase space integral comes from a large $q^2 \sim Q^2$ and, strictly speaking, the above reasoning makes no allowance for the collinear (DGLAP) evolution of two independent branches of the parton cascade (i.e., in the ladders $L1, L2, L1'$ and $L2'$) in the 2×2 term.

Formally in the framework of collinear approach this contribution should be considered as the result of interaction of *one* pair of partons with the $2 \to 4$ hard subprocess (*Blok, Dokshitzer, Frankfurt, Strikman; Gaunt, Stirling; Manohar, Waalewijn*).

On the boundary of phase space our formula reproduces naturally this result ($2 \to 4$) due to the specific δ-like initial conditions at $k^2 = Q^2$ for Green's functions.

Recall, however, that when estimating the phase space integral we neglect the anomalous dimension, γ, of the parton distributions

$D_j^k(x/z, k^2, Q^2) \propto (Q^2/k^2)^\gamma$. In collinear approach the anomalous dimensions $\gamma \propto \alpha_s << 1$ are assumed to be small. On the other hand, in a low x region the value of anomalous dimension is enhanced by the $\ln(1/x)$ logarithm and may be rather large numerically.

So the integral over q^2 is *slowly* convergent and the major contribution to the cross section is expected to come actually from some characteristic **intermediate region**, $m_g^2 << q^2 << Q_1^2$ $(Q_1 < Q_2)$.

Thus we do not expect such strong sensitivity to the upper limit of q-integration as in the case of the pure phase space integral.

Therefore it makes sense to consider the quantitative contribution of the 2×2 term even within the collinear approach as applied to the LHC kinematics, where the large (in comparison with m_g) available values of Q_1 and Q_2 provide wide enough integration region for the characteristic loop momenta q.

We demonstrate this fact by direct calculation in the double logarithm approximation. Let us put down the all integrations with splitting functions separately to make the analysis more transparent

$$D^{ij}_{h2}(x_1, x_2; q^2, Q_1^2, Q_2^2) = \sum_{j'j_1'j_2'} \int\limits_{q^2}^{\min(Q_1^2,Q_2^2)} dk^2 \frac{\alpha_s(k^2)}{2\pi k^2} \int\limits_{x_1}^{1-x_2} \frac{dz_1}{z_1} \int\limits_{x_2}^{1-z_1} \frac{dz_2}{z_2}$$

$$\times D^{j'}_h(z_1+z_2; \mu^2, k^2) \frac{1}{z_1+z_2} P_{j'\to j_1'j_2'}\left(\frac{z_1}{z_1+z_2}\right) D^i_{j_1'}\left(\frac{x_1}{z_1}; k^2, Q_1^2\right) D^j_{j_2'}\left(\frac{x_2}{z_2}; k^2, Q_2^2\right).$$

In the double logarithm approximation we can restrict ourselves to the gluon main contribution only and rewrite the integral under consideration in the following form

$$D^{gg}_{h2}(x_1, x_2; q^2, Q_1^2, Q_2^2) = \int\limits_{q^2}^{\min(Q_1^2,Q_2^2)} dk^2 \frac{\alpha_s(k^2)}{2\pi k^2} \int \frac{du}{u^2} D^g_h(u; \mu^2, k^2) \int \frac{dz}{z(1-z)}$$

$$\times P_{g\to gg}(z) D^g_g\left(\frac{x_1}{uz}; k^2, Q_1^2\right) D^g_g\left(\frac{x_2}{u(1-z)}; k^2, Q_2^2\right).$$

The Green's functions (gluon distributions at the parton level) in the double logarithm approximation read

$$xD_g^g(x,t) \simeq 4N_c t v^{-3/2} \exp\left[v - at\right]/\sqrt{2\pi},$$

where

$$v = \sqrt{8N_c t \ln(1/x)},\; a = \tfrac{11}{6}N_c + \tfrac{1}{3}n_f/N_c^2$$

$$t(Q^2) = \frac{2}{\beta}\ln\left[\frac{\ln(\frac{Q^2}{\Lambda^2})}{\ln(\frac{\mu^2}{\Lambda^2})}\right],$$

and where

$$\beta = (11N_c - 2n_f)/3$$

n_f is the number of active flavors, Λ is the dimensional QCD parameter, $N_c = 3$ is the color number and the one-loop running QCD coupling

$$\alpha_s(Q^2) = \frac{4\pi}{\beta\ln(Q^2/\Lambda^2)}$$

was used

After that the integral may be rewritten as

$$x_1 x_2 D_{h2}^{gg}(x_1, x_2; \tau, T_1, T_2)$$

$$\sim \int_{\tau}^{\min(T_1,T_2)} dt \int dz P_{g\to gg}(z) \int dy \exp\left[\sqrt{8N_c}\, d(t,y,z)\right],$$

where

$$d(t,y,z) = \sqrt{ty} + \sqrt{(T_1 - t)(Y_1 - y)} + \sqrt{(T_2 - t)(Y_2 - y)}$$

with

$$t = t(k^2),\ T_1 = t(Q_1^2),\ T_2 = t(Q_2^2),\ \tau = t(q^2)$$

and

$$y = \ln(1/u),\ Y_1 = \ln(1/x_1) - \ln(1/z),\ Y_2 = \ln(1/x_2) - \ln(1/(1-z)).$$

We keep the leading exponential terms only, which have the same structure both at the parton level and at the hadron one under the *smooth enough initial conditions* at the reference scale.

We are interested in the domain with large enough T_1, T_2, $\ln(1/x_1)$ and $\ln(1/x_2)$, when the exponential factors are large in comparison with **1** and where the approximations above are justified. In this case the integration over the rapidity y has the saddle point structure in the wide interval of z-integration not near the kinematic boundaries. The saddle-point equation reads

$$\frac{\sqrt{t}}{\sqrt{y_0}} - \frac{\sqrt{(T_1 - t)}}{\sqrt{(Y_1 - y_0)}} - \frac{\sqrt{(T_2 - t)}}{\sqrt{(Y_2 - y_0)}} = 0.$$

It may be solved explicitly in the simplest case of the two hard scales set equal $T_1 = T_2 = T$ and at $Y_1 \simeq Y_2 \simeq Y = \ln(1/x)$, i.e., in the z-region where $\ln(1/z) << \ln(1/x)$ and $\ln(1/(1-z)) << \ln(1/x)$ (*In spite of the large nonexponential factor like* $\ln(1/x)$ *(due to the singularity of the splitting function* $P_{g\to gg}(z)$*) the contribution from the integration region near the kinematical boundaries* $z \sim x$ *and* $1 - z \sim x$ *is not dominant, since in this case the obtained exponential factor* $\exp\left[\sqrt{8N_c}\sqrt{Y(T-\tau)}\right]$ *is not leading*).

Then the saddle-point is equal to

$$y_0 = Yt/(4T - 3t).$$

Thus, the splitting integrals reduce to

$$x^2 D_{h2}^{gg}(x, x; \tau, T, T) \sim \int_{\tau}^{T} dt \int_{x}^{1-x} dz P_{g\to gg}(z) \exp\left[\sqrt{8N_c}\sqrt{Y(4T-3t)}\right].$$

The t-integration is not a saddle-point type and therefore one of edges, namely $t \to \tau$, dominates. That is

$$x^2 D_{h2}^{gg}(x, x; \tau, T, T) \sim \exp\left[\sqrt{8N_c}\sqrt{Y(4T-3\tau)}\right].$$

What follows from our estimation of splitting integrals in the double logarithm approximation by the saddle-point method ?

For single splitting diagrams (1×2 contribution)

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the lower limit for the t-integration may be taken at the reference scale, i.e., $\tau = t(q^2)|_{q=\mu} = 0$ due to the strong suppression factor $F_{2g}^2(q)$. The characteristic value of q being of the order of "effective gluon mass" $m_g \sim \mu$ in the further q-integration. Thus one obtains for this contribution the following estimation

$$\boldsymbol{x^2 D_{h2}^{gg}(x, x; 0, T, T) \sim \exp\left[\sqrt{8N_c}(\sqrt{YT} + \sqrt{YT})\right].}$$

It means that the splitting takes place in the "characteristic point" with the scale k^2 close to μ^2 and with the longitudinal momentum fraction $u \sim 1$ (the saddle-point $y_0 \sim t \sim \tau \sim 0$ in this case).

After splitting one has the TWO independent ladders with the well-developed BFKL and DGLAP evolution. Every ladder contributes to the cross section with the large exponential factor, $\exp\left[\sqrt{8N_c}\sqrt{YT}\right]$, which is just the *same* as for single distributions.

Therefore in the double logarithm approximation single splitting diagrams have, in fact, the factorization property if one takes the leading exponential factors into consideration only.

For double splitting (2×2) diagrams
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the leading exponential contribution arises from the lower limits of t- and either lower or upper limits of q-integrations depending on the available rapidity interval Y.

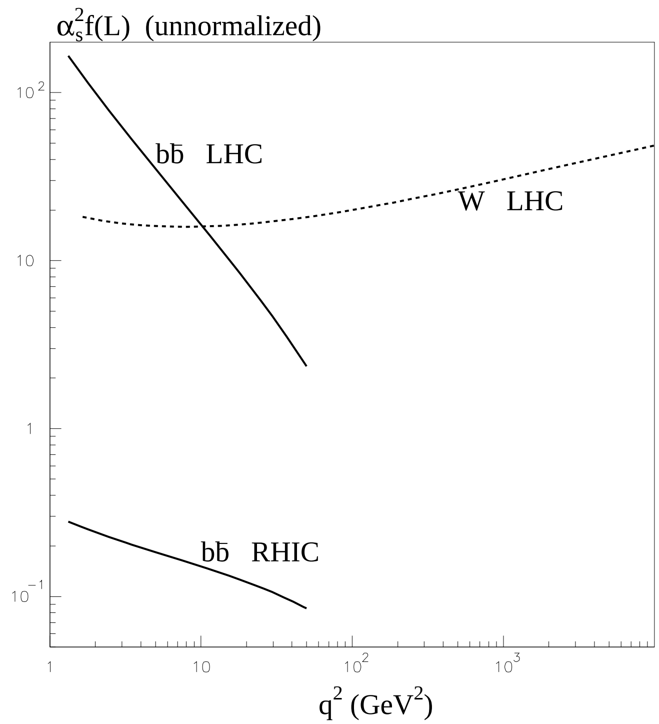
There is competition between the exponential factor caused by the evolution, which prefers a small τ, and the phase space factor in q^2-integral.

Due to the *non-logarithmic* character of the integration over d^2q for a *not sufficiently large* Y the contribution from the *upper limit* of q *may dominate*. Indeed, let us consider the production of two $b\bar{b}$ pairs in a central rapidity ($\eta \sim 0$) region. That is we take $T_1 = T_2 = T$, $Y_1 = Y_2 = Y$ and keep just the leading exponential factors in the double parton distributions

$$\boldsymbol{x^2 D_{h2}(x, x, q^2, Q^2, Q^2) \sim \exp(\sqrt{8N_c Y(4T - 3\tau)} - 2aT + a\tau).}$$

Thus the logarithmic dq^2/q^2 integral takes the form

$$\boldsymbol{\int \frac{dq^2}{q^2} \exp(2\sqrt{8N_c Y(4T - 3\tau)} - 4aT + 2a\tau) q^2.}$$



The q-dependence of the integrand $f(L)$ in the logarithmic scale

$$f(L) = exp\left(2\sqrt{8N_cY(4T-3\tau)} - 4aT + 2a\tau\right)\exp\left(\ln\left(\frac{\mu^2}{\Lambda^2}\right)e^{\beta\tau/2}\right)$$

with

$$L = \ln(q^2/\Lambda^2) = \ln(\mu^2/\Lambda^2)e^{\beta\tau/2}$$

For the DPS production of two $b\bar{b}$ pairs the major contribution comes from a low q^2 in the case of $Y = 5$ corresponding to the LHC energy $\sqrt{s} = 14$ TeV

That is the reaction may be effectively described by the 1×1 term; the formation of TWO parton branches (one to two splitting) takes place mainly at low scales.

However at the RHIC energy, when the available rapidity interval is not large ($Y = 2$), the q^2-dependence is not steep and the contribution caused by the splitting somewhere in the mid of evolution is still not negligible.

The same can be said about the DPS W-boson production at the LHC. Here the upper edge of the q^2-integral dominates. This part may be described as the collision of *one* pair of partons supplemented by a more complicated, $2 \to 4$ or $2 \to 2W$, hard matrix element. However, clearly we need to account also for contributions from the whole q^2-interval.

For the debatable double splitting diagrams,
depending on the precise kinematics, we may deal:

- either with a single parton pair collision (times the $2 \to 4$ hard subprocess) *in accordance with Blok, Dokshitzer, Frankfurt, Strikman;Gaunt, Stirling;Manohar, Waalewijn*

- or with the contribution of the 1×1 type where the formation of two parton branches (one to two splitting) takes place at low scales

- or with the 2×2 configuration where the splitting may happen EVERYWHERE (with more or less equal probabilities) during the evolution.

In order to probe the QCD evolution of the double distribution functions better we suggest also to investigate the processes with two quite different scales, in particular, production of a $b\bar{b}$ pair (or J/ψ) with W, which was estimated at the LHC kinematical conditions using the factorized component only.