Overview of General Relativity

Fawad Hassan

Stockholm University, Sweden

Spåtind 2018 - Nordic Conference on Particle Physics

Jan 2-7, 2018, Skeikampen, Norway

Improvised Talk, Will focus on the ideas and avoid details

Outline of the talk

Origins of General Relativity

Gravitational Waves

Blackholes

Cosmology and Evolution of the Universe

Spacetime in GR

Hamiltonian formulation

The algebra of constraints

Problem with Quantization

Outline of the talk

Origins of General Relativity

Gravitational Waves

Blackholes

Cosmology and Evolution of the Universe

Spacetime in GR

Hamiltonian formulation

The algebra of constraints

Problem with Quantization

Life before General Relativity

Newtonian Gravity:

$$\nabla^2\phi(x) = G_n\,\rho(x)\,, \qquad \not{m}\frac{d^2}{dt^2}\vec{x} = -\not{m}\,\vec{\nabla}\phi(x)$$

Riemannian Geometry:

In a space with coordinates x^i and metric $g_{ij}(x)$,

- Distance: $ds^2 = g_{ij}dx^i dx^j$,
- Connection: $\Gamma^i_{jk} = \frac{1}{2}g^{-1}(\partial g + \cdots)$
- Curvatures: $R^i_{jkl} = (\partial\Gamma + \Gamma\Gamma)$, R_{ij}, R

Insensetivity to coordinate choice: $x^i \to \tilde{x}^i(x) \Rightarrow$

$$g_{ij}(x) = \tilde{g}_{kl}(\tilde{x})\frac{\partial\tilde{x}^k}{\partial x^i}\frac{\partial\tilde{x}^l}{\partial x^j}\,, \quad \Gamma(g) = \tilde{\Gamma}(\tilde{g})\frac{\partial\tilde{x}}{\partial x}\frac{\partial\tilde{x}}{\partial x}\frac{\partial x}{\partial\tilde{x}} + \frac{\partial^2\tilde{x}}{\partial x\partial x}\frac{\partial x}{\partial\tilde{x}}$$

Origins of General Relativity (GR)

$\sim$ 1915:

No exprimental evidence for GR, but compelling theoretical reason:

Inconsistency of Newtonian gravity with Special Relativity.

Observations:

1. Particle in inertial frame $(x^\mu, \eta_{\mu\nu})$: $\frac{d^2}{d\tau^2} x^\mu(\tau) = 0$.

   Fictitious forces arise in non-interial frames $\tilde{x}^\mu(x)$,

$$\frac{d^2}{d\tau^2}\tilde{x}^\mu + (\Delta\Gamma^\mu_{\rho\sigma})\frac{d\tilde{x}^\rho}{d\tau}\frac{d\tilde{x}^\sigma}{d\tau} = 0$$

2. For free falling observers, gravitational force=0 locally.
3. There exist frames where locally $\Gamma^\mu_{\rho\sigma} \to 0$ & $g_{\mu\nu} \to \eta_{\mu\nu}$.

Similarity between $\Gamma^\mu_{\rho\sigma}$ and the gravitational force!

Einstein's theory of General Relativity

Postulate:

- Idenfity $g_{00} = -1 + 2\phi$ and replace $\frac{d^2}{dt^2}x^i = -\partial^i\phi$ by,

$$\frac{d^2}{d\tau^2}x^\mu + \Gamma^\mu_{\rho\sigma}\frac{dx^\rho}{d\tau}\frac{dx^\sigma}{d\tau} = 0$$

(soln: $x^\mu(\tau)$, $ds^2 \leq 0$; Geodesic equation)

- Generally, replace all $\eta_{\mu\nu}$ by $g_{\mu\nu}$ and $\int d^4x$ by $\int d^4x\sqrt{|\det g|}$

(Principle of Covariance: matter-gravity interaction)

- The covariant generalization of $\nabla^2\phi = G_n\,\rho$ is

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = G_n T_{\mu\nu}$$

(Einstein's equation, $\nabla^\mu T_{\mu\nu} = 0$)

The Einstein-Hilbert action:

$$S = \frac{1}{G_n} \int d^4x \sqrt{|\det g|}(R + 2\Lambda) + S_{matter}[g, \psi]$$

A very successful theory that has revolutionized our concept of spacetime and the Universe (although not free from troubles).

Predictions and tests:

- Bending of light and gravitational lensing (observed)
- Planetary orbits
- Shapiro time delay
- Gravitational waves (more later)
- Blackholes (more later)
- Cosmology and evolution of the Universe (more later)

Outline of the talk

Gravitational Waves

Weak field: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, + gauge fixing,

$$\Box h_{\mu\nu} = G_n T_{\mu\nu}$$

GW's: 2 propagating modes sourced by time dependent quadrupole moments of T_{00}.

Spin=2, m=0 $\Rightarrow$ helicities ± 2

Observed sources: binary blackholes/Neutron stars

- Implications for theories of blackhole formation
- Multimessenger cosmology
- Implications for modified gravity theories (speed of gravity = speed of light)
- Strong field tests of gravity/blackhole horizon

Outline of the talk

Origins of General Relativity

Gravitational Waves

Blackholes

Cosmology and Evolution of the Universe

Spacetime in GR

Hamiltonian formulation

The algebra of constraints

Problem with Quantization

Blackholes and related issues

"Vacuum" solutions of GR containing singularities hidden behind horizons.

Can be formed astrophysically by collapsing matter and can grow by accreting matter

- Superheavy blackholes $M_{bh} \sim 10^6 M_{sun}$: "observed" at galactic centers.
- Lower mass blackholes: Observed in binary systems and through gravitational waves.
- Primordial blackholes ? (not from star collapse)

Implications of the recent gravitational wave observations: Blackholes with $M_{bh} \gtrsim 5 M_{sun}$ observed much more frequently than expected! Formation mechanism?

Blackholes and related issues[cont.]

Fundamental unresolved issues:

- Breakdown of physics at the singularity, can it be real? (the Cosmic Censorship Hypothesis).
- Quantum problems: Hawking radiation and information loss at/near the horizon. Modifications of GR needed?
- Curious facts (some beyond blackholes):
  Connection between GR and Thermodynamics
  (Hawking-Bekenstein, Jacobson)

  AdS/CFT duality, Fluid/Gravity duality, etc

Outline of the talk

Cosmology and Evolution of the Universe

- FLRW Universe: $ds^2 = -dt^2 + a(t)^2(dx^i dx^i)$
- Early times: Inflation

  (imprints in CMBR, structure formation)

  Mechanism of inflation and reheating ?

  (beyond adhoc scalar fields)

  The transplanckian problem (Brandenburber)
- Late time: GR very successful but needs $\Lambda + CDM$

  (dark matter and dark energy problems)

Outline of the talk

Structure of Spacetime in GR

Natural formulation:
Covariant equations in a spacetime continuum $\{X^\mu\}$.

Physical requirement:
Time evolution equations for initial data specified over space.
Hence, the need for 3+1 (space+time) decomposition.

3+1 Decomposition of the metric:
Spatial hypersurfaces Σ_t at constant time $x^0 \equiv t$.

$$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{0j} \\ g_{i0} & g_{ij} \end{pmatrix} = \begin{pmatrix} -N^2 + N^K N_k & N_j \\ N_i & \gamma_{ij} \end{pmatrix}$$

N: Lapse, N_i: Shift, γ_{ij}: spatial metric on Σ_t

3+1 Decomposition

Light-cones (light cones):

$$0 = ds^2 = -N^2 dt^2 + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

Geometrical significance of N and N^i: (see figure)

(globally hyperbolic spacetimes)

Dynamical significance of N and N^i:
Time derivatives of N and N_i do not appear in the action!

Conjugate Momenta:

$$\pi^{ij} = \frac{\partial \mathcal{L}}{\partial \dot{\gamma}_{ij}}, \qquad \frac{\partial \mathcal{L}}{\partial \dot{N}_\mu} = 0$$

Outline of the talk

Hamiltonian formulation

$$\mathcal{L} = \pi^{ij}\dot{\gamma}_{ij} - NR^0 - N_i R^i \,, \quad \mathcal{H} = NR^0 + N_i R^i$$

N, N_i: Lagrange multipliers $\Rightarrow$ constraints

$$R^0 = 0 \,, R^i = 0$$

Holds for any generally covariant theory.

These along with the gauge conditions determine 8 out of 10 metric components $\Rightarrow$ 2-gravitational field degrees of freedom! (consistent with GW in the flat space limit)

Outline of the talk

The algebra of constraints

Algebra of General Coordinate Transformations (GCT):

$$\left\{R^0(x), R^0(y)\right\} = -\left[R^i(x)\frac{\partial}{\partial x^i}\delta^3(x-y) - R^i(y)\frac{\partial}{\partial y^i}\delta^3(x-y)\right]$$

$$\left\{R^0(x), R_i(y)\right\} = -R^0(y)\frac{\partial}{\partial x^i}\delta^3(x-y)$$

$$\left\{R_i(x), R_j(y)\right\} = -\left[R_j(x)\frac{\partial}{\partial x^i}\delta^3(x-y) - R_i(y)\frac{\partial}{\partial y^j}\delta^3(x-y)\right]$$

$R_i = \gamma_{ij}R^j$, γ_{ij}: metric of spatial 3-surfaces.

- Any generally covariant theory contains such an algebra.
- Hojman-Kuchar-Teitelboim Metric: The tensor that lowers the index on R^i is the physical metric of 3-surfaces.

Outline of the talk

- Origins of General Relativity
- Gravitational Waves
- Blackholes
- Cosmology and Evolution of the Universe
- Spacetime in GR
- Hamiltonian formulation
- The algebra of constraints
- Problem with Quantization

Problem with Quantization

- As a quantum field theory, GR is not renormalizable. Also, in the presence of quantum GR, the Standard Model is not renormalizable.
- Reason: $G_n = \frac{1}{M_p^2}$.

  Coupling constant in the QFT expansion is $g = \frac{1}{M_p}$, of inverse mass dimension.

  Infinite number of counter terms allowed.

Thank You!