Status of Bimetric Theory

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Based on work with:

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Outline of the talk

Motivation & Problems

Gravity with an extra spin-2 field: Bimetric Theory

Potential Issues

Uniqueness and the local structure of spacetime

Discussion

Bimetric gravity:

Why study a theory of gravity with two metrics?

(Gravitational metric coupled to extra spin-2 fields)

Outline of the talk

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- GR + Λ + CDM : Very successful

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- Observations and theory indicate new physics

  Dark matter, Dark energy (the cosmological constant problem),
  Origin of Inflation, The "trans-Planckian" problem,
  Quantum gravity, Matter-Antimatter asymmetry,
  Origin of Standard Model, $\cdots$

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Theoretical (top down) approaches:

GUTs, Susy, String Theory (with or without Multiverse),
Quantum Gravity, $\cdots$

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But why gravity with extra spin-2 fields?

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Spin (s) $\Rightarrow$ basic structure of field equations

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Unexplored corner: gravity with more spin-2 fields

Challenges in adding spin-2 fields to GR

- No multiple massless spin-2 fields
- Linear theory of massive spin-2 fields (5 helicities)

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  [Boulware, Deser (1972)]
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Can such theories exist or is GR unique?

Consequences?

The Ghost Problem

Ghost: A field with negative kinetic energy

Example:

$$\mathcal{L} = T - V = (\partial_t \phi)^2 \cdots \qquad (\textit{healthy})$$

But

$$\mathcal{L} = T - V = -(\partial_t \phi)^2 \cdots \qquad (\textit{ghostly})$$

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Consequences:

- Instability: unlimited energy transfer from ghost to other fields
- Negative probabilities, violation of unitarity in quantum theory

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A dynamical theory for the metric $g_{\mu\nu}$ & spin-2 field $f_{\mu\nu}$

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Overcoming the ghost in massive gravity:

[Creminelli, Nicolis, Papucci, Trincherini, (2005)]

[de Rham, Gabadadze, Tolley (2010)]

[SFH, Rosen (2011); SFH, Rosen, Schmidt-May (2011)]

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- what is $V(g^{-1} f)$?
- what is $\mathcal{L}(f, \nabla f)$?
- proof of absence of the Boulware-Deser ghost

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$V(g^{-1}f)$ for a ghost-free theory:

[SFH, Rosen (2011); de Rham, Gabadadze, Tolley (2010)]

Digression: elementary symmetric polynomials $e_n(S)$

Consider matrix S with eigenvalues $\lambda_1, \cdots, \lambda_4$.

$$
\begin{aligned}
&e_0(S) = 1\,, \qquad e_1(S) = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4\,,\\
&e_2(S) = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4,\\
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\begin{aligned}
e_0(S) &= 1\,,\\
e_1(S) &= \mathrm{Tr}(S) \equiv [S]\,,\\
e_2(S) &= \tfrac{1}{2}([S]^2 - [S^2]),\\
e_3(S) &= \tfrac{1}{6}([S]^3 - 3[S][S^2] + 2[S^3])\,,\\
e_4(S) &= \det(S)\,,\\
e_n(S) &= 0 \quad (\text{for} \quad n > 4)\,,
\end{aligned}
$$

$$\det(\mathbb{1} + S) = \sum_{n=0}^{4} e_n(S)$$

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$$V(S) = \sum_{n=0}^{4} \beta_n \, e_n(S)$$

Where:

$$S = \sqrt{g^{-1} f}$$

square root of the matrix $g^{\mu\lambda} f_{\lambda\nu}$

Real? Unique?

How to make sense of it? (to be answered later)

Ghost-free "bi-metric" theory

[SFH, Rosen (1109.3515,1111.2070)]

Ghost-free combination of *kinetic* and *potential* terms:

$$\mathcal{L} = m_g^2 \sqrt{-g} R_g - m^4 \sqrt{-g} \sum_{n=0}^{4} \beta_n \, e_n(\sqrt{g^{-1} f}) + m_f^2 \sqrt{-f} \, R_f$$

"bimetric" nature forced by the absence of ghost

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"bimetric" nature forced by the absence of ghost

Hamiltonian analysis:

$7 = 2 + 5$ nonlinear propagating modes, no BD ghost!

- $C_1 = 0, \qquad C_2 = \frac{d}{dt} C_1 = \{H, C\} = 0$

Detailed analysis of constraints *[SFH, Lundkvist (to appear)]*

Mass spectrum & Limits

$$\bar{f} = c^2 \bar{g}\,, \quad g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}\,, \quad f_{\mu\nu} = \bar{f}_{\mu\nu} + \delta f_{\mu\nu}$$

Linear modes:

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Linear modes:

Massless spin-2: $$\delta G_{\mu\nu} = \left(\delta g_{\mu\nu} + \frac{m_f^2}{m_g^2}\,\delta f_{\mu\nu}\right) \quad (2)$$

Massive spin-2 : $$\delta M_{\mu\nu} = \left(\delta f_{\mu\nu} - c^2 \delta g_{\mu\nu}\right) \quad (5)$$

$g_{\mu\nu},\ f_{\mu\nu}$ are mixtures of *massless* and *massive* modes

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The General Relativity limit: $m_g = M_P\,, \quad m_f/m_g \to 0$
(more later)

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The General Relativity limit: $m_g = M_P\,, \quad m_f/m_g \to 0$
(more later)

Massive gravity limit: $m_g = M_P\,, \quad m_f/m_g \to \infty$

Can Bimetric be a fundamental theory?

- Similar to Proca theory in curved background,

$$\sqrt{|\det g|}(F_{\mu\nu}F^{\mu\nu} - m^2\, g^{\mu\nu}A_\mu A_\nu + R_g)$$

- May need the equivalent of Higgs mechanism with the extra fields for better quantum or even classical behaviour

Some features

1) Ghost-free Matter couplings, same as in GR:

$$\mathcal{L}_g(g, \phi) + \mathcal{L}_f(f, \tilde{\phi})$$

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(protected by symmetry, like fermion masses)

3) Massive spin-2 particles: dark matter (stable enough)

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3) Massive spin-2 particles: dark matter (stable enough)

4) Some blackhole and cosmological solutions explored

5) Gravitational waves ?

6) Generalization to more than 2 fields

GR limit

The General Relativity limit:

$$m_g = M_P \,, \quad \alpha = m_f / m_g \to 0$$

Cosmological solutions in the GR limit:

$$3H^2 = \frac{\rho}{M_{Pl}^2} - \frac{2}{3}\frac{\beta_1^2}{\beta_2} m^2 - \alpha^2 \frac{\beta_1^2}{3\beta_2^2} H^2 + \mathcal{O}(\alpha^4)$$

The GR approximation breaks down at sufficiently strong fields

Outline of the talk

Potential Issues

- (1) g, f: incompatible notions of space and time?
- (2) nonunique $\sqrt{g^{-1}f}$: ambiguity in defining the action?

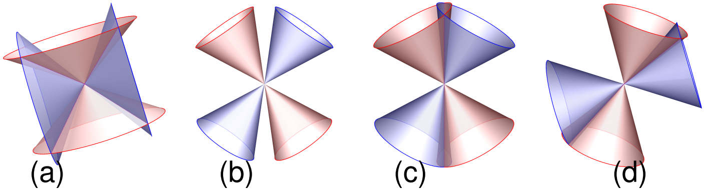
Other issues:

- Causality: local closed timelike curves (CTC's) ?
- The initial value problem?
- Faster than light possible in the gravitational sector (good or bad?)
- Analogue of "energy conditions" and global "bi"hyperbolicity?

Problem of incompatible spacetimes

Problem 1: $g_{\mu\nu}$ & $f_{\mu\nu}$ have Lorentzian signature $(1,3)$.

But may not admit compatible 3+1 decompositions



Then, no consistent time evolution equations,

no Hamiltonian formulation.

Nonuniqueness, Reality and Covariance

Problem 2:

- $S = \sqrt{g^{-1}f}$ is not unique,
- May not be real, covariant

To properly define the theory ($V(S)$):

(a) S^μ_ν needs to be specified uniquely,
(b) Restrict $g_{\mu\nu}, f_{\mu\nu}$ so that S is real, covariant

What are the restrictions on g & f ?
Can they be imposed meaningfully & consistent with dynamics?

Outline of the talk

Uniqueness and the local structure of spacetime

Natural requirements:

- General Covariance: $S^{\mu}_{\ \nu}$ *must be a (1,1) tensor*
- *S must be real*

Implication :

Problem 2 (*reality, uniqueness*) has a natural solution.

This also solves **Problem 1** (*compatible spacetimes*).

[SFH, M. Kocic (arXiv:1706.07806)]

Solution to the uniqueness problem of $V(S)$

Matrix square roots:

- Primary roots: Max 16 distinct roots, generic
- Nonprimary roots: Infinitely many, non-generic
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General Covariance: $A^{\mu}_{\ \nu} = g^{\mu\rho} f_{\rho\nu}$ is a (1,1) tensor,

$$x^{\mu} \to \tilde{x}^{\mu} \Rightarrow A \to Q^{-1} A Q \,, \quad \text{for} \quad Q^{\mu}_{\ \nu} = \frac{\partial x^{\mu}}{\partial \tilde{x}^{\nu}}$$

Uniqueness of S

$S^{\mu}_{\ \nu} = (\sqrt{A})^{\mu}_{\ \nu}$:

- Primary roots: $\sqrt{A} \rightarrow \sqrt{Q^{-1}AQ} = Q^{-1}\sqrt{A}\,Q$
- Nonprimary roots: $\sqrt{Q^{-1}AQ} \neq Q^{-1}\sqrt{A}\,Q$

Step 1:
General covariance $\Rightarrow$ only primary roots are allowed.

A Consequence: Examples of backgrounds with local CTC's correspond to nonprimary roots and are excluded

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Step 2:
Only the *principal root* is always primary. Hence, S must be a principal root.

(Nonprincipal roots degenerate to nonprimary roots when some eigenvalues coincide).

Reality of S

Real $S = \sqrt{g^{-1}f} \Rightarrow$ simple classification of allowed g, f configurations

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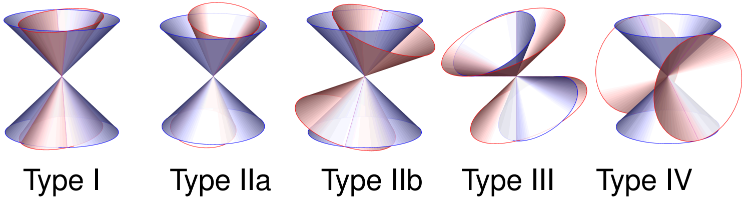
Theorem: Real $S = \sqrt{g^{-1}f}$ exist if and only if the null cones of g and f (i) intersect in open sets, or, (ii) have no common space nor common time directions (Type IV)



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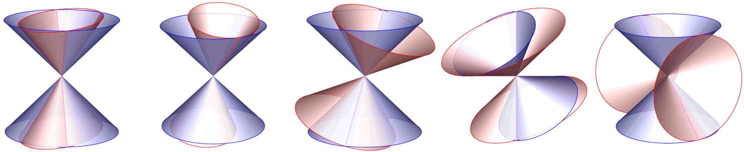


Type I Type IIa Type IIb Type III Type IV

*Types I-III: proper 3+1 decompositions, primary roots, allowed.

*Type IV: only nonprimary real roots! (excluded).

Choice of the square root

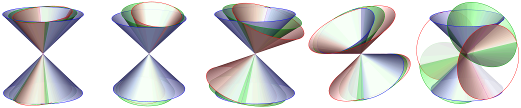
Reality + **General Covariance** $\Rightarrow$

* Real principal square root (unique),

* Compatible 3+1 decomposition

$$h_{\mu\nu} = g_{\mu\rho}(\sqrt{g^{-1}f}\,)^{\rho}{}_{\nu}$$

h null-cones for the principal root (except for the last one)



Useful for choosing good coordinate systems,
The specific, existing local CTC's in massive gravity rulled out.

Consistency with dynamics

Type IV metrics arise as a limit of Type IIb metrics. But in the limit, S has a branch cut. Then, for the principal root, the variation δS is not defined at the cut $\Rightarrow$ Eqns. of motion not valid for Type IV. Hence cannot arise dynamically.

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A simple mechanical example:

$$A = \int dt \left(\dot{x}^2/2 - \lambda \sqrt{x^2} \right) , \qquad \sqrt{x^2} = |x| \tag{1}$$

$$\ddot{x} = -\lambda (x > 0), \qquad \ddot{x} = \lambda (x < 0) \tag{2}$$

(no equation at $x = 0$)

Implication for some acausality arguments (CTC's) in massive gravity

Outline of the talk

Discussion

The beginning of understanding theories of spin-2 fields beyond General Relativity.

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The beginning of understanding theories of spin-2 fields beyond General Relativity.

- Superluminality? (yes, but not necessarily harmful, *replacement for inflation?*)
- Causality? (needs to be investigated further)
- Energy conditions, global “bi-hyperbolicity”
- A more fundamental formulation
- Application to cosmology, blackholes, GW, etc.
- Extra symmetries $\Rightarrow$ Modified kinetic terms? much less understood. *[SFH, Apolo (2016)]*

Thank you!

The Hojman-Kuchar-Teitelboim Metric

General Relativity in 3+1 decomposition ($g_{\mu\nu}$: γ_{ij}, N, N_i):

$$\sqrt{g}R \sim \pi^{ij}\partial_t\gamma_{ij} - NR^0 - N_iR^i$$

Constraints: $R^0 = 0\,, R^i = 0$.
Algebra of General Coordinate Transformations (GCT):

$$\{R^0(x), R^0(y)\} = -\left[R^i(x)\frac{\partial}{\partial x^i}\delta^3(x-y) - R^i(y)\frac{\partial}{\partial y^i}\delta^3(x-y)\right]$$

$$\{R^0(x), R_i(y)\} = -R^0(y)\frac{\partial}{\partial x^i}\delta^3(x-y)$$

$$\{R_i(x), R_j(y)\} = -\left[R_j(x)\frac{\partial}{\partial x^i}\delta^3(x-y) - R_i(y)\frac{\partial}{\partial y^j}\delta^3(x-y)\right]$$

$R_i = \gamma_{ij}R^j$, γ_{ij}: metric of spatial 3-surfaces.

- Any generally covariant theory contains such an algebra.
- HKT: The tensor that lowers the index on R^i is the physical metric of 3-surfaces.

The HKT metric in bimetric theory

Consider $g_{\mu\nu} = (\gamma_{ij}, N, N_i)$ and $f_{\mu\nu} = (\phi_{ij}, L, L_i)$,

$$\mathcal{L}_{g,f} \sim \pi^{ij}\gamma_{ij} + p^{ij}\phi_{ij} - M\tilde{R}^0 - M_i\tilde{R}^i$$

On the surface of second class Constraints.
GCT Algebra:

$$\left\{\tilde{R}^0(x), \tilde{R}^0(y)\right\} = -\left[\tilde{R}^i(x)\tfrac{\partial}{\partial x^i}\delta^3(x-y) - \tilde{R}^i(y)\tfrac{\partial}{\partial y^i}\delta^3(x-y)\right]$$

$$\left\{\tilde{R}^0(x), \tilde{R}_i(y)\right\} = -\tilde{R}^0(y)\tfrac{\partial}{\partial x^i}\delta^3(x-y)$$

$\tilde{R}_i = \phi_{ij}\tilde{R}^j$, ϕ_{ij}: the 3-metric of $f_{\mu\nu}$, or
$\tilde{R}_i = \gamma_{ij}\tilde{R}^j$, γ_{ij}: the 3-metric of $g_{\mu\nu}$.
The HKT metric of bimetric theory is $g_{\mu\nu}$ or $f_{\mu\nu}$, consistent with ghost-free matter couplings

[SFH, A. Lundkvist (to appear)]