MSSM HIGGS BOSON PRODUCTION : GLUON FUSION

Michael Spira (PSI)

I Introduction

II Higgs Production

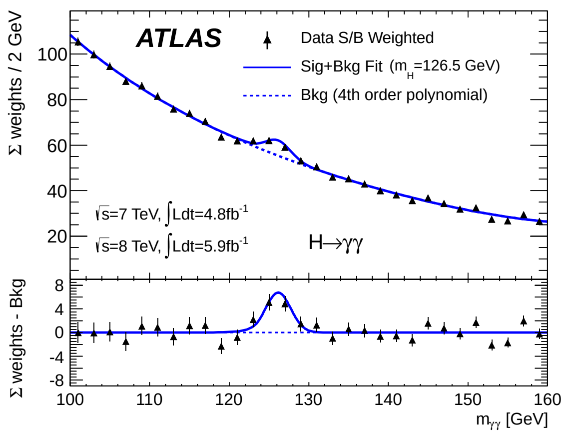
III Conclusions

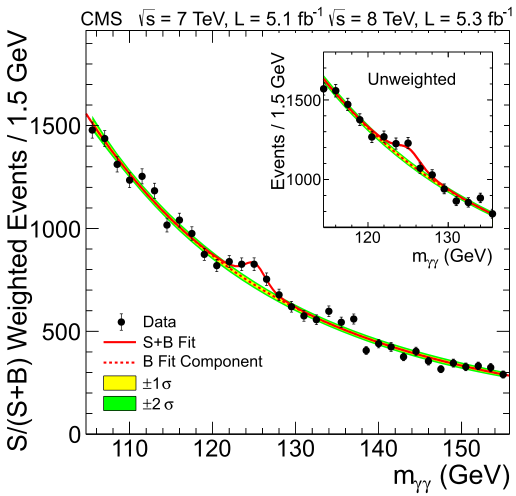
in collaboration with M. Mühlleitner and H. Rzehak

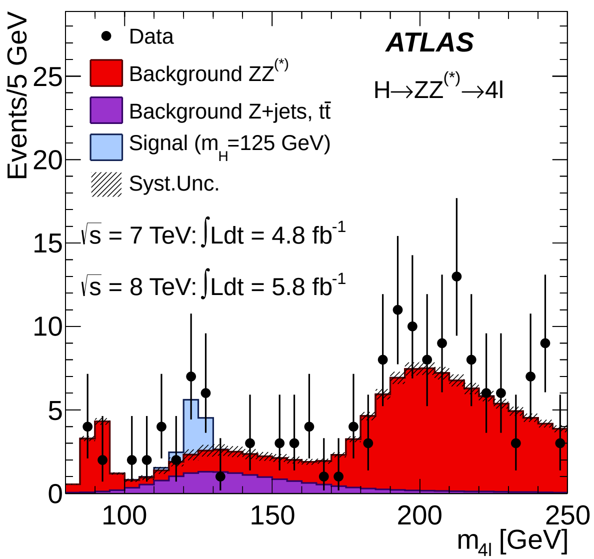
I INTRODUCTION

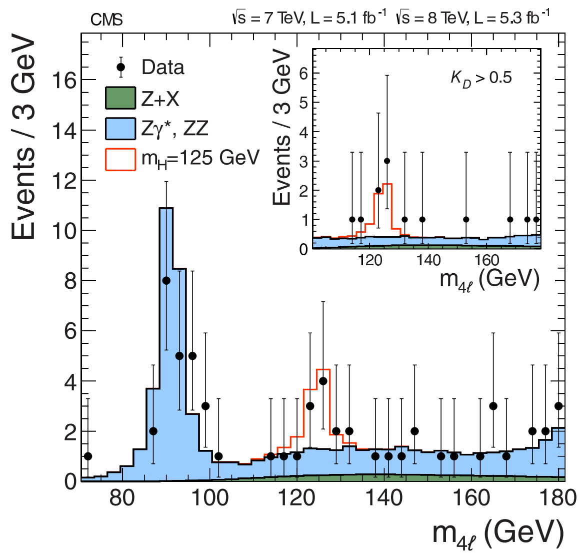
(i) Standard Model

- we have found the Higgs: $M_H \sim 125$ GeV
- $gg \to H$ dominant

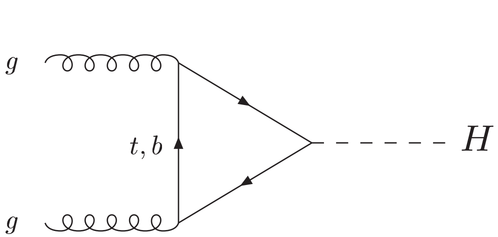


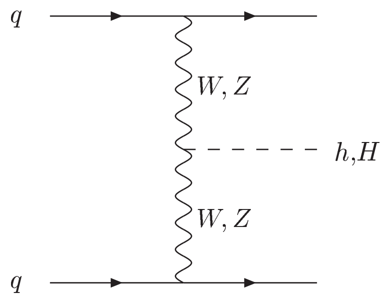


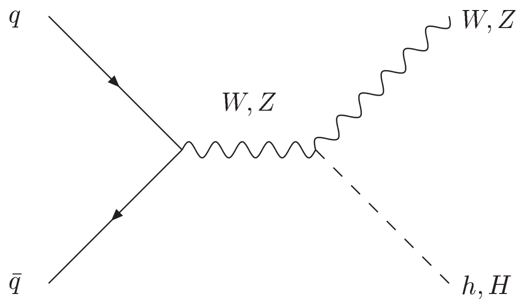


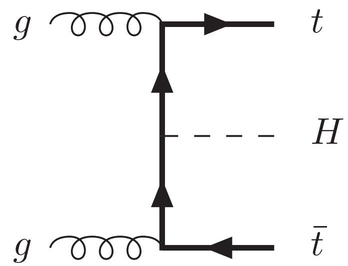


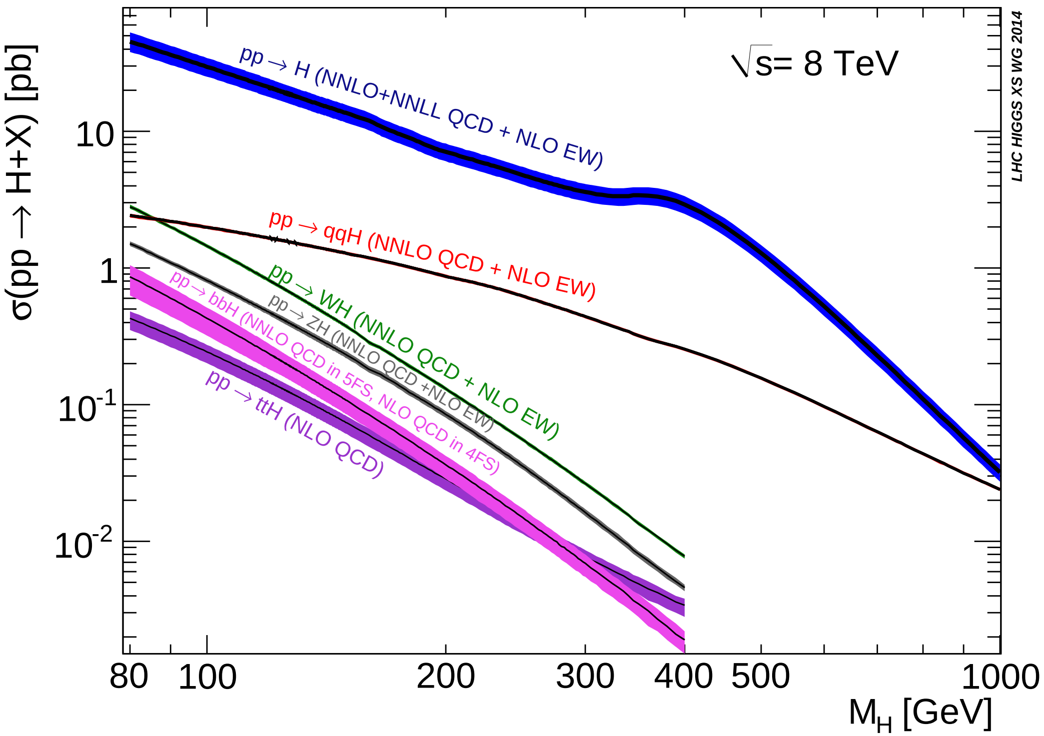
• Higgs Boson Production

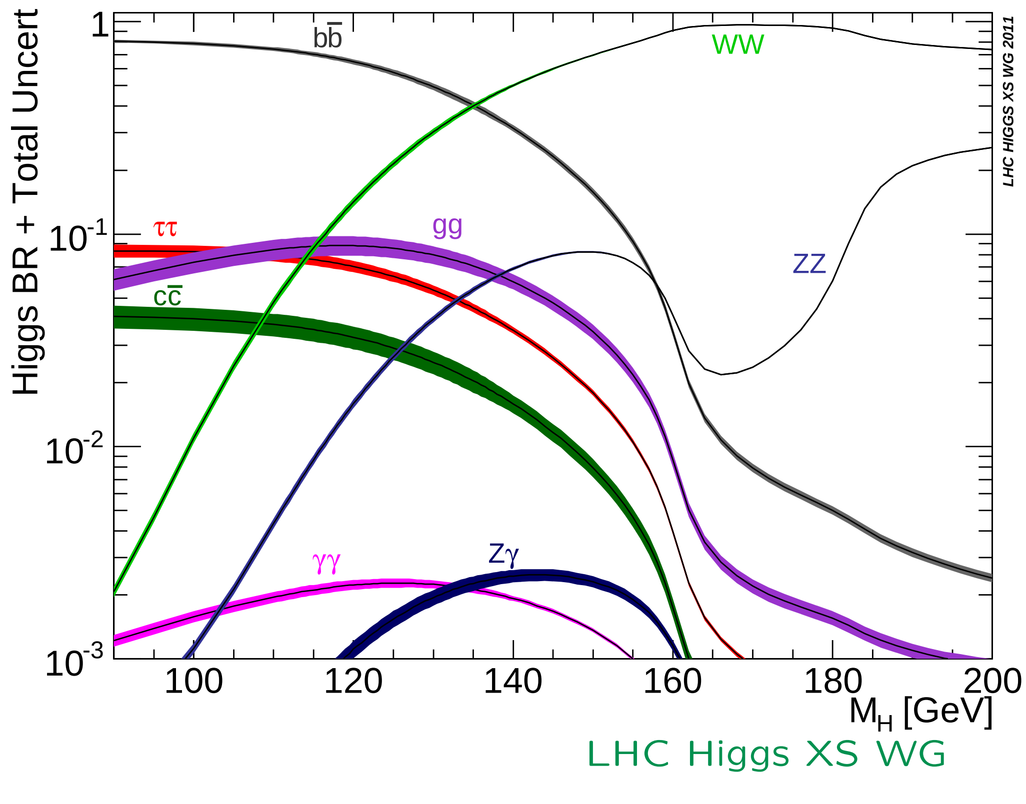












(ii) MSSM

- 2 Higgs doubletts $\stackrel{\text{ESB}}{\longrightarrow}$ 5 Higgs bosons: $h, H, A, H^\pm$

- LO: 2 input parameters: $M_A, \text{tg}\beta = \frac{v_2}{v_1}$

- radiative corrections $\propto m_t^4 \log \frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} \quad \rightarrow \quad \boxed{M_h \lesssim 135 \text{ GeV}}$

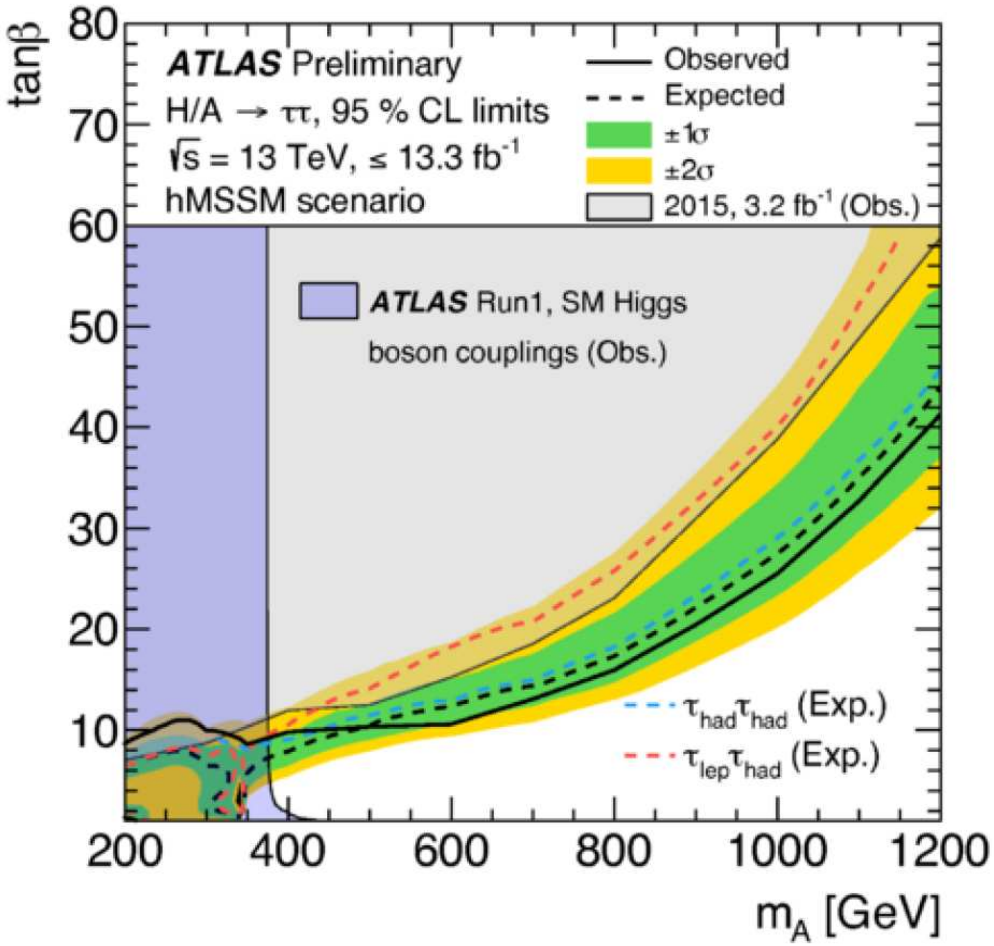
Haber
Carena,. . .
Heinemeyer,. . .
Zhang
Slavich,. . .
. . .

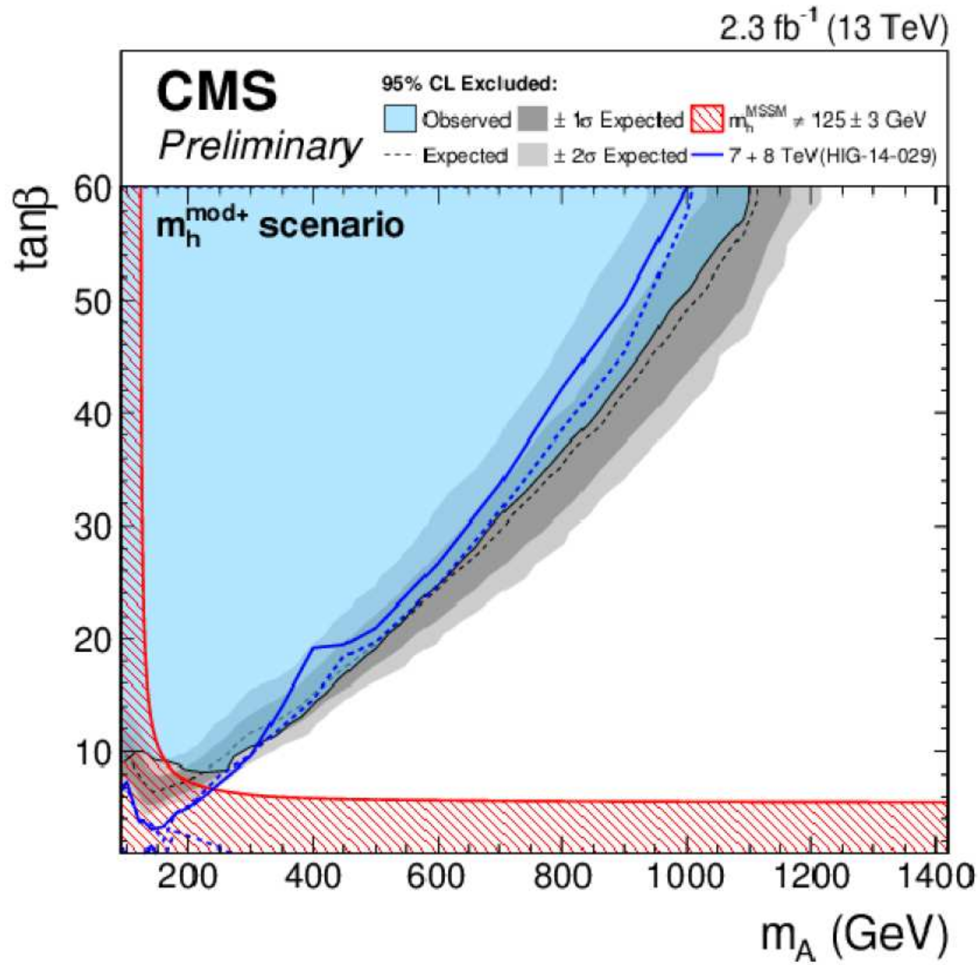
- Yukawa couplings: $\text{tg}\beta\uparrow \quad \Rightarrow \quad g_u^\phi\downarrow \quad g_d^\phi\uparrow \quad g_V^\phi\downarrow$

- LHC: $gg \rightarrow \phi$ dominant for $\text{tg}\beta \lesssim 10$
  $gg \rightarrow \phi b\bar{b}$ dominant for $\text{tg}\beta \gtrsim 10$

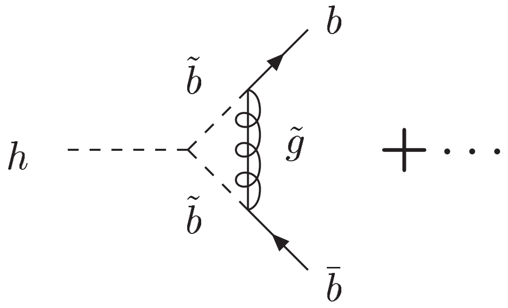
$gg \to b\bar{b}\phi^0, \ gg \to \phi^0 \qquad \phi^0 \to \tau^+\tau^-$

↓ ↓



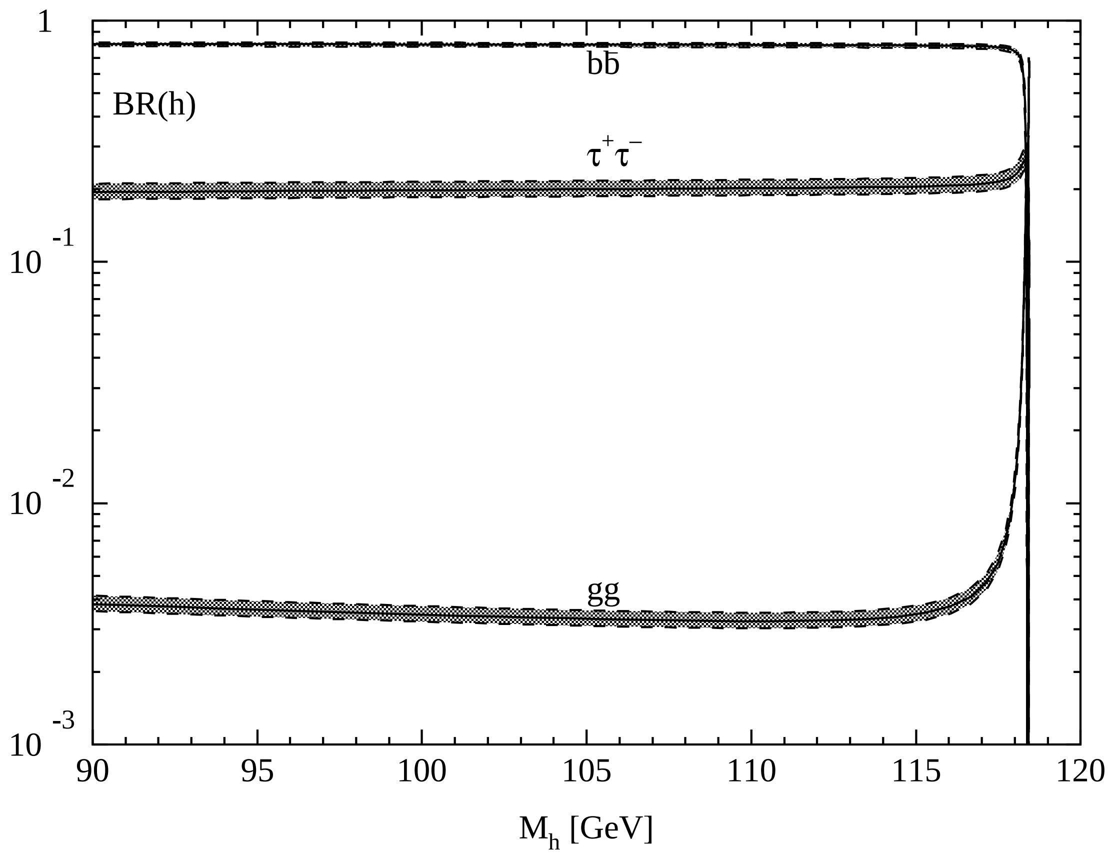


- large SUSY–QCD corrections to $\phi^0 \to b\bar{b}$



$$+ \cdots \qquad \propto \frac{\alpha_s}{\pi} \, \frac{m_{\tilde{g}} \mu \, \mathrm{tg}\beta}{m_{\tilde{b}}^2}$$

Hall,. . .
Carena,. . .
Nierste,. . .
Guasch,. . .
etc.



Guasch, Häfliger, S.

SUSY-QCD Corrections to $b\bar{b}\phi^0$ $[\Delta \lesssim 1\%]$

$$\mathcal{L}_{eff} = -\lambda_b \overline{b_R} \left[\phi_1^0 + \frac{\Delta_b}{\mathrm{tg}\beta}\phi_2^{0*}\right] b_L + h.c. \quad \text{valid to all orders in } \Delta_b$$

$$= -m_b \bar{b}\left[1 + i\gamma_5 \frac{G^0}{v}\right] b - \frac{m_b/v}{1+\Delta_b}\bar{b}\left[g_b^h\left(1 - \frac{\Delta_b}{\mathrm{tg}\alpha\ \mathrm{tg}\beta}\right) h + g_b^H\left(1 + \Delta_b \frac{\mathrm{tg}\alpha}{\mathrm{tg}\beta}\right) H - g_b^A\left(1 - \frac{\Delta_b}{\mathrm{tg}^2\beta}\right) i\gamma_5 A\right] b$$

$$\Delta_b = \Delta_b^{QCD(1)} + \Delta_b^{elw(1)}$$

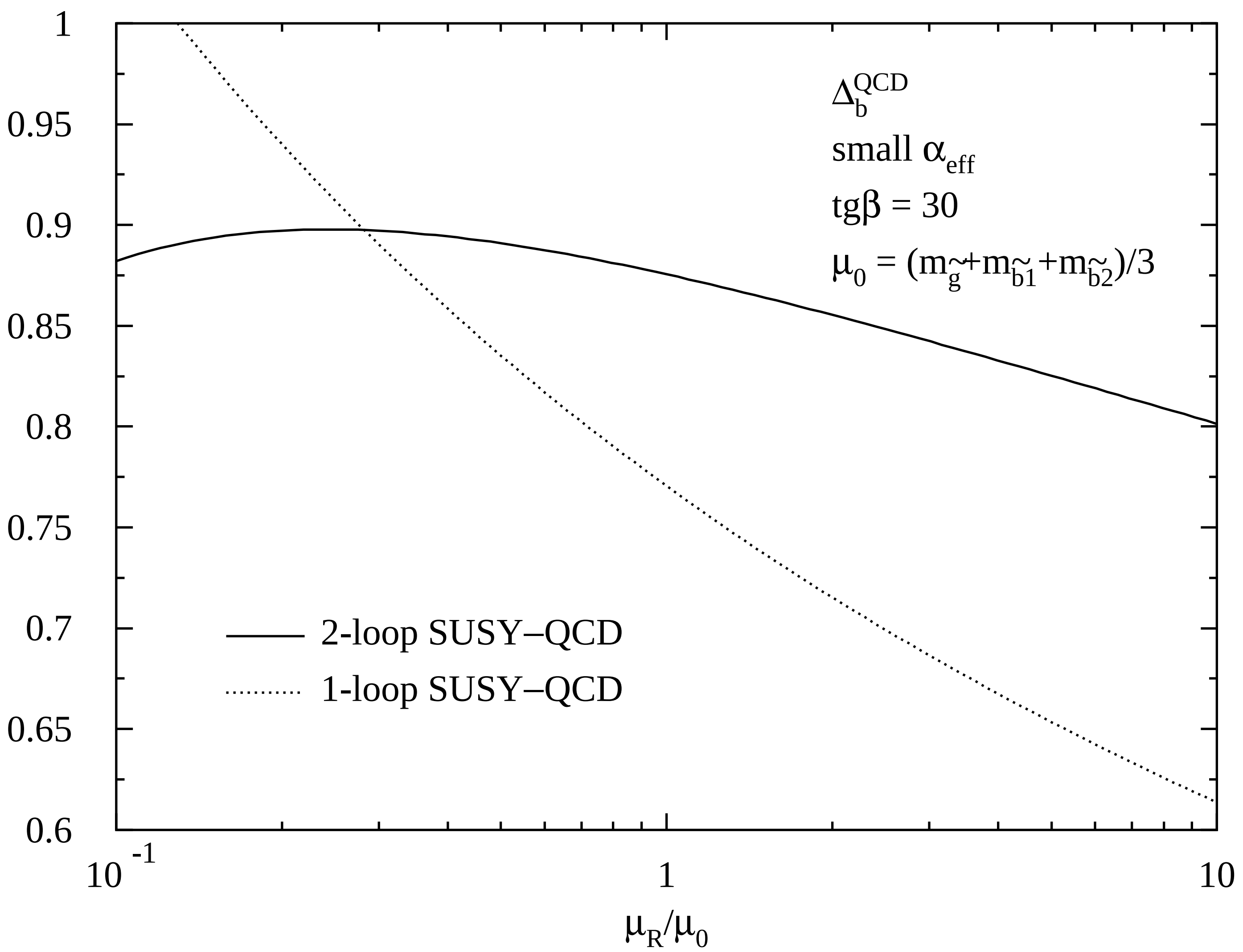
$$\Delta_b^{QCD(1)} = \frac{2}{3}\frac{\alpha_s(\mu_R)}{\pi} M_{\tilde{g}}\ \mu\ \mathrm{tg}\beta\ I(m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}, M^2_{\tilde{g}})$$

$$\Delta_b^{elw(1)} = \frac{\lambda_t^2(\mu_R)}{(4\pi)^2}\ \mu\ A_t\ \mathrm{tg}\beta\ I(m^2_{\tilde{t}_1}, m^2_{\tilde{t}_2}, \mu^2)$$

$$I(a,b,c) = -\frac{ab\log\frac{a}{b} + bc\log\frac{b}{c} + ca\log\frac{c}{a}}{(a-b)(b-c)(c-a)}$$

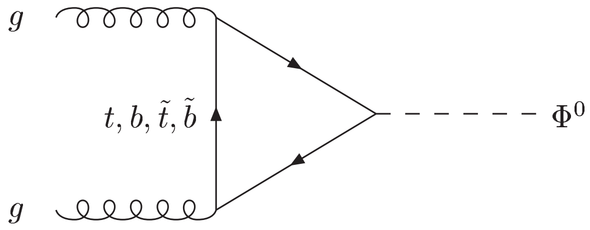
$\Rightarrow$ resummed Yukawa couplings

Carena, Garcia, Nierste, Wagner
Guasch, Häfliger, S.



Noth, S.
[Mihaila, Reisser]

II *HIGGS PRODUCTION* Gluon fusion: $pp \to gg \to h/H/A$



Georgi, Glashow, Machacek, Nanopoulos

- third generation dominant

- NLO QCD corrections: $\sim 10 \ldots 100\%$

S., Djouadi, Graudenz, Zerwas
Dawson, Kauffman

- elw. corrections: $\sim 5\%$

Aglietti,...
Degrassi, Maltoni
Actis, Passarino, Sturm, Uccirati

- NNLO QCD for $m_t \gg M_\phi \Rightarrow +$ 20–30%
  [mass effects small]

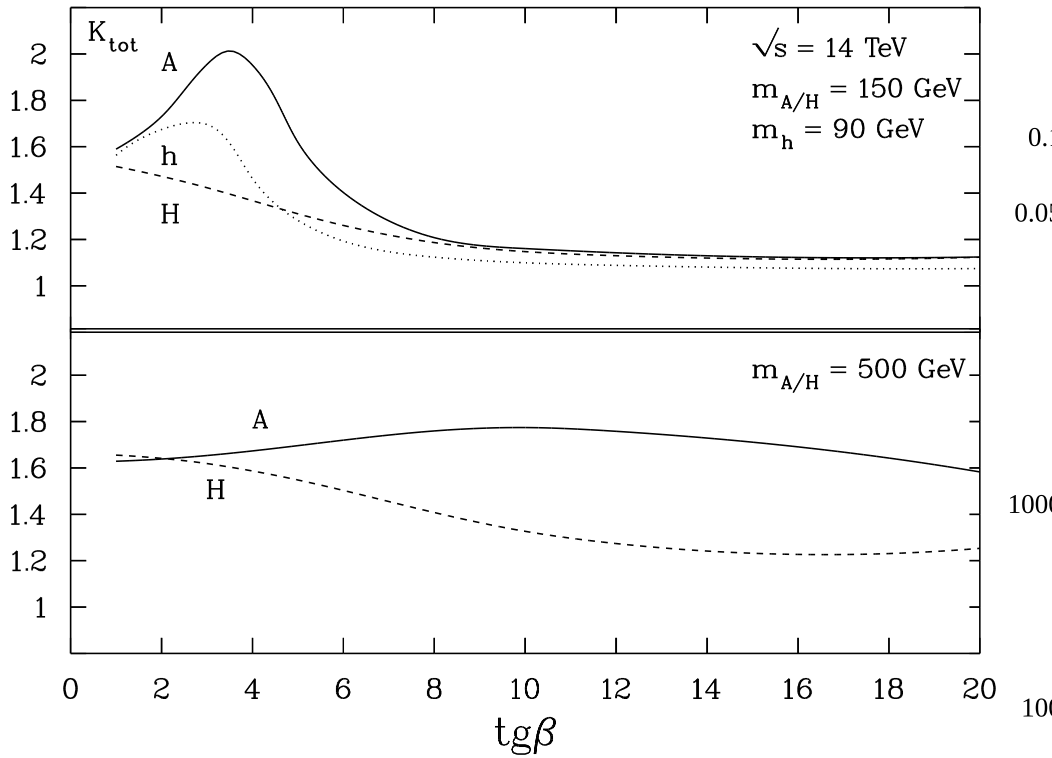
Harlander, Kilgore
Anastasiou, Melnikov
Ravindran, Smith, van Neerven
Marzani, Ball, Del Duca, Forte, Vicini
Harlander, Ozeren
Pak, Rogal, Steinhauser

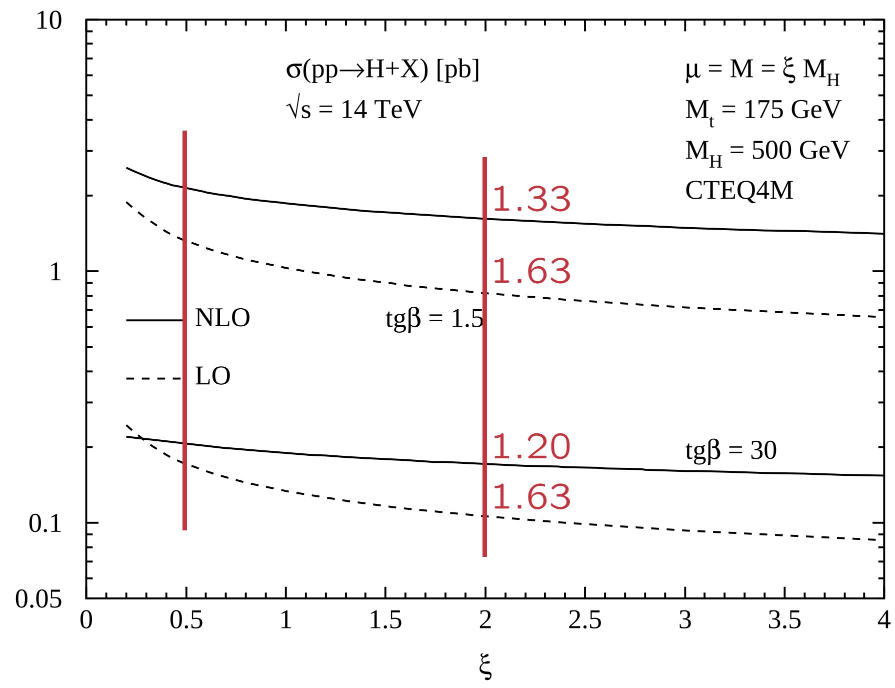
- N^{3}LO for $m_t \gg M_\phi \quad \Rightarrow$ scale stabilization

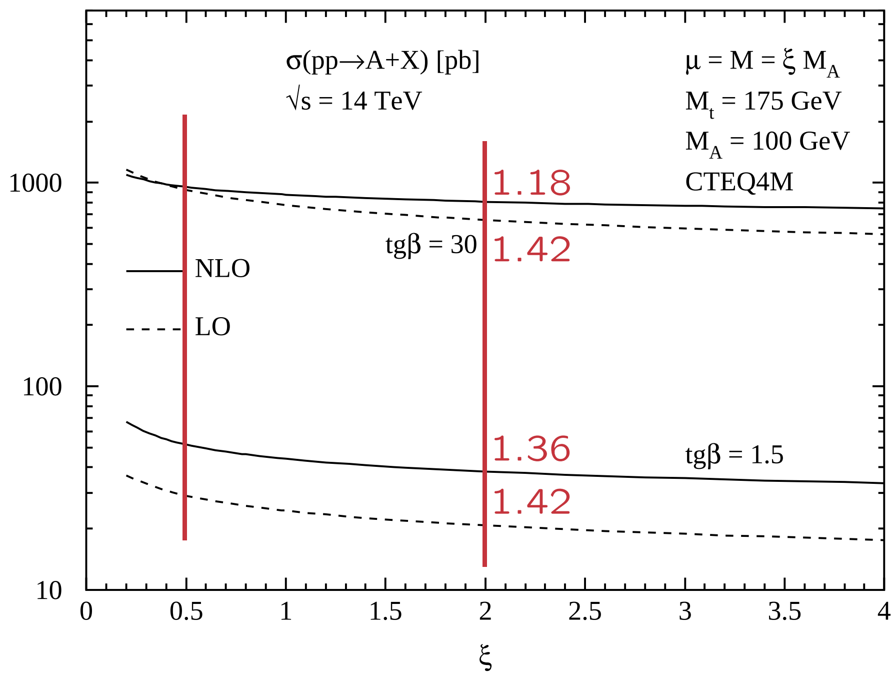
Moch, Vogt
Ravindran
de Florian, Mazzitelli, Moch, Vogt
Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger
Ball, Bonvini, Forte, Marzani, Ridolfi
Anastasiou, Duhr, Dulat, Herzog, Mistlberger
Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger

- N^{3}LL soft gluon resummation [$m_t \gg M_\phi$]: $\lesssim$ 5%

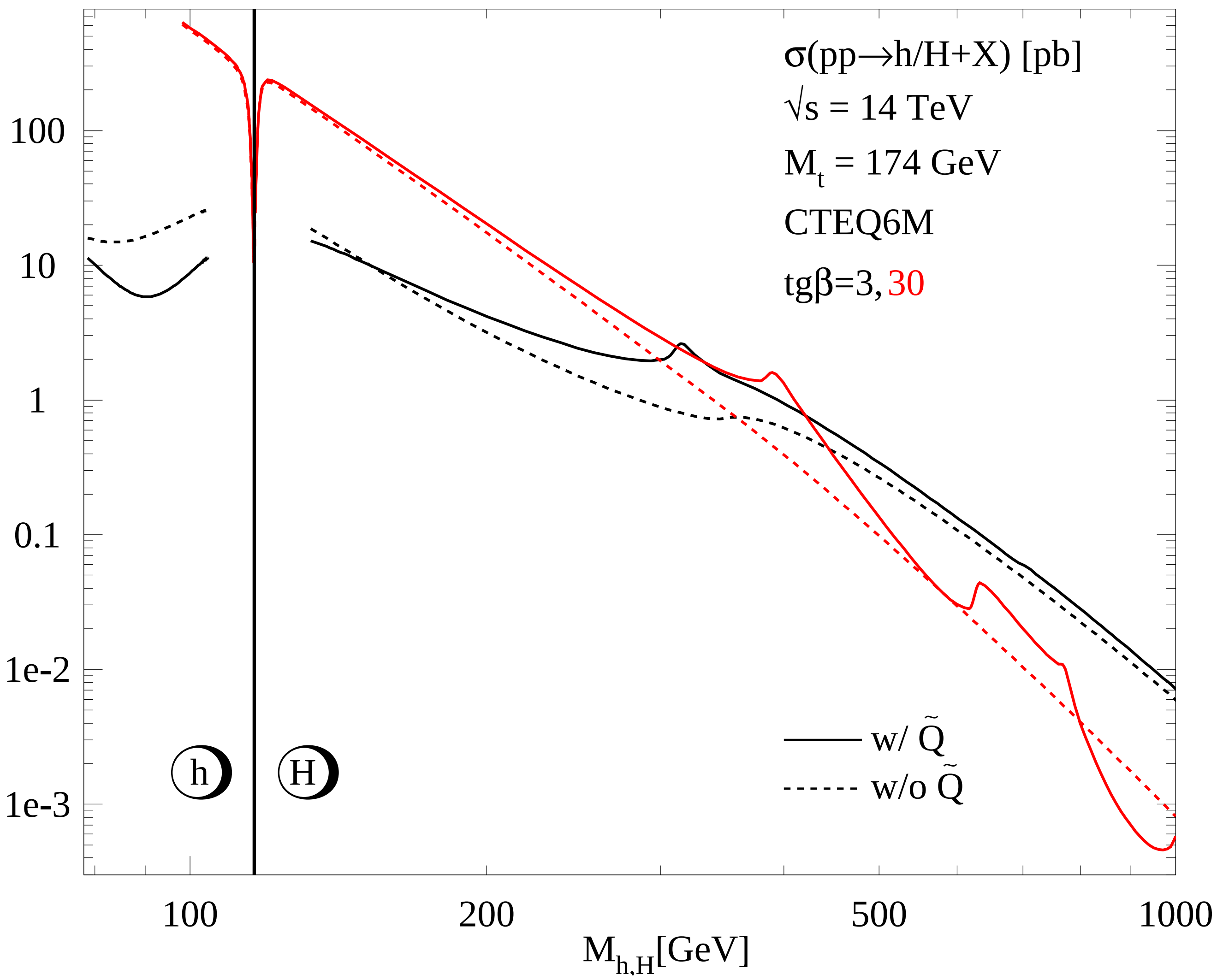
Catani, de Florian, Grazzini, Nason
Ravindran
Ahrens, Becher, Neubert, Yang
Ball, Bonvini, Forte, Marzani, Ridolfi
Schmidt, S.







S., Djouadi, Graudenz, Zerwas

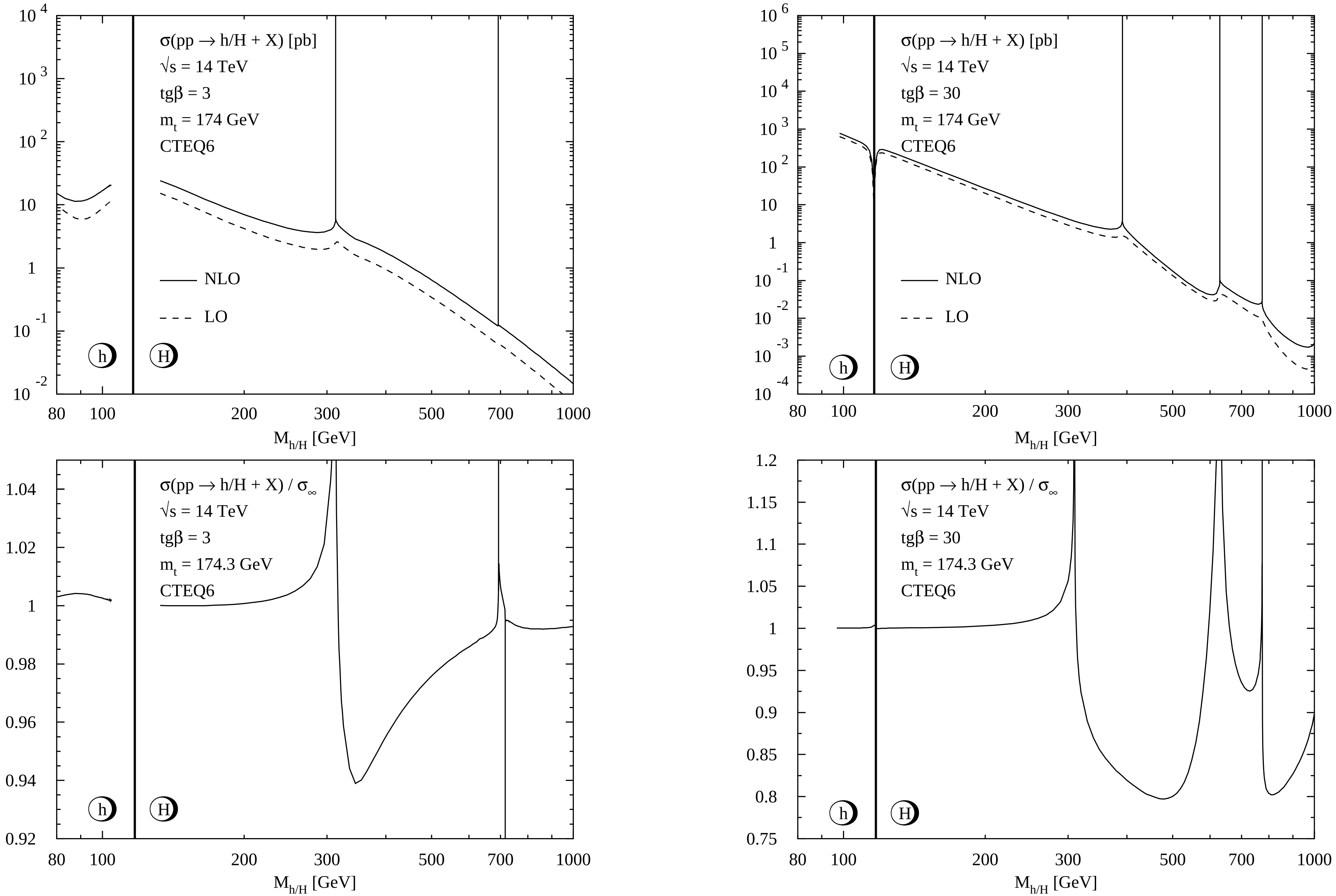


Mühlleitner, S.

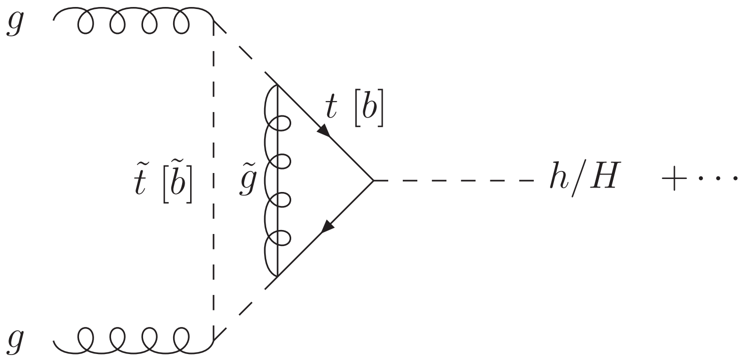
tgβ=3: $m_{\tilde{t}_1}$=156 GeV, $m_{\tilde{t}_2}$=516 GeV, $m_{\tilde{b}_1}$=346 GeV, $m_{\tilde{b}_2}$=358 GeV

tgβ=30: $m_{\tilde{t}_1}$=195 GeV, $m_{\tilde{t}_2}$=502 GeV, $m_{\tilde{b}_1}$=315 GeV, $m_{\tilde{b}_2}$=387 GeV

• QCD corrections to squark loops: Mühlleitner, S.



genuine SUSY–QCD corrections:



Harlander, Steinhauser, Hofmann
Degrassi, Slavich
Anastasiou, Beerli, Daleo
Mühlleitner, Rzehak, S.

$$\sigma_{LO}(pp \to \phi^0) = \sigma_0^\phi \tau_\phi \frac{d\mathcal{L}^{gg}}{d\tau_\phi}$$

$$\sigma_0^{h/H} = \frac{G_F \alpha_s^2}{288\sqrt{2}\pi} \left| \sum_Q g_Q^{h/H} A_Q^{h/H}(\tau_Q) + \sum_{\tilde{Q}} g_{\tilde{Q}}^{h/H} A_{\tilde{Q}}^{h/H}(\tau_{\tilde{Q}}) \right|^2 \qquad \sigma_0^A = \frac{G_F \alpha_s^2}{128\sqrt{2}\pi} \left| \sum_Q g_Q^A A_Q^A(\tau_Q) \right|^2$$

- numerical analysis: $A_Q^{h/H}(\tau_Q) \to A_Q^{h/H}(\tau_Q)\left[1 + C_{SUSY}^Q \frac{\alpha_s}{\pi}\right]$

• $m^2_{Q/\tilde{Q}} \to m^2_{Q/\tilde{Q}}(1 - i\epsilon)$

• 5-dimensional Feynman integrals $\to$ endpoint subtractions:

$$\int_0^1 dx \frac{f(x)}{x(1-x)} \;\to\; \int_0^1 dx \left\{ \frac{f(x)}{x(1-x)} - \frac{f(0)}{x} - \frac{f(1)}{1-x} \right\}$$

$\Rightarrow$ isolation of singularities

• thresholds for $M_H > 2m_Q$ $\to$ numerical instabilities $\to$ integration by parts:

$$\int_0^1 dz \frac{f(z)}{(a+bz)^2} = -\frac{f(z)}{b(a+bz)}\bigg|_0^1 + \int_0^1 dz \frac{f'(z)}{b(a+bz)}$$

$$\int_0^1 dz \frac{f(z)}{a+bz} = \frac{f(z)}{b} \log(a+bz)\bigg|_0^1 - \int_0^1 dz \frac{f'(z)}{b} \log(a+bz)$$

$\Rightarrow$ thresholds in reduced powers of denominators or in arguments of logs $\Rightarrow$ stabilization
[more involved for quadratic polynomials]

- α_s: $\overline{\text{MS}}$ scheme [5 flavours]

- $m_Q, m_{\tilde{Q}}$: on-shell

- A_t, A_b: $\overline{\text{MS}}$

- A_t, A_b: anomalous SUSY-restoring counter terms

- θ_Q: anti-Hermitean:

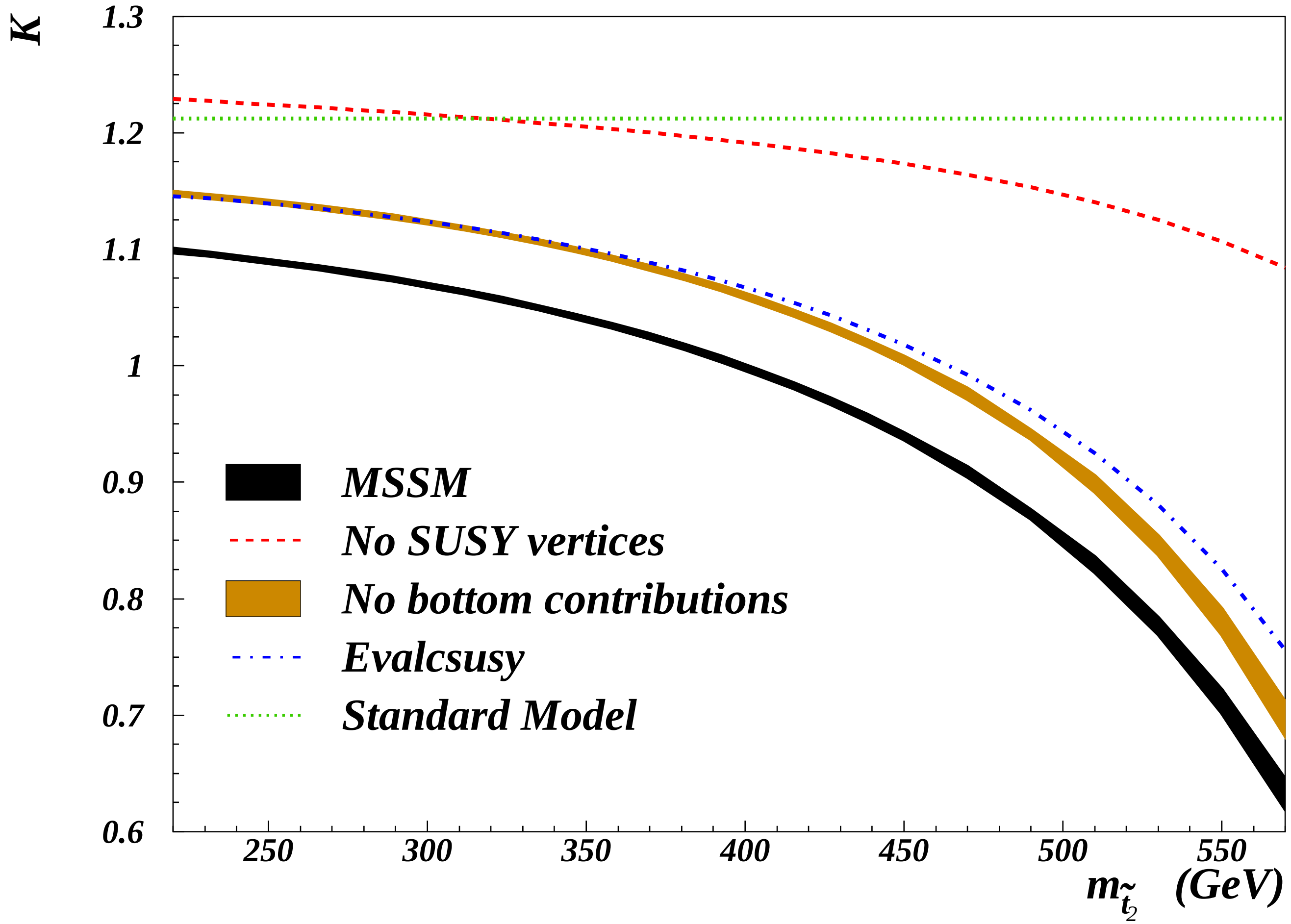
$$\delta\theta_t \;=\; \frac{1}{2}\,\Re e\frac{\Sigma_{12}(m^2_{\tilde{t}_1}) + \Sigma_{12}(m^2_{\tilde{t}_2})}{m^2_{\tilde{t}_2} - m^2_{\tilde{t}_1}}$$

- bottom Yukawa: resummation of Δ_b contributions

SUSY-QCD corrections:

$\alpha = 3^o, \mathrm{tg}\beta = 20, \mu = 300$ GeV, $M_{\tilde{g}} = 500$ GeV, $m_{\tilde{t}_1} = 150$ GeV

$m_{\tilde{t}_2} = 350$ GeV, $m_{\tilde{b}_1} = 350$ GeV, $m_{\tilde{b}_2} = 370$ GeV, $\theta_{\tilde{t}/\tilde{b}} = 40^o$



Anastasiou, Beerli, Daleo

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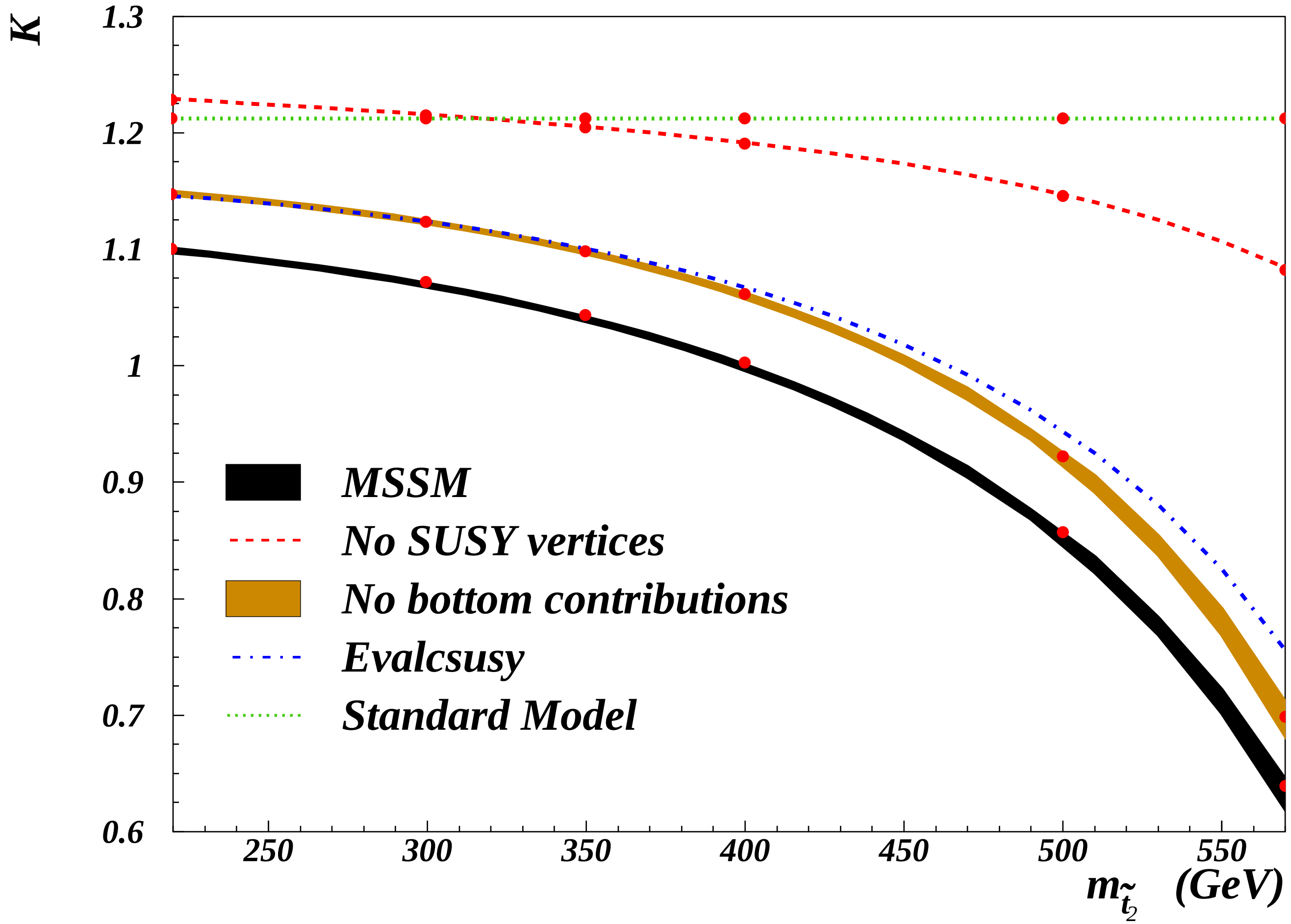

PRELIMINARY

Anastasiou, Beerli, Daleo
Mühlleitner, Rzehak, S.

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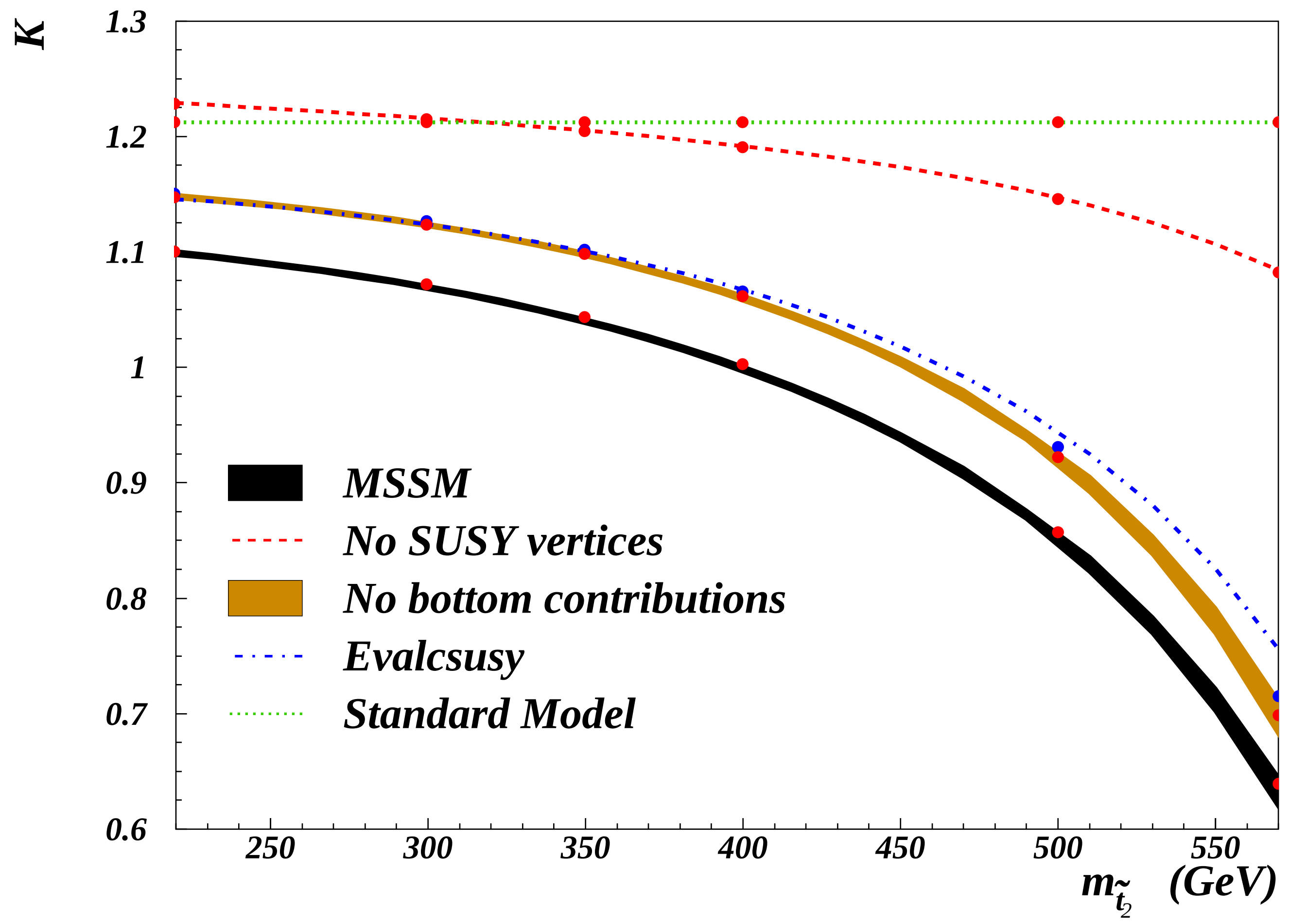

PRELIMINARY

Anastasiou, Beerli, Daleo
Mühlleitner, Rzehak, S.
Harlander, Steinhauser
Degrassi, Slavich

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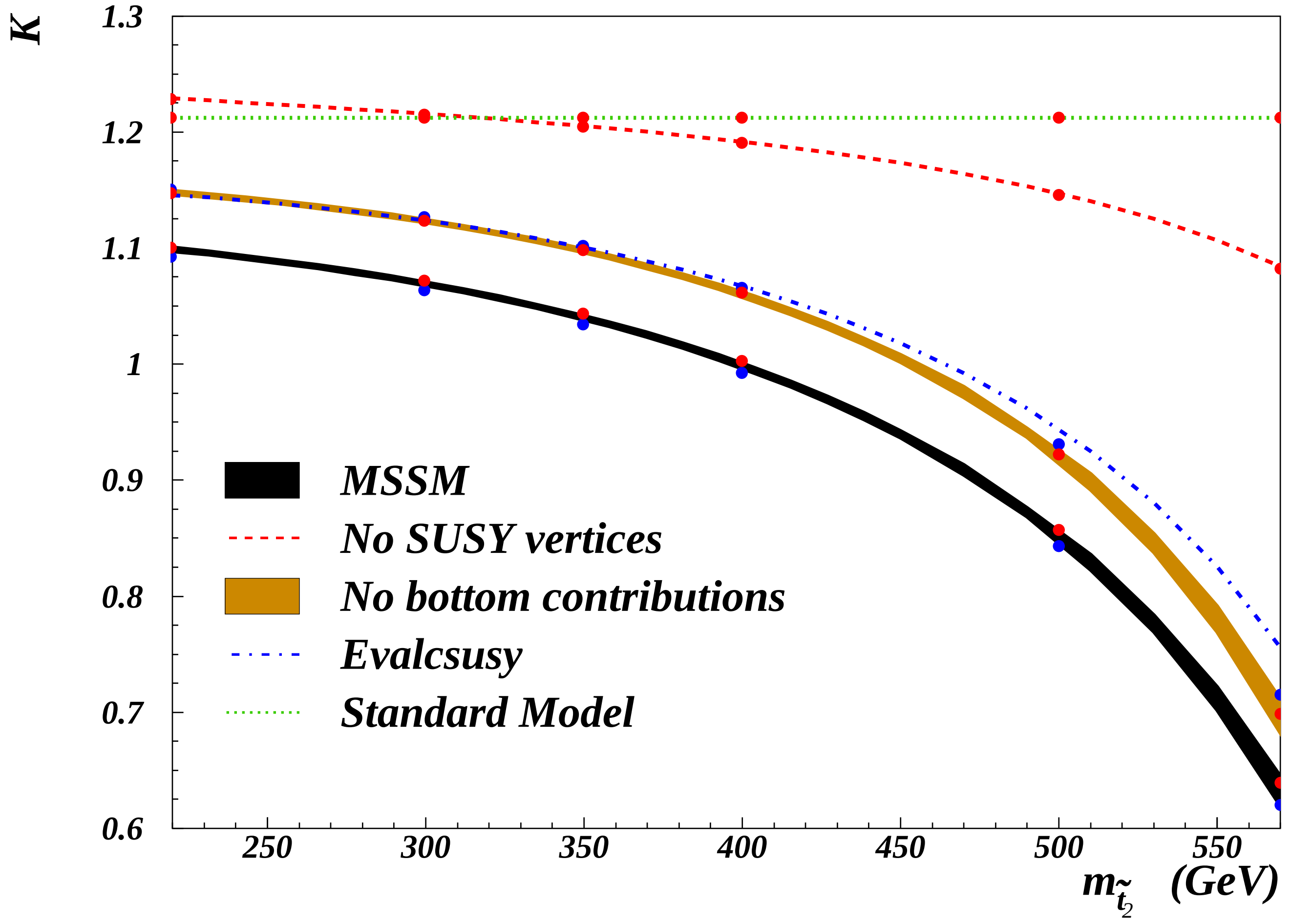

PRELIMINARY

Anastasiou, Beerli, Daleo
Mühlleitner, Rzehak, S.
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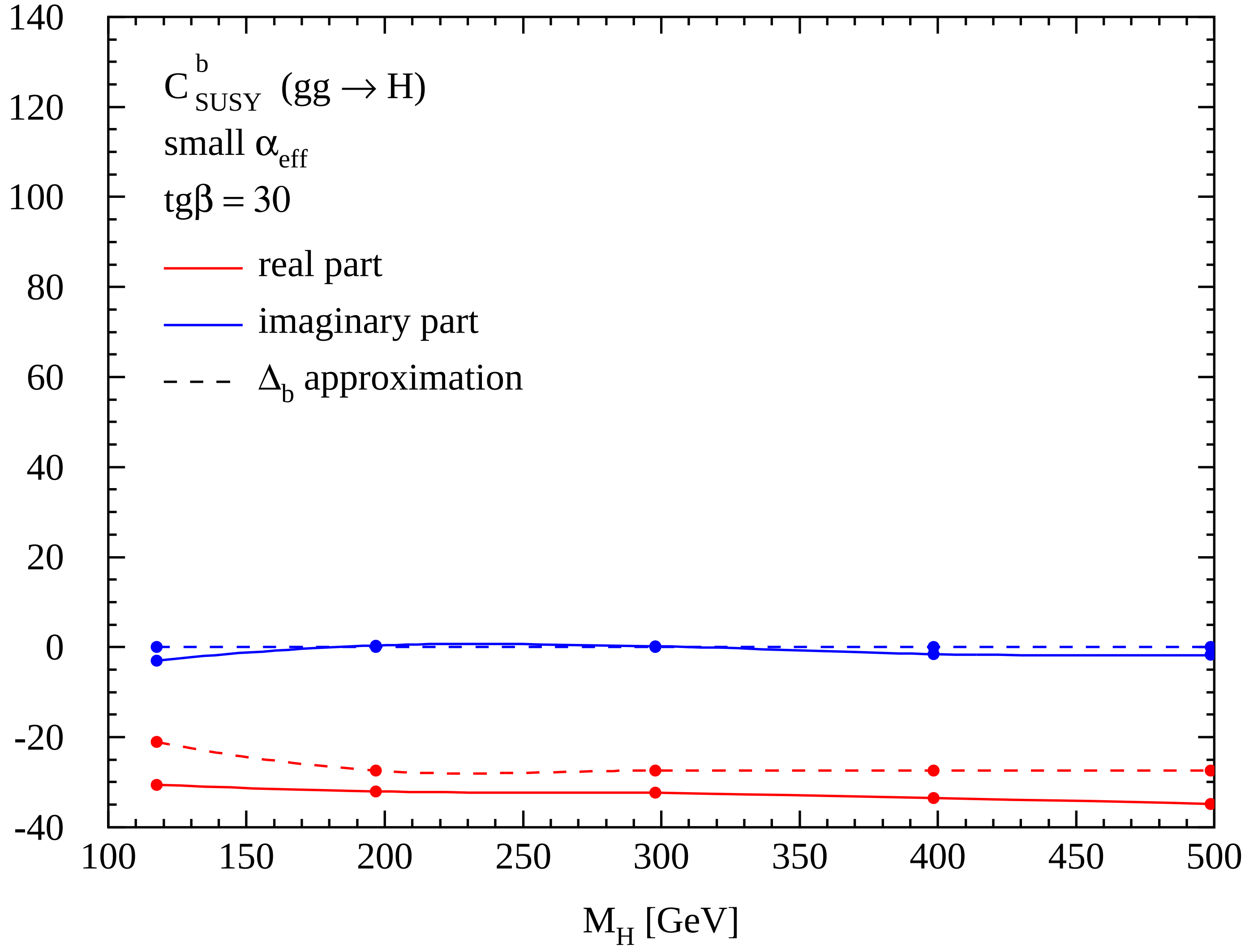
• 2009: PRELIMINARY



$$\Delta \sim -5\frac{\alpha_s}{\pi} \sim -15\%$$

Mühlleitner, Rzehak, S.

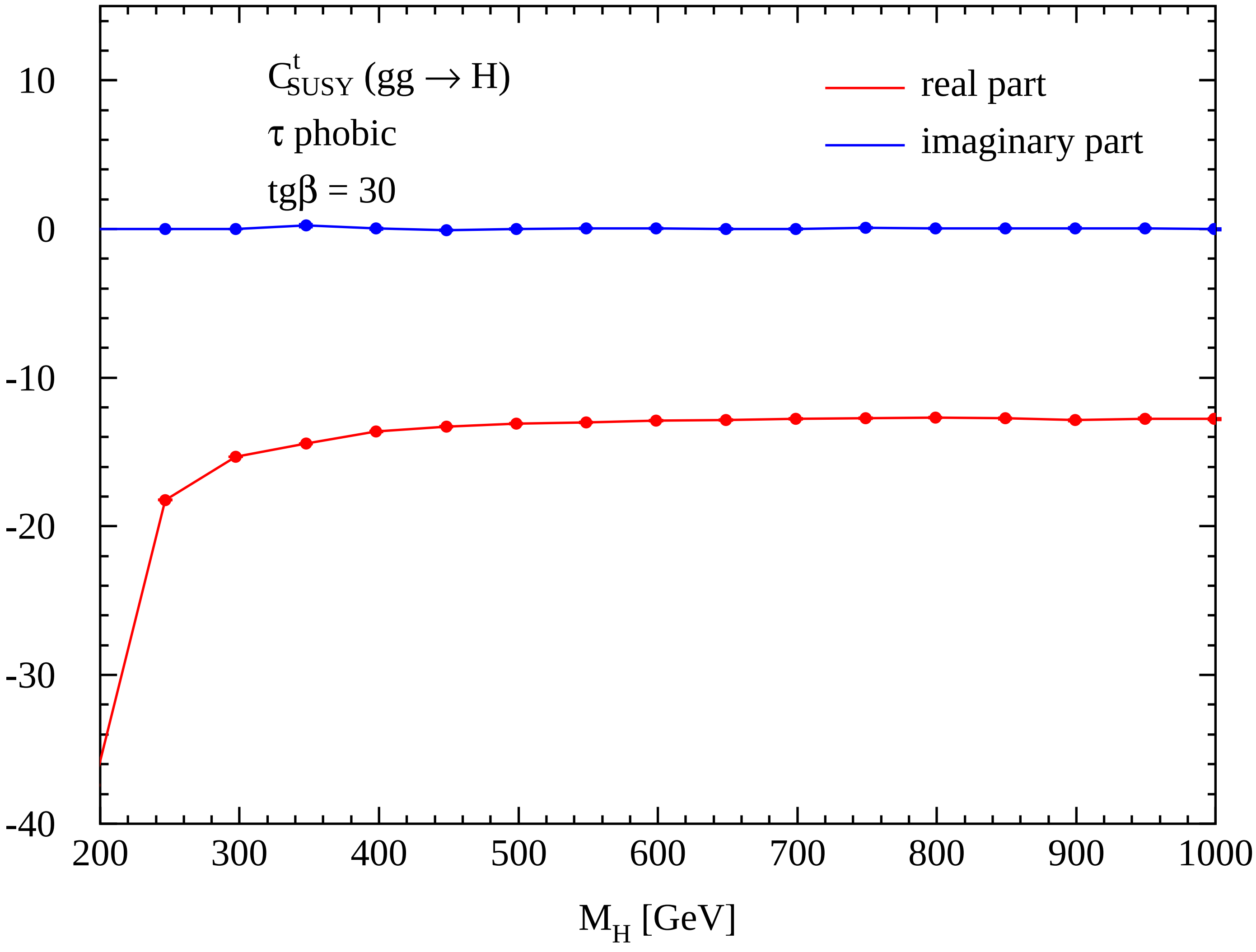
PAUL SCHERRER INSTITUT

τ-phobic scenario [scale = 1 TeV]

$$
\begin{aligned}
m_t &= 173.2 \text{ GeV} \\
\text{tg}\beta &= 30 \\
M_{\tilde{Q}} &= 1.5 \text{ TeV} \\
M_{\tilde{g}} &= 1.5 \text{ TeV} \\
M_2 &= 200 \text{ GeV} \\
A_b = A_t &= 4.417 \text{ TeV} \qquad [X_t = 2.9\ M_{\tilde{Q}}] \\
\mu &= 2 \text{ TeV} \\
M_{\tilde{\ell}_3} &= 500 \text{ GeV}
\end{aligned}
$$

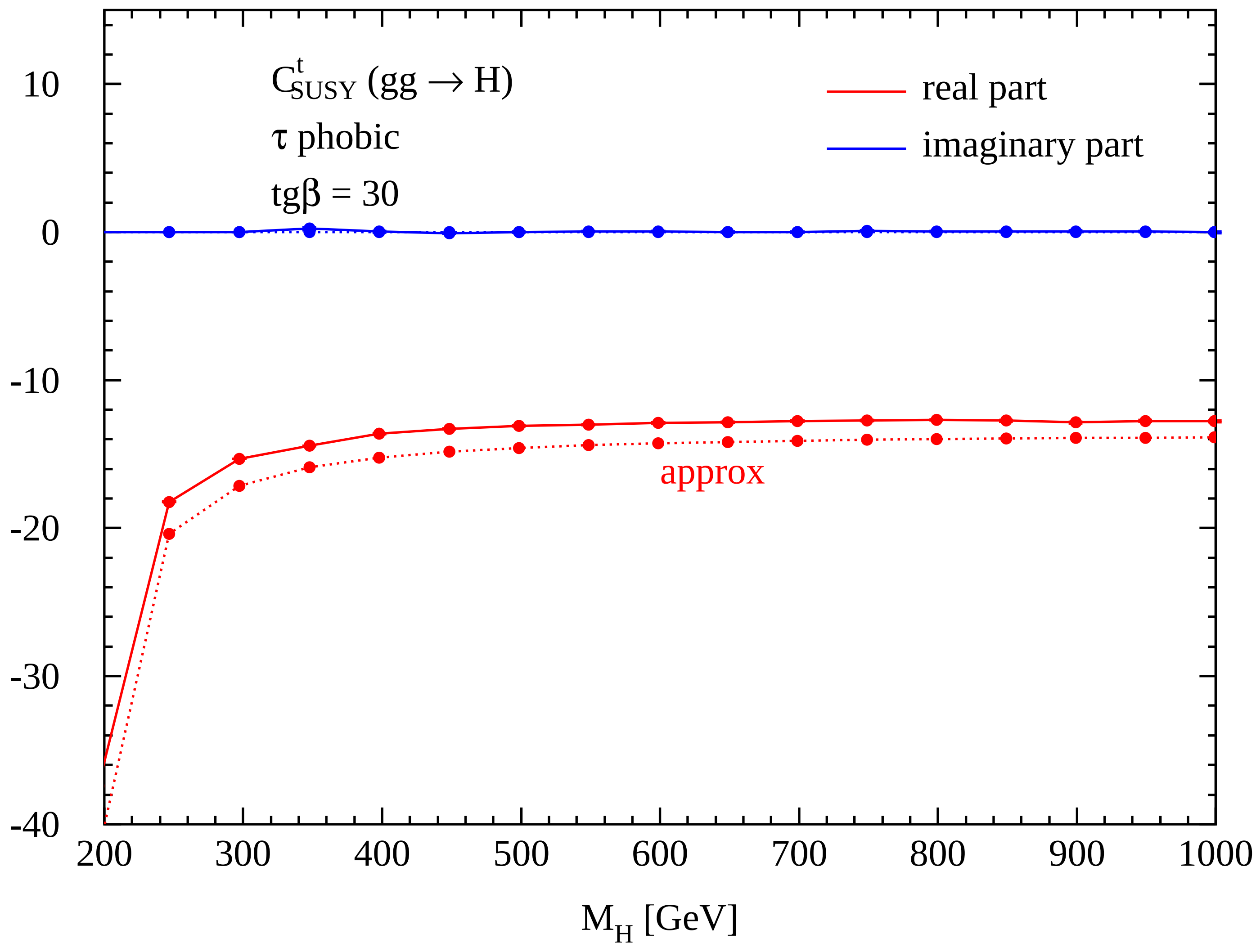
$$
\begin{aligned}
m_{\tilde{t}_1} &= 1.318 \text{ TeV} \qquad m_{\tilde{t}_2} = 1.726 \text{ TeV} \\
m_{\tilde{b}_1} &= 1.501 \text{ TeV} \qquad m_{\tilde{b}_2} = 1.565 \text{ TeV}
\end{aligned}
$$

PRELIMINARY



Mühlleitner, Rzehak, S.

PRELIMINARY



Mühlleitner, Rzehak, S.
Harlander, Steinhauser
Degrassi, Slavich

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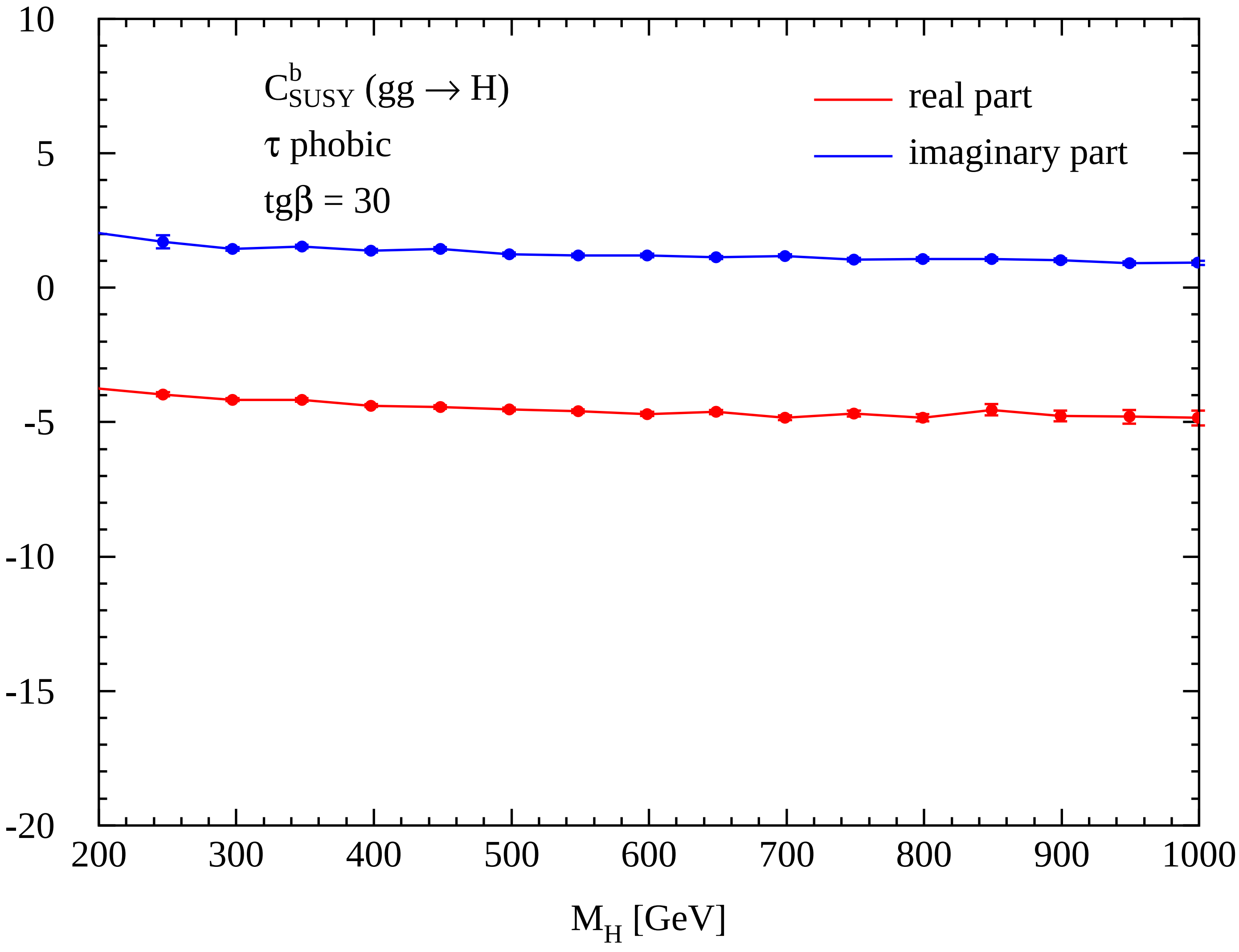
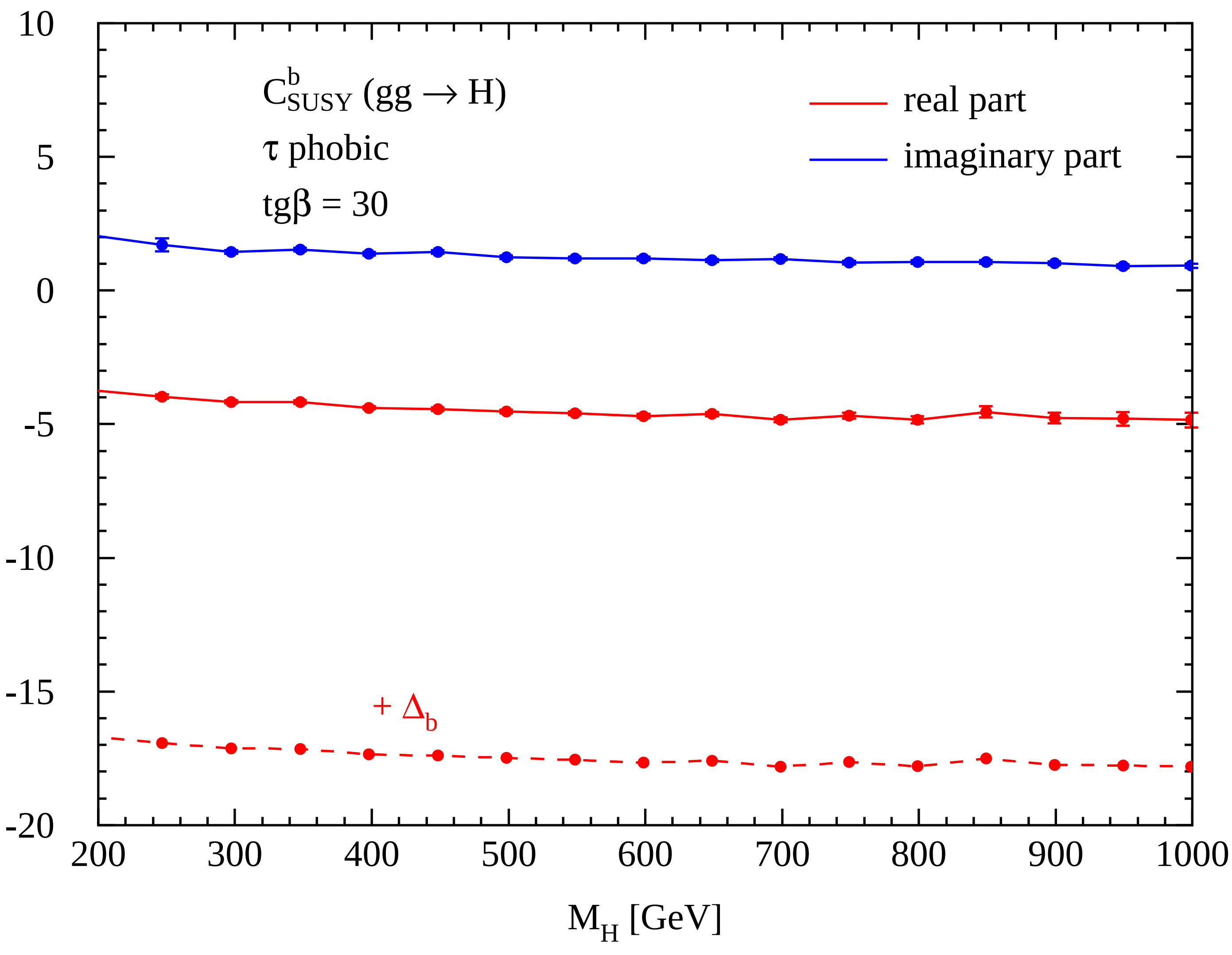


Mühlleitner, Rzehak, S.

PRELIMINARY



Mühlleitner, Rzehak, S.

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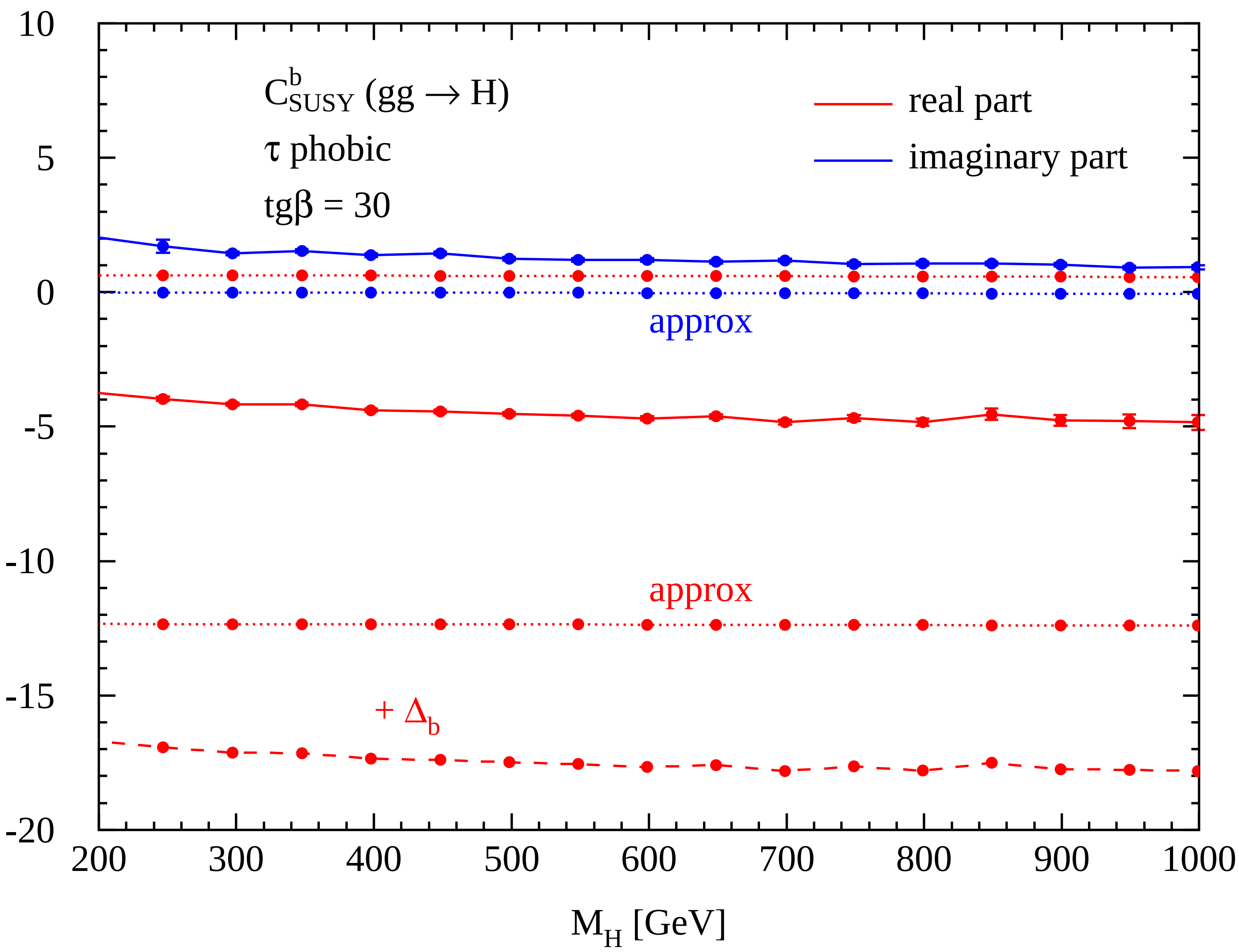

Mühlleitner, Rzehak, S.
Degrassi, Slavich

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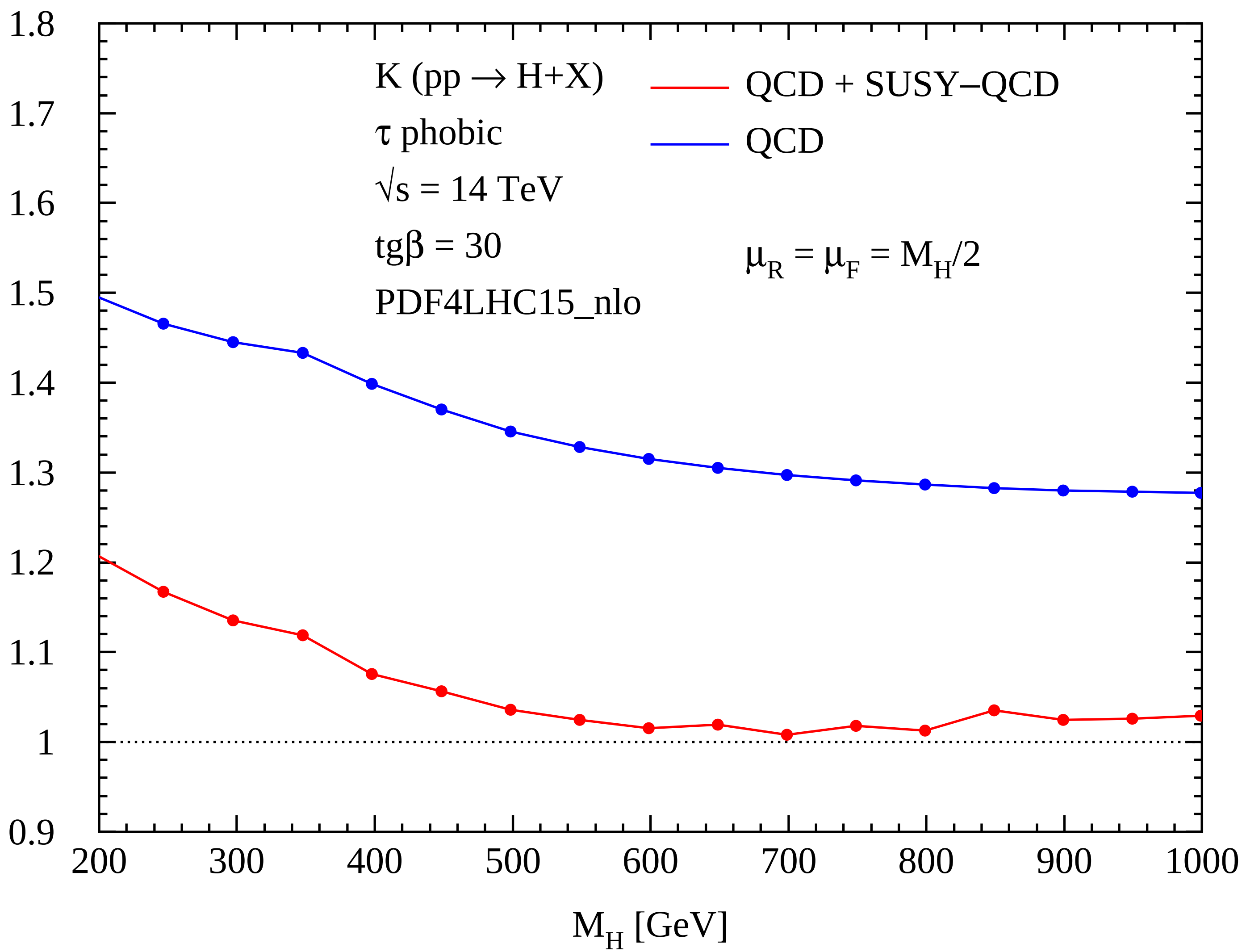
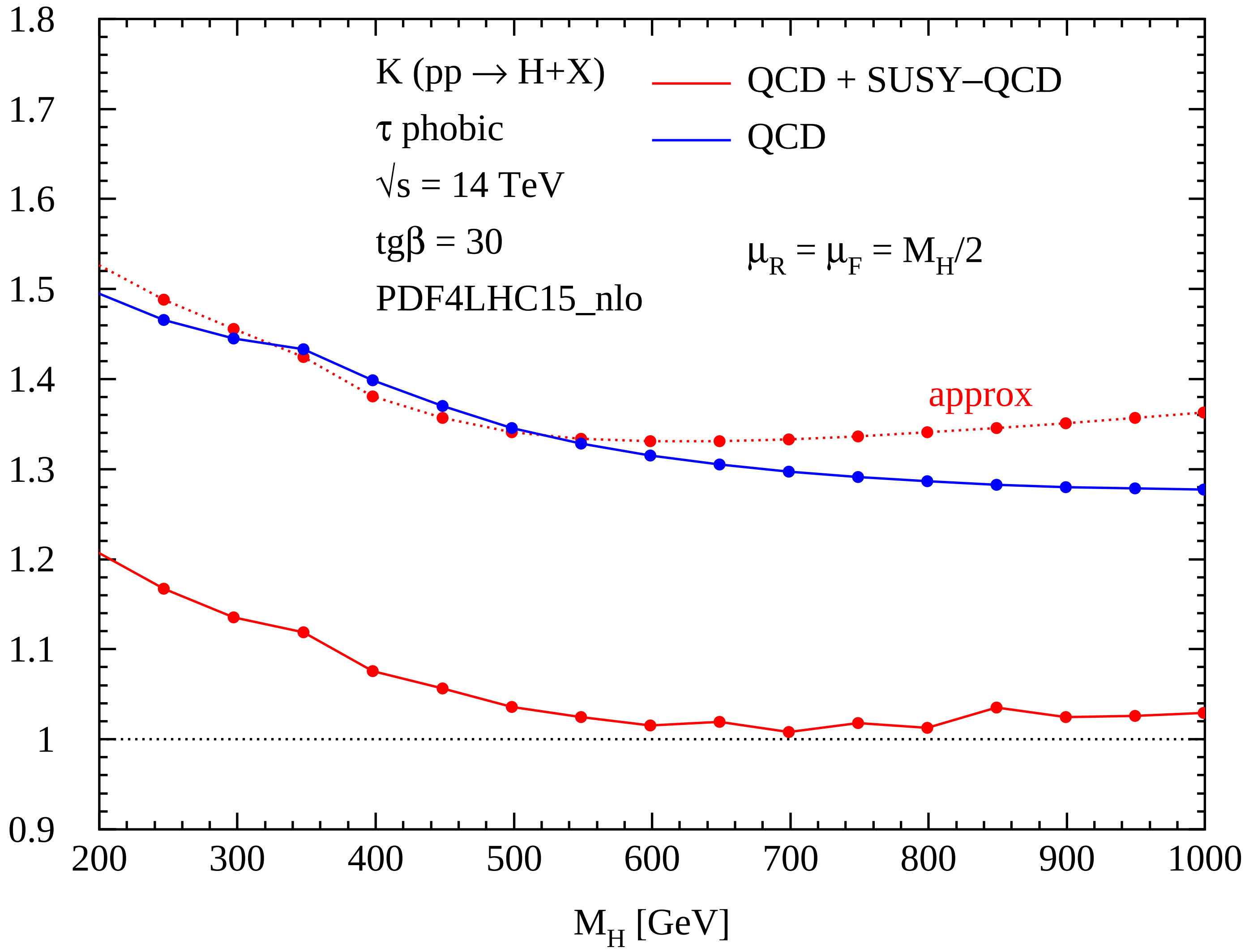


Mühlleitner, Rzehak, S.

PRELIMINARY



Mühlleitner, Rzehak, S.
Harlander, Steinhauser
Degrassi, Slavich

III CONCLUSIONS

- $gg \to h/H$: genuine SUSY–QCD corrections large

- sizeable corrections beyond Δ_b approximation
  $\Rightarrow$ sizeable differences to approximate calculations

- outlook: other scenarios
  pseudoscalar A
  application to HH @ NLO
  [results soon. . .]

BACKUP SLIDES

small α_{eff} scenario [modified]

$$
\begin{aligned}
\text{tg}\beta &= 30 \\
M_{\tilde{Q}} &= 800 \text{ GeV} \\
M_{\tilde{g}} &= 1000 \text{ GeV} \quad \longleftarrow \\
M_2 &= 500 \text{ GeV} \\
A_b = A_t &= -1.133 \text{ TeV} \\
\mu &= 2 \text{ TeV}
\end{aligned}
$$

$$
m_{\tilde{t}_1} = 679 \text{ GeV} \qquad m_{\tilde{t}_2} = 935 \text{ GeV}
$$

$$
m_{\tilde{b}_1} = 601 \text{ GeV} \qquad m_{\tilde{b}_2} = 961 \text{ GeV}
$$

$$\mathcal{L}_{eff} = -\lambda_b \overline{b_R}\left[\phi_1^0 + \frac{\Delta_b}{\mathrm{tg}\beta}\phi_2^{0*}\right] b_L + h.c. \quad \Rightarrow \quad \lambda_b = \frac{m_b}{v_1\ (1+\Delta_b)} = \frac{\widehat{m}_b}{v_1}$$

- input squark mass matrix: $\widehat{m}_b(Q_0) = \frac{\overline{m}_b(Q_0)}{1+\Delta_b}$ $\qquad \widehat{m}_t(Q_0) = \overline{m}_t(Q_0)$

$$\mathcal{M}_{\tilde{q}} = \begin{bmatrix} \tilde{M}^2_{\tilde{q}_L}(Q_0) + \widehat{m}^2_q(Q_0) + \Delta_{11} & \widehat{m}_q(Q_0)[A_q(Q_0) - \mu r_q] + \Delta_{12} \\ \widehat{m}_q(Q_0)[A_q(Q_0) - \mu r_q] + \Delta_{21} & \tilde{M}^2_{\tilde{q}_R}(Q_0) + \widehat{m}^2_q(Q_0) + \Delta_{22} \end{bmatrix}$$

$$m^2_{\tilde{q}_{1/2}} = \widehat{m}^2_q(Q_0) + \frac{1}{2}\Big[\tilde{M}^2_{\tilde{q}_L}(Q_0) + \tilde{M}^2_{\tilde{q}_R}(Q_0)$$
$$\mp\sqrt{[\tilde{M}^2_{\tilde{q}_L}(Q_0) - \tilde{M}^2_{\tilde{q}_R}(Q_0)]^2 + 4\widehat{m}^2_q(Q_0)[\bar{A}_q(Q_0) - \mu r_q]^2}\Big] + \Delta m^2_{\tilde{q}_{1/2}}$$

$$\tilde{q}_1 = \tilde{q}_L \cos\theta_q + \tilde{q}_R \sin\theta_q$$
$$\tilde{q}_2 = -\tilde{q}_L \sin\theta_q + \tilde{q}_R \cos\theta_q$$

- tree-level like mixing angle $\tilde{\theta}_q$ ($m_{\tilde{q}_i}$ pole masses):

$$\sin 2\tilde{\theta}_q = \frac{2\widehat{m}_q(Q_0)[A_q(Q_0) - \mu r_q]}{m^2_{\tilde{q}_1} - m^2_{\tilde{q}_2}} \qquad \cos 2\tilde{\theta}_q = \frac{\tilde{M}^2_{\tilde{q}_L}(Q_0) - \tilde{M}^2_{\tilde{q}_R}(Q_0)}{m^2_{\tilde{q}_1} - m^2_{\tilde{q}_2}}$$

• SUSY–QCD corr. Δ_{ij} significant $\rightarrow$ new diagonalization (no iteration)

• θ_q in physical amplitudes: remove artificial sing. for $m_{\tilde{q}_1} \sim m_{\tilde{q}_2}$

$$\rightarrow \qquad \delta\theta_q = \frac{1}{2}\,\frac{\Sigma_{12}(m_{\tilde{q}_1}) + \Sigma_{12}(m_{\tilde{q}_2})}{m^2_{\tilde{q}_1} - m^2_{\tilde{q}_2}}$$

• unique relation to tree-level like mixing angle $\tilde{\theta}_q \Rightarrow$ consistent scheme

$$\theta_q = \tilde{\theta}_q + \Delta\tilde{\theta}_q \qquad\qquad \Delta\tilde{\theta}_q = \delta\tilde{\theta}_q - \delta\theta_q$$

$$\delta\tilde{\theta}_q \;=\; \frac{\text{tg } 2\tilde{\theta}_q}{2}\left\{\frac{\delta\hat{m}_q}{\hat{m}_q(Q_0)} + \frac{\delta\bar{A}_q}{\bar{A}_q(Q_0) - \mu r_q} - \frac{\delta m^2_{\tilde{q}_1} - \delta m^2_{\tilde{q}_2}}{m^2_{\tilde{q}_1} - m^2_{\tilde{q}_2}}\right\}$$

• shift mass parameter: $M^2_{\tilde{q}_{L/R}}(Q_0) \rightarrow M^2_{\tilde{q}_{L/R}}(Q_0) + \Delta\overline{M}^2_{\tilde{q}_{L/R}}$

$$\begin{aligned}\tilde{M}^2_{\tilde{q}_L}(Q_0) &= M^2_{\tilde{q}_L}(Q_0) + D_{\tilde{q}_L} = m^2_{\tilde{q}_1}\cos^2\tilde{\theta}_q + m^2_{\tilde{q}_2}\sin^2\tilde{\theta}_q - \hat{m}^2_q(Q_0)\\ \tilde{M}^2_{\tilde{q}_R}(Q_0) &= M^2_{\tilde{q}_R}(Q_0) + D_{\tilde{q}_R} = m^2_{\tilde{q}_1}\sin^2\tilde{\theta}_q + m^2_{\tilde{q}_2}\cos^2\tilde{\theta}_q - \hat{m}^2_q(Q_0)\end{aligned}$$

$$\frac{\delta\widehat{m}_q}{\widehat{m}_q(Q)} = -C_F\frac{\alpha_s}{\pi}\Gamma(1+\epsilon)(4\pi)^\epsilon\frac{3}{4}\left\{\frac{1}{\epsilon}+\log\frac{\bar{\mu}^2}{Q^2}+\delta_{SUSY}\right\}+\Delta_q$$
$$- \quad C_F\frac{\alpha_s}{4\pi}\Big\{B_1[\widehat{m}_q^2(Q);M_{\tilde{g}},m_{\tilde{q}_1}]+B_1[\widehat{m}_q^2(Q);M_{\tilde{g}},m_{\tilde{q}_2}]$$
$$+2M_{\tilde{g}}(\bar{A}_q-\mu r_q)\frac{B_0[\widehat{m}_q^2(Q);M_{\tilde{g}},m_{\tilde{q}_1}]-B_0[\widehat{m}_q^2(Q);M_{\tilde{g}},m_{\tilde{q}_2}]}{m_{\tilde{q}_1}^2-m_{\tilde{q}_2}^2}\Bigg\}$$

$$\delta\bar{A}_q = C_F\frac{\alpha_s}{\pi}\Gamma(1+\epsilon)(4\pi)^\epsilon M_{\tilde{g}}\left\{\frac{1}{\epsilon}+\log\frac{\bar{\mu}^2}{\mu_R^2}\right\}$$

$$
\begin{aligned}
g^{h,H}_{\tilde f_1 \tilde f_1} &= g^{h,H}_{\tilde f_L \tilde f_L} \cos^2\theta_f + g^{h,H}_{\tilde f_R \tilde f_R} \sin^2\theta_f + g^{h,H}_{\tilde f_L \tilde f_R} \sin 2\theta_f \\
g^{h,H}_{\tilde f_2 \tilde f_2} &= g^{h,H}_{\tilde f_L \tilde f_L} \sin^2\theta_f + g^{h,H}_{\tilde f_R \tilde f_R} \cos^2\theta_f - g^{h,H}_{\tilde f_L \tilde f_R} \sin 2\theta_f \\
g^{h,H}_{\tilde f_1 \tilde f_2} &= g^{h,H}_{\tilde f_2 \tilde f_1} = \frac{1}{2}(g^{h,H}_{\tilde f_R \tilde f_R} - g^{h,H}_{\tilde f_L \tilde f_L}) \sin 2\theta_f + g^{h,H}_{\tilde f_L \tilde f_R} \cos 2\theta_f \\
g^{A}_{\tilde f_1 \tilde f_1} &= g^{A}_{\tilde f_2 \tilde f_2} = 0 \\
g^{A}_{\tilde f_1 \tilde f_2} &= -g^{A}_{\tilde f_2 \tilde f_1} = g^{A}_{\tilde f_L \tilde f_R}
\end{aligned}
$$

$$
\begin{aligned}
g^{\Phi}_{\tilde q_L \tilde q_L}(\mu_R) &= \widehat{m}_q^2(\mu_R) g_1^{\Phi} + M_Z^2 (I_{3q} - e_q \sin^2\theta_W) g_2^{\Phi} \\
g^{\Phi}_{\tilde q_R \tilde q_R}(\mu_R) &= \widehat{m}_q^2(\mu_R) g_1^{\Phi} + M_Z^2 e_q \sin^2\theta_W g_2^{\Phi} \\
g^{\Phi}_{\tilde q_L \tilde q_R}(\mu_R) &= -\frac{\widehat{m}_q(\mu_R)}{2} \left[\mu g_3^{\Phi} - \bar{A}_q(\mu_R) g_4^{\Phi} \right]
\end{aligned}
$$

$$\hat{m}_q(\mu_R) = \hat{m}_q(m_q)\,\frac{c[\alpha_s(\mu_R)/\pi]}{c[\alpha_s(m_q)/\pi]}$$

$$c(x) = \left(\frac{23}{6}x\right)^{\frac{12}{23}}\left[1+\frac{3731}{3174}x\right] \qquad \text{for } q=b \quad (N_F=5)$$

$$c(x) = \left(\frac{7}{2}x\right)^{\frac{4}{7}}\left[1+\frac{137}{98}x\right] \qquad \text{for } q=t \quad (N_F=6)$$

$$\bar{A}_q(\mu_R) = \bar{A}_q(Q_0) + M_3(Q_0)\left\{-\frac{16}{9}\left[\frac{\alpha_{s,SUSY}(\mu_R)}{\alpha_{s,SUSY}(Q_0)}-1\right]\left[1+\frac{1}{6}\frac{\alpha_{s,SUSY}(Q_0)}{\pi}\right]\right.$$
$$\left. -\frac{16}{27}\frac{\alpha_{s,SUSY}(Q_0)}{\pi}\left[\frac{\alpha^2_{s,SUSY}(\mu_R)}{\alpha^2_{s,SUSY}(Q_0)}-1\right]\right\}$$

$$\alpha_{s,SUSY}(\mu) = \frac{12\pi}{9\log(\mu^2/\Lambda^2_{SUSY})}\left\{1-\frac{14}{9}\frac{\log\log(\mu^2/\Lambda^2_{SUSY})}{\log(\mu^2/\Lambda^2_{SUSY})}\right\}$$

$$\alpha_{s,SUSY}(Q_0) = \alpha_s(Q_0)\left\{1 + \frac{\alpha_s(Q_0)}{\pi}\left[\frac{1}{6}\log\frac{Q_0^2}{m_t^2} + \frac{1}{2}\log\frac{Q_0^2}{M_{\tilde{g}}^2} + \frac{1}{24}\sum_{\tilde{q}_i}\log\frac{Q_0^2}{m_{\tilde{q}_i}^2}\right]\right\}$$

$$\begin{aligned} M_3(Q_0) \;&=\; M_{\tilde{g}}\left\{1 - \frac{\alpha_s(M_{\tilde{g}})}{\pi}\left[C_A + \frac{3}{4}C_A\log\frac{Q_0^2}{M_{\tilde{g}}^2}\right.\right. \\ &\quad + \frac{1}{4}\sum_{q,i}\left(B_1(M_{\tilde{g}}^2; m_q, m_{\tilde{q}_i}) + \frac{\Gamma(1+\epsilon)(4\pi)^\epsilon}{2\epsilon} + \frac{1}{2}\log\frac{\bar{\mu}^2}{Q_0^2}\right. \\ &\quad \left.\left.\left. - (-1)^i\frac{m_q}{M_{\tilde{g}}}\sin 2\theta_q B_0(M_{\tilde{g}}^2; m_q, m_{\tilde{q}_i})\right)\right]\right\} \end{aligned}$$