Surrogate models for Direct Detection

By Andrew Cheek

Based on arXiv:1802.03174 (https://arxiv.org/abs/1802.03174) with D. Cerdeño, E. Reid and H. Schulz

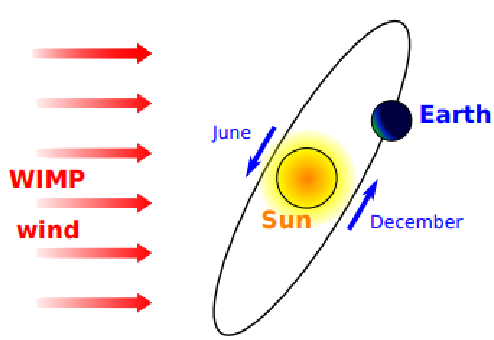
Surrogate models for Direct Detection

- A surrogate model refers to emulators of the true calculations.
- We have developed RAPIDD, a working surrogate model for general DD responses.
- First I will motivate the need for RAPIDD.
- Then I will review the how/if/when RAPIDD works.
- Finally I will (very briefly) talk about current work and status of Rapidd.

Dark Matter Direct Detection Calculation

- Direct Detection exploits the relative velocity of us and Dark Matter to tell us something about the possible interactions DM has with ordinary matter.
- In order to calculate the number of recoils in a given energy bin, one typically needs to evaluate these nested integrals.

$$N_k = \frac{\rho_0 \epsilon}{m_T\, m_\chi} \int_{E_k}^{E_{k+1}} dE_R\; \varepsilon(E_R) \int_{E_R'} dE_R'\; Res(E_R', E_R) \int_{v_{min}} d\vec{v}\; v f(\vec{v})\, \frac{d\sigma_{\chi T}}{dE_R'}$$



Halo Integrals

- The energy deposited in the experiment is dependent on the relative velocity of DM and the target $E_{\mathrm{R}} = 2\frac{\mu_{T\chi}}{m_T} v_{\mathrm{min}}^2$.
- The velocity distribution of incident DM particles is often assumed to be maxwellian $f(v) = \left(\frac{1}{N}\right) \exp(-v^2/v_0^2)\Theta(v_{esc} - v)$, which can be integrated analytically.
- However, to account for uncertainties in halo parameters, and unknowns about the shape of this distribution, one could take results from simulations or data.
- One could also use a general form which can recreate the general shape that we expect with

$$f(v) = N_k^{-1}\left[e^{-v^2/kv_0^2} - e^{-v_{esc}^2/kv_0^2}\right]^k \Theta(v_{esc} - v)$$

Cross-Section

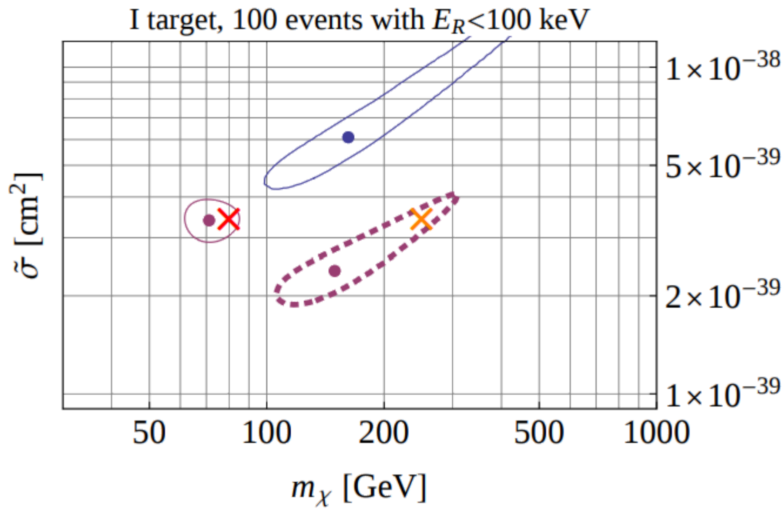
- Typically the cross-section is given by either the spin independent or spin dependent cross-sections.
- Both of which exhibit similar behaviour with changing energy,

$$\frac{d\sigma^{WT}}{dE_{\mathrm{R}}} = \frac{m_T}{2\mu_{\chi T}^2 v^2}\left(\sigma_0^{SI,T} F_{SI}^2(E_R) + \sigma_0^{SD,T} F_{SD}^2(E_R)\right)$$

- The behaviour being $\propto 1/v^2$ has important consequences for the halo integral.
- Particle theories that are missing from this description,
  - Anapole (plus other poles)
  - Composites
  - Vector DM
  - Others ...

Parameter Reconstruction

- In [arXiv:1401.3739] it was shown that fitting data to different models will give not only different values for couplings, but also different values for dark matter mass.
- This is basically because the nuclear responses are different for different particle models.



NREFT

- General particle interactions are not fully encapsulated by the canonical spin-(in-)dependent parametrisation and misrepresent the physics of DM.
- A Non-Relavistic Effective Field Theory has been developed for the 4 point DM-Nucleon interaction,

$$\mathcal{L}_{\text{int}} = \chi \mathcal{O}_\chi \chi N \mathcal{O}_N N = \sum_{N=n,p} \sum_i c_i^{(N)} \mathcal{O}_i \chi^+ \chi^- N^+ N^-$$

- Like all EFTs they describe the physics by only using the relevant degrees of freedom.
- In Direct Detection these quantities that are relavent are velocity v, the tranfer momentum q and the spins of DM and the nucleon S_χ and S_N.

Introducing Complexity

- The more complex NREFT basis will allow analysis to be more general and model independent.
- By widening the parameter space, we can test what DD experiments could tell us thing about the particle nature of Dark Matter in general.
- A computational drawback to the NREFT is that we're going from σ_0^{SI} and σ_0^{SD} to

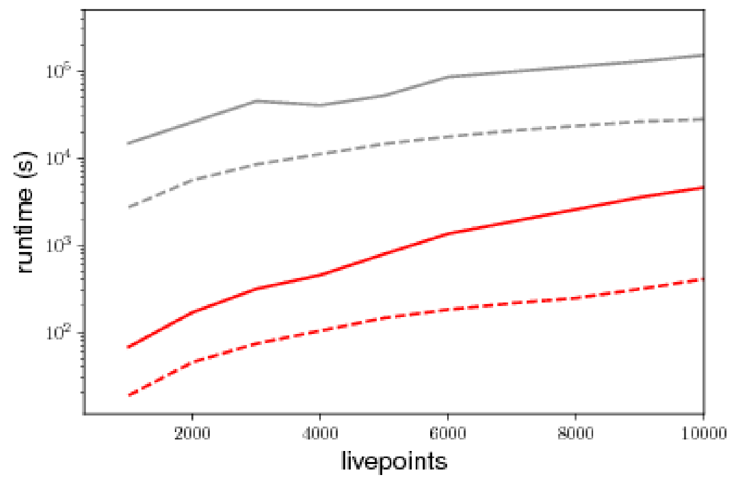
$$\hat{\mathcal{O}}_1 = \mathbb{1}_\chi \mathbb{1}_N$$
$$\hat{\mathcal{O}}_3 = i\hat{\mathbf{S}}_N \cdot \left(\frac{\hat{\mathbf{q}}}{m_N} \times \hat{\mathbf{v}}^\perp\right) \mathbb{1}_\chi$$
$$\hat{\mathcal{O}}_4 = \hat{\mathbf{S}}_\chi \cdot \hat{\mathbf{S}}_N$$
$$\hat{\mathcal{O}}_5 = i\hat{\mathbf{S}}_\chi \cdot \left(\frac{\hat{\mathbf{q}}}{m_N} \times \hat{\mathbf{v}}^\perp\right) \mathbb{1}_N$$
$$\hat{\mathcal{O}}_6 = \left(\hat{\mathbf{S}}_\chi \cdot \frac{\hat{\mathbf{q}}}{m_N}\right)\left(\hat{\mathbf{S}}_N \cdot \frac{\hat{\mathbf{q}}}{m_N}\right)$$
$$\hat{\mathcal{O}}_7 = \hat{\mathbf{S}}_N \cdot \hat{\mathbf{v}}^\perp \mathbb{1}_\chi$$
$$\hat{\mathcal{O}}_8 = \hat{\mathbf{S}}_\chi \cdot \hat{\mathbf{v}}^\perp \mathbb{1}_N$$
$$\hat{\mathcal{O}}_9 = i\hat{\mathbf{S}}_\chi \cdot \left(\hat{\mathbf{S}}_N \times \frac{\hat{\mathbf{q}}}{m_N}\right)$$
$$\hat{\mathcal{O}}_{10} = i\hat{\mathbf{S}}_N \cdot \frac{\hat{\mathbf{q}}}{m_N} \mathbb{1}_\chi$$
$$\hat{\mathcal{O}}_{11} = i\hat{\mathbf{S}}_\chi \cdot \frac{\hat{\mathbf{q}}}{m_N} \mathbb{1}_N$$
$$\hat{\mathcal{O}}_{12} = \hat{\mathbf{S}}_\chi \cdot \left(\hat{\mathbf{S}}_N \times \hat{\mathbf{v}}^\perp\right)$$
$$\hat{\mathcal{O}}_{13} = i\left(\hat{\mathbf{S}}_\chi \cdot \hat{\mathbf{v}}^\perp\right)\left(\hat{\mathbf{S}}_N \cdot \frac{\hat{\mathbf{q}}}{m_N}\right)$$
$$\hat{\mathcal{O}}_{14} = i\left(\hat{\mathbf{S}}_\chi \cdot \frac{\hat{\mathbf{q}}}{m_N}\right)\left(\hat{\mathbf{S}}_N \cdot \hat{\mathbf{v}}^\perp\right)$$
$$\hat{\mathcal{O}}_{15} = -\left(\hat{\mathbf{S}}_\chi \cdot \frac{\hat{\mathbf{q}}}{m_N}\right)\left[\left(\hat{\mathbf{S}}_N \times \hat{\mathbf{v}}^\perp\right) \cdot \frac{\hat{\mathbf{q}}}{m_N}\right]$$
$$\hat{\mathcal{O}}_{17} = i\frac{\hat{\mathbf{q}}}{m_N} \cdot \mathcal{S} \cdot \hat{\mathbf{v}}^\perp \mathbb{1}_N$$
$$\hat{\mathcal{O}}_{18} = i\frac{\hat{\mathbf{q}}}{m_N} \cdot \mathcal{S} \cdot \hat{\mathbf{S}}_N$$

Caveat to Complexity

- Just like in the canonical case, $\mathcal{O}_1$, the spin independent response is usually dominant (enhanced by A^2).
- This enhancement can lead to loop generated $\mathcal{O}_1$ responses being the dominant contribution in DD.
- Its been shown for certain simplified models, running from LHC scales to DD scales, operators that aren't present at tree level will be at DD. [arXiv:1605.04917]
- In the similar vain, a full UV complete pseudo-scalar dark matter model has been studied at 1-loop in [arXiv:1803.01574]. An $\mathcal{O}_1$ response is generated and becomes dominant.

We have developed RAPIDD for fast and general analysis

- Reconstruction Analysis using Polynomials In Direct Detection (RAPIDD) is a surrogate model for DD experiments.
- It enables fast evaluation of predicted Direct Detection responses with all operator responses and non-standard halo models.
- RAPIDD at worst sees a speed up factor of $\sim$ 20. At best above 200.



How does RAPIDD work?

- Instead of using the physics code to produce a result for a given energy bin N_k^a we call a polynomial $\mathcal{P}_k^a$.
- To do so we first choose a polynomial order $\mathcal{O}$ appropriate for the physics problem at hand. With $\mathcal{O}$ and the parameter point $\mathbf{\Theta}$ given, the structure of the polynomial is fixed. What remains to be done is to determine the coefficients, $d_{k,l}^a$, that allow to approximate the true behaviour of $N_k^a(\mathbf{\Theta})$ such that

$$N_k^a(\mathbf{\Theta}) \approx \mathcal{P}_k^a(\mathbf{\Theta}) = \sum_{l=1}^{N_{\text{coeffs}}} d_{k,l}^a \, \tilde{\Theta}_l \equiv \mathbf{d_k^a} \cdot \tilde{\mathbf{\Theta}}$$

- For example, for a quadratic polynomial in a two dimensional parameter space $\mathbf{\Theta} = (m_\chi, \, c_1) = (x, y)$, the coefficients take on the form $\mathbf{d_k^a} = (\alpha, \beta_x, \beta_y, \gamma_{xx}, \gamma_{xy}, \gamma_{yy})$

How does RAPIDD work?

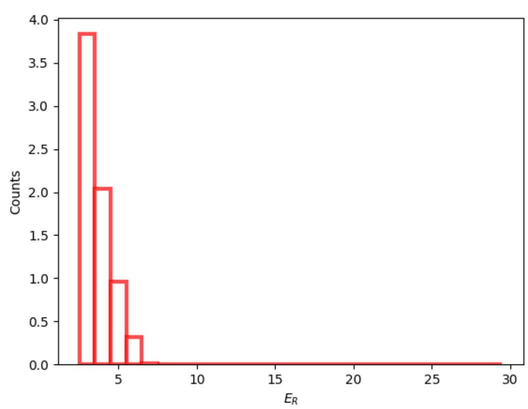
- This is done by collecting each $N_k^a(\mathbf{\Theta})$ for the set of sample points and solving this matrix equation

$$\overrightarrow{N_k^a} = M_{\tilde{\mathbf{\Theta}}} \cdot \mathbf{d_k^a}$$

- Where $M_{\tilde{\mathbf{\Theta}}}$ is a quantity similar to a Vandermorde matrix where each row contains the values of $\tilde{\mathbf{\Theta}}$ for each sampled point, and $\overrightarrow{N_k^a}$ is a vector of the resulting number of events. This allows us to solve for $\mathbf{d_k^a}$ using the (pseudo-) inverse of $M_{\tilde{\mathbf{\Theta}}}$, which in the PROFESSOR program is evaluated by means of a singular value decomposition.

Instant issue for low masses

- In the situation where we model a DM with low enough mass the spectrum is discontinuous.

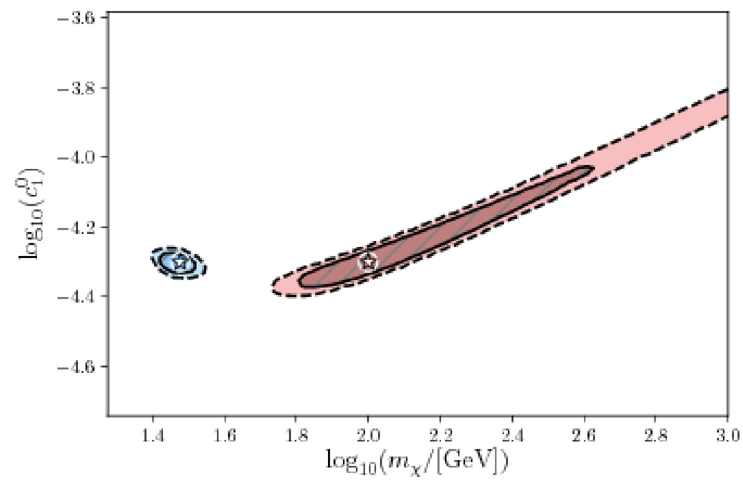


- This is simply from the kinematic relations $v_{\text{min}} = \sqrt{m_T E_R/(2\mu_T)^2}$
- And the fact that the DM distribution has a upper limit in velocity v_{esc}.
- We take this into account in the determination of polynomials by not using bins which return zero in the training.
- When we call the polynomial later, we also evaluate where the discontinuity should be and impose it by hand.

Tests

- In order to test our code we used RAPIDD and the physics code for some canonical examples.
- The first of wich was to test in 2-D, scanning in the (m_χ, c_1^0) plane, which is just the NREFT equivalent to the spin independent case, where there's this weird conversion,

$$\sigma_{\chi N} = \frac{\mu_{\chi N}^2}{\pi\, m_v^4} \left(c_1^0\right)^2$$

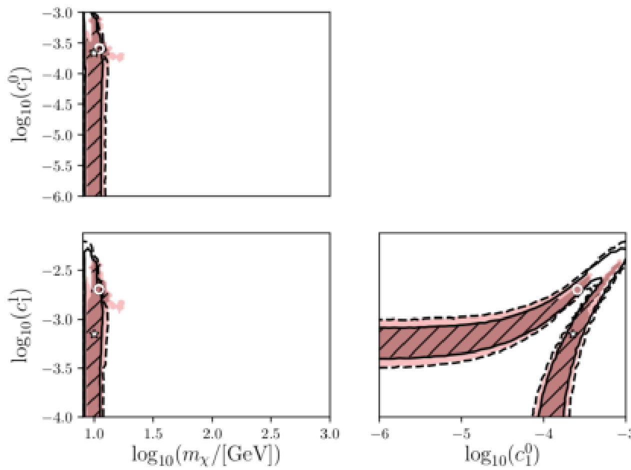


3-D Test 2 (Cancellation)

- We also wanted to test RAPIDD in specific cases where finely tuned cancellations were possible.
- This inspired us to build the different polynomials contributions seperately

$$N_k^a(\mathbf{\Theta}) \approx \sum_{ij} \sum_{\tau,\tau'=0,1} \mathcal{P}_k^{a,i,j,\tau,\tau'}(\mathbf{\Theta})$$

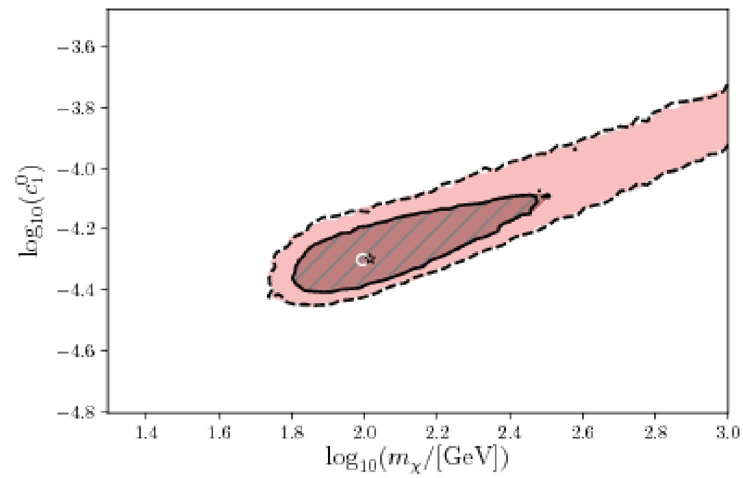
- For example when isoscalar and isovector couplings are free (would cause problems with quark universality).



6-D Test (with Halo)

- Finally we tested how RAPIDD works with the general halo function

$$f(v) = N_k^{-1}\left[e^{-v^2/kv_0^2} - e^{-v_{esc}^2/kv_0^2}\right]^k \Theta(v_{esc} - v)$$



Using RAPIDD to Constrain Models

- We wanted to provide a case study of how our code could be used in future analysis.
- We took the following detector variables

Target	Exposure	Energy window	Bin No
Xe	5.6×10^6 kg days	3-30 keV	27
Ge	91250 kg days	0.35-50 keV	49
Ar	7.3×10^6 kg days	5.0-30 keV	24

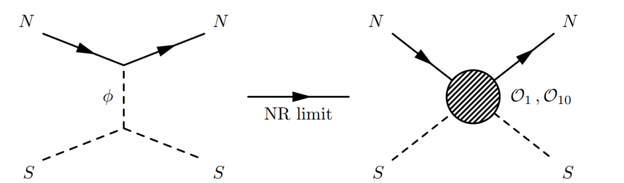
- Then we took three benchmark points, which are accessible by future detectors.

Name	Model	DM Parameters	$N_{\rm Xe}$	$N_{\rm Ge}$	$N_{\rm Ar}$
BP1	SS	$m_\chi = 10$ GeV $c_1 = 1 \times 10^{-4}$ $c_{10} = 5$	93	10	50
BP2	SS	$m_\chi = 100$ GeV $c_1 = 3 \times 10^{-5}$ $c_{10} = 5 \times 10^{-1}$	206	2	30
BP3	FS	$m_\chi = 30$ GeV $c_1 = 0.0$ $c_6 = 60$ $c_{10} = 0.0$ $c_{11} = 0.0$	256	1	0

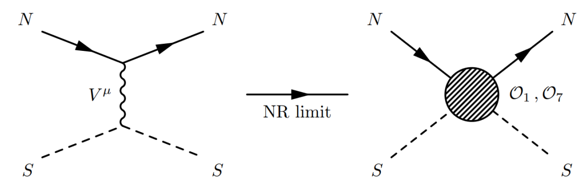
Analysing with simplified models

- We can use a set of simplified models and try and match them to the data.
- If you treat each operator coefficient as a free parameter

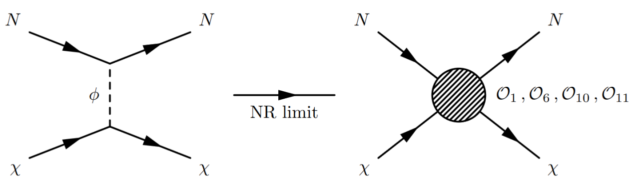
Scalar - Scalar



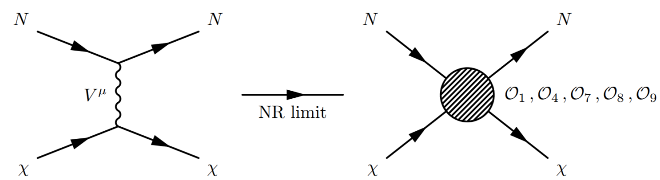
Scalar- Vector



Fermion- Scalar



Fermion- Vector



Result for BP3

- Setting up a Poissonnian log-likelihood

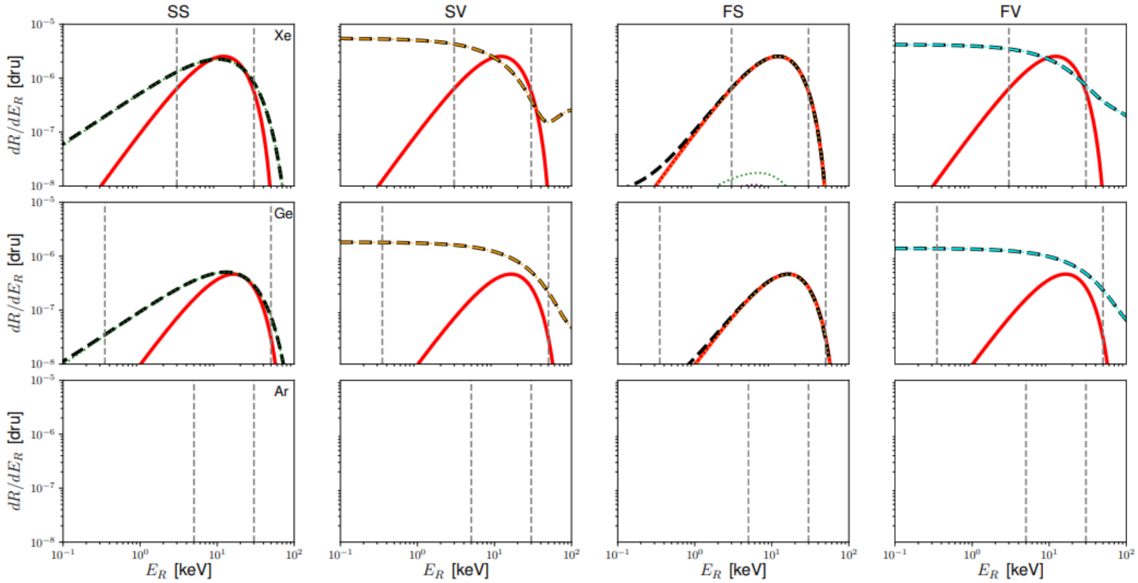
$$\mathcal{L}(\mathbf{\Theta}) = \prod_a \mathcal{L}^a(\mathbf{\Theta}) = \prod_a \prod_k \frac{N_k^a(\mathbf{\Theta})^{\lambda_k^a} e^{N_k^a(\mathbf{\Theta})}}{\lambda_k^a!}$$

- Where a runs over experiments and k runs over bins. $N_k^a(\Theta)$ is the counts given by the theoretical expectation at a parameter point Θ and λ_k^a is the data result.

$m_\chi = 47.0$ GeV $c_1 = 1.06 \times 10^{-6}$ $c_{10} = 1.39$		$m_\chi = 9920$ GeV $c_1 = 2.48 \times 10^{-6}$ $c_7 = 4.91 \times 10^2$		$m_\chi = 30.9$ GeV $c_1 = 1.00 \times 10^{-6}$ $c_6 = 59,8$ $c_{10} = 0.126$ $c_{11} = 1.02 \times 10^{-4}$		$m_\chi = 9966$ GeV $c_1 = 2.35 \times 10^{-6}$ $c_4 = 0.469$ $c_7 = 1.40$ $c_8 = 1.21 \times 10^{-2}$ $c_9 = 1.20 \times 10^{-2}$
$\log \mathcal{L}(SS) = -62.6$		$\log \mathcal{L}(SV) = -103$		$\log \mathcal{L}(FS) = -58.9$		$\log \mathcal{L}(FV) = -84.4$

Result for BP3

- We can see how the unbinned spectra shape up here, and can also see which contributions are dominant.



SS	SV	FS	FV
$m_\chi = 47.0$ GeV $c_1 = 1.06 \times 10^{-6}$ $c_{10} = 1.39$	$m_\chi = 9920$ GeV $c_1 = 2.48 \times 10^{-6}$ $c_7 = 4.91 \times 10^2$	$m_\chi = 30.9$ GeV $c_1 = 1.00 \times 10^{-6}$ $c_6 = 59,8$ $c_{10} = 0.126$ $c_{11} = 1.02 \times 10^{-4}$	$m_\chi = 9966$ GeV $c_1 = 2.35 \times 10^{-6}$ $c_4 = 0.469$ $c_7 = 1.40$ $c_8 = 1.21 \times 10^{-2}$ $c_9 = 1.20 \times 10^{-2}$
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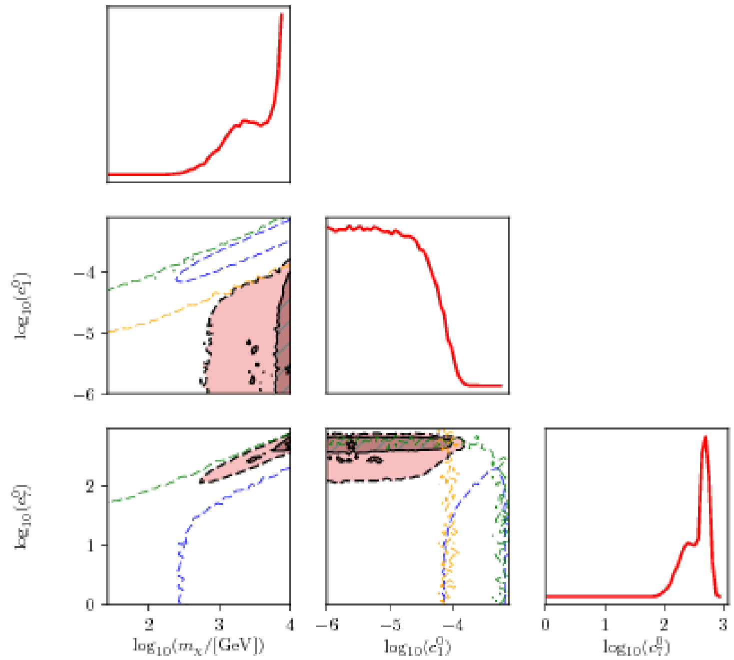
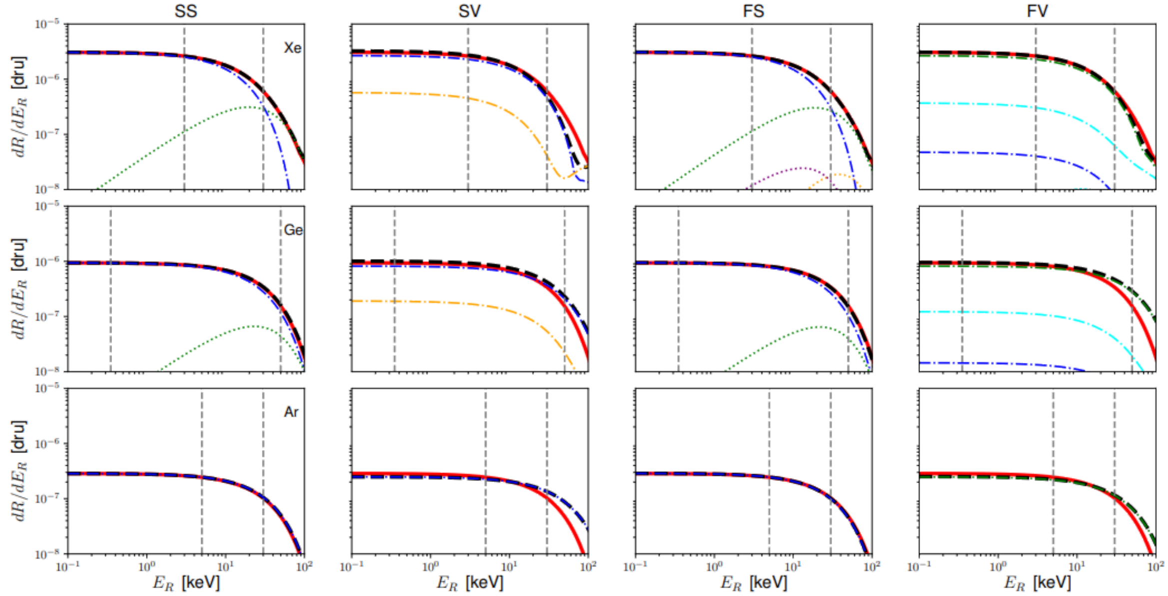
Scalar-Vector Profile likelihoods



Result for BP2

- Here is a less illuminating result.



SS	SV	FS	FV
$m_\chi = 105$ GeV $c_1 = 4.11 \times 10^{-5}$ $c_{10} = 0.595$	$m_\chi = 989$ GeV $c_1 = 3.79 \times 10^{-4}$ $c_7 = 161$	$m_\chi = 95.7$ GeV $c_1 = 3.90 \times 10^{-5}$ $c_6 = 3.22$ $c_{10} = 0.570$ $c_{11} = 1.85 \times 10^{-4}$	$m_\chi = 1403$ GeV $c_1 = 1.81 \times 10^{-5}$ $c_4 = 5.14 \times 10^{-2}$ $c_7 = 1.69$ $c_8 = 3.04 \times 10^{-1}$ $c_9 = 0.303$
$\log \mathcal{L}(SS) = -85.2$	$\log \mathcal{L}(SV) = -86.0$	$\log \mathcal{L}(FS) = -85.2$	$\log \mathcal{L}(FV) = -85.4$

Current Work and Conclusion

- Rapidd, is a new tool that enables more general analysis in DD experiments.
- Along with N. Bozorgnia, D.G. Cerdeño, and B Penning [arXiv:1807.xxxx], I am exploring the experimental parameters and how that effects parameter reconstruction in the NREFT basis as well as information on the halo.
- Rapidd is not public yet, we've been delayed by advances in the methods we use to build the polynomial.
- We want this to be as useful as possible. So you have any ideas of how to make RAPIDD more useful for you. Let me know!
- Thank you