Systematic uncertainty of first-principle calculations of the radiation energy emitted by EAS

Marvin Gottowik, Christian Glaser, Tim Huege and Julian Rautenberg

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Energy Estimation

- Estimate CR energy based on radio emission
  1) Signal at reference distance
  2) Total emitted radio signal (radiation energy)
- $E_{\text{rad}}$ proportional to electromagnetic energy
- Can be calculated using first principles
- Shower property, i.e. detector independent
Energy Scale

- Determination of energy scale with radio
- „Radio yield“ given by first principle calculation
- Estimate systematic uncertainty of „radio yield“ with CoREAS and ZHAireS simulations

@Auger: Phys. Rev. D 93, 122005 (2016)

\[ E_{30-80 \text{ MHz}} = (15.8 \pm 0.7(\text{stat}) \pm 6.7(\text{sys})) \text{ MeV} \]

\[ \times \left( \sin \alpha \frac{E_{\text{CR}}}{10^{18} \text{ eV}} \frac{B_{\text{Earth}}}{0.24 \text{ G}} \right)^2 \]

- 6.7 (sys): 28% on radio, 16% on SD energy scale
### Systematics at Auger

<table>
<thead>
<tr>
<th>source of uncertainty</th>
<th>$\sigma_{\vec{E}}$</th>
<th>$\sigma_{S_{\text{radio}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>event-by-event</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>temperature dependence</td>
<td>4%</td>
<td>8%</td>
</tr>
<tr>
<td>angular dependence of antenna response pattern</td>
<td>5%</td>
<td>10%</td>
</tr>
<tr>
<td>reconstructed direction</td>
<td>negligible</td>
<td>negligible</td>
</tr>
<tr>
<td>LDF fit uncertainty</td>
<td>-</td>
<td>error propagation of fit parameters</td>
</tr>
<tr>
<td><strong>total event-by-event uncertainty</strong></td>
<td>6.4%</td>
<td>12.8% $\oplus$ fit uncertainty</td>
</tr>
<tr>
<td><strong>absolute scale</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>absolute scale of antenna response pattern</td>
<td>12.5%</td>
<td>25%</td>
</tr>
<tr>
<td>analog signal chain</td>
<td>Phys. Rev. D 93, 122005 (2016)</td>
<td>6%</td>
</tr>
<tr>
<td>LDF model</td>
<td>$&lt;2.5%$</td>
<td>$&lt;5%$</td>
</tr>
<tr>
<td><strong>total absolute scale uncertainty</strong></td>
<td>14%</td>
<td>28%</td>
</tr>
</tbody>
</table>

- Antenna response reduced $\sim 10\%$
- $\rightarrow$ total uncertainty $\sim 16\%$
Simulation Setup

• Air Shower
  - CORSIKA 7.4100 (Oct. 12, 2015), Aires 2.8.4a (Dec. 12, 2006)
  - SIBYLL 2.1 as hadronic interaction model
  - Endpoint formalism vs. ZHS algorithm

• Simulation
  - Exp. Scaling of refractive index
  - Proton and iron primary with energy in 100 PeV – 10 EeV
  - Azimuth in 0° - 360°, zenith 0° - 75°

• Technical Parameters:
  - Low E had. interaction model, thinning, trace lengths
Radiation Energy

- Electrical field only in shower plane
- Computation of energy fluence $f$ on $\mathbf{v} \times (\mathbf{v} \times \mathbf{B})$ axis
- Radiation energy

$$E_{\text{rad}} = 2\pi \int dr \, r \cdot f(r, \varphi = 90^\circ)$$

- Assumptions:
  1) ce and geo are in phase
  2) Individual LDFs are radially symmetric
- Simplification from star shape: $\sim$1.5% overestimation
Corsika STEPFC parameter

- STEPFC: "Factor by which the multiple scattering length for electrons and positrons in EGS4 simulations is elongated"

- 11% impact on radiation energy

- Lower STEPFC (0.05) value needed for convergence, but increase in computing time (~ x 4)
Charge-Excess Fraction $a$

- **Definition**
  
  $$a = \sin \alpha \sqrt{E_{\text{rad}}^{\text{ce}} / E_{\text{rad}}^{\text{geo}}}$$

- **$a$ depends on the density at shower maximum**

- **Differences at high densities and small $\sin \alpha$**

\[ a = q_0 + q_1 \exp(q_2(\rho - \langle \rho \rangle)) \]
\[ q_0 = -0.175, \quad q_1 = 0.373, \quad q_2 = 1.302 \]
Radiation vs. Electromagnetic Energy

- Energy estimator

\[ S_{\text{rad}} = \frac{E_{\text{rad}}}{a(\rho(X_{\text{max}}))^2 + (1-a(\rho(X_{\text{max}}))^2) \sin^2 \alpha} \frac{1}{(1-p_0 + p_0 \exp(p_1(\rho(X_{\text{max}}) - \langle \rho(X_{\text{max}}) \rangle))^2} \]

- Scatter: \( \sim 7\% \)
- \( A_{\text{CoREAS}} / A_{\text{ZHAireS}} = 4\% \)
- On \( E_{\text{emag}} \):
  - \( \sqrt{7}\% \) stat.
  - \( 2\% \) sys.

\[ S_{\text{rad}} = A' \cdot E_{\text{emag}}^2 \]

\[ S_{\text{rad}} = A \cdot 10^7 \cdot \left( E_{\text{emag}} / 10^{18} \right)^b \]

\[ A = 1.680 \pm 0.004 \]

\[ B = 2.000 \pm 0.001 \]
Combined Corrections

- Fit constants $q_i$ (charge excess fraction) and $p_i$ (energy estimator) on a combined data set
- Only fit power law individually with combined corrections
- **5.2% more radiation in CoREAS**
- Check stepsizes for atmospheric heights
  - ZHAireS slightly larger at lower heights
  - Possible ~ 1% increase for ZHAireS with reduced stepsize based on CoREAS STEPFC study
Other simulation uncertainties

<table>
<thead>
<tr>
<th>source of uncertainty</th>
<th>uncertainty of radiation energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>hadronic interaction models [12]</td>
<td>0.3%</td>
</tr>
<tr>
<td>approximations in the air-shower simulation</td>
<td></td>
</tr>
<tr>
<td>particle thinning [12]</td>
<td>&lt; 0.3%</td>
</tr>
<tr>
<td>energy thresholds of shower particles [12]</td>
<td>&lt; 1%</td>
</tr>
<tr>
<td>state of the atmosphere [12]</td>
<td>&lt; 3%</td>
</tr>
<tr>
<td>underlying first principle calculations (this work)</td>
<td>&lt; 5.2%</td>
</tr>
</tbody>
</table>

Table 1: Systematic uncertainties of the calculation of the radiation energy from the electromagnetic shower energy.

- First principle calculation biggest uncertainty
- compare 16% new systematic of Auger
- CoREAS ZHAireS differences important
Additional Checks

• Refractivities at sea level
  - no significant differences for CoREAS and ZHAireS

• Magnetic Field Strength
  - no significant differences for CoREAS and ZHAireS

• Primary particles
  - 3% more radiation for p than Fe shower with same electromagnetic energy
  - Additional composition sensitivity of radio emission
Summary

- CoREAS and ZHAireS consistent
- $S_{rad}$ well suited estimator of CR energy
- Technical and physical effects studied
- Small STEPFC value suggest for CoREAS simulations
- Final difference in radiation energy 5.2%
  $\rightarrow$ 2.6% difference on electromagnetic energy
**Thinning Algorithm**

- Thin shower if sum of energy of all secondary particles in an interaction below threshold
- Thinning level of $10^{-5}$ sufficient

<table>
<thead>
<tr>
<th>Thinning Level</th>
<th>Rad. E / MeV</th>
<th>#Sim</th>
</tr>
</thead>
<tbody>
<tr>
<td>1e-3</td>
<td>10.2(2)</td>
<td>40</td>
</tr>
<tr>
<td>1e-4</td>
<td>8.5(1)</td>
<td>40</td>
</tr>
<tr>
<td><strong>1e-5</strong></td>
<td>8.3(1)</td>
<td>40</td>
</tr>
<tr>
<td>1e-6</td>
<td>8.3(1)</td>
<td>40</td>
</tr>
<tr>
<td>1e-7</td>
<td>8.4(1)</td>
<td>21</td>
</tr>
</tbody>
</table>
Low Energy Hadronic Model

No significant influence

\[ f(x) = ax \]
\[ a = 1.001 \pm 0.002 \]
\[ \chi^2 / \text{ndf} = 94.498016 / 34 \]
**LDF Fit**

- **Sum of 2 gaussians**

\[
f(x, y) = A_+ \exp\left(-\frac{(x - X_+)^2 + (y - Y_+)^2}{\sigma_+^2}\right) - A_- \exp\left(-\frac{(x - X_-)^2 + (y - Y_-)^2}{\sigma_-^2}\right)
\]

- **Reduced LDF**

\[
f(\vec{r}) = A \left(\exp\left(-\frac{\vec{r} + C_1 \vec{e}_{v \times B} - \vec{r}_{\text{core}}}{\sigma^2}\right) - C_0 \exp\left(-\frac{\vec{r} + C_2 \vec{e}_{v \times B} - \vec{r}_{\text{core}}}{(C_3 \exp(C_4 \cdot \sigma))^2}\right)\right)
\]

- **\(C_i\) determined by simulations**

- **Limit relevance:**
  - Detector dependent
  - Here: Auger vs. sea level
  - In general good agreement
FFT & Bandpass Filtering

- Filter radio emission to 30 – 80 MHz
  - CoREAS: 28.79 – 81.57 MHz
  - ZHAireS: 30.00 – 80.17 MHz
- ZHAireS trace longer than CoREAS → finer frequency resolution
- 5 % more effective bandwith used in CoREAS
- 10 % more radiation energy