Auto-stabilized Electron

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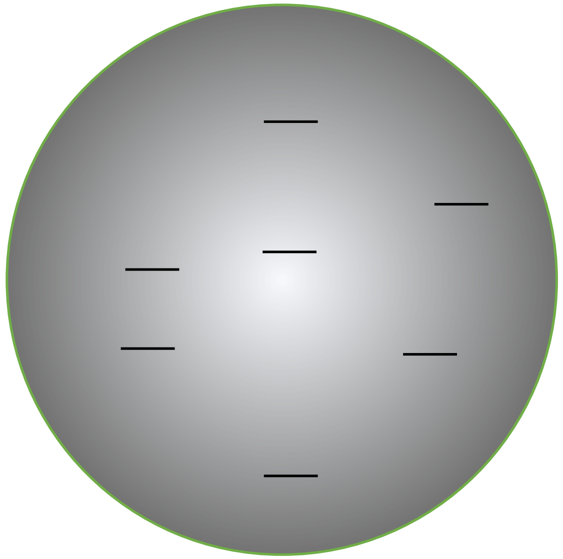
Why is the electron stable?

- Negatively charged sphere
- Mutual repulsion

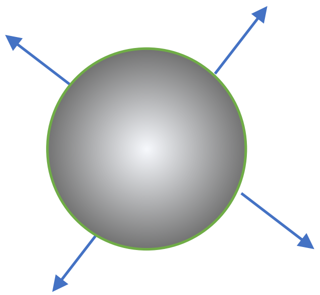


What holds the electron together?

- Puzzle since 1895 (J J Thomson)
- Yet unsolved in spite of heroic attempts over 123 years
- Solutions sought in quantum electrodynamics (QED)
- All approaches lead to infinite outward pressure
- And unstable electron – electron should explode
- But the electron is stable!

What is the outward pressure?

- Energy per unit volume: $\left[\left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{e^2}{r}\right)\right]\left(\frac{1}{\frac{4}{3}\pi r^3}\right)\frac{J}{m^3}$
- $r = \frac{\hbar}{mc}$ (Compton wavelength of electron)
- $\frac{e^2}{4\pi\epsilon_0}\frac{1}{(\frac{4}{3})\pi(\frac{\hbar}{mc})^4}$ $=10^{18}\ \frac{J}{m^3}$



- 10^{13} atmospheres!!

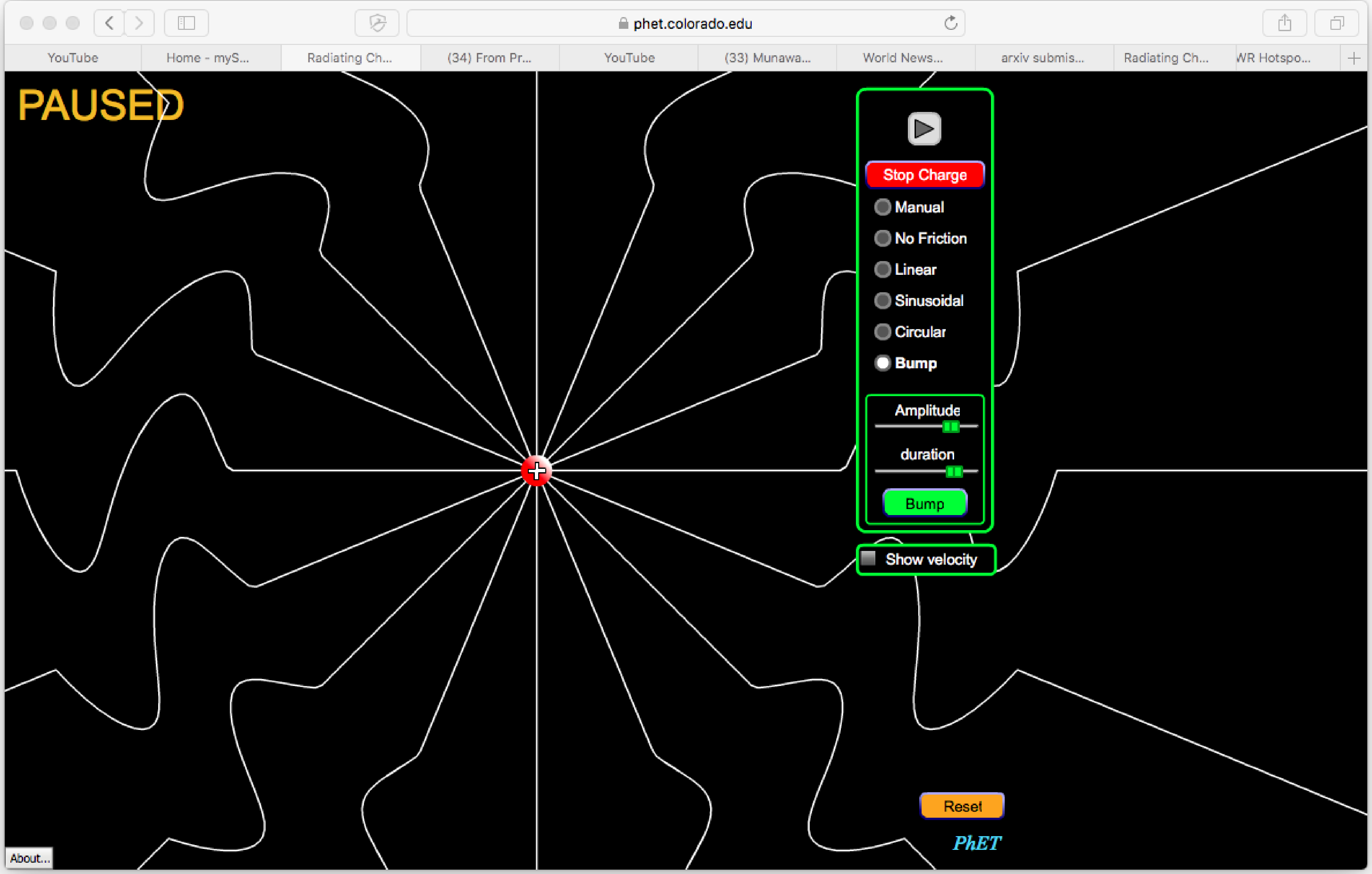
At these pressures space-time is not flat

When energy density is high space-time geometry is not flat

Geometry $\leftrightarrow$ Energy density (Einstein equation)

Applet for radiating charge

https://phet.colorado.edu/sims/radiating-charge/radiating-charge_en.html



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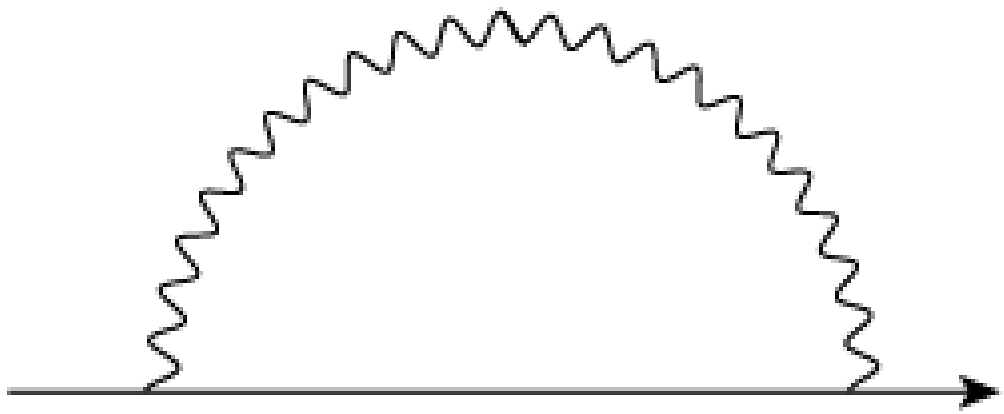
About...

Near zone and Radiation zone

- Retardation effects due to finite speed of light
- Near zone extends to $r = \frac{\lambda}{2\pi}$
- As λ decreases (or frequency increases) so does r
- As *r* decreases so does volume
- Energy density increases as r^4
- Space-time curvature increases

Field energy in the near zone

- Energy is exchanged between accelerating charge and near zone field
- Energy is stored in near zone: adds to inertia of accelerating charge
- Shows up as increase in mass of electron
- Since there is no lower limit on λ current theories predict the mass increases to infinity



“Emission and absorption of virtual photons”

The mass correction is a sum over all allowed photon momenta k^{μ}

$$\Delta m = \int_{-\infty}^{\infty} \frac{\tilde{u}\left(2m + 2k^{\mu}\gamma_{\mu}\right)u}{k^2 - 2\vec{p}\cdot\vec{k}} \frac{d^4k}{(2\pi)^4} \frac{1}{k^4}$$

$$\mu \equiv \Delta m$$

This integral is infinite

This has been the situation since 1945

Attempts to truncate the integral

- Abraham-Lorentz self-force (1904)
- Cut off the upper limit to Compton wavelength (Bethe 1947)
- Alter the laws of electromagnetism at short distances (Tomonaga, Schwinger, Feynman 1949)
- Assume dimensions of space time are continuous (T'ooft, Giambiagi 1972)
- None of these works: integral remains infinite
- None of these assumptions is justified

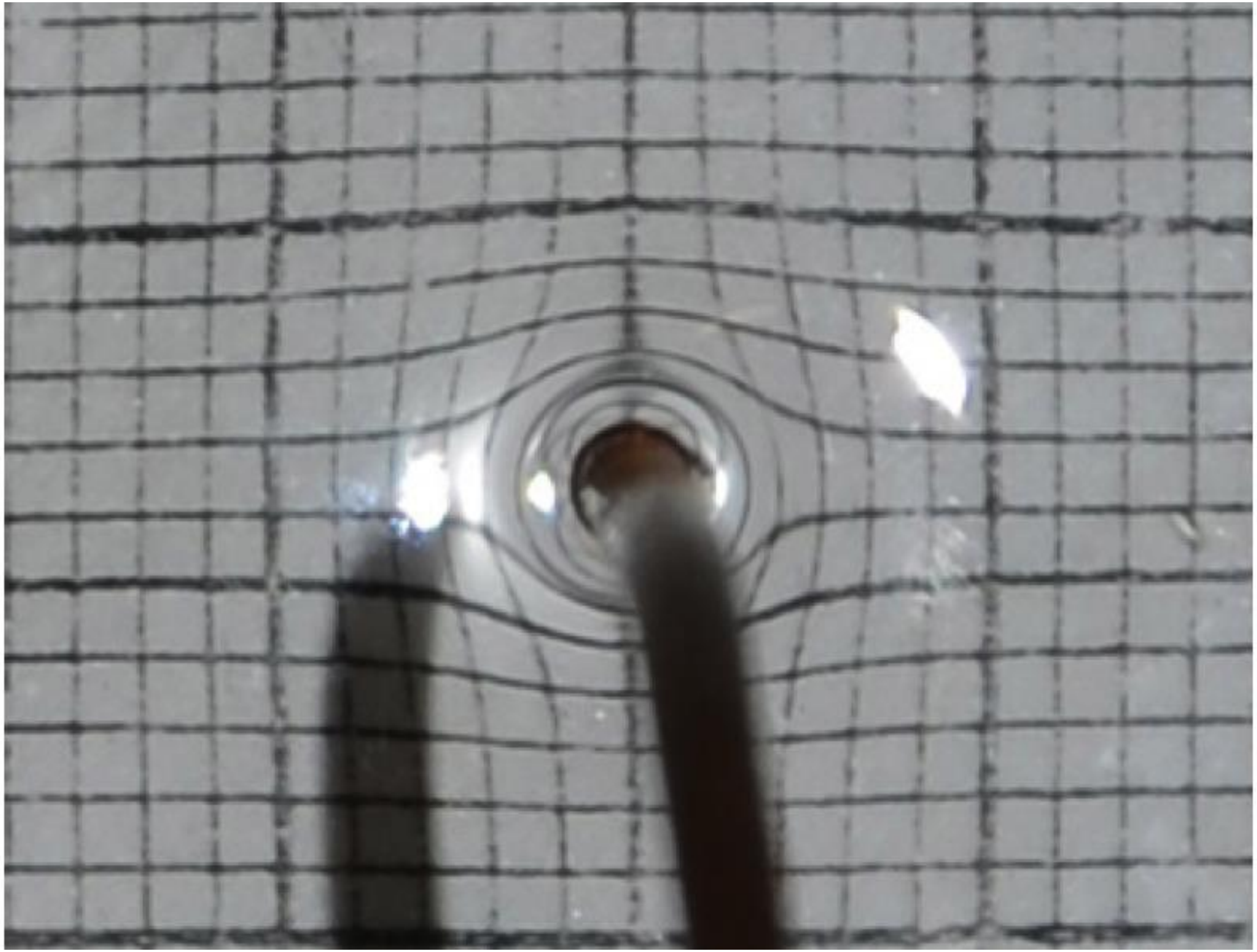
“Since the mathematicians have invaded the theory of relativity,
I do not understand it myself anymore.”

A. Einstein (1949)

In the near zone if the energy density is high space-time is not flat

Geometry is altered: must introduce gravitation

Example of curved metric



In order to include gravitation need to solve Einstein equation

Geometry $\leftrightarrow$ Energy density (Einstein equation)

$$G^{\hat{0}\hat{0}} = 8\pi T^{\hat{0}\hat{0}} = 8\pi\rho$$

$$G^{\hat{1}\hat{1}} = 8\pi T^{\hat{1}\hat{1}} = 8\pi P$$

Relate Pressure P and density ρ

Equation of state

- Relate pressure and density
- $dE = -PdV$; ${}^{dE}/_{dV} = -P$
- $E = \rho V; dE = Vd\rho + \rho dV$
- $P = -\rho - r\frac{d\rho}{dr}$
- $\rho = \frac{1}{\sqrt{g_{11}}4\pi r^2}\frac{d\mu}{dr}$
- $\Delta m = \mu = \int_{-\infty}^{\infty}\frac{\tilde{u}\left(2m+2k^{\mu}\gamma_{\mu}\right)u}{k^2-2\vec{p}\cdot\vec{k}}\frac{d^4k}{(2\pi)^4}\frac{1}{k^4}$

Steps needed

- Introduce a dimensionless variable $\eta = \frac{\hbar}{2mc}\frac{1}{r}$; value to be determined
- Integrate to an upper limit *k* (yet unknown)
- Integral is then
- $\Delta m \equiv \mu(\eta) = \frac{\alpha m}{2\pi}\left[-\frac{\eta}{2}\sqrt{1+\frac{1}{\eta}} + \eta + \eta \ln\left\{\sqrt{\eta}\left(\sqrt{1+\frac{1}{\eta}}\right) + 1\right\}\right]$
- The density is $\rho = \frac{1}{\sqrt{g_{11}}4\pi r^2}\frac{d\mu}{dr}$

Equation of hydrostatic equilibrium

- Insert P and ρ
- $G^{\hat{0}\hat{0}} = 8\pi T^{\hat{0}\hat{0}} = 8\pi\rho$
- $G^{\hat{1}\hat{1}} = 8\pi T^{\hat{1}\hat{1}} = 8\pi P$
- $\frac{dP}{dr} = -[\rho(r) + P(r)]\frac{[\mu(r)+4\pi r^3 P(r)]}{r^2\left(1-\frac{2\mu(r)}{r}\right)}$
- Solve equation of hydrostatic equilibrium for r when $P(r)=0$

Conditions for equilibrium

- Energy is stored in the metric
- Metric pushes inwards, electron field pushes outwards
- Until both inward and outward pressures are equal at $P(r)=0$
- Equilibrium is stable
- This occurs at $r = \frac{1}{2\sqrt{2}}\sqrt{\frac{\alpha}{4\pi}\frac{\hbar G}{c^3}} = \frac{1}{2\sqrt{2}}\sqrt{\frac{\alpha}{4\pi}}\ell_P = 10^{-37}m$
- *This is the radius of the electron*
- Inward and outward pressures are 10^{109} atmospheres

Conditions at equilibrium

- At *r* the two competing pressures are equal
- The equilibrium is stable
- If the momentum *k* increases so does the opposing pressure
- The system is self-correcting
- The electron is auto-stabilized

$$\alpha = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{e^2}{\hbar c}$$

The corresponding energy is $\sqrt{\frac{\alpha}{8\pi}\frac{\hbar c}{2G}} = \sqrt{\frac{\alpha}{16\pi}}\, m_P = 10^{17}$ GeV

At this energy all forces merge (Grand Unified Theory energy)

Results are independent of $\hbar$

Conclusion: from first principles

- We have explained why the electron is stable: “auto-stabilized”
- We have calculated the radius of the electron
- We have calculated the GUT energy $10^{17} GeV$
- Shown that the unified field is continuous
- Successful theory of quantum fields in curved space time

“You know it would be sufficient to really understand the electron”

A. Einstein

THANK YOU!!