# From Relativistic Heavy Ion Collisions to Neutron Star Mergers - the Equation of State of Dense Matter as signalled by Gravitational Waves

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*Nelson Mandela University*
*Port Elizabeth, South Africa*

THE FIFTH BIENNIAL

AFRICAN SCHOOL OF FUNDAMENTAL PHYSICS AND APPLICATIONS

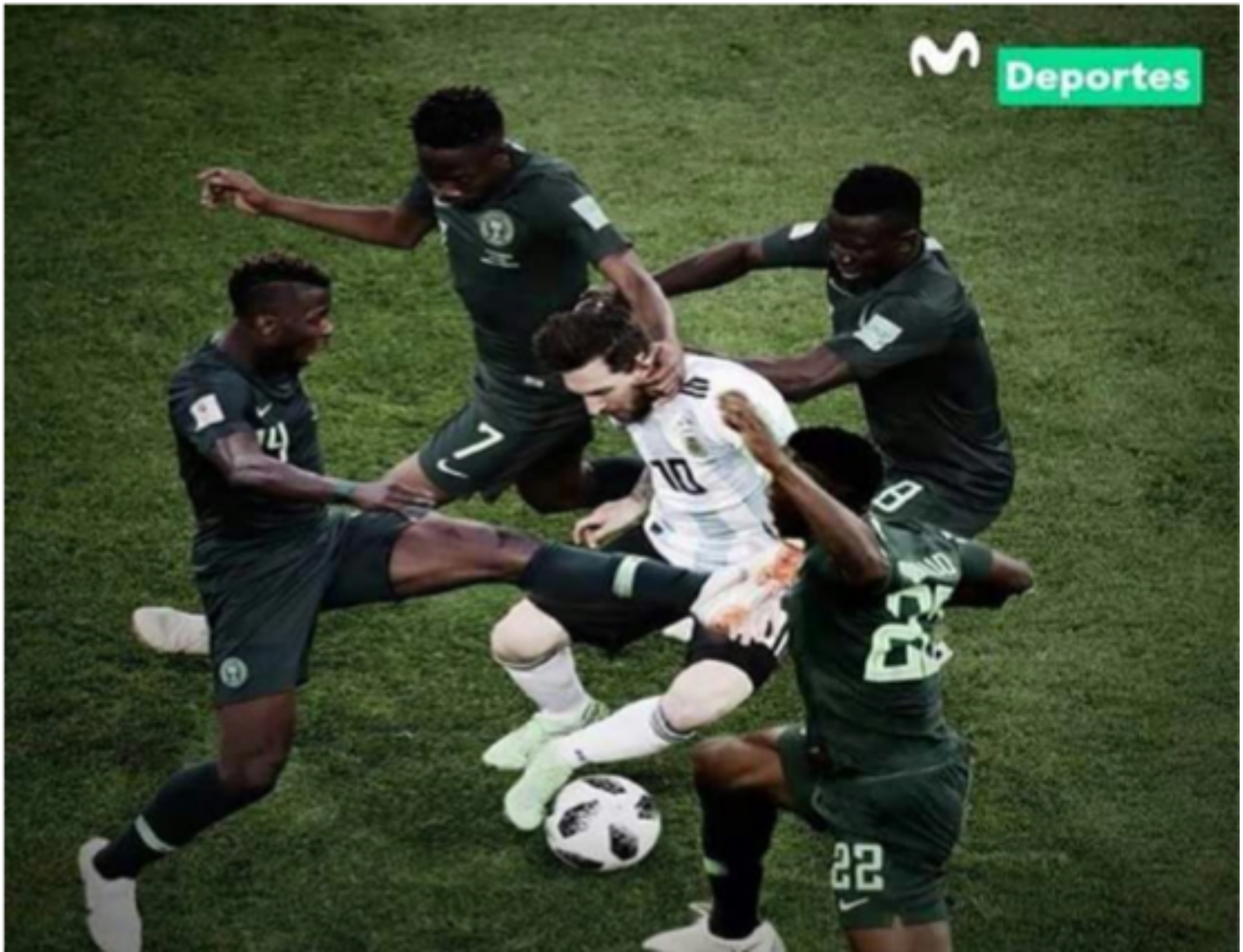
University of Namibia, and
Namibia University of Science and Technology
Windhoek, Namibia
June 24 - July 14, 2018

# Summerstrand, Port Elizabeth

# Outline

- Relativistic Heavy Ion Collisions
- Neutron Stars
- Numerical Relativity
- Equation of State
- Gravitational Waves breakthrough
- HIC and Astrophysics connection

Beyond the scope of this talk ! And beyond standard model !



# New developments

- **Matthias Hanauske, Kentaro Takami, Luke Bovard, Luciano Rezzolla, José A. Font, Filippo Galeazzi, and Horst Stöcker**
- **Phys. Rev. D 96, 043004 (2017) – Published 7 August 2017**

- B. P. Abbott et al. (Virgo, LIGO Scientific), (2018), arXiv:1805.11581 [gr-qc].

- E. R. Most, L. R. Weih, L. Rezzolla, and J. Schaffner- Bielich, (2018), arXiv:1803.00549 [gr-qc].

- E. Annala, T. Gorda, A. Kurkela, and A. Vuorinen, Phys. Rev. Lett. 120, 172703 (2018), arXiv:1711.02644 [astro-ph.HE].

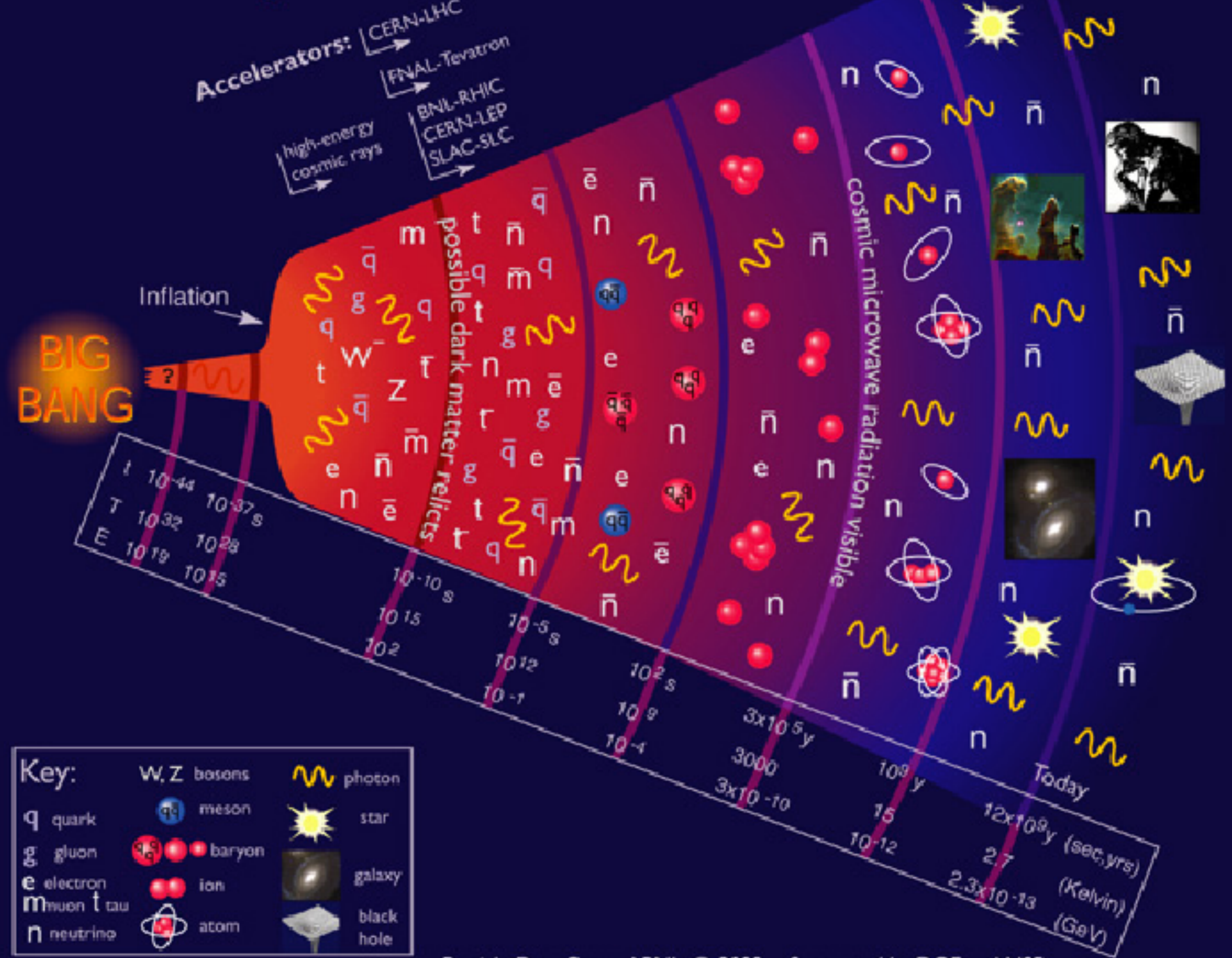
- S. De, D. Finstad, J. M. Lattimer, D. A. Brown, E. Berger, and C. M. Biwer, (2018), arXiv:1804.08583 [astro-ph.HE].

- B. Kumar, B. K. Agrawal, and S. K. Patra, Phys. Rev. C97, 045806 (2018), arXiv:1711.04940 [nucl-th].

- F. J. Fattoyev, J. Piekarewicz, and C. J. Horowitz, Phys. Rev. Lett. 120, 172702 (2018), arXiv:1711.06615 [nuclth].

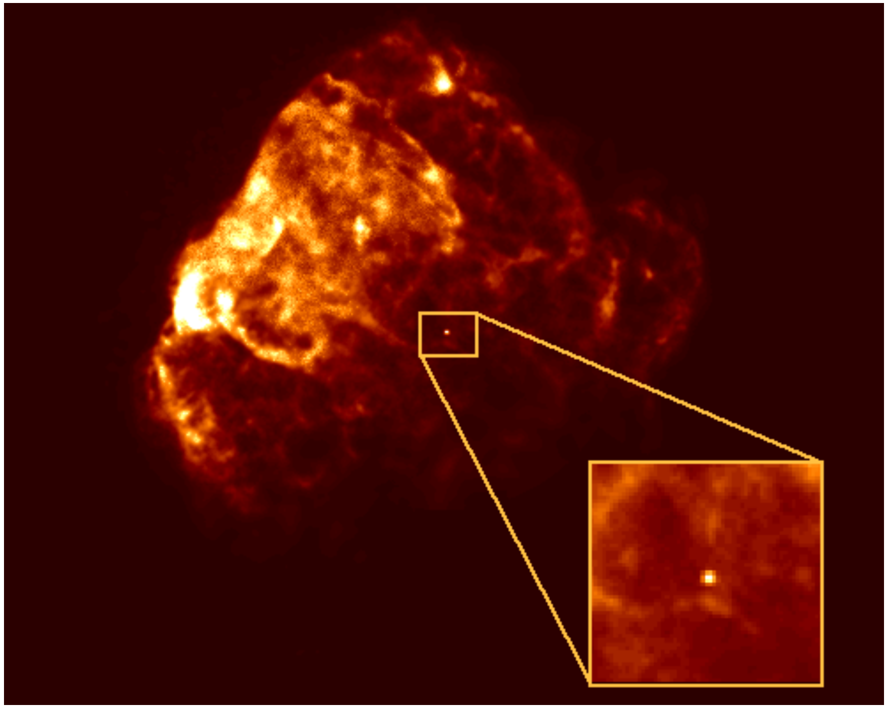
- T. Malik, N. Alam, M. Fortin, C. Providencia, B. K. Agrawal, T. K. Jha, B. Kumar, and S. K. Patra, (2018), arXiv:1805.11963 [nucl-th].

# History of the Universe



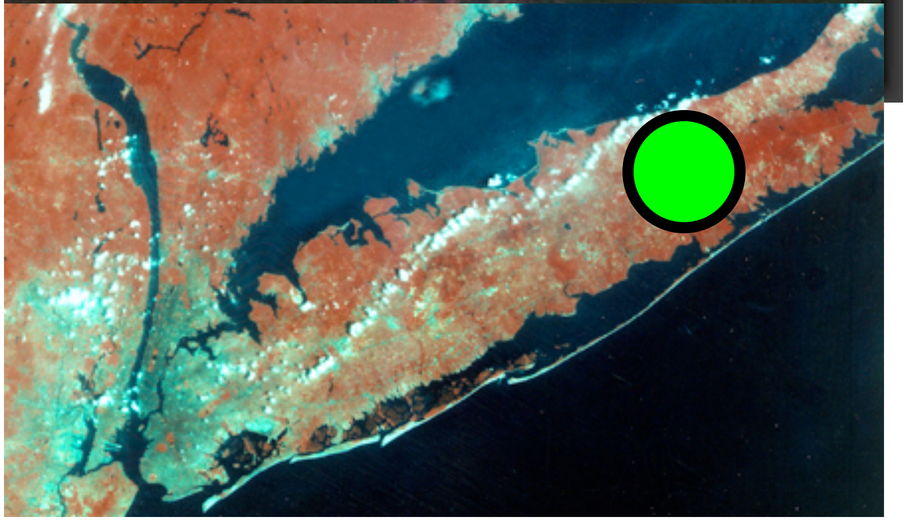
Particle Data Group, LBNL, © 2000. Supported by DOE and NSF

# Other possible QGP hide-outs

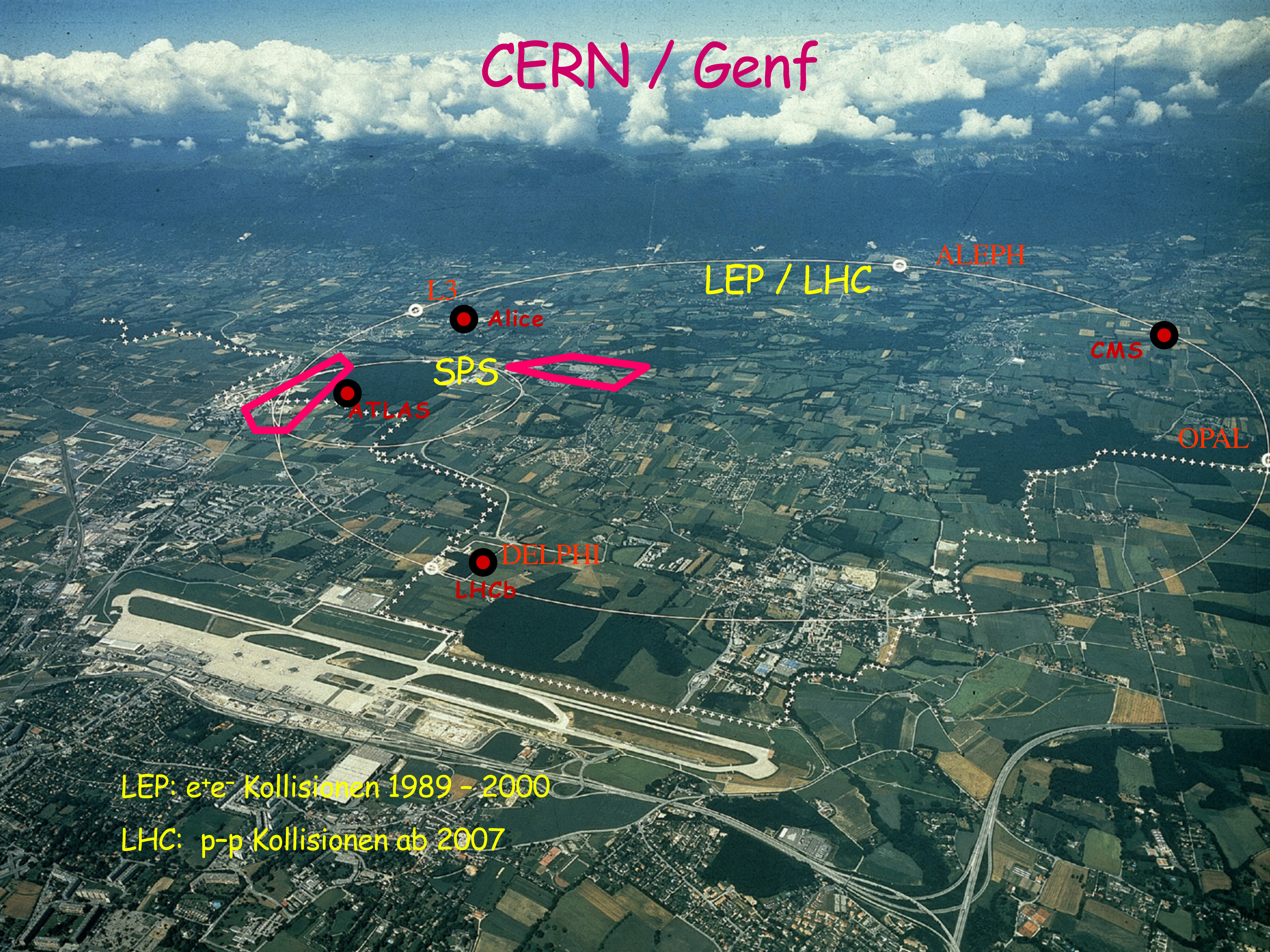


Neutron stars are the collapsed cores of a massive star.

They pack the mass of the sun into the size of a city.



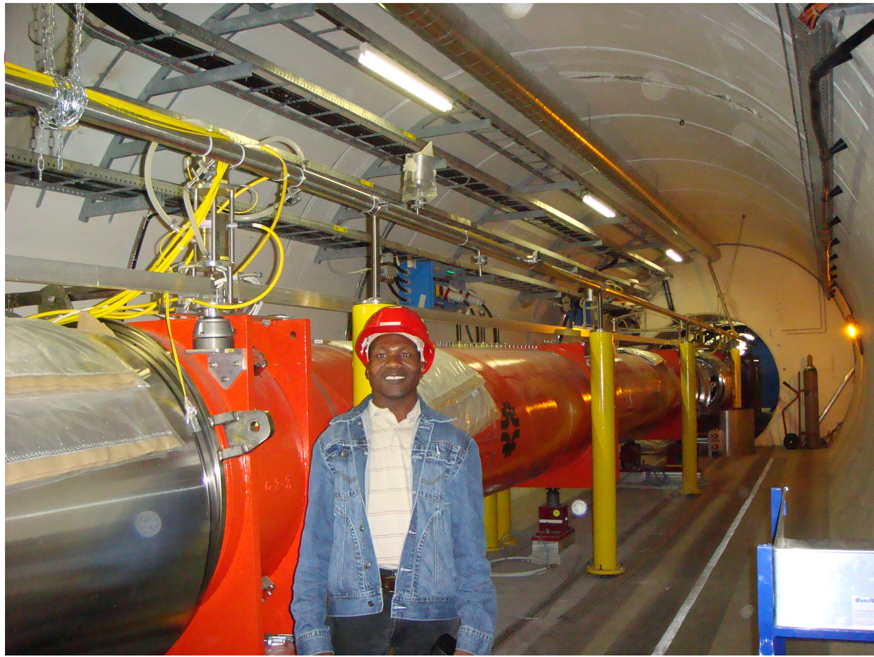
# CERN / Genf



LEP: $e^+e^-$ Kollisionen 1989 – 2000

LHC: p–p Kollisionen ab 2007


ATTENTION

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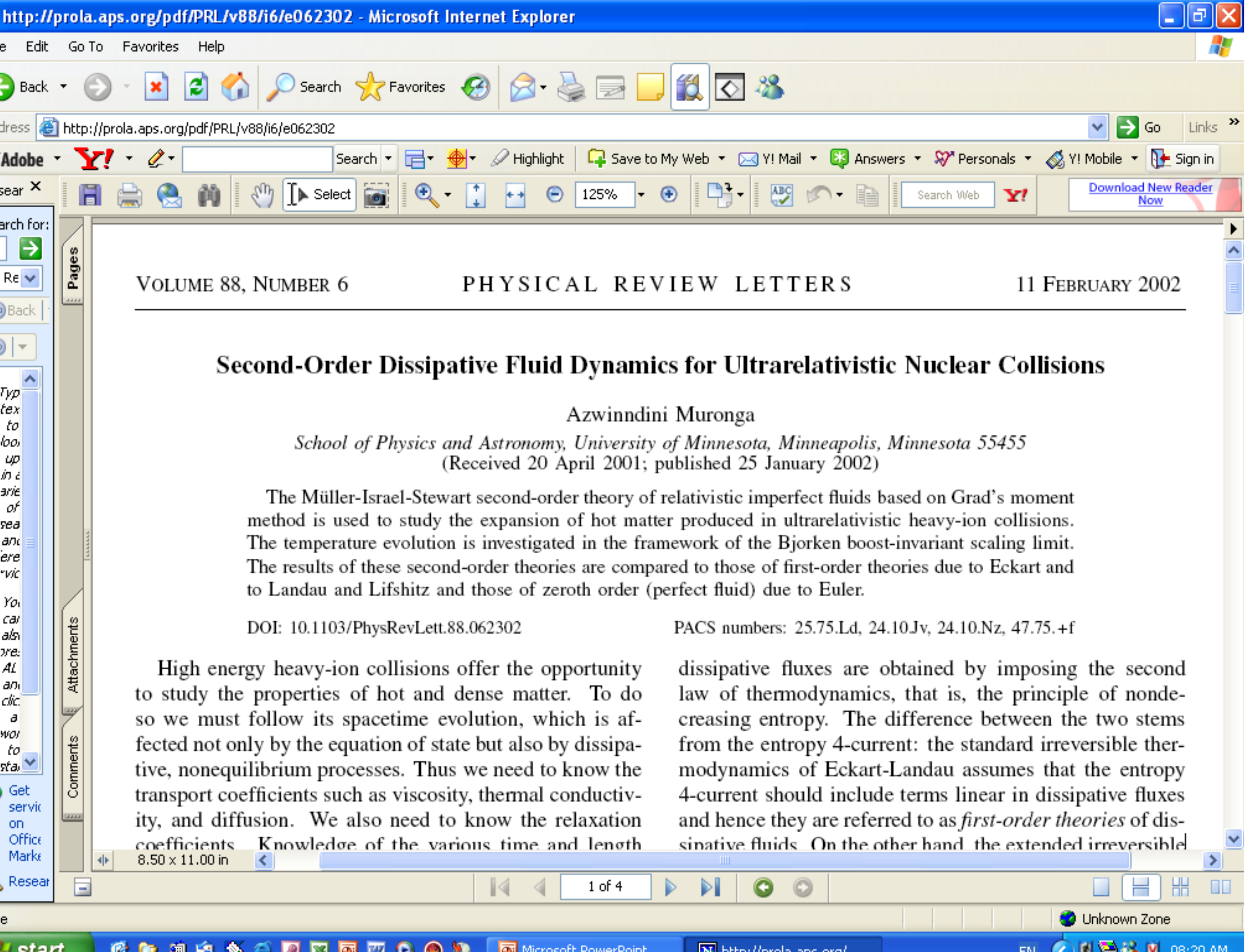
NO VISIT
IN THIS AREA



# Second-Order Dissipative Fluid Dynamics for Ultrarelativistic Nuclear Collisions

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(Received 20 April 2001; published 25 January 2002)

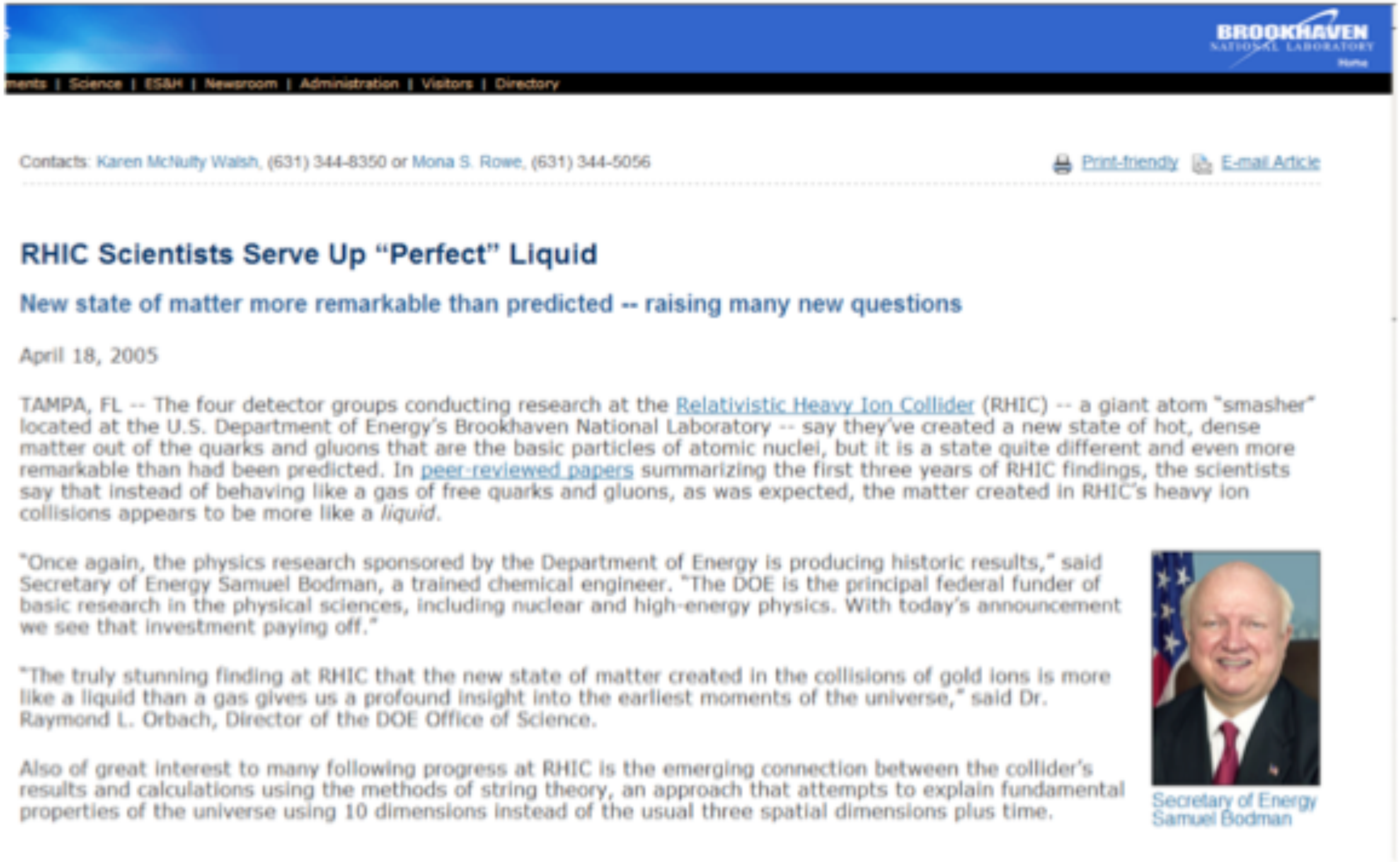
The Müller-Israel-Stewart second-order theory of relativistic imperfect fluids based on Grad's moment method is used to study the expansion of hot matter produced in ultrarelativistic heavy-ion collisions. The temperature evolution is investigated in the framework of the Bjorken boost-invariant scaling limit. The results of these second-order theories are compared to those of first-order theories due to Eckart and to Landau and Lifshitz and those of zeroth order (perfect fluid) due to Euler.

DOI: 10.1103/PhysRevLett.88.062302 PACS numbers: 25.75.Ld, 24.10.Jv, 24.10.Nz, 47.75.+f

High energy heavy-ion collisions offer the opportunity to study the properties of hot and dense matter. To do so we must follow its spacetime evolution, which is affected not only by the equation of state but also by dissipative, nonequilibrium processes. Thus we need to know the transport coefficients such as viscosity, thermal conductivity, and diffusion. We also need to know the relaxation coefficients. Knowledge of the various time and length

dissipative fluxes are obtained by imposing the second law of thermodynamics, that is, the principle of nondecreasing entropy. The difference between the two stems from the entropy 4-current: the standard irreversible thermodynamics of Eckart-Landau assumes that the entropy 4-current should include terms linear in dissipative fluxes and hence they are referred to as *first-order theories* of dissipative fluids. On the other hand, the extended irreversible

# And then the need to qualify the nature of matter produced in relativistic HIC



Contacts: Karen McNulty Walsh, (631) 344-8350 or Mona S. Rowe, (631) 344-5056

## RHIC Scientists Serve Up "Perfect" Liquid

### New state of matter more remarkable than predicted -- raising many new questions

April 18, 2005

TAMPA, FL -- The four detector groups conducting research at the Relativistic Heavy Ion Collider (RHIC) -- a giant atom "smasher" located at the U.S. Department of Energy's Brookhaven National Laboratory -- say they've created a new state of hot, dense matter out of the quarks and gluons that are the basic particles of atomic nuclei, but it is a state quite different and even more remarkable than had been predicted. In peer-reviewed papers summarizing the first three years of RHIC findings, the scientists say that instead of behaving like a gas of free quarks and gluons, as was expected, the matter created in RHIC's heavy ion collisions appears to be more like a *liquid*.

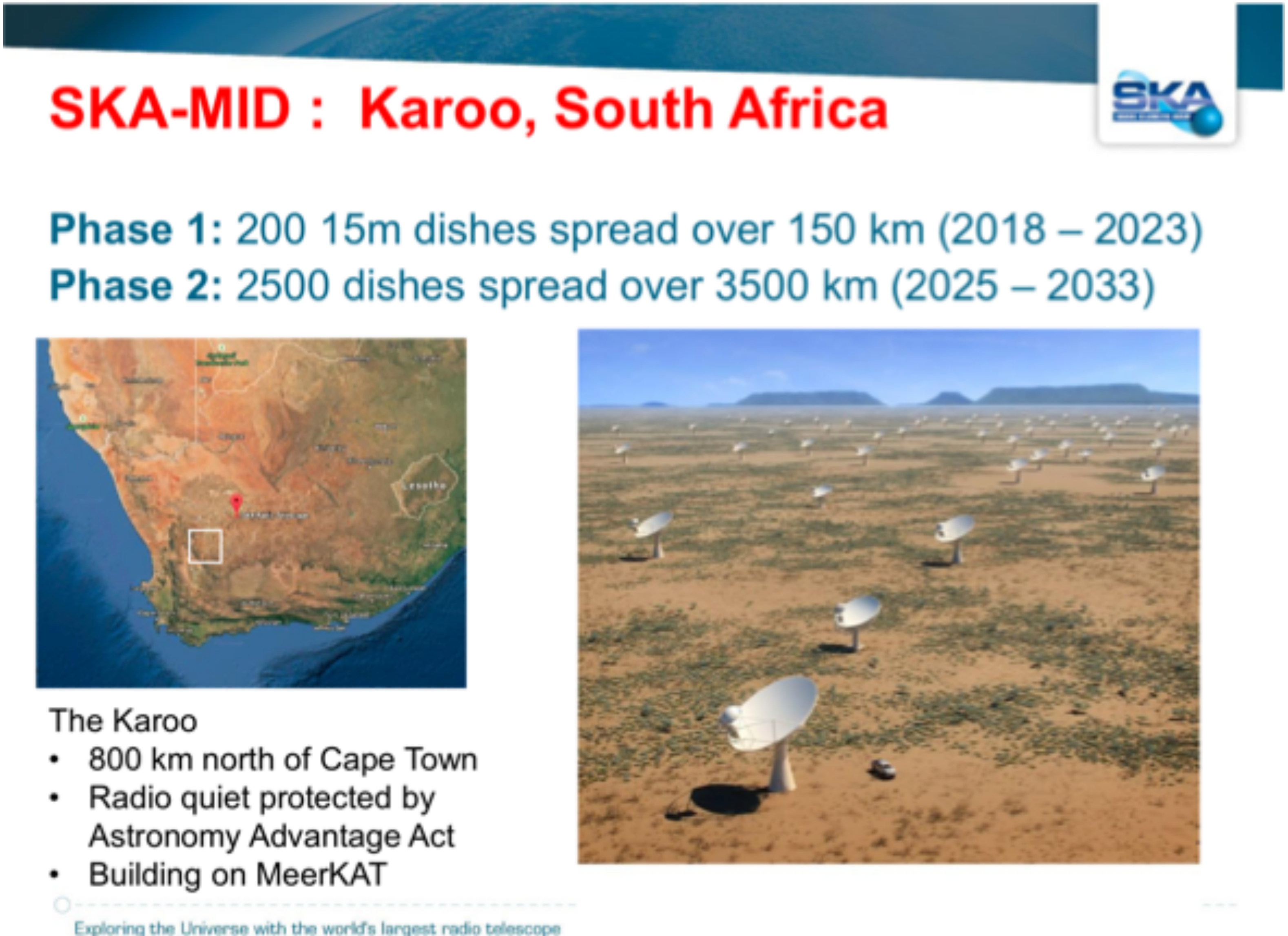
"Once again, the physics research sponsored by the Department of Energy is producing historic results," said Secretary of Energy Samuel Bodman, a trained chemical engineer. "The DOE is the principal federal funder of basic research in the physical sciences, including nuclear and high-energy physics. With today's announcement we see that investment paying off."

"The truly stunning finding at RHIC that the new state of matter created in the collisions of gold ions is more like a liquid than a gas gives us a profound insight into the earliest moments of the universe," said Dr. Raymond L. Orbach, Director of the DOE Office of Science.

Also of great interest to many following progress at RHIC is the emerging connection between the collider's results and calculations using the methods of string theory, an approach that attempts to explain fundamental properties of the universe using 10 dimensions instead of the usual three spatial dimensions plus time.

Secretary of Energy Samuel Bodman

# The new African connection

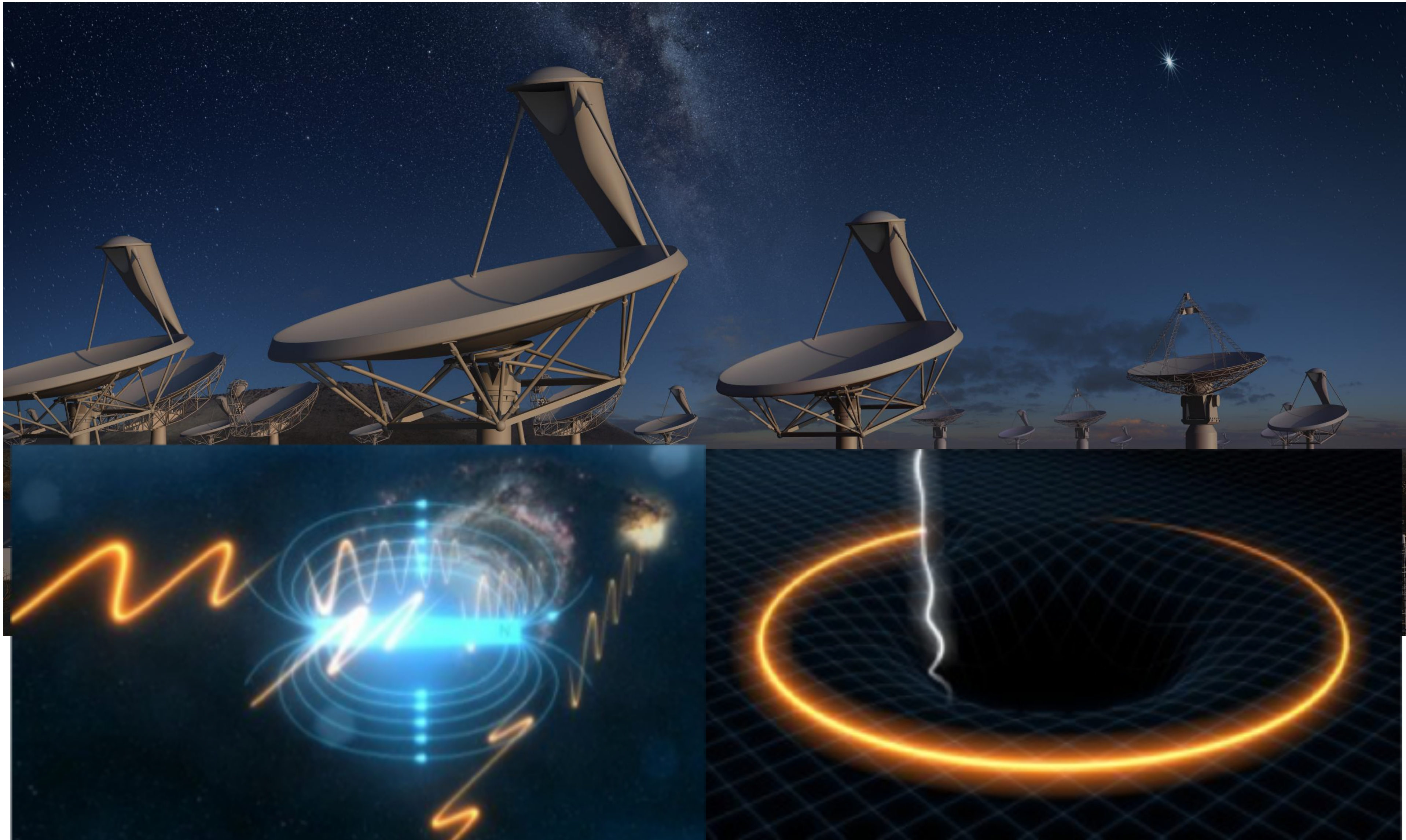


## SKA-MID : Karoo, South Africa

**Phase 1:** 200 15m dishes spread over 150 km (2018 – 2023)
**Phase 2:** 2500 dishes spread over 3500 km (2025 – 2033)

The Karoo
- 800 km north of Cape Town
- Radio quiet protected by Astronomy Advantage Act
- Building on MeerKAT

# SKA Science



# Origins of Ultrarelativstic Heavy Ion Collisions:

## Workshop on BeV Collisions of Heavy Ions: How and Why

Nov 29 - Dec 1 1974

Bear Mountain New York

**Introduction and Summary:**

**The history of physics teaches us that profound revolutions arise from a gradual perception that certain observations can be accommodated only by radical departures from current thinking.  The workshop addressed itself to the intriguing question of the possible existence of a nuclear world quite different from the one we have learned to accept as familiar and stable.**

**Leon Lederman and Joseph Weneser**

**It would be interesting to explore new phenomena by distributing high energy or high nuclear density over a relatively large volume.**

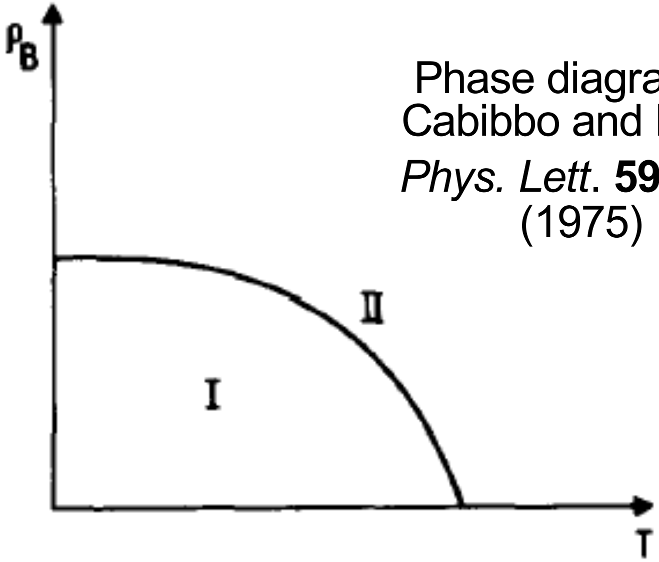
**T. D. Lee**

# Early Work on the Phase Diagram of QCD

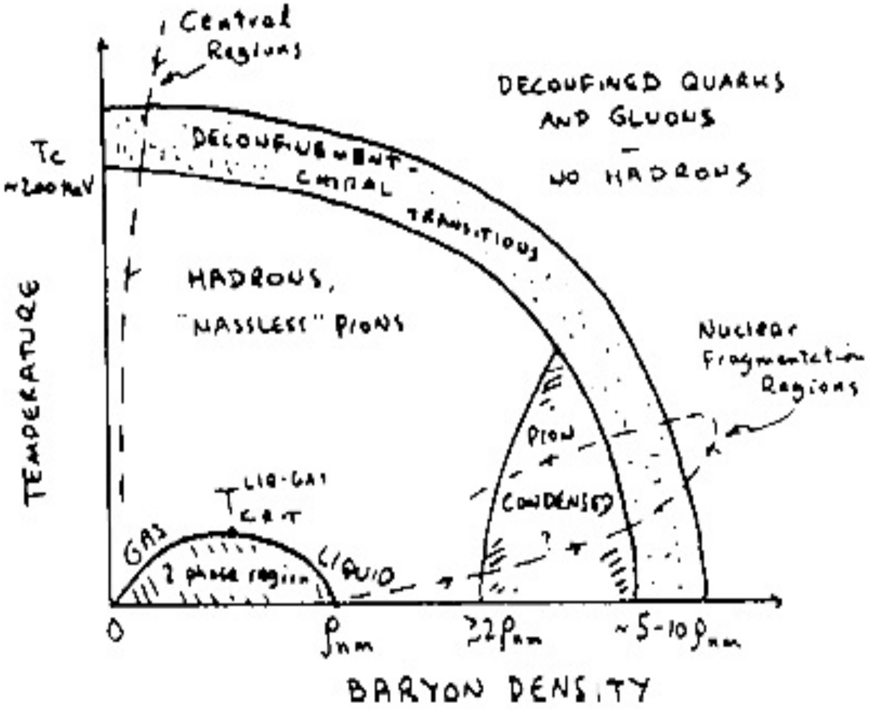
N. Itoh, *Prog. Theor. Phys*. **44**, 291 (1970)

P. Carruthers, *Coll. Phenom*. **1**, 147 (1973)

Arguments using asymptotic freedom by J. Collins and M. Perry, *Phys. Rev. Lett*. , **34**, 1353 (1975)



Phase diagram of Cabibbo and Parisi *Phys. Lett*. **59B**, 67 (1975)



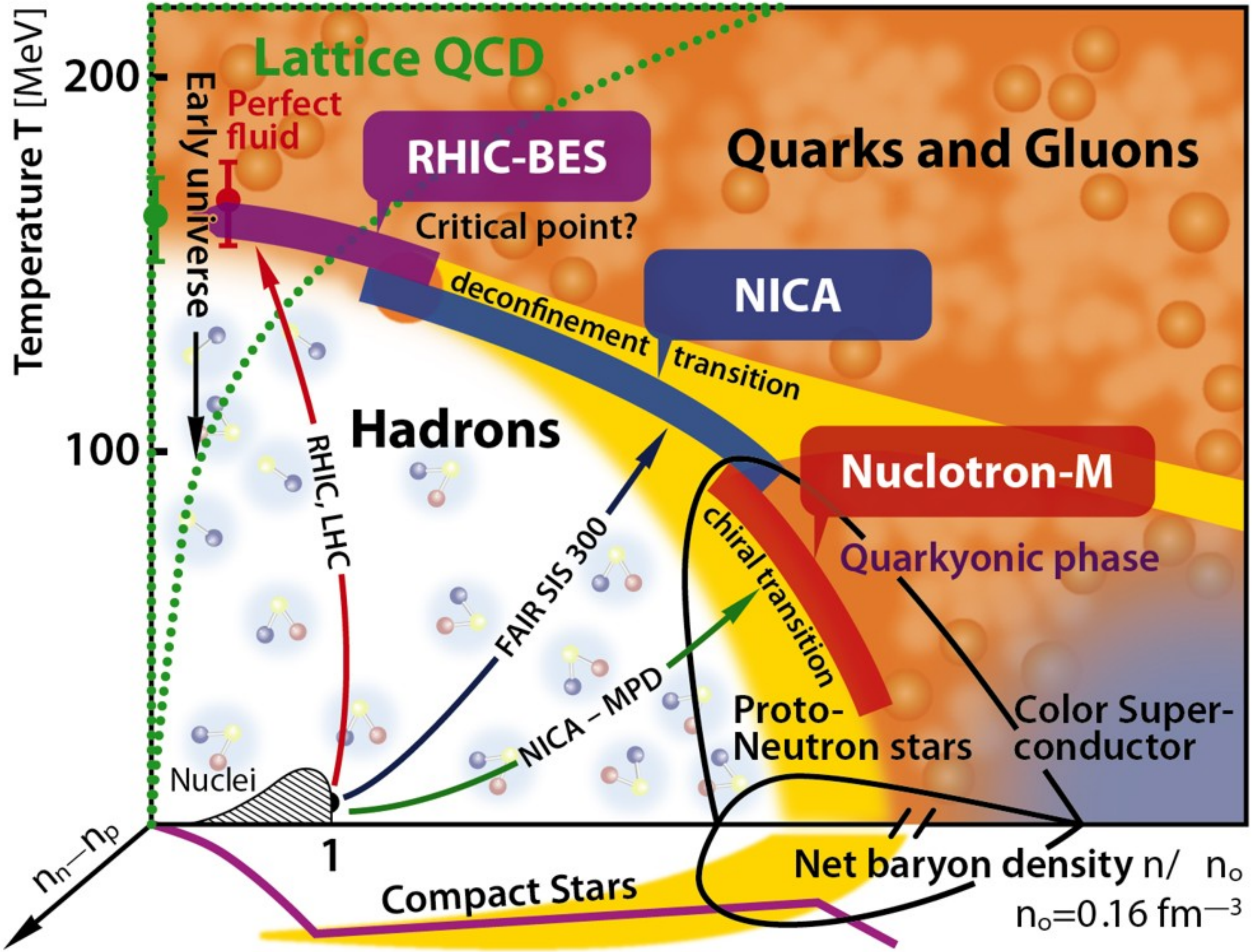
Higher order computations by Baym and Chin 1976; McLerran and Freedman 1977;

Finite T and name Quark Gluon Plasma by Shuryak 1978;

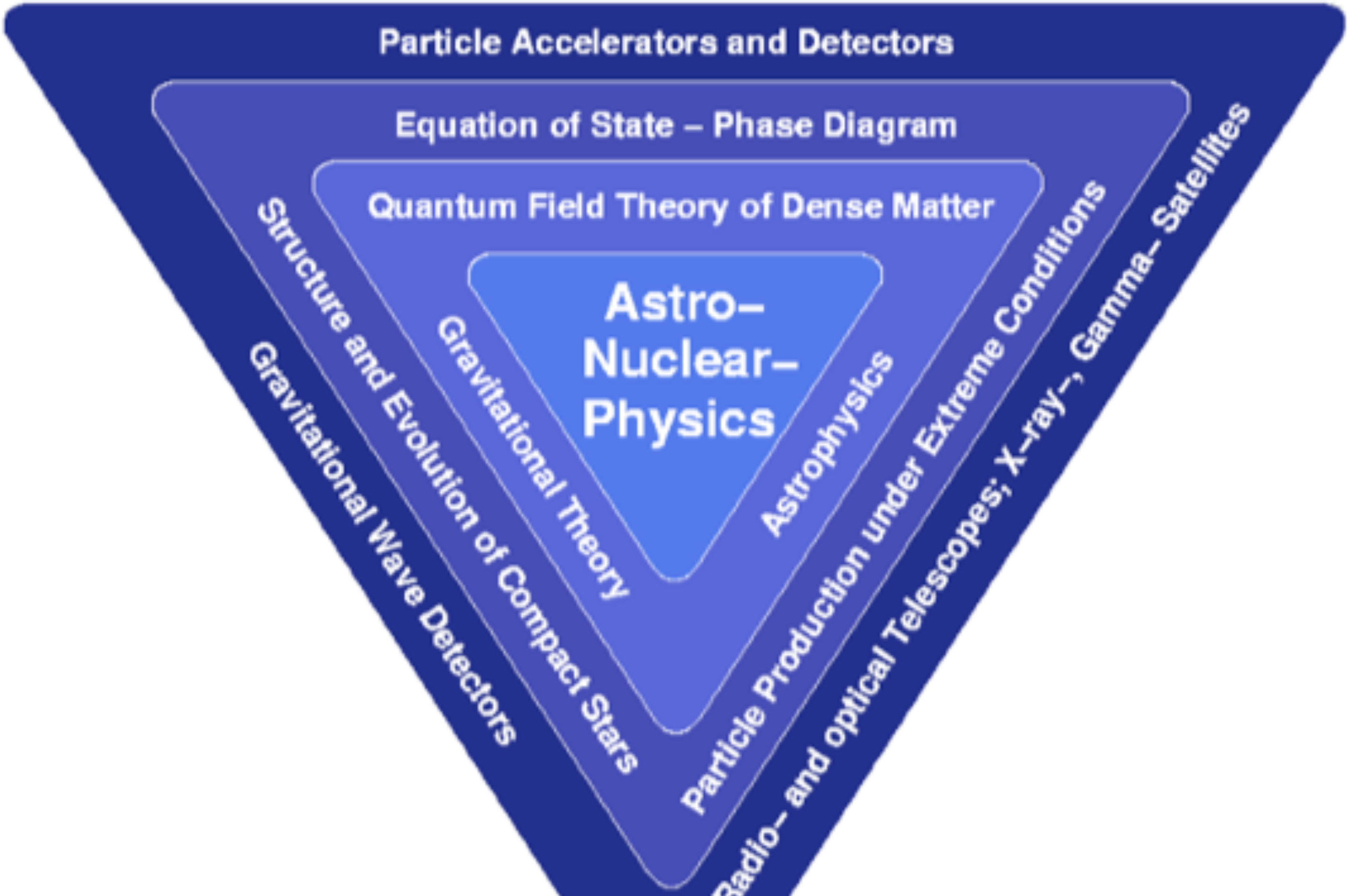
Kapusta 1979

Phase diagram of Baym from 1983 NSAC Long Range Plan

# Nuclear Matter Phase Diagram

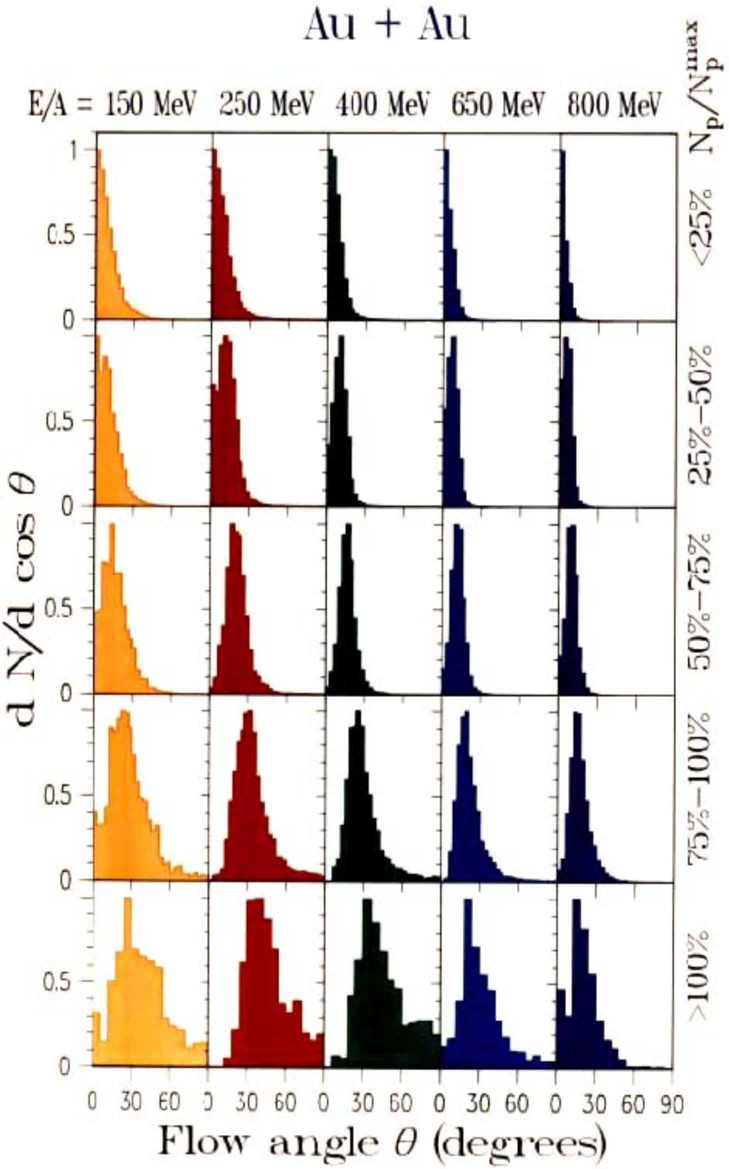


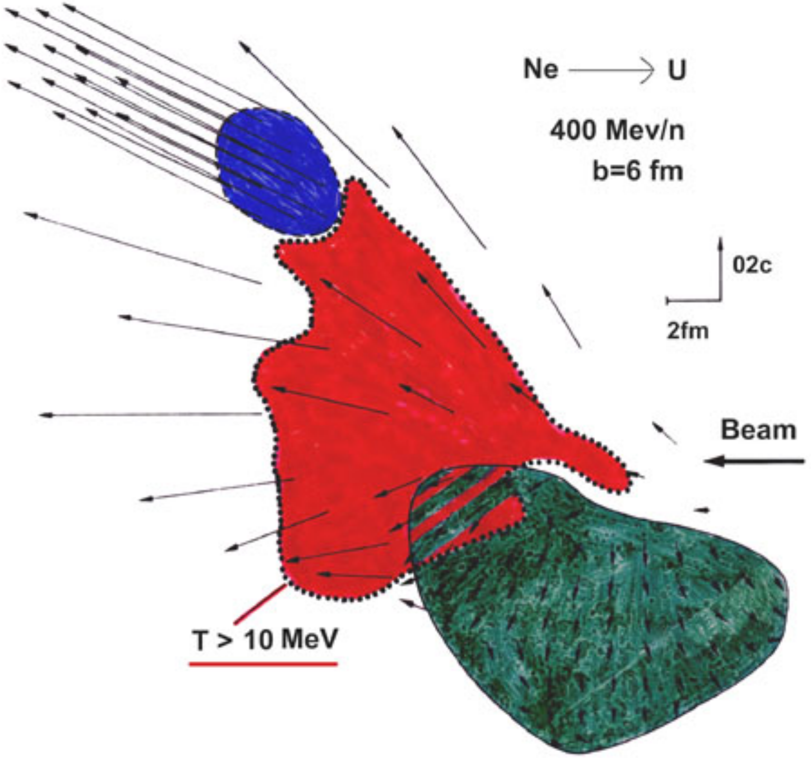
# Intersection of nuclear physics, particle physics, astrophysics, and cosmology



# Discovery of Flow at the Bevalac

## Plastic Ball and Streamer Chamber



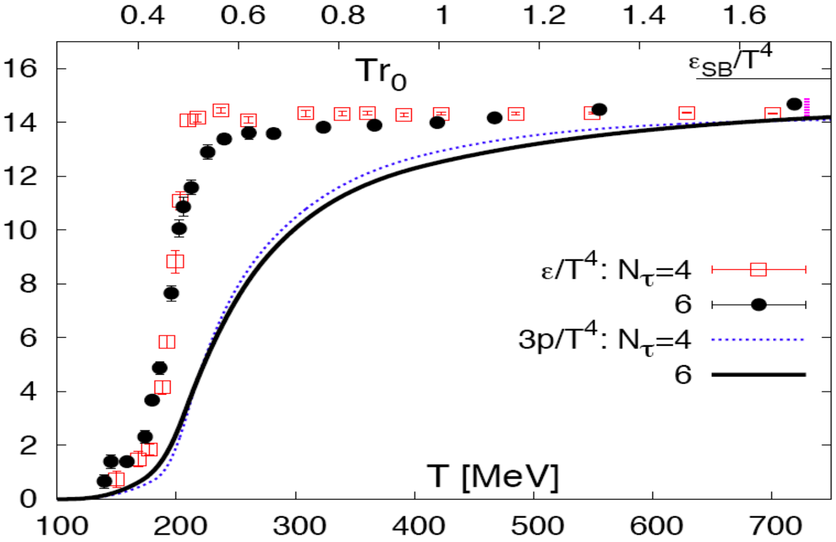


# Lattice Gauge Theory and Deconfinement:

L is similar to a spin variable
=> Confinement-
Deconfinement transition

Polyakov 1978 Susskind 1979

$$e^{-\beta F_q} =< L >$$

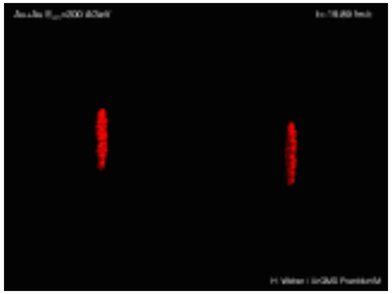


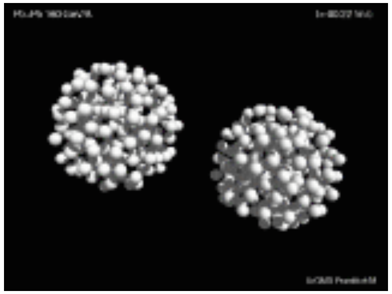
Wuppertal, Bielefeld,BNL, MILC, Mumbai …

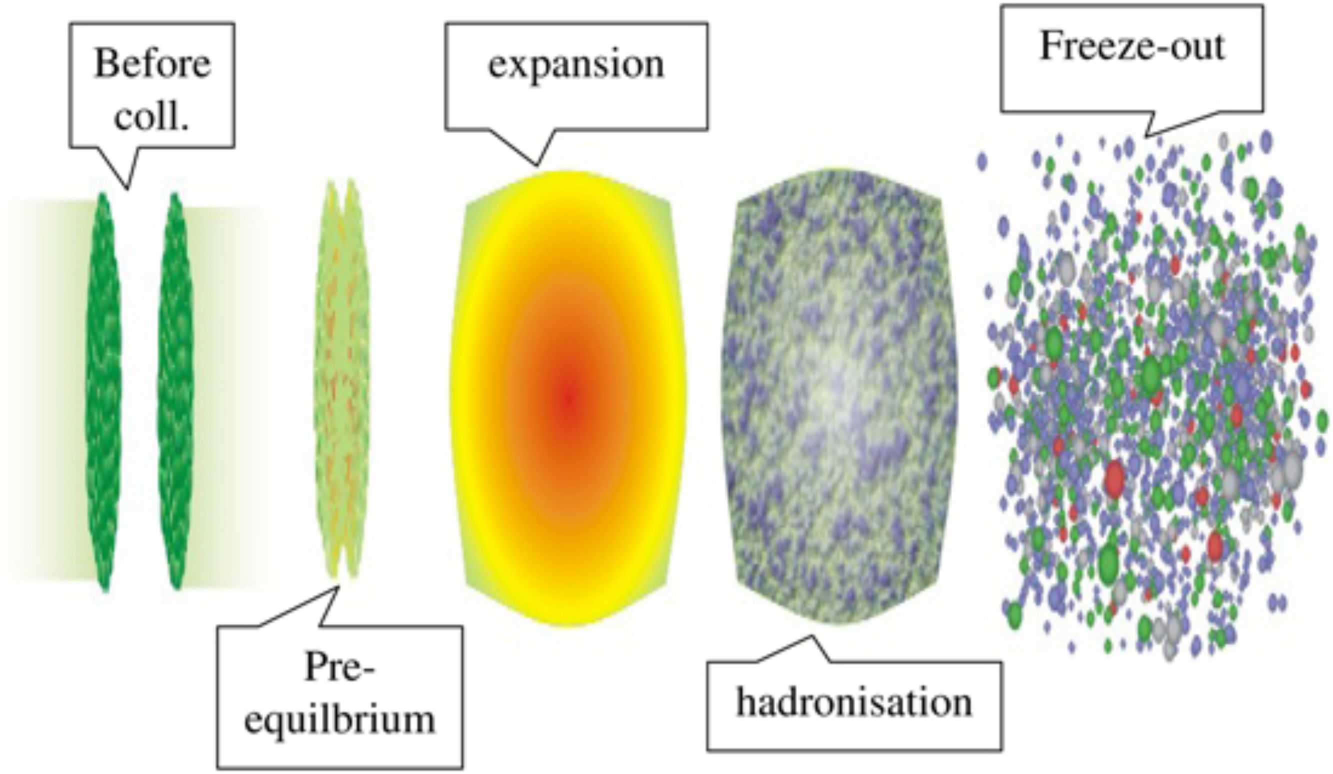
First lattice computations at finite T; Kuti, Polonyi and Szlachanyi; McLerran and Svetitsky

Beginning of Bielefeld lattice gauge theory effort: Engels, Gavai, Karsch, Montvay and Satz

# Relativistic Dynamics in Heavy Ion Collisions







# Space-Time Picture:

Early work on energy densities:

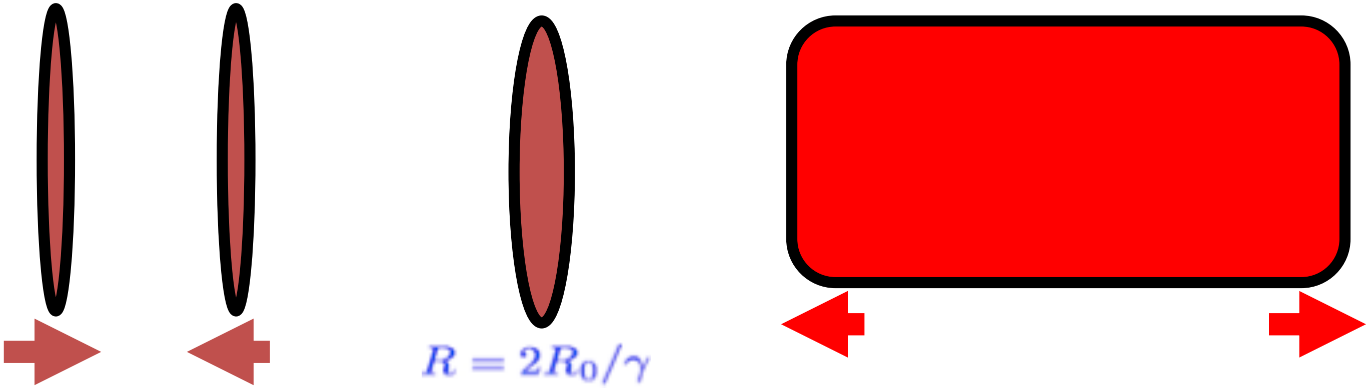
Shuryak 1974

Ansiehtty et. al. 1980

$$\tau = \sqrt{t^2 - z^2}$$

$$\eta = \frac{1}{2} ln \left( \frac{t+z}{t-z} \right)$$

$$\epsilon = \frac{1}{2}\gamma^2\rho_0$$



$$R = 2R_0/\gamma$$

$$E = \gamma M_N V_0 \rho_0$$

Landau Hydrodynamics

Colliding Nuclei

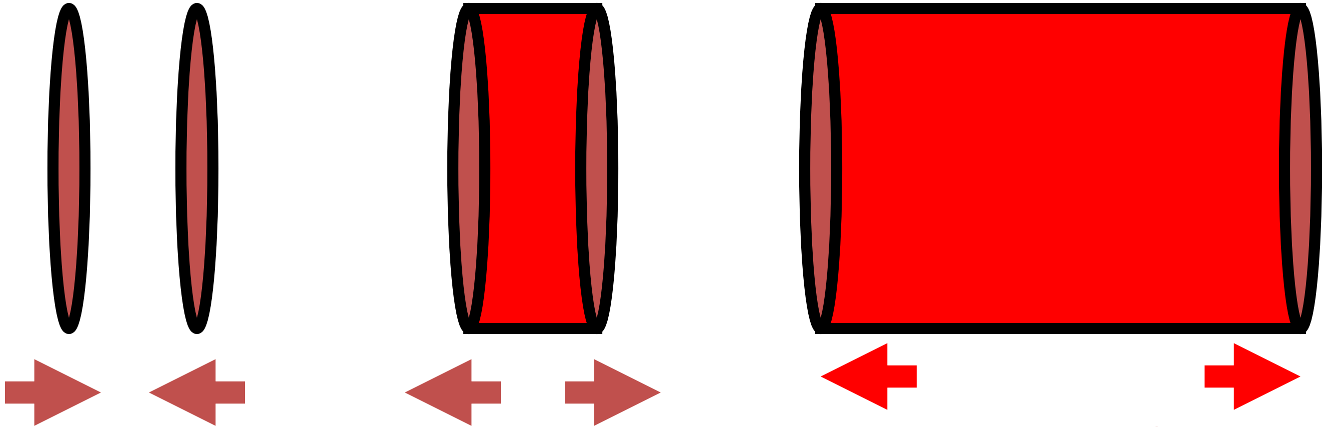
Collision

Expanding Fireball

Landau

Feynman

Bjorken



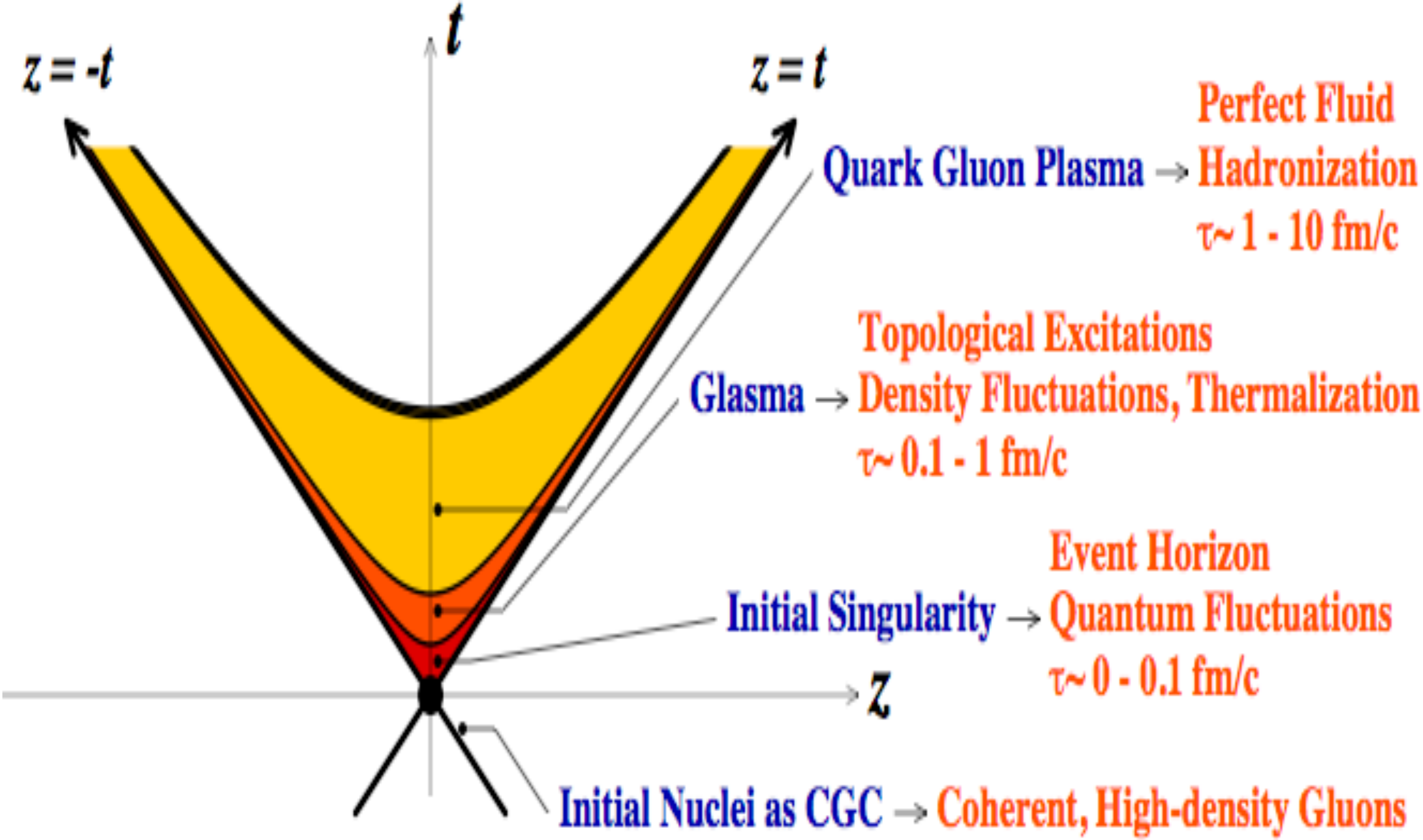
Longitudinal flow

Bjorken Hydrodynamics

$$\epsilon_0 = \frac{1}{\pi R^2 \tau_0} \frac{dE_T}{dy}$$

$$y = \eta$$

# Space-time Descriptions:



**Quark Gluon Plasma** → Perfect Fluid
Hadronization
τ~ 1 - 10 fm/c

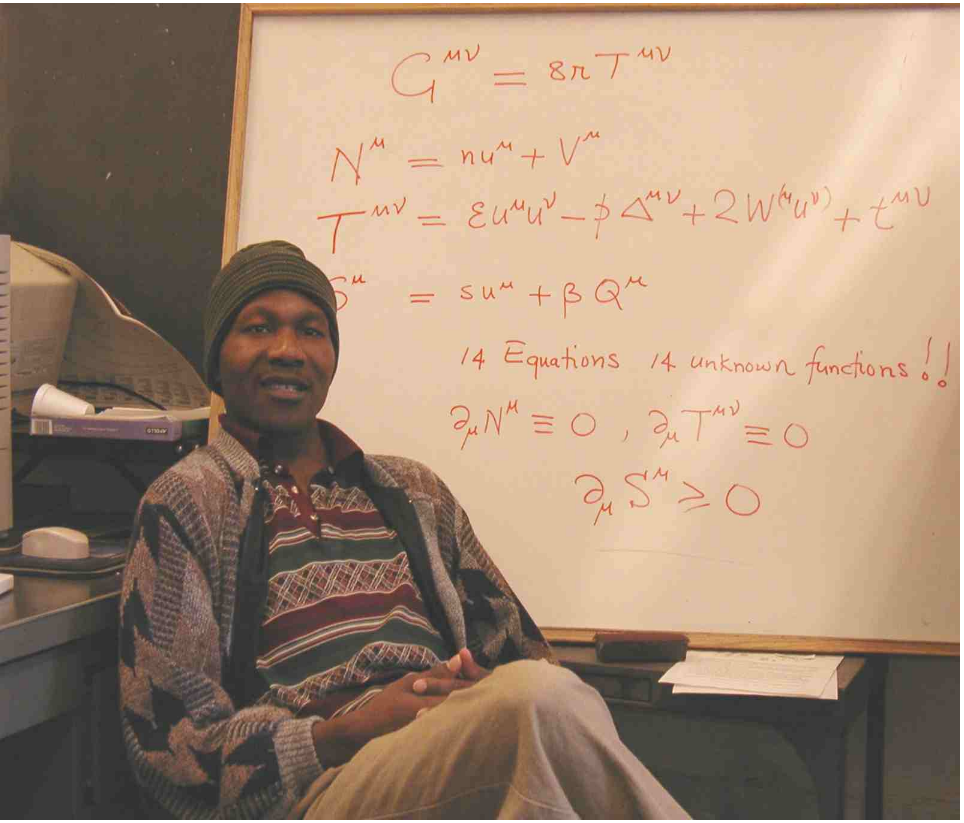
**Glasma** → Topological Excitations
Density Fluctuations, Thermalization
τ~ 0.1 - 1 fm/c

**Initial Singularity** → Event Horizon
Quantum Fluctuations
τ~ 0 - 0.1 fm/c

**Initial Nuclei as CGC** → Coherent, High-density Gluons

# The Development of Ideas:

## A Story about People

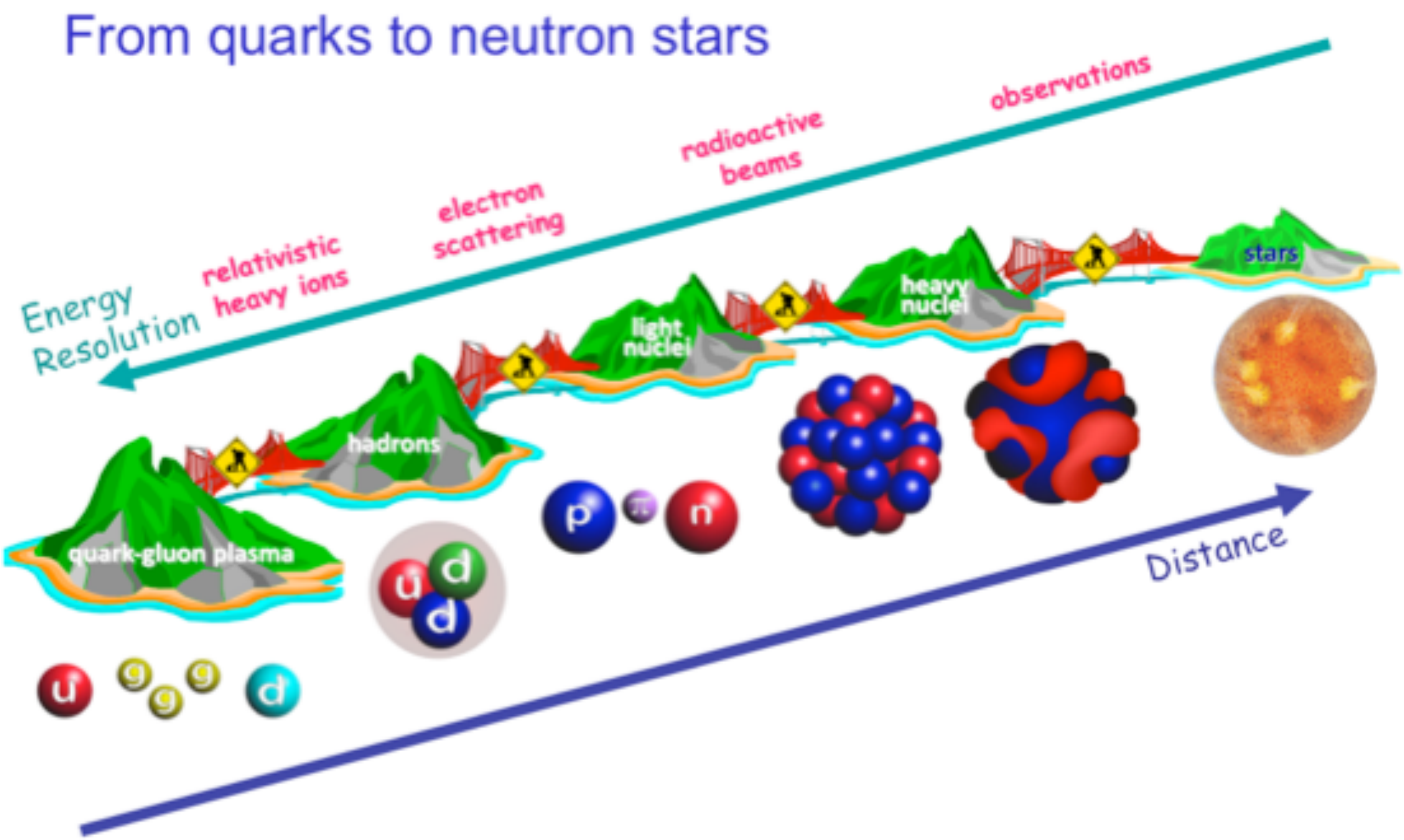


## Ideas and their Realization

The Space-Time Picture of Heavy Ion Collisions

Non-equilibrium Fluid Dynamics and Transport Theory Description

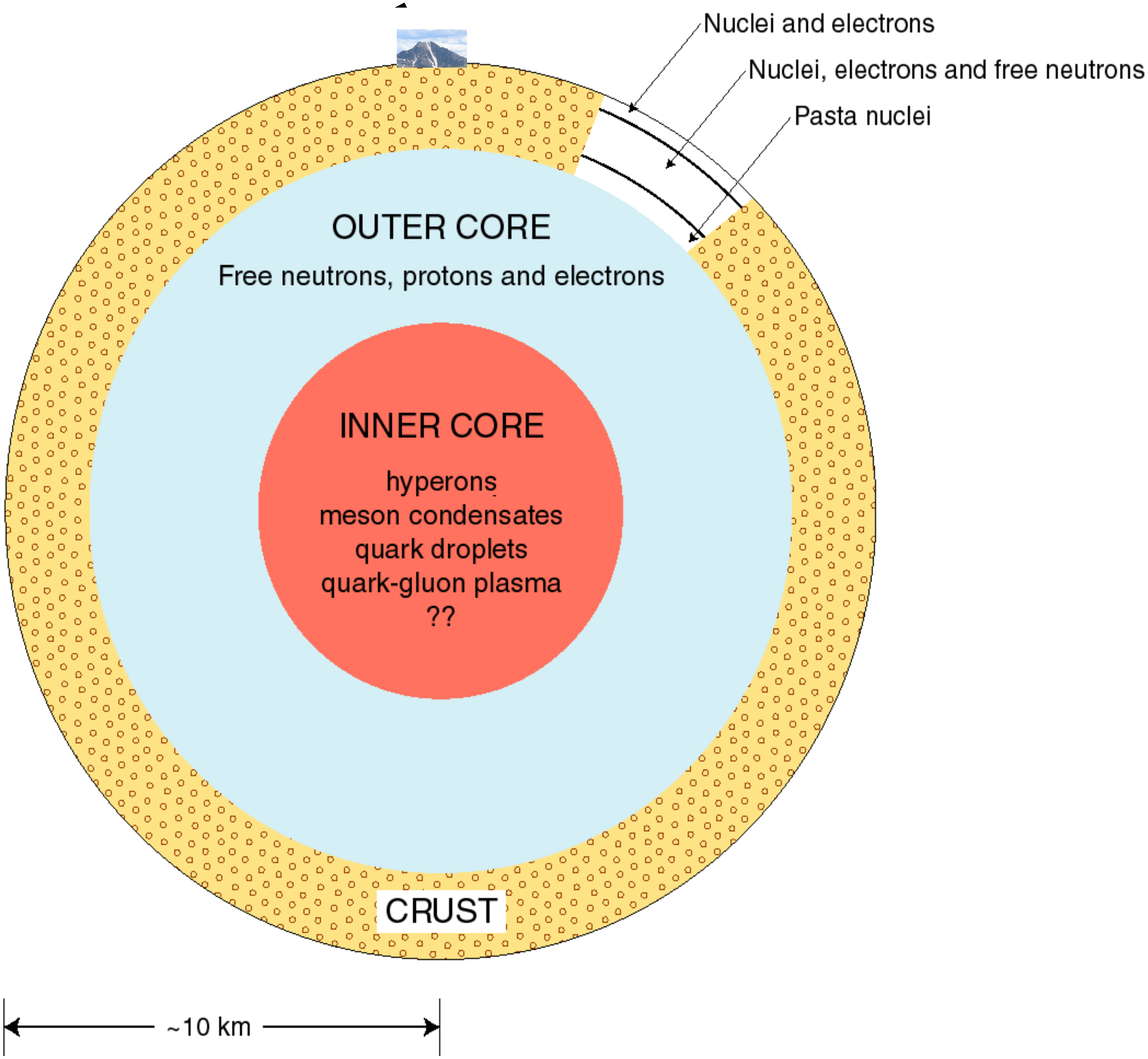
# Neutron stars and the properties of matter at high density

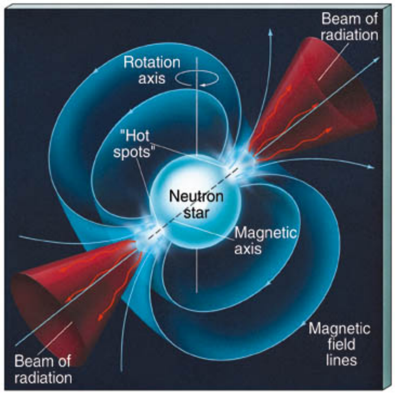


# Neutron star interior

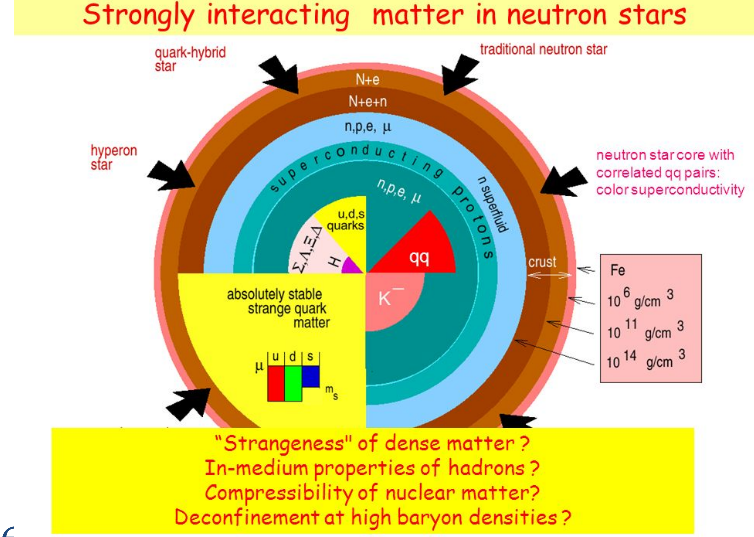
Mass ~ 1.4-2 $M_{sun}$
Radius ~ 10-12 km
Temperature
~ $10^6$-$10^9$ K

Surface gravity
~$10^{14}$ that of Earth
Surface binding
~ 1/10 $mc^2$





Baryon number ~ $10^{57}$


Strongly interacting matter in neutron stars

"Strangeness" of dense matter ?
In-medium properties of hadrons ?
Compressibility of nuclear matter?
Deconfinement at high baryon densities ?

Made in gravitational collapse
of massive stars (supernovae)

Central element in variety of compact energetic systems:
pulsars, binary x-ray sources, soft gamma repeaters
Merging neutron star-neutron star and neutron star-black hole
sources of gamma ray bursts

Matter in neutron stars is densest in universe:
$\rho$ up to ~ 5-10 $\rho_0$ ($\rho_0 = 3\text{X}10^{14}$g/cm$^3$ = density of matter in atomic nuclei)
[cf. white dwarfs: $\rho \sim 10^5$-$10^9$ g/cm$^3$]

Supported against gravitational collapse by nucleon degeneracy pressure

Astrophysical laboratory for study of high density matter
complementary to accelerator experiments

*What are states in interior? Onset of quark degrees of freedom!*
*Do quark stars, as well as strange stars exist?*

# The liquid interior

| | |
|---|---|
| Neutrons (likely superfluid) ~ 95% | Non-relativistic |
| Protons (likely superconducting) ~ 5% | Non-relativistic |
| Electrons (normal, $T_c \sim T_f e^{-137}$) ~ 5% | Fully relativistic |

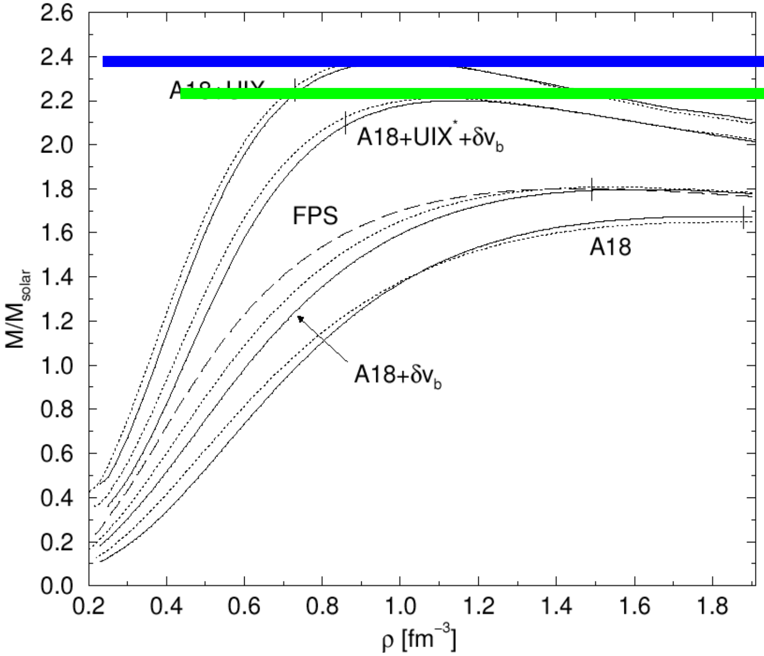
Eventually muons, hyperons, and possibly exotica:

- pion condensation
- kaon condensation
- quark droplets
- bulk quark matter

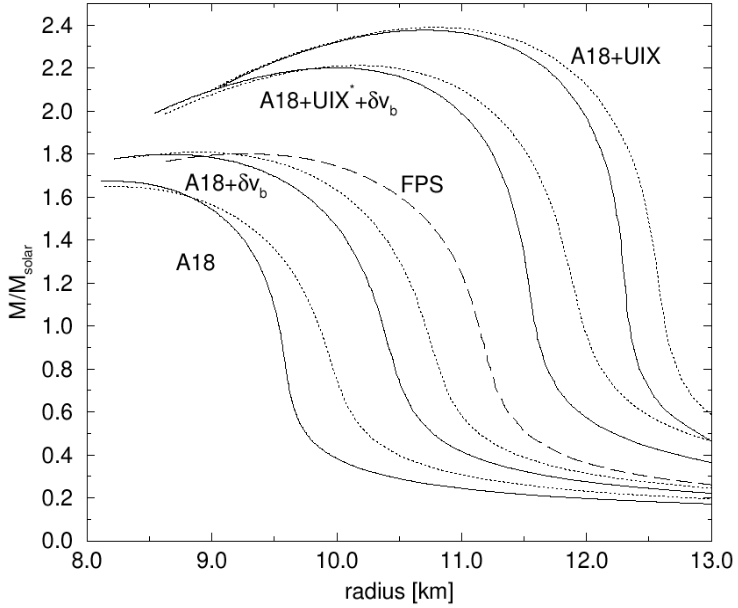
$n_0$ = baryon density
in large nuclei $\simeq 0.16\ \mathrm{fm}^{-3}$
$1\,\mathrm{fm} = 10^{-13}\,\mathrm{cm}$

Phase transition from crust to liquid at $n_b \cong 0.7 n_0 \cong 0.09\ \mathrm{fm}^{-3}$
or $\rho$ = mass density ~ $2 \times 10^{14}\ \mathrm{g/cm^3}$

# Maximum neutron star mass



**Mass vs. central density**



**Mass vs. radius**

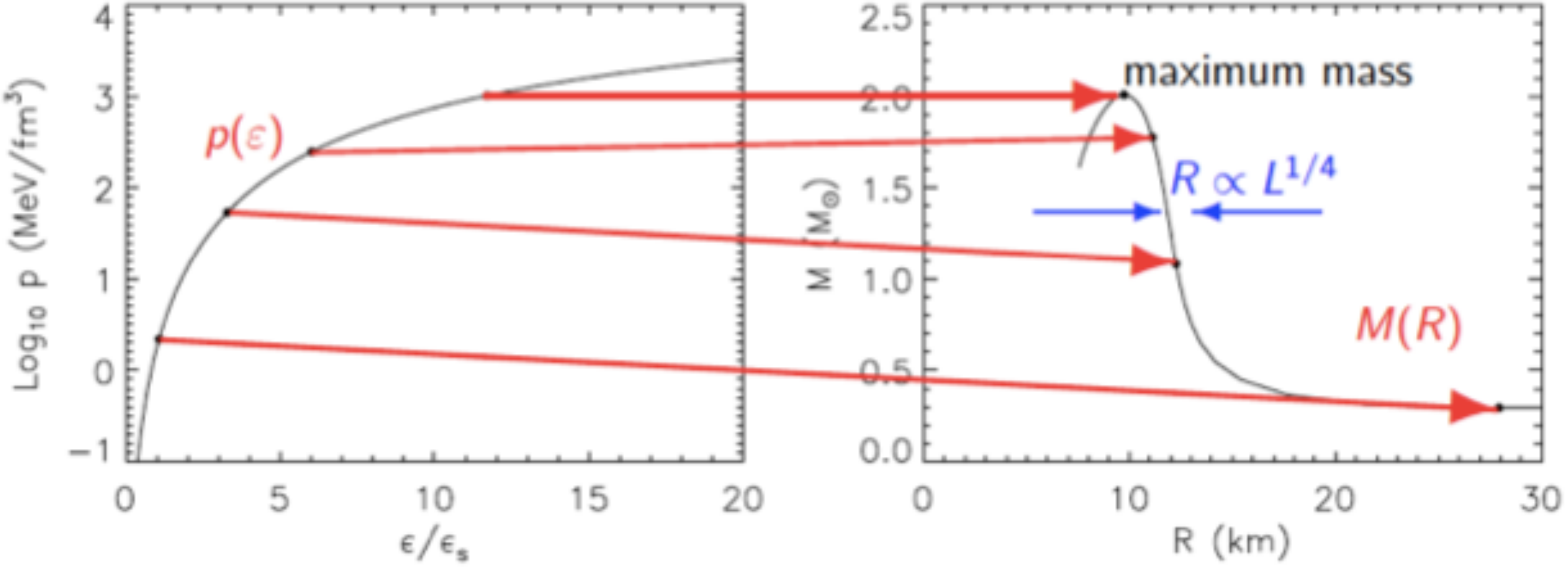
*Akmal, Pandharipande and Ravenhall, 1998*

# Equation of state vs. neutron star structure

Tolman-Oppenheimer-Volkov equations

$$\frac{dp}{dr} = -\frac{G}{c^4}\frac{(mc^2 + 4\pi pr^3)(\varepsilon + p)}{r(r - 2Gm/c^2)}$$

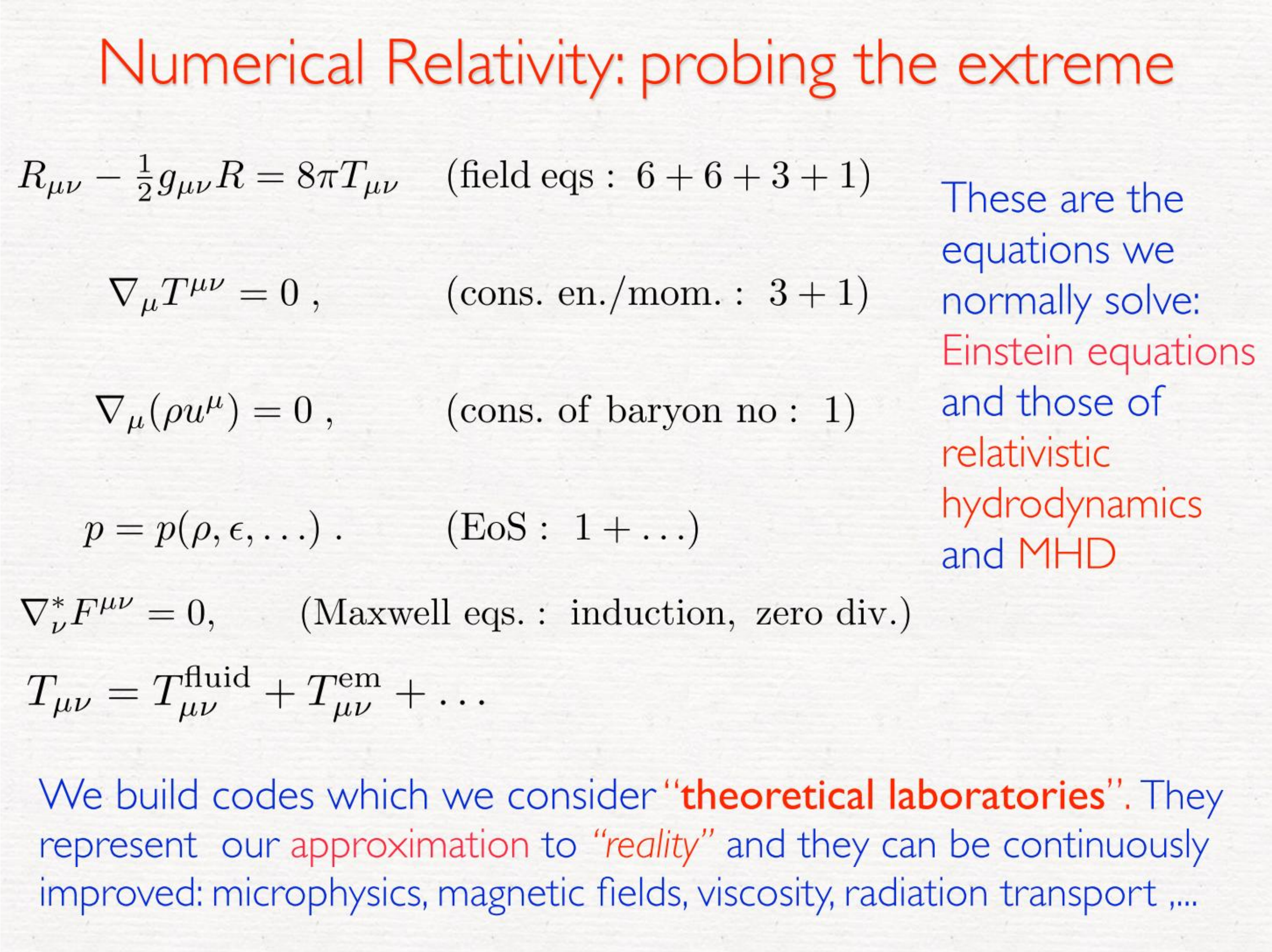
$$\frac{dm}{dr} = 4\pi\frac{\varepsilon}{c^2}r^2$$



Equation of State ← Observations

*from J. Lattimer*

# Numerical Relativity



## Numerical Relativity: probing the extreme

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu} \qquad (\text{field eqs}: \ 6+6+3+1)$$

$$\nabla_\mu T^{\mu\nu} = 0 \ , \qquad (\text{cons. en./mom.}: \ 3+1)$$

$$\nabla_\mu (\rho u^\mu) = 0 \ , \qquad (\text{cons. of baryon no}: \ 1)$$

$$p = p(\rho, \epsilon, \ldots) \ . \qquad (\text{EoS}: \ 1 + \ldots)$$

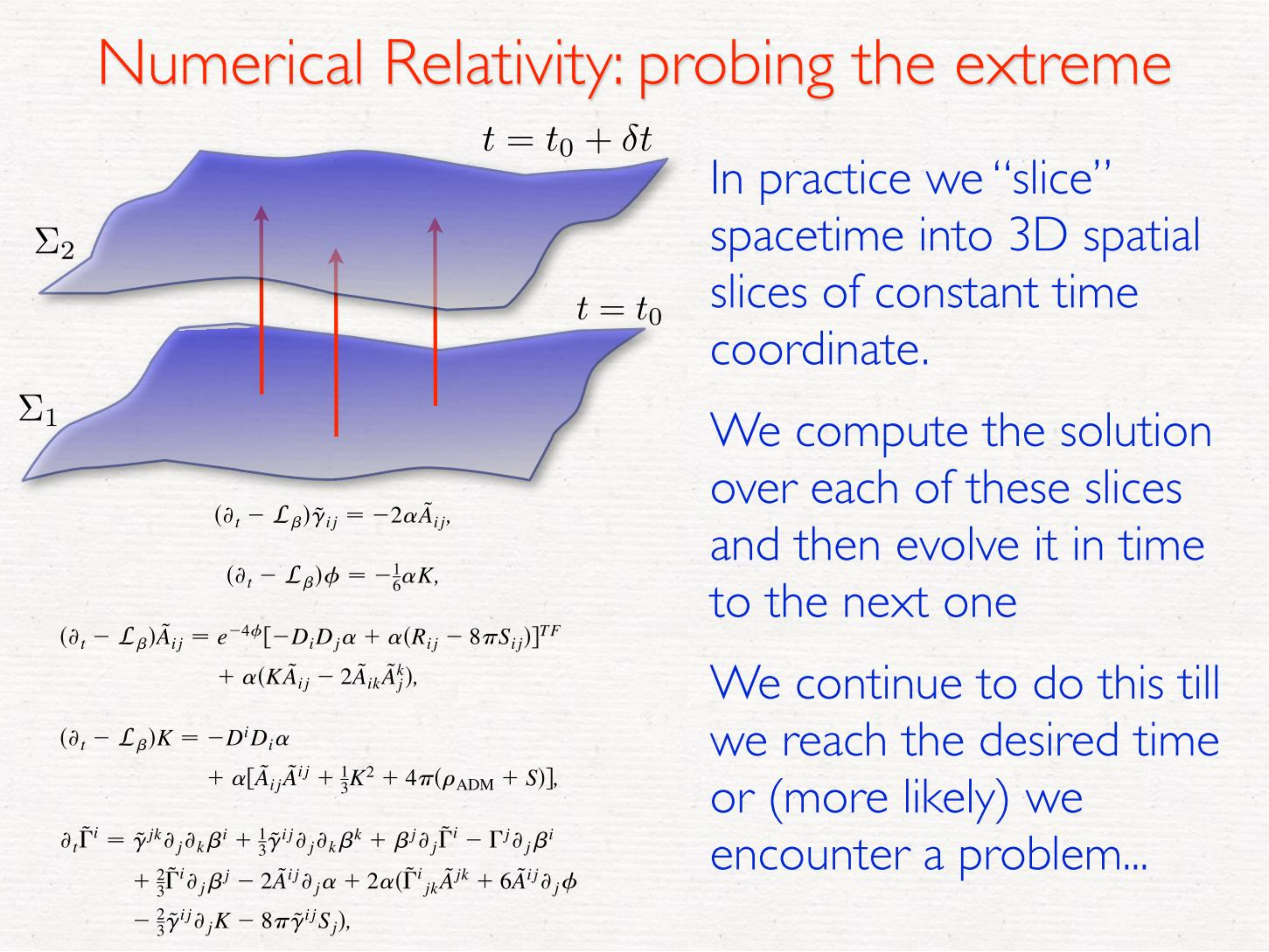
$$\nabla^*_\nu F^{\mu\nu} = 0, \qquad (\text{Maxwell eqs.}: \ \text{induction, zero div.})$$

$$T_{\mu\nu} = T^{\text{fluid}}_{\mu\nu} + T^{\text{em}}_{\mu\nu} + \ldots$$

These are the equations we normally solve: Einstein equations and those of relativistic hydrodynamics and MHD

We build codes which we consider "**theoretical laboratories**". They represent our approximation to *"reality"* and they can be continuously improved: microphysics, magnetic fields, viscosity, radiation transport ,...

# Numerical Relativity



## Numerical Relativity: probing the extreme

$$(\partial_t - \mathcal{L}_\beta)\tilde{\gamma}_{ij} = -2\alpha\tilde{A}_{ij},$$

$$(\partial_t - \mathcal{L}_\beta)\phi = -\tfrac{1}{6}\alpha K,$$

$$\begin{aligned}(\partial_t - \mathcal{L}_\beta)\tilde{A}_{ij} = {} & e^{-4\phi}[-D_i D_j \alpha + \alpha(R_{ij} - 8\pi S_{ij})]^{TF} \\ & + \alpha(K\tilde{A}_{ij} - 2\tilde{A}_{ik}\tilde{A}^k_j),\end{aligned}$$

$$\begin{aligned}(\partial_t - \mathcal{L}_\beta)K = {} & -D^i D_i \alpha \\ & + \alpha[\tilde{A}_{ij}\tilde{A}^{ij} + \tfrac{1}{3}K^2 + 4\pi(\rho_{\mathrm{ADM}} + S)],\end{aligned}$$

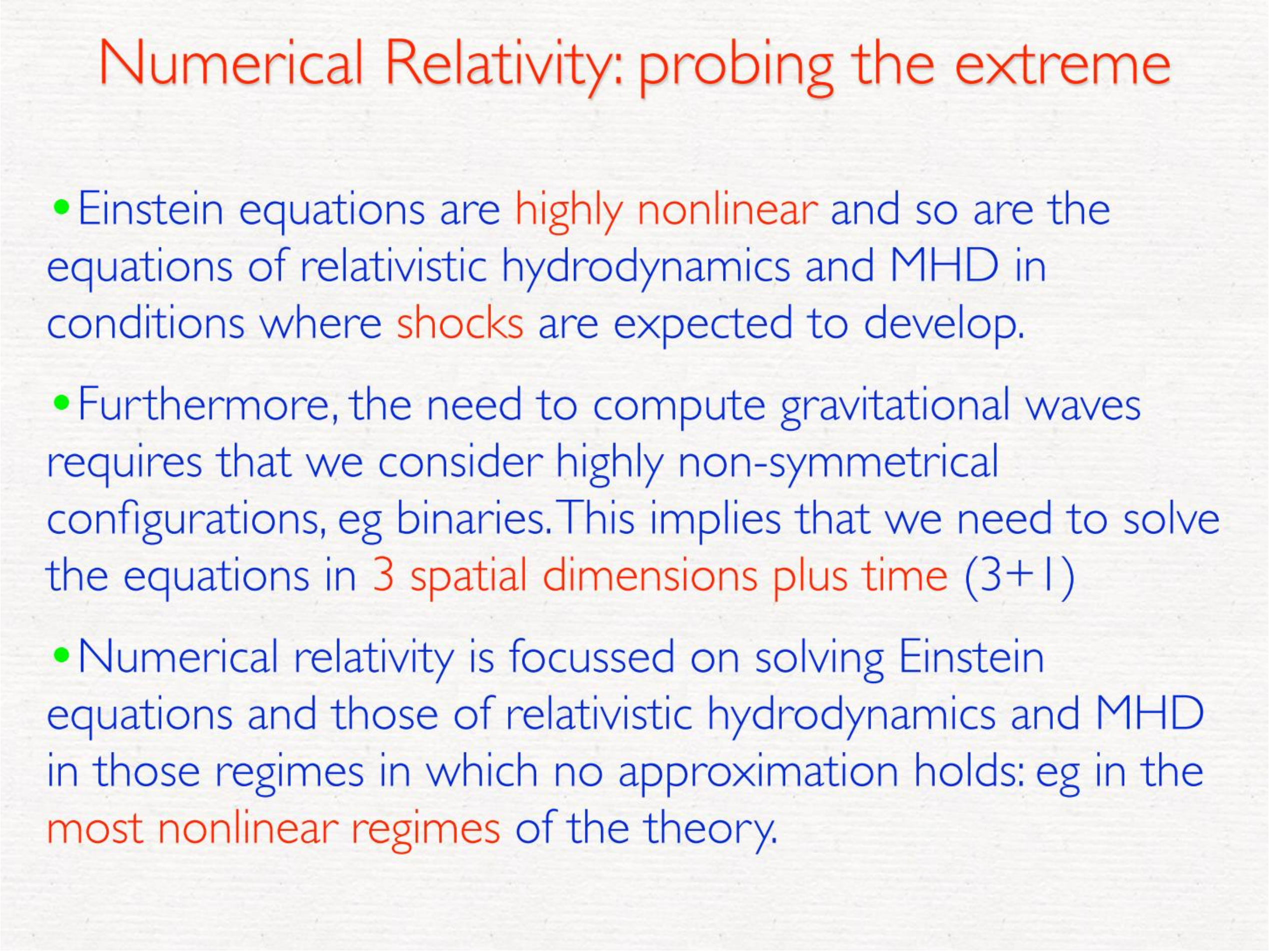
$$\begin{aligned}\partial_t\tilde{\Gamma}^i = {} & \tilde{\gamma}^{jk}\partial_j\partial_k\beta^i + \tfrac{1}{3}\tilde{\gamma}^{ij}\partial_j\partial_k\beta^k + \beta^j\partial_j\tilde{\Gamma}^i - \Gamma^j\partial_j\beta^i \\ & + \tfrac{2}{3}\tilde{\Gamma}^i\partial_j\beta^j - 2\tilde{A}^{ij}\partial_j\alpha + 2\alpha(\tilde{\Gamma}^i{}_{jk}\tilde{A}^{jk} + 6\tilde{A}^{ij}\partial_j\phi \\ & - \tfrac{2}{3}\tilde{\gamma}^{ij}\partial_j K - 8\pi\tilde{\gamma}^{ij}S_j),\end{aligned}$$

In practice we "slice" spacetime into 3D spatial slices of constant time coordinate.

We compute the solution over each of these slices and then evolve it in time to the next one

We continue to do this till we reach the desired time or (more likely) we encounter a problem...

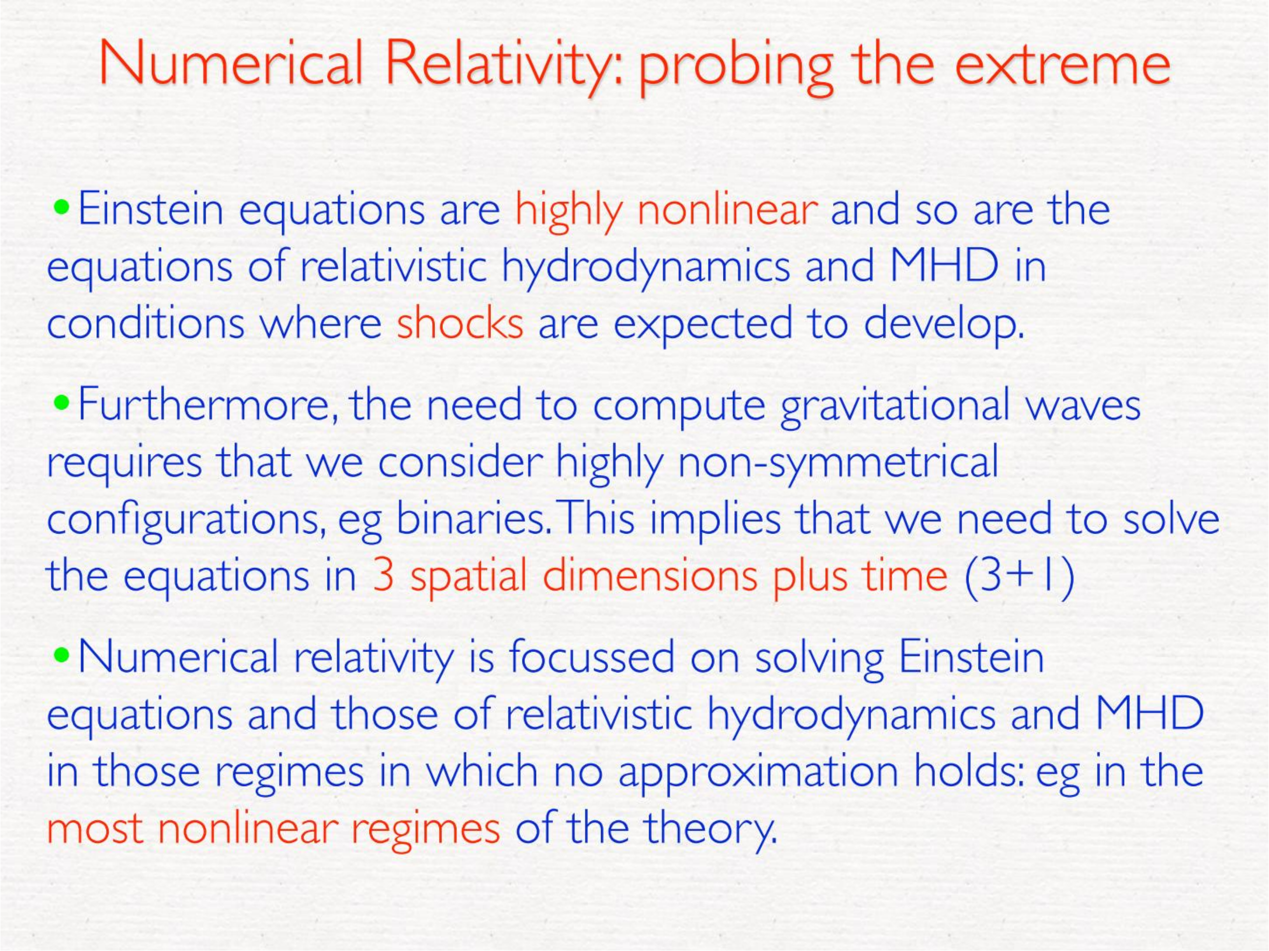
# Numerical Relativity



## Numerical Relativity: probing the extreme

- Einstein equations are highly nonlinear and so are the equations of relativistic hydrodynamics and MHD in conditions where shocks are expected to develop.
- Furthermore, the need to compute gravitational waves requires that we consider highly non-symmetrical configurations, eg binaries. This implies that we need to solve the equations in 3 spatial dimensions plus time (3+1)
- Numerical relativity is focussed on solving Einstein equations and those of relativistic hydrodynamics and MHD in those regimes in which no approximation holds: eg in the most nonlinear regimes of the theory.

# Numerical Relativity



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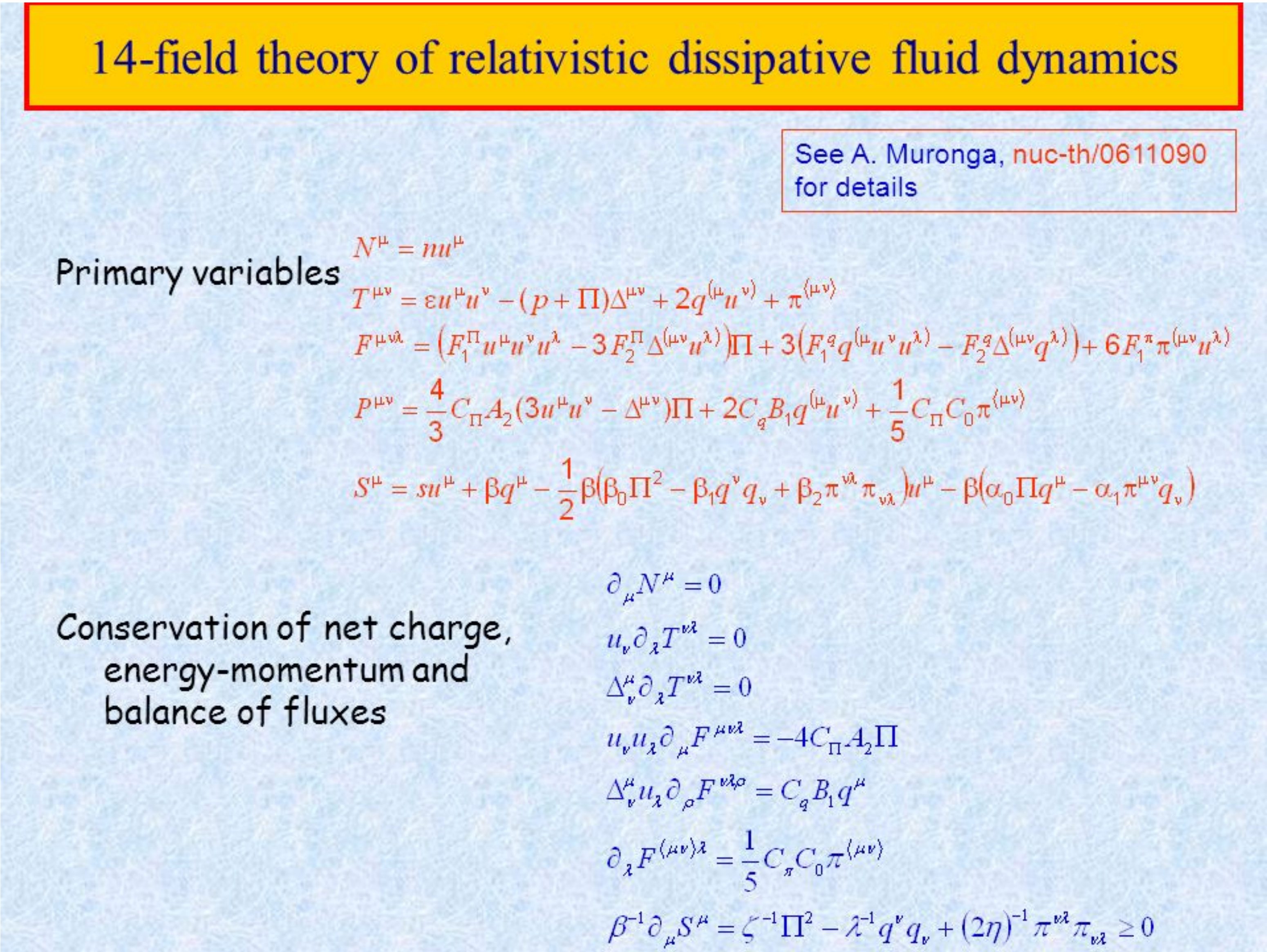
- Numerical relativity is focussed on solving Einstein equations and those of relativistic hydrodynamics and MHD in those regimes in which no approximation holds: eg in the most nonlinear regimes of the theory.

# Numerical Relativity



## 14-field theory of relativistic dissipative fluid dynamics

See A. Muronga, nuc-th/0611090
for details

Primary variables

$$N^{\mu} = n u^{\mu}$$

$$T^{\mu\nu} = \varepsilon u^{\mu} u^{\nu} - (p + \Pi)\Delta^{\mu\nu} + 2q^{(\mu} u^{\nu)} + \pi^{\langle\mu\nu\rangle}$$

$$F^{\mu\nu\lambda} = \left(F_1^{\Pi} u^{\mu} u^{\nu} u^{\lambda} - 3F_2^{\Pi}\Delta^{(\mu\nu} u^{\lambda)}\right)\Pi + 3\left(F_1^{q} q^{(\mu} u^{\nu} u^{\lambda)} - F_2^{q}\Delta^{(\mu\nu} q^{\lambda)}\right) + 6F_1^{\pi}\pi^{(\mu\nu} u^{\lambda)}$$

$$P^{\mu\nu} = \frac{4}{3} C_{\Pi} A_2 (3u^{\mu} u^{\nu} - \Delta^{\mu\nu})\Pi + 2C_q B_1 q^{(\mu} u^{\nu)} + \frac{1}{5} C_{\Pi} C_0 \pi^{\langle\mu\nu\rangle}$$

$$S^{\mu} = s u^{\mu} + \beta q^{\mu} - \frac{1}{2}\beta\left(\beta_0 \Pi^2 - \beta_1 q^{\nu} q_{\nu} + \beta_2 \pi^{\nu\lambda}\pi_{\nu\lambda}\right) u^{\mu} - \beta\left(\alpha_0 \Pi q^{\mu} - \alpha_1 \pi^{\mu\nu} q_{\nu}\right)$$

Conservation of net charge,
energy-momentum and
balance of fluxes

$$\partial_{\mu} N^{\mu} = 0$$

$$u_{\nu}\partial_{\lambda} T^{\nu\lambda} = 0$$

$$\Delta^{\mu}_{\nu}\partial_{\lambda} T^{\nu\lambda} = 0$$

$$u_{\nu} u_{\lambda}\partial_{\mu} F^{\mu\nu\lambda} = -4C_{\Pi} A_2 \Pi$$

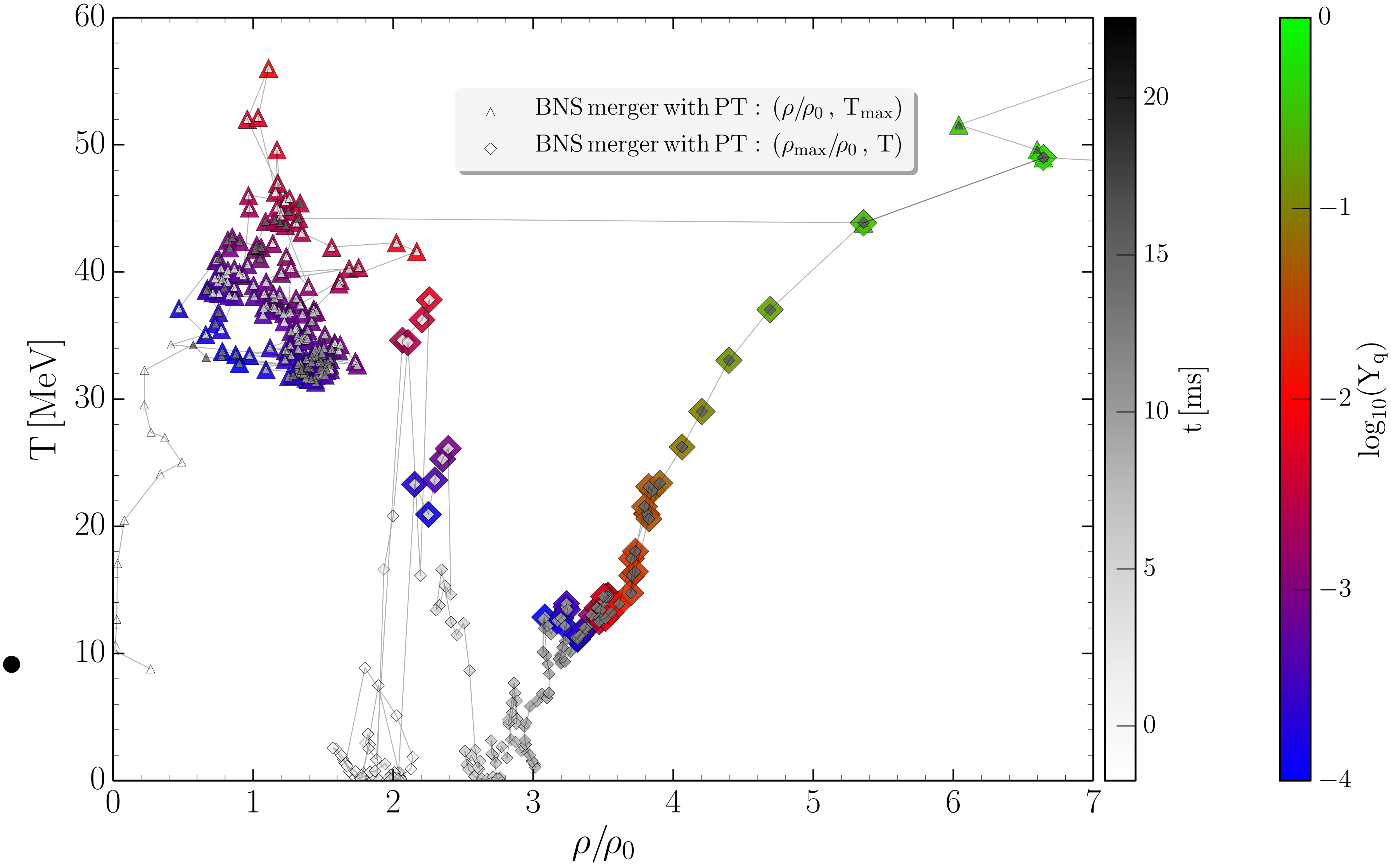
$$\Delta^{\mu}_{\nu} u_{\lambda}\partial_{\rho} F^{\nu\lambda\rho} = C_q B_1 q^{\mu}$$

$$\partial_{\lambda} F^{\langle\mu\nu\rangle\lambda} = \frac{1}{5} C_{\pi} C_0 \pi^{\langle\mu\nu\rangle}$$

$$\beta^{-1}\partial_{\mu} S^{\mu} = \zeta^{-1}\Pi^2 - \lambda^{-1} q^{\nu} q_{\nu} + (2\eta)^{-1}\pi^{\nu\lambda}\pi_{\nu\lambda} \geq 0$$

# Simulation with PT EoS

Hanauske et. al,. (2017)



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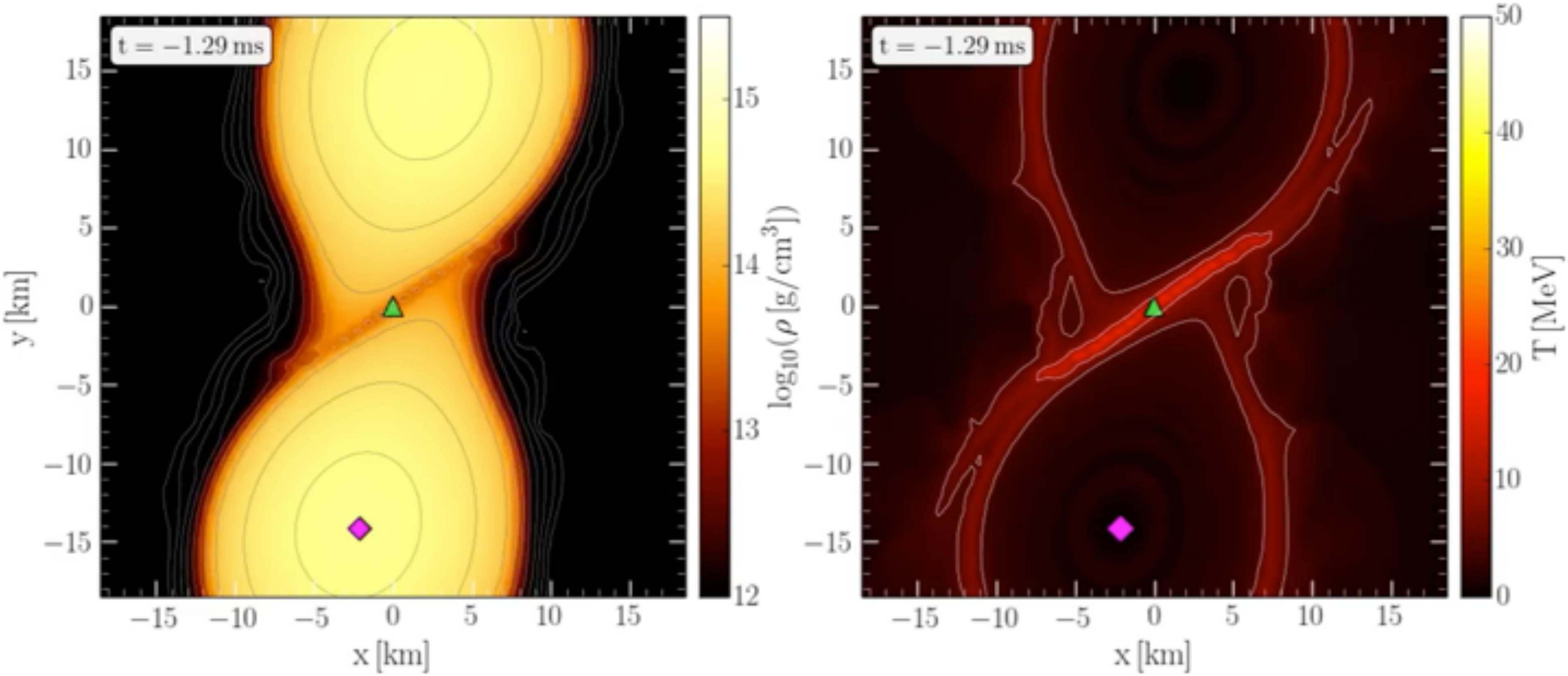
# Animation from numerical simulation

-with Horst Stoecker et. al. Frankfurt (2017)



# Fundamental limitations of equation of state based on nucleon-nucleon interactions alone:

Accurate for $n \sim n_0$.

$n >> n_0$:

-can forces be described with static few-body potentials?

-Force range $\sim 1/2m_\pi$ => relative importance of
3 (and higher) body forces $\sim n/(2m_\pi)^3 \sim 0.4n$ $\mathrm{fm}^{-3}$

-No well defined expansion in terms of 2,3,4,...body forces.

# Well beyond nuclear matter density

Onset of new degrees of freedom: mesonic, Δ's, quarks and gluons, ...

Properties of matter in this extreme regime determine maximum neutron star mass.

Large uncertainties!

Hyperons: Σ, Λ, ...
Meson condensates: $\pi^-$, $\pi^0$, $K^-$
Quark matter
  in droplets
  in bulk
Color superconductivity
Strange quark matter
  absolute ground state of matter??

# Equation of State

$$\begin{aligned}
\mathcal{L}_{\mathrm{RMF}} = & \sum_{i=p,n} \bar{\psi}_i \Big[ i\gamma_\mu \partial^\mu - M - g_\sigma \sigma - g_\omega \gamma_\mu \omega^\mu \\
& - g_\rho \gamma_\mu \tau_a \rho^{a\mu} - e\gamma_\mu \frac{1+\tau_3}{2} A^\mu \Big] \psi_i \\
& + \bar{\psi}_e \left[ i\gamma_\mu \partial^\mu - m_e + e\gamma_\mu A^\mu \right] \psi_e \\
& + \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 \\
& - \frac{1}{4} W_{\mu\nu} W^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu + \frac{1}{4} c_3 (\omega_\mu \omega^\mu)^2 \\
& - \frac{1}{4} R^a_{\mu\nu} R^{a\mu\nu} + \frac{1}{2} m_\rho^2 \rho^a_\mu \rho^{a\mu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \qquad (1)
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_H = & \sum_N \bar{\psi}_N \Big[ \gamma^\mu \Big( \mathrm{i}\partial_\mu - g_{\omega N} \omega_\mu - \frac{1}{2} g_{\rho N} \boldsymbol{\tau} \cdot \boldsymbol{\rho}_\mu \Big) \\
& -(m_N - g_{\sigma N} \sigma) \Big] \psi_N + \frac{1}{2} \partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 \\
& -\frac{1}{4} \omega^{\mu\nu} \omega_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu - \frac{1}{4} \boldsymbol{\rho}^{\mu\nu} \boldsymbol{\rho}_{\mu\nu} + \frac{1}{2} m_\rho^2 \boldsymbol{\rho}^\mu \cdot \boldsymbol{\rho}_\mu \\
& + \sum_\lambda \bar{\psi}_\lambda (\mathrm{i}\gamma^\mu \partial_\mu - m_\lambda) \psi_\lambda - \frac{1}{4} F^{\mu\nu} F_{\mu\nu},
\end{aligned}$$

$$\begin{aligned}
\varepsilon = & \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{3} g_2 \sigma^3 + \frac{1}{4} g_3 \sigma^4 + \frac{1}{2} m_\omega^2 \omega_0^2 \\
& + \frac{1}{2} m_\rho^2 \rho_{03}^2 + \frac{1}{2} m_{\sigma^*}^2 \sigma^{*2} + \frac{1}{2} m_\phi^2 \phi_0^2 \\
& + \sum_B \frac{2J_B + 1}{2\pi^2} \int_0^\infty \sqrt{\kappa^2 + (m_B - g_{\sigma B} \sigma - g_{\sigma^* B} \sigma^*)^2} \\
& \times (\exp[(\varepsilon_B(\kappa) - \mu_B)/T + 1)^{-1} \kappa^2 d\kappa \\
& + \sum_{\lambda = e,\mu} \frac{1}{\pi^2} \int_0^\infty \sqrt{\kappa^2 + m_\lambda^2}\, n_\lambda(\kappa) \kappa^2 d\kappa \\
& + \sum_\nu \left( \frac{7\pi^2 T^4}{120} + \frac{\mu_\nu^2}{12} - \frac{\mu_\nu^4}{24\pi^2} \right)
\end{aligned}$$

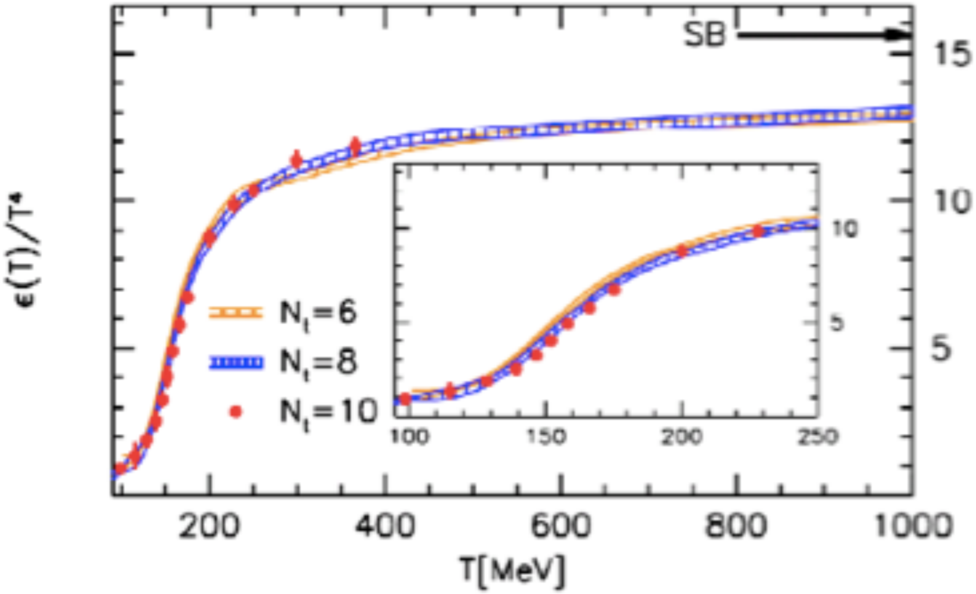
(6)

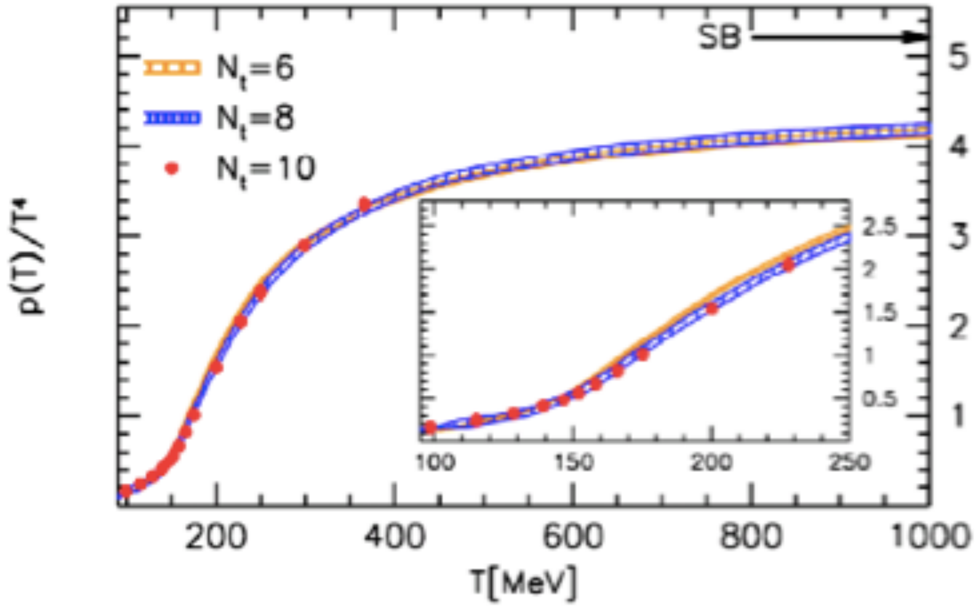
$$\begin{aligned}
P = & -\frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 + \frac{1}{2} m_\omega^2 \omega_0^2 \\
& + \frac{1}{2} m_\rho^2 \rho_{03}^2 - \frac{1}{2} m_{\sigma^*}^2 \sigma^{*2} + \frac{1}{2} m_\phi^2 \phi_0^2 \\
& + \frac{1}{3} \sum_B \frac{2J_B + 1}{2\pi^2} \int_0^\infty \frac{\kappa^2}{\sqrt{\kappa^2 + (m_B - g_{\sigma B} \sigma - g_{\sigma^* B} \sigma^*)^2}} \\
& \times (\exp[\varepsilon_B(\kappa) - \mu_B)/T + 1)^{-1} \kappa^2 d\kappa \\
& + \frac{1}{3} \sum_{\lambda = e,\mu} \frac{1}{\pi^2} \int_0^\infty \frac{\kappa^4}{\sqrt{\kappa^2 + m_\lambda^2}} n_\lambda(\kappa) d\kappa \\
& + \sum_\nu \frac{1}{360} \left( 7\pi^2 T^4 + 30\mu_\nu^4 T^2 + \frac{15\mu_\nu^4}{\pi^2} \right),
\end{aligned}$$

(7)

Lattice gauge theory calculations of equation of state of QGP

Not useful yet for realistic chemical potentials

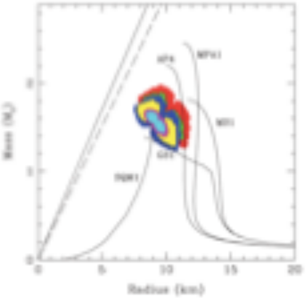




# Learning about dense matter from neutron star observations

# Learning about dense matter from neutron star observations

## Masses of neutron stars



Binary systems: stiff eos
Thermonuclear bursts in X-ray binaries => Mass vs. Radius, strongly constrains eos

Glitches: probe n,p superfluidity and crust

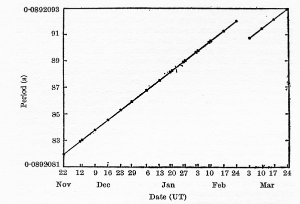
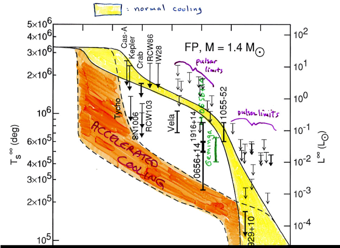


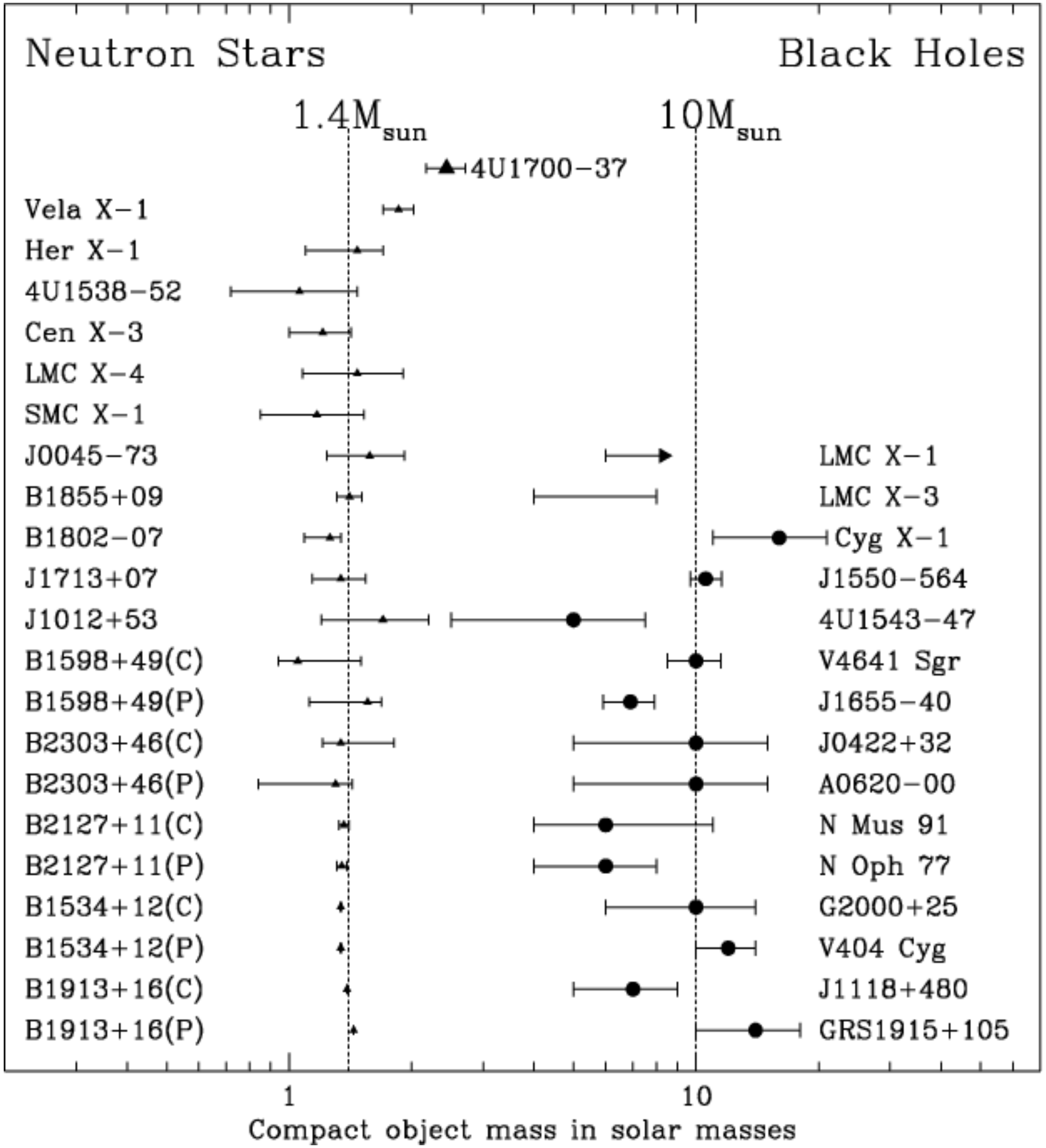
Fig. 1. The barycentric period of *PSR* 0833–45 as observed from November 22, 1968, to March 24, 1969, showing the 134 ns decrease between February 24 and March 3.

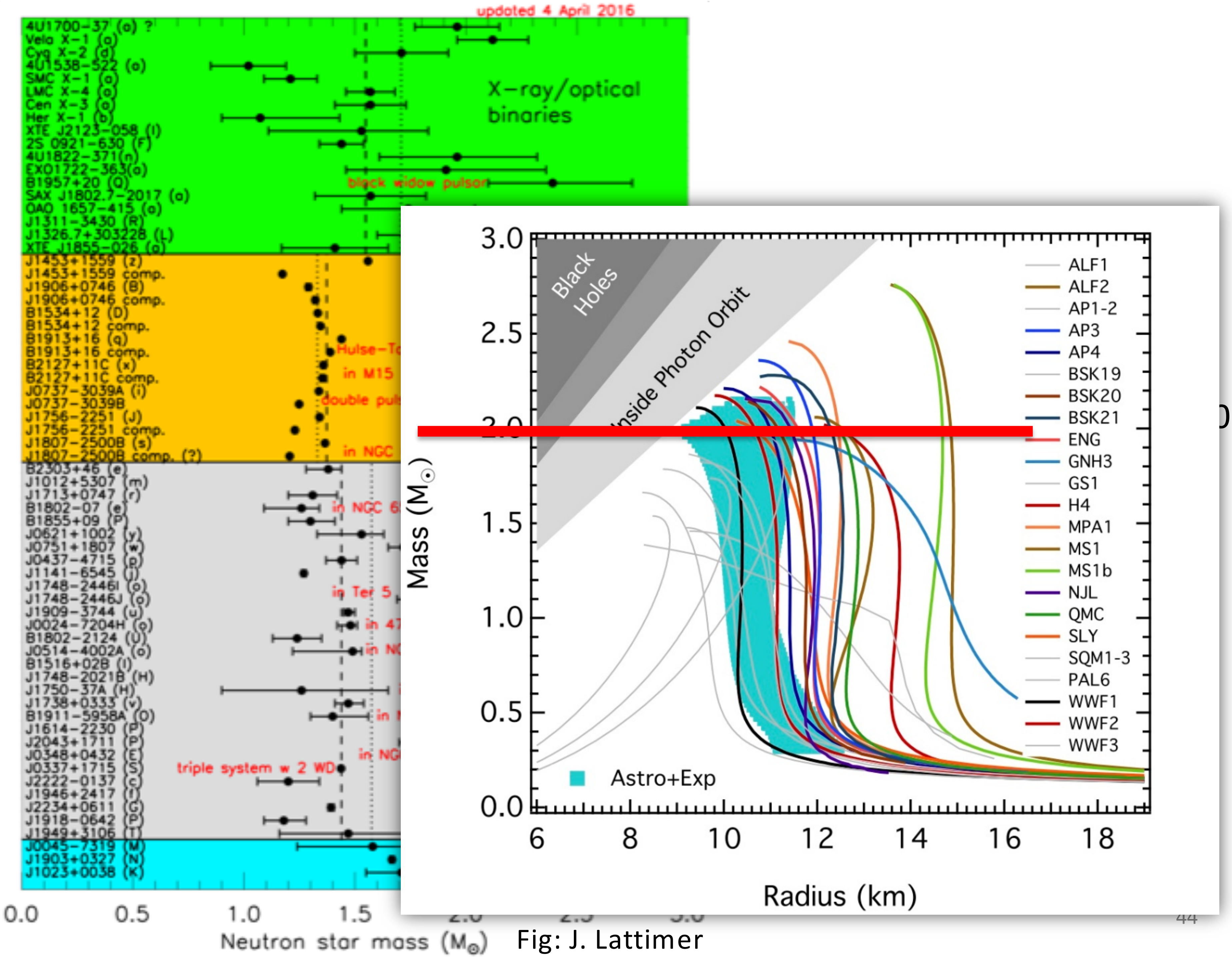
Cooling of n-stars: search for exotic particles



[redacted]

# Neutron Star Masses ca. 2007



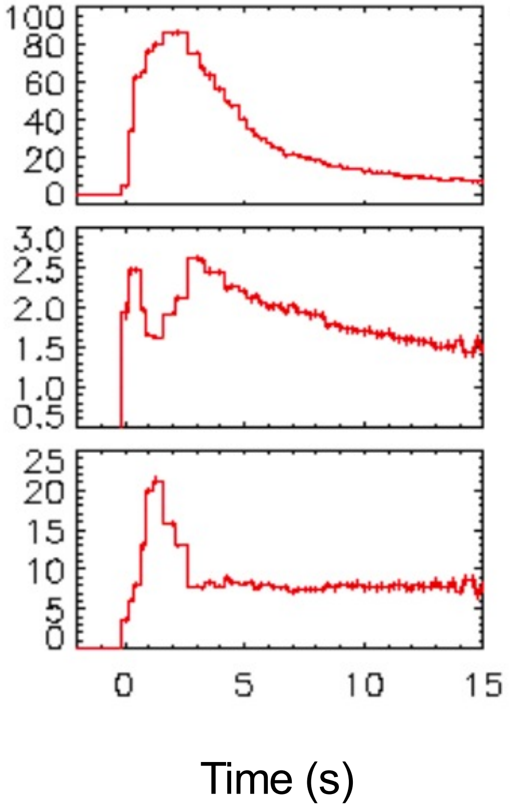


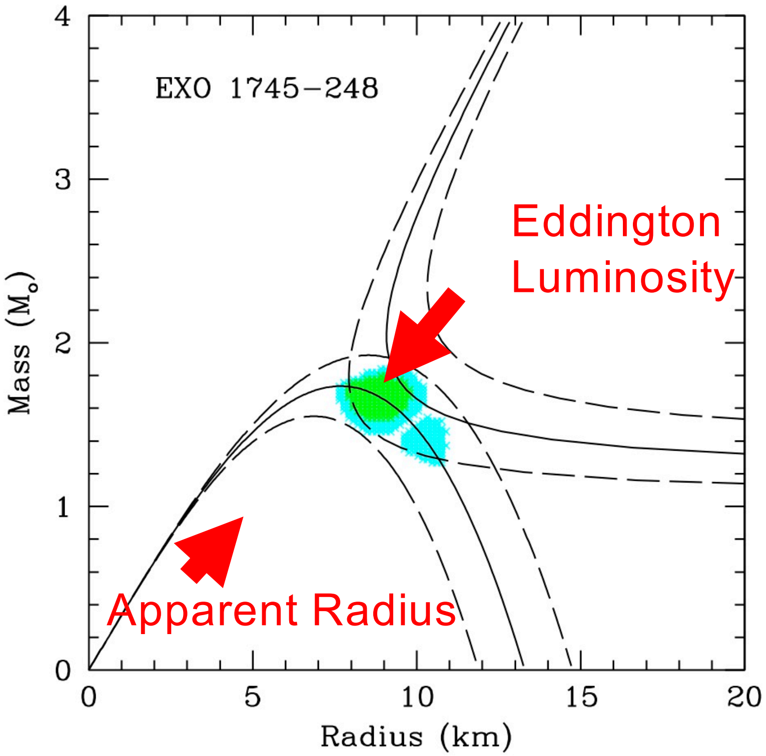
0)

Fig: J. Lattimer

# Measuring masses and radii of neutron stars in thermonuclear bursts in X-ray binaries

*Ozel et al., 2006-20l2*





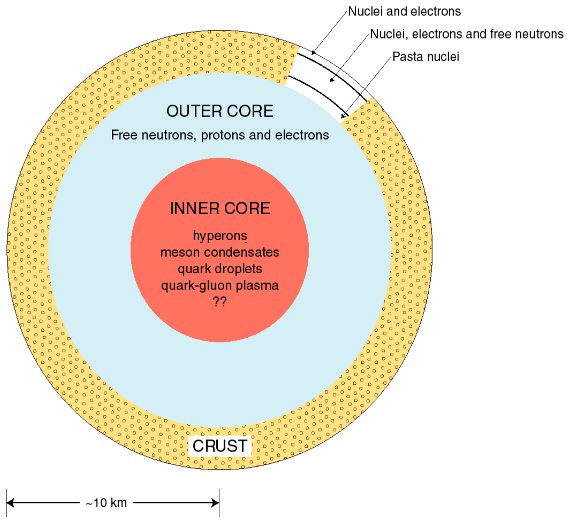
Eddington Luminosity

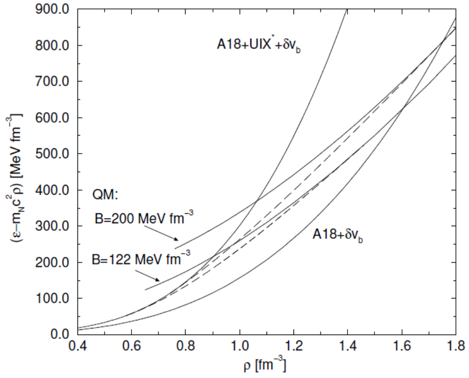
Apparent Radius

Measurements of *apparent* surface area, & flux at Eddington limit (radiation pressure = gravity), combined with distance to star constrains M and R.

# Quark matter cores in neutron stars

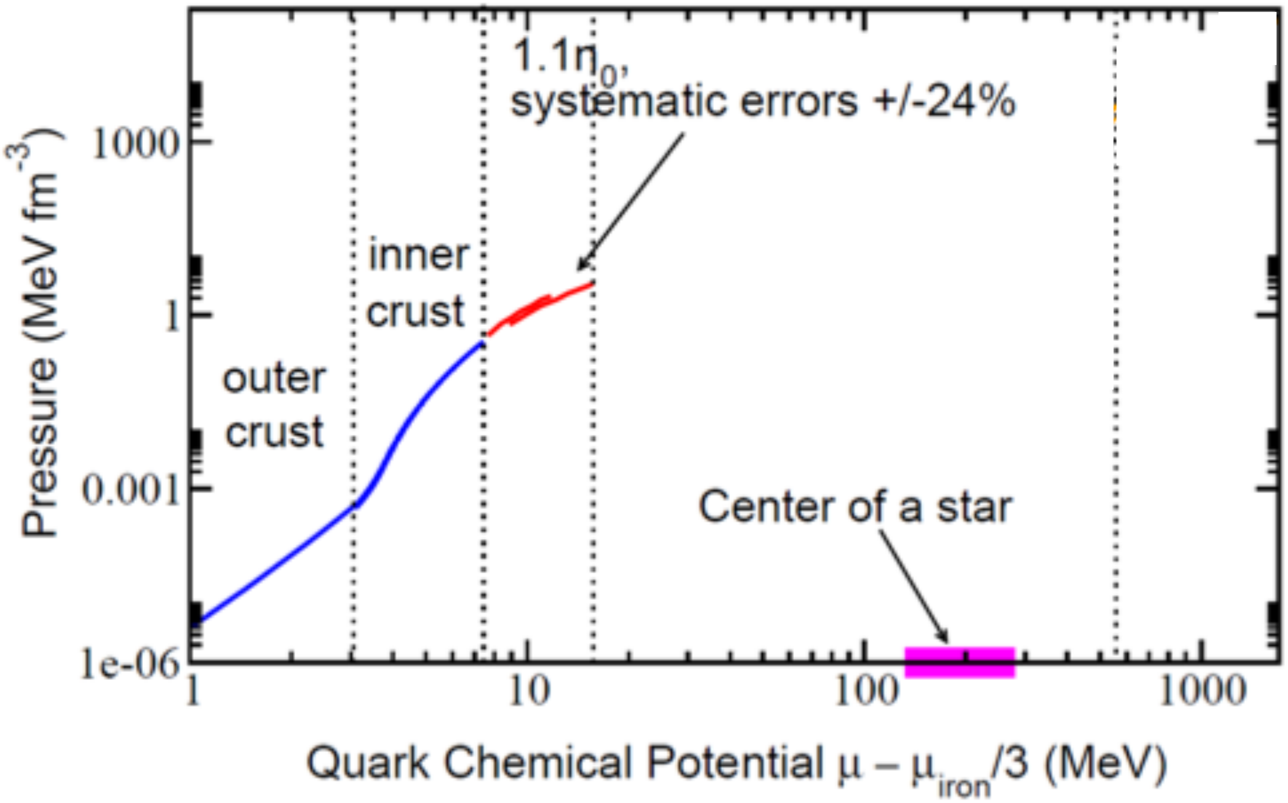
Canonical picture: compare calculations of eqs. of state of hadronic matter and quark matter.
Crossing of thermodynamic potentials
=> first order phase transition.





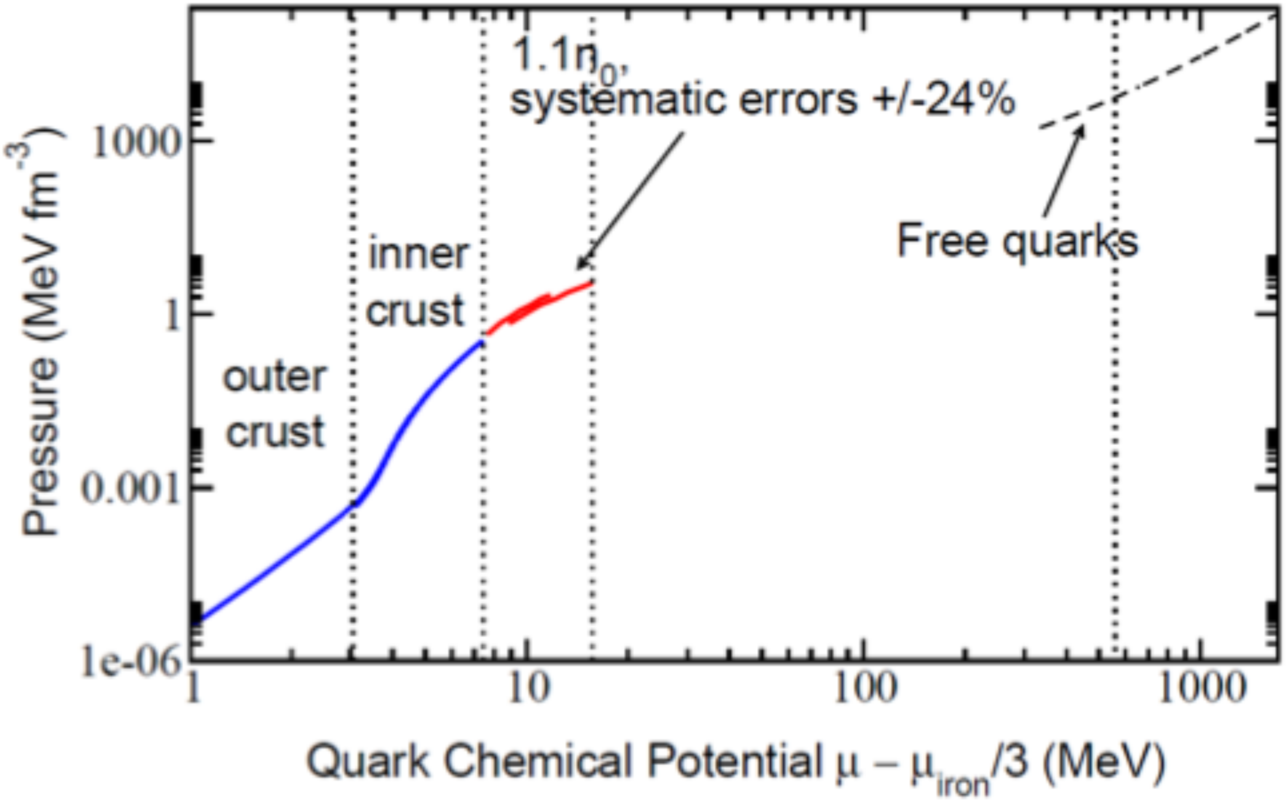
ex. nuclear matter using 2 & 3 body interactions, vs. pert. expansion or bag models. *Akmal, Pandharipande, Ravenhall 1998*

Typically conclude transition at $\rho \sim 10\rho_{nm}$ -- would not be reached in neutron stars given observation of high mass PSR J1614-2230 with M = 1.97 => no quark matter cores



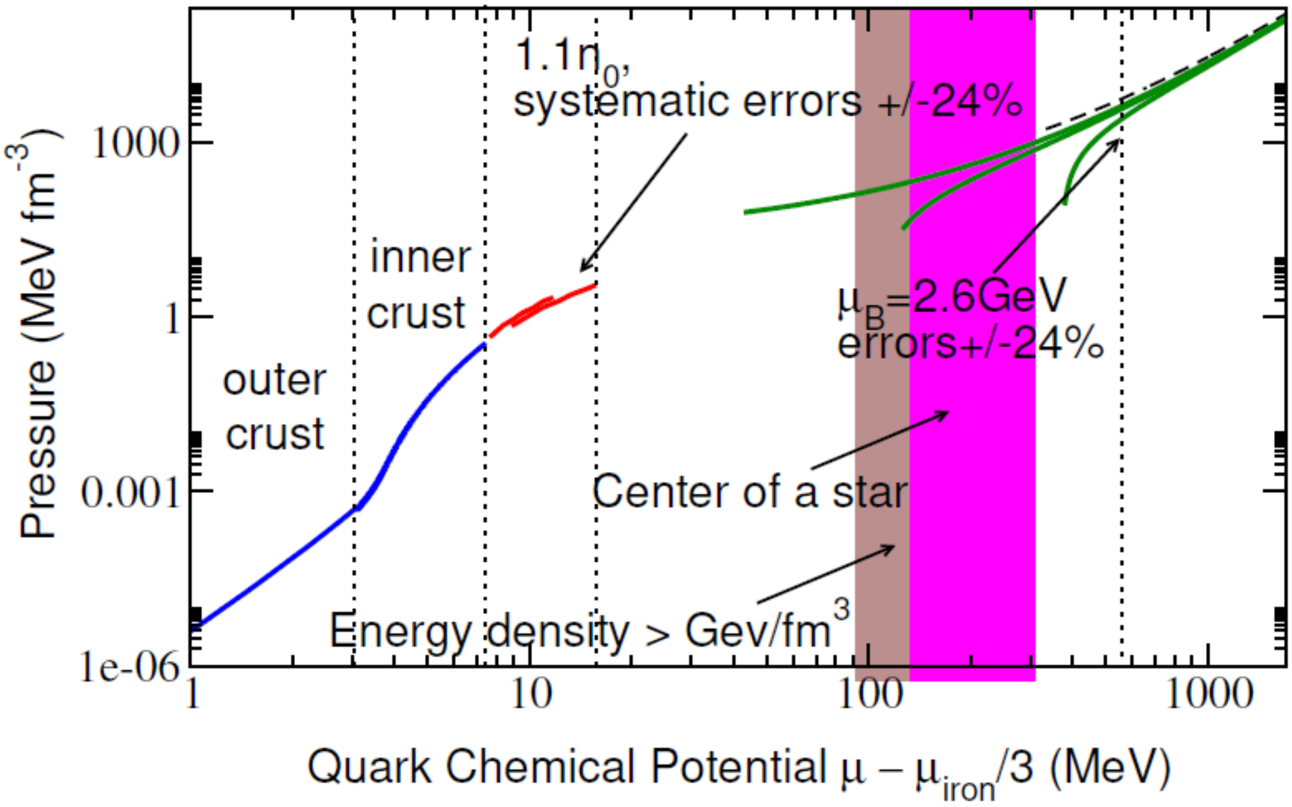
Low-density behavior of EoS well known from nuclear theory side. Challenges begin close to saturation density:

- At $1.1n_s$, current errors in Chiral Effective Theory EoS $\pm 24\%$ - mostly due to uncertainties in effective theory parameters
- State-of-the-art EoS NNNLO in chiral perturbation theory power counting [Tews et al., PRL 110 (2013), Hebeler et al., ApJ 772 (2013)]



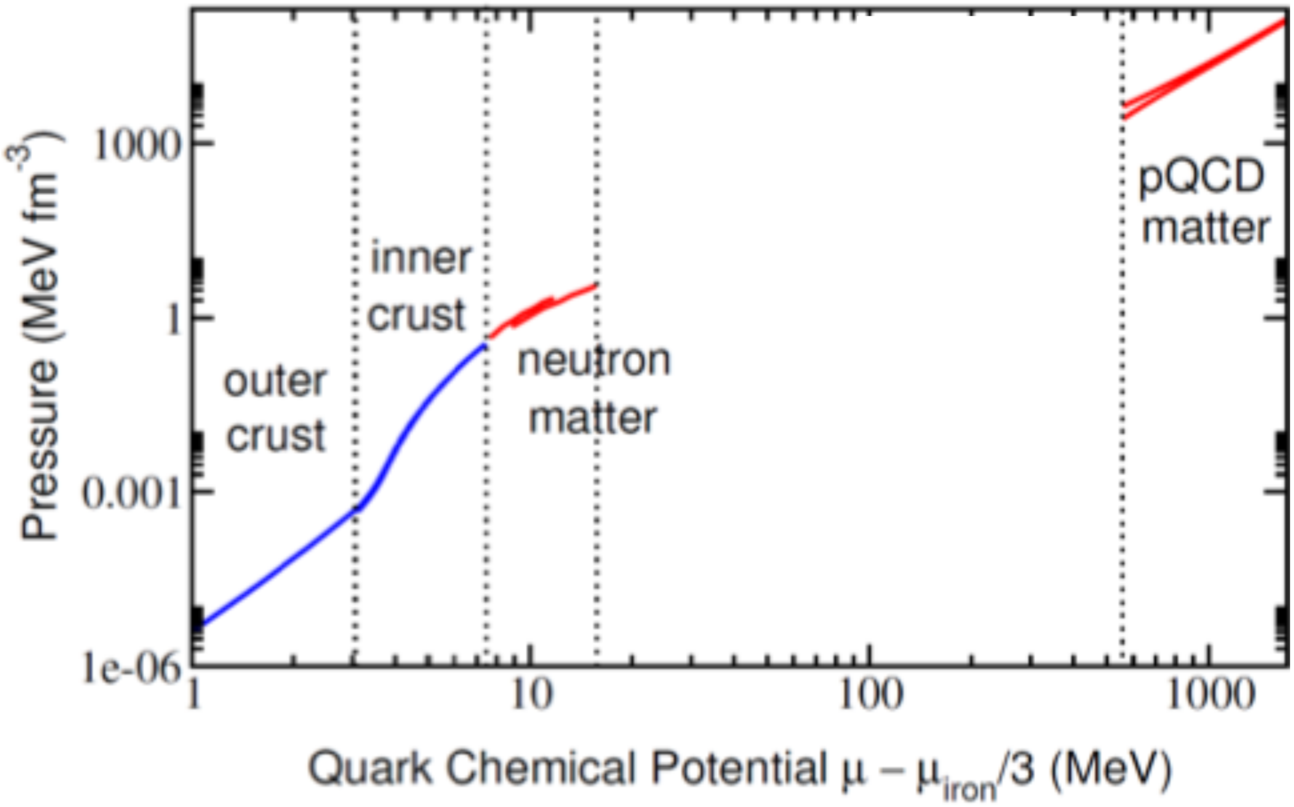
Asymptotic freedom of QCD ⇒ High-density limit from a non-interacting theory. However,…

- At interesting densities $(1-10)n_s$ system strongly interacting but no nonperturbative methods available
- Naïve expectation: Weak coupling methods only useful at very high densities



Three-loop result with nonzero quark masses [Kurkela, Romatschke, Vuorinen, PRD 81 (2009)]

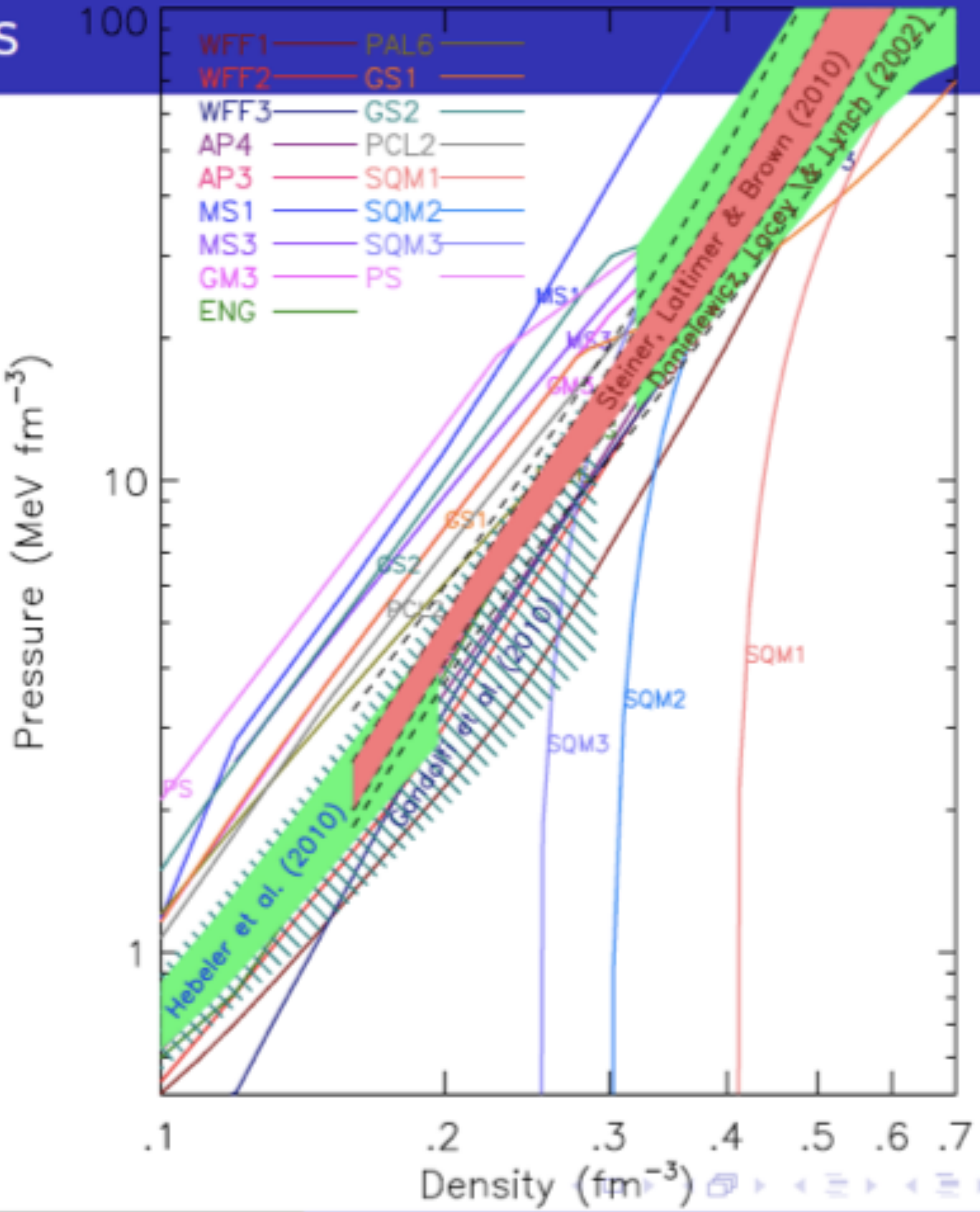
- Uncertainty of result at $\pm 24\%$ level around $40 n_s$
- Main uncertainty from renormalization scale dependence
- Pairing contributions to EoS subdominant at relevant densities



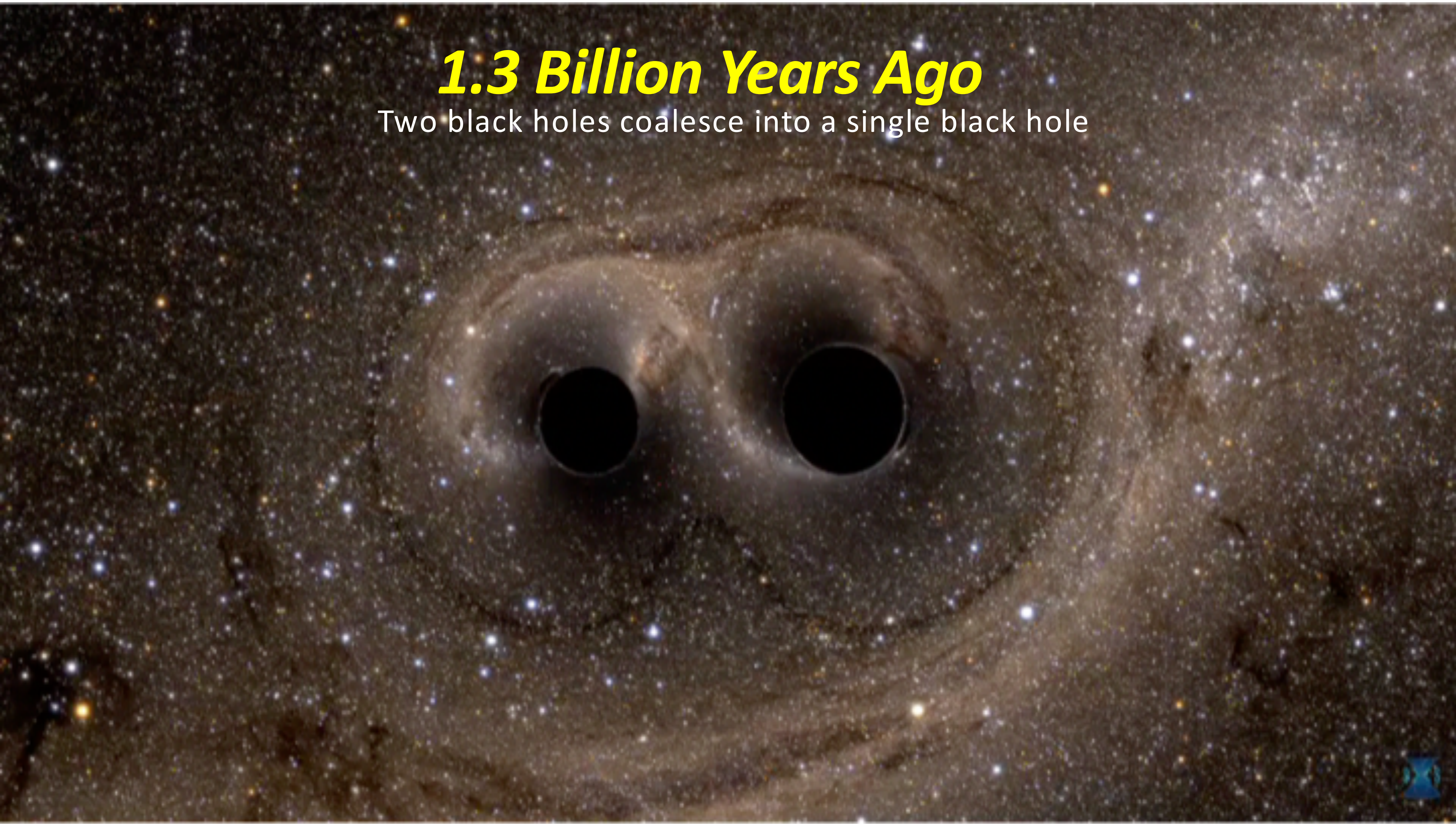
Conclusion: Sizable no man's land extending from outer core to densities not realized inside physical neutron stars

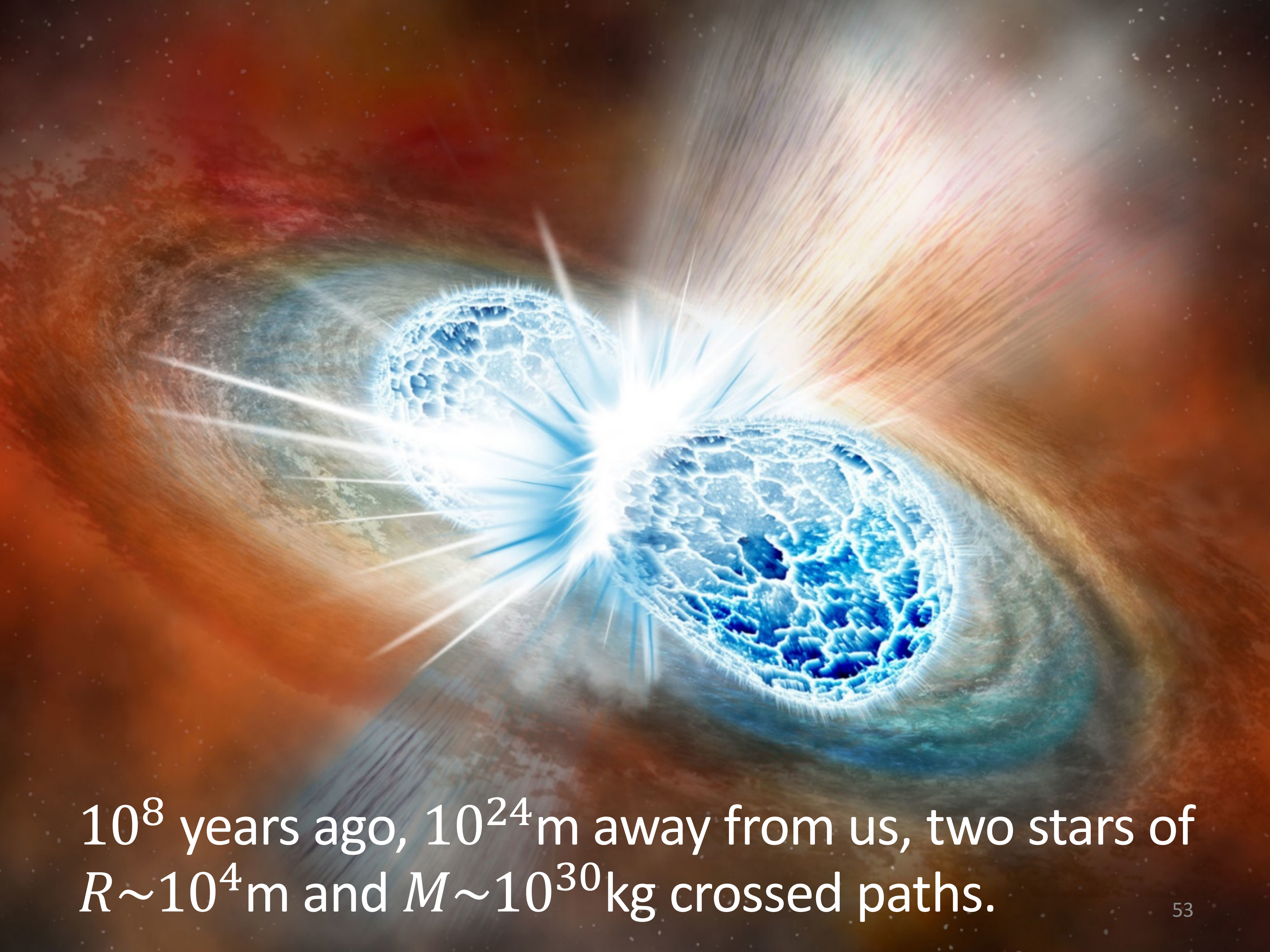
Options: Use models, deform theory, or interpolate EoS between known limits and use astrophysical constraints

# Astrophysical Consistency with Neutron Matter and Heavy-Ion Collisions



*from of LOGO collaboration*

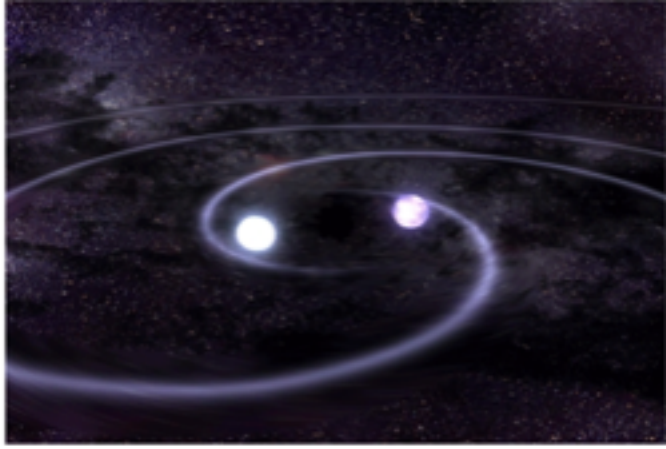


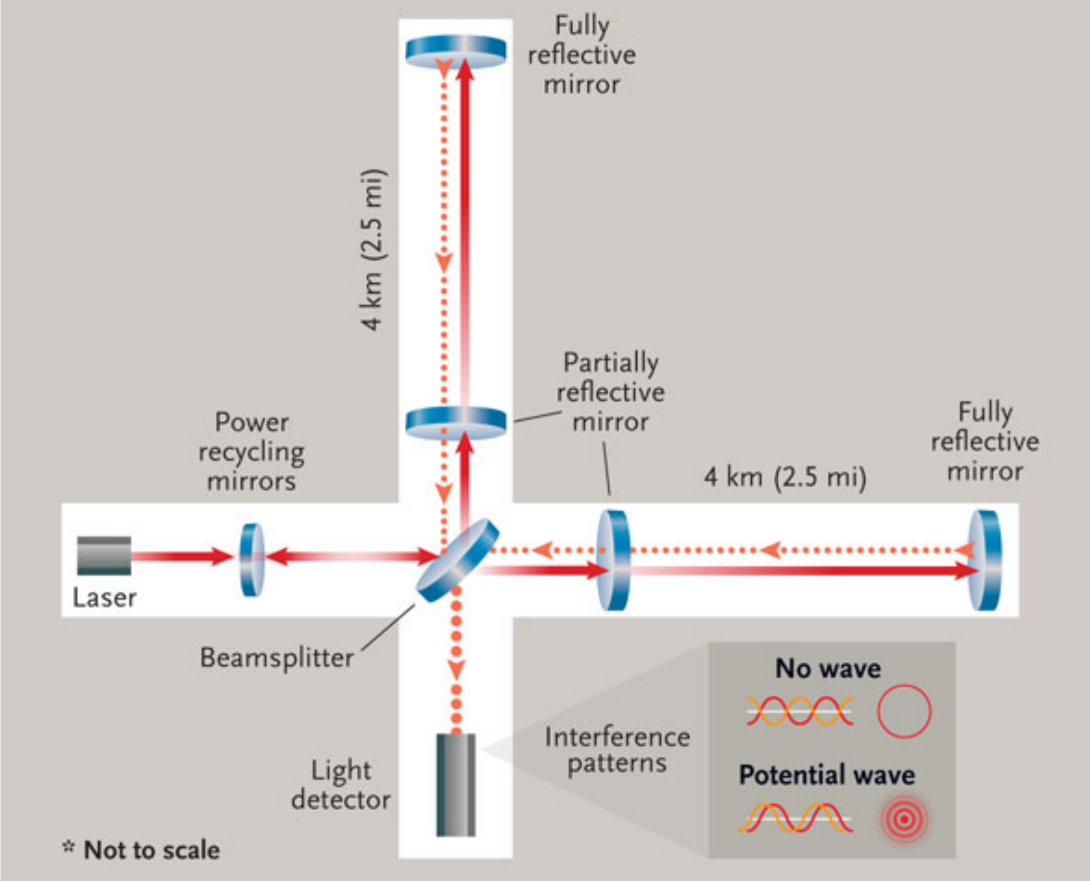


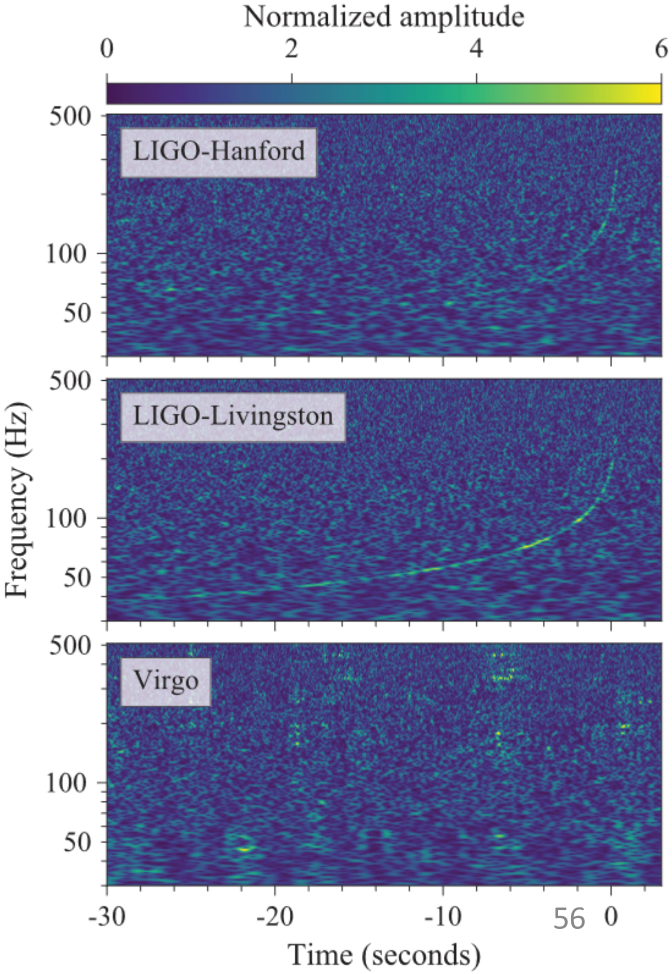
$10^8$ years ago, $10^{24}$m away from us, two stars of $R{\sim}10^4$m and $M{\sim}10^{30}$kg crossed paths.

On August 17th 2017, LIGO measured a $10^{-17}$m oscillation in the length of its $10^{3}$m arms.

# Gravitational wave breakthrough: LIGO and Virgo observation of NS merger 130 million ly away!





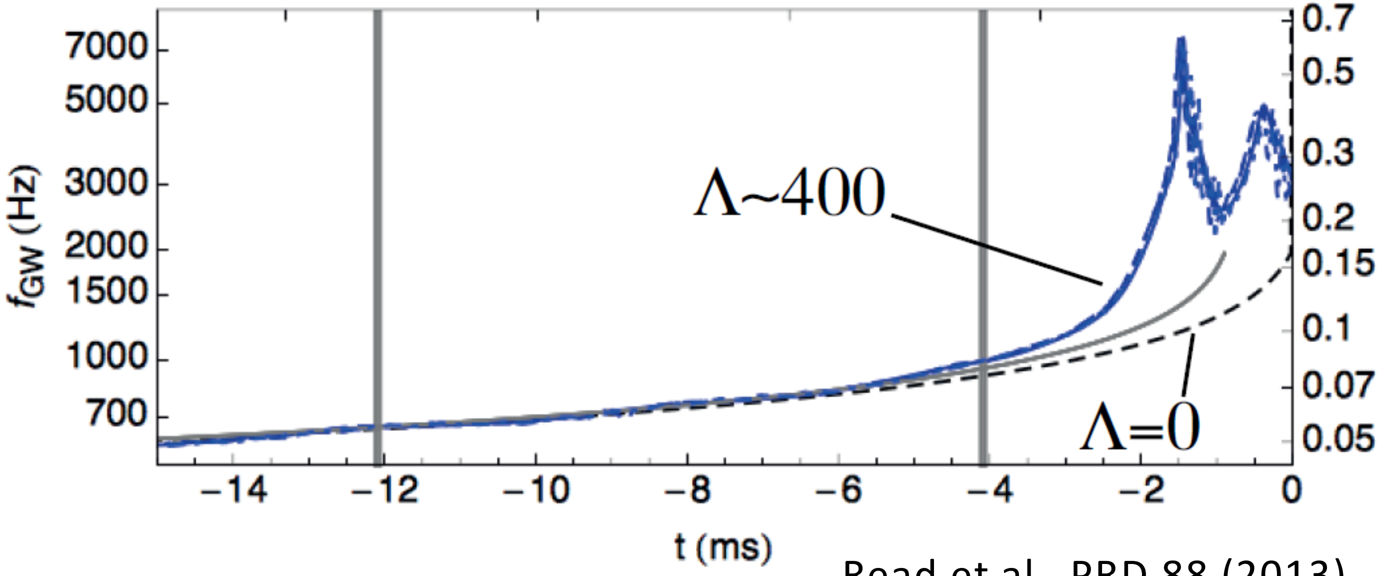


LIGO and Virgo collaborations, PRL 119 (2017)

Tidal deformability: How large a quadrupolar moment a star's gravitational field develops due to an external quadrupolar field

$$Q_{ij} = -\Lambda \mathcal{E}_{ij}$$

Substantial effect on observed GW waveform during inspiral phase

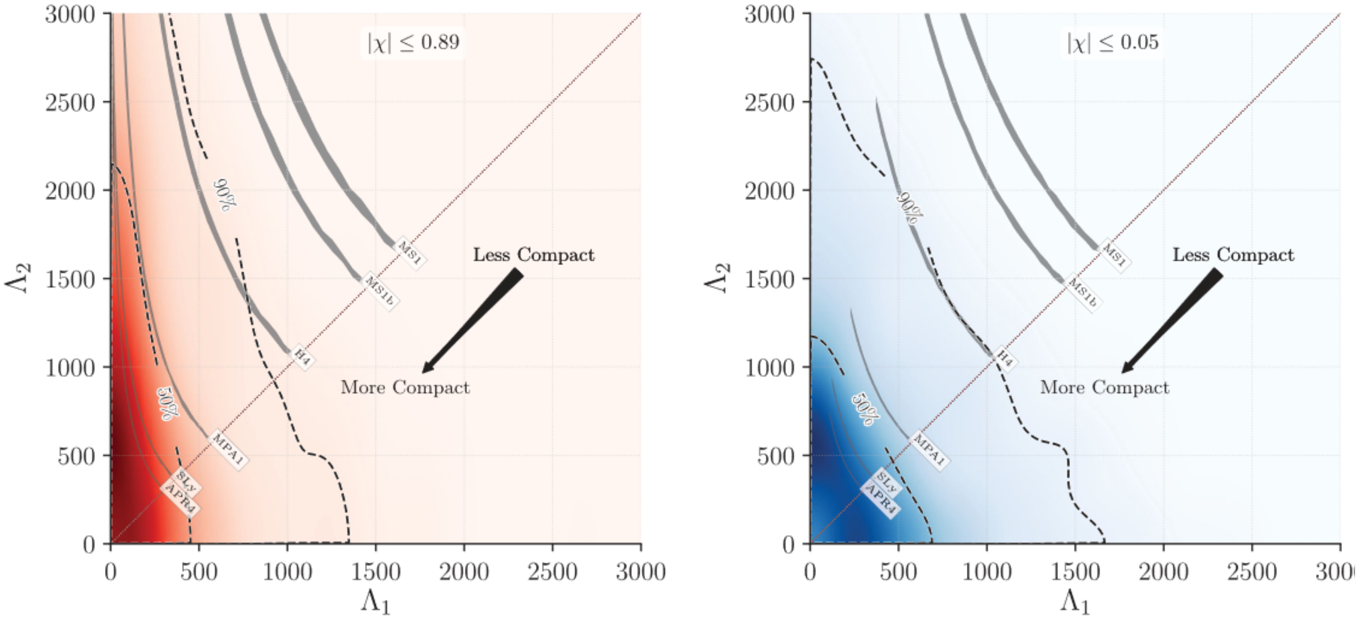


Read et al., PRD 88 (2013)

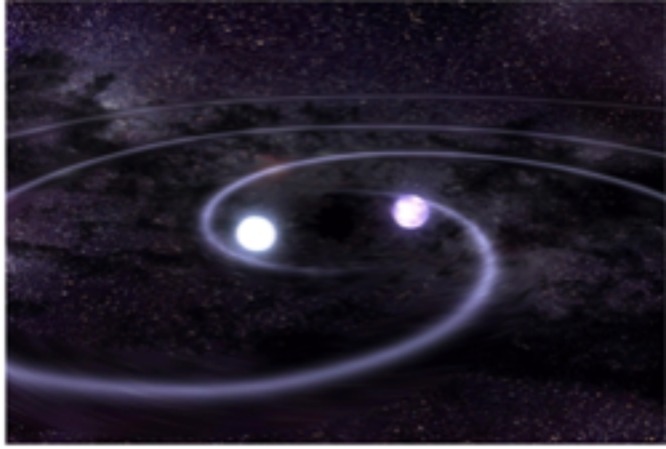
Tidal deformability: How large a quadrupolar moment a star’s gravitational field develops due to an external quadrupolar field

$$Q_{ij} = -\Lambda \mathcal{E}_{ij}$$

However, no detection by LIGO → Upper limit $\Lambda(1.4 M_\odot) < 800$ at 90% credence (low spin prior)

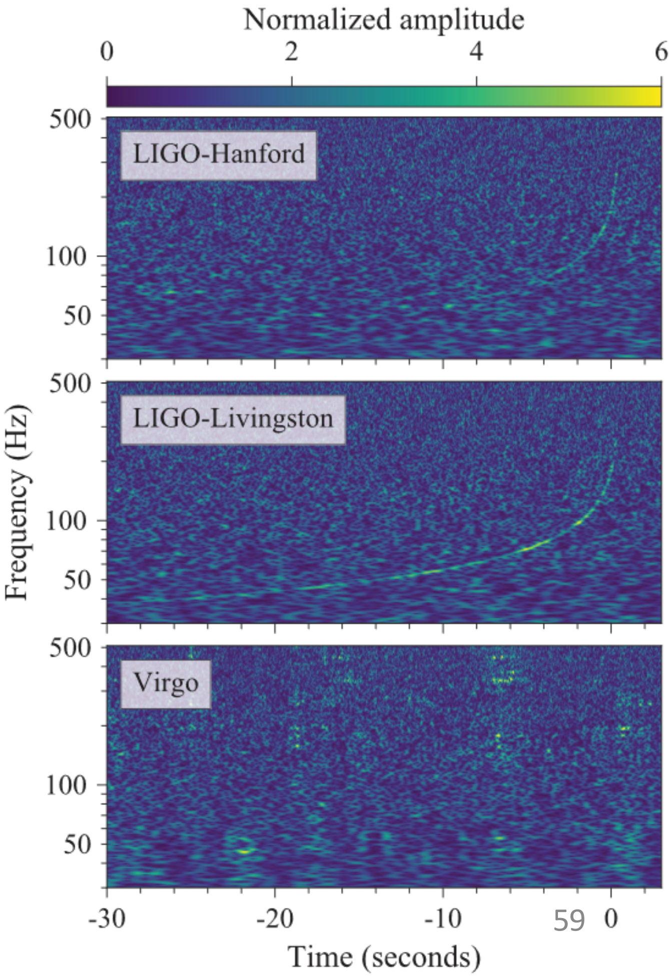


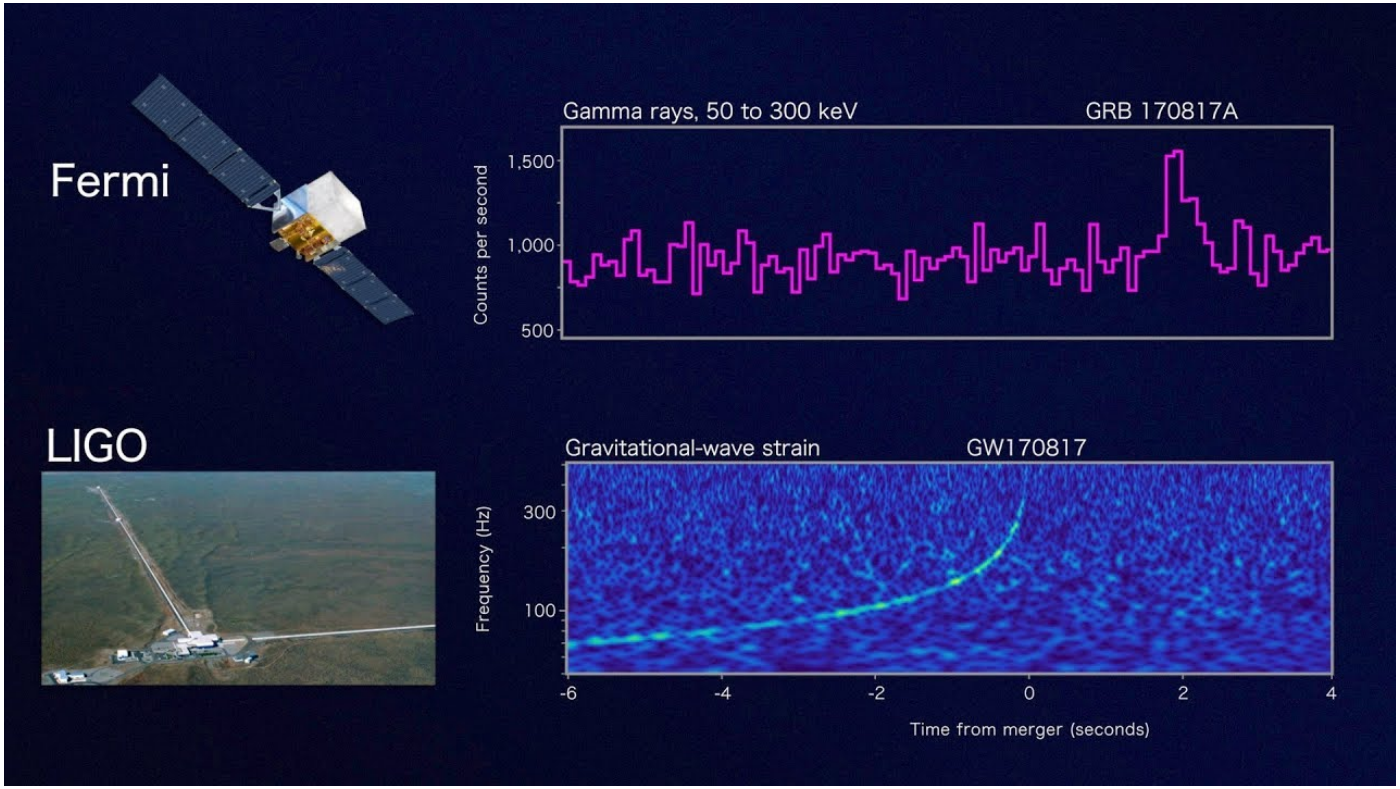
Gravitational wave breakthrough: LIGO and VIRGO observation of NS merger 130 million ly away!



Three types of potential inputs:

1) Tidal deformabilities of the NSs during inspiral – good measure of stellar compactness
2) EM signatures – present if no immediate collapse to a BH
3) Ringdown pattern – sensitive to EoS (also at $T \neq 0$), but freq. too high for LIGO

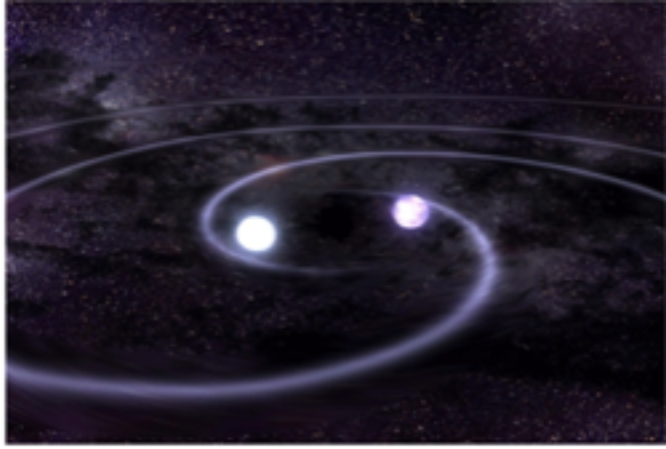




EM counterpart: short gamma ray burst detected 1.7s after GW measurement, followed by an optical signal

- Kilonova: Decay of heavy r-process elements
- GRB → Proposed upper limit for the maximal mass of NSs: $M_{\max} \leq 2.16^{+0.17}_{-0.15} M_{\odot}$ [Rezzolla, Most, Weih, ApJ 852 (2018)]

Gravitational wave breakthrough: LIGO and VIRGO observation of NS merger 130 million ly away!



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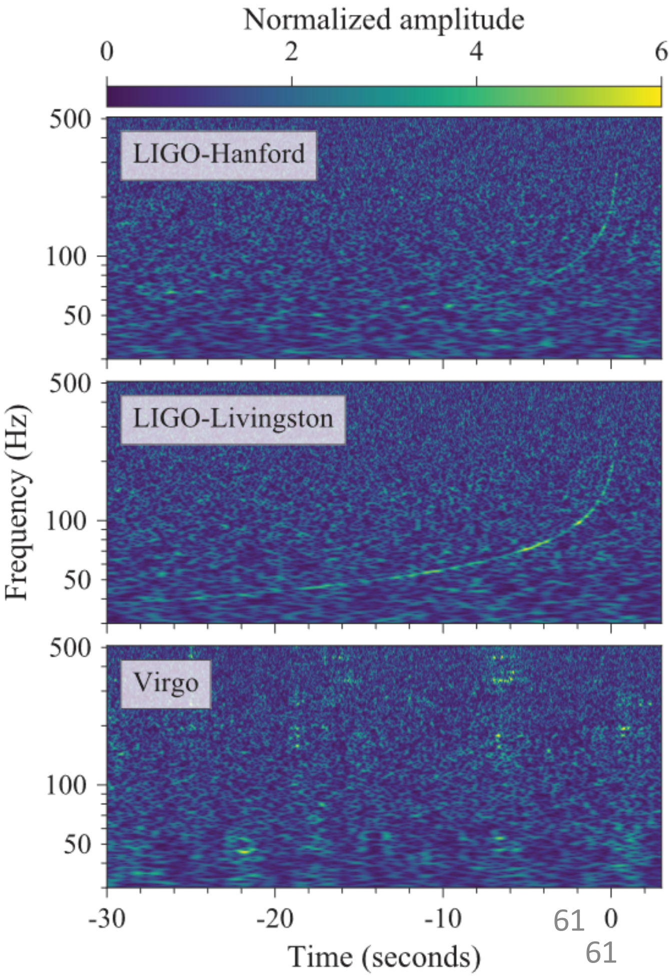
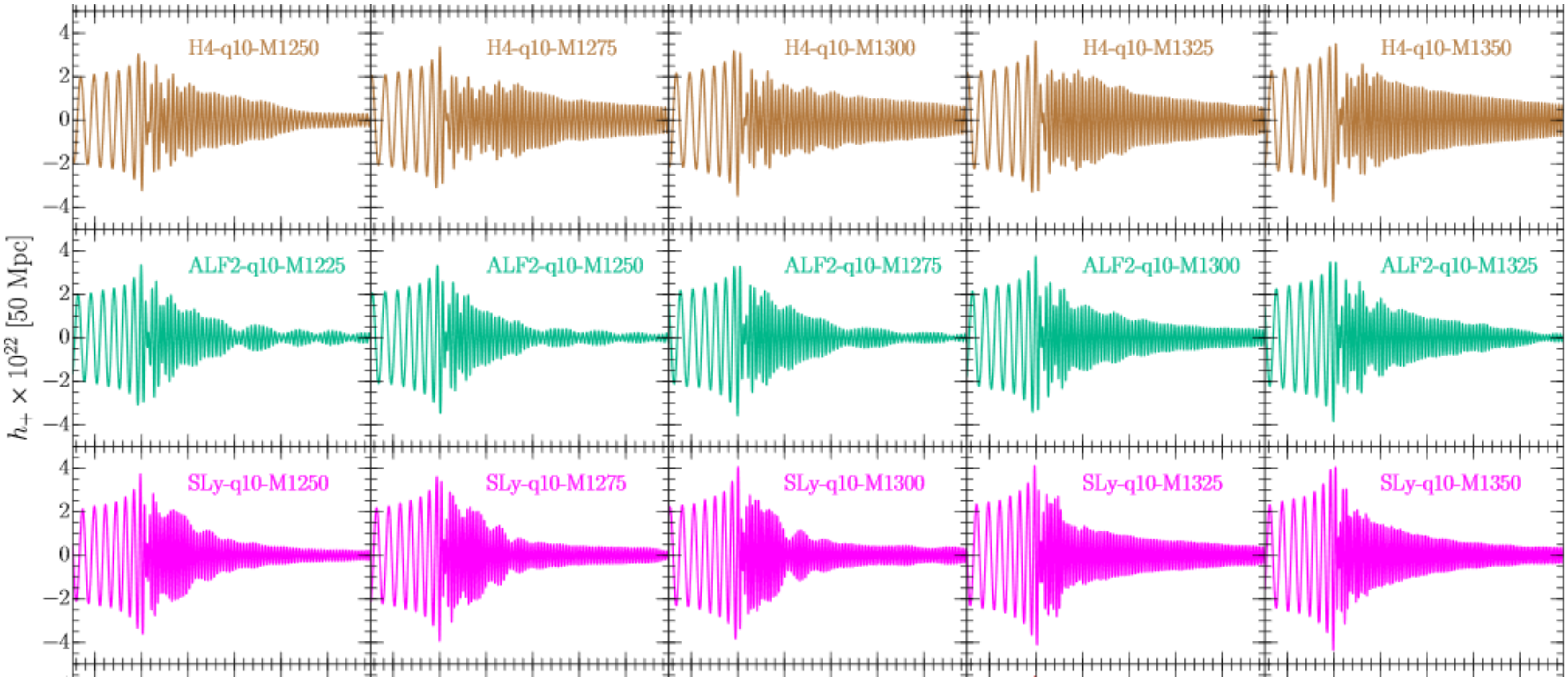


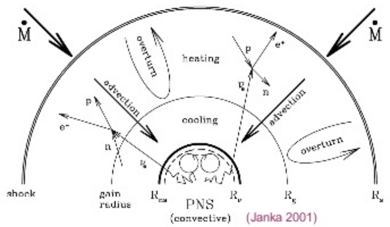
# Post-merger dynamics can be studied with relativistic hydrodynamics, showing marked sensitivity to EoS, but frequency range (currently) too high for LIGO and Virgo



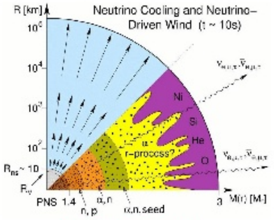
Takami, Rezzolla, Baiotti, PRD 91 (2015)

# Relativistic Dynamics in Heavy Ion Physics and Astrophysics

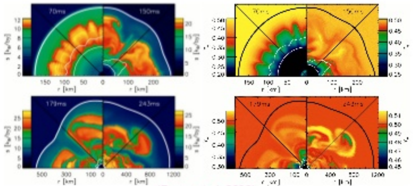
## Core Collapse Supernovae



(Janka 2001)

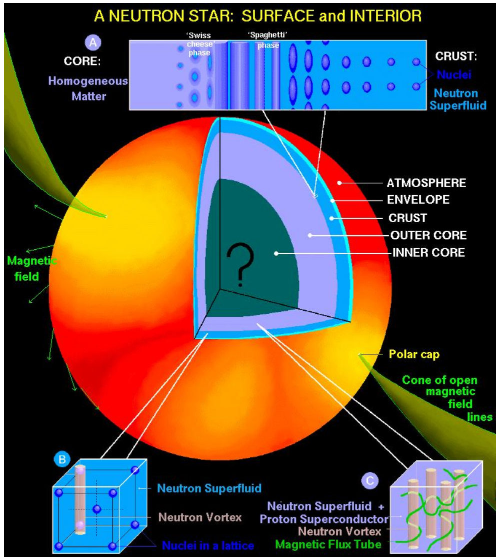


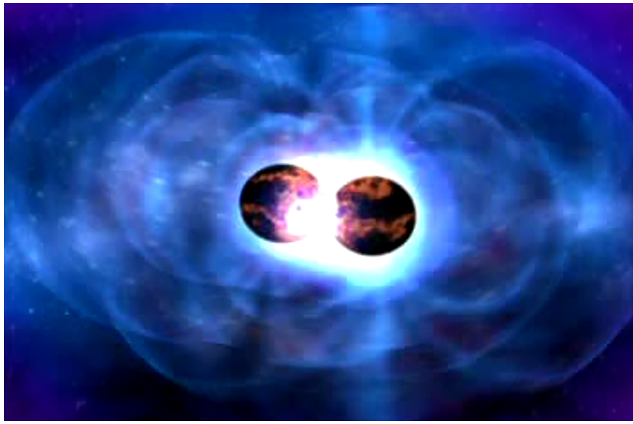
(Woosley & Janka 2006)

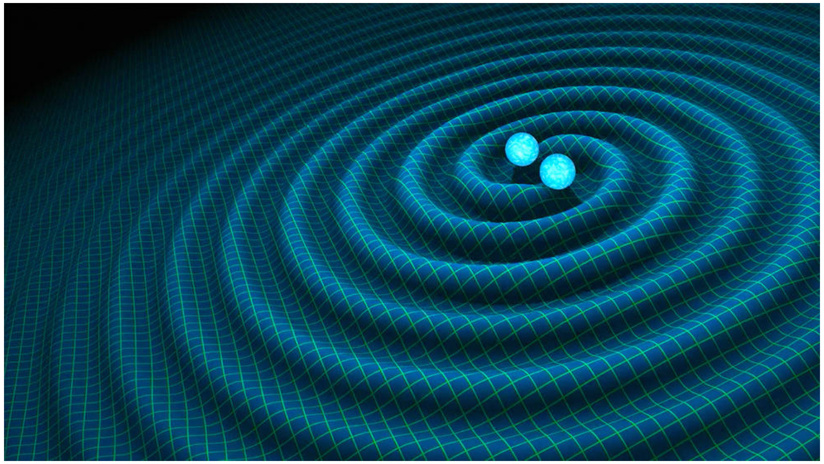


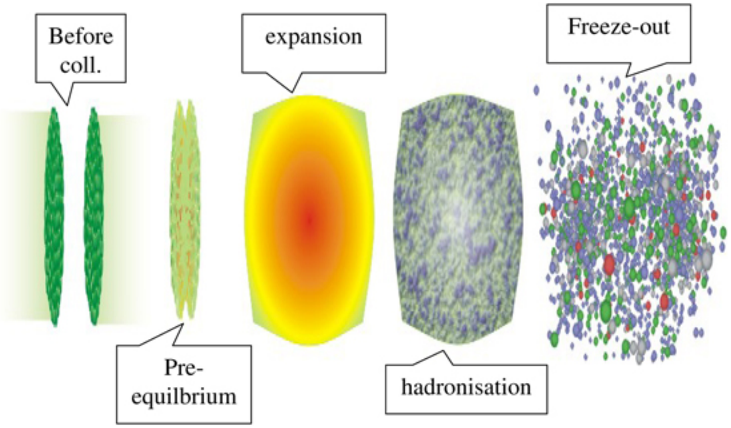
(Buras *et al.* 2006)

←Entropy and electron per baryon ($Y_e$) at different time snapshots in a core collapse supernova (simulation: equatorial band)

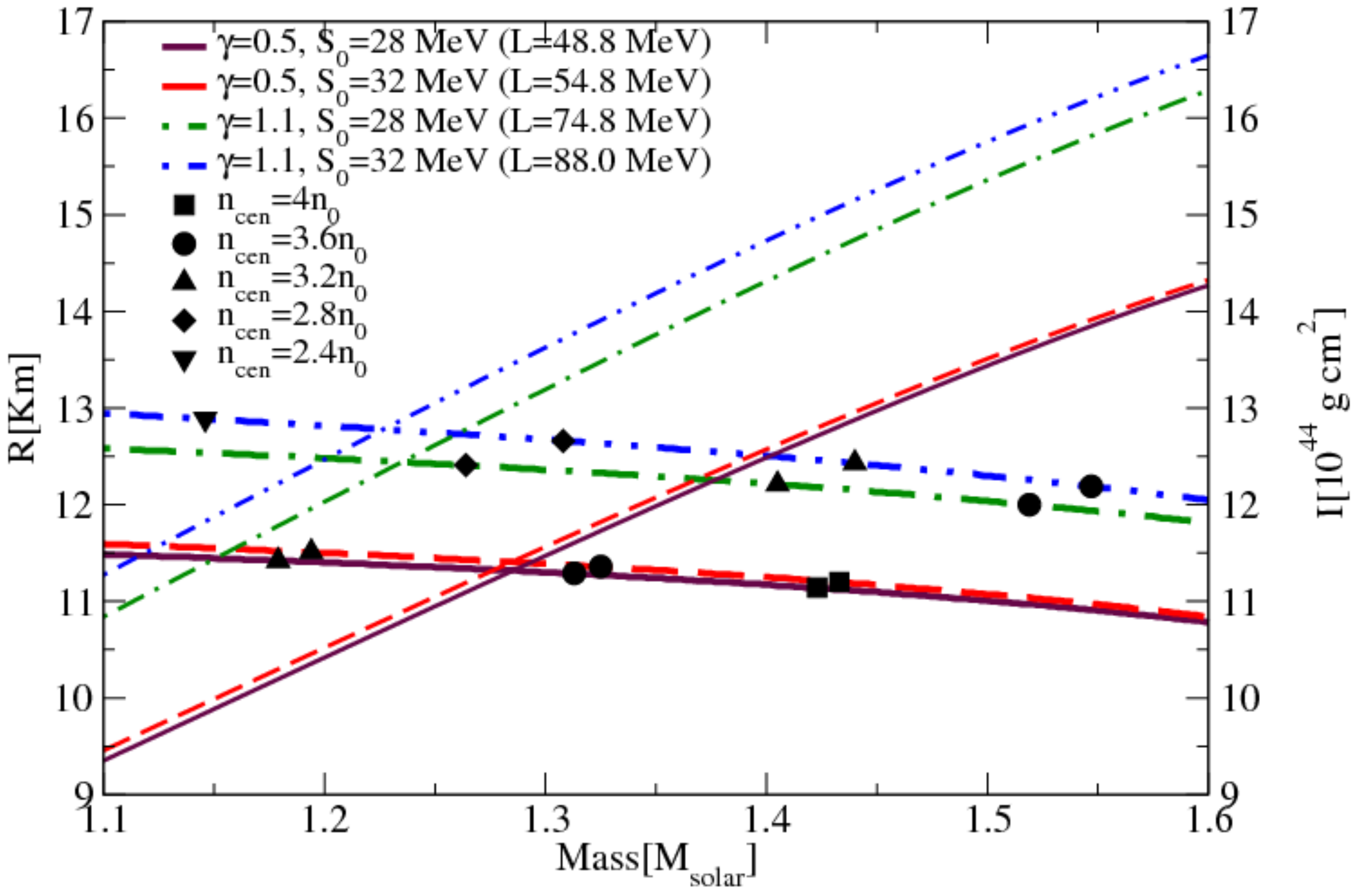




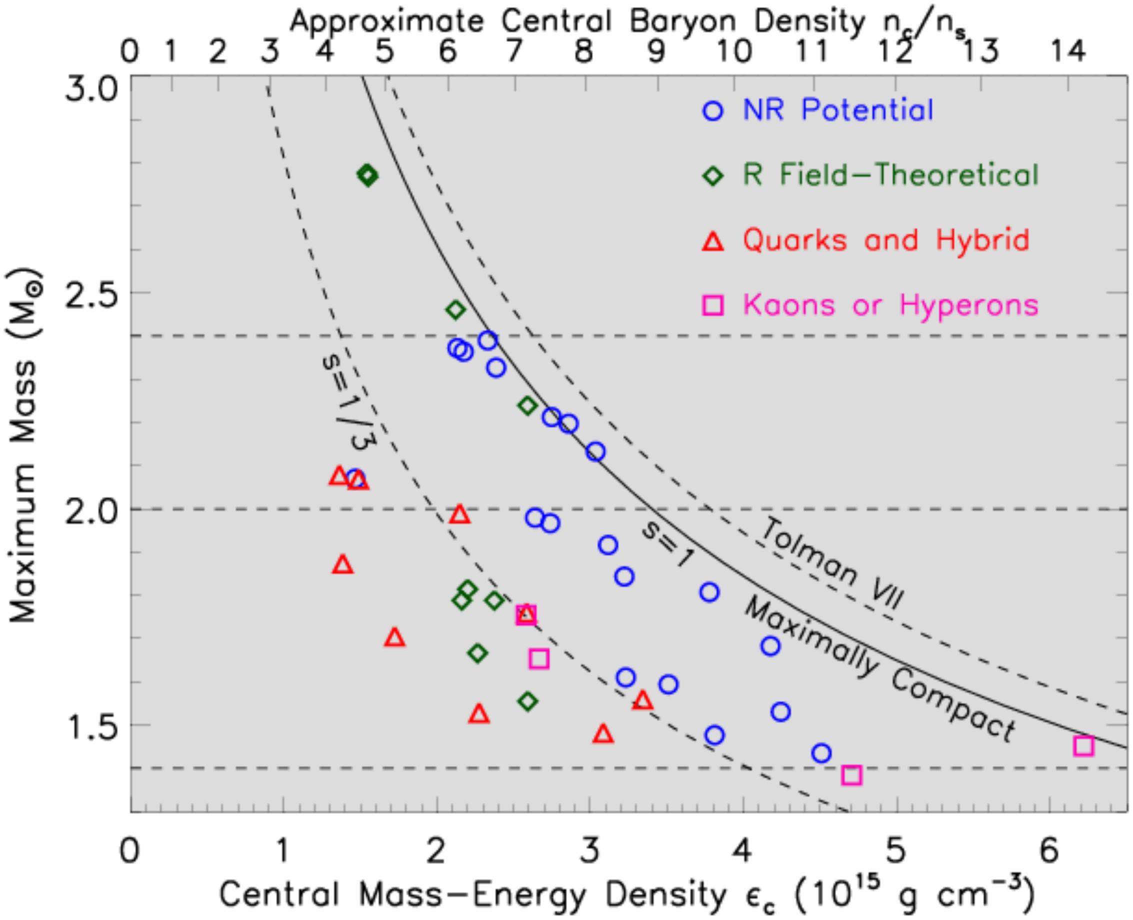




# Neutron Star Radii and the Asymmetry Potential (Sagert, Tolos, Chatterjee, JSB, Sturm 2012)



# Maximum central density of a compact star (Lattimer and Prakash 2011)



# Neutron star merger vs. heavy ion collisions Differences in dynamical description.

- System Size: Kilometers vs. Femtometers
- Evolution time: Milliseconds vs. fm/c
- Equilibrium: Chemical + Phase-Equilibrium vs. Non-Equilibrium
- Gravity is relevant - or not ?
- Yet : hydrodynamics seems to work for both!

  ➢ **Importance of the equation of state as input for hydrodynamics**

# Neutron star merger vs. heavy ion collisions: *Which densities and Temperatures can we expect?*

- Compare central heavy ion collisions with head-on neutron star collisions
- Coarse grained UrQMD simulation input for hydrodynamical evolution; Jan Steinheimer *et al*
- Estimate using the relativistic Rankine Hugoniot Taub Adiabate: conserved baryon number and energy momentum current densities across shock front yields 1-Dim, stationary hydrodynamical equation

Big open questions:

- Can QCD theorists predict neutron star measurements?
  - ➢ Not there yet – need fundamentally new machinery

- Can we infer the QCD matter EoS from observations?
  - ➢ Looks very promising, fast progress with GWs

- **Can deconfined matter be found inside the stars?**
  - ➢ **Tough question, but we’re on the right path!**

# Conclusions and future work

- Analysis of heavy ion collisions provides constraints on the nuclear matter EOS
- This has implications on the physics of compact stars, supernovae, and neutron star mergers
- Strong interplay between heavy ion physics and astrophysics
- Connecting Relativistic Heavy Ion Collisions and Neutron Star Mergers by the Equation of State of Dense Hadron- and Quark Matter as signaled by Gravitational Waves
- It has happened over and over in the history of astronomy: as a new “window” has been opened, a “new”, universe has been revealed.
- GWs will reveal Einstein’s universe of black holes and neutron stars
- Transport coefficients next – the shear viscosity to entropy ratio!
- MHD in the simulations next