The three FR(L)DM 1981, 1995, and 2012 mass/nuclear structure models

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PRESENTATION, CERN OCTOBER 2, 2017

Collaborators on this and other projects,
see coauthors on papers posted on URL below.

More details about masses, other projects (beta-decay,fission), associated ASCII data files and figures are at

http://t2.lanl.gov/nis/molleretal/

A I M

- BRIEF OUTLINE OF MODELS:
  1) MACROSCOPIC MODEL
  2) SINGLE-PARTICLE MODEL
  3) STRUTINSKY SHELL CORRECTION METHOD
  4) MACROSCOPIC-MICROSCOPIC METHOD

- GLOBAL NUCLEAR DATA PROVIDED:
  MASSES
  SHAPES (Q_2, Q_3, Q_4, ETC)
  GS SPINS
  FISSION BARRIERS
  BETA-DECAY SPECTRA
  BETA-DELAYED NEUTRON PROBABILITIES
  BETA-DELAYED FISSION PROBABILITIES
  FISSION-FRAGMENT YIELDS $Y(Z, N)$ (NEW!)

- OTHER:
  EQS PARAMETERS J and L

REALITY CHECK

- IN SOLID STATE PHYSICS THE POTENTIAL (COULOMB) IS KNOWN AND CONSEQUENTLY THE EQUATIONS DESCRIBING THE SYSTEM. YOU CAN MAKE INFORMED DECISIONS ON HOW TO SOLVE THEM.

- IN (GLOBAL) NUCLEAR STRUCTURE CALCULATIONS WE USE “TOY” OR “EFFECTIVE” MODELS (EQUATIONS). THE POTENTIALS ARE THEREFORE ALSO “EFFECTIVE”, AND THERE IS NO EFFECTIVE POTENTIAL THAT CAN BE USED WITH ALL (EFFECTIVE) MODELS; A DIFFERENT ONE IS NEEDED WITH EACH EFFECTIVE MODEL. HOW DO YOU DERIVE “EFFECTIVE” (MODEL, POTENTIAL)

- FEYNMAN: THERE IS NO METHOD TO DERIVE. YOU USE INTUITION AND TRIAL AND ERROR. HE SAYS ABOUT SCHRÖDINGER EQ: HOW DID HE DERIVE HIS EQ.? HE DID NOT, IT JUST POPPED INTO HIS HEAD. (SO IT IS ALSO “EFFECTIVE”, BUT WORKS SO WELL IT HAS BECOME “FUNDAMENTAL”).

- IN SCIENCE IT IS GENERALLY AGREED THAT TO BE A SCIENTIFIC MODEL ALL OF THIS SHOULD BE FULFILLED:
  IT SHOULD BE WELL SPECIFIED AND CAN BE EXPLAINED. (DUFLO-ZUCKER FAILED IN THIS RESPECT)
  IT SHOULD DESCRIBE KNOWN DATA GROUP (EG MASSES)
  IT SHOULD DESCRIBE NEW MEASUREMENTS THAT BECOME AVAILABLE
  (SO FAR 1936 LDM MODEL IS OK)
  IT SHOULD BE ABLE TO EVENTUALLY DESCRIBE OTHER TYPES OF DATA AND IDEALLY LEAD TO NEW DISCOVERIES
  (1936 LDM EXCELLENT EXAMPLE, MEITNER AND FRISCH INTERPRETED HAHN AND STRASS-MAN EXP. AS FISSION, NO CALCULATION THOUGH. BOHR AND WHEELER 1939 QUANTITATIVE DE-SCRIPTION OF FISSION)

M O D E L S P E C

OUR MODEL IS COMPLETELY SPECIFIED IN THE FOLLOWING PAPERS:

Nucl. Phys. **A135** (1969) 432 (some math needed)

Phys. Rev. C **5** (1972) 1050 (fY sing. model, shell corr)

Nucl. Phys. **A229** (1974) 292 (spin-orbit fttting, benchmarks)

Phys. Rev. C **14** (1976) 1977 (folding models)

Phys. Rev. C **20** (1979) 992 (folding model math, FRLDM)

Nucl. Phys. **A361** (1981) 117 (first (limited) FRDLM)

ADNDT **26** (1981) 165" (mass table)

Proc. 7th Int. Conf. on nuclear masses and fundamental constants, Darmstadt-Seeheim, 1984 (Lehrdruckerei, Darmstadt, 1984) (first limited discussion of FRDM)

ADNDT **39** (1988) 213 (how to calculate model error) p. 457

Nucl. Phys. **A536** (1992) 20 (Pairing)

ADNDT **59** (1995) 185 (masses FRDM(1992), major ref)

ADNDT **66** (1997) 131 (benchmarks mass table)

ADNDT **109-110** (2016) 1 (FRDM(2012), new enhanced mass table)

FEYNMAN REALITY CHECK

Feynman:

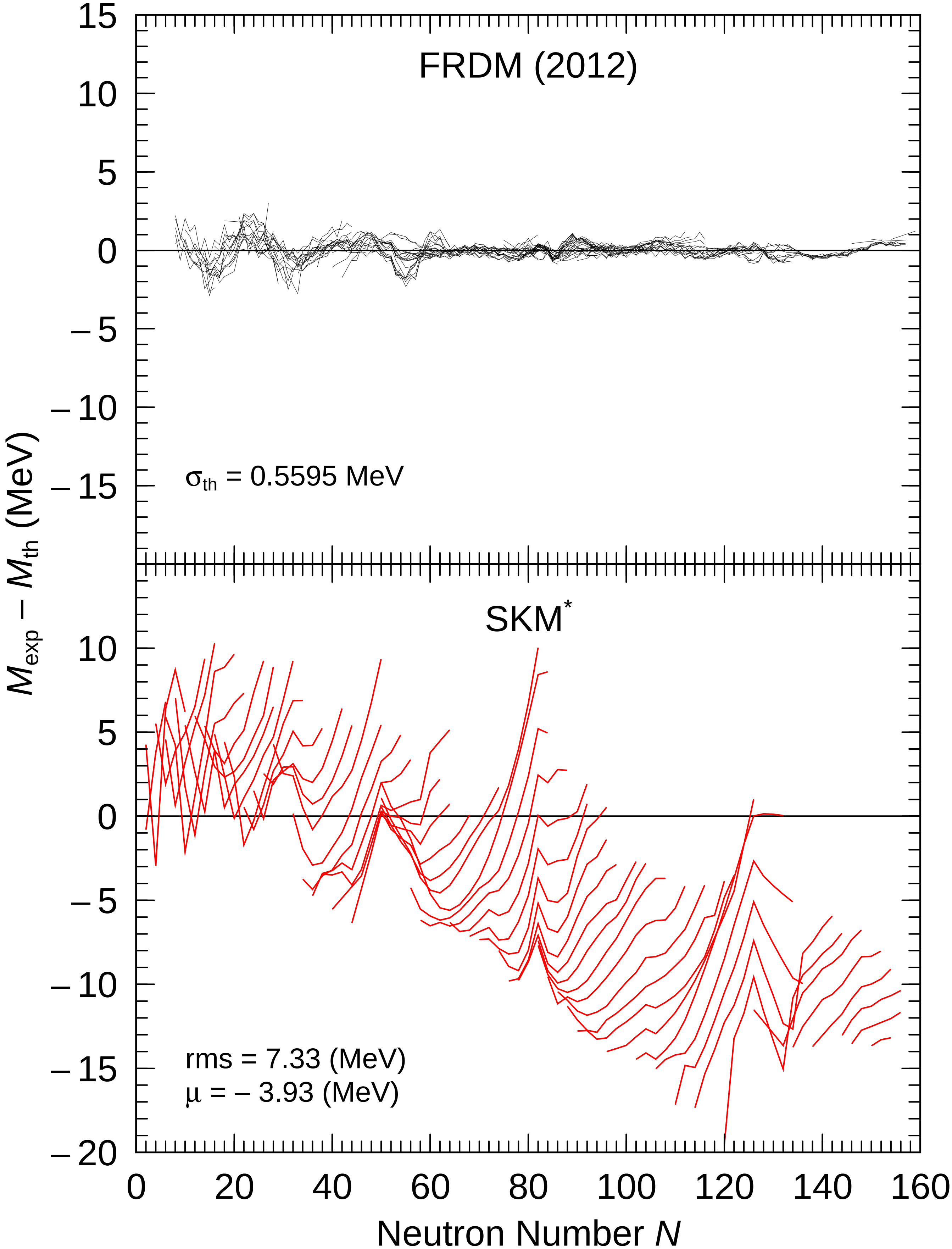
- I do not care how smart you are
- or how complicated your model is
- If it does not agree with experimental measurements it is wrong!

REFERENCES:

Feynman, R. P. What Is Science. Phys. Teach. 7, 313320 (1969).

and further discussion at:

https://en.wikiquote.org/wiki/Richard_Feynman#Disputed


FRDM (2012)
σ_th = 0.5595 MeV
SKM*
rms = 7.33 (MeV)
μ = – 3.93 (MeV)
M_exp – M_th (MeV)
Neutron Number N
15
10
5
0
–5
–10
–15
–20
0
20
40
60
80
100
120
140
160

Nuclear Binding Energy BW (1935)

The nuclear binding energy according to BW is given by

$$B(N,Z) =$$

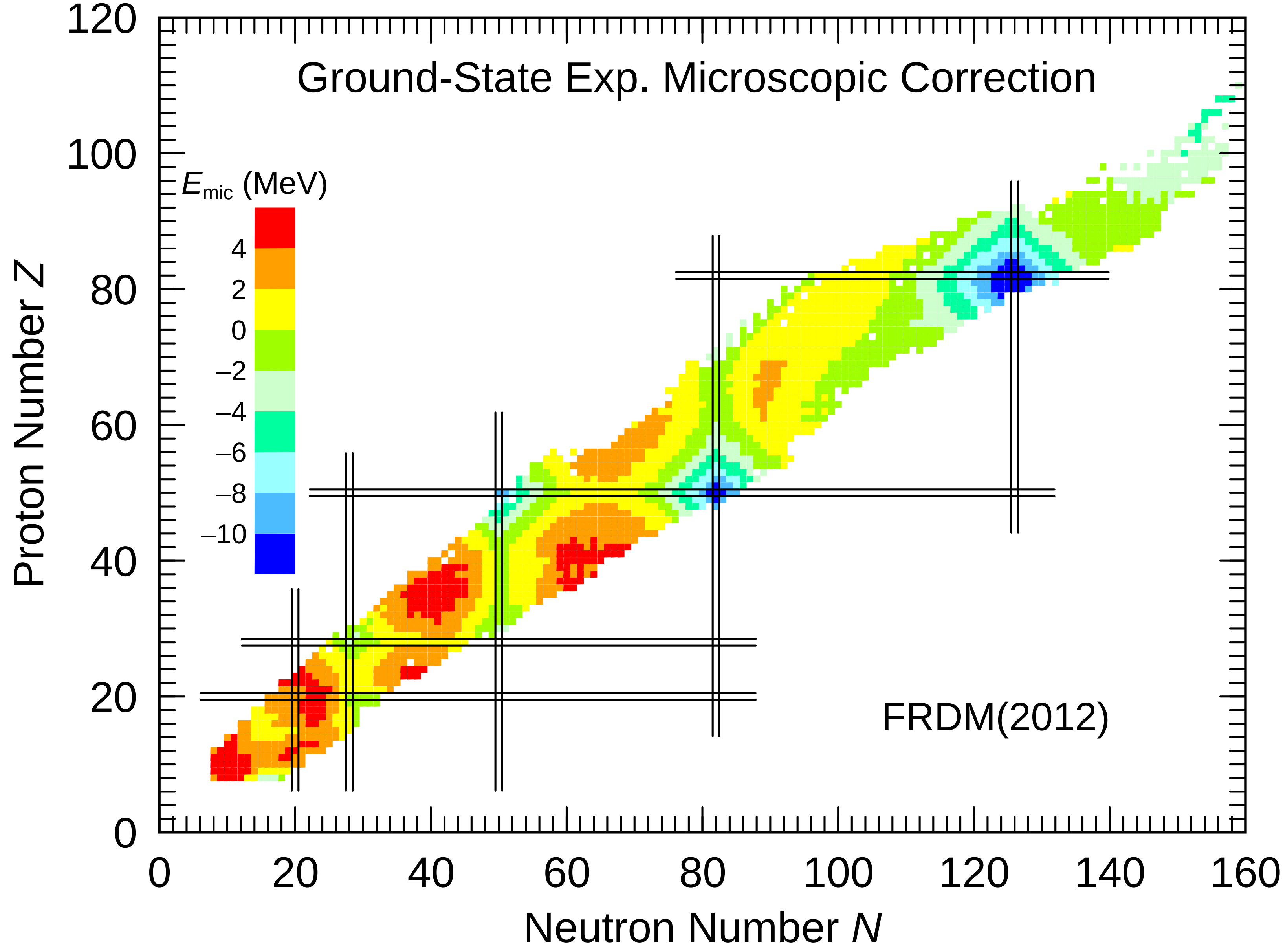
$$+a_{\mathrm{v}} A \qquad \text{(Volume energy)}$$

$$-a_{\mathrm{s}} A^{2/3} \qquad \text{(Surface energy)}$$

$$-a_{\mathrm{C}} \frac{Z^2}{A^{1/3}} \qquad \text{(Coulomb energy)}$$

$$-a_I \frac{(N-Z)^2}{A} \qquad \text{(Symmetry energy)}$$

$$-\delta(A) \qquad \text{(Pairing energy)}$$



Ground-State Exp. Microscopic Correction
$E_{\rm mic}$ (MeV)
4
2
0
−2
−4
−6
−8
−10
FRDM(2012)
Proton Number Z
Neutron Number N
0
20
40
60
80
100
120
140
160

Nuclear POTENTIAL ENERGY BW (1939)

$$B(N,Z) =$$

$$+a_{\mathrm{V}}A \qquad \text{(Volume energy)}$$

$$-a_{\mathrm{S}}A^{2/3}B_s(\beta) \qquad \text{(Surface energy)}$$

$$-a_{\mathrm{C}}\frac{Z^2}{A^{1/3}}B_C(\beta) \qquad \text{(Coulomb energy)}$$

$$-a_I\frac{(N-Z)^2}{A} \qquad \text{(Symmetry energy)}$$

$$-\delta(A) \qquad \text{(Pairing energy)}$$

Nuclear Deformation Energy

Let the nuclear surface be described by

$$r(\theta, \phi) = R_0 \left[1 + \alpha_2 P_2(\cos\theta)\right]$$

The surface energy lowest order Taylor expansion:

$$E_\mathrm{s} = E_\mathrm{s}^0 (1 + \frac{2}{5}{\alpha_2}^2)$$

The Coulomb energy lowest order Taylor expansion

$$E_\mathrm{C} = E_\mathrm{C}^0 (1 - \frac{1}{5}{\alpha_2}^2)$$

The energy at deformation α_2 relative to spherical shape

$$E_\mathrm{def}(\alpha_2) = E_\mathrm{C}(\alpha_2) + E_\mathrm{s}(\alpha_2) - (E_\mathrm{C}^0 + E_\mathrm{s}^0)$$

If E_def is negative then the system has no barrier wrt fission

$$E_\mathrm{def}(\alpha_2) = \frac{2}{5}{\alpha_2}^2 E_\mathrm{s}^0 - \frac{1}{5}{\alpha_2}^2 E_\mathrm{C}^0 < 0$$

$$1 < \frac{E_\mathrm{C}^0}{2E_\mathrm{s}^0} = x$$

The surface energy for a sphere

$$E_{\mathrm{s}}^{0} = 17.80A^{2/3}$$

The Coulomb energy for a sphere

$$E_{\mathrm{C}}^{0} = 0.7103\frac{Z^2}{A^{1/3}}$$

The fissility parameter x:

$$x = \frac{Z^2}{50.13A}$$

Z	A	x
50	124	0.402
82	208	0.645
92	138	0.709
100	252	0.792
114	298	0.870
125	328	0.950
130	335	1.006

Potential Energy of Deformation

We use the macroscopic-microscopic method introduced by Swiatecki and Strutinsky:

$$E_{\mathrm{pot}}(\mathrm{shape}) = E_{\mathrm{macr}}(\mathrm{shape}) + E_{\mathrm{micr}}(\mathrm{shape}) \quad (1)$$

The macroscopic term is calculated in a liquid-drop type model (for a specific deformed shape).

The microscopic correction is determined in the following steps

1. A shape is prescribed
2. A single-particle potential with this shape is generated. A spin-orbit term is included.
3. The Schrödinger equation is solved for this deformed potential and single-particle levels and wave-functions are obtained
4. The shell correction is calculated by use of Strutinsky's method.
5. The pairing correction is calculated in the BCS or Lipkin-Nogami method.

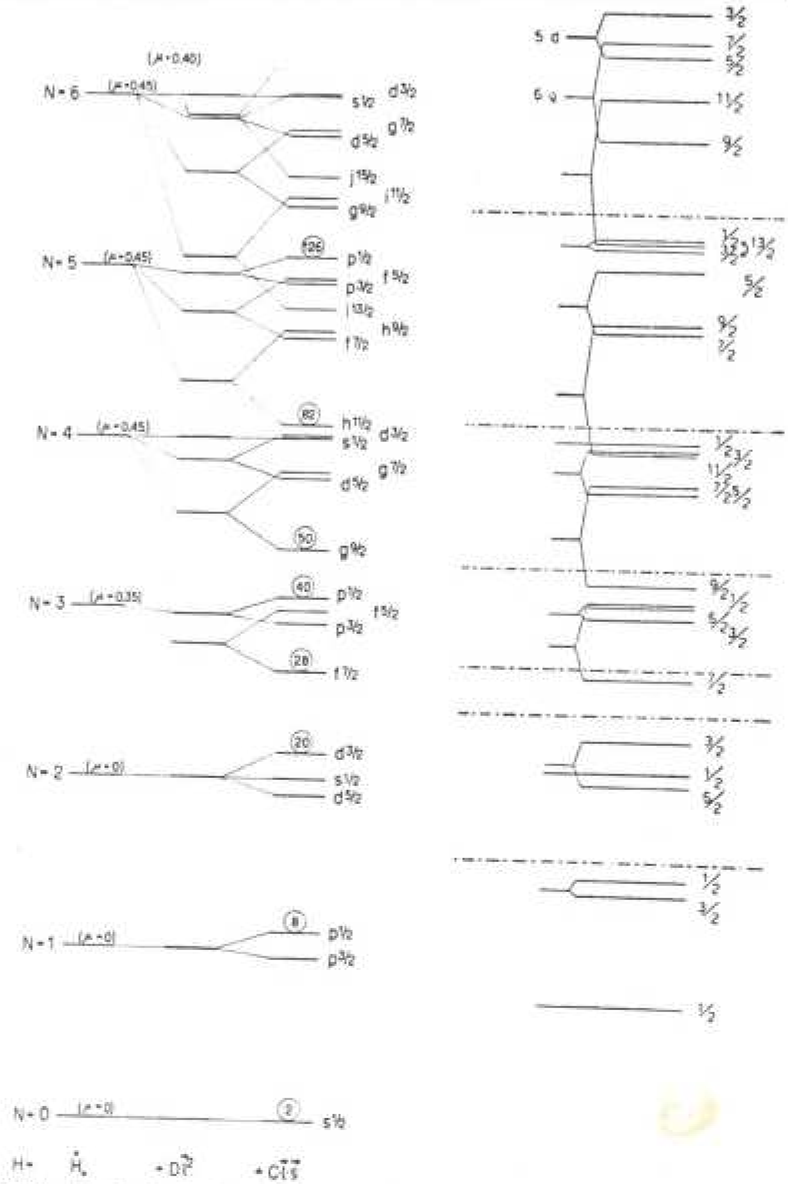


Fig. 2. *Level order for the spherical case compared with the shell model level order.*

Energy levels of the potential assumed in formula (2) for the spherical case ($\delta = 0$) are plotted to the left. The right part of the figure shows the level scheme proposed by P. KLINKENBERG, which he has obtained from empirical data interpreted according to the shell model. The level scheme of Dr. KLINKENBERG is reproduced by his kind permission from Reviews of Modern Physics.

Det Kongelige Danske Videnskabernes Selskab
Matematisk-fysiske Meddelelser, bind **29**, nr. 16

Dan. Mat. Fys. Medd. **29**, no. 16 (1955)

BINDING STATES OF INDIVIDUAL NUCLEONS IN STRONGLY DEFORMED NUCLEI

BY

SVEN GÖSTA NILSSON

København 1955
i kommission hos Ejnar Munksgaard

Matematisk-fysiske Skrifter
udgivet af
Det Kongelige Danske Videnskabernes Selskab
Bind **1**, nr. 8

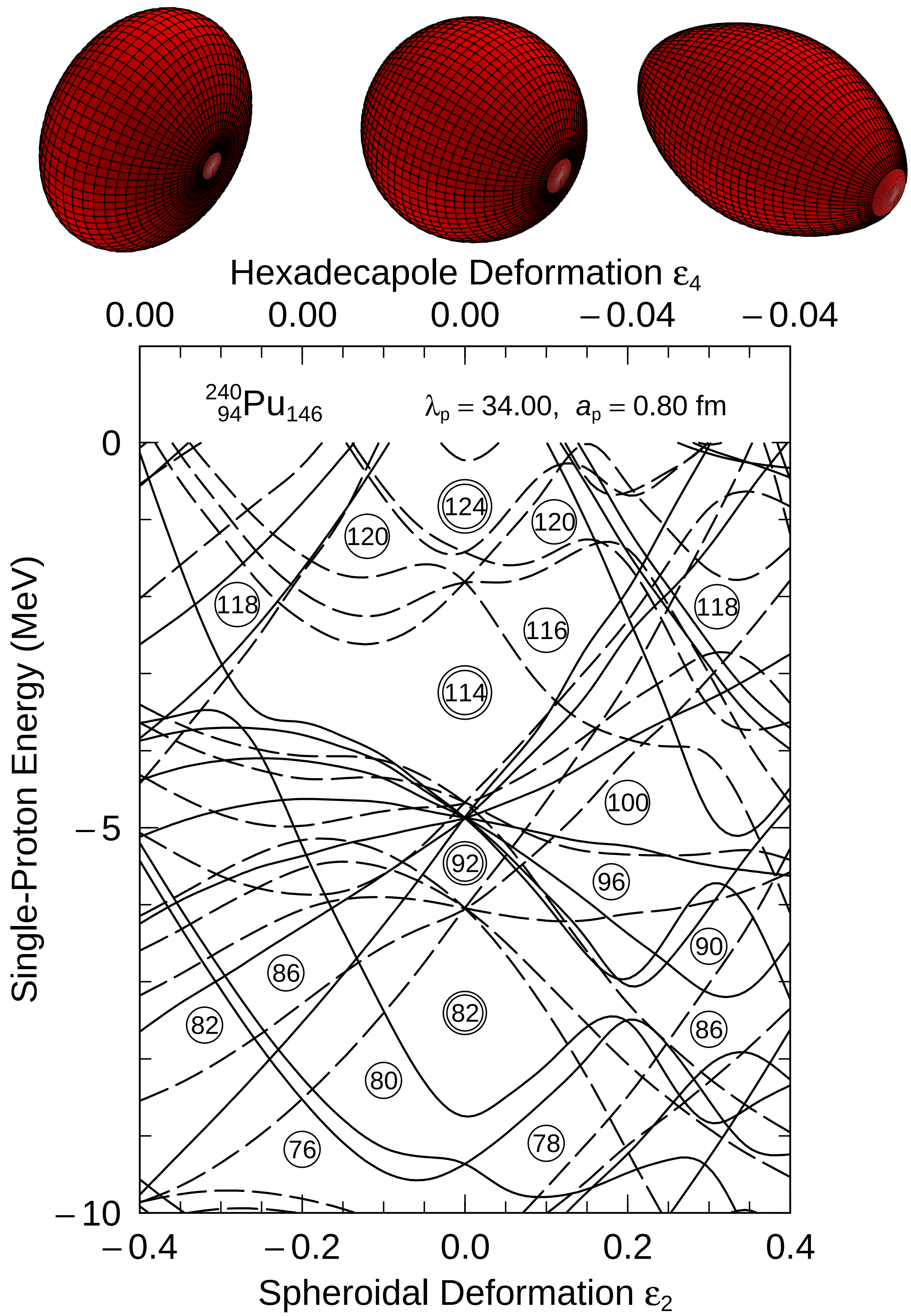
Mat. Fys. Skr. Dan. Vid. Selsk. **1**, no. 8 (1959)

THE INTRINSIC STATES OF ODD-A NUCLEI HAVING ELLIPSOIDAL EQUILIBRIUM SHAPE

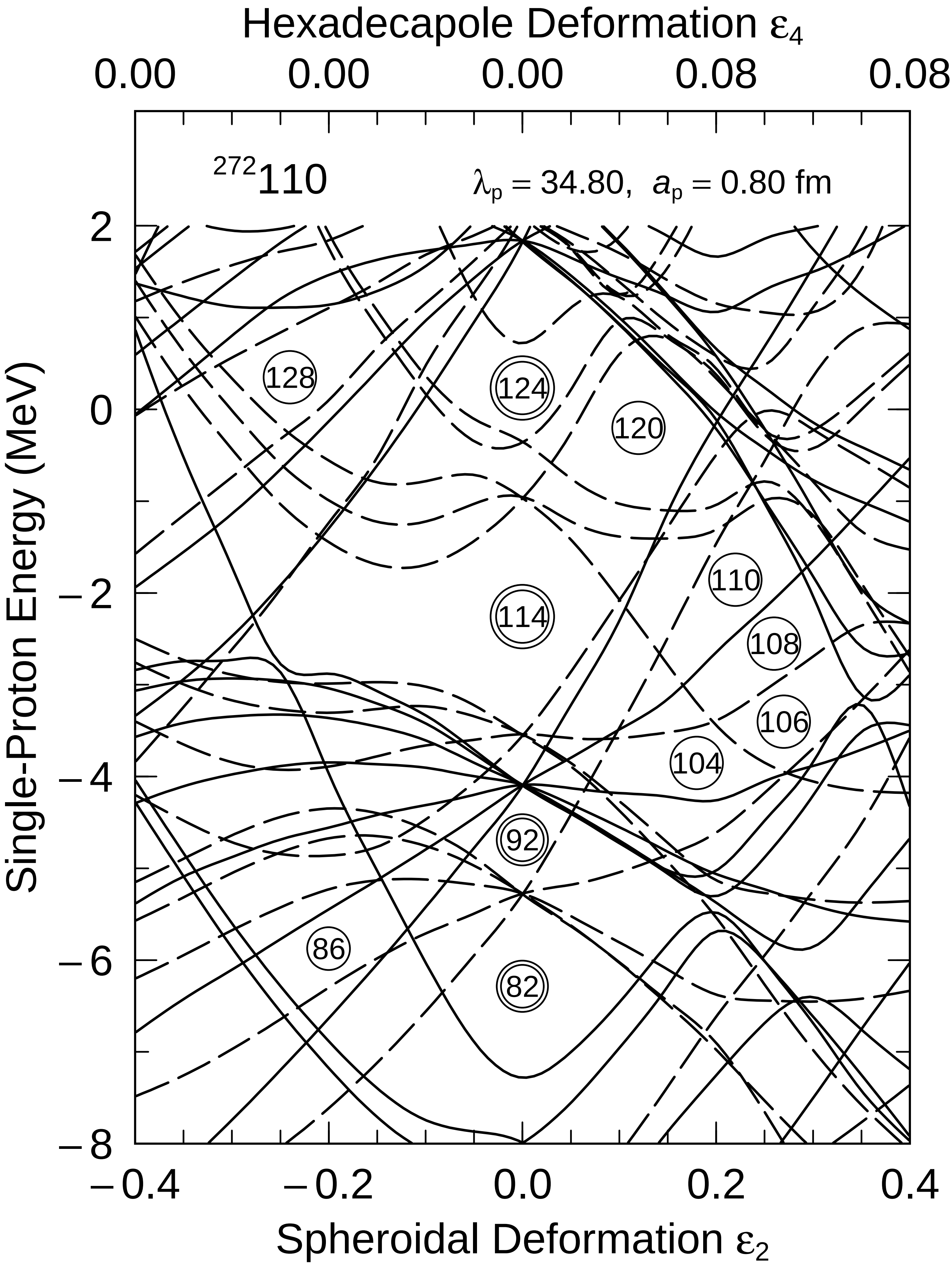
BY

BEN R. MOTTELSON AND SVEN GÖSTA NILSSON

København 1959
i kommission hos Ejnar Munksgaard



Hexadecapole Deformation ε4
0.00
0.00
0.00
−0.04
−0.04
$^{240}_{94}Pu_{146}$
λ_p = 34.00, a_p = 0.80 fm
124
120
120
118
118
116
114
100
96
92
90
86
86
82
82
80
78
76
Single-Proton Energy (MeV)
0
−5
−10
−0.4
−0.2
0.0
0.2
0.4
Spheroidal Deformation ε2


Hexadecapole Deformation ε4
0.00
0.00
0.00
0.08
0.08
272110
λp = 34.80, ap = 0.80 fm
2
0
−2
−4
−6
−8
Single-Proton Energy (MeV)
128
124
120
114
110
108
106
104
92
86
82
−0.4
−0.2
0.0
0.2
0.4
Spheroidal Deformation ε2

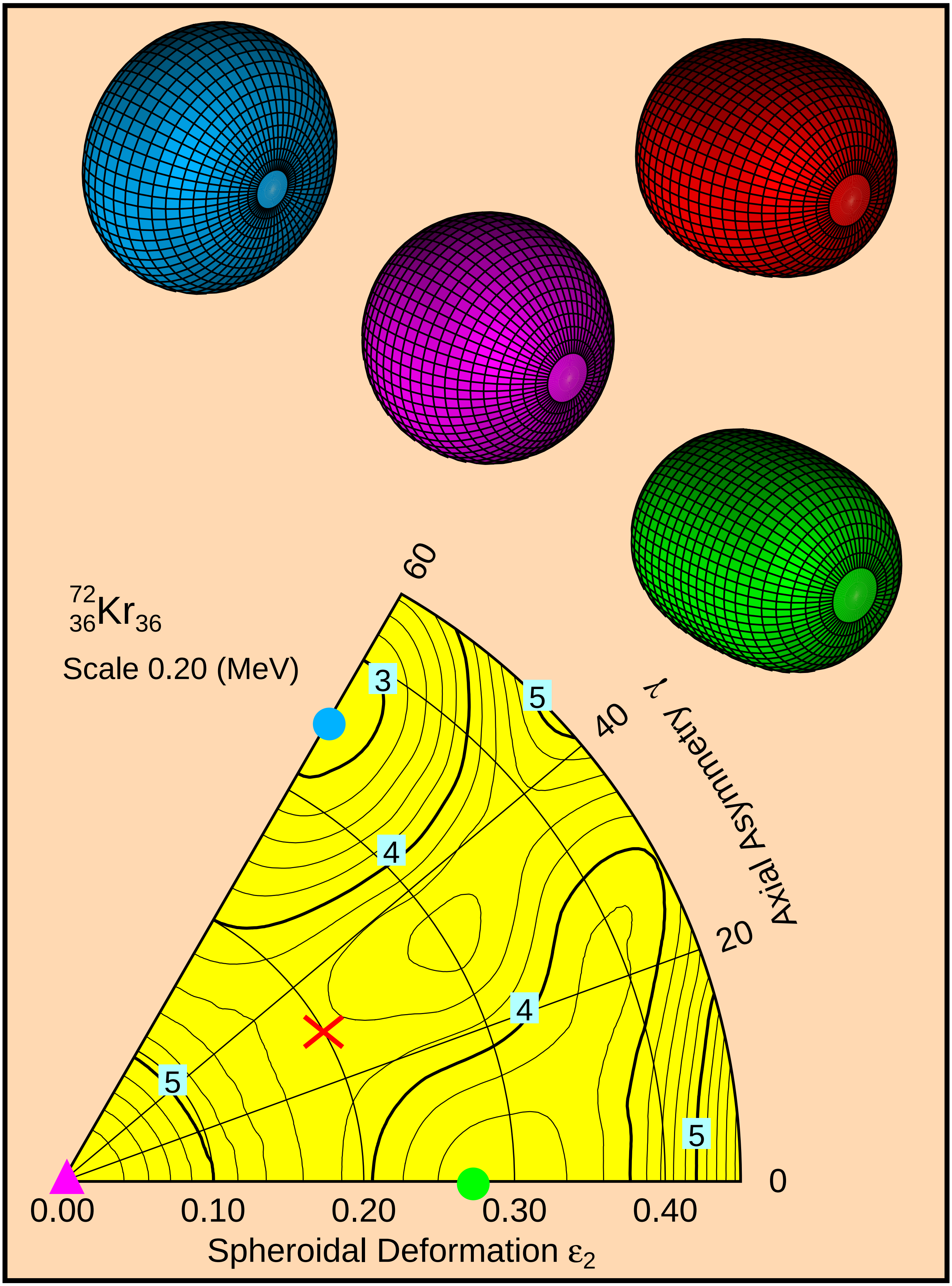
Shape Parameterizations

For small distortions we use multipole expansions, for example the β parameterization:

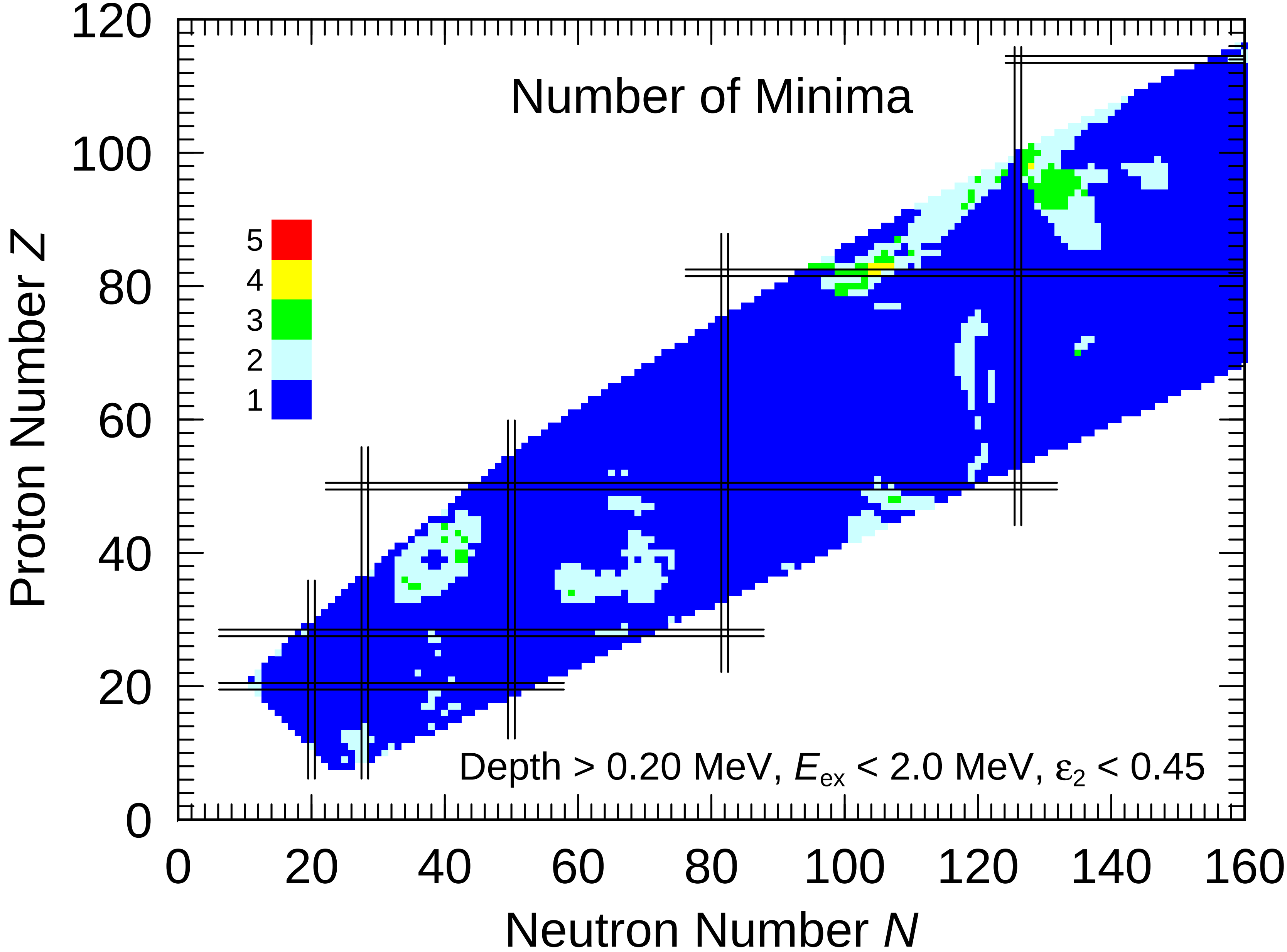
$$r(\theta,\phi) = R_0(1 + \sum_{l=1}^{\infty} \sum_{m=-l}^{l} \beta_{lm} Y_l^m)$$

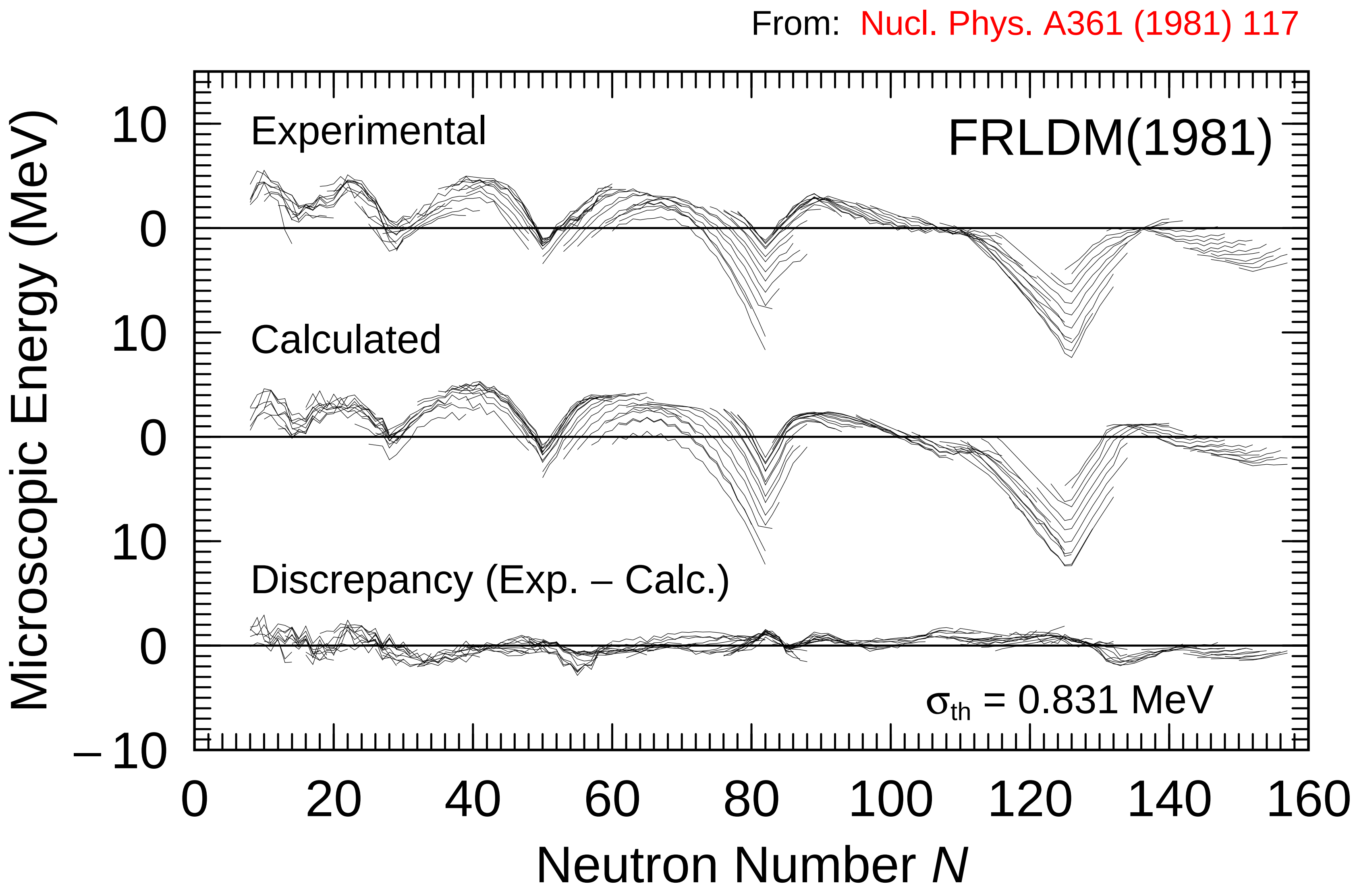
For large deformations near the outer saddle in the actinide region or beyond we use the three-quadratic-surface parameterization:

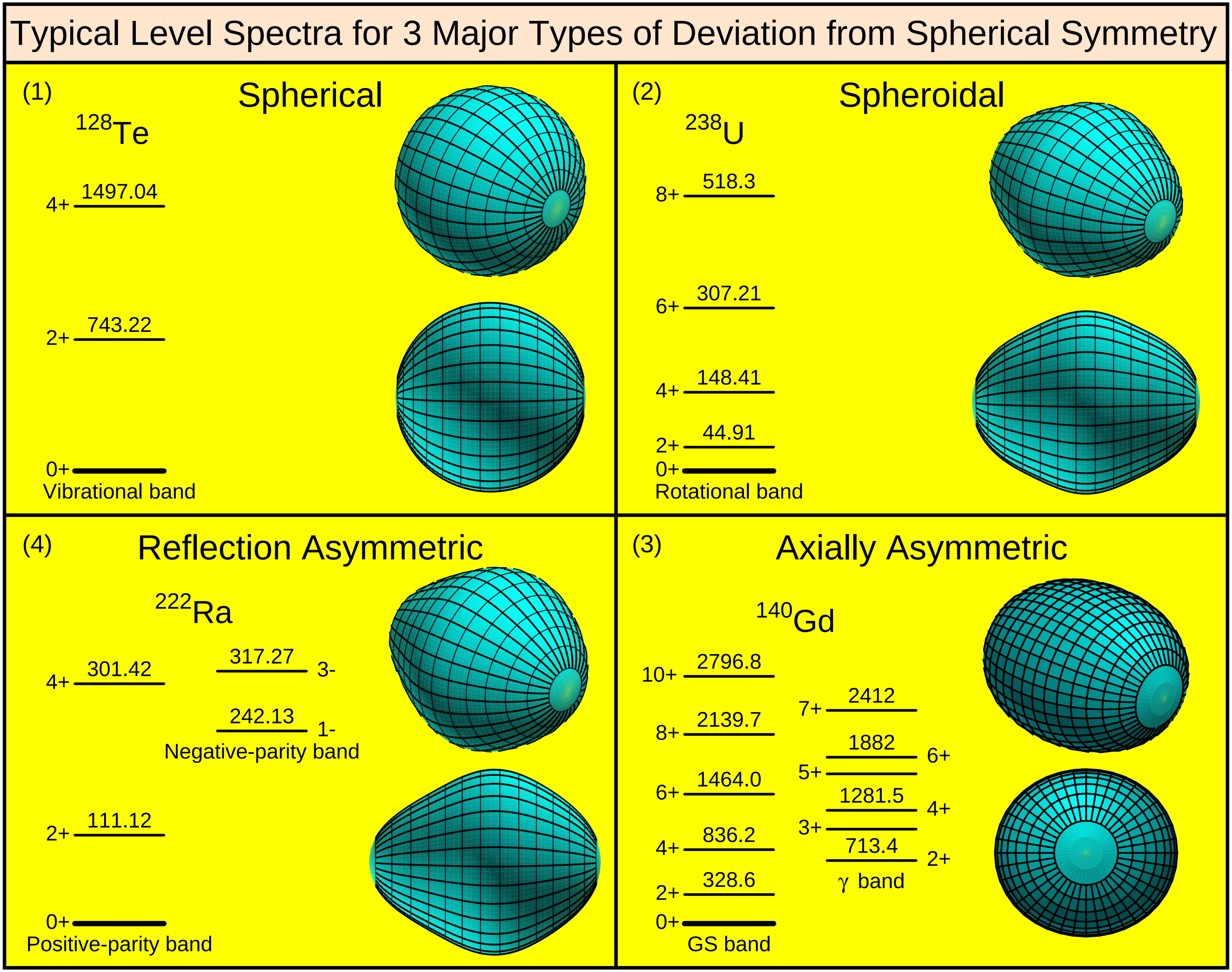
$$\rho(z)^2 = \begin{cases} {a_1}^2 - \frac{{a_1}^2}{{c_1}^2}(z-l_1)^2 \quad , \quad l_1 - c_1 \le z \le z_1 \\ {a_2}^2 - \frac{{a_2}^2}{{c_2}^2}(z-l_2)^2 \quad , \quad z_2 \le z \le l_2 + c_2 \\ {a_3}^2 - \frac{{a_3}^2}{{c_3}^2}(z-l_3)^2 \quad , \quad z_1 \le z \le z_2 \end{cases}$$



60
$^{72}_{36}Kr_{36}$
Scale 0.20 (MeV)
3
5
40
Axial Asymmetry γ
4
20
4
5
5
0
0.00
0.10
0.20
0.30
0.40
Spheroidal Deformation ε_2

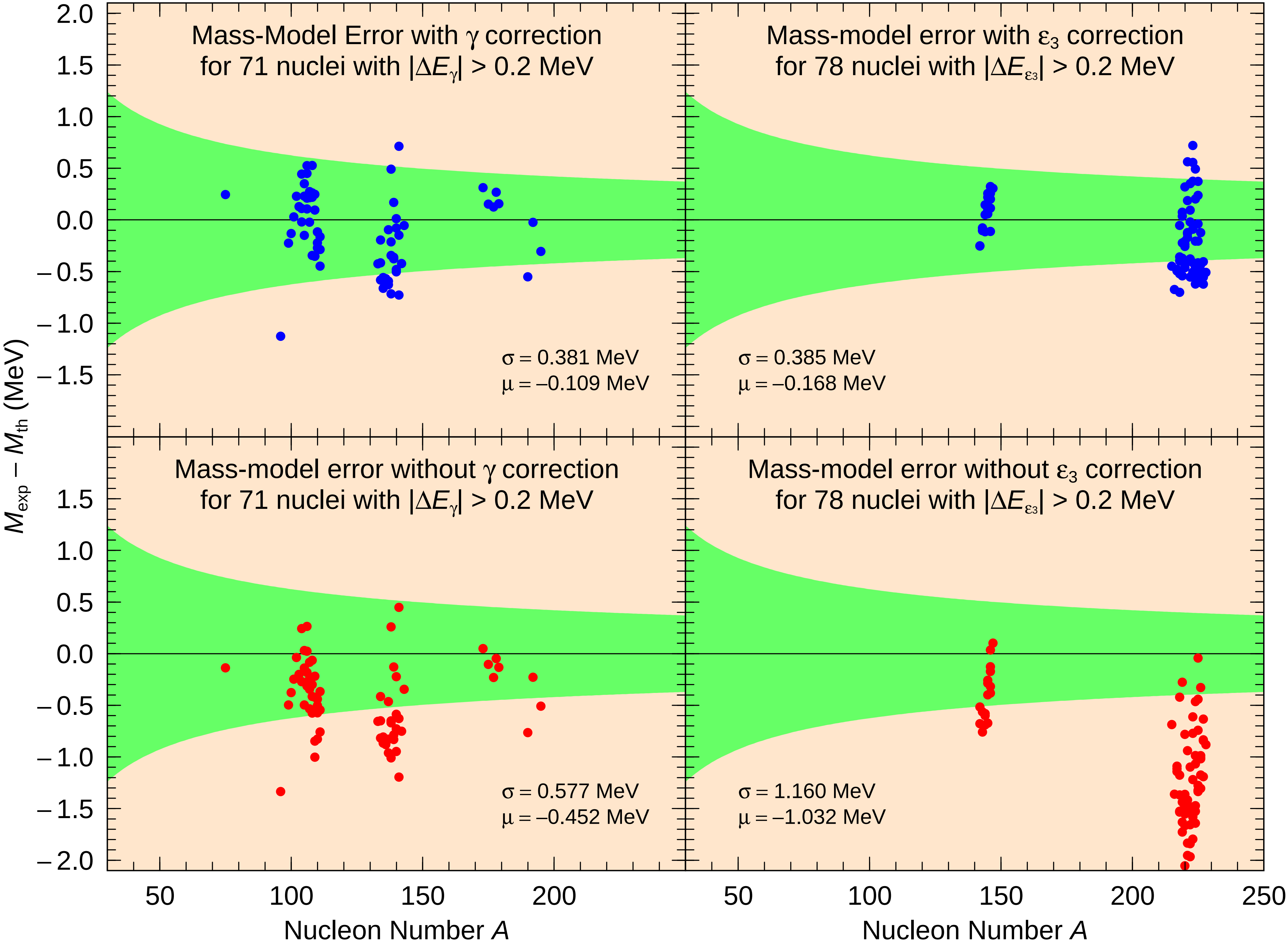

Number of Minima
5
4
3
2
1
Depth > 0.20 MeV, E_{ex} < 2.0 MeV, ε_2 < 0.45
Proton Number Z
Neutron Number N
0
20
40
60
80
100
120
140
160

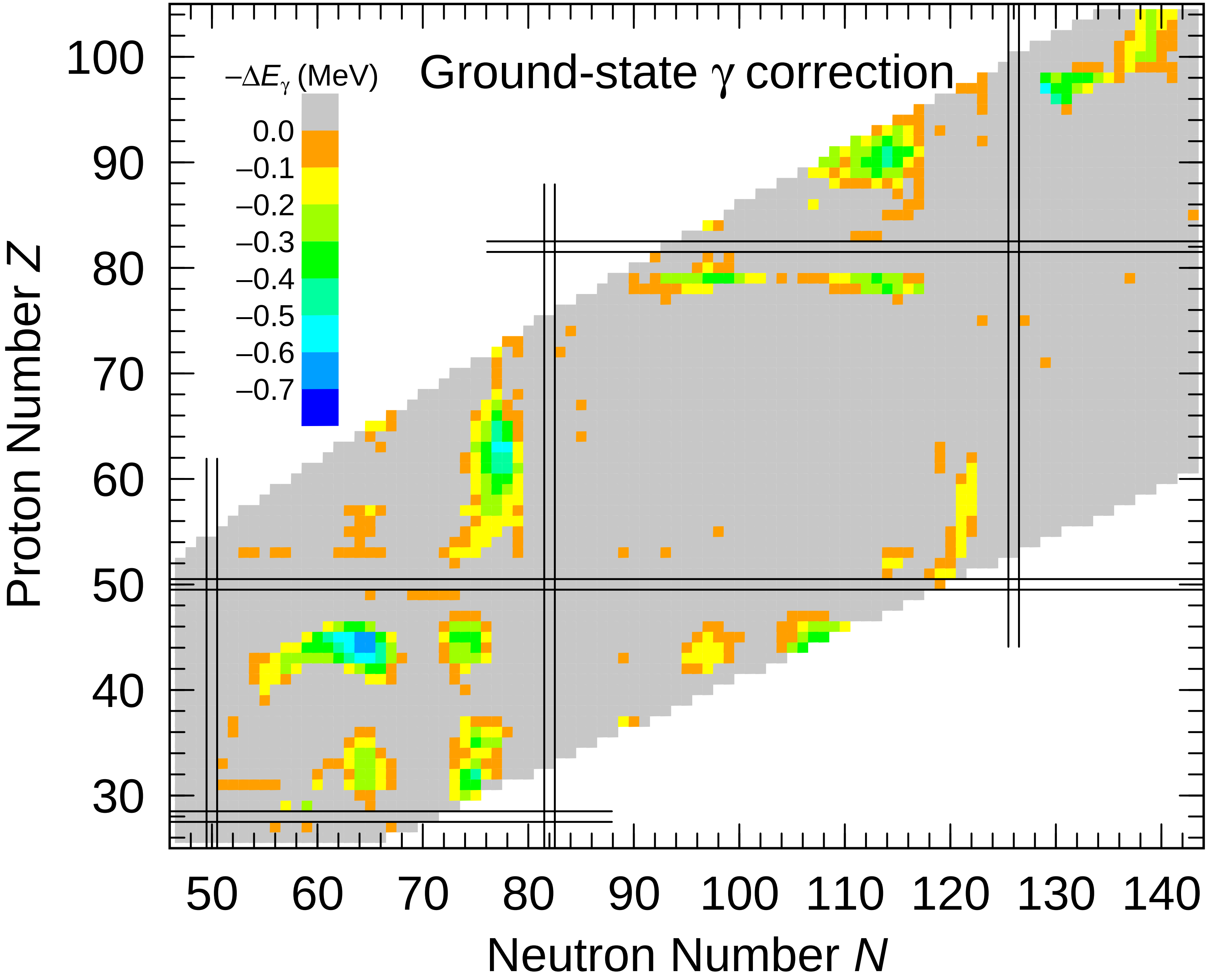

From: Nucl. Phys. A361 (1981) 117
Experimental
FRLDM(1981)
Calculated
Discrepancy (Exp. – Calc.)
σth = 0.831 MeV
Microscopic Energy (MeV)
Neutron Number N
10
0
10
0
10
0
–10
0
20
40
60
80
100
120
140
160



Typical Level Spectra for 3 Major Types of Deviation from Spherical Symmetry
(1)
Spherical
128Te
4+ 1497.04
2+ 743.22
0+
Vibrational band
(2)
Spheroidal
238U
8+ 518.3
6+ 307.21
4+ 148.41
2+ 44.91
0+
Rotational band
(4)
Reflection Asymmetric
222Ra
4+ 301.42
317.27 3-
242.13 1-
Negative-parity band
2+ 111.12
0+
Positive-parity band
(3)
Axially Asymmetric
140Gd
10+ 2796.8
8+ 2139.7
6+ 1464.0
4+ 836.2
2+ 328.6
0+
GS band
7+ 2412
1882 6+
5+
1281.5 4+
3+
713.4 2+
γ band

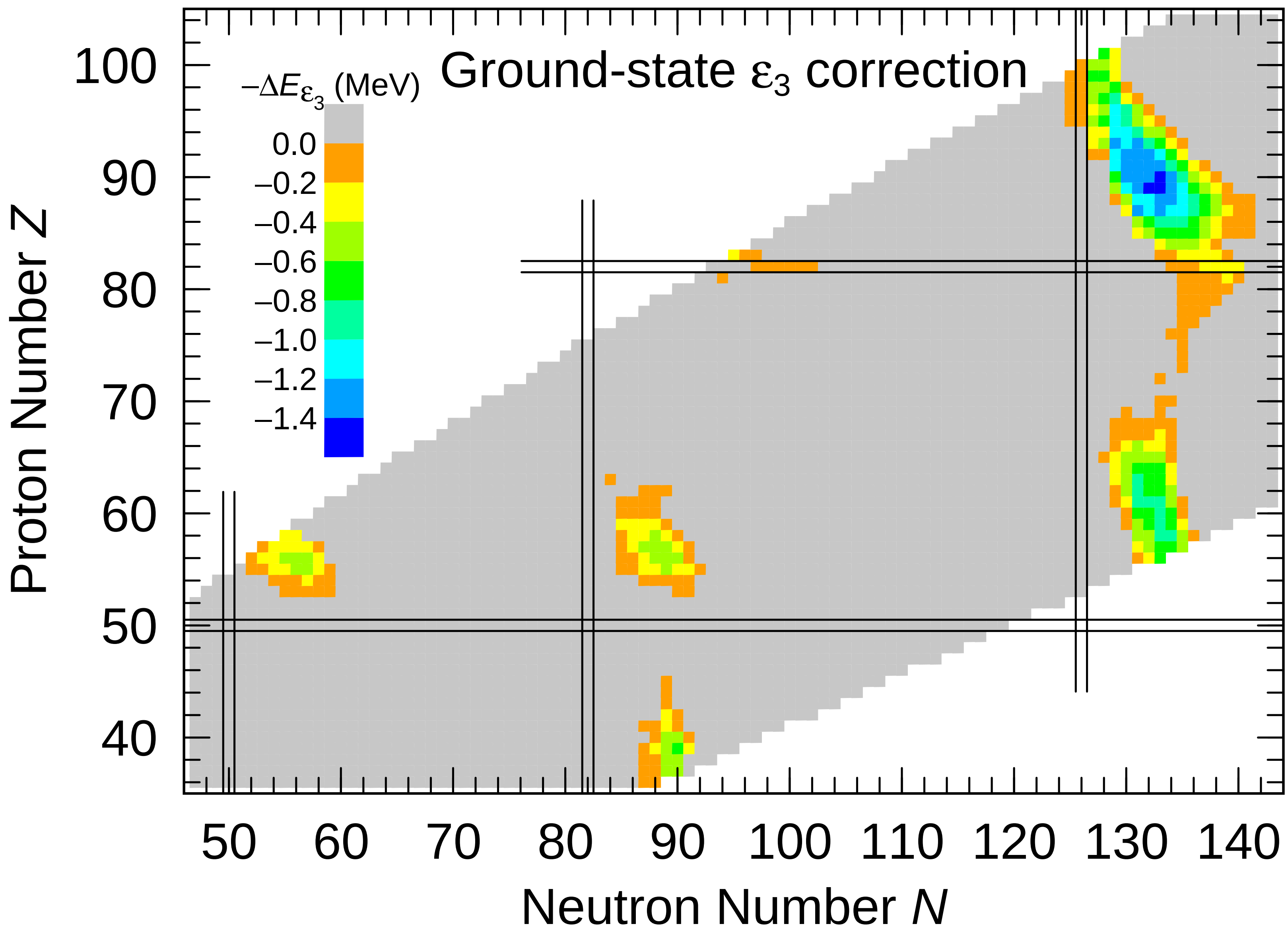
Graph 2


Mass-Model Error with γ correction
for 71 nuclei with |ΔE_γ| > 0.2 MeV
σ = 0.381 MeV
μ = –0.109 MeV
Mass-model error with ε_3 correction
for 78 nuclei with |ΔE_ε3| > 0.2 MeV
σ = 0.385 MeV
μ = –0.168 MeV
Mass-model error without γ correction
for 71 nuclei with |ΔE_γ| > 0.2 MeV
σ = 0.577 MeV
μ = –0.452 MeV
Mass-model error without ε_3 correction
for 78 nuclei with |ΔE_ε3| > 0.2 MeV
σ = 1.160 MeV
μ = –1.032 MeV
M_exp – M_th (MeV)
Nucleon Number A
Nucleon Number A



Ground-state γ correction
−ΔE_γ (MeV)
0.0
−0.1
−0.2
−0.3
−0.4
−0.5
−0.6
−0.7
Proton Number Z
100
90
80
70
60
50
40
30
50
60
70
80
90
100
110
120
130
140
Neutron Number N

Graph 2



Ground-state ε_3 correction
$-\Delta E_{\varepsilon_3}$ (MeV)
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
−1.2
−1.4
Proton Number Z
Neutron Number N
100
90
80
70
60
50
40
50
60
70
80
90
100
110
120
130
140

Graph 3

THÈSE PRÉSENTÉE

POUR OBTENIR LE GRADE DE

DOCTEUR DE L'UNIVERSITÉ DE BORDEAUX

ÉCOLE DOCTORALE DES SCIENCES PHYSIQUES ET DE L'INGENIEUR

SPÉCIALITÉ ASTROPHYSIQUE, PLASMAS, NUCLÉAIRE

Par Pierre TAMAGNO

CHALLENGING FISSION CROSS SECTION SIMULATION WITH LONG STANDING MACRO-MICROSCOPIC MODEL OF NUCLEUS POTENTIAL ENERGY SURFACE

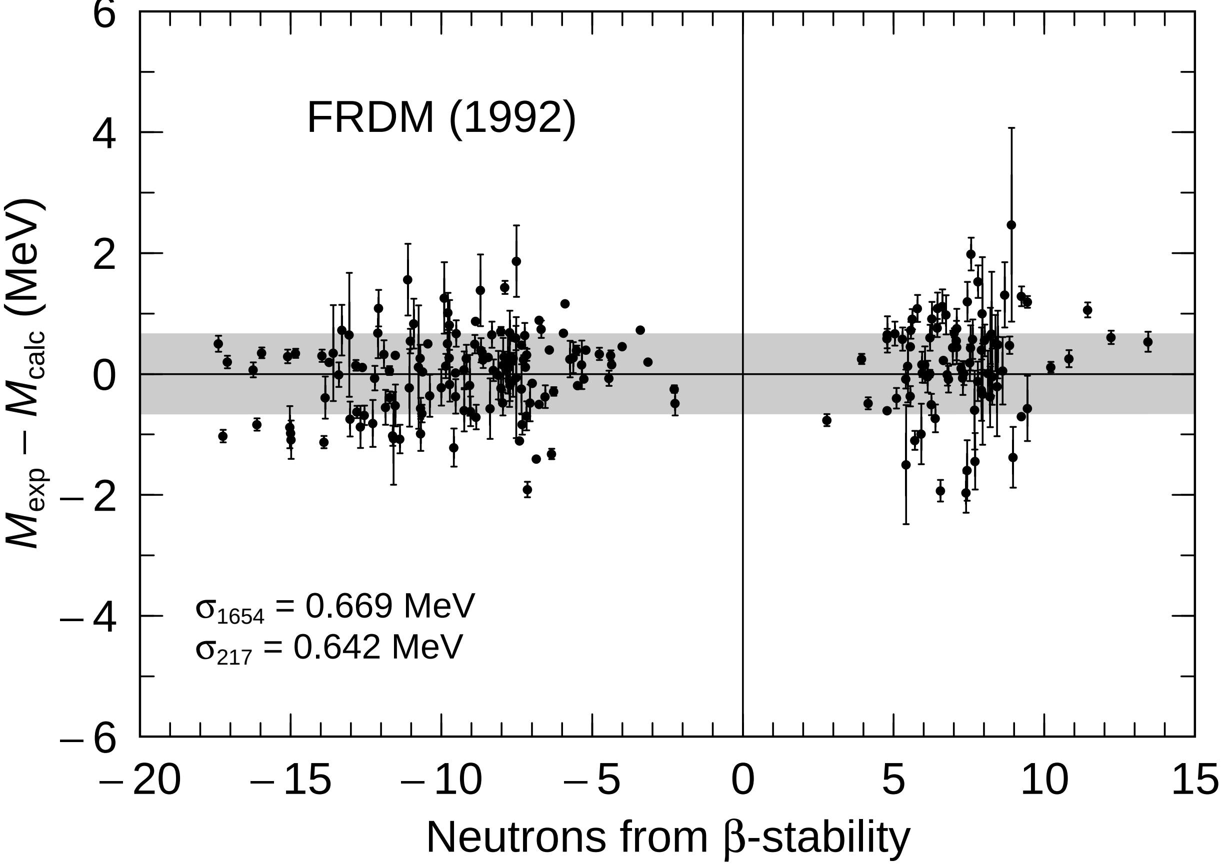
Sous la direction de Mourad AÏCHE

Soutenue le ...

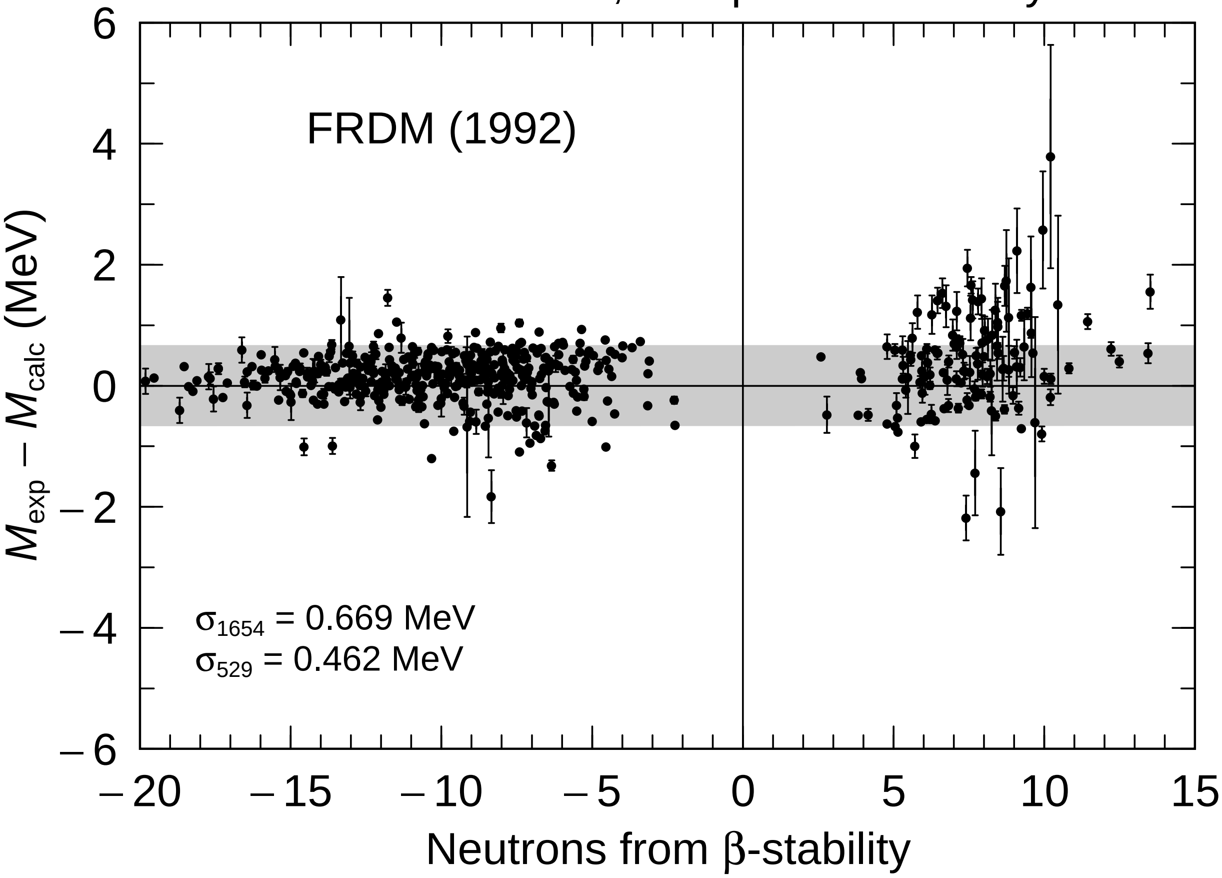
Membres du jury :

M. TSEKHANOVICH, Igor Professeur Université Bordeaux I (CENBG) Examinateur
Mme GOUTTE, Héloise Dr, HDR, Chef de Service au CEA-DSM (Saclay) Rapporteure
M. KONING, Arjan Dr, Professeur, Physicien à NRG (Petten) Rapporteur
M. MÖLLER, Peter Dr, Physicien au Los Alamos National Laboratory (USA) Examinateur
M. QUENTIN, Philippe Dr, Professeur émérite au CENBG Examinateur
M. AÏCHE, Mourad Dr, CR au CNRS (CENBG) Directeur de Thèse
M. BOULAND, Olivier Dr, HDR, Physicien au CEA-DEN (Cadarache) Encadrant
M. SEROT, Olivier Dr, HDR, Physicien au CEA-DEN (Cadarache) Encadrant

New Masses in Audi 1993 Evaluation,
Relative to 1989, Compared to Theory



New Masses in Audi 2003 Evaluation,
Relative to 1989, Compared to Theory



Successive FRDM enhancements

Optimization (2006)

Better search for optimum FRDM parameters.

Accuracy improvement: 0.01 MeV

New mass data base (AME2003) (2006)

Better agreement than with AME1989.

Accuracy improvement: 0.04 MeV

Full 4D energy minimization (2006–2008)

Full 4D minimization($\epsilon_2, \epsilon_3, \epsilon_4, \epsilon_6$) step=0.01.

Accuracy improvement: 0.02 MeV

Axial asymmetry (2002–2006)

Also yields correct SHE gs assignments.

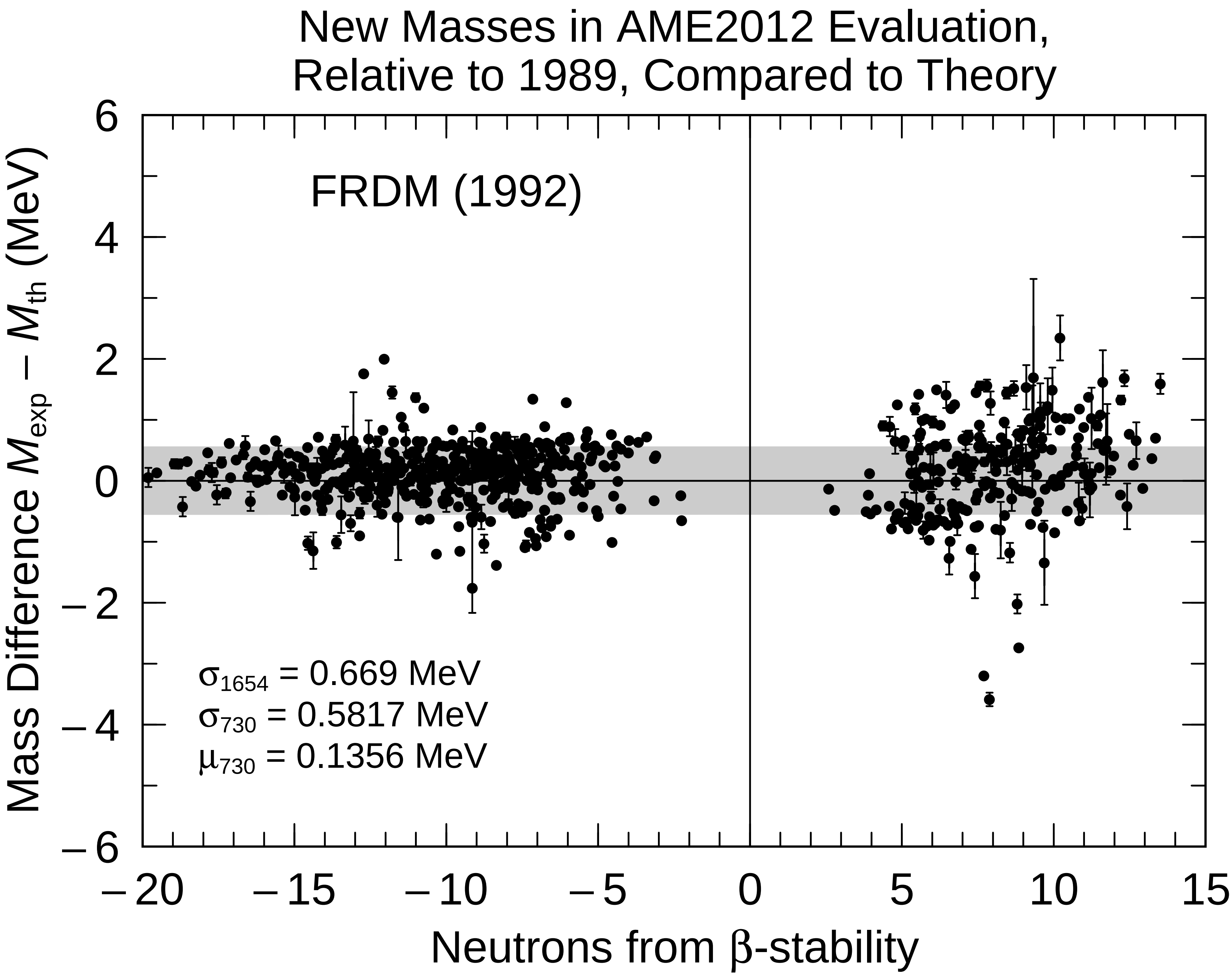
Accuracy improvement: 0.01 MeV

L **variation** (2009–2011)

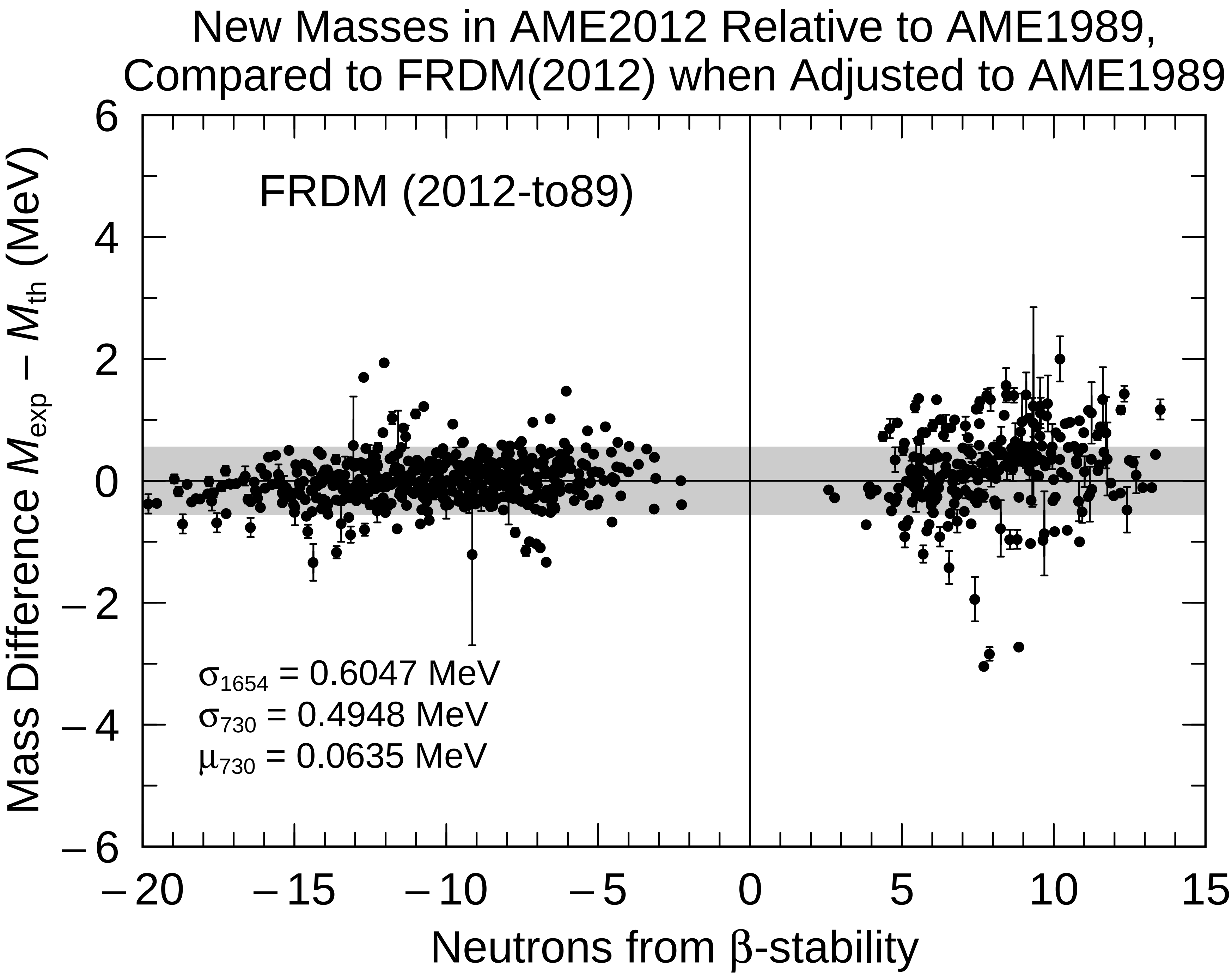
Accuracy improvement: 0.02 MeV

Improved gs correlation energies (2012)

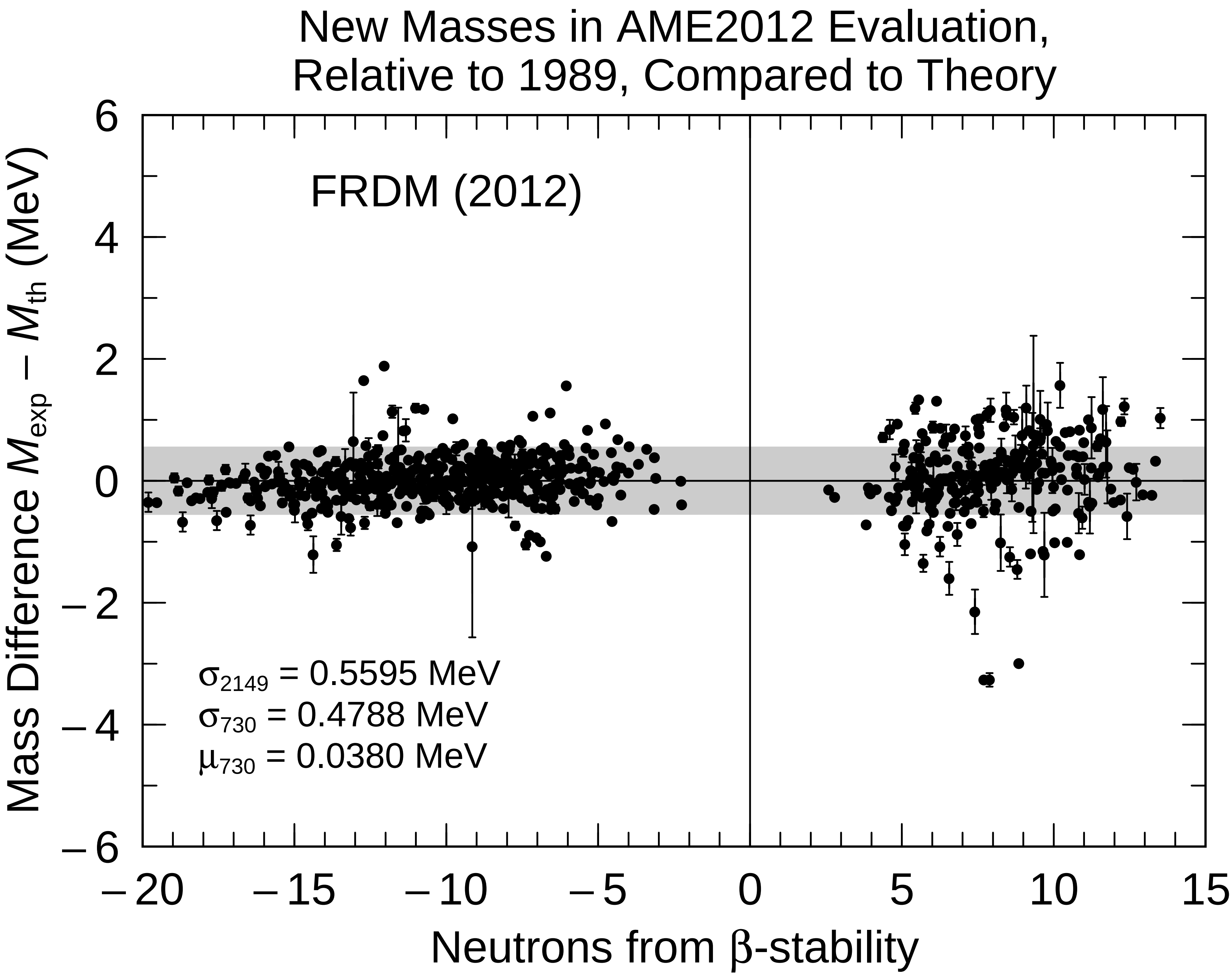
Accuracy improvement: 0.01 MeV



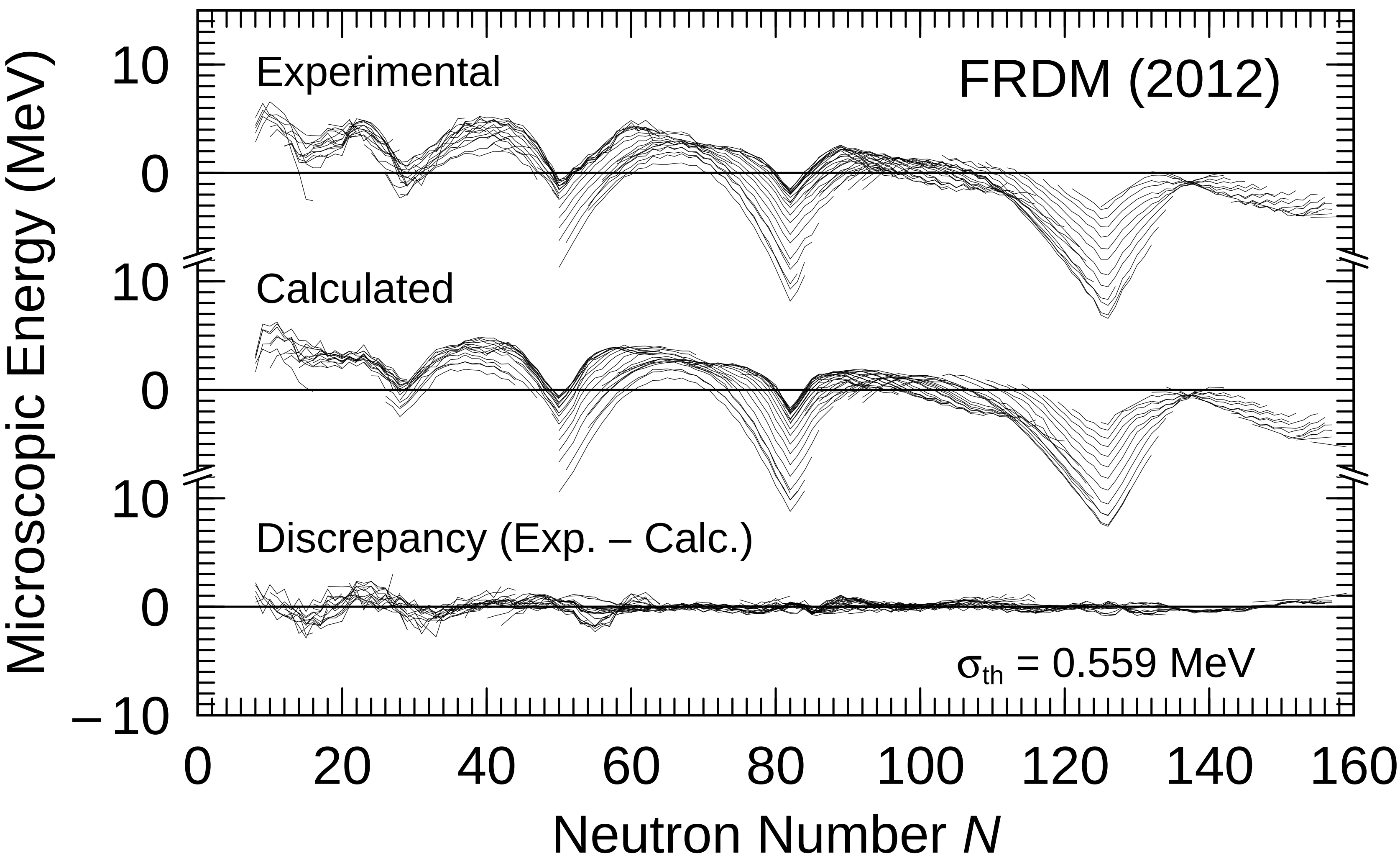
New Masses in AME2012 Evaluation,
Relative to 1989, Compared to Theory
FRDM (1992)
σ_{1654} = 0.669 MeV
σ_{730} = 0.5817 MeV
μ_{730} = 0.1356 MeV
Mass Difference $M_{exp} - M_{th}$ (MeV)
Neutrons from β-stability
6
4
2
0
−2
−4
−6
−20
−15
−10
−5
0
5
10
15

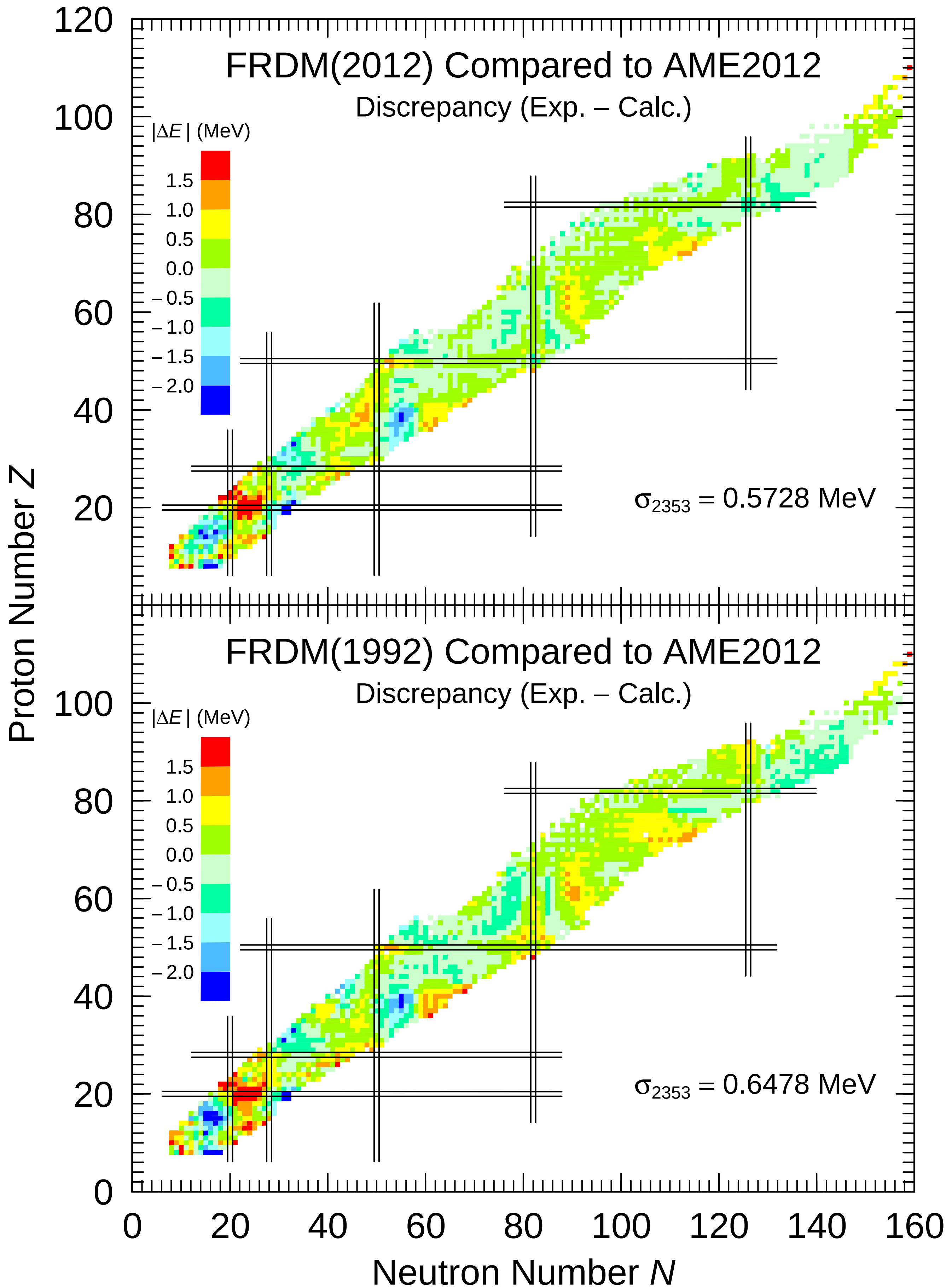


New Masses in AME2012 Relative to AME1989,
Compared to FRDM(2012) when Adjusted to AME1989
FRDM (2012-to89)
σ_{1654} = 0.6047 MeV
σ_{730} = 0.4948 MeV
μ_{730} = 0.0635 MeV
Mass Difference $M_{\rm exp} - M_{\rm th}$ (MeV)
Neutrons from β-stability
6
4
2
0
−2
−4
−6
−20
−15
−10
−5
0
5
10
15

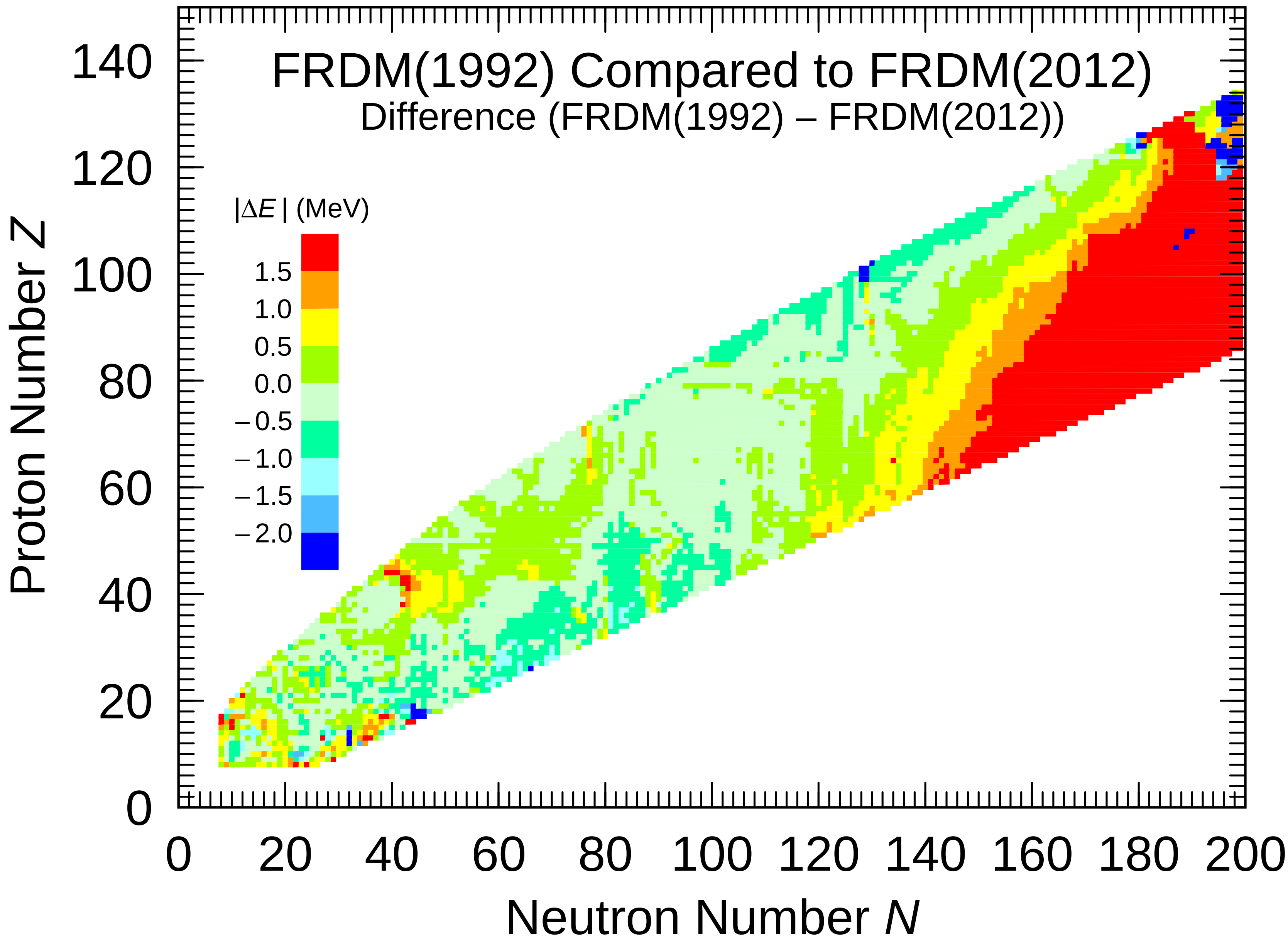


New Masses in AME2012 Evaluation,
Relative to 1989, Compared to Theory
FRDM (2012)
Mass Difference $M_{exp} - M_{th}$ (MeV)
Neutrons from β-stability
σ_{2149} = 0.5595 MeV
σ_{730} = 0.4788 MeV
μ_{730} = 0.0380 MeV
6
4
2
0
−2
−4
−6
−20
−15
−10
−5
0
5
10
15

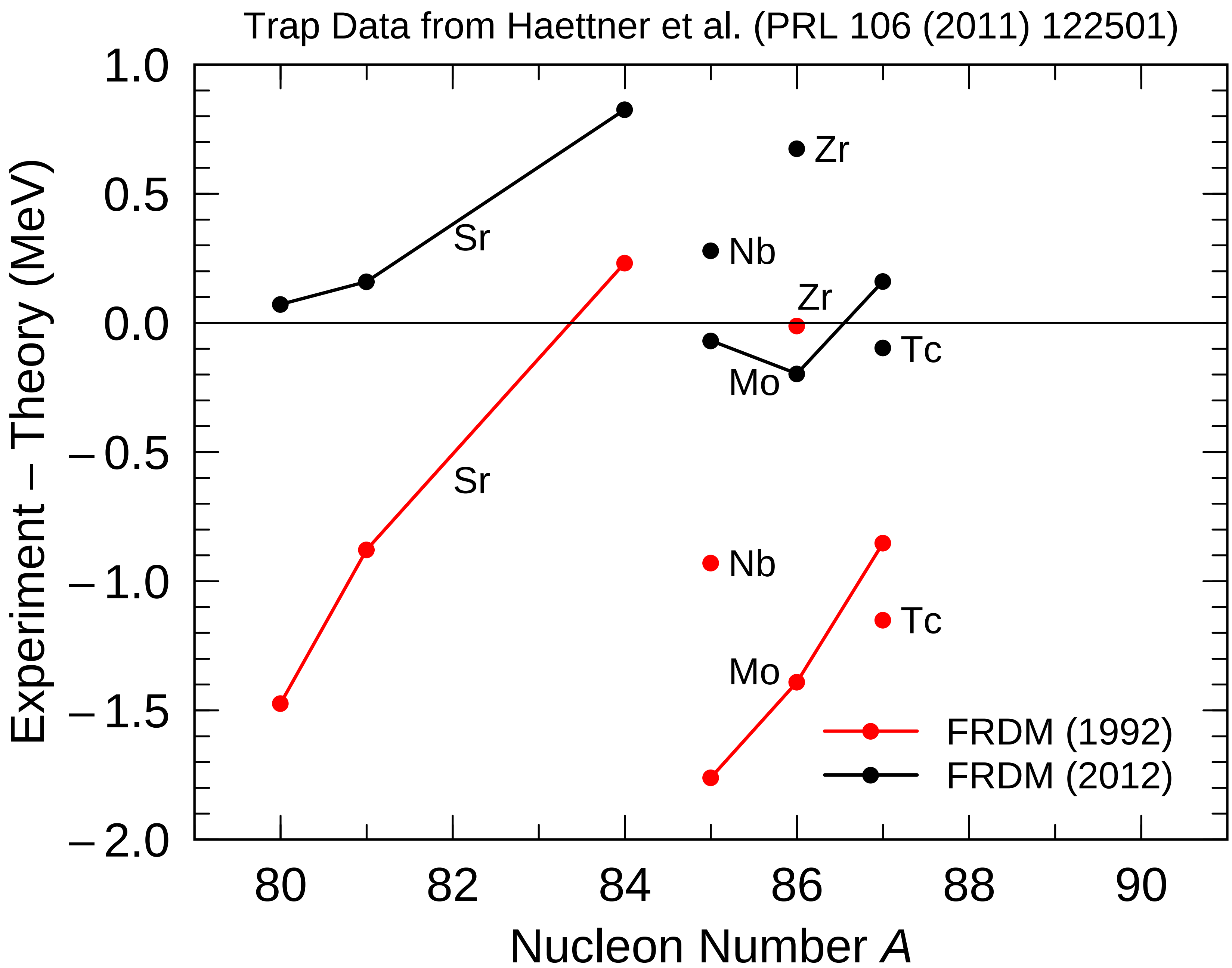

Experimental
FRDM (2012)
Calculated
Discrepancy (Exp. – Calc.)
σth = 0.559 MeV
Microscopic Energy (MeV)
Neutron Number N
10
0
10
0
10
0
–10
0
20
40
60
80
100
120
140
160

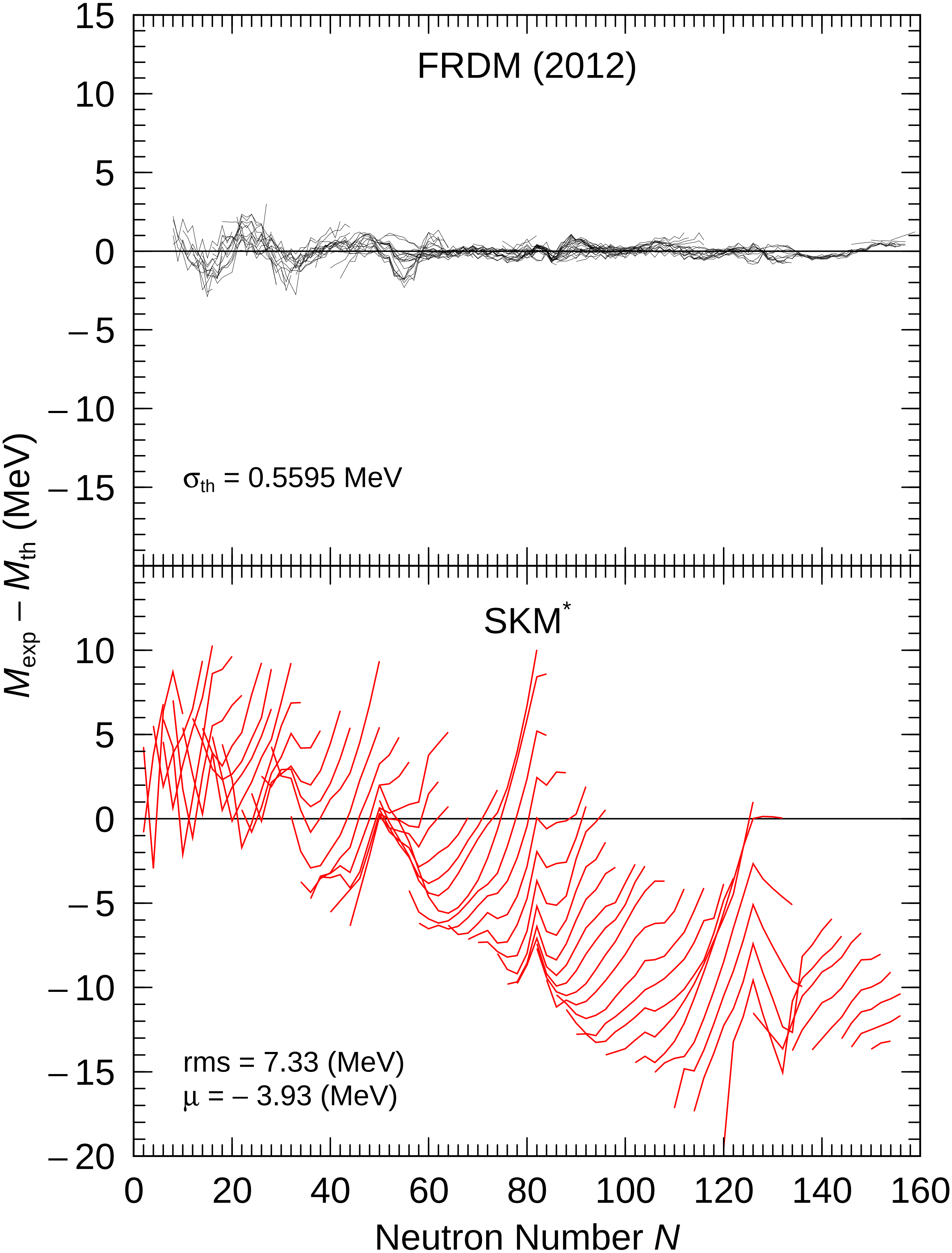


FRDM(2012) Compared to AME2012
Discrepancy (Exp. – Calc.)
|ΔE| (MeV)
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
σ2353 = 0.5728 MeV
FRDM(1992) Compared to AME2012
Discrepancy (Exp. – Calc.)
|ΔE| (MeV)
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
σ2353 = 0.6478 MeV
Proton Number Z
Neutron Number N
0
20
40
60
80
100
120
140
160



FRDM(1992) Compared to FRDM(2012)
Difference (FRDM(1992) – FRDM(2012))
|ΔE| (MeV)
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
Proton Number Z
Neutron Number N
0
20
40
60
80
100
120
140
160
180
200


Trap Data from Haettner et al. (PRL 106 (2011) 122501)
Experiment – Theory (MeV)
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
80
82
84
86
88
90
Nucleon Number A
Sr
Sr
Zr
Zr
Nb
Nb
Mo
Mo
Tc
Tc
FRDM (1992)
FRDM (2012)



FRDM (2012)
σth = 0.5595 MeV
SKM*
rms = 7.33 (MeV)
μ = – 3.93 (MeV)
Mexp – Mth (MeV)
Neutron Number N
15
10
5
0
–5
–10
–15
–20
0
20
40
60
80
100
120
140
160

TABLE II. Saturation properties of all Skyrme parametrizations used in this work. All entries are in MeV, except for the saturation density ρ_o in fm^{-3} and the dimensionless effective mass $m^* = M^*/M$.

Model	ρ_o	E_o	K_o	K'	J	L	K_{sym}	Q_{sym}	$K_{\tau,v}$	m^*
BSk1 [160]	0.157	−15.81	231.31	385.59	27.81	7.19	−281.83	606.46	−312.97	1.05
BSk2 [161]	0.157	−15.80	233.65	380.07	28.00	7.98	−296.98	557.91	−331.87	1.04
BSk2′ [161]	0.157	−15.79	233.32	380.86	28.00	7.79	−298.02	558.62	−332.04	1.05
BSk3 [162]	0.157	−15.81	234.81	380.83	27.93	6.78	−306.90	550.34	−336.59	1.12
BSk4 [163]	0.157	−15.77	236.84	367.17	28.00	12.54	−265.93	558.40	−321.74	0.92
BSk5 [163]	0.157	−15.80	237.19	367.86	28.70	21.41	−240.30	499.92	−335.56	0.92
BSk6 [163]	0.157	−15.75	229.14	370.64	28.00	16.84	−215.19	603.54	−289.01	0.80
BSk7 [163]	0.157	−15.76	229.26	370.92	28.00	17.99	−209.35	598.16	−288.18	0.80
BSk8 [164]	0.159	−15.83	230.31	372.39	28.00	14.85	−220.88	624.89	−285.98	0.80
BSk9 [165]	0.159	−15.92	231.32	374.67	30.00	38.29	−153.70	482.61	−321.44	0.80
BSk10 [166]	0.159	−15.91	238.83	370.34	30.00	37.24	−194.90	396.99	−360.60	0.92
BSk11 [166]	0.159	−15.86	238.09	369.18	30.00	38.36	−189.81	390.14	−360.48	0.92
BSk12 [166]	0.159	−15.86	238.06	369.11	30.00	38.01	−191.35	392.53	−360.47	0.92
BSk13 [166]	0.159	−15.86	238.09	369.17	30.00	38.82	−187.90	386.57	−360.65	0.92
BSk14 [167]	0.159	−15.85	239.33	358.67	30.00	43.91	−152.02	388.27	−349.68	0.80
BSk15 [168]	0.159	−16.04	241.56	363.14	30.00	33.60	−194.35	466.51	−345.43	0.80
BSk16 [169]	0.159	−16.05	241.67	363.58	30.00	34.88	−187.37	461.89	−344.17	0.80
BSk17 [170]	0.159	−16.06	241.69	363.62	30.00	36.29	−181.84	450.48	−344.97	0.80
BSk18 [52]	0.159	−16.06	241.79	363.82	30.00	36.22	−180.90	454.52	−343.71	0.80
BSk19 [130]	0.160	−16.08	237.33	297.89	30.00	31.90	−191.44	472.94	−342.79	0.80
BSk20 [130]	0.160	−16.08	241.39	282.26	30.00	37.38	−136.49	549.73	−317.05	0.80
BSk21 [130]	0.158	−16.05	245.80	274.09	30.00	46.56	−37.20	709.66	−264.62	0.80
E [171]	0.159	−16.13	333.46	63.72	27.66	−31.27	−570.73	448.61	−389.09	0.87
Es [171]	0.163	−16.02	248.60	352.41	26.44	−36.86	−457.76	880.01	−288.86	0.84
f_- [153]	0.162	−16.02	230.01	404.93	32.00	43.78	−105.08	654.90	−290.70	0.70
f_+ [153]	0.162	−16.04	230.01	406.17	32.00	41.54	−117.98	661.07	−293.85	0.70
f_0 [153]	0.162	−16.03	230.01	405.45	32.00	42.41	−113.41	657.36	−293.11	0.70
FPLyon [172]	0.162	−15.92	217.03	399.45	30.93	42.76	−135.60	485.79	−313.47	0.84
Gs [171]	0.158	−15.59	237.29	348.79	31.13	93.31	14.07	−26.92	−408.61	0.78
GS1 [154]	0.159	−16.03	235.09	812.19	28.86	50.22	−58.00	965.37	−185.83	0.60
GS2 [154]	0.159	−16.01	300.14	321.65	25.96	30.27	−188.78	467.72	−337.95	0.60
GS3 [154]	0.159	−16.00	399.91	−428.61	21.49	−0.44	−389.11	−293.19	−386.02	0.60
GS4 [154]	0.158	−15.96	235.15	846.53	12.83	−18.70	−161.42	1121.05	−116.53	0.80
GS5 [154]	0.158	−15.91	299.20	358.53	18.70	−12.14	−290.66	625.40	−232.35	0.80
GS6 [154]	0.159	−16.04	400.86	−383.63	14.33	−42.98	−492.21	−125.65	−193.21	0.80
GSkI [51]	0.159	−16.02	230.21	405.58	32.03	63.45	−95.29	293.44	−364.19	0.78
GSkII [51]	0.159	−16.12	233.40	398.73	30.49	48.63	−157.83	310.27	−366.54	0.79
KDE [173]	0.164	−15.99	223.90	381.81	31.97	41.42	−141.83	543.33	−319.71	0.76
KDE0v [173]	0.161	−16.10	228.71	373.39	32.98	45.21	−144.78	523.27	−342.24	0.72
KDE0v1 [173]	0.165	−16.23	227.54	384.86	34.58	54.69	−127.12	484.45	−362.78	0.74
LNS [118]	0.175	−15.32	210.78	382.55	33.43	61.45	−127.36	302.46	−384.55	0.83
MSk1 [174]	0.157	−15.83	233.73	379.97	30.00	33.92	−200.02	448.66	−348.39	1.00
MSk2 [174]	0.157	−15.83	231.65	386.21	30.00	33.35	−203.44	449.71	−347.94	1.05
MSk3 [174]	0.157	−15.79	233.25	379.01	28.00	7.04	−283.52	615.65	−314.33	1.00
MSk4 [174]	0.157	−15.79	231.17	385.26	28.00	7.20	−284.05	610.93	−315.24	1.05
MSk5 [174]	0.157	−15.79	231.17	385.26	28.00	7.57	−282.55	607.93	−315.36	1.05
MSk5* [119]	0.156	−15.78	243.74	346.15	28.00	7.02	−290.66	595.12	−322.81	0.80
MSk6 [174]	0.157	−15.79	231.17	385.26	28.00	9.63	−274.33	591.49	−316.05	1.05
MSk7 [175]	0.157	−15.80	231.22	385.36	27.95	9.40	−274.63	592.08	−315.38	1.05
MSk8 [175]	0.157	−15.80	229.31	391.01	27.93	8.26	−280.01	597.59	−315.49	1.10
MSk9 [175]	0.157	−15.80	233.33	379.16	28.00	10.36	−270.23	589.06	−315.57	1.00
MSkA [176]	0.153	−15.99	313.33	138.15	30.35	57.17	−135.34	197.74	−453.13	0.79
MSL0 [101]	0.160	−16.00	230.00	380.32	30.00	60.00	−99.33	224.29	−360.11	0.80
NRAPR [177]	0.161	−15.85	225.65	362.54	32.78	59.63	−123.32	311.61	−385.32	0.69
PRC45 [178]	0.145	−15.82	367.58	−165.69	51.01	141.52	−23.01	92.05	−935.89	1.00

TABLE II. (*Continued.*)

Model	ρ_o	E_o	K_o	K'	J	L	K_{sym}	Q_{sym}	$K_{\tau,v}$	m^*
RATP [179]	0.160	−16.05	239.52	349.83	29.26	32.39	−191.23	440.70	−338.28	0.67
Rs [171]	0.158	−15.59	237.42	348.46	30.82	86.39	−9.21	22.41	−400.74	0.78
Sefm068 [180]	0.160	−15.92	240.11	347.11	88.57	254.43	−32.10	59.40	−1190.85	0.68
Sefm074 [180]	0.160	−15.81	240.10	350.15	33.40	88.73	−33.14	58.41	−436.12	0.74
Sefm081 [180]	0.161	−15.69	237.04	356.66	30.76	79.39	−39.54	66.74	−396.41	0.81
Sefm09 [180]	0.161	−15.55	240.06	349.75	27.78	69.96	−40.80	70.63	−358.63	0.90
Sefm1 [180]	0.161	−15.40	240.07	346.34	24.81	59.55	−46.89	81.53	−318.28	1.00
SGI [181]	0.154	−15.89	261.75	297.93	28.33	63.86	−51.99	194.46	−362.49	0.61
SGII [181]	0.158	−15.60	214.65	380.91	26.83	37.63	−145.90	330.41	−304.90	0.79
SGOI [182]	0.168	−16.63	361.59	−37.36	45.20	99.76	−155.64	144.36	−764.53	0.61
SGOII [182]	0.168	−16.70	253.28	346.18	93.98	246.02	−119.57	272.39	−1259.44	0.61
SI [27]	0.155	−15.99	370.38	−152.32	29.24	1.22	−461.84	141.44	−469.66	0.91
SII [27]	0.148	−15.99	341.40	−15.76	34.16	50.02	−265.72	104.75	−568.17	0.58
SIII [183]	0.145	−15.85	355.37	−101.38	28.16	9.91	−393.73	130.45	−456.01	0.76
SIII* [184]	0.148	−16.07	361.15	−107.94	31.97	28.70	−358.37	84.84	−539.13	0.78
SIV [183]	0.151	−15.96	324.55	68.84	31.22	63.50	−136.72	79.45	−504.22	0.47
Sk1′ [185]	0.155	−15.99	370.38	−152.32	29.35	35.34	−259.16	141.44	−485.71	0.91
SK255 [68]	0.157	−16.33	254.93	350.09	37.40	95.05	−58.33	94.23	−498.11	0.80
SK272 [68]	0.155	−16.28	271.51	305.31	37.40	91.67	−67.78	134.36	−514.70	0.77
SkA [186]	0.155	−15.99	263.16	300.13	32.91	74.62	−78.46	174.54	−441.08	0.61
Ska25s20 [187]	0.161	−16.07	220.75	413.45	33.78	63.81	−118.22	314.13	−381.56	0.98
Ska35s15 [187]	0.158	−16.01	238.89	378.88	30.56	30.60	−222.90	481.99	−357.96	1.01
Ska35s20 [187]	0.158	−16.08	240.27	378.65	33.57	64.83	−120.32	284.54	−407.11	1.00
Ska35s25 [187]	0.158	−16.14	241.30	378.94	36.98	98.89	−23.57	97.46	−461.60	0.99
Ska45s20 [187]	0.156	−16.08	260.21	330.55	33.39	66.21	−119.99	251.77	−433.13	1.02
SkB [186]	0.155	−15.99	263.16	300.13	23.88	47.54	−78.46	174.54	−309.50	0.61
SkI1 [188]	0.160	−15.95	242.75	346.14	37.53	161.05	234.67	−328.02	−502.01	0.69
SkI2 [188]	0.158	−15.78	240.93	339.70	33.37	104.33	70.69	51.62	−408.21	0.68
SkI3 [188]	0.158	−15.98	258.19	303.86	34.83	100.53	73.04	211.54	−411.80	0.58
SkI4 [188]	0.160	−15.95	247.95	331.21	29.50	60.39	−40.56	351.16	−322.23	0.65
SkI5 [188]	0.156	−15.85	255.79	301.95	36.64	129.33	159.57	11.73	−463.74	0.58
SkI6 [189]	0.159	−15.89	248.17	326.58	29.90	59.24	−46.77	378.12	−324.26	0.64
SkM [122]	0.160	−15.77	216.61	386.09	30.75	49.34	−148.81	323.34	−356.91	0.79
SkM* [190]	0.160	−15.77	216.61	386.09	30.03	45.78	−155.94	330.47	−349.00	0.79
SkM1 [191]	0.160	−15.77	216.61	386.09	25.17	−35.37	−388.89	912.87	−239.72	0.79
SkMP [192]	0.157	−15.56	230.87	338.05	29.89	70.31	−49.82	159.44	−368.73	0.65
SkO [193]	0.160	−15.84	223.34	392.86	31.97	79.14	−43.17	131.13	−378.80	0.90
SkO′ [193]	0.160	−15.75	222.36	390.83	31.95	68.94	−78.82	223.37	−371.29	0.90
SkP [194]	0.163	−15.95	200.97	435.43	30.00	19.68	−266.60	508.35	−342.04	1.00
SKRA [195]	0.159	−15.78	216.98	378.76	31.32	53.04	−139.28	310.84	−364.92	0.75
SkS1 [196]	0.161	−15.86	228.43	382.76	28.75	30.52	−218.69	379.24	−350.66	0.86
SkS2 [196]	0.161	−15.89	229.02	382.73	29.23	37.84	−218.07	270.03	−381.86	0.85
SkS3 [196]	0.161	−15.88	228.83	382.62	28.84	51.74	−157.38	154.06	−381.30	0.85
SkS4 [196]	0.163	−15.88	228.08	385.45	28.35	23.28	−238.42	438.06	−338.77	0.87
SkSC1 [197]	0.161	−15.85	234.58	380.50	28.10	0.13	−312.03	673.32	−312.62	1.00
SkSC2 [197]	0.161	−15.90	235.13	381.60	24.74	11.00	−228.22	505.69	−276.35	1.00
SkSC3 [197]	0.161	−15.85	234.49	380.32	27.01	0.81	−296.20	641.65	−299.75	1.00
SkSC4 [198]	0.161	−15.87	234.72	380.79	28.80	−2.12	−329.49	708.23	−320.20	1.00
SkSC4o [199]	0.161	−15.87	234.74	380.79	27.00	−9.67	−338.03	725.33	−295.70	1.00
SkSC5 [200]	0.161	−15.85	234.50	380.34	30.99	−6.97	−375.08	799.41	−344.58	1.00
SkSC6 [200]	0.161	−15.92	235.41	382.13	24.57	11.00	−226.26	501.80	−274.39	1.00
SkSC10 [200]	0.161	−15.96	235.89	383.08	22.83	19.13	−172.77	394.81	−256.47	1.00
SkSC11 [201]	0.161	−15.87	234.72	380.79	28.80	−2.12	−329.49	708.23	−320.20	1.00
SkSC14 [199]	0.161	−15.92	235.41	382.13	30.00	33.13	−202.83	454.93	−347.84	1.00
SkSC15 [199]	0.161	−15.88	234.93	381.17	28.00	6.72	−284.47	618.21	−313.89	1.00
SkSP.1 [119]	0.162	−15.90	230.02	502.64	28.00	7.17	−289.55	662.66	−316.92	0.80
SkT [202]	0.148	−15.40	333.36	−29.01	33.66	80.83	−78.93	69.87	−570.95	0.60
SkT1 [113]	0.161	−15.98	236.16	383.52	32.02	56.18	−134.83	318.99	−380.68	1.00

TABLE II. (*Continued.*)

Model	ρ_o	E_o	K_o	K'	J	L	K_{sym}	Q_{sym}	$K_{\tau,v}$	m^*
SkT2 [113]	0.161	−15.94	235.73	382.67	32.00	56.16	−134.67	318.66	−380.48	1.00
SkT3 [113]	0.161	−15.95	235.74	382.70	31.50	55.31	−132.05	313.43	−374.14	1.00
SkT4 [113]	0.159	−15.96	235.50	382.94	35.24	93.49	−24.46	97.84	−433.36	1.00
SkT5 [113]	0.164	−16.00	201.69	436.81	37.00	98.53	−24.97	99.88	−402.76	1.00
SkT6 [113]	0.161	−15.96	235.95	383.15	29.97	30.85	−211.53	472.36	−346.54	1.00
SkT7 [113]	0.161	−15.94	235.64	372.22	29.52	31.12	−209.85	439.35	−347.42	0.83
SkT8 [113]	0.161	−15.94	235.70	372.37	29.92	33.72	−187.52	476.25	−336.59	0.83
SkT9 [113]	0.160	−15.88	234.91	370.97	29.76	33.74	−185.62	471.98	−334.76	0.83
SkT1* [113]	0.162	−16.20	238.95	388.75	32.31	56.58	−136.66	322.86	−384.07	1.00
SkT3* [113]	0.162	−16.20	238.95	388.76	31.97	56.32	−133.65	316.82	−379.93	1.00
SkT1a [180]	0.161	−15.98	236.16	383.52	32.02	56.18	−134.83	318.99	−380.68	1.00
SkT2a [180]	0.161	−15.94	235.73	382.67	32.00	56.16	−134.67	318.66	−380.48	1.00
SkT3a [180]	0.161	−15.95	235.74	382.70	31.50	55.31	−132.05	313.43	−374.14	1.00
SkT4a [180]	0.159	−15.96	235.50	382.94	35.45	94.13	−24.46	97.84	−436.19	1.00
SkT5a [180]	0.164	−16.00	201.69	436.81	37.00	98.53	−24.97	99.88	−402.76	1.00
SkT6a [180]	0.161	−15.96	235.95	383.15	29.97	30.85	−211.53	472.36	−346.54	1.00
SkT7a [180]	0.161	−15.94	235.64	372.22	29.52	31.12	−209.85	439.35	−347.42	0.83
SkT8a [180]	0.161	−15.94	235.70	372.37	29.92	33.72	−187.52	476.25	−336.59	0.83
SkT9a [180]	0.160	−15.88	234.91	370.97	29.76	33.74	−185.62	471.98	−334.76	0.83
SkTK [203]	0.168	−16.70	253.28	346.18	35.57	41.59	−221.79	527.94	−414.46	0.61
SKX [204]	0.155	−16.05	271.06	297.42	31.10	33.18	−252.12	379.69	−414.81	0.99
SKXce [204]	0.155	−15.86	268.19	294.59	30.15	33.48	−238.39	356.93	−402.51	1.01
SKXm [204]	0.159	−16.04	238.09	380.38	31.20	32.08	−242.76	428.73	−384.00	0.97
Skxs15 [205]	0.161	−15.76	201.10	424.57	31.88	34.79	−197.10	516.30	−332.38	0.97
Skxs20 [205]	0.162	−15.81	201.95	425.56	35.50	67.06	−122.31	328.52	−383.37	0.96
Skxs25 [205]	0.161	−15.87	202.92	−440.88	39.60	100.10	−50.28	145.99	−440.88	0.96
Skz-1 [128]	0.160	−16.01	230.08	365.25	32.00	54.14	−184.08	217.03	−422.99	0.70
Skz0 [128]	0.160	−16.01	230.08	365.24	32.00	35.10	−242.20	405.16	−397.08	0.70
Skz1 [128]	0.160	−16.01	230.08	365.25	32.01	27.67	−242.40	535.38	−364.50	0.70
Skz2 [128]	0.160	−16.01	230.07	365.23	32.01	16.81	−259.66	682.63	−333.83	0.70
Skz3 [128]	0.160	−16.01	230.09	365.26	32.01	12.96	−241.91	794.95	−299.08	0.70
Skz4 [128]	0.160	−16.01	230.08	365.26	32.01	5.75	−240.86	923.89	−266.24	0.70
SLy0 [206]	0.160	−15.97	229.66	364.01	31.98	47.11	−116.23	508.68	−324.23	0.70
SLy1 [206]	0.160	−15.99	229.81	364.35	31.99	47.07	−116.49	509.36	−324.27	0.70
SLy2 [206]	0.161	−15.99	229.92	364.21	32.00	47.46	−115.13	506.52	−324.69	0.70
SLy230a [45]	0.160	−15.99	229.89	364.18	31.99	44.32	−98.22	602.87	−293.91	0.70
SLy230b [45]	0.160	−15.97	229.91	363.10	32.01	45.97	−119.72	521.50	−322.92	0.69
SLy3 [206]	0.160	−15.94	229.51	362.56	31.97	45.36	−121.90	524.75	−322.39	0.70
SLy4 [207]	0.160	−15.97	229.91	363.11	32.00	45.94	−119.73	521.53	−322.83	0.69
SLy5 [207]	0.161	−15.99	229.92	364.16	32.01	48.15	−112.76	500.67	−325.38	0.70
SLy6 [207]	0.159	−15.92	229.86	360.24	31.96	47.45	−112.71	510.63	−323.03	0.69
SLy7 [207]	0.158	−15.90	229.75	359.22	31.99	46.94	−114.34	517.14	−322.60	0.69
SLy8 [206]	0.160	−15.97	229.89	363.27	32.00	47.18	−115.59	509.88	−324.09	0.70
SLy9 [206]	0.151	−15.80	229.84	350.42	31.98	54.86	−81.42	462.35	−326.92	0.67
SLy10 [207]	0.156	−15.90	229.68	358.32	31.90	38.51	−142.18	591.23	−313.17	0.68
SQMC1 [156]	0.137	−14.00	328.76	−143.78	29.68	−6.70	−504.25	218.08	−461.10	0.93
SQMC2 [156]	0.140	−14.29	330.10	−121.75	28.70	8.67	−408.41	145.55	−463.63	0.83
SQMC3 [156]	0.161	−15.98	366.97	−130.22	45.78	91.80	−210.95	163.48	−794.33	0.82
SQMC600 [157]	0.174	−15.74	217.00	388.62	34.38	46.38	−215.16	396.85	−410.40	0.81
SQMC650 [157]	0.172	−15.57	218.11	376.75	33.65	52.92	−173.15	349.74	−399.28	0.78
SQMC700 [157]	0.171	−15.49	222.20	369.94	33.47	59.06	−140.84	313.84	−396.85	0.76
SQMC750 [157]	0.171	−15.60	222.86	365.83	33.75	64.67	−117.51	288.41	−399.38	0.74
SSk [51]	0.161	−16.16	229.31	375.38	33.50	52.78	−119.15	482.24	−349.42	0.72
SV [183]	0.155	−16.05	305.70	175.78	32.82	96.09	24.17	48.00	−497.11	0.38
SV-bas [115]	0.160	−15.91	233.45	379.28	30.00	32.37	−221.75	410.93	−363.36	0.90
SV-min [115]	0.161	−15.91	221.76	403.08	30.66	44.81	−156.57	389.56	−343.99	0.95
SVI [183]	0.143	−15.76	363.64	−153.50	26.88	−7.34	−471.30	146.04	−424.18	0.95
SVII [184]	0.143	−15.79	366.44	−164.51	26.96	−10.16	−488.90	149.74	−423.36	1.00

TABLE II. (*Continued.*)

Model	ρ_o	E_o	K_o	K'	J	L	K_{sym}	Q_{sym}	$K_{\tau,v}$	m^*
SV-K218 [115]	0.161	−15.90	218.23	403.15	30.00	34.62	−206.87	401.58	−350.65	0.90
SV-K226 [115]	0.160	−15.90	225.82	392.14	30.00	34.09	−211.92	401.84	−357.27	0.90
SV-K241 [115]	0.159	−15.91	241.07	364.54	30.00	30.95	−230.77	416.01	−369.66	0.90
SV-kap00 [115]	0.160	−15.90	233.44	379.15	30.00	39.44	−161.78	446.94	−334.34	0.90
SV-kap02 [115]	0.160	−15.90	233.44	379.21	30.00	35.54	−193.19	431.91	−348.69	0.90
SV-kap06 [115]	0.160	−15.91	233.45	379.33	30.00	29.33	−249.75	388.84	−378.10	0.90
SV-mas07 [115]	0.160	−15.89	233.54	356.93	30.00	52.15	−98.77	365.68	−331.96	0.70
SV-mas08 [115]	0.160	−15.90	233.13	371.28	30.00	40.15	−172.38	397.44	−349.35	0.80
SV-mas10 [115]	0.159	−15.91	234.33	383.22	30.00	28.04	−252.50	408.07	−374.87	1.00
SV-sym28 [115]	0.163	−16.47	240.86	392.55	28.47	6.29	−305.94	584.47	−333.41	0.90
SV-sym32 [115]	0.159	−15.94	233.81	380.11	32.00	57.07	−148.79	257.70	−398.44	0.90
SV-sym34 [115]	0.159	−15.97	234.07	380.82	34.00	80.95	−79.08	111.28	−433.08	0.90
SV-tls [115]	0.160	−15.89	233.30	379.03	30.00	33.22	−218.42	403.90	−363.79	0.90
T [171]	0.161	−15.93	235.66	382.44	28.35	27.18	−206.76	462.91	−325.76	1.00
T11 [152]	0.161	−16.01	230.01	365.75	32.00	49.46	−108.76	486.98	−326.88	0.70
T12 [152]	0.161	−16.00	230.01	365.11	32.00	49.38	−108.75	488.50	−326.63	0.70
T13 [152]	0.161	−16.00	230.01	364.78	32.00	49.53	−108.06	487.57	−326.69	0.70
T14 [152]	0.161	−15.99	230.01	364.48	32.00	49.48	−108.12	488.35	−326.57	0.70
T15 [152]	0.161	−16.01	230.01	365.32	32.00	49.65	−107.91	485.83	−326.95	0.70
T16 [152]	0.161	−16.01	230.01	365.68	32.00	49.45	−108.75	487.24	−326.83	0.70
T21 [152]	0.161	−16.03	230.01	366.49	32.00	49.77	−108.03	483.25	−327.37	0.70
T22 [152]	0.161	−16.02	230.01	365.95	32.00	49.57	−108.50	485.74	−327.04	0.70
T23 [152]	0.161	−16.01	230.01	365.63	32.00	49.59	−108.27	485.95	−326.97	0.70
T24 [152]	0.161	−16.01	230.01	365.37	32.00	49.85	−107.22	484.00	−327.14	0.70
T25 [152]	0.161	−15.99	230.01	364.24	32.00	49.12	−109.21	491.85	−326.16	0.70
T26 [152]	0.161	−15.98	230.01	363.48	32.00	48.76	−110.15	495.92	−325.64	0.70
T31 [152]	0.161	−16.02	230.01	366.28	32.00	49.75	−108.00	483.82	−327.27	0.70
T32 [152]	0.161	−16.03	230.01	366.39	32.00	50.28	−106.20	478.97	−327.80	0.70
T33 [152]	0.161	−16.02	230.01	366.10	32.00	49.66	−108.23	484.88	−327.13	0.70
T34 [152]	0.161	−16.02	230.01	366.28	32.00	50.10	−106.81	480.71	−327.60	0.70
T35 [152]	0.161	−16.00	230.01	364.84	32.00	49.59	−107.85	487.05	−326.74	0.70
T36 [152]	0.161	−15.99	230.01	364.51	32.00	49.05	−109.62	491.98	−326.20	0.70
T41 [152]	0.162	−16.06	230.01	368.36	32.00	50.60	−106.02	473.67	−328.60	0.70
T42 [152]	0.162	−16.05	230.01	368.04	32.00	50.70	−105.51	473.28	−328.59	0.70
T43 [152]	0.162	−16.04	230.01	367.39	32.00	50.57	−105.66	475.23	−328.31	0.70
T44 [152]	0.161	−16.02	230.01	365.91	32.00	50.05	−106.76	481.62	−327.45	0.70
T45 [152]	0.161	−16.02	230.01	366.10	32.00	49.66	−108.24	484.73	−327.16	0.70
T46 [152]	0.161	−16.00	230.01	364.75	32.00	49.93	−106.59	484.25	−327.00	0.70
T51 [152]	0.162	−16.05	230.01	367.96	32.00	50.69	−105.52	473.55	−328.55	0.70
T52 [152]	0.161	−16.06	230.01	368.07	32.00	50.68	−105.55	473.55	−328.55	0.70
T53 [152]	0.161	−16.02	230.01	366.21	32.00	50.03	−106.99	481.50	−327.50	0.70
T54 [152]	0.161	−16.03	230.01	366.73	32.00	50.27	−106.36	478.71	−327.85	0.70
T55 [152]	0.161	−16.03	230.01	366.66	32.00	50.24	−106.49	479.02	−327.83	0.70
T56 [152]	0.161	−16.01	230.01	365.26	32.00	50.13	−106.19	481.83	−327.34	0.70
T61 [152]	0.162	−16.07	230.01	368.76	32.00	50.79	−105.56	471.67	−328.85	0.71
T62 [152]	0.162	−16.07	230.01	368.93	32.00	50.33	−107.25	475.46	−328.49	0.71
T63 [152]	0.162	−16.06	230.01	368.30	32.00	51.07	−104.36	469.72	−329.00	0.70
T64 [152]	0.162	−16.03	230.01	366.74	32.00	50.49	−105.65	476.73	−328.08	0.70
T65 [152]	0.162	−16.04	230.01	367.37	32.00	50.50	−105.90	475.82	−328.25	0.70
T66 [152]	0.161	−16.02	230.01	366.04	32.00	50.30	−105.96	479.25	−327.72	0.70
v070 [208]	0.157	−15.78	230.99	384.93	27.98	−3.45	−361.15	591.72	−346.20	1.05
v075 [208]	0.157	−15.80	231.29	385.51	28.00	−0.31	−341.88	587.67	−340.52	1.05
v080 [208]	0.157	−15.79	231.17	385.26	28.00	2.23	−325.61	585.53	−335.29	1.05
v090 [208]	0.157	−15.79	231.17	385.26	28.00	5.04	−304.26	593.46	−326.10	1.05
v100 [208]	0.157	−15.79	231.17	385.26	28.00	8.73	−281.39	588.25	−319.22	1.05
v105 [208]	0.157	−15.79	231.17	385.26	28.00	7.08	−284.51	611.85	−315.20	1.05
v110 [208]	0.157	−15.79	231.17	385.26	28.00	7.51	−279.62	617.86	−312.19	1.05
Z [171]	0.159	−15.97	330.30	64.98	26.82	−49.70	−657.85	495.24	−369.43	0.84

A LIST OF SOME MASS MODELS

Myers-Swiatecki (1966-1967)
Seeger-Howard (1974)
Hilf-vonGroote (1976)
Garvey-Kelson type
Liran-Zeldes (1976)
Spanier and Johansson (1988)
Neural Net (≈ 1990)
FDSM (Fermion Dynamical Symmetry Model, 1990)
ETFSI-1 (1992)
Myers-Swiatecki (1994) (“TF”)
Audi systematics (?)
Duflo-Zucker (1995)
Lublin-Strassbourg
Koura (2000)
Goriely HFB01–HFB27 (2000–now)
“Chinese Mass Model” (2010)

“Chinese mass Model”, Wang et al

(e.g. PRC **81** (2010) 044322
84 (2011) 014333

$$E(A,Z,\beta) = E_{\mathsf{LD}}(A,Z)\prod_{k\geq 2}(1+b_k\beta_k{}^1)+\Delta E(A,Z,\beta)$$

with

$$E_{\mathsf{LD}}(A,Z) = a_{\mathsf{v}}+a_{\mathsf{s}}A^{2/3}+E_{\mathsf{C}}+a_{\mathsf{sym}}I^2+\ldots \quad (1)$$

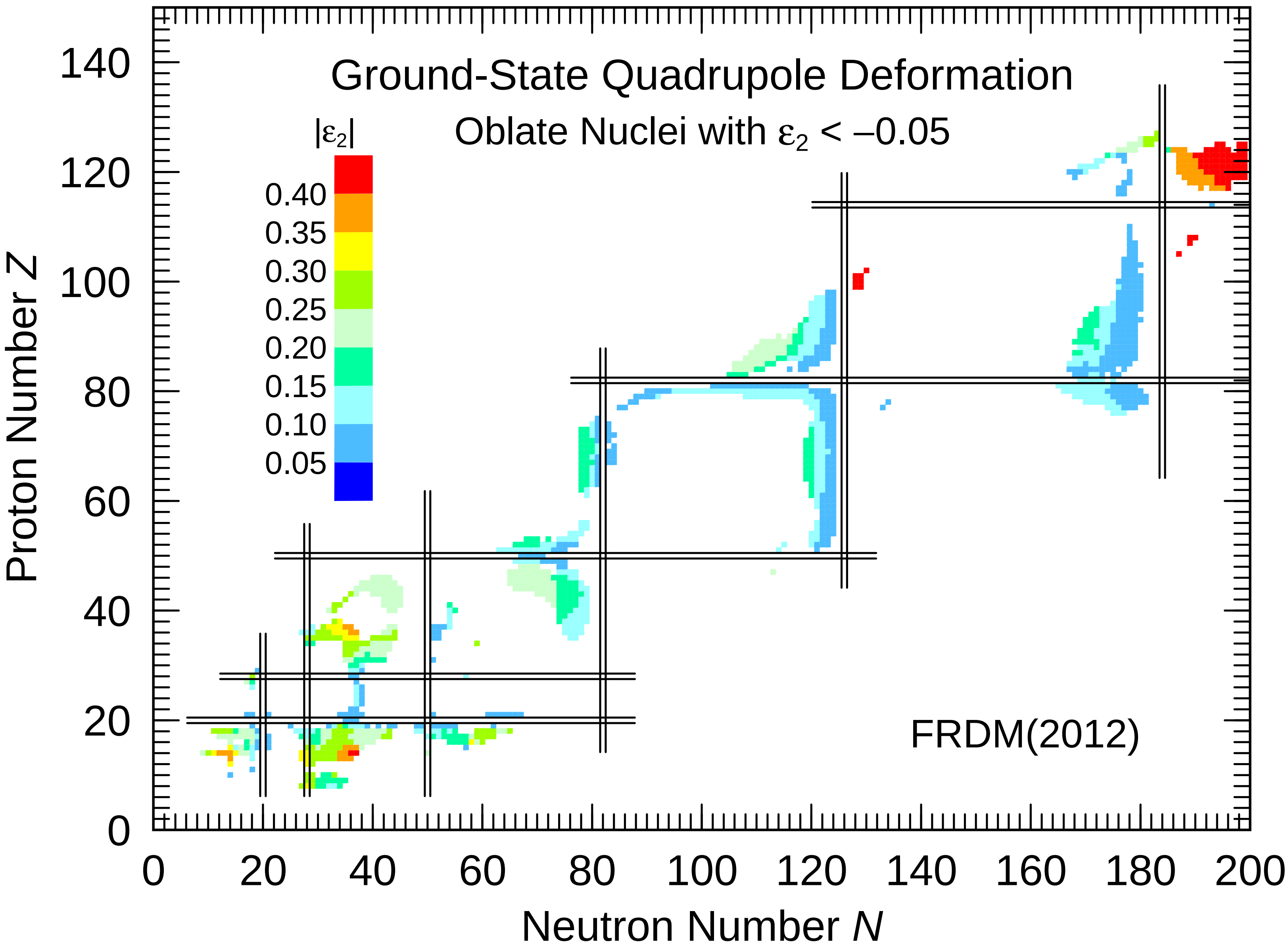
HFB (Goriely)“phenomenological” corrections:

WIGNER:

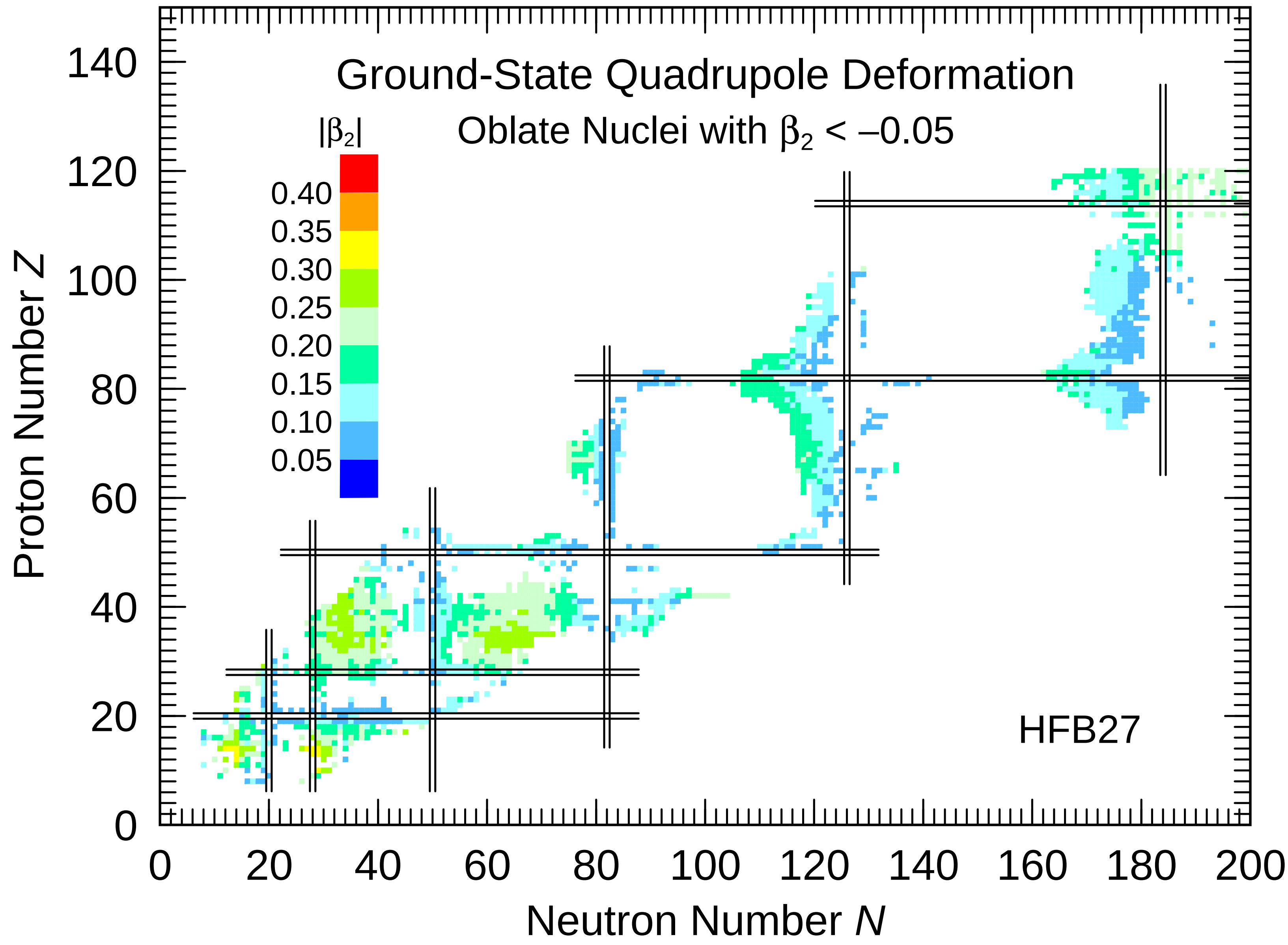
$$E_{\mathrm{W}} = V_{\mathrm{W}} \exp\left[-\lambda\left(\frac{N-Z}{A}\right)^2\right] + V_{\mathrm{W}}'|N-Z| \exp\left[-\left(\frac{A}{A_0}\right)^2\right]$$

spurious collective energy:

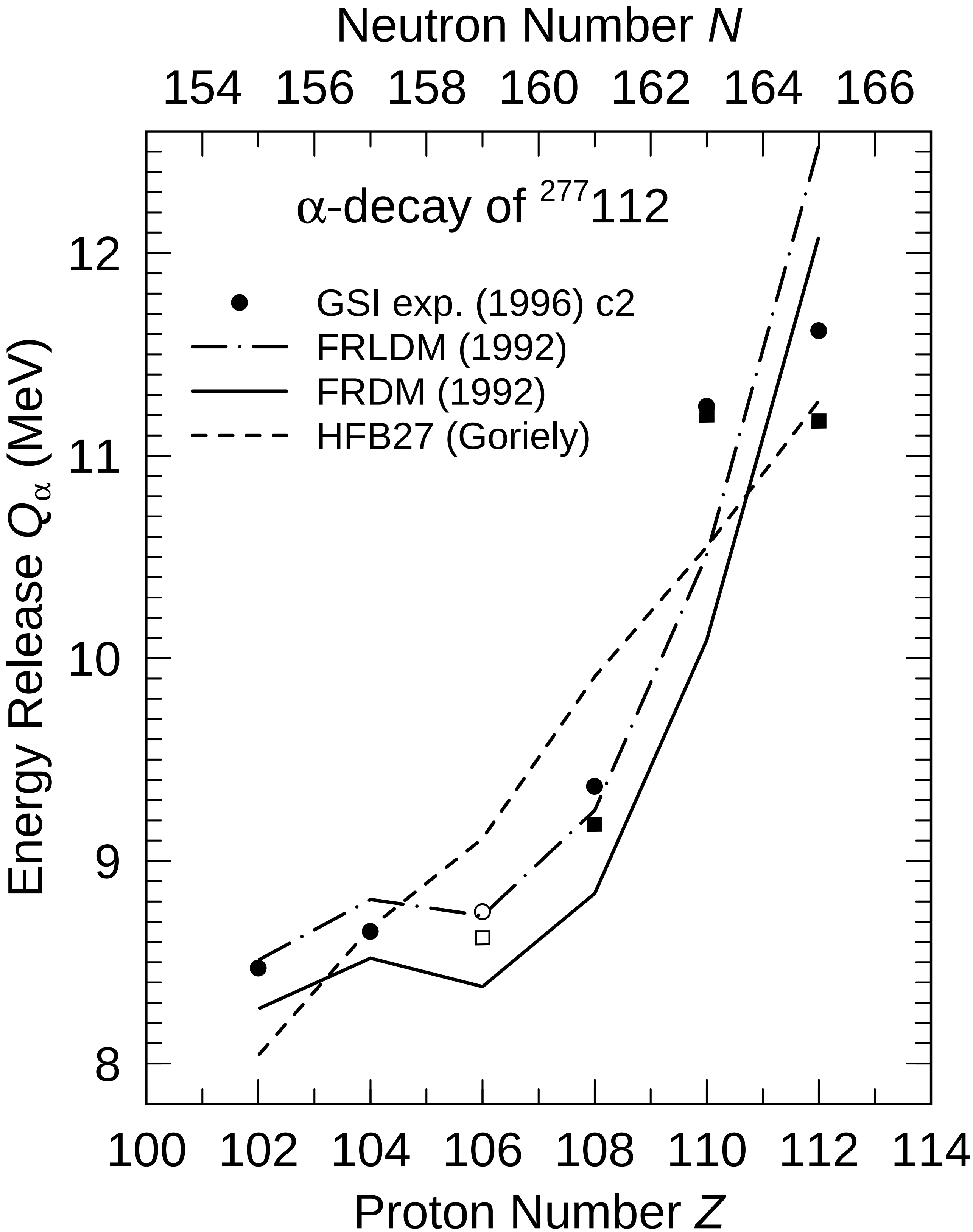
$$E_{\mathrm{coll}} = E_{\mathrm{rot}}^{\mathrm{crank}}\left[b \tanh(c|\beta_2|) + d|\beta_2| \exp\left(-l(|\beta_2| - \beta_2^0)^2\right)\right]$$



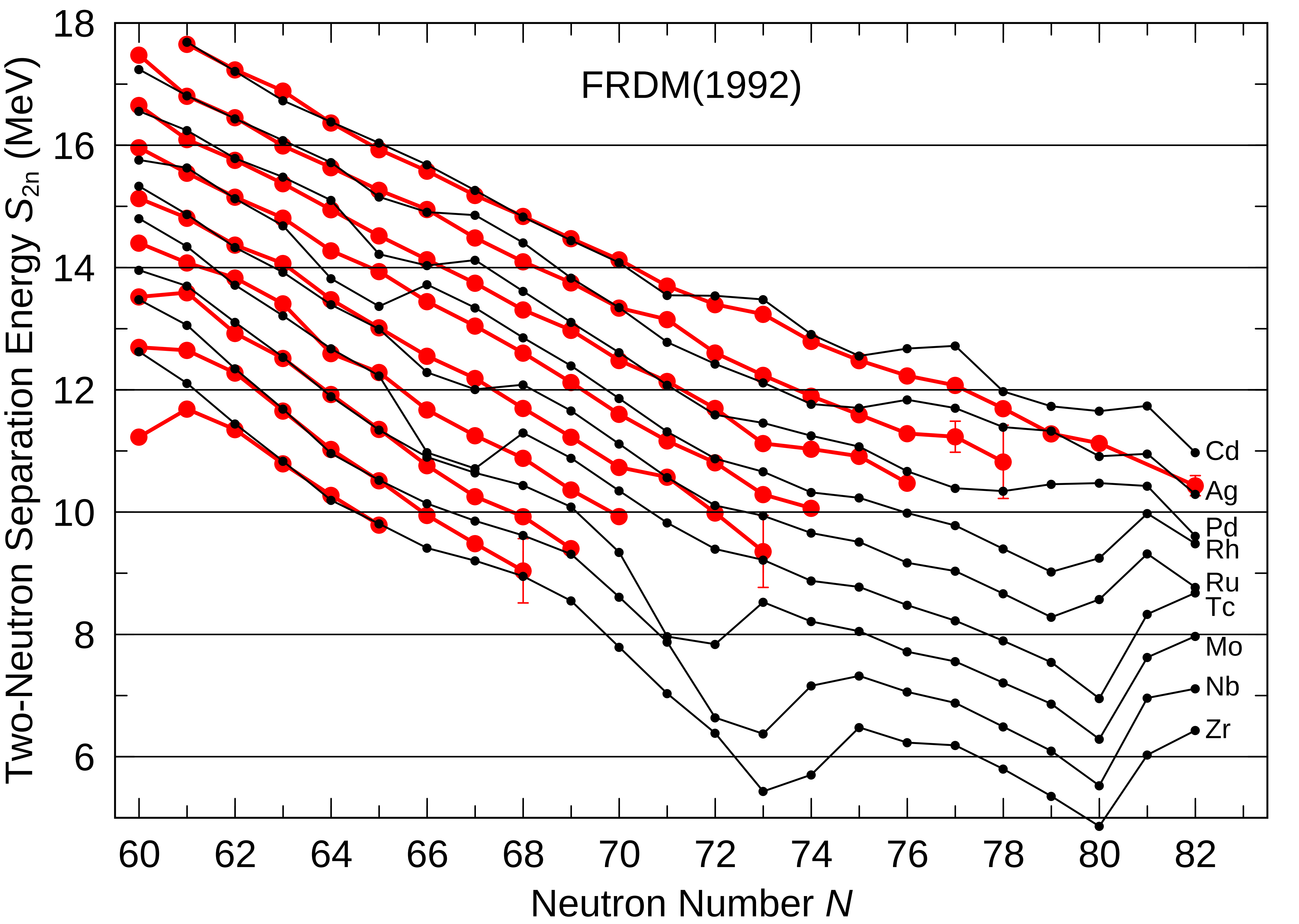
Ground-State Quadrupole Deformation
Oblate Nuclei with ε2 < –0.05
|ε2|
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
FRDM(2012)
Proton Number Z
Neutron Number N
0
20
40
60
80
100
120
140
160
180
200



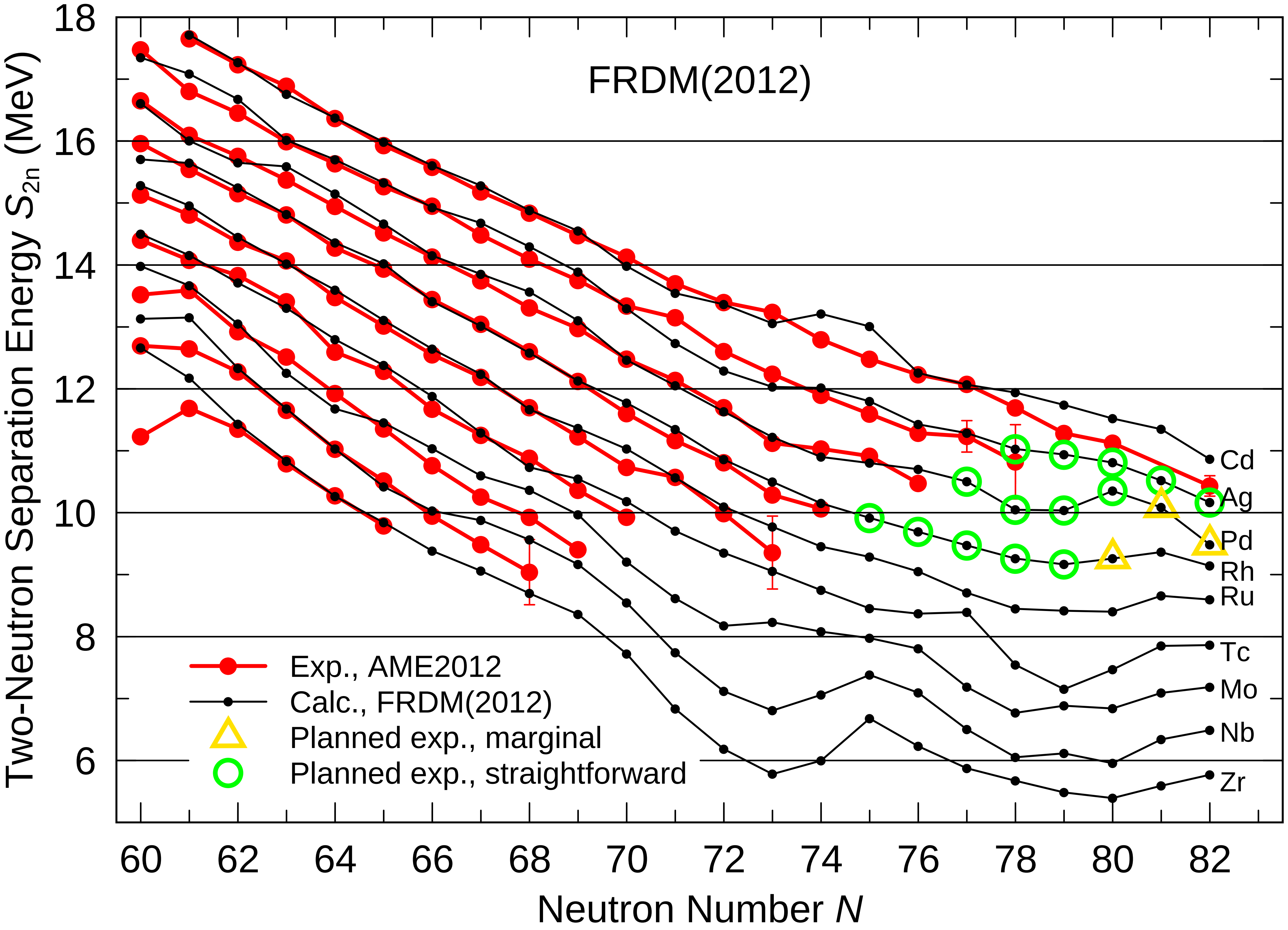
Ground-State Quadrupole Deformation
Oblate Nuclei with $\beta_2 < -0.05$
$|\beta_2|$
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
HFB27
Proton Number Z
Neutron Number N
0
20
40
60
80
100
120
140
160
180
200



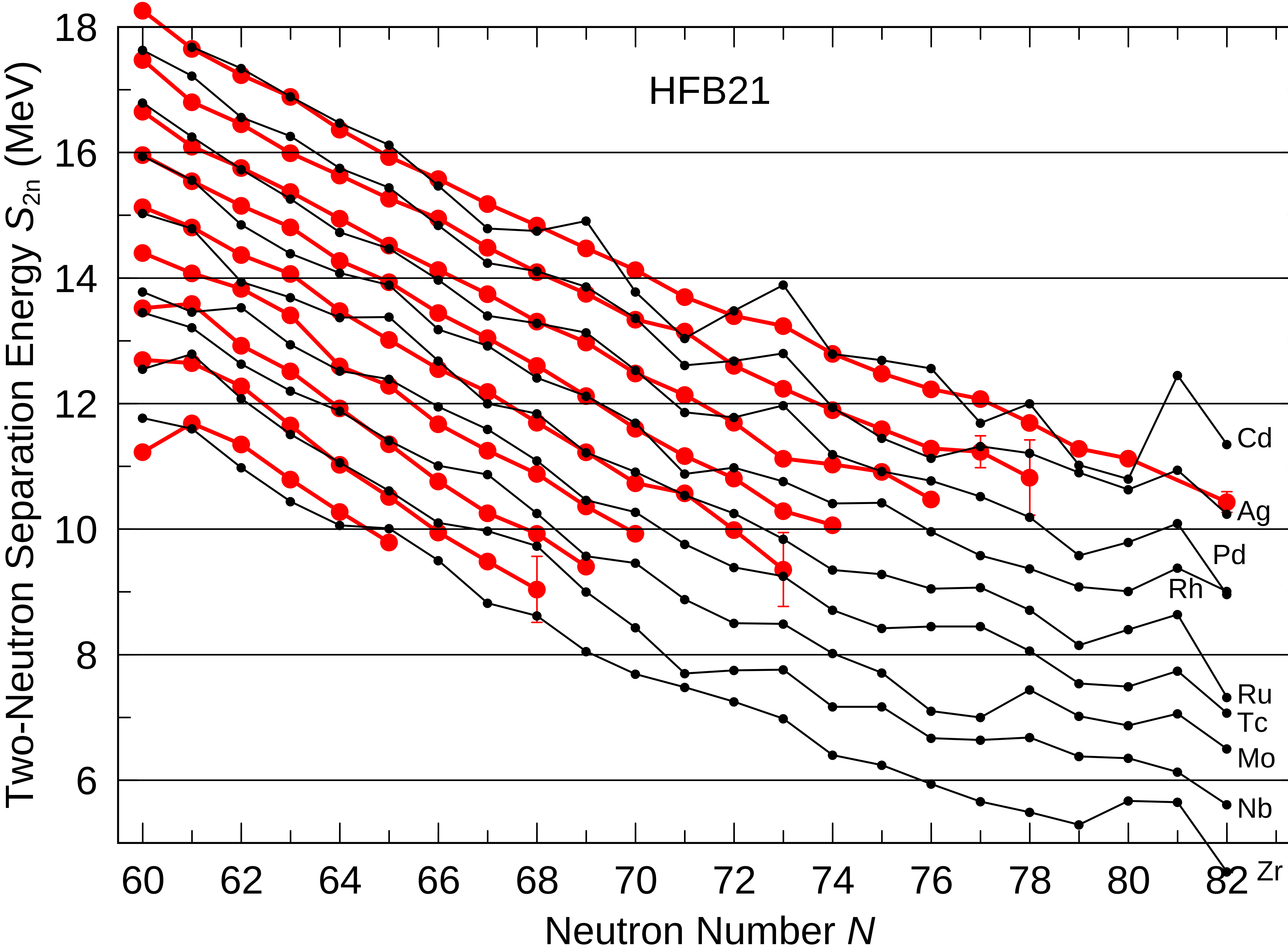
Neutron Number N
154
156
158
160
162
164
166
α-decay of 277112
GSI exp. (1996) c2
FRLDM (1992)
FRDM (1992)
HFB27 (Goriely)
Energy Release Qα (MeV)
12
11
10
9
8
100
102
104
106
108
110
112
114
Proton Number Z

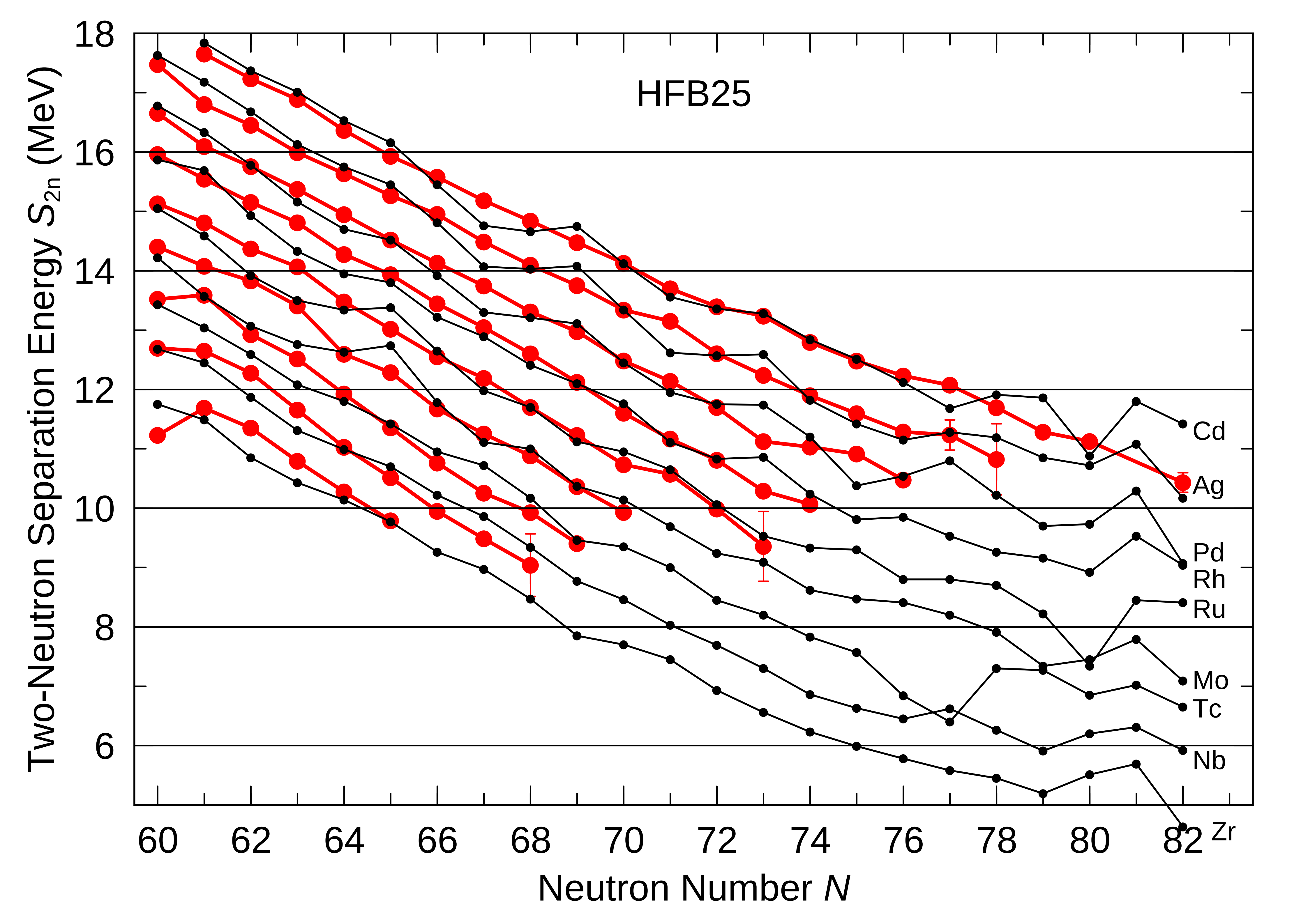


FRDM(1992)
Two-Neutron Separation Energy S_{2n} (MeV)
Neutron Number N
18
16
14
12
10
8
6
60
62
64
66
68
70
72
74
76
78
80
82
Cd
Ag
Pd
Rh
Ru
Tc
Mo
Nb
Zr

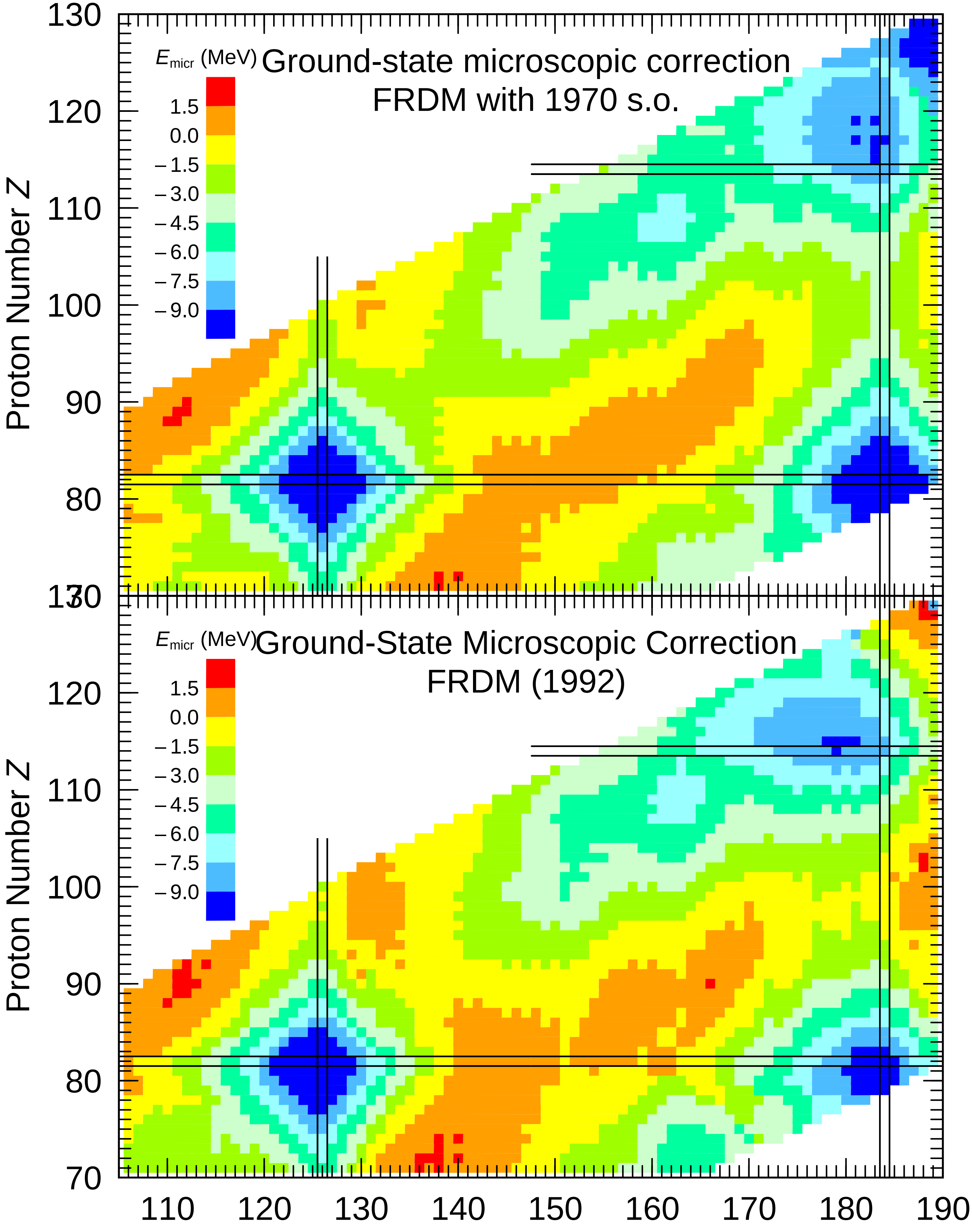


FRDM(2012)
Two-Neutron Separation Energy S_{2n} (MeV)
Neutron Number N
18
16
14
12
10
8
6
60
62
64
66
68
70
72
74
76
78
80
82
Cd
Ag
Pd
Rh
Ru
Tc
Mo
Nb
Zr
Exp., AME2012
Calc., FRDM(2012)
Planned exp., marginal
Planned exp., straightforward

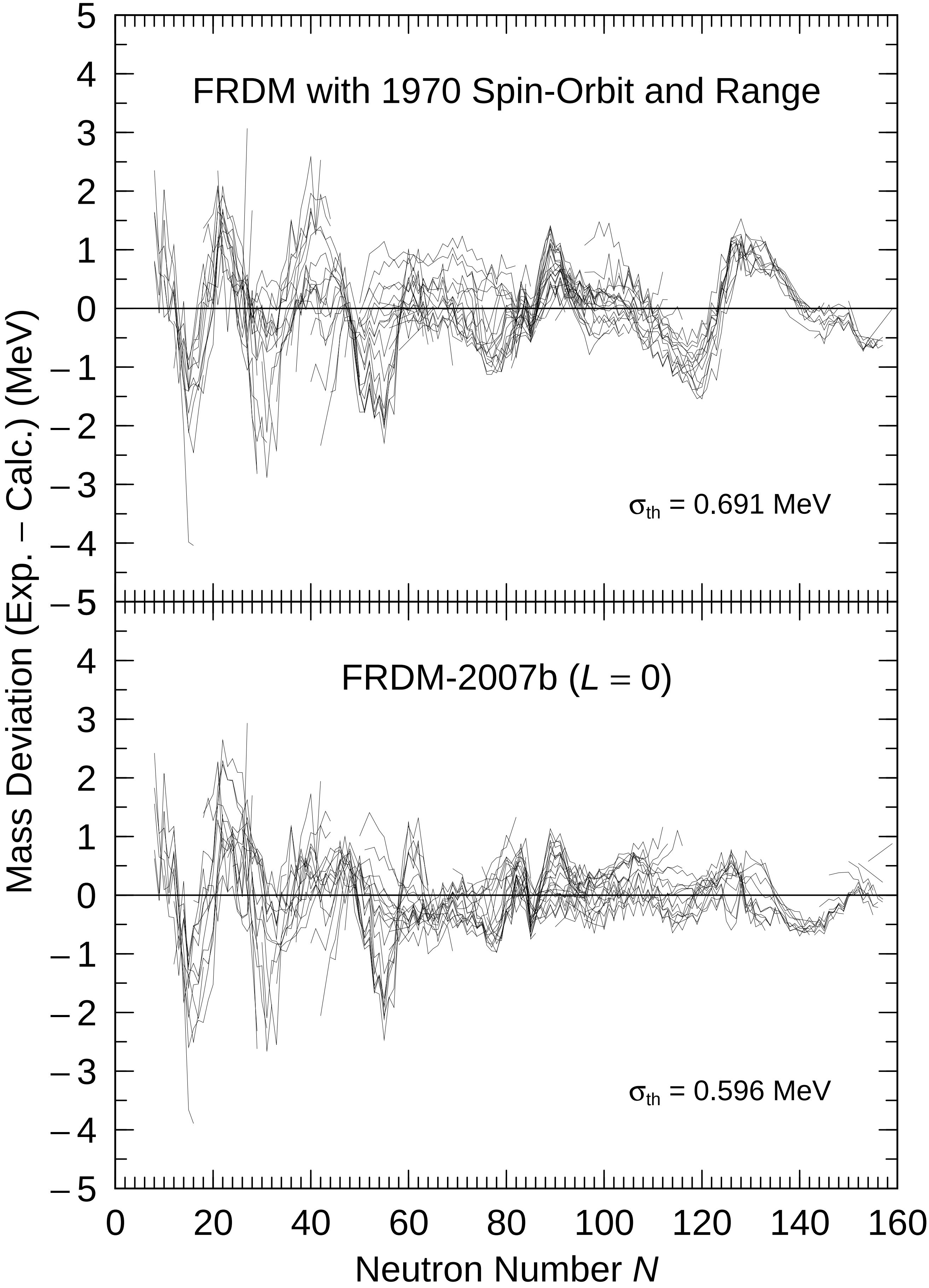

HFB21
Two-Neutron Separation Energy S_{2n} (MeV)
Neutron Number N
18
16
14
12
10
8
6
60
62
64
66
68
70
72
74
76
78
80
82
Cd
Ag
Pd
Rh
Ru
Tc
Mo
Nb
Zr

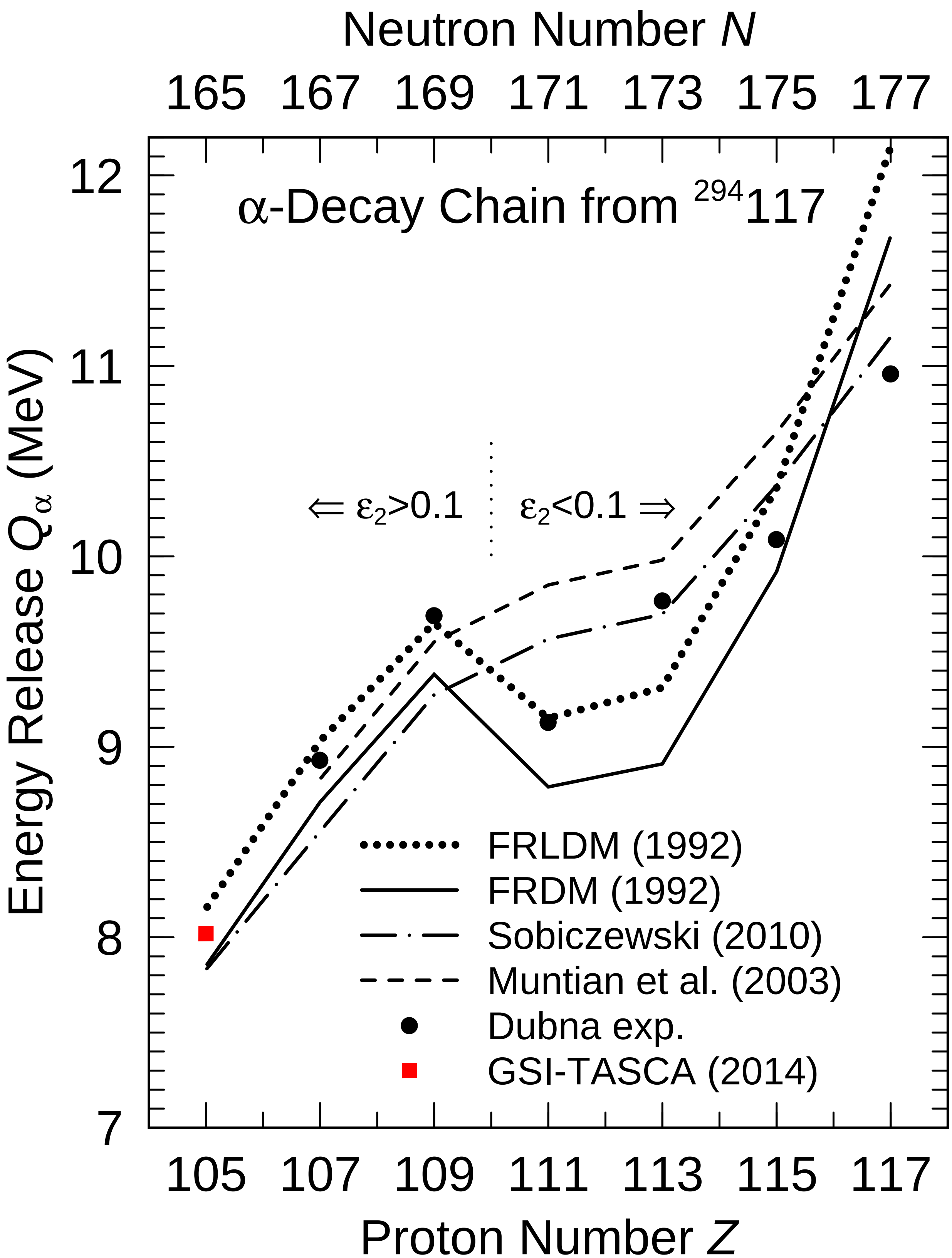


HFB25
Two-Neutron Separation Energy S_{2n} (MeV)
Neutron Number N
18
16
14
12
10
8
6
60
62
64
66
68
70
72
74
76
78
80
82
Cd
Ag
Pd
Rh
Ru
Mo
Tc
Nb
Zr

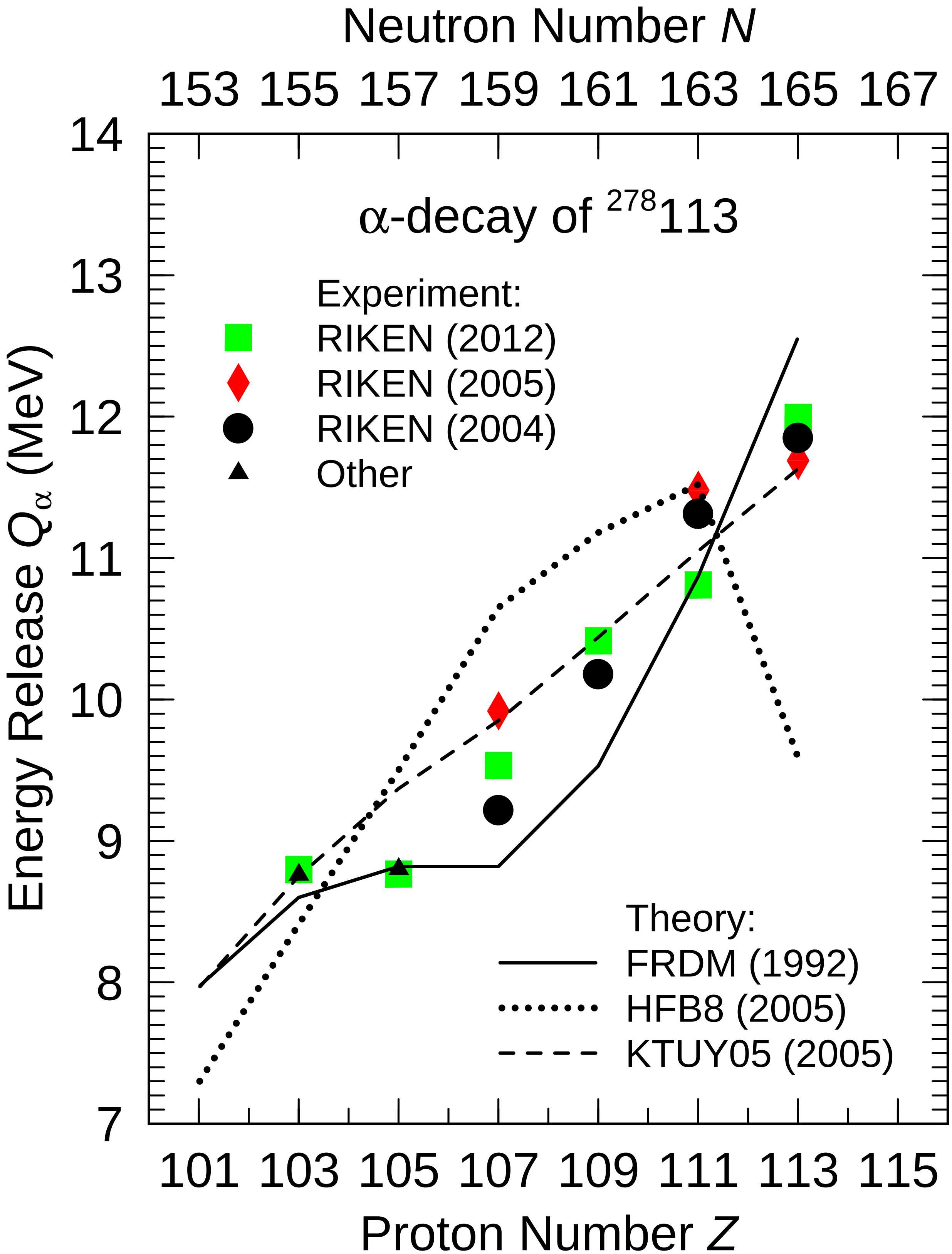


Ground-state microscopic correction
FRDM with 1970 s.o.
Ground-State Microscopic Correction
FRDM (1992)
E_{micr} (MeV)
1.5
0.0
−1.5
−3.0
−4.5
−6.0
−7.5
−9.0
Proton Number Z
Neutron Number N

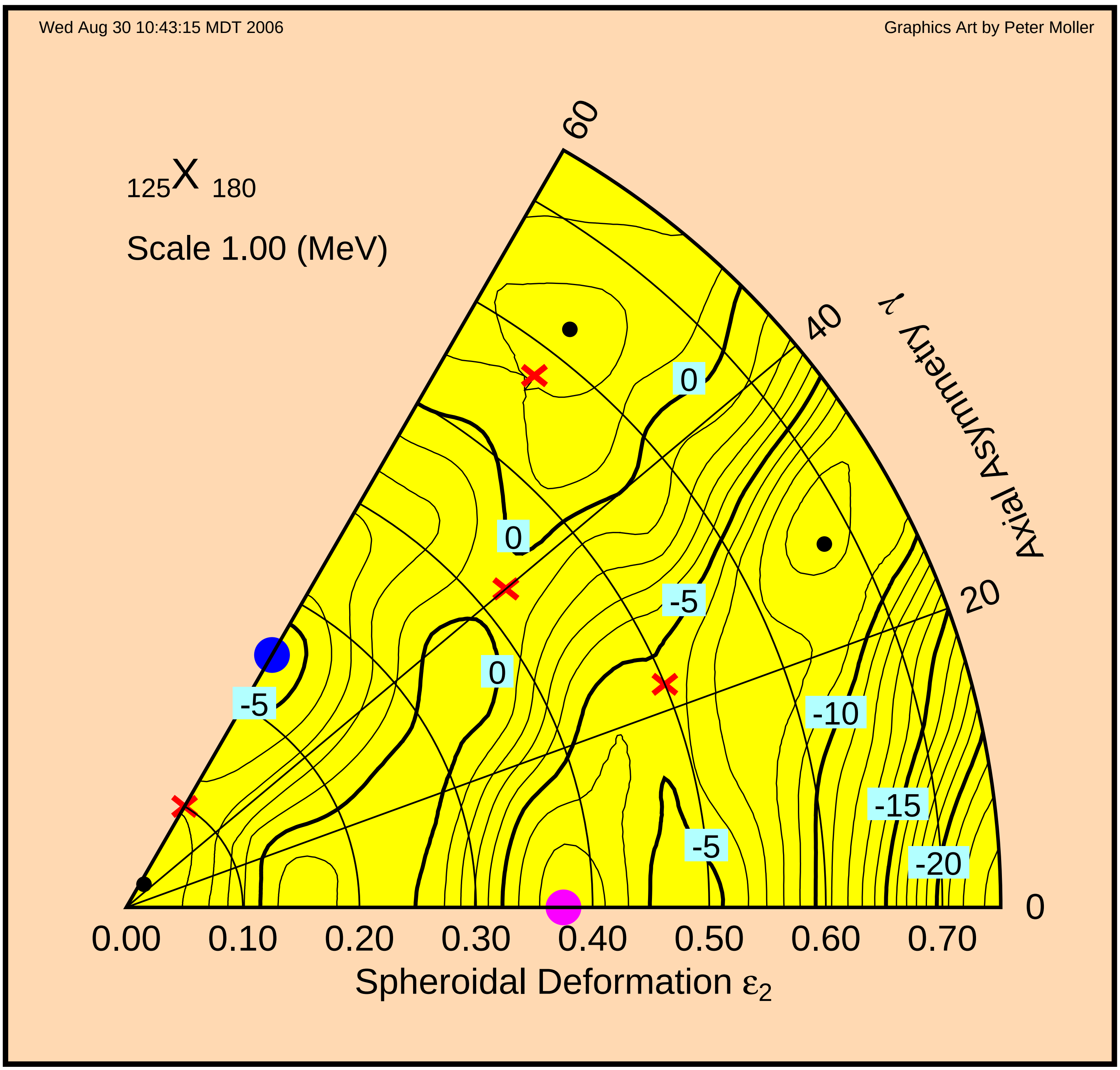

FRDM with 1970 Spin-Orbit and Range
σ_{th} = 0.691 MeV
FRDM-2007b ($L = 0$)
σ_{th} = 0.596 MeV
Mass Deviation (Exp. − Calc.) (MeV)
Neutron Number N
5
4
3
2
1
0
−1
−2
−3
−4
−5
0
20
40
60
80
100
120
140
160



Neutron Number N
165 167 169 171 173 175 177
α-Decay Chain from 294117
⇐ ε2>0.1
ε2<0.1 ⇒
Energy Release Qα (MeV)
12 11 10 9 8 7
FRLDM (1992)
FRDM (1992)
Sobiczewski (2010)
Muntian et al. (2003)
Dubna exp.
GSI-TASCA (2014)
105 107 109 111 113 115 117
Proton Number Z



Neutron Number N
153 155 157 159 161 163 165 167
α-decay of 278113
Experiment:
RIKEN (2012)
RIKEN (2005)
RIKEN (2004)
Other
Theory:
FRDM (1992)
HFB8 (2005)
KTUY05 (2005)
Energy Release Qα (MeV)
14 13 12 11 10 9 8 7
101 103 105 107 109 111 113 115
Proton Number Z



Wed Aug 30 10:43:15 MDT 2006
Graphics Art by Peter Moller
125X 180
Scale 1.00 (MeV)
60
40
20
0
Axial Asymmetry γ
0
0
0
-5
-5
-5
-10
-15
-20
0.00
0.10
0.20
0.30
0.40
0.50
0.60
0.70
Spheroidal Deformation ε2

Results by Peter Möller and Collaborators

Interactive access to data here

P U B L I C A T I O N S

- [Abstract and link to full paper]
  Nuclear Ground-State Masses and Deformations: FRDM(2012)
  Submitted to Atomic Data and Nuclear Data Tables
  and
  --> *Los Alamos Preprint LA-UR-15-26310.*

- [Abstract and link to full paper]
  A Method to Calculate Fission-Fragment Yields *Y(Z,N)* versus Proton and Neutron Number in the Brownian Shape-Motion Model;
  Application to Calculations of U and Pu Charge Yields
  Accepted by European Physics Journal A
  and
  --> *Los Alamos Preprint LA-UR-15-26634.*

- [Abstract and link to full paper]
  Calculated Fission-Fragment Yield Systematics in the Region 74 ≤ Z ≤ 94 and 90 ≤ N ≤ 150
  *Published in Physical Review C **91** 044316 (2015)*
  and
  --> *Los Alamos Preprint LA-UR-14-24794.*

- [Abstract and link to full paper]

Nuclear ground-state masses and deformations: FRDM(2012)

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ATOMIC DATA AND NUCLEAR DATA TABLES
Submitted August 13, 2015
It has been assigned Los Alamos National Laboratory Preprint No LA-UR-15-26310.

Abstract:

We tabulate the atomic mass excesses and binding energies, ground-state shell-plus-pairing corrections, ground-state microscopic corrections, and nuclear ground-state deformations of 9318 nuclei ranging from ^{16}O to A=339. The calculations are based on the finite-range droplet macroscopic model and the folded-Yukawa single-particle microscopic model. Relative to our FRDM(1992) mass table in [ATOMIC DATA AND NUCLEAR DATA TABLES **59**,185 (1995)], the results are obtained in the same model, but with considerably improved treatment of deformation and fewer of the approximations that were necessary earlier, due to limitations in computer power. The more accurate execution of the model and the more extensive and more accurate experimental mass data base now available allows us to determine one additional macroscopic-model parameter, the density-symmetry coefficient L, which was not varied in the previous calculation, but set to zero. Because we now realize that the FRDM is inaccurate for some highly deformed shapes occurring in fission, because some effects are derived in terms of perturbations around a sphere, we only adjust its macroscopic parameters to ground-state masses. The values of ten constants are

determined directly from an optimization to fit ground-state masses of 2149 nuclei ranging from ^{16}O to ^{265}Sg and ^{264}Hs. The error of the mass model is 0.5595 MeV for the entire region of nuclei included in the adjustment, but is only 0.3549 MeV for the region $N \geq 65$.

We also provide masses in the FRLDM, which in the more accurate treatments now has an error of 0.6618 MeV, with 0.5181 MeV for nuclei with $N \geq 65$, both somewhat larger than in the FRDM. But in contrast to the FRDM, it is suitable for studies of fission and has been extensively so applied elsewhere, with FRLDM(2002) constants. The FRLDM(2012) fits 31 fission barrier heights from ^{70}Se to ^{252}Cf with a root-mean-square deviation of 1.052 MeV.

The complete manuscript in color is available for download.

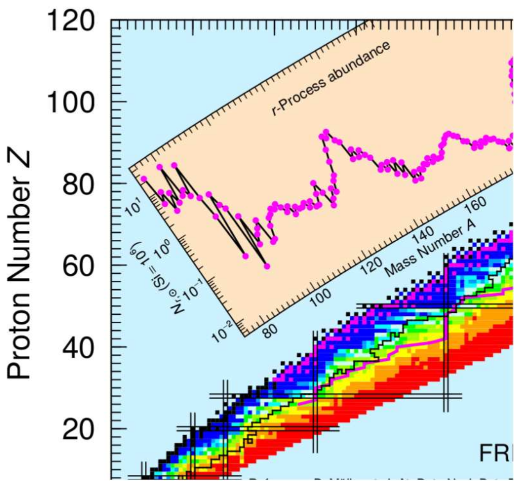
The table ADNDT-FRDM2012-TABLE.dat , in computer-readable format, is available for download.
The format is (3I5,4F10.2,4F10.3,5F10.2,F10.3,2F10.2)
The variables are explained in Explanation of Table on page 67 of the manuscript.

We provide the 41 figures as individual .ps.gz and .pdf files files:

Figure Fig-01-FRDM2012.ps.gz in format .ps.gz and Fig-01-FRDM2012.pdf in .pdf format is available for download.
Figure Fig-02-FRDM2012.ps.gz in format .ps.gz and Fig-02-FRDM2012.pdf in .pdf format is available for download.
Figure Fig-03-FRDM2012.ps.gz in format .ps.gz and Fig-03-FRDM2012.pdf in .pdf format is available for download.
Figure Fig-04-FRDM2012.ps.gz in format .ps.gz and Fig-04-FRDM2012.pdf in .pdf format is available for download.
Figure Fig-05-FRDM2012.ps.gz in format .ps.gz and Fig-05-FRDM2012.pdf in .pdf format is available for download.
Figure Fig-06-FRDM2012.ps.gz in format .ps.gz and Fig-06-FRDM2012.pdf in .pdf format is available for download.
Figure Fig-07-FRDM2012.ps.gz in format .ps.gz and Fig-07-FRDM2012.pdf in .pdf format is available for download.
Figure Fig-08-FRDM2012.ps.gz in format .ps.gz and Fig-08-FRDM2012.pdf in .pdf format is available for download.
Figure Fig-09-FRDM2012.ps.gz in format .ps.gz and Fig-09-FRDM2012.pdf in .pdf format is available for download.
Figure Fig-10-FRDM2012.ps.gz in format .ps.gz and Fig-10-FRDM2012.pdf in .pdf format is available for download.
Figure Fig-11-FRDM2012.ps.gz in format .ps.gz and Fig-11-FRDM2012.pdf in .pdf format is available for download.

Nuclear Properties for Astrophysics

Interactive access to nuclear properties of interest for astrophysics for almost 9000 nuclei. Get your experimental masses and calculated masses, deformations, decay Q-values, half-lives, spins, and separation energies here! Access to potential-energy contour maps versus ε_2 and γ shape degrees of freedom is also provided.



Plot of R-Process Abundances

The magenta line shows a typical r-process path, and the small magenta squares are the nuclei produced when the nuclides along the path decay back to stability after the supernova neutron flux ends. The insert shows the solar r-process abundances as a function of nucleon number *A* The *A* axis of the insert curves so that a line from a β-stable nucleus (black square at the upper edge of the chart plot), which is perpendicular to the line of β-stability in the chart, will intersect the *A* axis at the value corresponding to the *A* value of the black square of the β-stable nucleus where the (imaginary) line originates. An extension of the line upward will intersect with the appropriate data point. In the reverse, a line through, for example the upper and lower *A*=120 tic mark continuing towards the chart plot will intersect the black square (β-stable nucleus) with *A*=120.

Access to the data:

- Ground-state masses and deformations
- Alpha-decay *Q* values and half-lives
- Ground-state odd-proton and odd-neutron spins
- Ground-state proton and neutron pairing gaps
- Ground-state proton separation energies
- Ground-state neutron separation energies
- Beta-decay *Q*-values for beta-minus and EC decay
- Beta-decay half-lives and beta-delayed neutron-emission probabilities
- Potential-energy surfaces versus ε_2 and γ, for ground-state shapes, largest ε_2 is 0.45.
- Fission-potential-energy surfaces versus ε_2 and γ, largest ε_2 is 0.75.

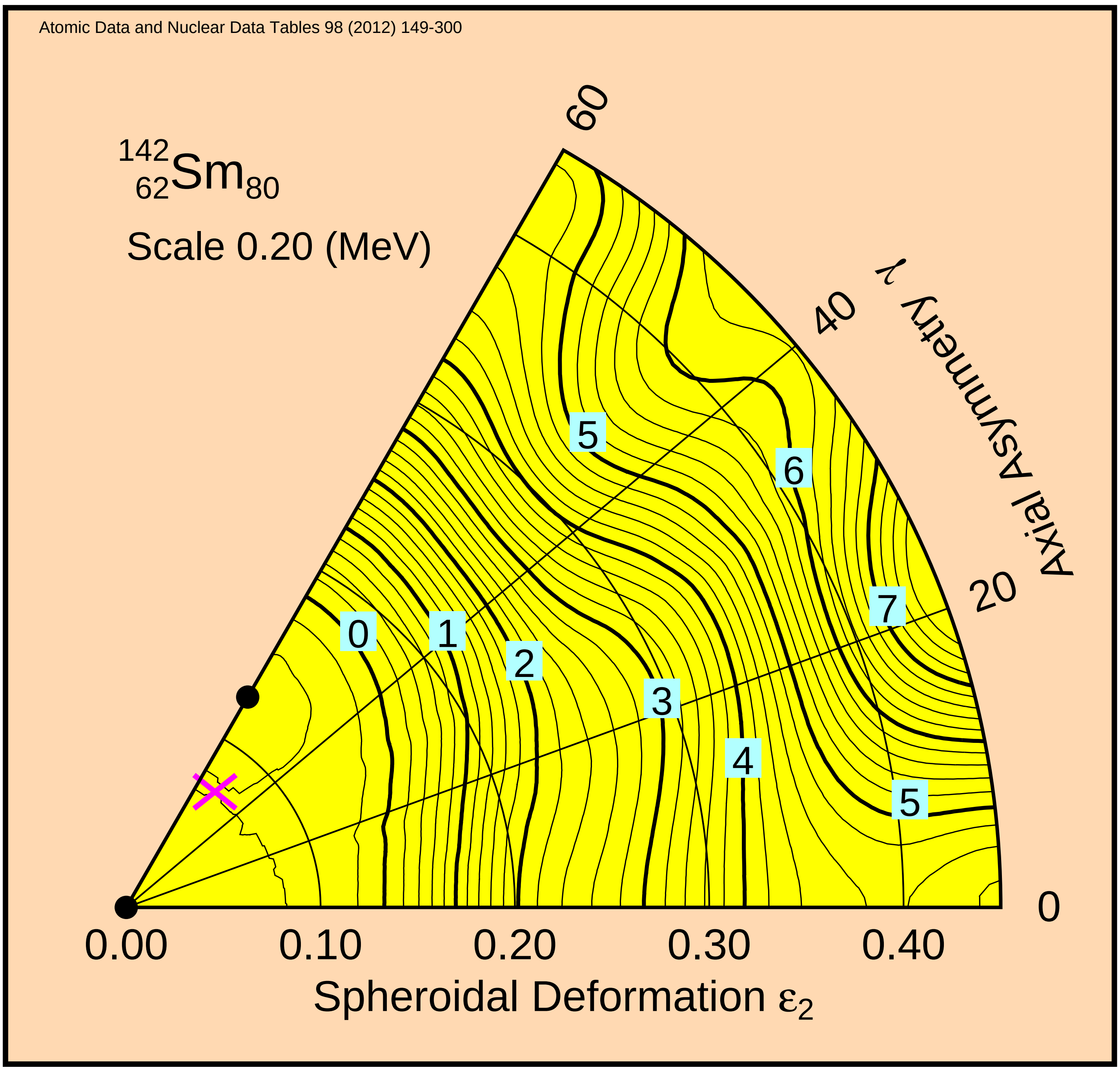
The calculated masses and deformations are from
Nuclear Ground-State Masses and Deformations, by P. Möller, J. R. Nix, W. D. Myers, and W. J. Swiatecki
Atomic Data Nucl. Data Tables **59** (1995), 185-381.

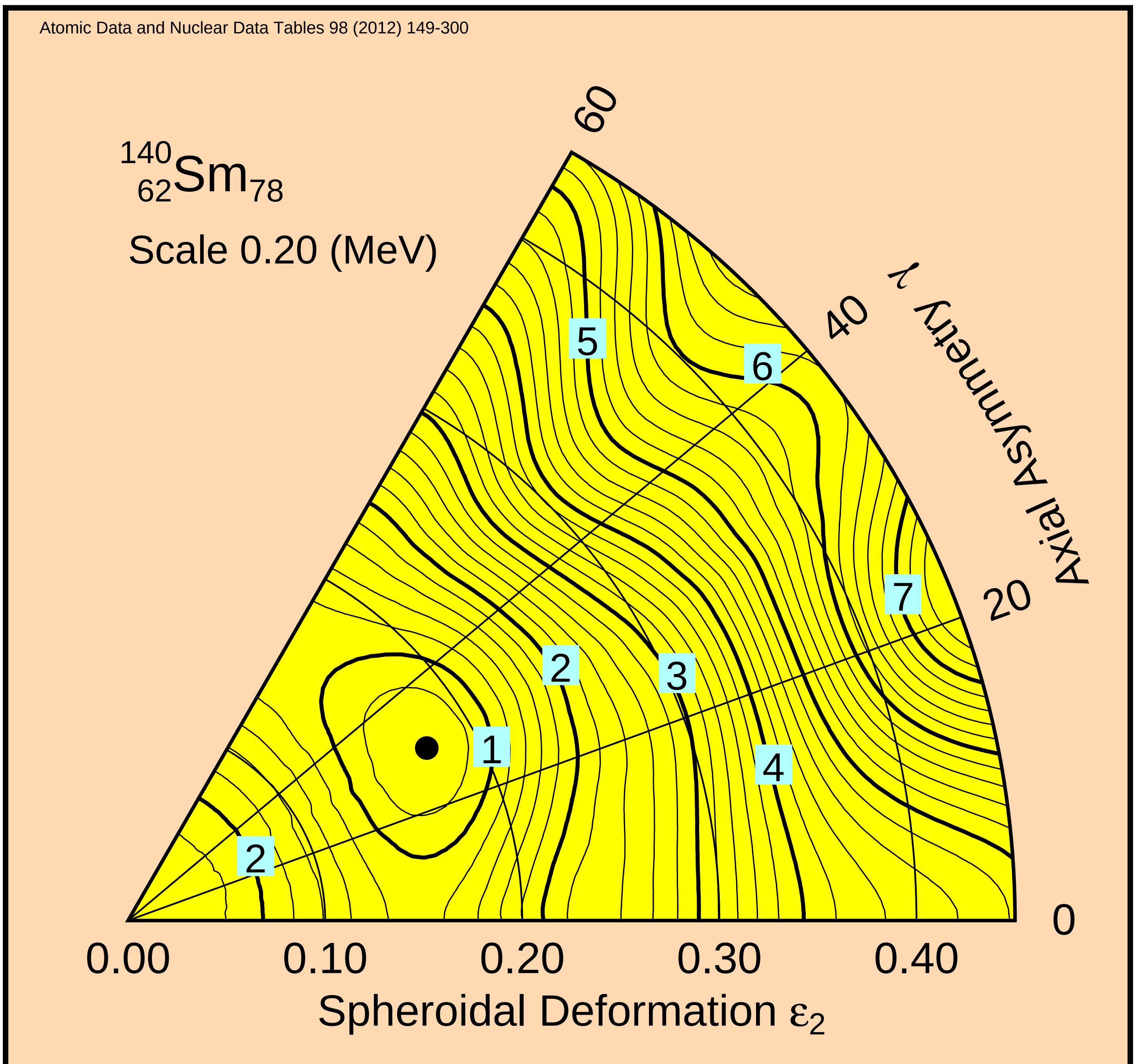
The other calculated nuclear properties are from
Nuclear Properties for Astrophysical and Radioactive-Ion-Beam Applications,
by P. Möller, J. R. Nix, and K.-L. Kratz,
Atomic Data Nucl. Data Tables **66** (1997), 131-343.

The reference to the calculated ground-state potential-energy surfaces is
NUCLEAR SHAPE ISOMERS
P. Möller, A. J. Sierk, R. Bengtsson, H. Sagawa, T. Ichikawa
Atomic Data and Nuclear Data Tables **98** (2012) 149-300.
This publication presents potential-energy surfaces of most even-even nuclei, here we provide access to stand-alone potential-energy surfaces of all 7206 nuclides we calculated. In addition the surfaces here are in color. A preprint of the ADNDT publication with associated figures and tables is available on our web site. But for the complete publication we recommend the published version available on the ADNDT web site.

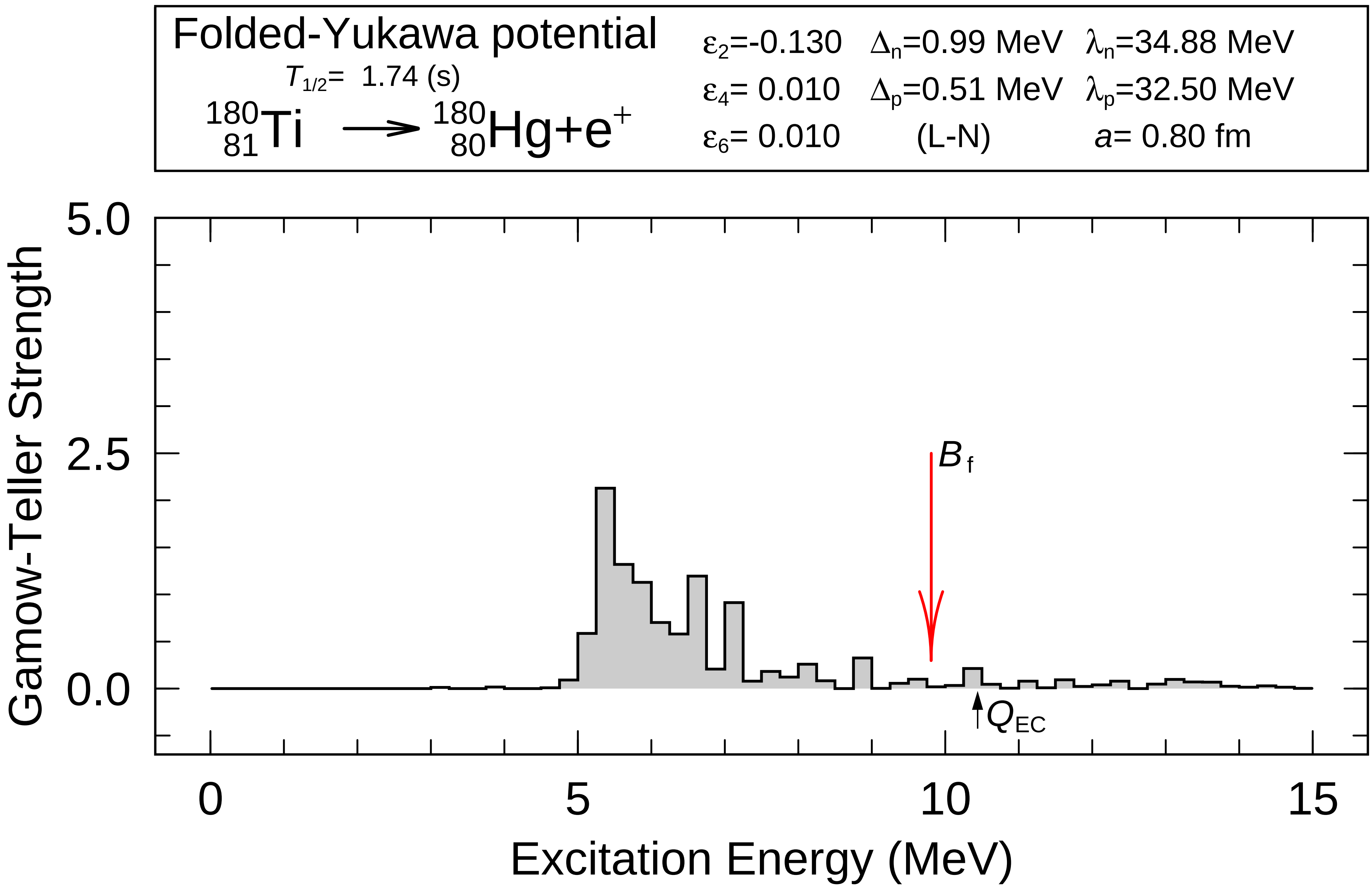
The reference to the fission potential-energy surfaces calculated in three dimensions (ε_2,ε_4,&gamma) posted on this web site, simplified to 2D contour diagrams versus ε_2 and &gamma, is
HEAVY-ELEMENT FISSION BARRIERS
P. Möller, A. J. Sierk, T. Ichikawa, A. Iwamoto,R. Bengtsson, H. Uhrenholt, and S. Åberg,
Physical Review C **79** (2009) 064304. Please note that a more complete characterization of the fission barriers also requires calculations in 5 dimensions in the three-quadratic-surface parameterization for several million different nuclear shapes. Such calculations are also carried out in the above reference, but the complexity of such surfaces prevents them to be simplified to two-dimensional contour drawings.

The most recent experimental mass evaluation is
Wapstra, Audi, and Thibault (A. H. Wapstra, G. Audi, and C. Thibault, Nucl. Phys. **A729** (2003), 129). Another repository of this evaluation and related data can be found here . We use here recent data from the interim evaluation:
G. Audi and W. Meng, Private Communication, April 2011.

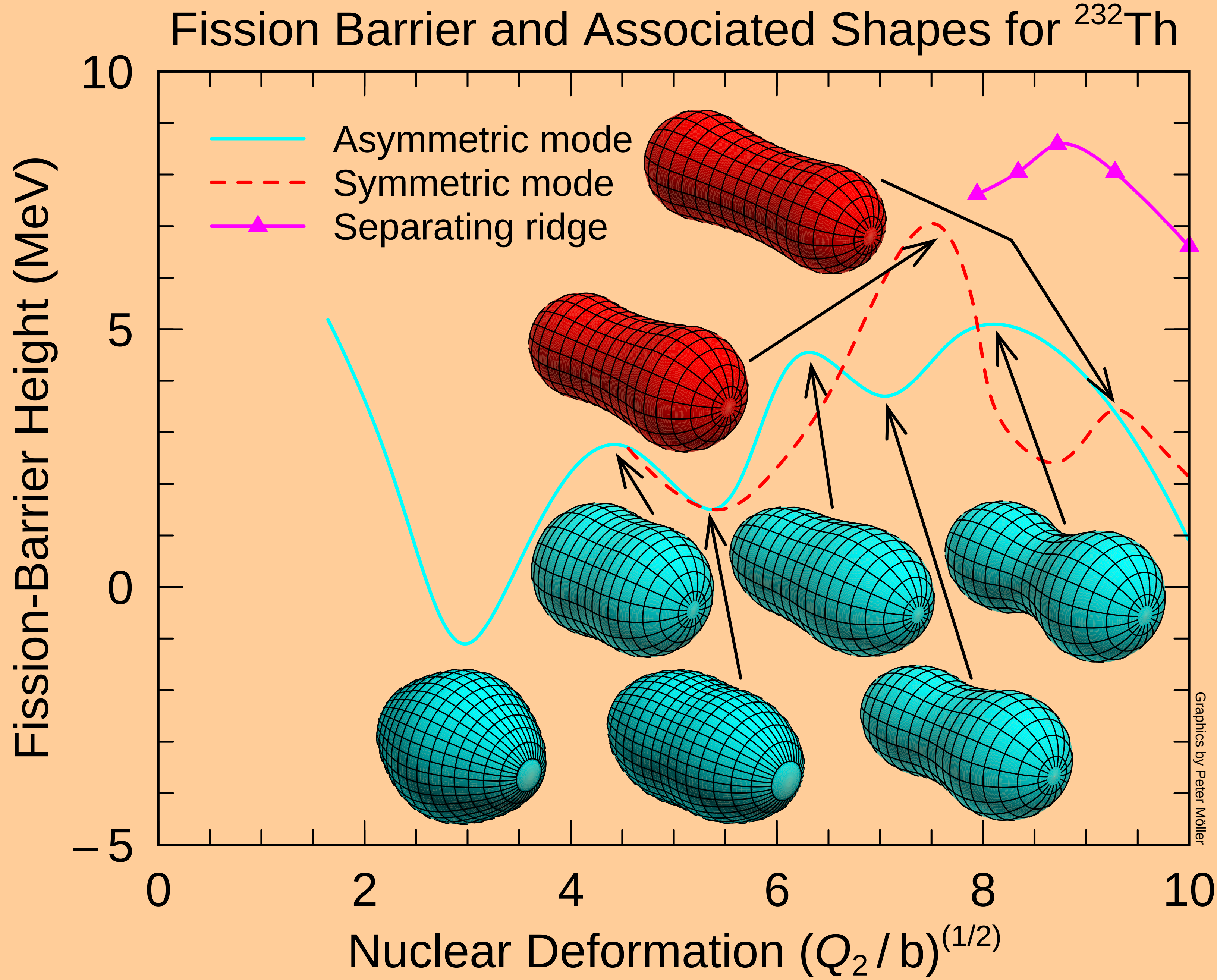

Atomic Data and Nuclear Data Tables 98 (2012) 149-300
$^{142}_{62}Sm_{80}$
Scale 0.20 (MeV)
60
40
20
0
Axial Asymmetry γ
0.00
0.10
0.20
0.30
0.40
Spheroidal Deformation ε_2
0
1
2
3
4
5
6
7
5



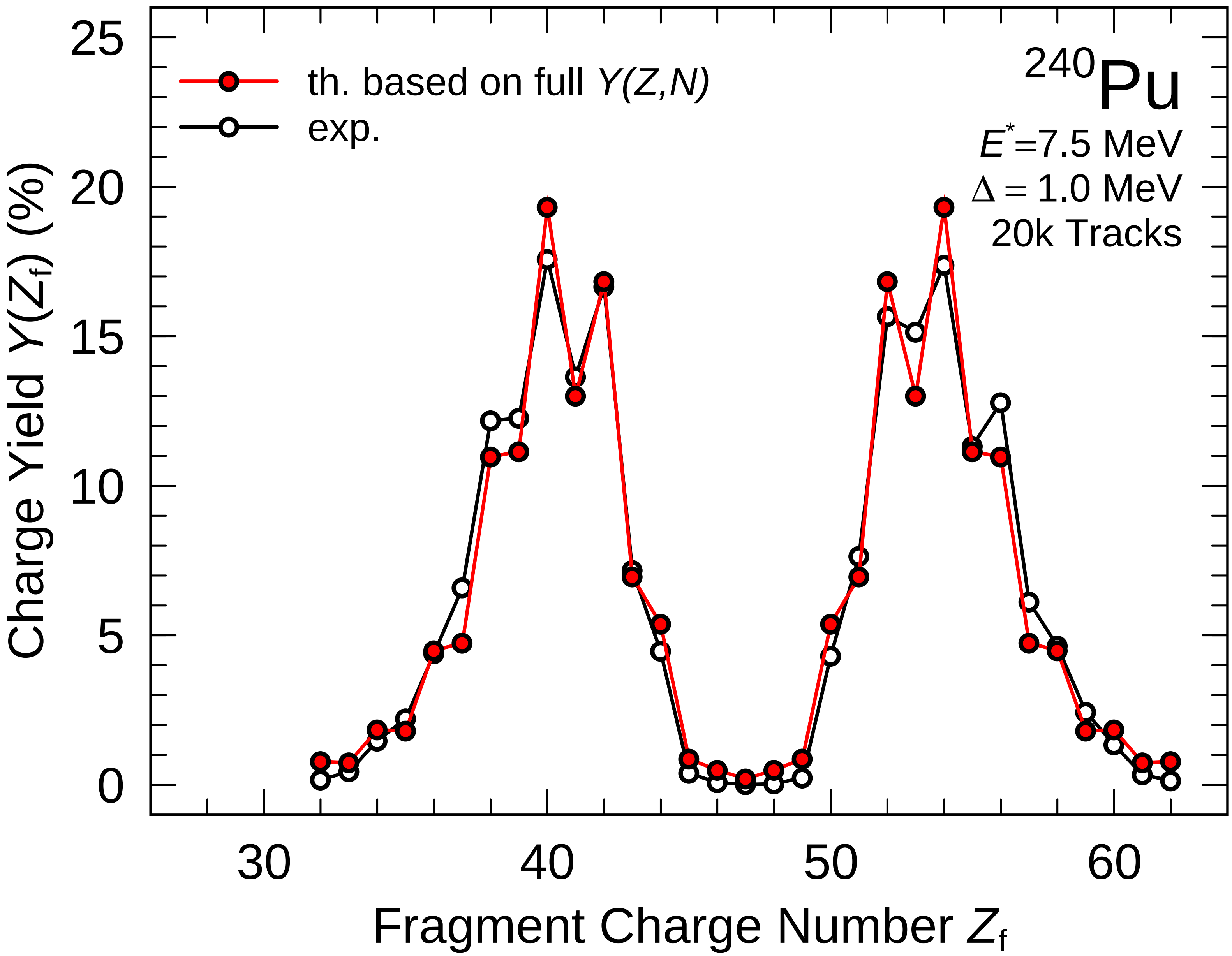
Atomic Data and Nuclear Data Tables 98 (2012) 149-300
$^{140}_{62}Sm_{78}$
Scale 0.20 (MeV)
60
40
20
0
Axial Asymmetry γ
1
2
2
3
4
5
6
7
0.00
0.10
0.20
0.30
0.40
Spheroidal Deformation ε_2



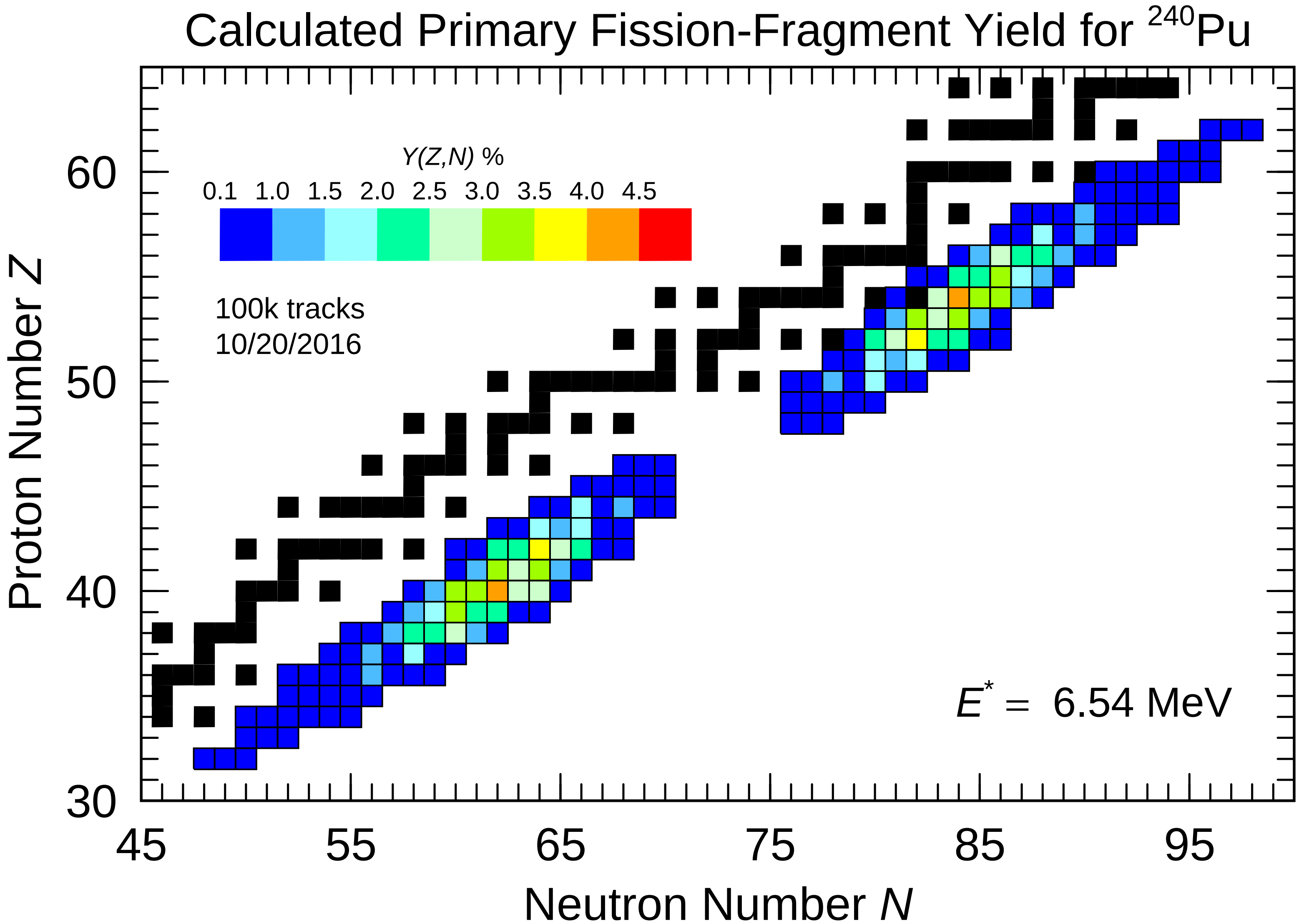
Folded-Yukawa potential
$T_{1/2}$= 1.74 (s)
$^{180}_{81}$Ti ⟶ $^{180}_{80}$Hg+e^{+}
ε_2=-0.130 Δ_n=0.99 MeV λ_n=34.88 MeV
ε_4= 0.010 Δ_p=0.51 MeV λ_p=32.50 MeV
ε_6= 0.010 (L-N) a= 0.80 fm
5.0
2.5
0.0
Gamow-Teller Strength
B_f
Q_{EC}
0
5
10
15
Excitation Energy (MeV)



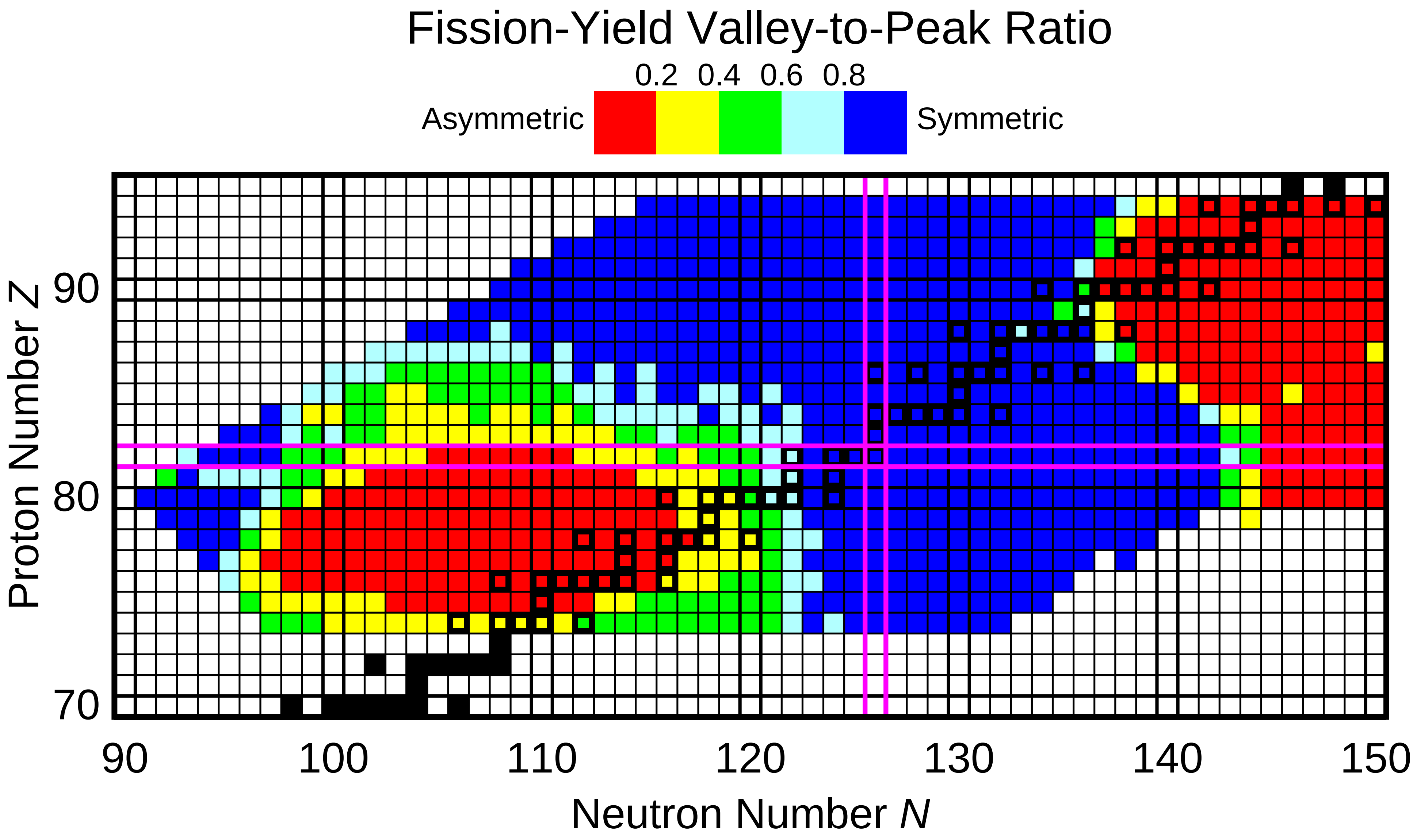
Fission Barrier and Associated Shapes for 232Th
Asymmetric mode
Symmetric mode
Separating ridge
Fission-Barrier Height (MeV)
Nuclear Deformation (Q2 / b)(1/2)
10
5
0
−5
0
2
4
6
8
10
Graphics by Peter Möller



th. based on full Y(Z,N)
exp.
240Pu
E*=7.5 MeV
Δ = 1.0 MeV
20k Tracks
Charge Yield Y(Zf) (%)
Fragment Charge Number Zf
0
5
10
15
20
25
30
40
50
60



Calculated Primary Fission-Fragment Yield for 240Pu
Y(Z,N) %
0.1 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5
100k tracks
10/20/2016
E* = 6.54 MeV
Proton Number Z
30
40
50
60
Neutron Number N
45
55
65
75
85
95


Fission-Yield Valley-to-Peak Ratio
0.2
0.4
0.6
0.8
Asymmetric
Symmetric
Proton Number Z
90
80
70
90
100
110
120
130
140
150
Neutron Number N

S U M M A R Y and O U T L O O K

SPECIFICALLY MASSES:

- FRDM2012 mass/deformation table published in ADNDT **109-110** (2016) 1, for 9318 nuclides.
- Associated EQS ($J = 32.3$ MeV and $L = 53.5$ MeV) parameters.
- The many tests on predictive qualities are encouraging.

“RELATED” RESULTS:

- A microscopic (non-phenomenological) fission-Yield model that can be applied to all nuclei at all energies above the barrier.

- Fission barriers for more than 5000 nuclei published

- Calculated β-decay half-lives and delayed neutron-emission probabilities to be submitted shortly to ADNDT.