
In this unit we are going to do some problems in electricity and magnetism. We will do a few problems in calculating static fields, and then a couple of circuits.

As usual, units are very important and so we need to setup the unit system first.

I prefer to use, Volts, Coulombs, Meters, and Seconds, and Tesla

Electric field is in units of Volts/Meter or

Newton/Coulomb. Current is Amperes = Coulombs/sec.

Two charges q and $-q$ are at locations $(0, +1/2)$ and $(0, -1/2)$ meters. Calculate the electric field.

First we calculate the potential and then take its gradient.

Setup the constants. μ_0 is the Magnetic Permeability and k_{coulomb} is Coulombs constant. $k_{\text{coulomb}} = 1/(4 \pi \epsilon_0)$

```
In[ ]:= mu0 = QuantityMagnitude[
      UnitConvert[Quantity[1, "MagneticConstant"], "Tesla Meter/Ampere"]]
```

```
Out[ ]:=  $\frac{\pi}{2500000}$ 
```

```
In[ ]:= kcoulomb = N[QuantityMagnitude[
      UnitConvert[Quantity[1, "CoulombsConstant"], "Newton * meter^2/Coulomb^2"]]]
```

```
Out[ ]:=  $8.98755 \times 10^9$ 
```

Let's check that these constants are correctly defined. $1/\text{Sqrt}[\epsilon_0 * \mu_0]$ should be the speed of light. This should be in meters/second.

```
 $\epsilon_0 = 1 / (4 \text{ Pi} * \text{k coulomb});$   
 $\text{c light} = 1 / \text{Sqrt}[\mu_0 * \epsilon_0]$ 
```

Out[]= 2.99792×10^8

Let's do a problem of the electric field between two charges.

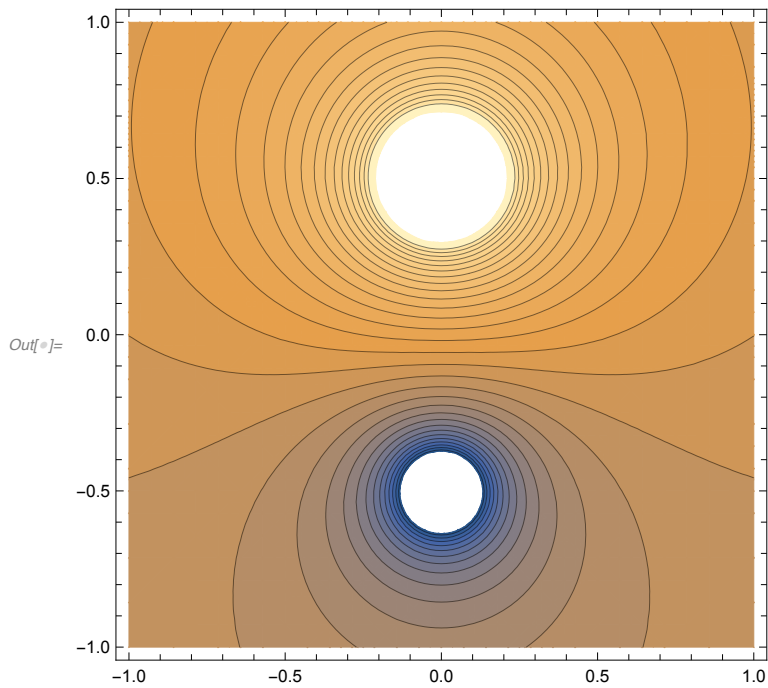
```
In[ ]:=  $q_1 = 2 * 10^{-6};$  (* units are Coulomb *)  
 $q_2 = -10^{-6};$   
 $p_1 = \{0, +1/2, 0\};$  (* make it 3 dimensional*)  
 $p_2 = \{0, -1/2, 0\};$  (* units are in meters *)  
 $\text{dipole}[x_, y_, z_] :=$   
   $\text{k coulomb} * (q_1 / \text{Sqrt}[(x - p_1[[1]])^2 + (y - p_1[[2]])^2 + (z - p_1[[3]])^2] +$   
     $q_2 / \text{Sqrt}[(x - p_2[[1]])^2 + (y - p_2[[2]])^2 + (z - p_2[[3]])^2]);$   
  (* Potential *)  
 $r = \{0, 2, 0\};$   
 $\text{dipole}[r[[1]], r[[2]], r[[3]]]$ 
```

Out[]= 8388.38

In[]:= ? ContourPlot

ContourPlot[f, {x, x_{min}, x_{max}}, {y, y_{min}, y_{max}}] generates a contour plot of f as a function of x and y.
ContourPlot[f == g, {x, x_{min}, x_{max}}, {y, y_{min}, y_{max}}] plots contour lines for which f = g.
ContourPlot[{f₁ == g₁, f₂ == g₂, ...}, {x, x_{min}, x_{max}}, {y, y_{min}, y_{max}}] plots several contour lines.
ContourPlot[... , {x, y} ∈ reg] takes the variables {x, y} to be in the geometric region reg. >>

```
In[ ]:= c1 = ContourPlot[dipole[x, y, 0], {x, -1, 1}, {y, -1, 1}, Contours -> 30]
```



```
In[ ]:= ? Grad
```

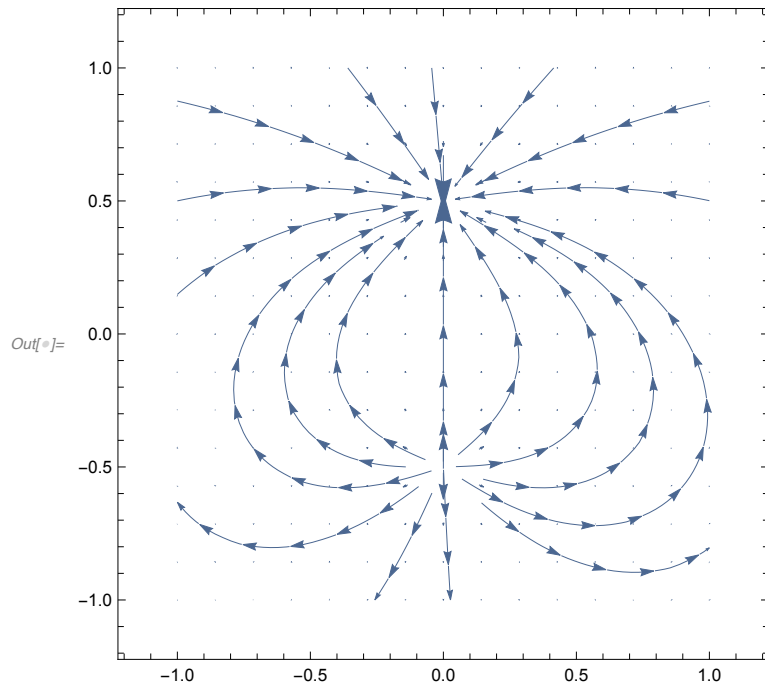
Grad[f , { x_1, \dots, x_n }] gives the gradient $(\partial f / \partial x_1, \dots, \partial f / \partial x_n)$.
 Grad[f , { x_1, \dots, x_n }, *chart*] gives the gradient in the coordinates *chart*. >>

```
In[ ]:= efield[x_, y_] := Grad[dipole[x, y, 0], {x, y}] // Evaluate
```

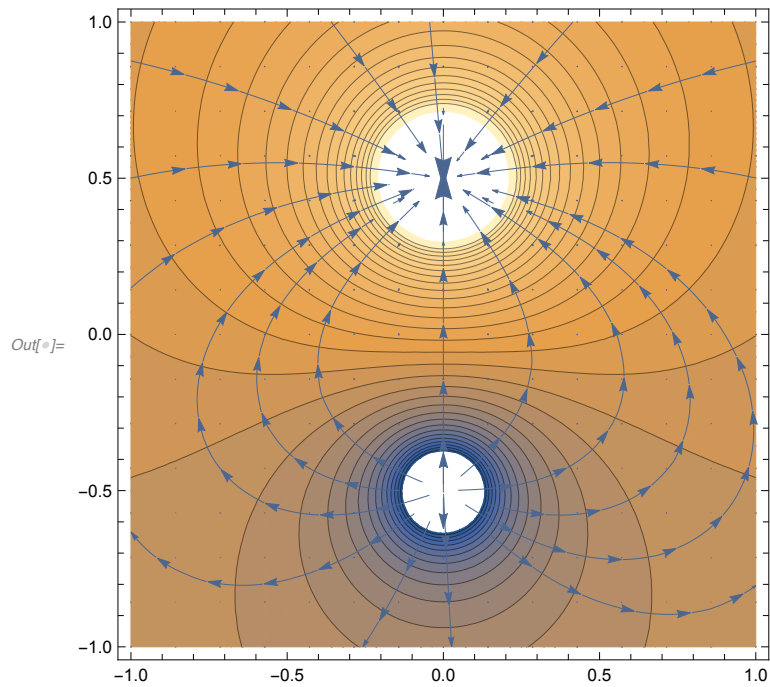
```
In[ ]:= efield[0, 0]
```

```
Out[ ]:= {0., 107851.}
```

```
In[ ]:= v1 = VectorPlot[efield[x, y], {x, -1, 1},  
{y, -1, 1}, VectorPoints -> Automatic, StreamPoints -> 20]
```



```
In[ ]:= Show[c1, v1]
```



What about the strength of the electric field between the two charges along the line in x ?

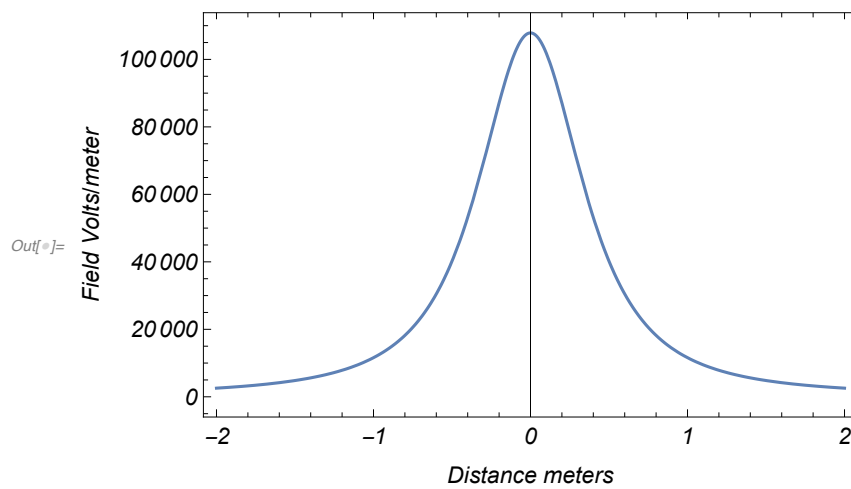
```
In[ ]:= Norm[efield[0, 0]]
```

```
Out[ ]:= 107 851.
```

```
In[ ]:= Norm[efield[-1, 0]]
```

```
Out[ ]:= 11 593.6
```

```
In[ ]:= Plot[Norm[efield[x, 0]], {x, -2, 2}, Frame → True,  
FrameLabel → {"Distance meters", "Field Volts/meter"},  
LabelStyle → {Medium, Italic}]
```



Magnetic Field calculations are more difficult because they need calculus and complicated integrals. But let's just calculate the field of a wire using basic formulas.

The magnetic field of a wire is in a circle around the wire with the field dropping as $1/\text{distance}$ from wire. How can we represent this? Easiest to do this in cylindrical coordinates.

```
In[ ]:= mu0 (* the magnetic permeability *)
```

```
Out[ ]:=  $\frac{\pi}{25000000}$ 
```

```
In[ ]:= I0 = 1; (* current in the wire *)
```

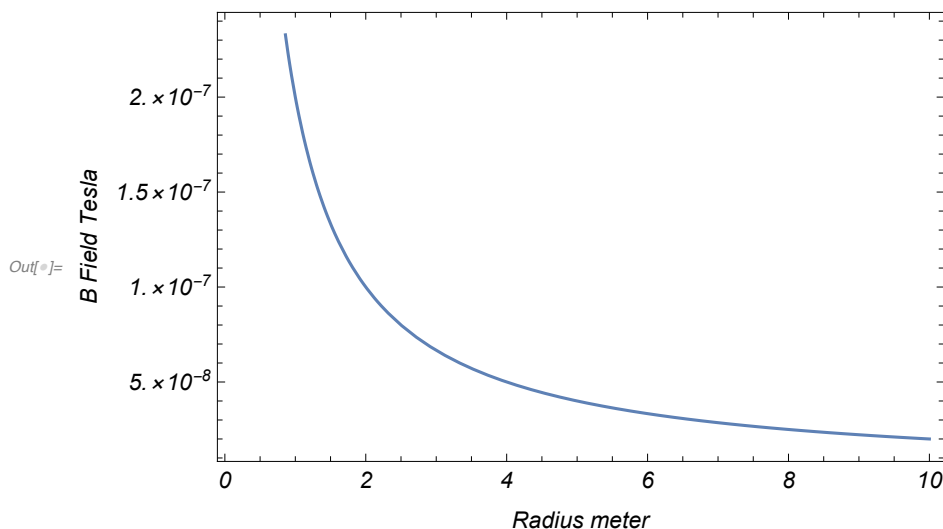
```
B0 = mu0 * I0 / (2 Pi); (* this is in Tesla * meter *)
```

```
In[ ]:= BwireField[x_, y_, z_] := {B0 * y / (x^2 + y^2), -B0 * x / (x^2 + y^2), 0}
```

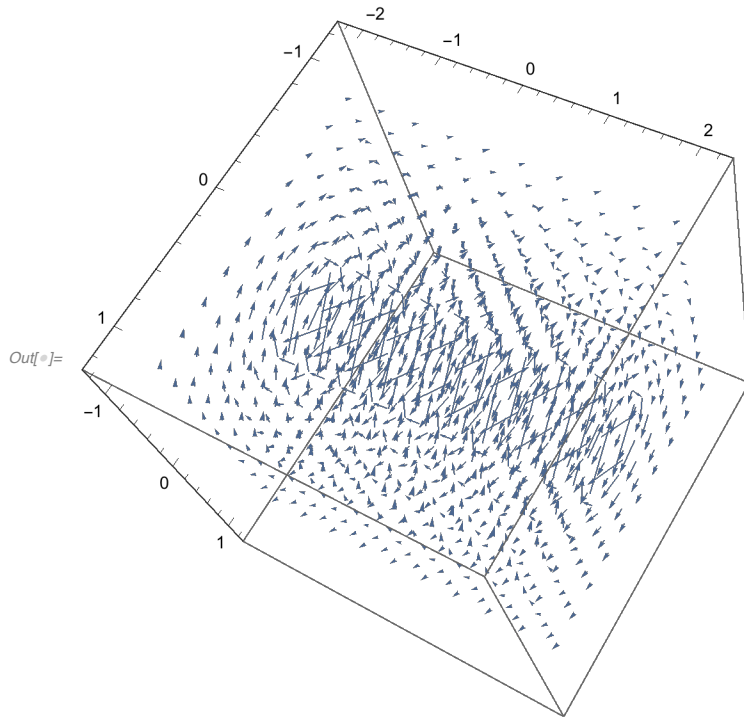
```
In[ ]:= BwireField[10, 0, 0]
```

```
Out[ ]:= {0, - $\frac{1}{50000000}$ , 0}
```

```
In[ ]:= Plot[Norm[BwireField[x, 0, 0]], {x, 0.1, 10}, Frame -> True,
  FrameLabel -> {"Radius meter", "B Field Tesla"}, LabelStyle -> {Medium, Italic}]
```

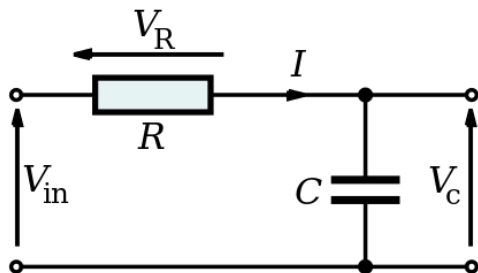


```
In[ ]:= VectorPlot3D[BwireField[x, y, z], {x, -1, 1.}, {y, -1, 1}, {z, -2, 2},
  VectorPoints -> 10, VectorStyle -> Arrowheads[0.01], VectorScale -> Large ]
```



Now we will do some circuits.

The most important non-trivial circuit is an RC filter.
This is where a resistor and capacitor are in series and we have to measure the voltage in between.



The units are Volts, Amperes, Ohms, and Farads.

We will setup the input voltage to be an oscillating voltage

This is a little more advanced for High School, but it is not difficult to teach with Mathematica.

```
In[ ]:= Clear[R1]; Clear[C1]; Clear[V0]; Clear[V];
R1 = 1000; (* Ohm *)
C1 = 10^-6; (* Farad *)
V0 = 1; (* Volts *)
omega = 2 * Pi * 600; (* The frequency is 600 Hz *)
V[t_] := V0 * Sin[omega * t];
```

The charge on the capacitor of $q[t]$ changes according to the current. The input voltage must be equal to the sum of the voltages across R and C.

```
In[ ]:= eq1 = V[t] == R1 * D[q[t], t] + q[t] / C1;
In[ ]:= soln = DSolve[eq1 && q[0] == 0, q[t], t] // Flatten // Simplify
```

```
Out[ ]:= {q[t] -> 
$$\frac{6 e^{-1000 t} \pi - 6 \pi \cos[1200 \pi t] + 5 \sin[1200 \pi t]}{200000 (25 + 36 \pi^2)}$$
}
```


This looks complicated. But it is easy to see what is happening. It is in this form

Constant * (Exp[-t/T] + A Cos[w t + phi])

The time constant $T = R \cdot C$.

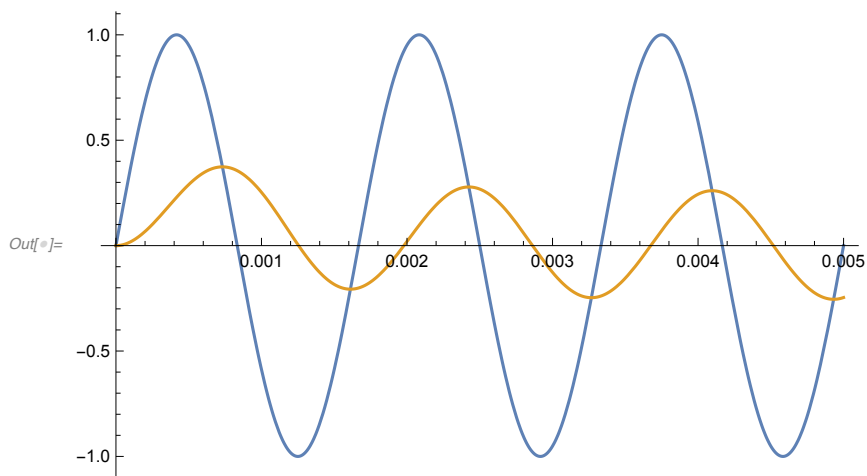
This solution is only useful for $t > 0$. For backwards time it will give nonsensical answers. It is as if the voltage was turned on at $t = 0$.

The first part slowly dies out as the capacitor charges up and then we just have a slightly shifted oscillation of the same frequency.

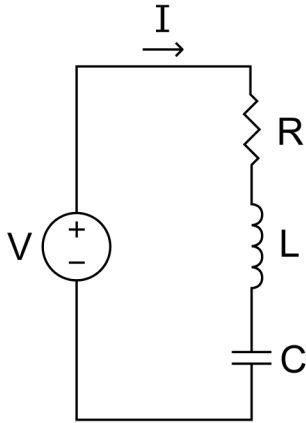
The voltage on the capacitor is $q[t]/C$

Try changing the frequency to see what happens. As the frequency goes higher the amplitude of the voltage on the capacitor falls.

```
In[ ]:= Plot[{V[t], (1/C1) * q[t] /. soln}, {t, 0., 0.005}]
```



What about a RLC circuit. What does that do ?



```
In[ ]:= Clear[R1]; Clear[C1]; Clear[V0]; Clear[V];
R1 = 1000; (* Ohm *)
C1 = 10^-6; (* Farad *)
L1 = 10^-1; (* Henry *)
V0 = 1; (* Volts *)
omega = 2 * Pi * 2500; (* The frequency is 600 Hz *)
omega = 1/Sqrt[L1 * C1];
V[t_] := V0 * Sin[omega * t];
```

The input voltage must be equal to the sum of the voltages across R and C and L

Voltage across R is just current * R

Voltage across C is just charge/C

Voltage across L is (change-in-current)*L

We need some initial conditions: charge on C is 0 at time = 0 and current at t=0 is zero

```
In[ ]:= eq2 = V[t] == R1 * D[q[t], t] + q[t] / C1 + L1 * D[q[t], {t, 2}];
```

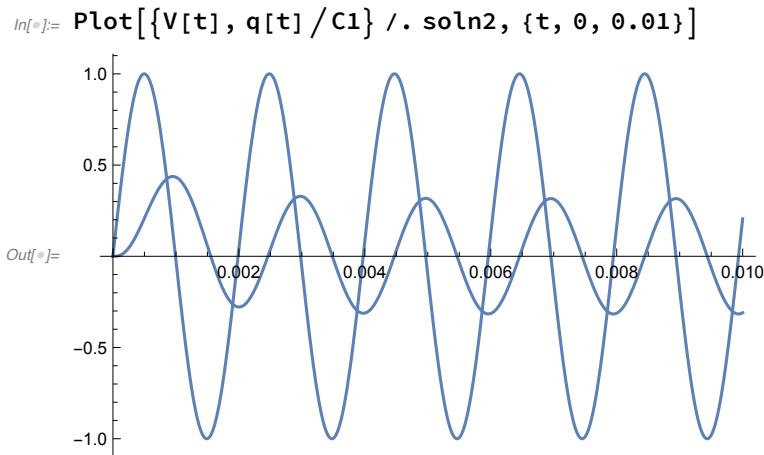
```
In[ ]:= soln2 = DSolve[eq2 && q[0] == 0 && q'[0] == 0, q[t], t] // Flatten // Simplify
```

```
Out[ ]:= {q[t] -> -\frac{1}{30000000\sqrt{2}} \left( e^{-1000(5+\sqrt{15})t} (5\sqrt{3} - 3\sqrt{5} - (5\sqrt{3} + 3\sqrt{5}) e^{2000\sqrt{15}t}) + 6\sqrt{5} \cos[1000\sqrt{10}t] \right)}
```

This is very difficult to understand from this complicated equation. We will not try here. For a proper understanding we need to use Fourier or Laplace

transforms. But Mathematica lets us explore this. Let's just plot it without too much concern.

The circuit displays resonance characteristics.



We will finish with a demonstration of waves on a string.

Imagine a string with length L .

When it is plucked, vibrations are excited with fundamental wavelength of $2L$.

frequency $\omega = v \cdot 2\pi / (2L)$ where v is the velocity of the wave.

Higher vibrations are also excited with different amplitudes.

$$\omega_i = i \cdot \omega$$

This creates a pattern that moves back and forth on the string.

Setup frequency and wave number parameters.

```
In[106]:= L = 10; V = 1.;
Kn = Table[i * Pi / L, {i, 1, 11}];
 $\omega$ n = Table[i * V * Pi / L, {i, 1, 11}];
Bn1 = {1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0};
Bn = {1, 0, 1/3, 0, 1/5, 0, 1/7, 0, 1/9, 0, 1/11};
```

setup the wavefunction as a function of time and space. It is a sum of sines and cosines.

```
In[105]:= Pckt[x_, t_] := Sum[Bn[[i]] * Cos[ $\omega$ n[[i]] * t] * Sin[Kn[[i]] * x], {i, 1, 11}];
```

We are now going to introduce how to animate a plot. We are going to plot the wavefunction over the length of the string, but then move the time variable from 1 to 20 and see how the wave changes. You can change the Bn parameters and see what the wave looks like.

```
In[ ]:= Animate[Plot[Pckt[x, t], {x, 0, 10}, PlotRange -> {{0, 10}, {-2., 2.}}], {t, 1, 20}]
```

