SM predictions for $R(D)$ and $R(D^*)$

1606.08030 & 1707.09509 with D.Bigi and S. Schacht

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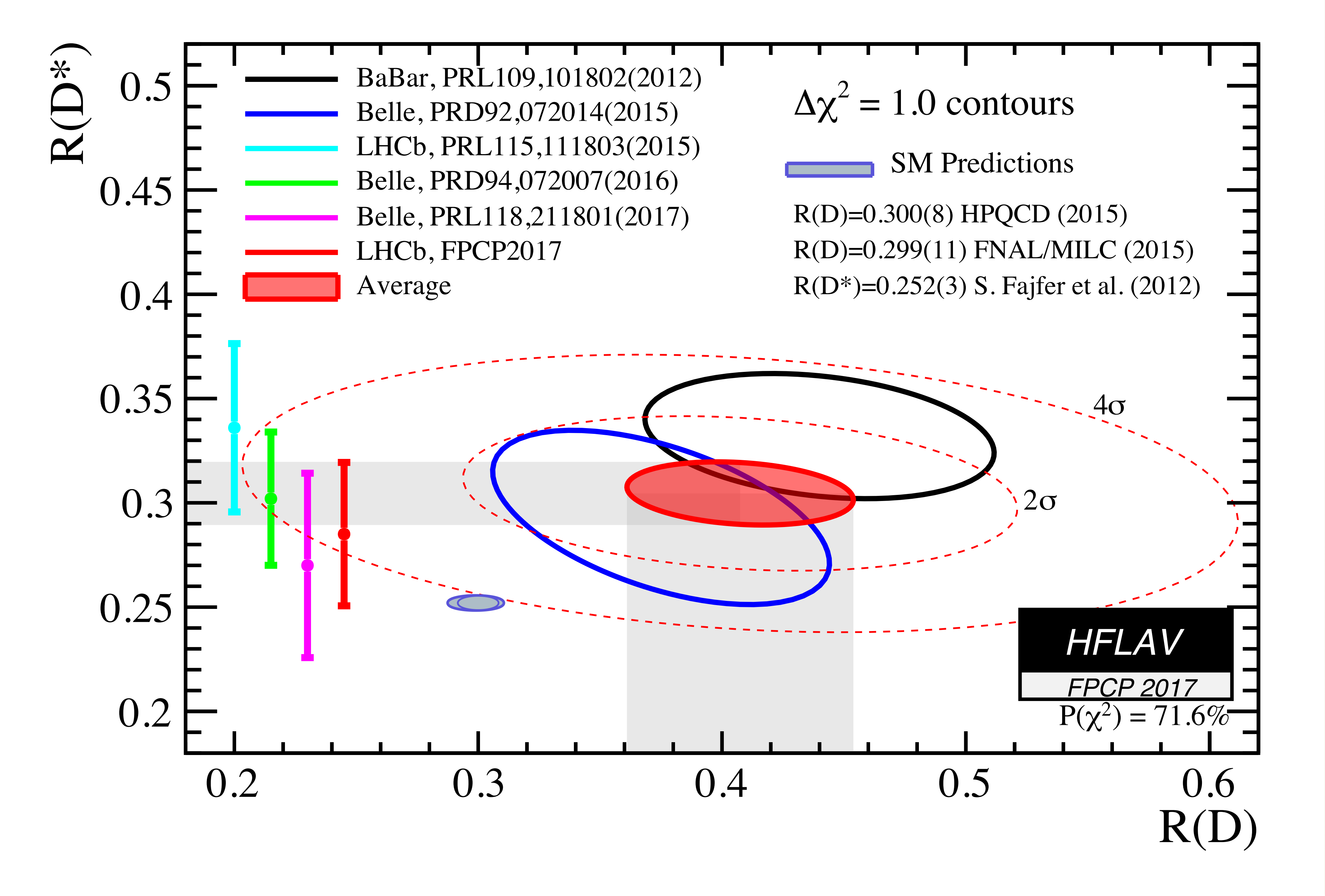
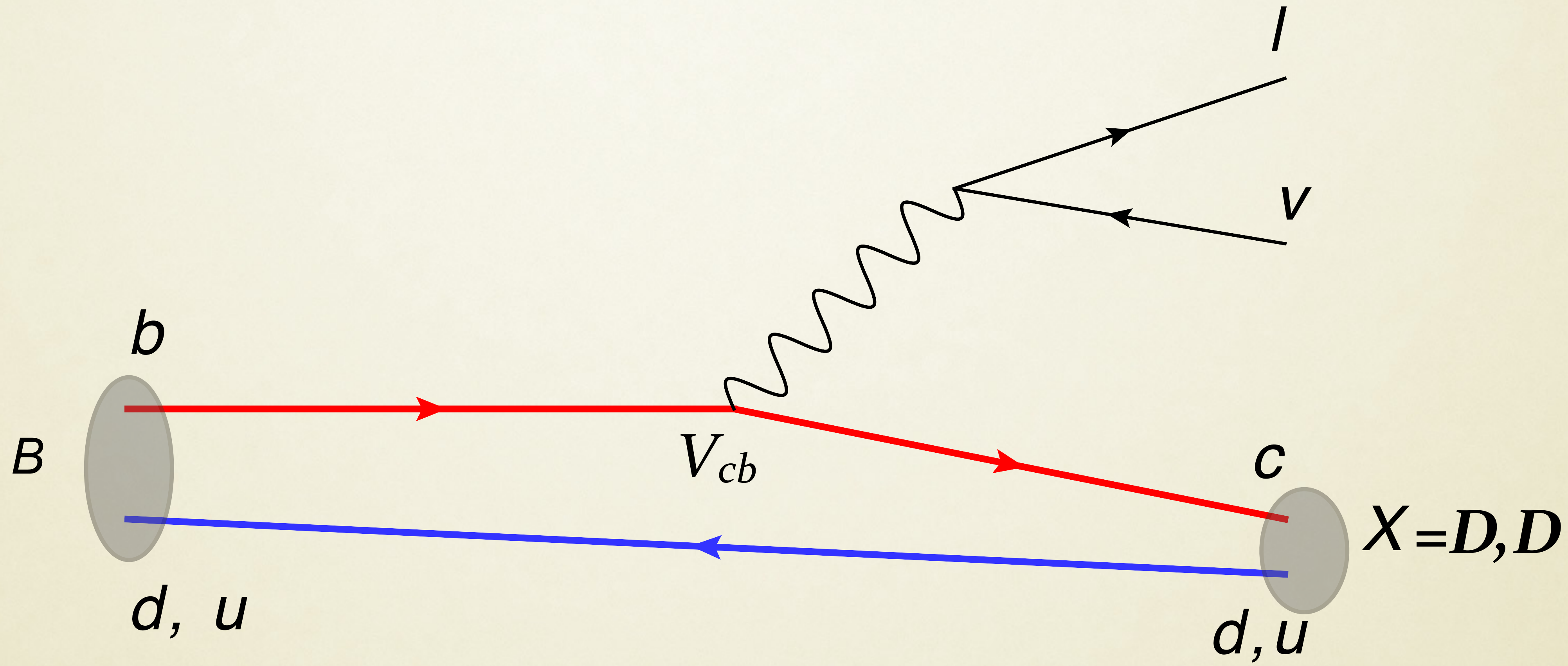
LEPTON FLAVOUR UNIVERSALITY VIOLATION?

$$R(D^{(*)}) = \frac{\mathcal{B}(B \to D^{(*)}\tau\nu)}{\mathcal{B}(B \to D^{(*)}\mu\nu)}$$



SEMILEPTONIC B DECAYS

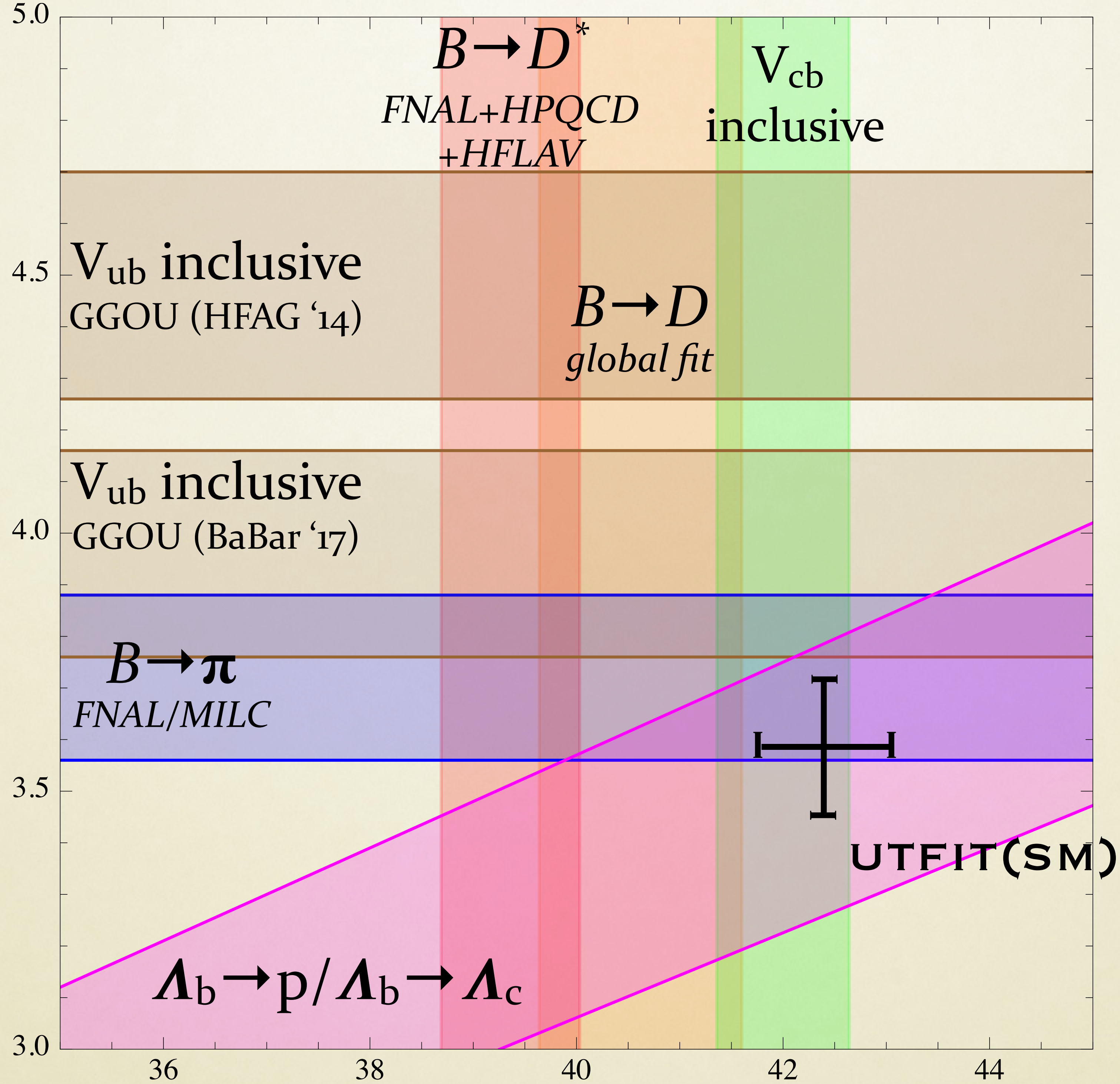


Allow for the determination of V_{cb} , which drops out of $R(D,D^*)$.
There are 1(2) and 3(4) FFs for D and D^* for light (heavy) leptons, for instance

$$\langle D|\bar{c}\gamma^\mu b|B\rangle \propto f_{+,0}(q^2)$$

V_{cb} and V_{ub} status

1σ ranges



$10^3\ V_{ub}$

$10^3\ V_{cb}$

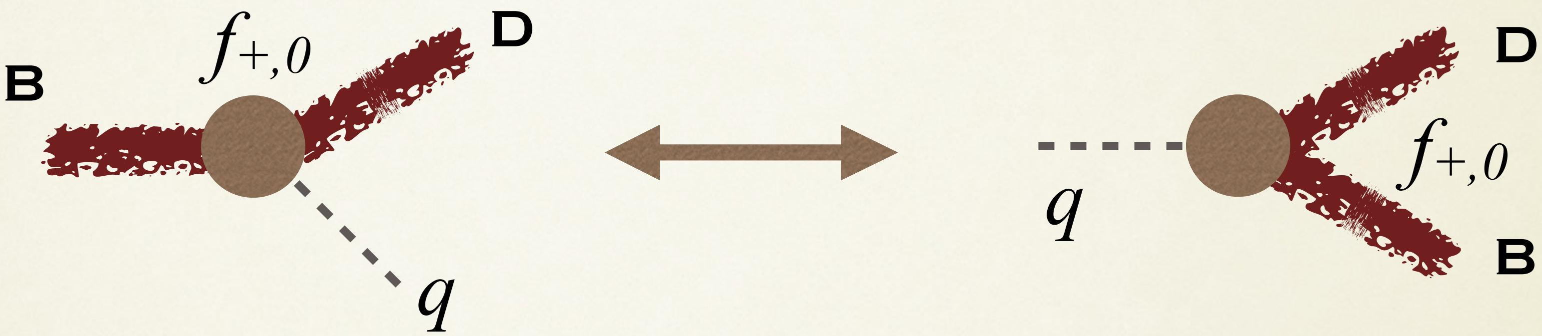
New V_{ub} incl
by Babar
in agreement
with exclusive
but needs checks
PRD 95 (2017) 7, 072001

New HPQCD
$B \to D^*$ result
at zero recoil

New Belle $B \to D^*$
result: with FNAL
V_{cb}=37.4(1.3) 10^{-3}

Model independent FF parametrization

crossing + analitycity

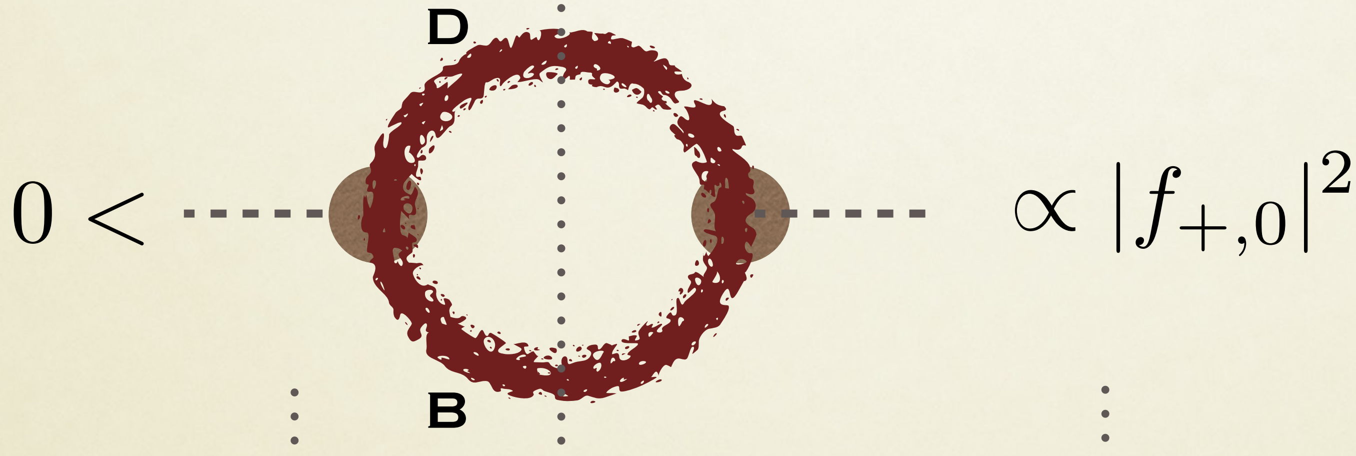


physical semileptonic region

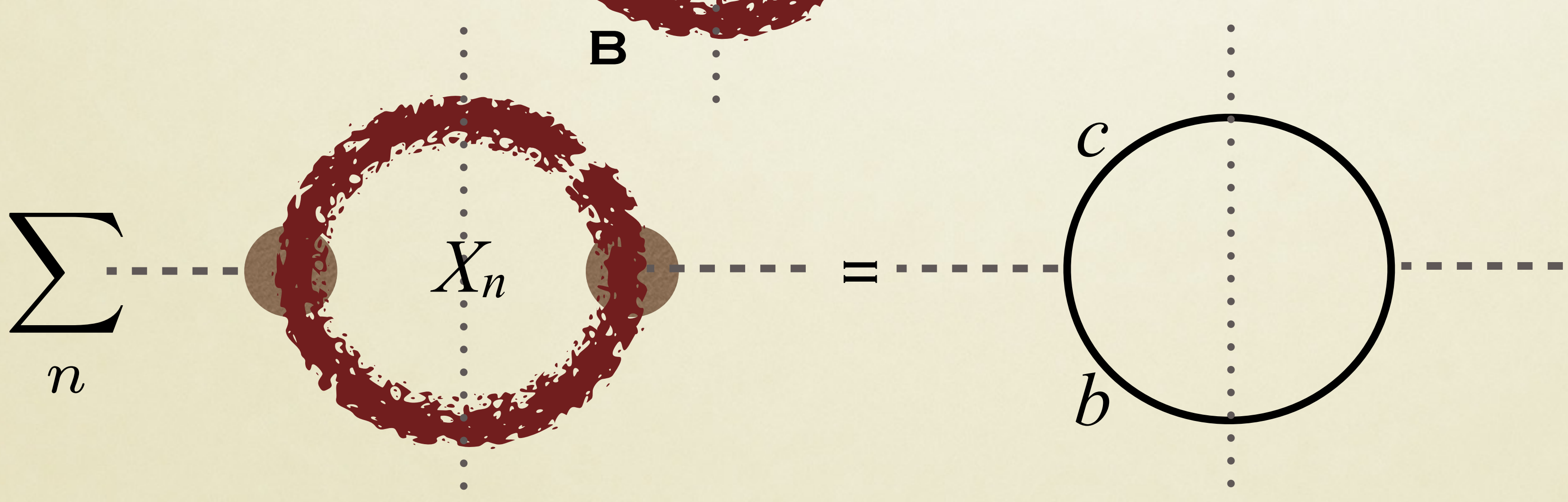
$m_\ell^2 \leq q^2 \leq (m_B - m_D)^2$

2-point correlator cuts

$q^2 \geq (m_B + m_D)^2$



poles at $q^2=m^2{}_{Bc}$ etc



+pert corr

(+condensates)

using quark-hadron duality. dispersion relations→ global QHD

UNITARITY CONSTRAINTS

$$\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)\Pi^T(q^2) + \frac{q^\mu q^\nu}{q^2}\Pi^L(q^2) \equiv i\int d^4x\, e^{iqx}\langle 0|T J^\mu(x) J^{\dagger\nu}(0)|0\rangle$$

$$\chi^L(q^2) = \frac{\partial \Pi^L}{\partial q^2}, \qquad \chi^T(q^2) = \frac{1}{2}\frac{\partial^2 \Pi^T}{\partial (q^2)^2}$$

SATISFY UNSUBTRACTED DISP REL, PERT CALCULATION FOR $q^2=0$ Boyd, Grinstein, Lebed 1995

$$\chi^T_V(0) = [5.883 + 0.552_{\alpha_s} + 0.050_{\alpha_s^2}]\ 10^{-4}\,\text{GeV}^{-2} = 6.486(48)\ 10^{-4}\,\text{GeV}^{-2}$$

$$\chi^L_V(0) = [5.456 + 0.782_{\alpha_s} - 0.034_{\alpha_s^2}]\ 10^{-3} = 6.204(81)\ 10^{-3}$$ & analogous for axial etc

USING UP-TO-DATE QUARK MASSES AND 3LOOP CALCULATION Grigo et al 2012

$$\tilde{\chi}^T(0) = \chi^T(0) - \sum_{n=1,2} \frac{f_n^2(B_c^*)}{M_n^4(B_c^*)}$$

SUBTRACT BOUND STATE CONTRIBUTIONS

Type	Mass (GeV)	Decay constants (GeV)
1^-	6.329(3)	0.422(13)
1^-	6.920(20)	0.300(30)
1^-	7.020	
1^-	7.280	
0^+	6.716	
0^+	7.121	

UNITARITY CONSTRAINTS

$$z = \frac{\sqrt{1+w} - \sqrt{2}}{\sqrt{1+w} + \sqrt{2}} \qquad w = \frac{m_B^2 + m_{D^*}^2 - q^2}{2 m_B m_{D^*}} \qquad 0 < z < 0.056$$

$$f_i(z) = \frac{\sqrt{\chi_i}}{P_i(z)\phi_i(z)} \sum_{n=0}^{\infty} a_n^i z^n$$

BGL Boyd Grinstein Lebed 1997

BLASCHKE FACTORS REMOVE POLES BELOW THRESHOLD

PHASE SPACE FACTORS

TRUNCATED AT ORDER N

$$\sum_{n=0}^{N} (a_n^i)^2 < 1$$

WEAK UNITARITY CONSTRAINTS

assuming saturation by single hadron channel

For massless leptons only 3 form factors $A_{1,5}$ V_4 contribute to $B \to D^* l\nu$

$$\sum_{n=0}^{N} (a_n^{V_4})^2 < 1, \qquad \sum_{n=0}^{N} \left[(a_n^{A_1})^2 + (a_n^{A_5})^2 \right] < 1$$

vector current　　　　axial vector current

STRONG UNITARITY CONSTRAINTS

Information on other channels makes the constraints tighter.
HQS implies that all $B^{(*)}\to D^{(*)}$ ff either vanish or are prop to the Isgur-Wise function: any ff F_j can be expressed as

$$F_j(z) = \left(\frac{F_j}{F_i}\right)_{\rm HQET} F_i(z)$$

which leads to (hyper)ellipsoids in the a_i space for S, P, V, A currents

$$\sum_{i=1}^{H}\sum_{n=0}^{N} b_{in}^2 < 1$$

CLN exploit NLO HQET relations between form factors + QCD sum rules to reduce parameters for ff... **up to < 2% uncertainty (?), never included in exp analysis.**

CAPRINI
LELLOUCH
NEUBERT
CLN
1998

$$f_+(z) \simeq f_+(0)\left[1 - 8\rho_1^2 z + (51\rho_1^2 - 10)z^2 - (252\rho_1^2 - 84)z^3)\right]$$

$$\frac{f_0(z)}{f_+(z)} \simeq \left(\frac{2\sqrt{r}}{1+r}\right)^2 \frac{1+w}{2}\, 1.0036\left[1 - 0.0068 w_1 + 0.0017 w_1^2 - 0.0013 w_1^3\right]$$

$$h_{A1}(z) = h_{A1}(1)\left[1 - 8\rho^2 z + (53\rho^2 - 15)z^2 - (231\rho^2 - 91)z^3\right]$$

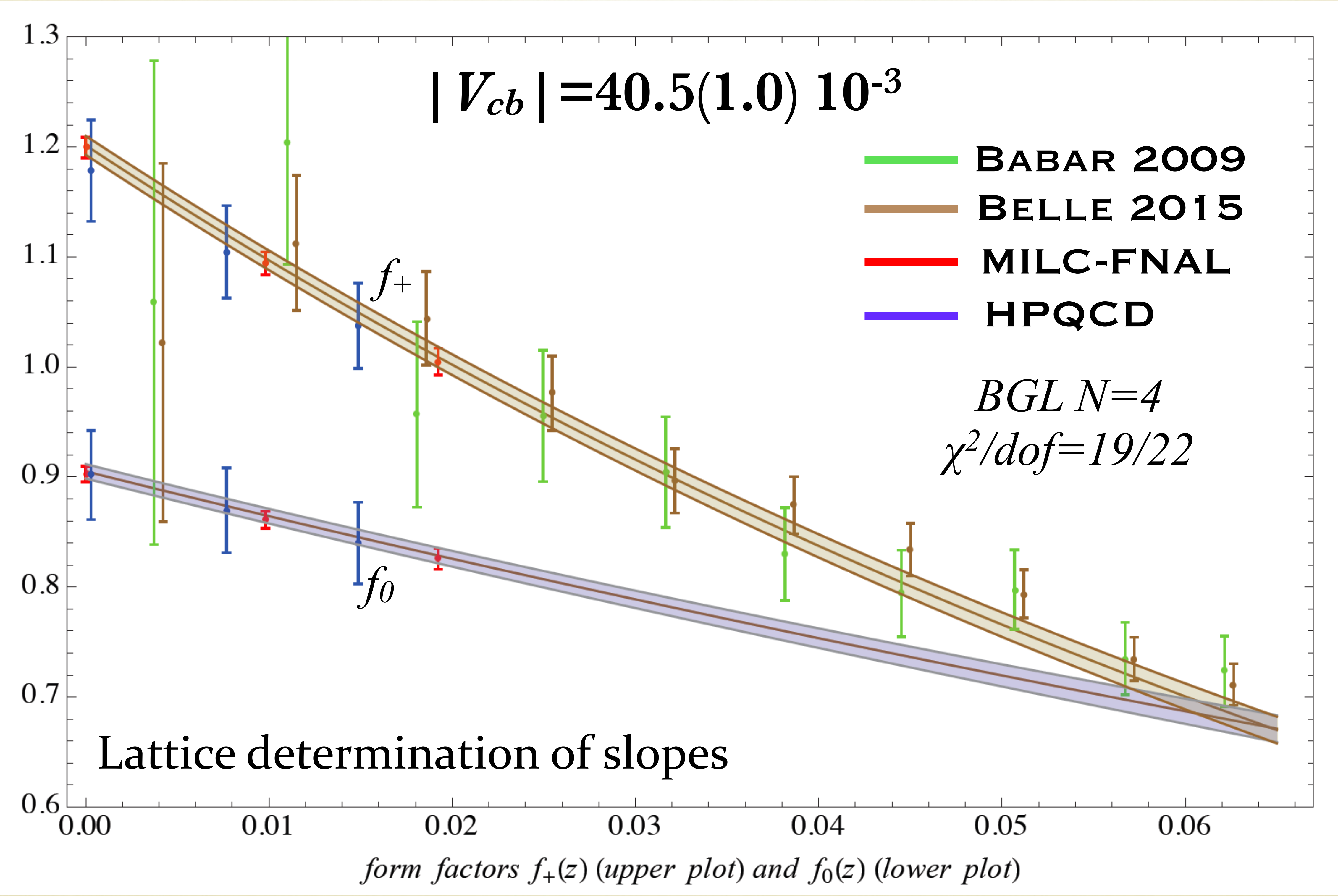
$$R_1(w) = R_1(1) - 0.12 w_1 + 0.05 w_1^2$$

$$R_2(w) = R_2(1) + 0.11 w_1 - 0.06 w_1^2$$

$$w_1 = w - 1$$

Global fit to $B \to Dl\nu$

D.Bigi, PG
arXiv:1606.08030



Global fit to $B\rightarrow Dl\nu$

- ***$|V_{cb}|$=40.49(0.97) 10^{-3}*** compatible with inclusive, same for BGL, BCL parametrizations
- Constrained fit with **strong unitarity bounds** (weak bounds lead to similar results with slightly larger errors)
- CLN relies heavily on HQET: its intrinsic uncertainties can no longer be neglected
- fit assumes no correlation between FNAL and HPQCD, 3% syst error on Babar data, correct treatment of last bin, no finite size bin effect.
- **Non-zero recoil lattice results are <u>crucial</u>:** only zero recoil leads to $|V_{cb}|$=39.6(2.0) 10^{-3} (BGL)
- Possible improvements from more precise data (Belle-II, reanalysis of Babar data), lattice calculations, QED corrections
- ***R(D)=0.299(3)*** ***2.4***σ from HFLAV average 0.407(46)

Ref.	$R(D)$	Deviation
Experiment [HFLAV update]	0.407(39)(24)	—
2016/17 theory results, using new lattice and exp. data:		
[Bigi Gambino 1606.08030]	0.299(3)	2.4σ
[Bernlochner Ligeti Papucci Robinson 1703.05330]	0.299(3)	2.4σ
[Jaiswal Nandi Patra 1707.09977]	0.302(3)	2.3σ
2012 theory results:		
[Fajfer Kamenik Nisandzic 1203.2654]	0.296(16)	2.3σ
[Celis Jung Li Pich 1210.8443]	$0.296\binom{8}{6}(15)$	2.3σ
[Tanaka Watanabe 1212.1878]	0.305(12)	2.2σ

LATTICE ONLY RESULTS

HPQCD 2015: 0.300(8), FNAL/MILC 2015: 0.299(11)

$|V_{cb}|$ from $B \to D^* \ell \nu$

So far LQCD gives only light lepton FF at zero recoil, w=1, where rate vanishes and the FF is

$$\mathcal{F}(1) = \eta_A \left[1 + O\left(\frac{1}{m_c^2}\right) + ...\right]$$

Exp error only ~1.3%: $\mathcal{F}(1)\eta_{ew}|V_{cb}|$ =35.61(45) x10^{-3} (extrapolation to zero recoil with CLN parameterization)

Two unquenched calculations

$\mathcal{F}(1)$ =0.906(13)

Bailey et al 1403.0635 (FNAL/MILC)

$\mathcal{F}(1)$ =0.881(22)

Harrison et al 1711.11013 (HPQCD)

Using their average 0.900(11):

$|V_{cb}|$= 39.31(72) 10^{-3}

~ 2.8σ or ~ 7% from inclusive determination 42.00(65) 10^{-3}

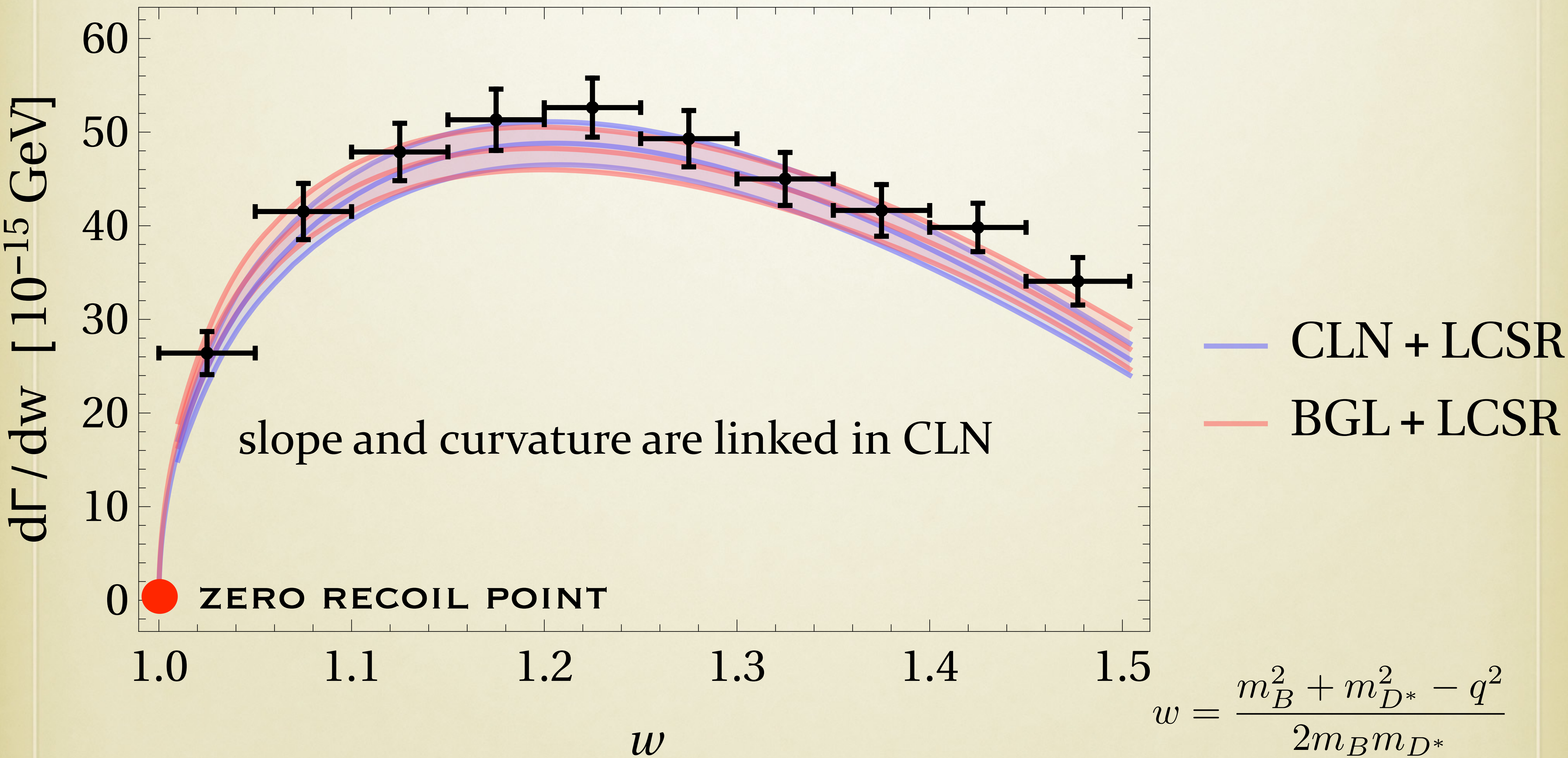
PG,Healey,Turczyk 2016

NB Heavy Quark Sum Rules estimate F(1)=0.86(2)

PG,Mannel,Uraltsev 2012

Preliminary Belle analysis of $B \to D^* l\nu$ 1702.01521

for the first time w and angular deconvoluted distributions independent of parameterization. All previous analyses are CLN based.



$$w = \frac{m_B^2 + m_{D^*}^2 - q^2}{2 m_B m_{D^*}}$$

Bands show two parametrizations both fitting data well, with 6% different V_{cb}

FITS WITH WEAK CONSTRAINTS

1703.06124

CLN

CLN Fit:	Data + lattice	Data + lattice + LCSR
χ^2/dof	34.3/36	34.8/39
$\lvert V_{cb}\rvert$	0.0382 (15)	0.0382 (14)
$\rho^2_{D^*}$	1.17 $(^{+15}_{-16})$	1.16 (14)
$R_1(1)$	1.391 $(^{+92}_{-88})$	1.372 (36)
$R_2(1)$	0.913 $(^{+73}_{-80})$	0.916 $(^{+65}_{-70})$
$h_{A_1}(1)$	0.906 (13)	0.906 (13)

reproduces Belle's deconvoluted results. Best CLN analysis V_{cb}=0.0374(13)

BGL (N=2)

BGL Fit:	Data + lattice	Data + lattice + LCSR
χ^2/dof	27.9/32	31.4/35
$\lvert V_{cb}\rvert$	0.0417 $(^{+20}_{-21})$	0.0404 $(^{+16}_{-17})$
a_0^f	0.01223(18)	0.01224(18)
a_1^f	−0.054 $(^{+58}_{-43})$	−0.052 $(^{+27}_{-15})$
a_2^f	0.2 $(^{+7}_{-12})$	1.0 $(^{+0}_{-5})$
$a_1^{\mathcal{F}_1}$	−0.0100 $(^{+61}_{-56})$	−0.0070 $(^{+54}_{-52})$
$a_2^{\mathcal{F}_1}$	0.12 (10)	0.089 $(^{+96}_{-100})$
a_0^g	0.012 $(^{+11}_{-8})$	0.0289 $(^{+57}_{-37})$
a_1^g	0.7 $(^{+3}_{-4})$	0.08 $(^{+8}_{-22})$
a_2^g	0.8 $(^{+2}_{-17})$	−1.0 $(^{+20}_{-0})$

see also Grinstein & Kobach, 1703.08170
Jaiswal et al., 1707.09977
Bernlochner et al. 1708.07134

9% and 6% (with LCSR) difference in V_{cb}

LCSR: Light Cone Sum Rule results from Faller et al, 0809.0222

$$h_{A_1}(w_{max}) = 0.65(18),$$
$$R_1(w_{max}) = 1.32(4), \quad R_2(w_{max}) = 0.91(17)$$

Lattice will clarify...

Future lattice fits	χ^2/dof	$\lvert V_{cb}\rvert$
CLN	56.4/37	0.0407 (12)
CLN+LCSR	59.3/40	0.0406 (12)
BGL	28.2/33	0.0409 (15)
BGL+LCSR	31.4/36	0.0404 (13)

assuming Lattice QCD will provide an estimate of the slope with 5% accuracy

$$\frac{\partial \mathcal{F}}{\partial w}\Big|_{w=1} = -1.44 \pm 0.07$$

HQS breaking in FF relations

HQET: $F_i(w) = \xi(w)\left[1 + c^i_{\alpha_s}\frac{\alpha_s}{\pi} + c^i_b\,\epsilon_b + c^i_c\,\epsilon_c + \dots\right]$ $\quad \epsilon_{b,c} = \overline{\Lambda}/2m_{b,c}$

$$\eta(1) = 0.62 \pm 0.20\,, \qquad \eta'(1) = 0.0 \pm 0.2\,,$$
$$\hat\chi_2(1) = -0.06 \pm 0.02 \qquad \hat\chi'_2(1) = 0 \pm 0.02$$
$$\hat\chi'_3(1) = 0.04 \pm 0.02\,.$$

Subleading IW functions from QCD sumrules

Neubert, Ligeti, Nir 1992-93

Bernlochner et al 1703.05330

RATIOS $\frac{F_j(w)}{V_1(w)} = A_j\left[1 + B_j\,w_1 + C_j\,w_1^2 + D_j\,w_1^3 + \dots\right]$ $\quad w_1 = w - 1$

Roughly $\epsilon_c \sim 0.25,$ $\quad \epsilon_c^2 \sim 0.06$ $\quad$ but coefficients?

$$\left.\frac{S_1(w)}{V_1(w)}\right|_{\rm LQCD} = 0.975(6) + 0.055(18)(w-1) + \dots \qquad \left.\frac{S_1(w)}{V_1(w)}\right|_{\rm HQET} = 1.021(30) - 0.044(64)(w-1) + \dots$$

$$\left.\frac{A_1(1)}{V_1(1)}\right|_{\rm LQCD} = 0.857(15), \qquad \left.\frac{A_1(1)}{V_1(1)}\right|_{\rm HQET} = 0.966(28)$$

$$\left.\frac{S_1(1)}{A_1(1)}\right|_{\rm LQCD} = 1.137(21). \qquad \left.\frac{S_1(1)}{A_1(1)}\right|_{\rm HQET} = 1.055(2),$$

5-13% differences, always > NLO correction

The size of NLO corrections varies strongly. Some ff are protected by Luke's theorem (no 1/m corrections at zero recoil), others are linked by kinematic relations at max recoil to those protected

NNLO corrections can be sizeable and are naturally O(10-20)%

$$\frac{F_j(w)}{V_1(w)} = A_j \left[1 + B_j\, w_1 + C_j\, w_1^2 + D_j\, w_1^3 + \dots\right]$$

F_j	A_j	B_j	C_j	D_j
S_1	1.0208	−0.0436	0.0201	−0.0105
S_2	1.0208	−0.0749	−0.0846	0.0418
S_3	1.0208	0.0710	−0.1903	0.0947
P_1	1.2089	−0.2164	0.0026	−0.0007
P_2	0.8938	−0.0949	0.0034	−0.0009
P_3	1.0544	−0.2490	0.0030	−0.0008
V_1	1	0	0	0
V_2	1.0894	−0.2251	0.0000	0.0000
V_3	1.1777	−0.2651	0.0000	0.0000
V_4	1.2351	−0.1492	−0.0012	0.0003
V_5	1.0399	−0.0440	−0.0014	0.0004
V_6	1.5808	−0.1835	−0.0009	0.0003
V_7	1.3856	−0.1821	−0.0011	0.0003
A_1	0.9656	−0.0704	−0.0580	0.0276
A_2	0.9656	−0.0280	−0.0074	0.0023
A_3	0.9656	−0.0629	−0.0969	0.0470
A_4	0.9656	−0.0009	−0.1475	0.0723
A_5	0.9656	0.3488	−0.2944	0.1456
A_6	0.9656	−0.2548	0.0978	−0.0504
A_7	0.9656	−0.0528	−0.0942	0.0455

Fit to new Belle's data + total branching ratio (world average)

with strong unitarity bounds (with uncertainties & LQCD inputs)

for reference CLN fit: $|V_{cb}|$=0.0392(12)

BGL Fit:	Data + lattice	Data + lattice + LCSR	Data + lattice	Data + lattice + LCSR
unitarity	weak	weak	strong	strong
χ^2/dof	28.2/33	32.0/36	29.6/33	33.1/36
$\|V_{cb}\|$	0.0424 (18)	0.0413 (14)	0.0415 (13)	$0.0406\,\left(^{+12}_{-13}\right)$
$a_0^{A_1}$	0.01218(16)	0.01218(16)	0.01218(16)	0.01218(16)
$a_1^{A_1}$	$-0.053\,\left(^{+56}_{-44}\right)$	$-0.052\,\left(^{+25}_{-14}\right)$	$-0.046\left(^{+34}_{-18}\right)$	$-0.029\left(^{+21}_{-13}\right)$
$a_2^{A_1}$	$0.2\,\left(^{+8}_{-12}\right)$	$0.99\,\left(^{+0}_{-46}\right)$	$0.48\left(^{+2}_{-92}\right)$	$0.5\left(^{+0}_{-3}\right)$
$a_1^{A_5}$	$-0.0101\,\left(^{+59}_{-55}\right)$	$-0.0072\,\left(^{+52}_{-50}\right)$	$-0.0063\left(^{+36}_{-11}\right)$	$-0.0051\left(^{+49}_{-13}\right)$
$a_2^{A_5}$	0.12 (10)	$0.092\,\left(^{+92}_{-95}\right)$	$0.062\left(^{+4}_{-64}\right)$	$0.065\,\left(^{+9}_{-89}\right)$
$a_0^{V_4}$	$0.011\,\left(^{+10}_{-8}\right)$	$0.0286\,\left(^{+55}_{-36}\right)$	$0.0209\left(^{+44}_{-0}\right)$	$0.0299\left(^{+53}_{-35}\right)$
$a_1^{V_4}$	$0.7\,\left(^{+3}_{-4}\right)$	$0.08\,\left(^{+8}_{-22}\right)$	$0.33\left(^{+4}_{-17}\right)$	$0.04\left(^{+7}_{-20}\right)$
$a_2^{V_4}$	$0.7\,\left(^{+2}_{-17}\right)$	$-1.0\,\left(^{+20}_{-0}\right)$	$0.6\left(^{+2}_{-13}\right)$	$-0.9\left(^{+18}_{-0}\right)$

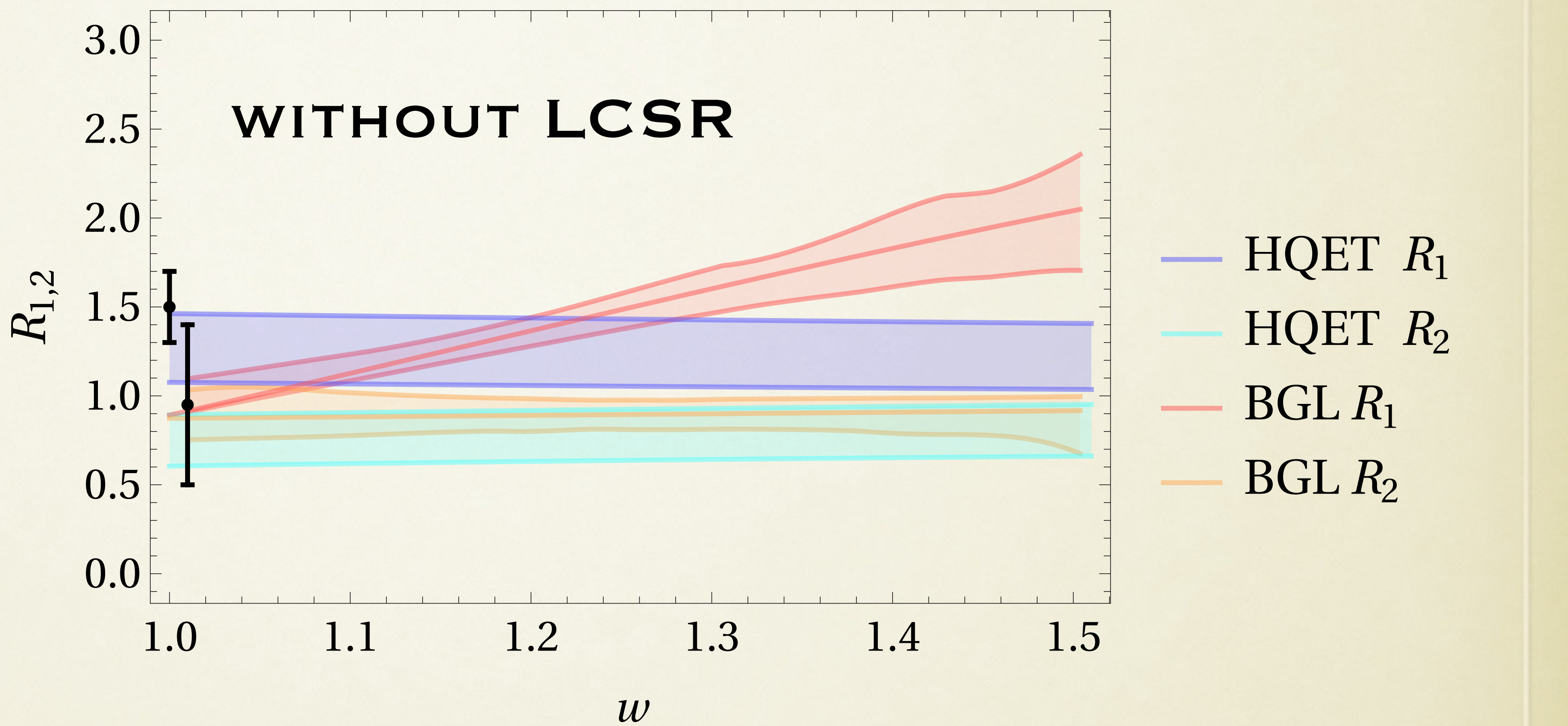
1707.09509

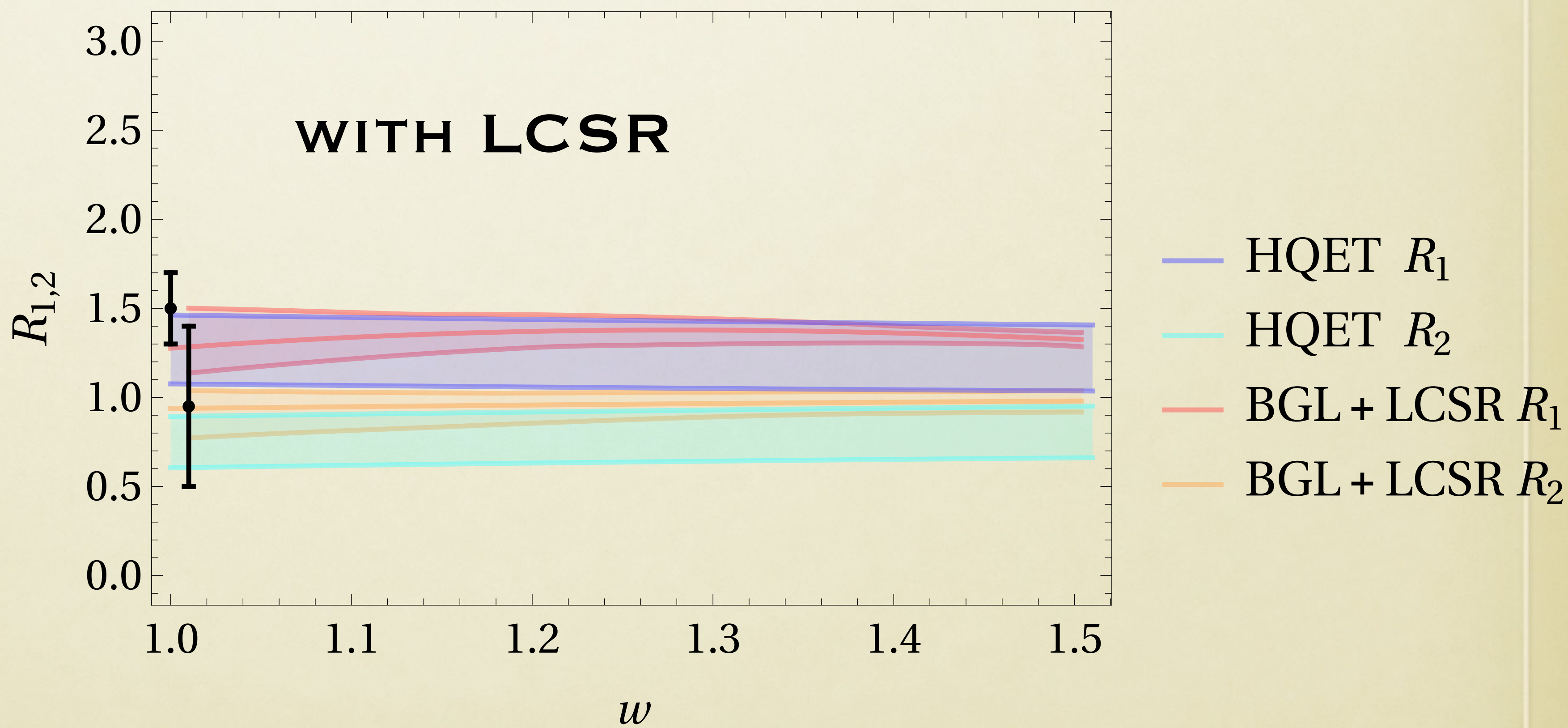
Using strong unitarity bounds brings BGL closer to CLN and reduce uncertainties but 3.5-5% difference persists

Consistency with HQS

Comparison of $R_{1,2}$ from BGL fit vs HQET+QCD sum rule predictions (with parametric + 15% th uncertainty)

black points from preliminary FNAL-MILC calculation according to Bernlochner et al 1708.07134 (before continuum and chiral extrapolations...)





Calculation of $R(D^*)$

$$\frac{d\Gamma_\tau}{dw} = \frac{d\Gamma_{\tau,1}}{dw} + \frac{d\Gamma_{\tau,2}}{dw} \qquad \begin{cases} \dfrac{d\Gamma_{\tau,1}}{dw} = \left(1 - \dfrac{m_\tau^2}{q^2}\right)^2 \left(1 + \dfrac{m_\tau^2}{2q^2}\right) \dfrac{d\Gamma}{dw}, \\ \dfrac{d\Gamma_{\tau,2}}{dw} = k \dfrac{m_\tau^2 (m_\tau^2 - q^2)^2 r^3 (1+r)^2 (w^2-1)^{\frac{3}{2}} P_1(w)^2}{(q^2)^3} \end{cases}$$

$\pm 30\%$!!

$$R(D^*) = R_{\tau,1}(D^*) + R_{\tau,2}(D^*)$$

$$R_{\tau,1}(D^*) = \frac{\int_1^{w_{\tau,\max}} dw \, d\Gamma_{\tau,1}/dw}{\int_1^{w_{\max}} dw \, d\Gamma/dw}$$

$$R_{\tau,2}(D^*) = \frac{\int_1^{w_{\tau,\max}} dw \, d\Gamma_{\tau,2}/dw}{\int_1^{w_{\max}} dw \, d\Gamma/dw}$$

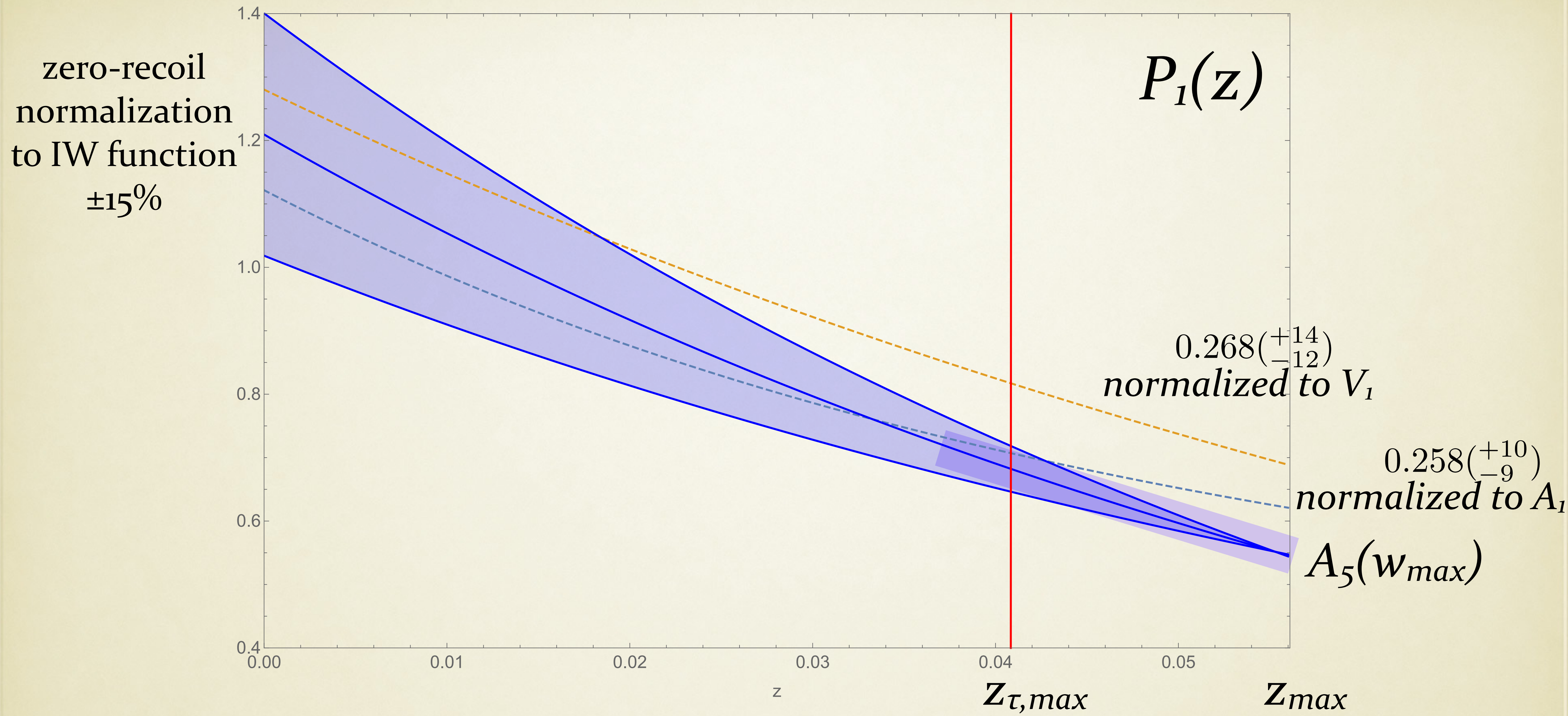
$$w_{\max} \approx 1.56, \qquad w_{\tau,\max} \approx 1.35$$

P_1 is a new FF, for which no lattice calculation is yet available, but its contribution is only ~10%

$$R_{\tau,1} \sim 90\% R_\tau \quad R_{\tau,2} \sim 10\% R_\tau$$

Again, normalize P_1 to one of the FF with proper uncertainties

$$P_1 = (P_1/V_1)_{\rm HQET} V_1^{exp} \qquad P_1 = (P_1/A_1)_{\rm HQET} A_1^{exp} \qquad P_1 = \xi(w)(1 + \ldots)_{\rm HQET}$$



Important endpoint constraint

$$P_1(w_{max}) = A_5(w_{max}) = 0.545 \pm 0.025$$

$$R(D^*) = 0.260(5)(6) = 0.260(8)$$

2.6σ from exp

Consistent with previous estimates but with larger uncertainty

Ref.	$R(D^*)$	Deviation
Experiment [HFLAV update]	0.304(13)(7)	—
2017 theory results, using new lattice and exp. data:		
[Bernlochner Ligeti Papucci Robinson 1703.05330]	0.257(3)	3.1σ
Our result [Bigi Gambino Schacht 1707.09509]	0.260(8)	2.6σ
[Jaiswal Nandi Patra 1707.09977]	0.257(5)	3.0σ
2012 theory results:		
[Fajfer Kamenik Nisandzic 1203.2654]	0.252(3)	3.5σ
[Celis Jung Li Pich 1210.8443]	0.252(2)(3)	3.4σ
[Tanaka Watanabe 1212.1878]	0.252(4)	3.4σ

Summary

- The SM prediction for *R(D)* has 1% accuracy and appears reliable (there are two lattice calculations at non-zero recoil of all relevant FFs)
- For $R(D^*)$ we only have the FF at zero recoil for light leptons and we have to rely on HQET + QCD sum rules. Hence larger uncertainty, but the anomaly persists. A LQCD determination of P_1 at zero recoil would cut the uncertainty by almost 2.
- Is the V_{cb} puzzle resolved? not quite yet… but a few pieces fit together. The uncertainty in $B \to D^* l\nu$ *was* underestimated, old data should be reanalised.
- We revisited main ideas behind CLN, using LQCD & exp results and conservative theory uncertainties, and obtained *strong* unitarity bounds on BGL coefficients. We do *not* give a simplified parametrization. Our results provide a good framework for future exp analyses.

Importance of $|V_{xb}|$

The most important CKM unitarity test is the Unitarity Triangle (UT)

V_{cb} plays an important role in UT

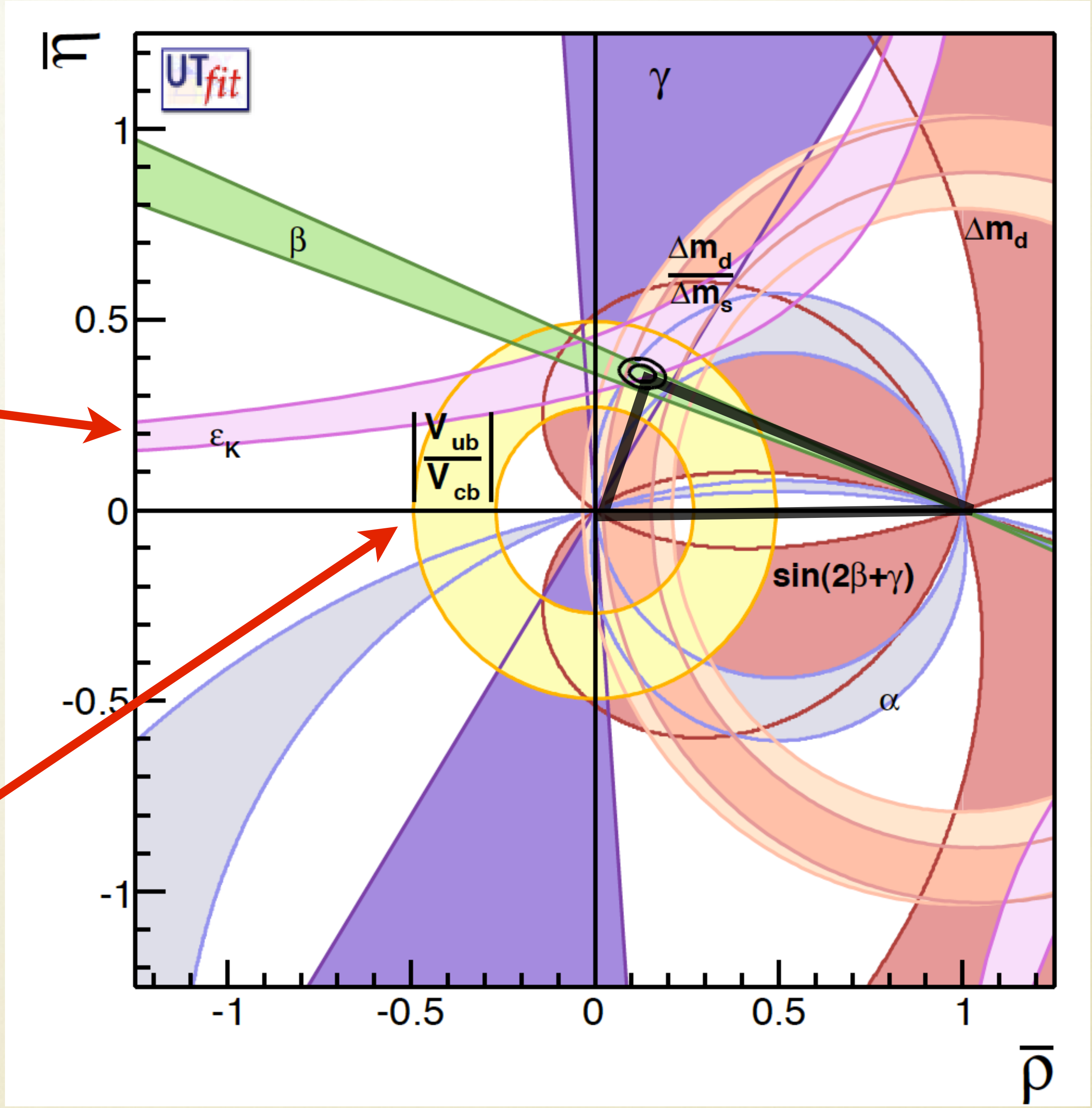
$$\varepsilon_K \approx x|V_{cb}|^4 + \ldots$$

and in the prediction of FCNC:

$$\propto |V_{tb}V_{ts}|^2 \simeq |V_{cb}|^2\left[1 + O(\lambda^2)\right]$$

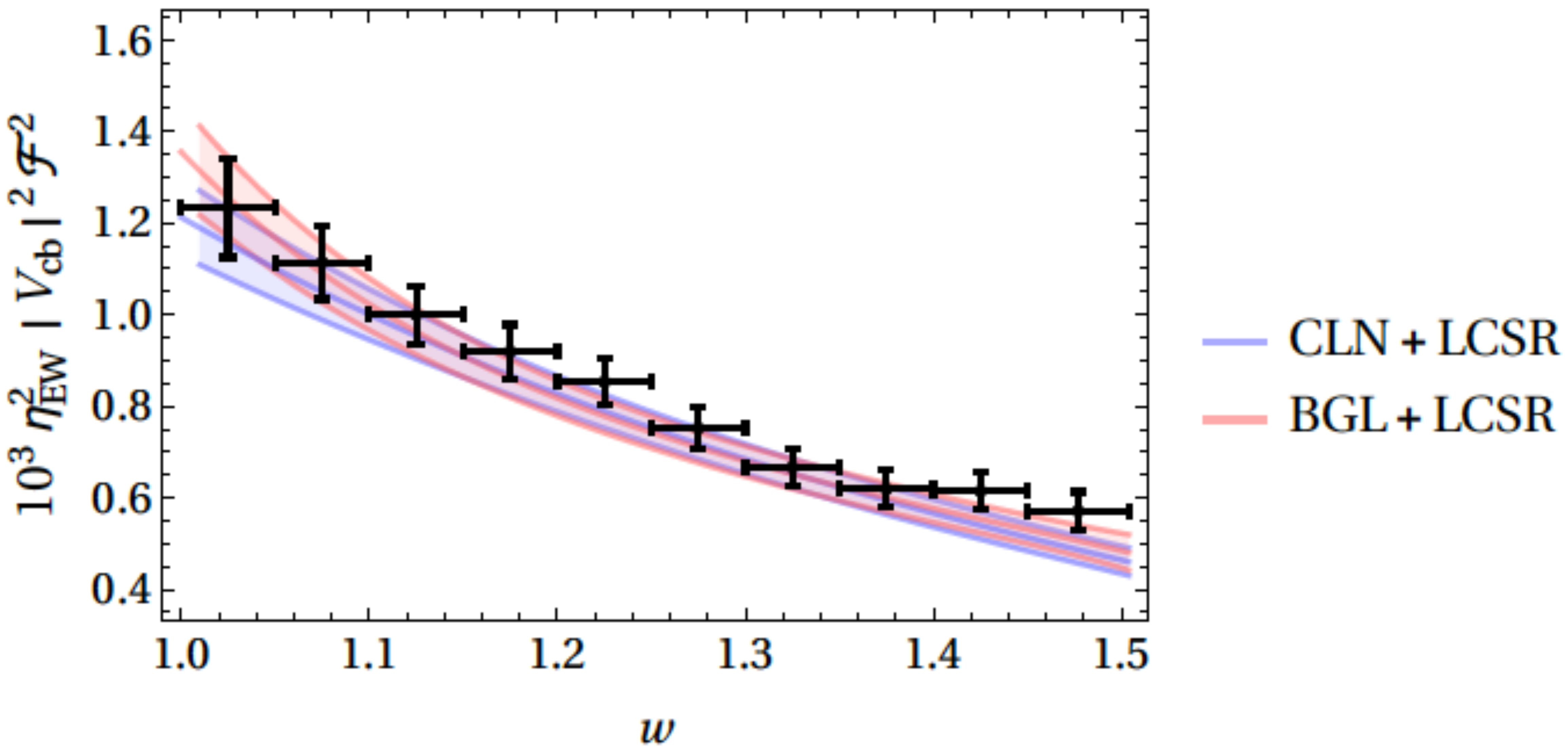
where it often dominates the theoretical uncertainty.

V_{ub}/V_{cb} constrains directly the UT



Since several years, exclusive decays prefer smaller $|V_{ub}|$ and $|V_{cb}|$

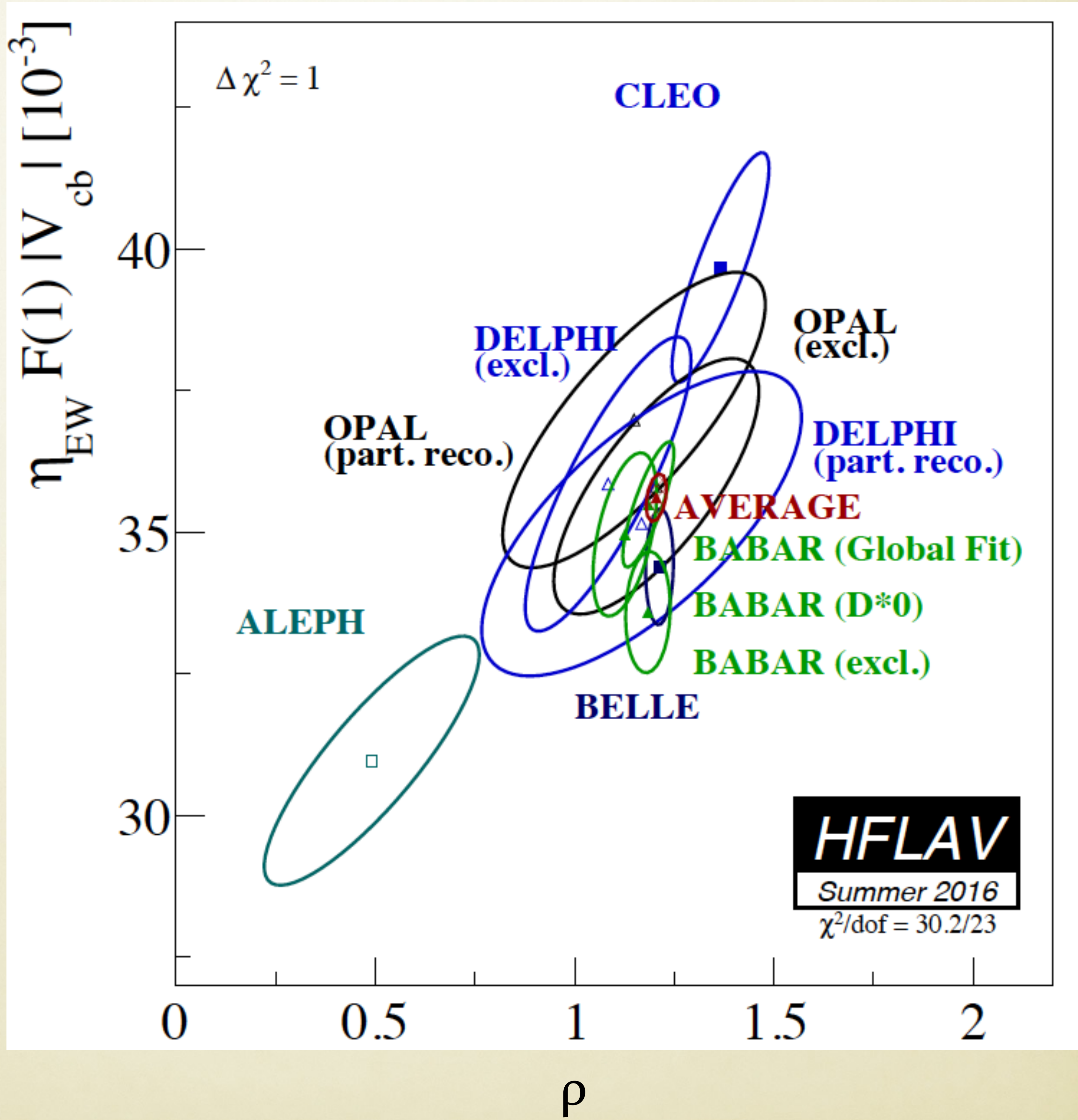
Main reason for deviation



- CLN fit has limited flexibility of slope.
  ➡ CLN band underestimates all three low recoil points.
- Extrapolation near $w = 1$ crucial: Lattice input for V_{cb} extraction.
- CLN fit with free floating $R_{1,2}$ slopes (wo LCSR): $|V_{cb}| = 0.0415(19)$.
- Intrinsic uncertainties of CLN fit can no longer be neglected.

Since almost 20 years experimental & theory analyses of B->D(*)lν are based on the CLN (Caprini,Lellouch, Neubert, 1998) parametrization of the form factors.

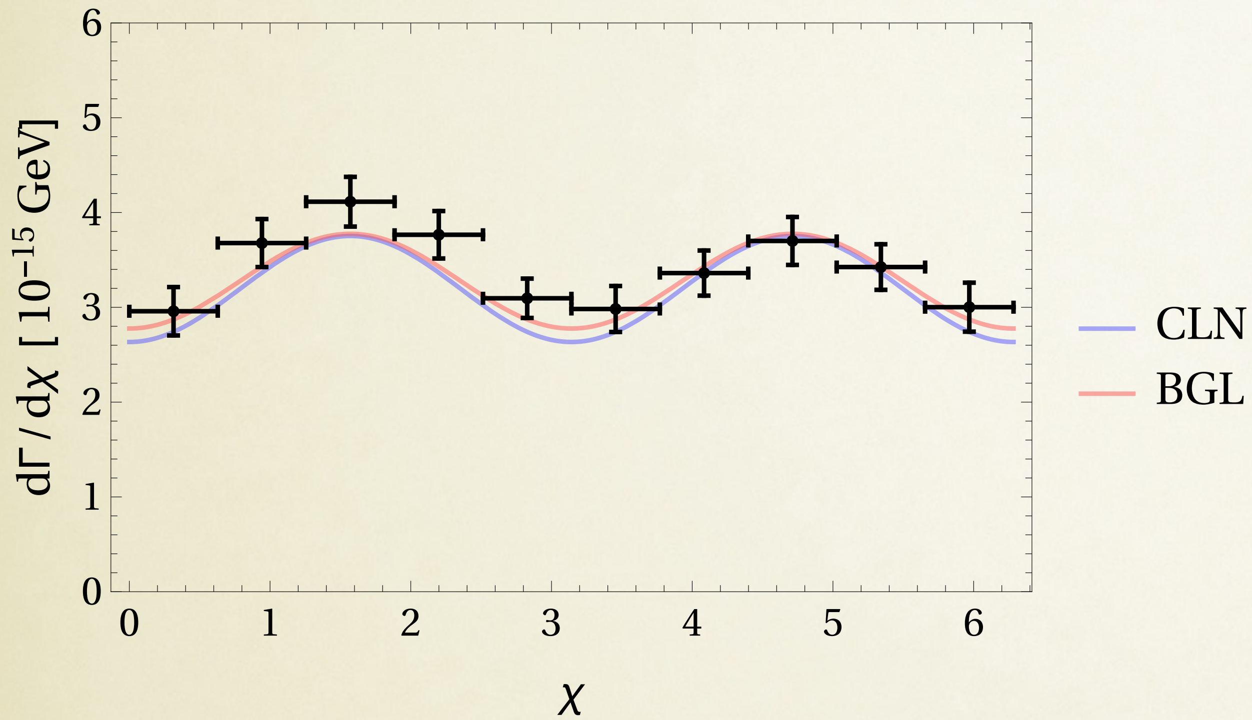
In view of the long-standing discrepancy between inclusive and exclusive determinations of V_{cb}, Belle has released deconvoluted B->D(*)lν spectra that can be analysed with other parametrizations

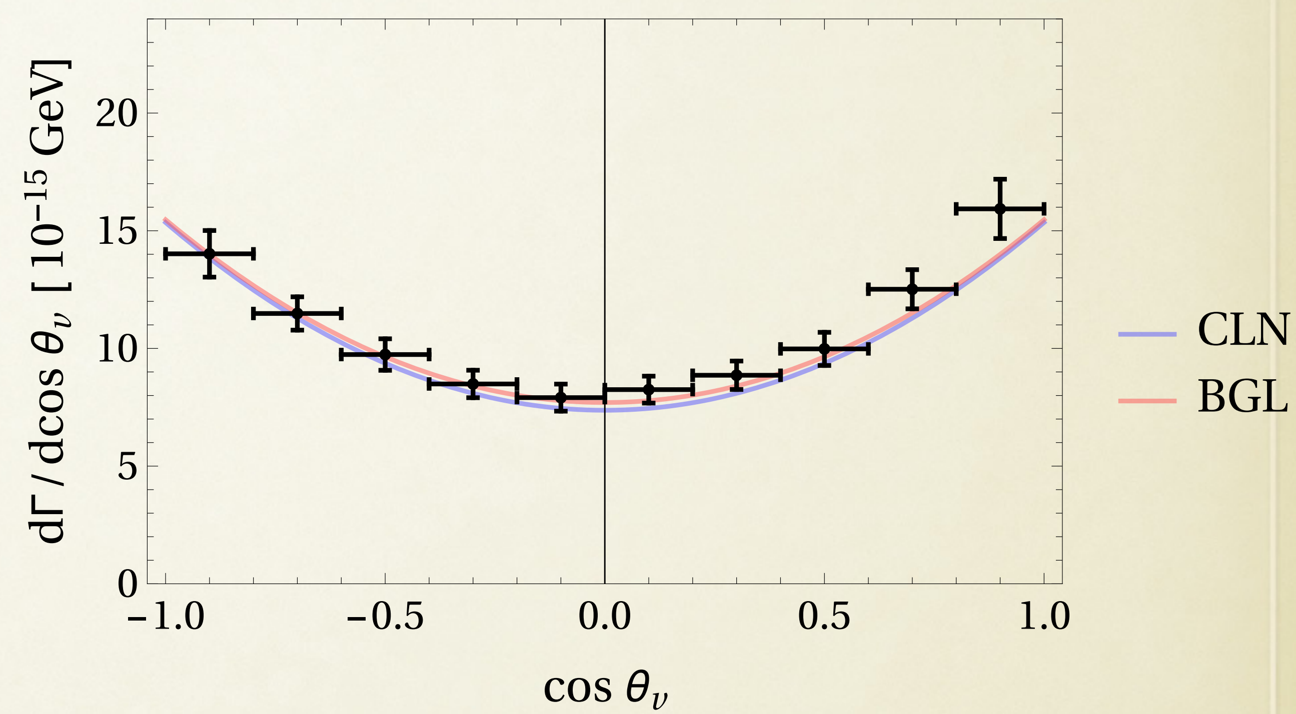


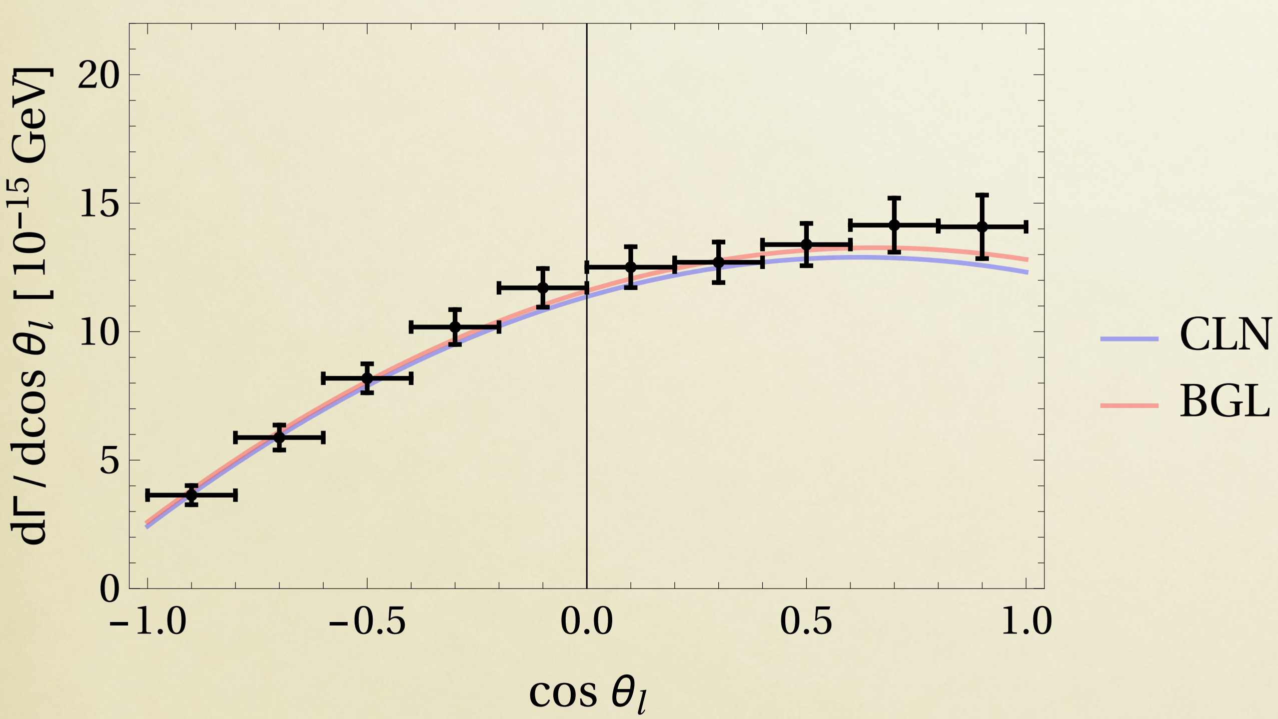
QUESTIONS

- **Why do CLN and BGL fits differ so much?** because BGL is more flexible: slight modifications to CLN lead to same V_{cb}
- **What's the basic difference between CLN and BGL?** They are based on the same dispersive (or unitarity) bounds, but CLN employs HQET relations + QCD sum rules to reduce number of parameters.
- **Are theory uncertainties included in the CLN approach?** The experimental analyses have systematically neglected the uncertainty estimated by CLN. Moreover, we need to check that the assumptions made by CLN in 1998 are consistent with what we know now.

Angular dependence



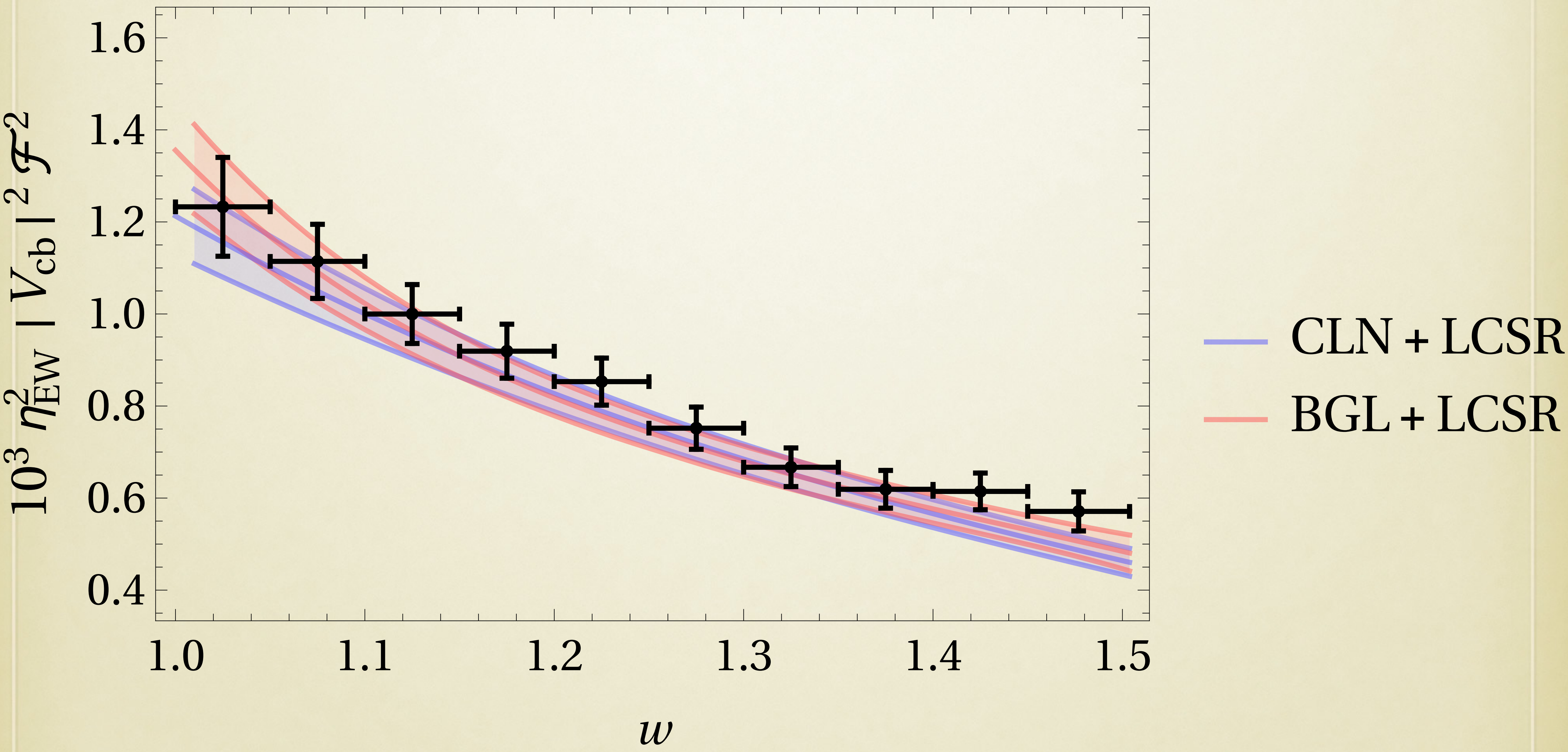


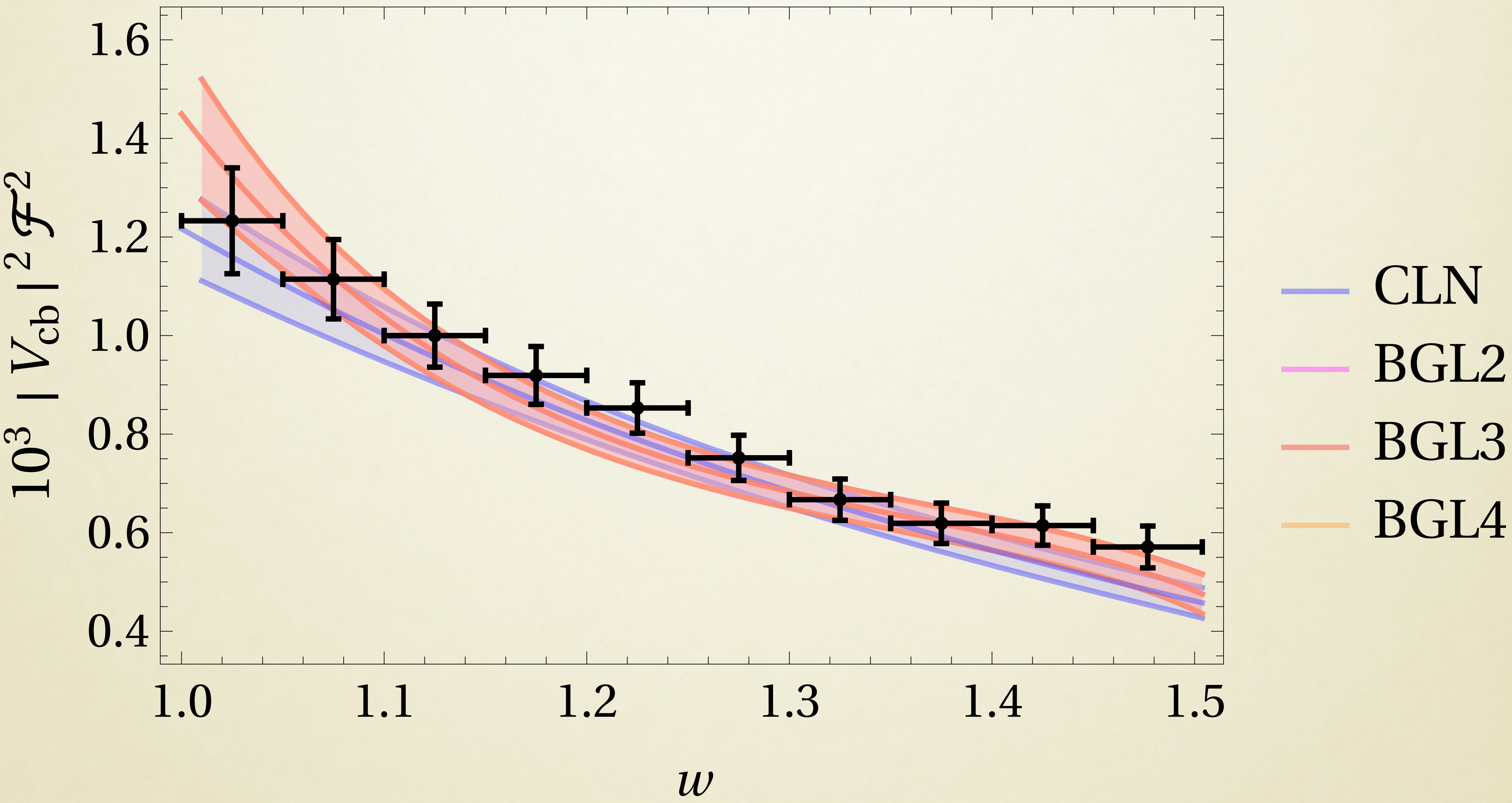


Angular bins are very little sensitive to the low recoil region. Effectively, they dilute the information of the first bins in the w spectrum

CLN fit *without* angular variables gives $|V_{cb}|$=0.0409(16)

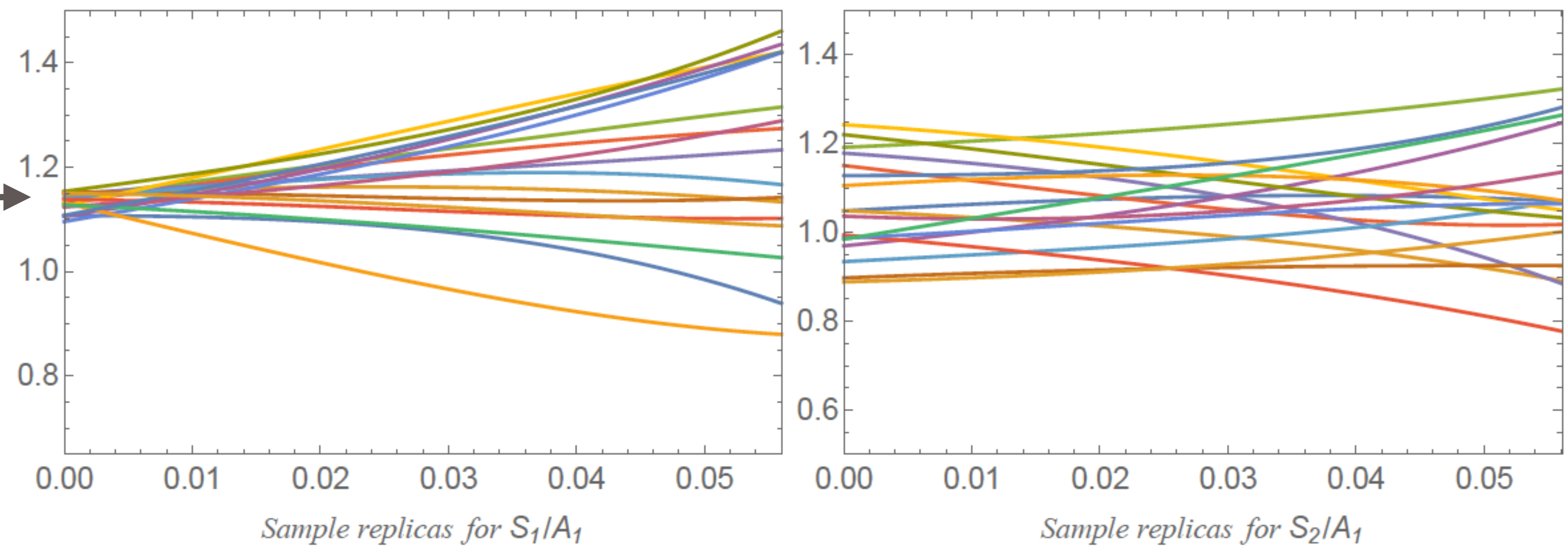
with LCSR constraints





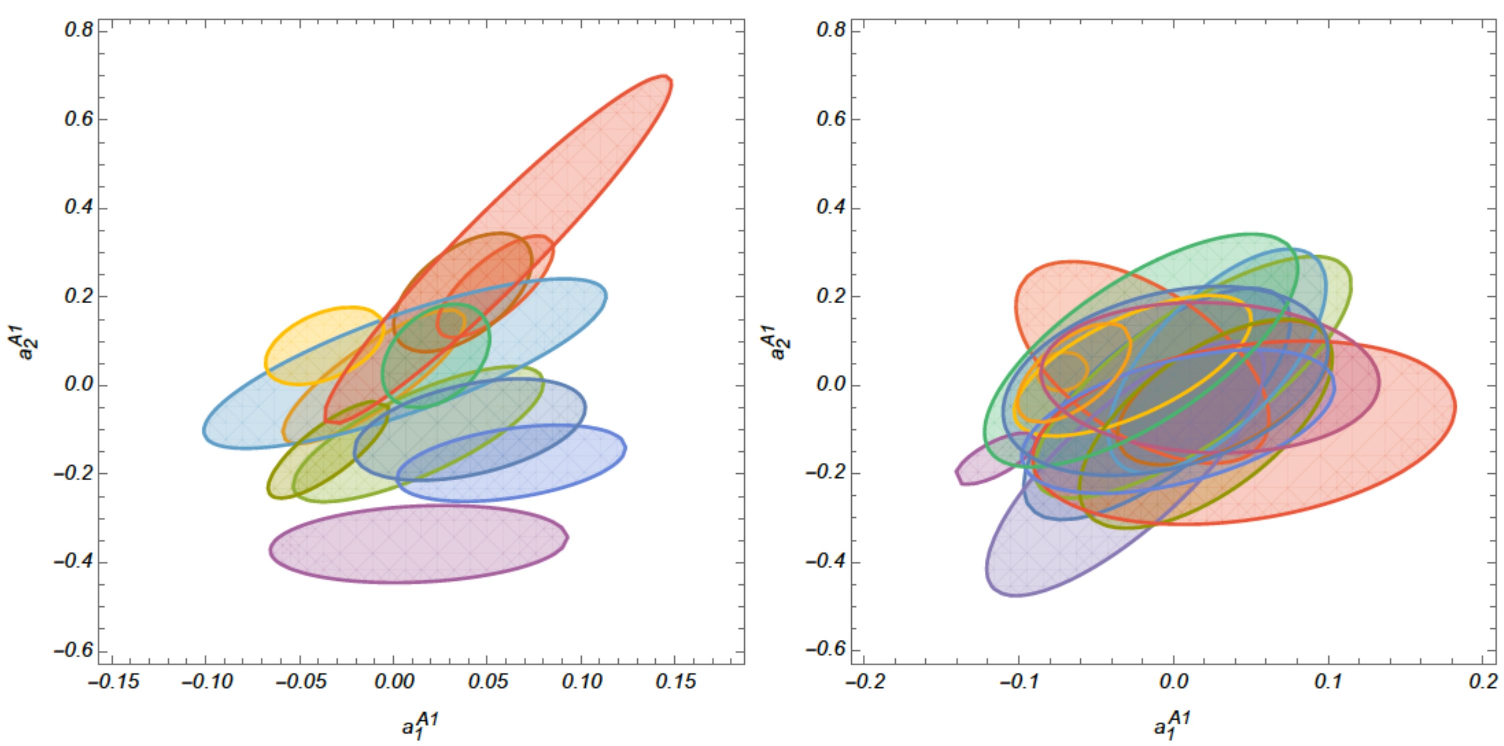
1.6
1.4
1.2
1.0
0.8
0.6
0.4
$10^3\,|V_{cb}|^2\,\mathcal{F}^2$
1.0
1.1
1.2
1.3
1.4
1.5
w
CLN
BGL2
BGL3
BGL4

Lattice input

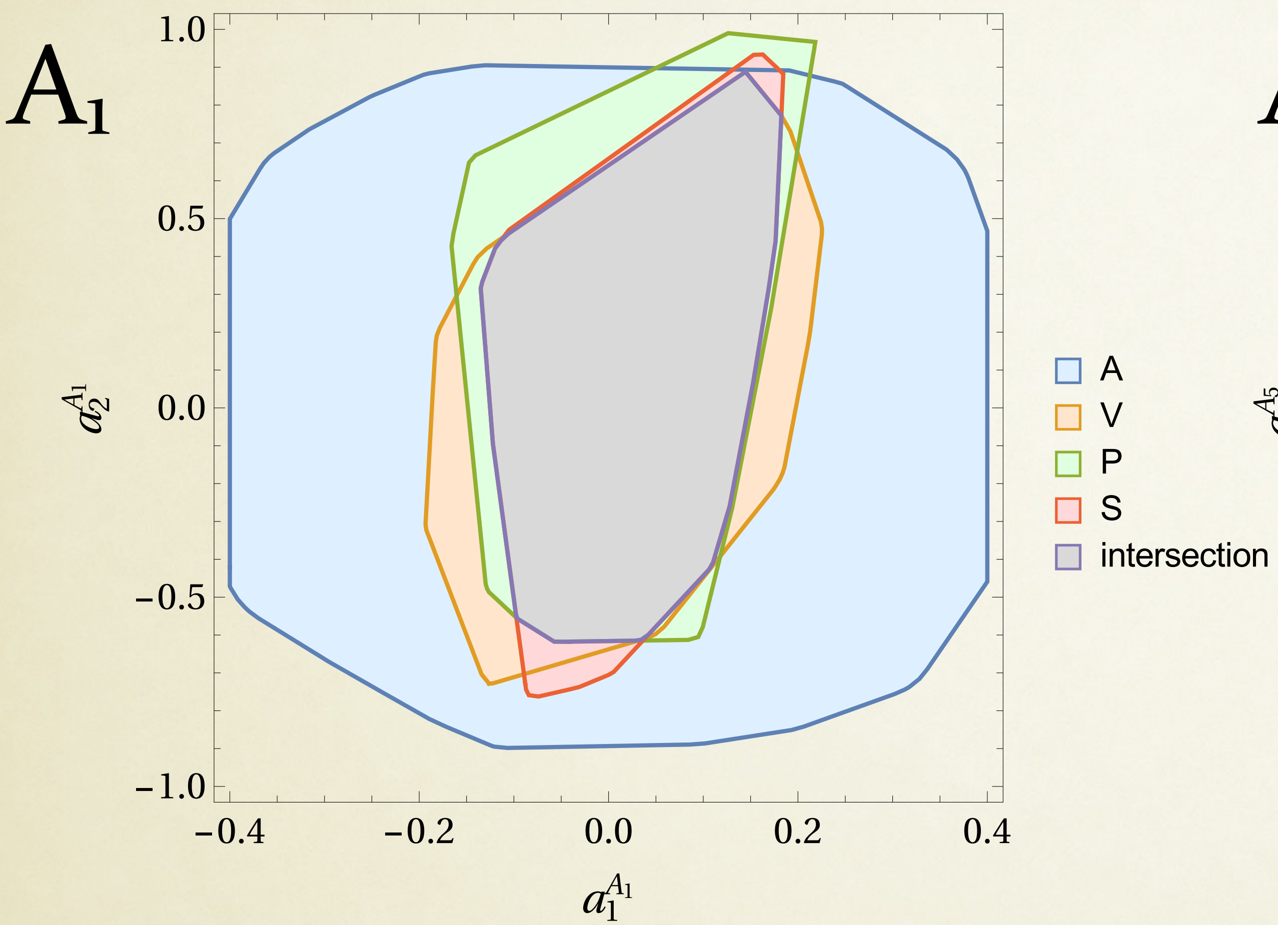


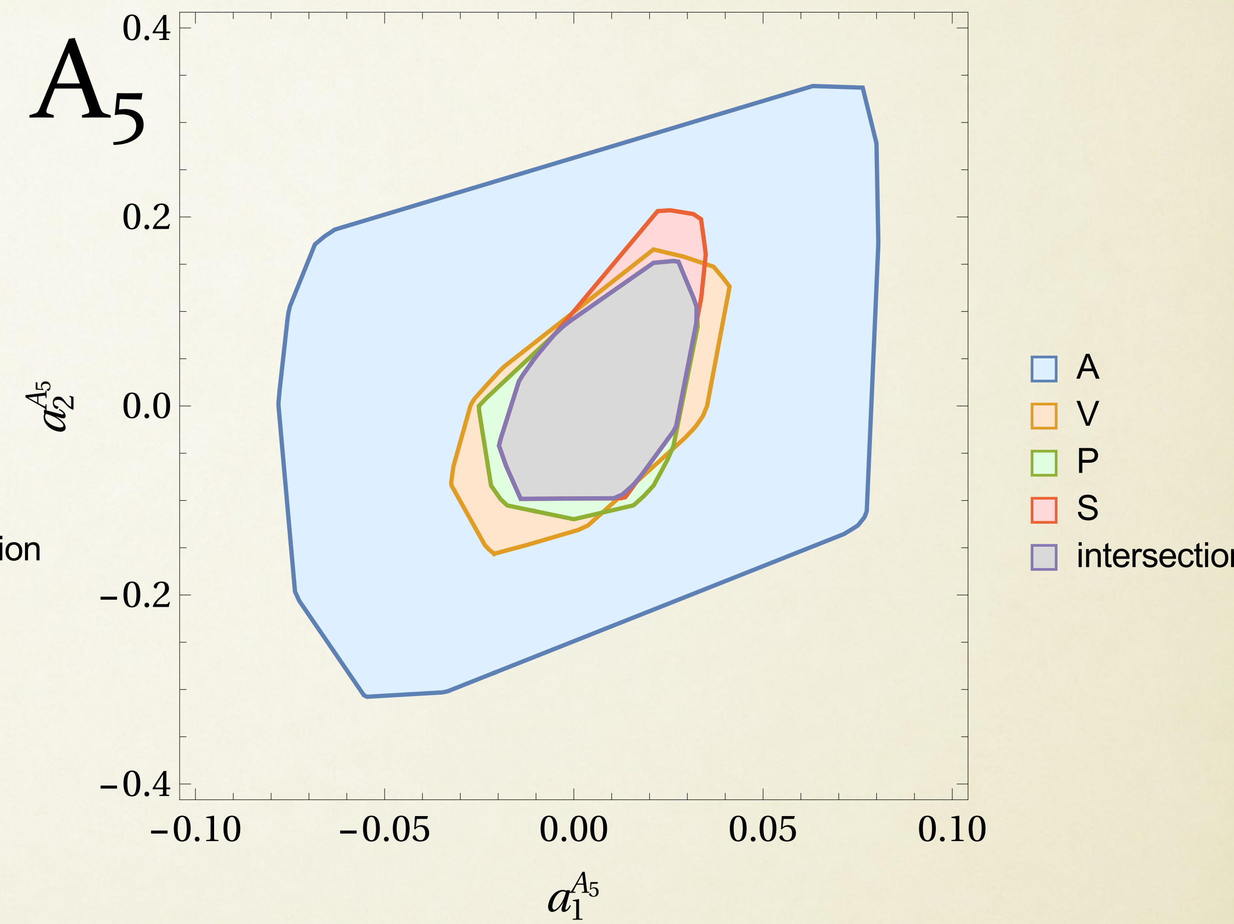
Each replica is a viable model of a f.f. complying with existing lattice and experimental results, and within a band centered in the HQET expectation: ~±25(30)% at zero (maximal) recoil.

a_0 fixed by LQCD in relevant cases: constraints are ellipses in (a_1, a_2) plane



Constraints in the a_1-a_2 planes





Envelopes formed by a large number of ellipses represent allowed regions in *(a_1,a_2)* planes

One gets different (but consistent) constraints from the S,P,V,A channels: take intersection