Flavor Physics from Lattice QCD

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Flavors: Light, Heavy, and Dark | January 15, 2018

IAS TUM Institute for Advanced Study

QCD

- SU(3) gauge symmetry and $1 + n_f + 1$ parameters:

$$
\begin{aligned}
\mathcal{L}_{\mathrm{QCD}} &= \frac{1}{g_0^2}\,\mathrm{tr}[F_{\mu\nu}F^{\mu\nu}] \\
&- \sum_f \bar{\psi}_f(\not{D} + m_f)\psi_f \\
&+ \frac{i\theta}{32\pi^2}\varepsilon^{\mu\nu\rho\sigma}\,\mathrm{tr}[F_{\mu\nu}F_{\rho\sigma}]
\end{aligned}
$$

- Ultraviolet regulator necessary; use spacetime lattice.

- Infrared regulator helpful; use finite lattice ⇒ # d.o.f. is finite ⇒ computer.

- Compute $M(L, a(g_0^2); \{m_{\mathrm{sea}}\}; \{m_{\mathrm{val}}\}; \boldsymbol{p})$.

QCD

- SU(3) gauge symmetry and $1 + n_f + 1$ parameters:

$$\mathcal{L}_{\mathrm{QCD}} = \frac{1}{g_0^2}\mathrm{tr}[F_{\mu\nu}F^{\mu\nu}]$$ f_{p4s} & f_π, or w_0 & M_Ω, or ....

$$- \sum_f \bar{\psi}_f(\not{D} + m_f)\psi_f$$ M_π, M_K, M_{Ds}, M_{Bs}, ...

$$+ \frac{i\theta}{32\pi^2}\varepsilon^{\mu\nu\rho\sigma}\mathrm{tr}[F_{\mu\nu}F_{\rho\sigma}]$$ $\theta = 0 \Leftarrow$ neutron EDM.

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- Compute $M(L, a(g_0^2); \{m_{\mathrm{sea}}\}; \{m_{\mathrm{val}}\}; \boldsymbol{p})$.

Effective Field Theory

Effective
Field
Theory
Gauge
Lattice

$$M(L, a(g_0^2); \{m_{\rm sea}\}; \{m_{\rm val}\}; \boldsymbol{p})$$

Effective Field Theory

RG trajectory:

$$a^{-1}e^{-2\pi/\beta_0 g_0^2} = \Lambda_{\text{lat}} \propto \Lambda_{\overline{\text{MS}}}$$

O(a^2) ⇐ Symanzik EFT

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sea quarks:

$\{m'_l, m'_l, m'_s\}$ = 2+1

$m'_l \to m_l \Leftarrow$ χPT

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$\{m_Q, m_q\}$ for meson

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nonzero $2\pi\boldsymbol{n}/L$ for, *e.g.*,
form factors

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often exponential, but can be power-law, *e.g.*, scattering or frozen Q

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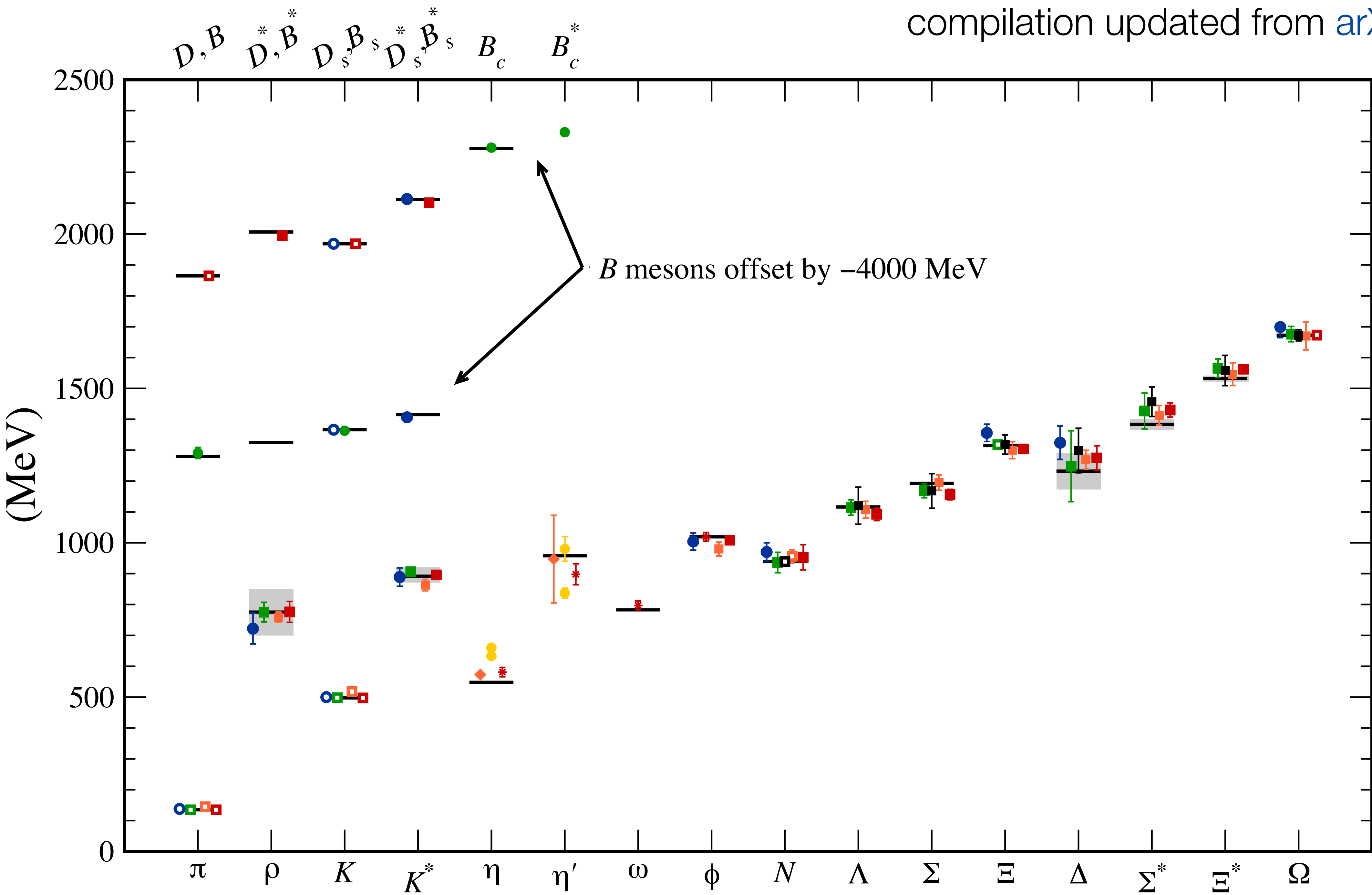
sea quarks: $\{m_l', m_l', m_s', m_c'\}$ = 2+1+1

QCD Hadron Spectrum

π…Ω: BMW, MILC, PACS-CS, QCDSF; ETM (2+1+1);
η-η′: RBC, UKQCD, Hadron Spectrum (ω);
D, *B*: Fermilab, HPQCD, Mohler&Woloshyn

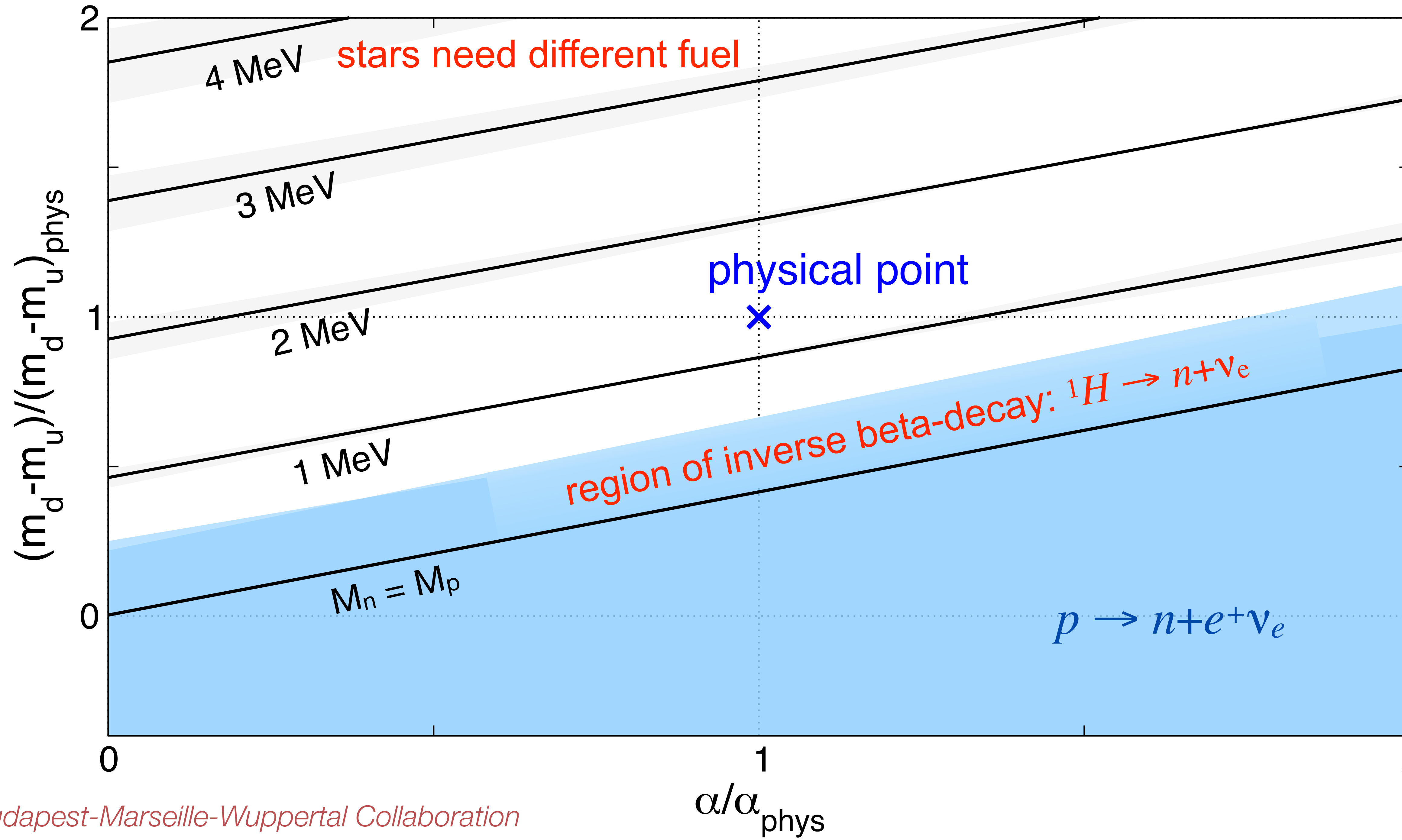
compilation updated from arXiv:1203.1204



numerous quarkonium omitted

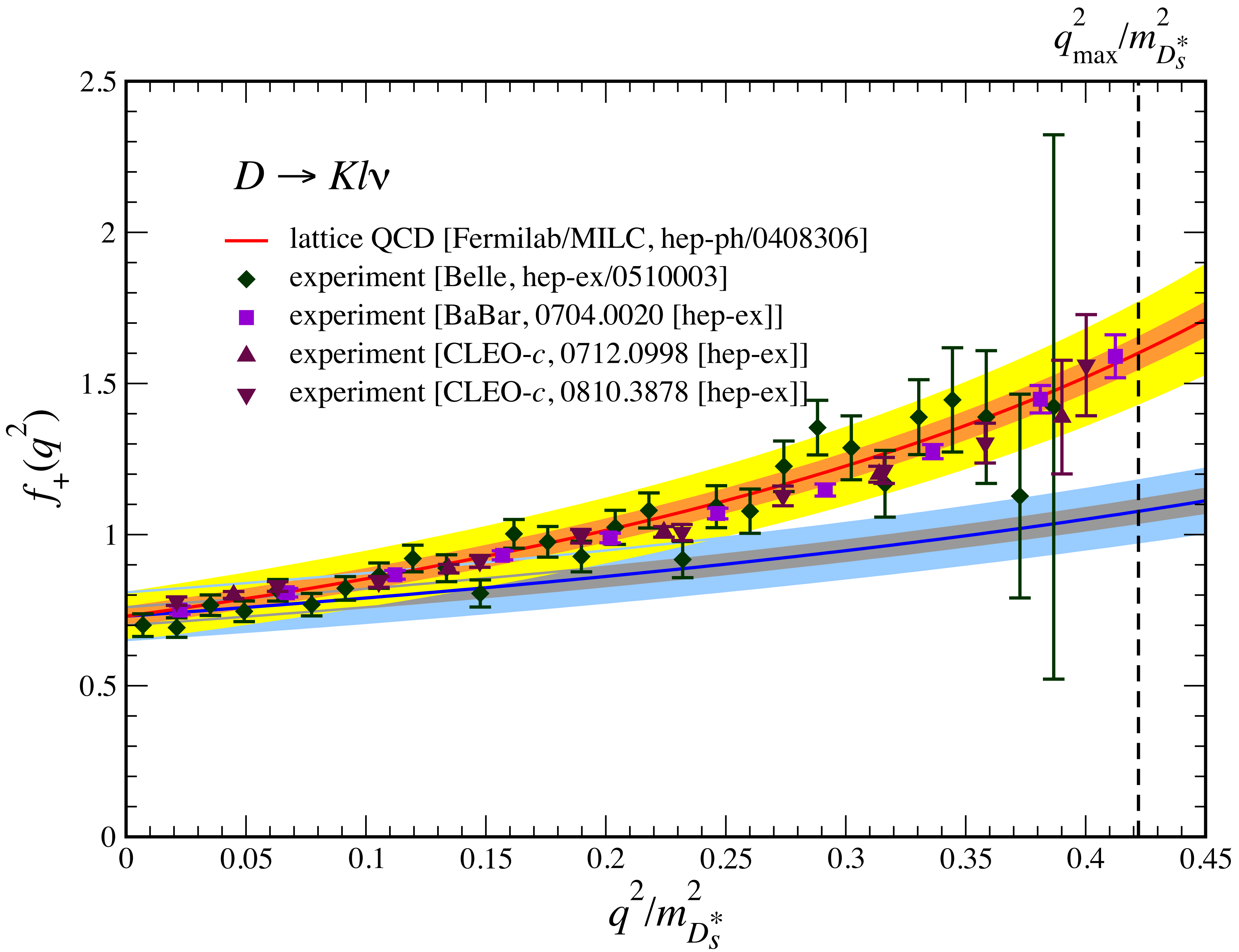
Neutron-Proton Mass Difference

Science 347 (2015) 1452

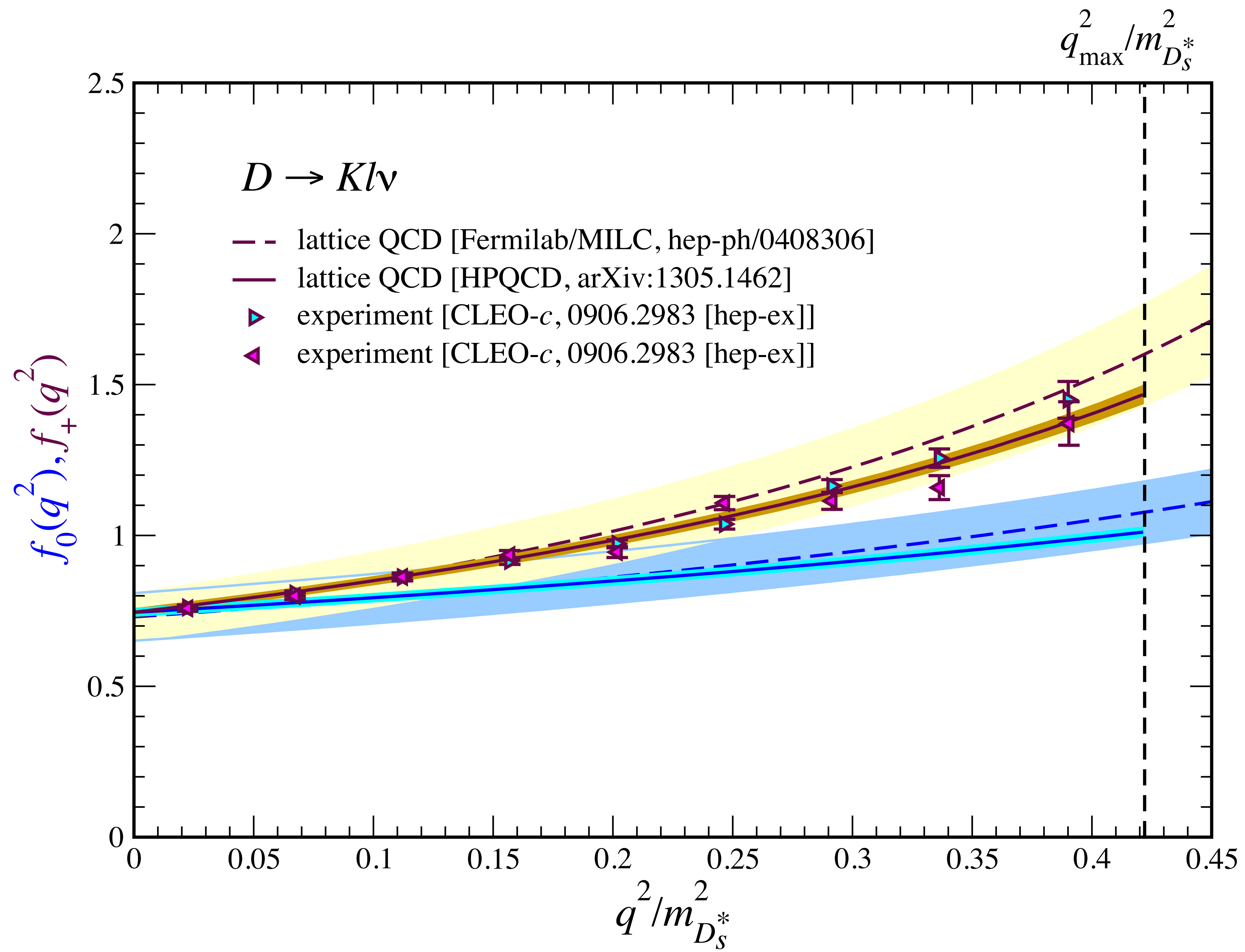


Budapest-Marseille-Wuppertal Collaboration

Predictions



Predictions & Progress



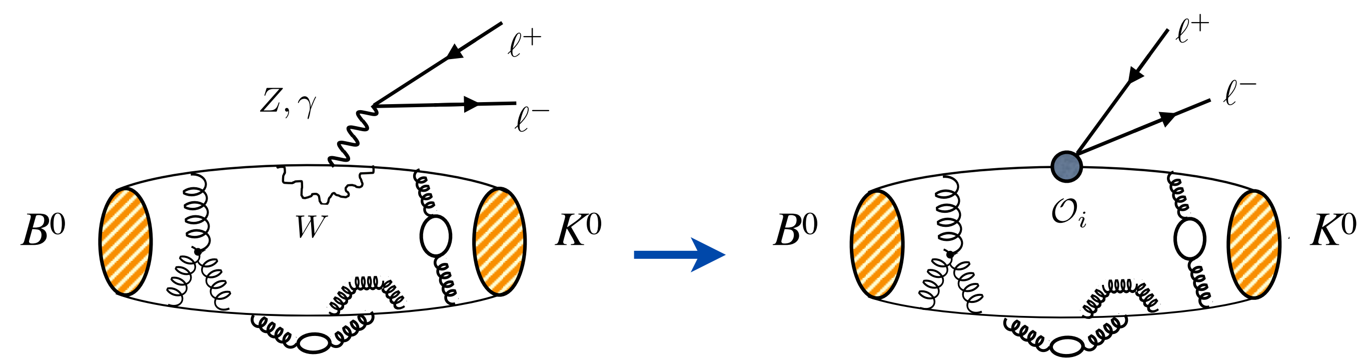
Outline

- Lattice QCD
  - What is it?
  - What has it done to make America great again?
- Selected Calculations
  - FCNC & $B^0_{(s)}$-$\bar{B}^0_{(s)}$ Mixing — One constraint to kill them all!
  - Charged-current anomalies: $R(D)$ and $R(D^*)$; $B \to \tau\nu$ (new!).
  - Quark masses (brand new!)

QCD for Neutral-Current Anomalies

Flavor-Changing Neutral Currents

- Rare processes are sensitive to non-Standard physics: leptoquarks, Z', 4th generation, non-Standard Higgs bosons, supersymmetry.
- Several “tensions”:
  - deficits in $B \to K^{(*)}\mu\mu$;
  - $B^0_{(s)}$-$\bar{B}^0_{(s)}$ mixing;
  - excess in $B \to D^{(*)}\tau\nu$ (even if it is a charged current).



- Experimental results available; more on the way, e.g., $B \to K^{(*)}ee$.
- Nonperturbative hadronic matrix elements available (with full error budgets).

Basic Formulas for $B \to \pi l^+ l^-$, $K l^+ l^-$

cf., arXiv:1510.02349, Sec. 2 & Appendix B

- One-loop effective Hamiltonian contains many operators ($q = d, s$):

$$Q_1^u = (\bar{q}_L \gamma_\mu T^a u_L)(\bar{u}_L \gamma^\mu T^a b_L) \qquad Q_2^u = (\bar{q}_L \gamma_\mu u_L)(\bar{u}_L \gamma^\mu b_L)$$

$$Q_1 = (\bar{q}_L \gamma_\mu T^a c_L)(\bar{c}_L \gamma^\mu T^a b_L) \qquad Q_2 = (\bar{q}_L \gamma_\mu c_L)(\bar{c}_L \gamma^\mu b_L)$$

$$Q_3 = (\bar{q}_L \gamma_\mu b_L) \sum_{q'} (\bar{q}' \gamma^\mu q') \qquad Q_4 = (\bar{q}_L \gamma_\mu T^a b_L) \sum_{q'} (\bar{q}' \gamma^\mu T^a q')$$

$$Q_5 = (\bar{q}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} b_L) \sum_{q'} (\bar{q}' \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} q') \qquad Q_6 = (\bar{q}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} T^a b_L) \sum_{q'} (\bar{q}' \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} T^a q')$$

$$Q_7 = \frac{e}{16\pi^2} m_b (\bar{q}_L \sigma^{\mu\nu} b_R) F_{\mu\nu} \qquad Q_8 = \frac{g}{16\pi^2} m_b (\bar{q}_L \sigma^{\mu\nu} T^a b_R) G^a_{\mu\nu}$$

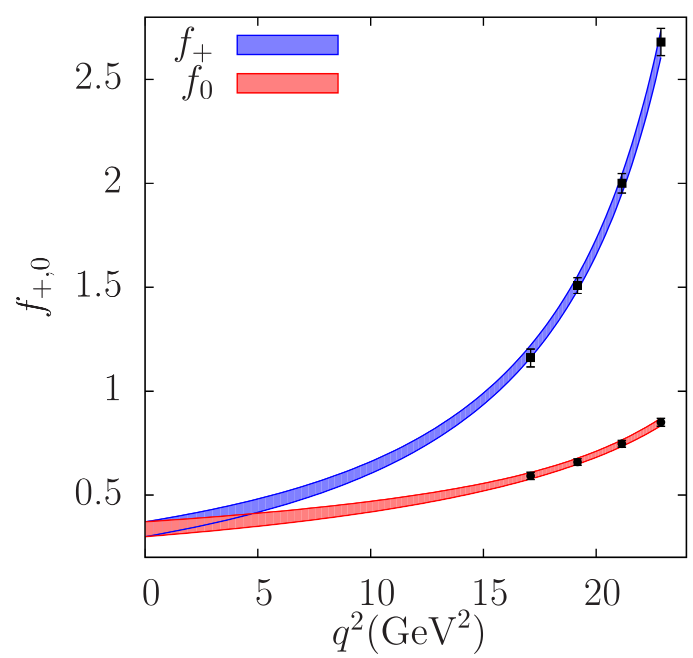
$$Q_9 = (\bar{q}_L \gamma_\mu b_L) \sum_{\ell} (\bar{\ell} \gamma^\mu \ell) \qquad Q_{10} = (\bar{q}_L \gamma_\mu b_L) \sum_{\ell} (\bar{\ell} \gamma^\mu \gamma_5 \ell)$$

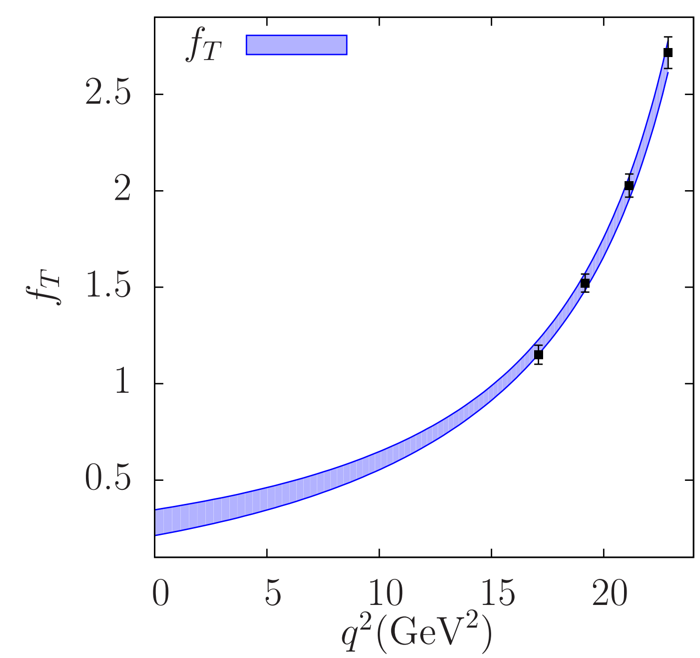
- Matrix elements of Q_7, Q_9, Q_{10} yield form factors, including tensor f_T.

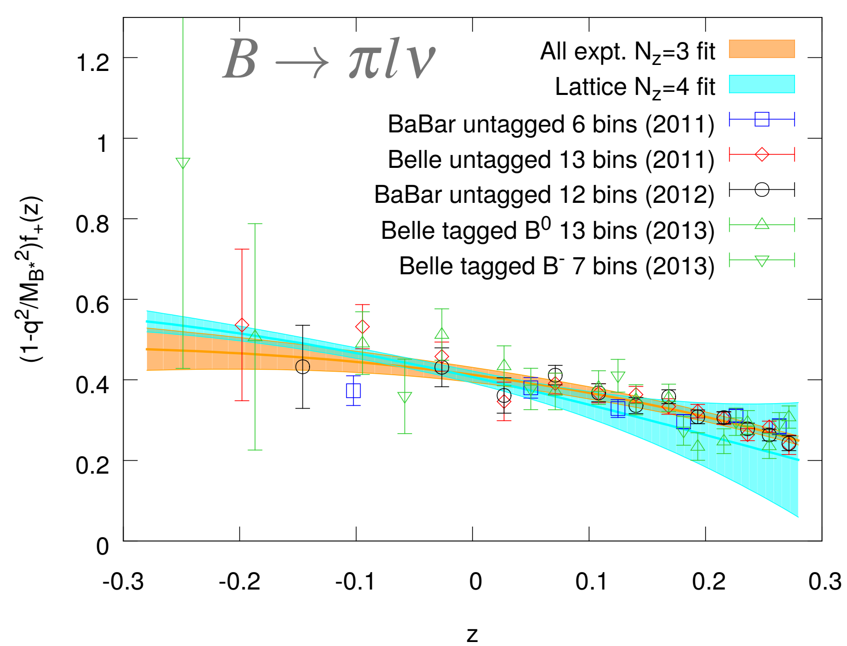
Semileptonic $B \to Kll$, πll

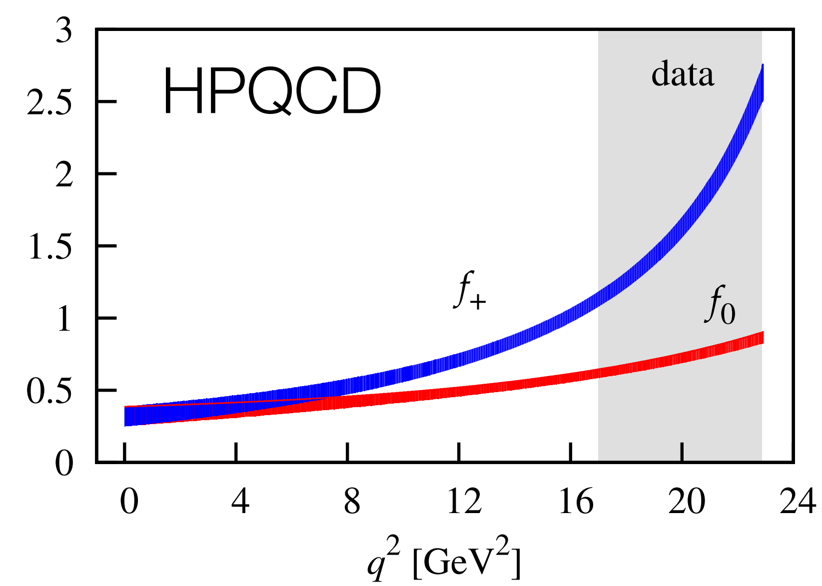
arXiv:1509.06235, arXiv:1306.2384

- Compute $f(\boldsymbol{k}, m_s, m_l, a)$.
- Combine data with Symanzik EFT & χPT:
  - $m_l \to \frac{1}{2}(m_u+m_d)$;
  - $a \to 0$.
- Limited range: $|\boldsymbol{k}|a \ll 1$.
- Extend range with z expansion $|z| < 0.3$.

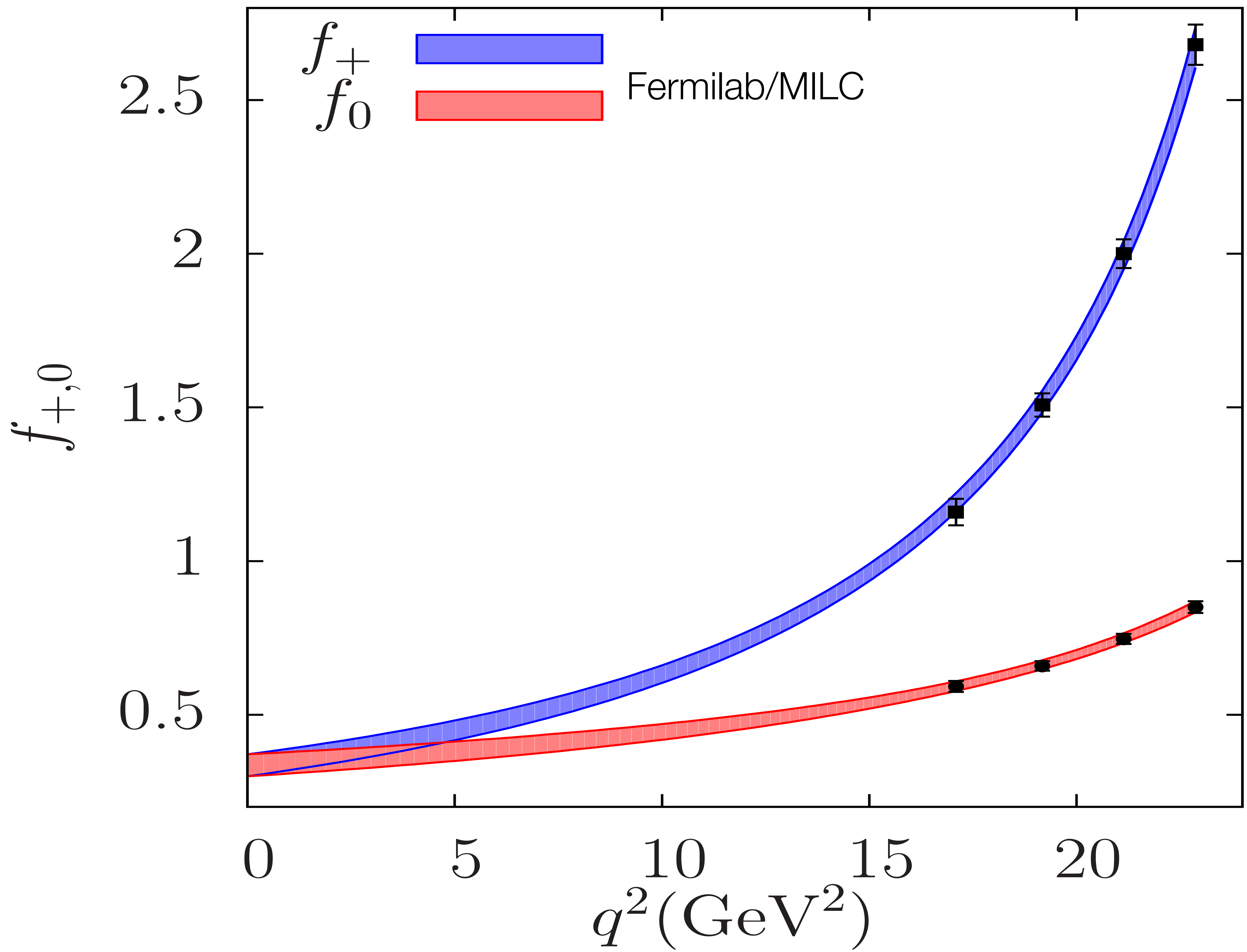








Comparison



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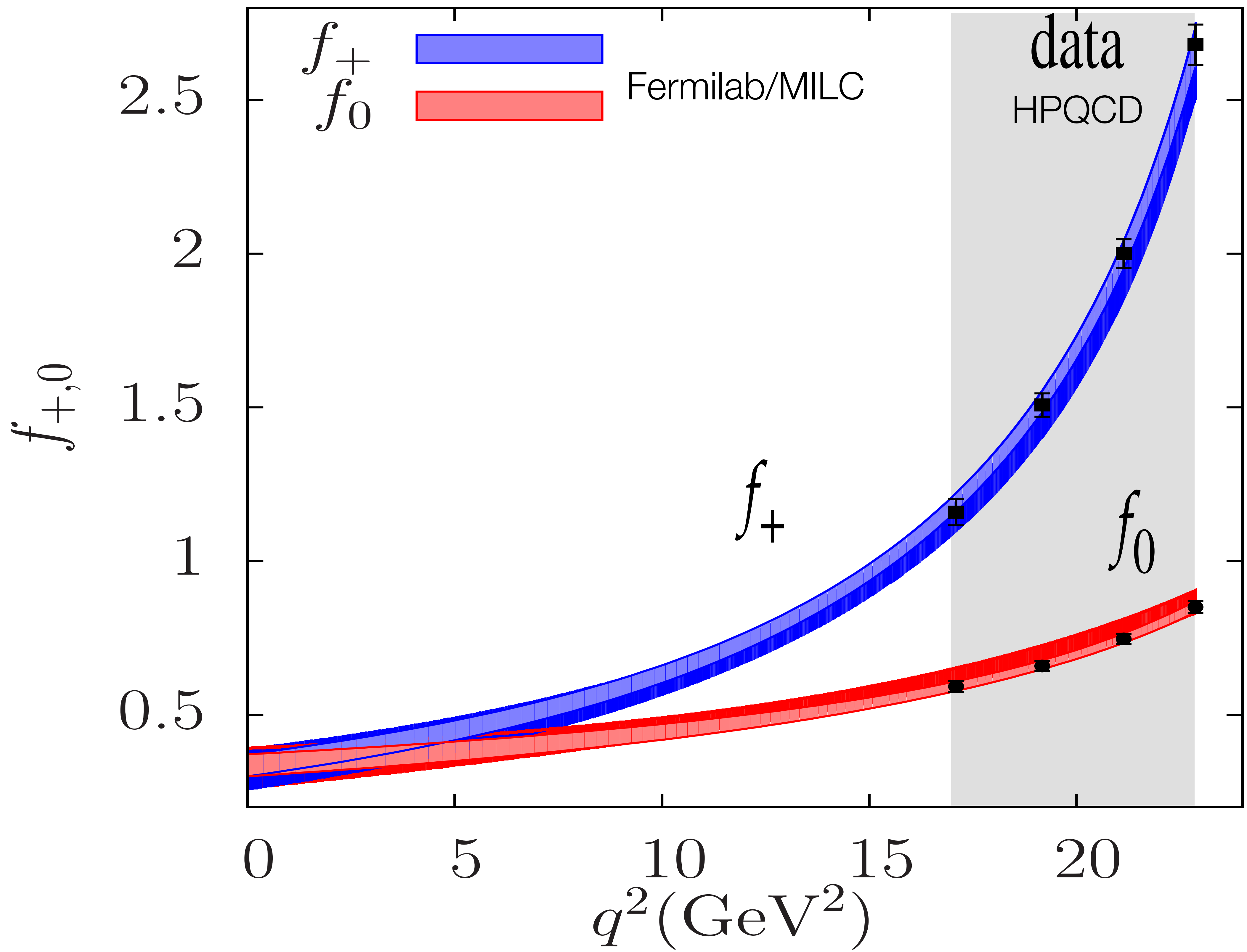
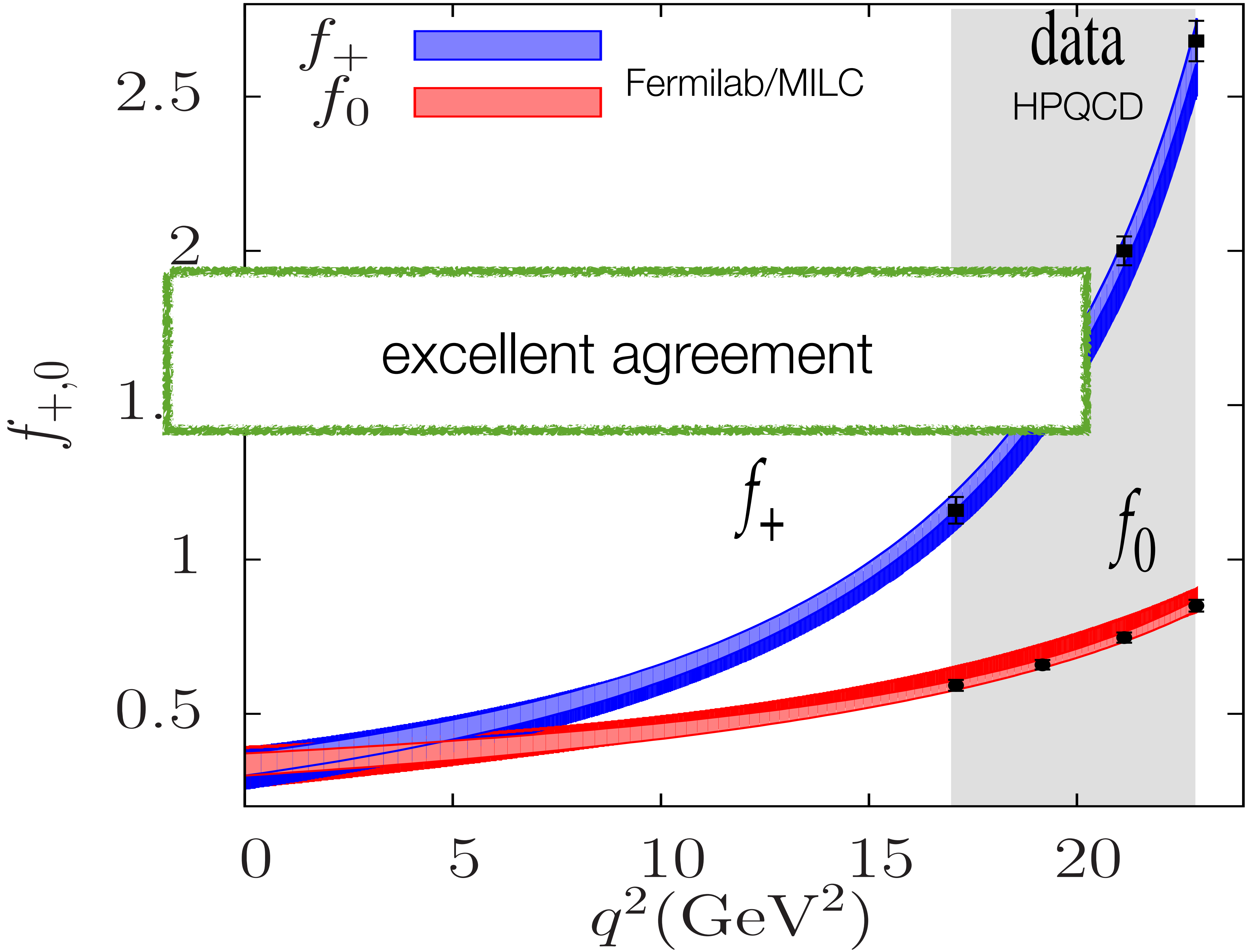


f_+
f_0
Fermilab/MILC
data
HPQCD
2.5
2
1.5
1
0.5
$f_{+,0}$
f_+
f_0
0
5
10
15
20
$q^2(\mathrm{GeV}^2)$

Comparison



excellent agreement

Coefficients and Correlations: $B \to Kl^+l^-$

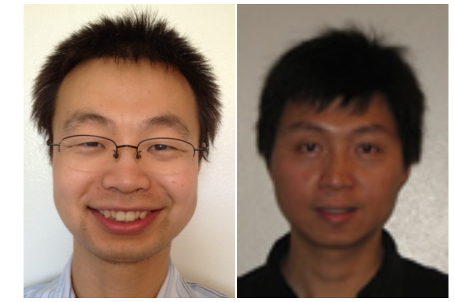
arXiv:1509.06235

- Form factors expressed as polynomial in $z = \dfrac{\sqrt{t_{\rm cut} - q^2} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - q^2} + \sqrt{t_{\rm cut} - t_0}}$

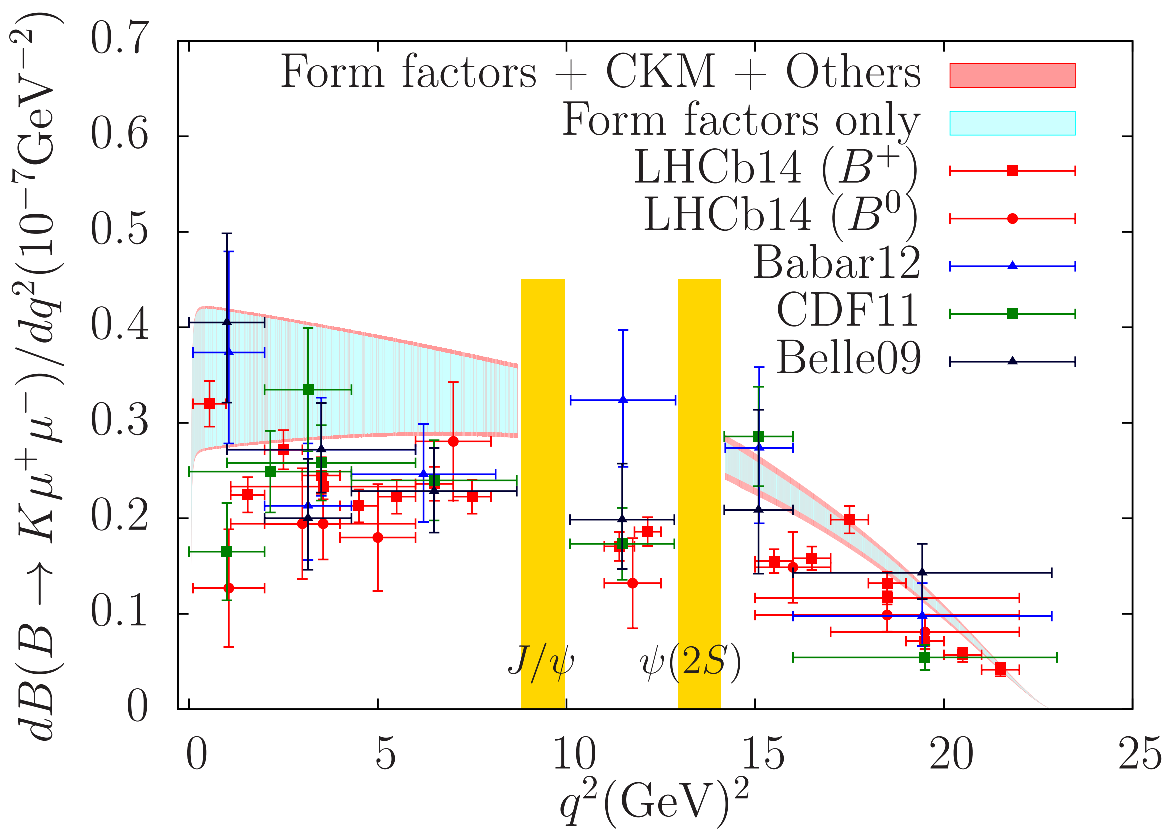
	b_0^+	b_1^+	b_2^+	b_0^0	b_1^0	b_2^0	b_0^T	b_1^T	b_2^T
mean	0.466	−0.885	−0.213	0.292	0.281	0.150	0.460	−1.089	−1.114
error	0.014	0.128	0.548	0.010	0.125	0.441	0.019	0.236	0.971
b_0^+	1	0.450	0.190	0.857	0.598	0.531	0.752	0.229	0.117
b_1^+		1	0.677	0.708	0.958	0.927	0.227	0.443	0.287
b_2^+			1	0.595	0.770	0.819	−0.023	0.070	0.196
b_0^0				1	0.830	0.766	0.582	0.237	0.192
b_1^0					1	0.973	0.324	0.372	0.272
b_2^0						1	0.268	0.332	0.269
b_0^T							1	0.590	0.515
b_1^T								1	0.897
b_2^T									1

- Correlations for $B \to \pi l^+l^-$ in arXiv:1503.07839 and arXiv:1507.01618.

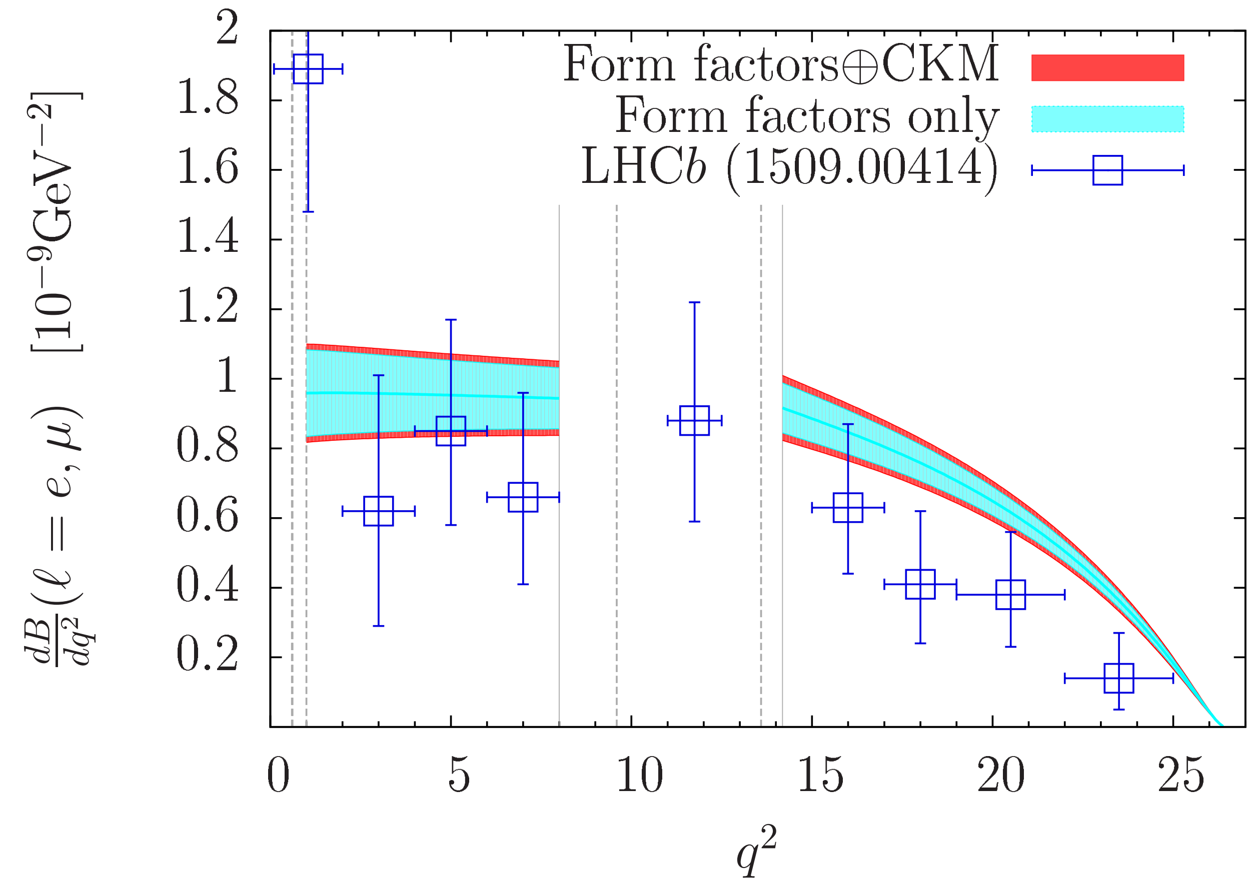
Kinematic Distributions



- Experimental data from LHCb [arXiv:1403.8044, arXiv:1509.00414] and earlier experiments; right plot's theory **preceded** measurement:

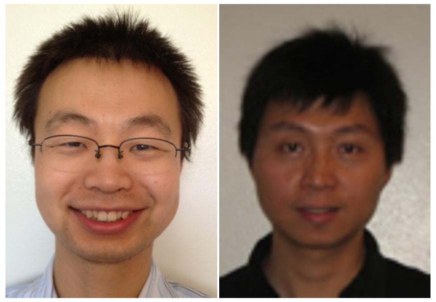


arXiv:1510.02349



arXiv:1507.01618

Kinematic Distributions



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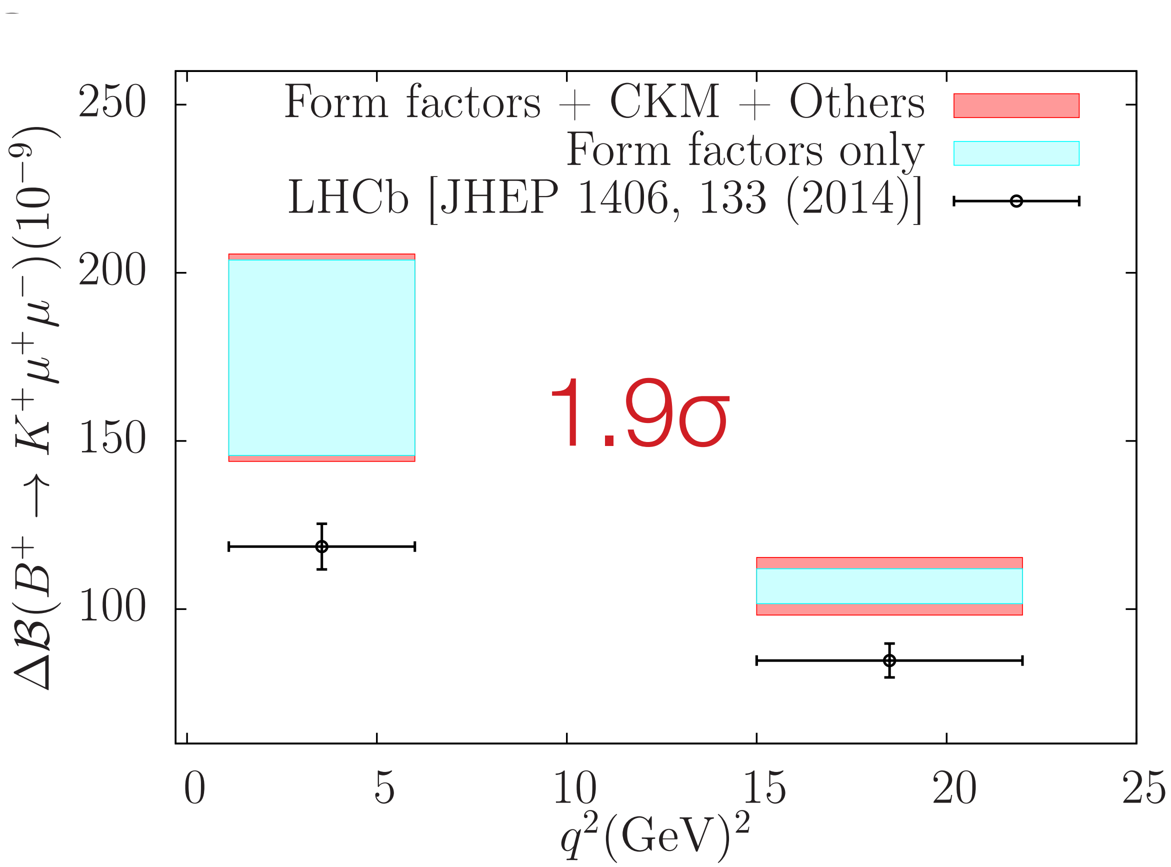
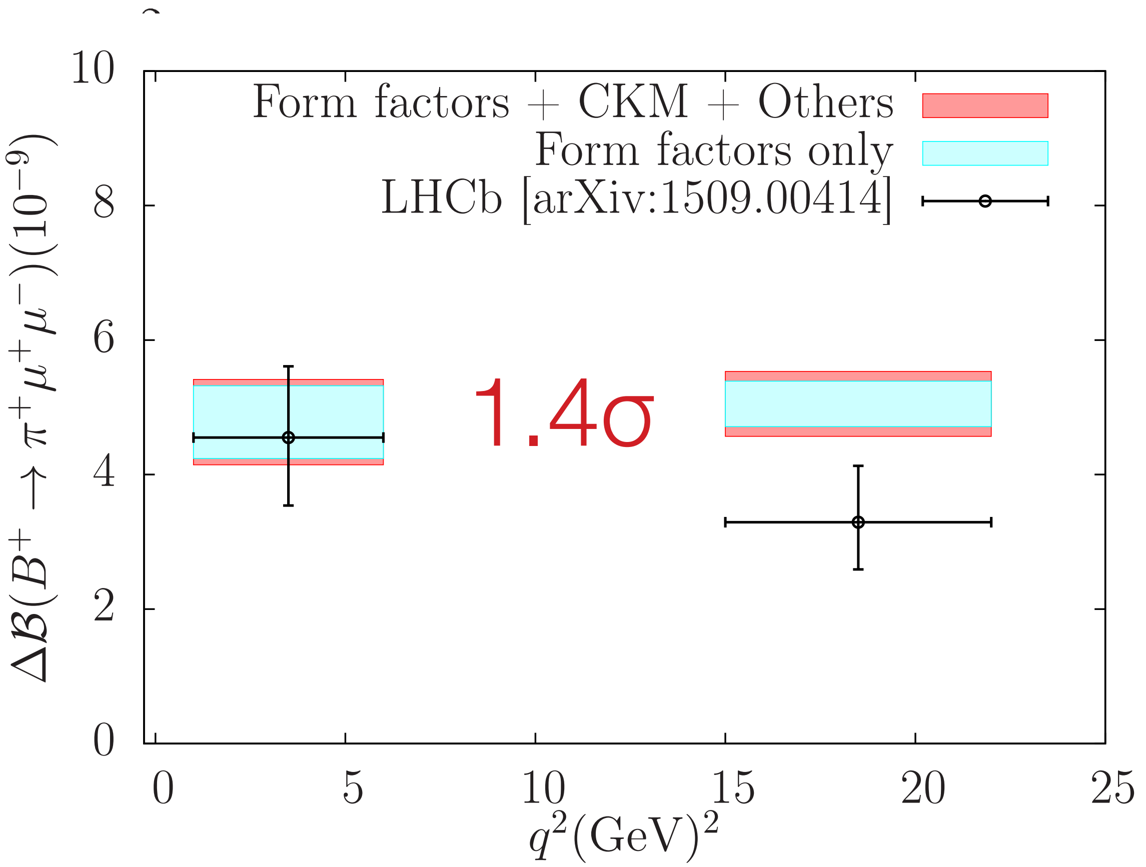


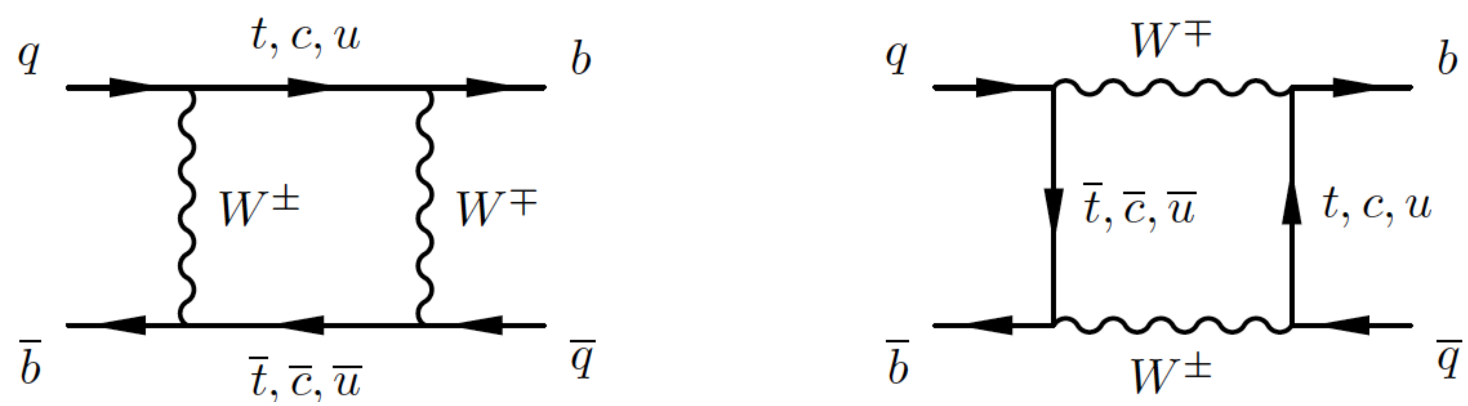
arXiv:1510.02349



arXiv:1507.01618

Neutral-Meson Mixing

- In the Standard Model, neutral mesons can oscillate into their antiparticles:



- In extensions of the SM, other particles
  - could appear in the boxes;
  - could appear at the tree level: flavor-changing neutral current.
- Observed for all neutral mesons: K^0, D^0, B^0, B_s.

Basic Observables

- The particle and antiparticle evolve in time via

$$i\frac{d}{dt}\left|\begin{array}{c} B^0 \\ \bar{B}^0 \end{array}\right\rangle = \left(M - i\frac{\Gamma}{2}\right)\left|\begin{array}{c} B^0 \\ \bar{B}^0 \end{array}\right\rangle$$

where M and Γ are 2×2 Hermitian matrices, with $M_{11} = M_{22}$, $\Gamma_{11} = \Gamma_{22}$.

- Mass difference: $\Delta M \approx 2|M_{12}|$.
- Width difference: $\Delta\Gamma \approx 2|\Gamma_{12}|\cos\phi, \quad \phi = \arg[-M_{12}/\Gamma_{12}]$
- CP asymmetry of flavor-specific decays: $a_{\rm fs} = \frac{|\Gamma_{12}|}{|M_{12}|}\sin\phi$

Effective Hamiltonian

- After integrating out heavy particles:

$$\mathscr{L} = \mathscr{L}_{\text{kin}}[\ell, q, \gamma, g] + \sum_i \mathscr{C}_i(\alpha, \alpha_s, G_F, \sin^2\theta, m_\ell, m_q, V; \text{NP})\mathscr{L}_i[\ell, q, \gamma, g]$$

- For $\Delta F = 2$ processes, discrete symmetries and Fierz rearrangement reduces the list of all possible operators to 8 = 5 + 3:

$$\mathscr{O}_1 = \bar{b}\gamma^\mu Lq\,\bar{b}\gamma^\mu Lq \qquad \tilde{\mathscr{O}}_1 = \bar{b}\gamma^\mu Rq\,\bar{b}\gamma^\mu Rq$$

$$\mathscr{O}_2 = \bar{b}Lq\,\bar{b}Lq \qquad \tilde{\mathscr{O}}_2 = \bar{b}Rq\,\bar{b}Rq$$

$$\mathscr{O}_3 = \bar{b}^\alpha Lq^\beta\,\bar{b}^\beta Lq^\alpha \qquad \tilde{\mathscr{O}}_3 = \bar{b}^\alpha Rq^\beta\,\bar{b}^\beta Rq^\alpha$$

$$\mathscr{O}_4 = \bar{b}Lq\,\bar{b}Rq$$

$$\mathscr{O}_5 = \bar{b}^\alpha Lq^\beta\,\bar{b}^\beta Rq^\alpha$$

By parity in QCD: $\langle \bar{B}^0 | \mathscr{O}_i | B^0 \rangle = \langle \bar{B}^0 | \tilde{\mathscr{O}}_i | B^0 \rangle$

- The off-diagonal terms in the mass and width matrices are related to the $\langle \bar{B}_q | \mathscr{O}_i | B_q \rangle$.

- In the Standard Model, the mass difference (aka oscillation frequency)

$$\Delta M_q = \frac{G_F^2 m_W^2}{4\pi^2 M_{B_q}} \left| V_{tq}^* V_{tb} \right|^2 S_0(m_t^2/m_W^2) \eta_{2B} \langle \bar{B}_q | \mathscr{O}_1 | B_q \rangle, \quad q = d, s$$

where S_0 comes from the box diagrams, η_{2B} from pQCD corrections.

- The product $\eta_{2B} \langle \bar{B}_q | \mathscr{O}_1 | B_q \rangle$ is scheme and scale independent.

- In extensions of the Standard Model, any or all of the 5+3 operators could appear.

- arXiv:1602.03560 has first unquenched lattice QCD calculation of all five matrix elements: dominate FLAG average of November 2017.

Oscillation Frequencies


- Taking CKM from tree-only inputs (from CKMfitter):

$$\Delta M_d^{\mathrm{SM}} = 0.639(50)(36)(5)(13)\ \mathrm{ps}^{-1}$$
$$\Delta M_s^{\mathrm{SM}} = 19.8(1.1)(1.0)(0.2)(0.4)\ \mathrm{ps}^{-1}$$
$$\frac{\Delta M_d^{\mathrm{SM}}}{\Delta M_s^{\mathrm{SM}}} = 0.0323(9)(9)(0)(3)$$

- Contrast with the measured frequencies:

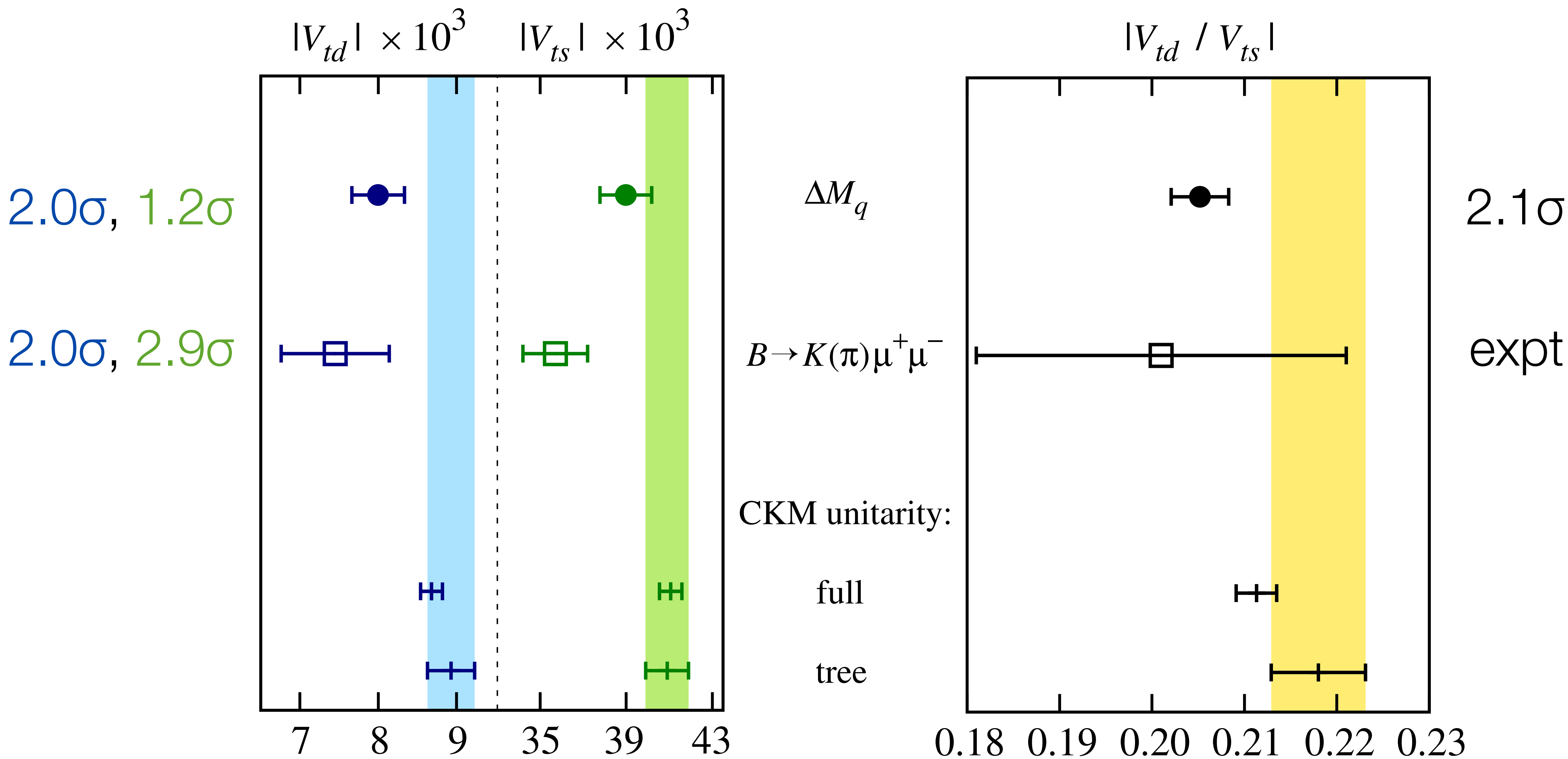
$$\Delta M_d^{\mathrm{expt}} = (0.5055 \pm 0.0020)\ \mathrm{ps}^{-1}$$
$$\Delta M_s^{\mathrm{expt}} = (17.757 \pm 0.021)\ \mathrm{ps}^{-1}$$

- These amount to discrepancies of 2.1σ, 1.3σ, and 2.9σ, respectively.

- Examine these tensions with those in other FCNC processes, casting each one as a “CKM determination”.

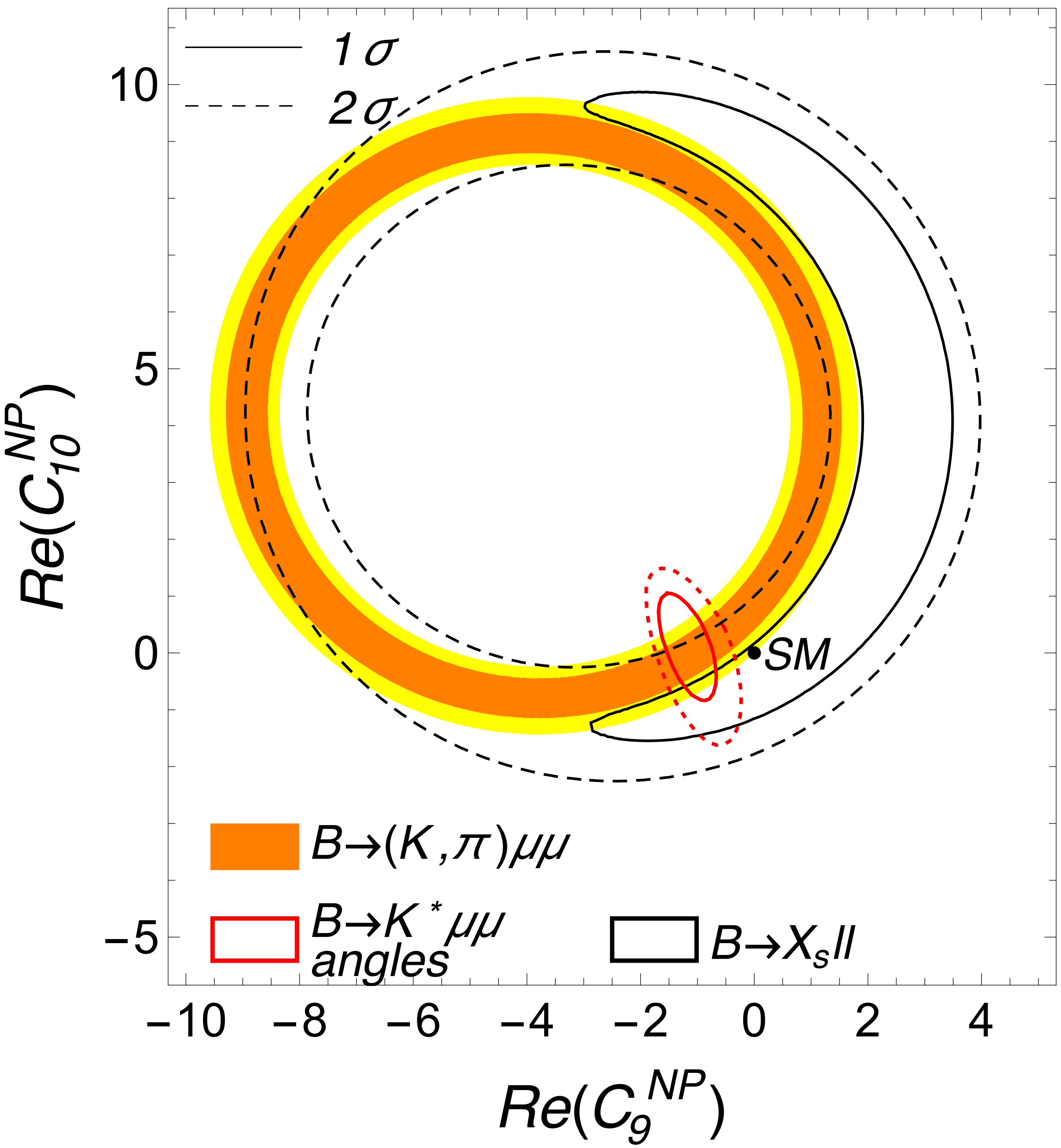
CKM Comparison

- CKM from FCNC are lower than determinations from trees and unitarity.



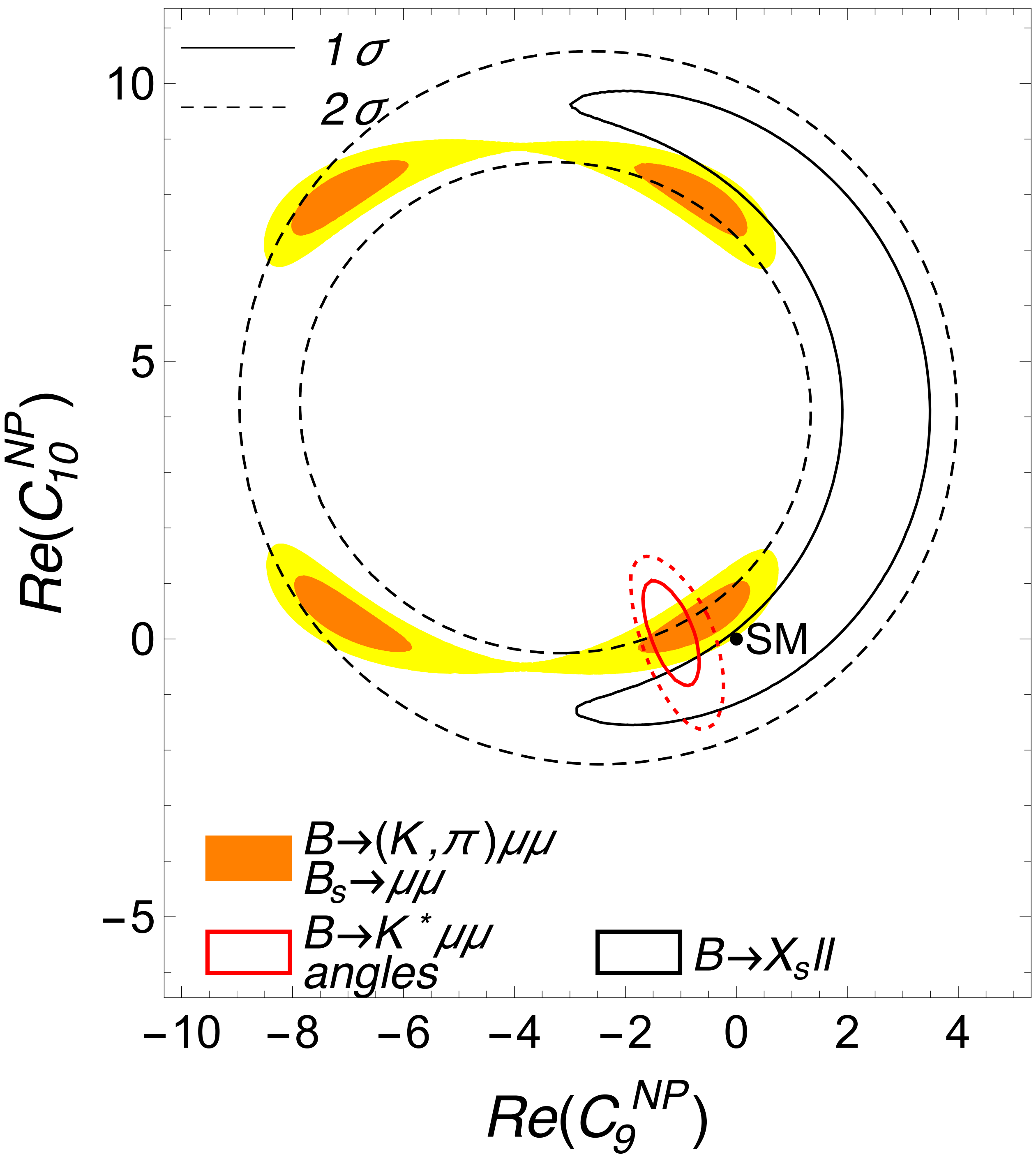
Wilson Coefficients

- It's sad to assume no new physics:
  - take the CKM matrix from a global fit;
  - determine best fit to Wilson coefficients C_9 and C_{10}.
- From the observables considered here, the SM is 2σ away from the best fit.
- Comparable but complementary to angular observables in $B \to K^* \mu\mu$.



Wilson Coefficients 2

- Add $B_s \to \mu\mu$, which also relies on lattice QCD—f_{B_s}.
- (Decay constant of 2015.)
- Favored region shrinks but only away from SM point.
- NB: assuming no new CPV and avoiding $b \to s\gamma$ constraints.

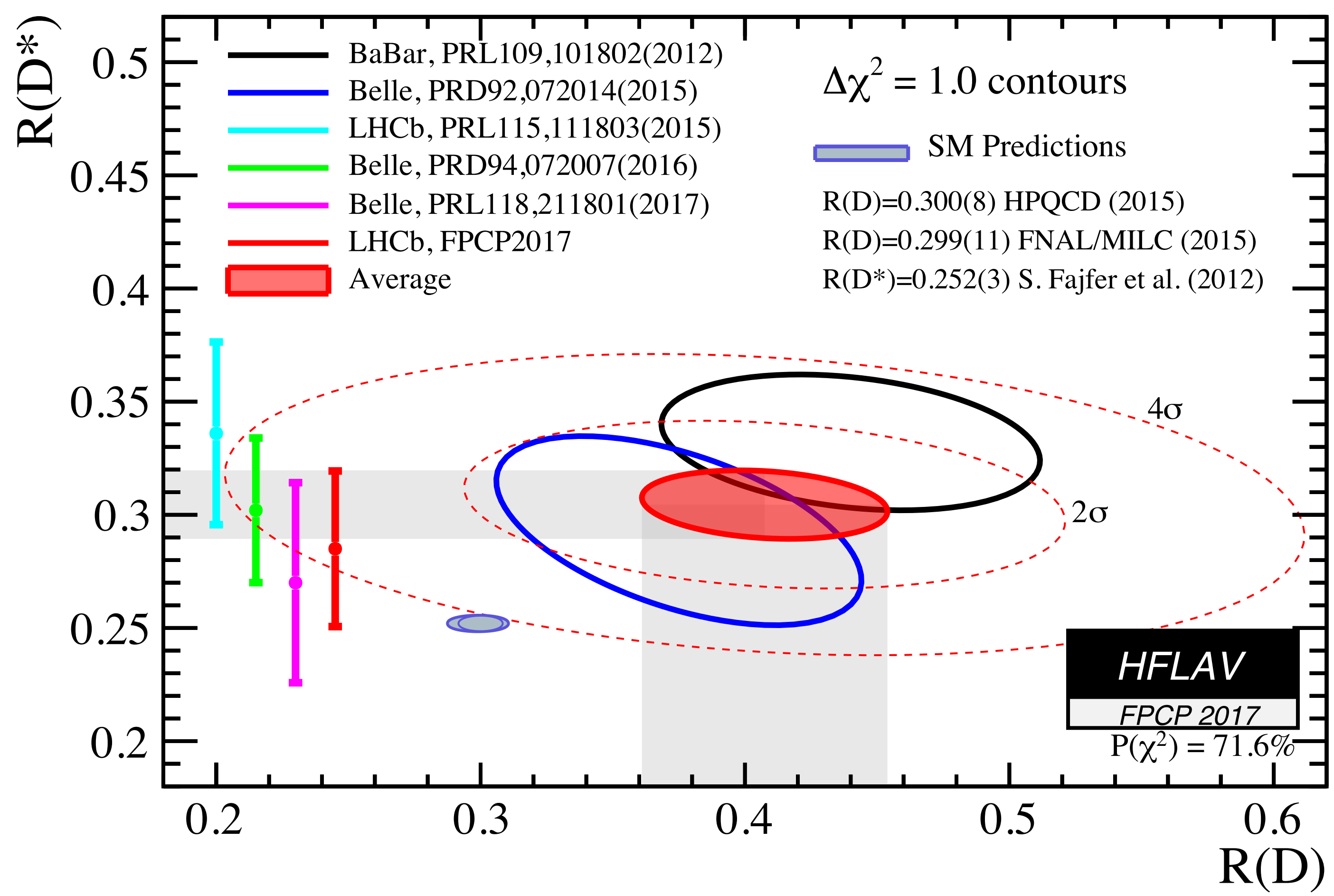


QCD for Charged-Current Anomalies

New Physics in $B \to D^{(*)}\tau\nu$?

BaBar, arXiv:1205.5442; Belle, arXiv:1507.03233; LHCb, arXiv:1506.08614

- BaBar presented evidence for an excess in both channels:
  - 2.0σ for $R(D)$; 2.7σ for $R(D^*)$; 3.4σ combined.
- With Belle & LHCb:
  - 3.9σ combined.
- Estimated form factors w/
  - HQET;
  - quenched QCD.

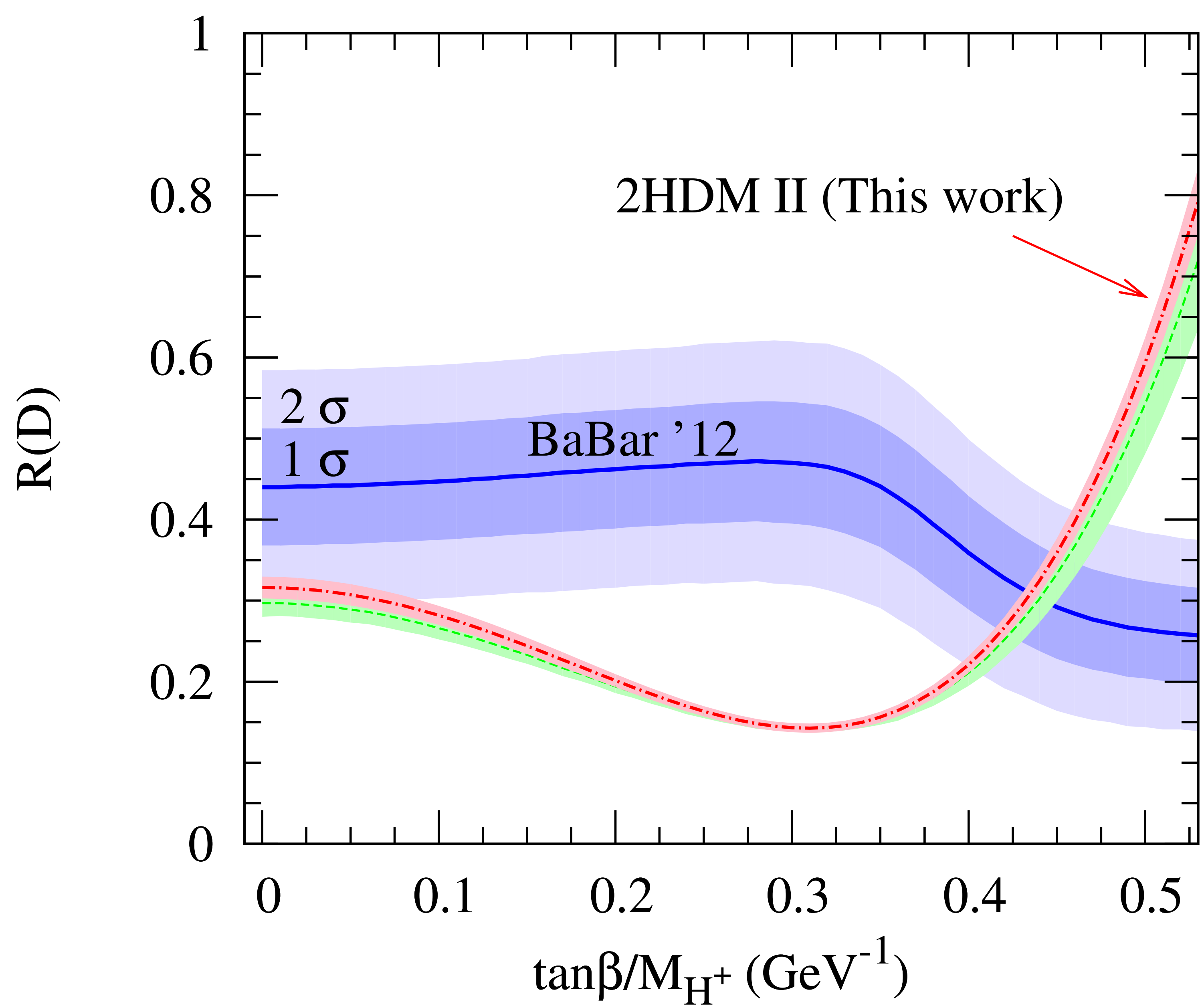


Form Factors for $B \to D^{(*)}\tau\nu$

Fermilab/MILC, arXiv:1206.4992, arXiv:1503.07237; HPQCD, arXiv:1505.03925

see also arXiv:1206.4977.

- $R(D)$ values:
  - 0.297 ± 0.017 (est.);
  - 0.316 ± 0.014 (F/M '12);
  - 0.299 ± 0.011 (F/M '15);
  - 0.300 ± 0.008 (HPQCD).
- Lattice QCD work for $R(D^*)$ underway (see below).



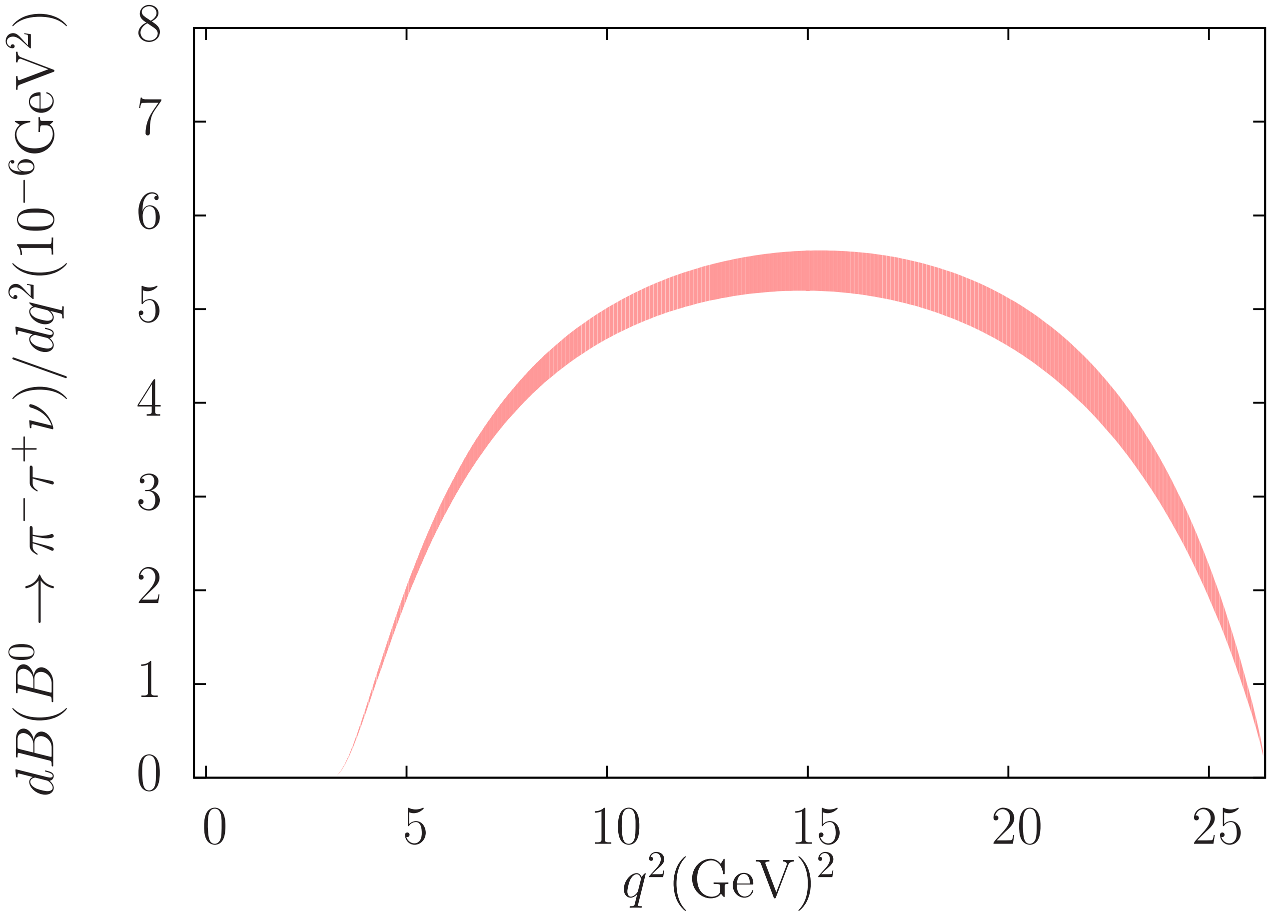
New Physics in $B \to \pi\tau\nu$?

arXiv:1510.02349

- A charged Higgs boson mediating $b \to c$ could also mediate $b \to u$.
- SM prediction, including term $\sim m_\tau^2 |f_0|^2$.
- With the Fermilab/MILC form factors, we find

$$R(\pi) \equiv \frac{\mathscr{B}(B \to \pi\tau\nu_\tau)}{\mathscr{B}(B \to \pi\ell\nu_\ell)} = 0.641(17)$$

- Awaits LHCb, Belle 2

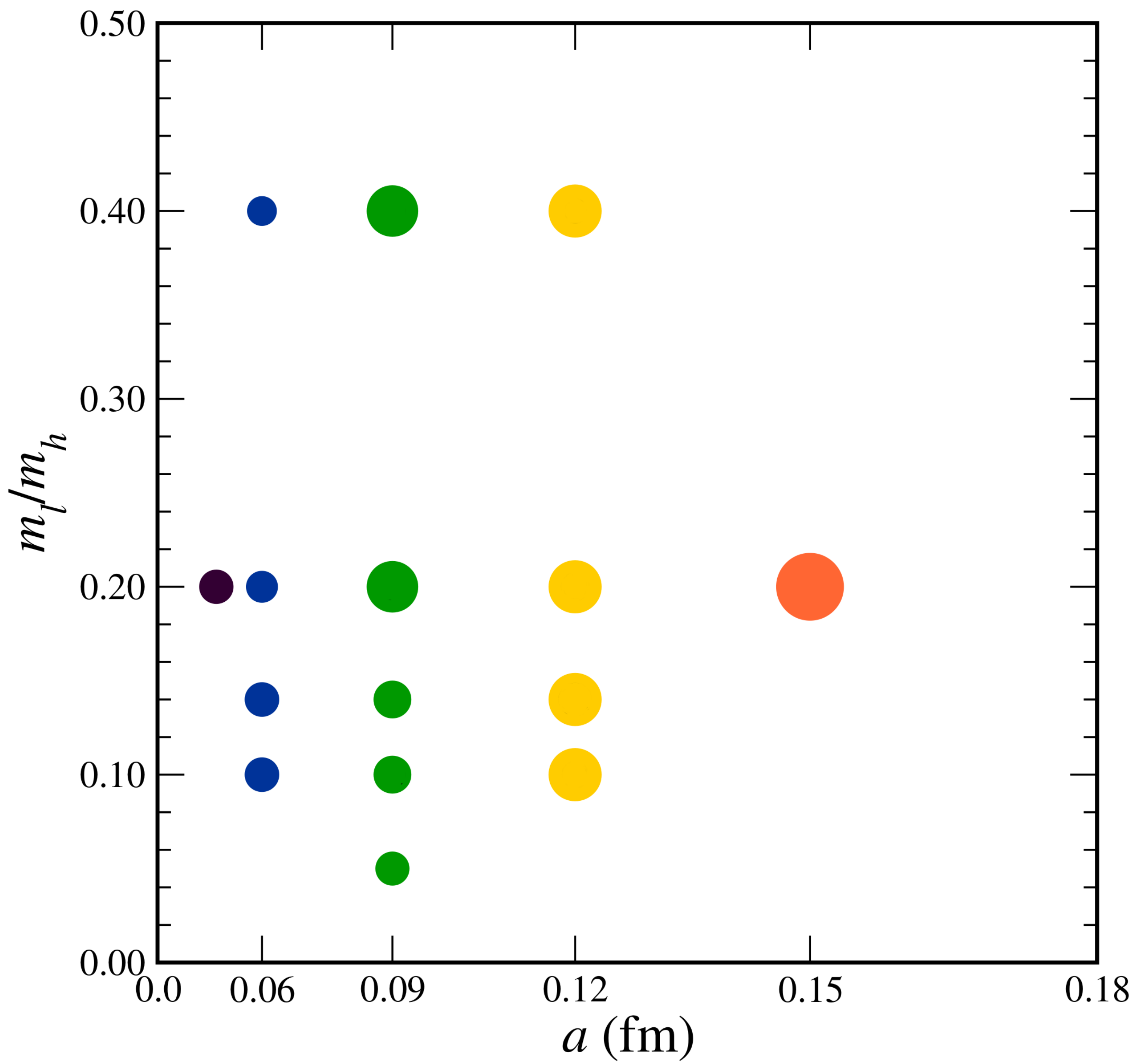


see also arXiv:1501.05373

Progress Report on $R(D^*)$

arXiv:1710.09817

- Fermilab/MILC $B \to D^*$:
  - all four form factors;
  - MILC's asqtad ensembles;
  - 2+1 sea quarks;
  - (same as previous slides).
- Lattice 2017 proceedings: Alejandro Vaquero



Form Factors Defined

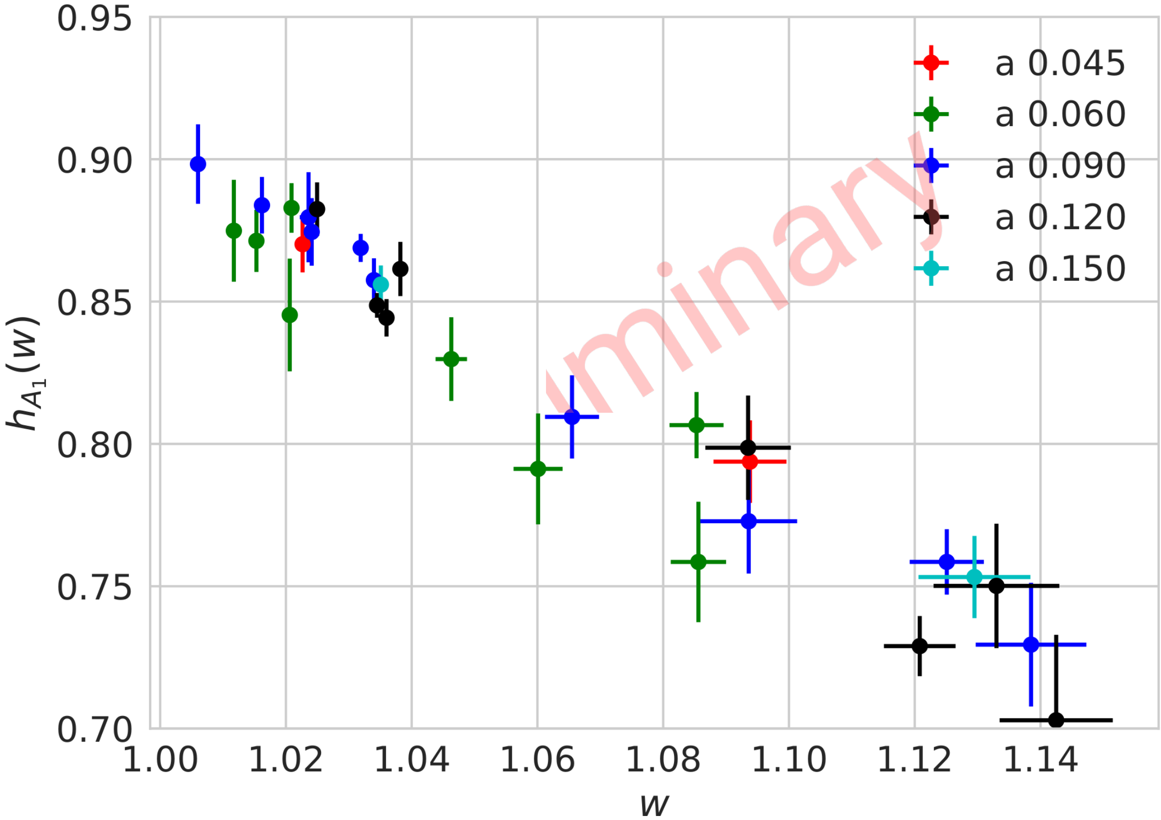
- Electroweak charged current ($w = v_B \cdot v_D$):

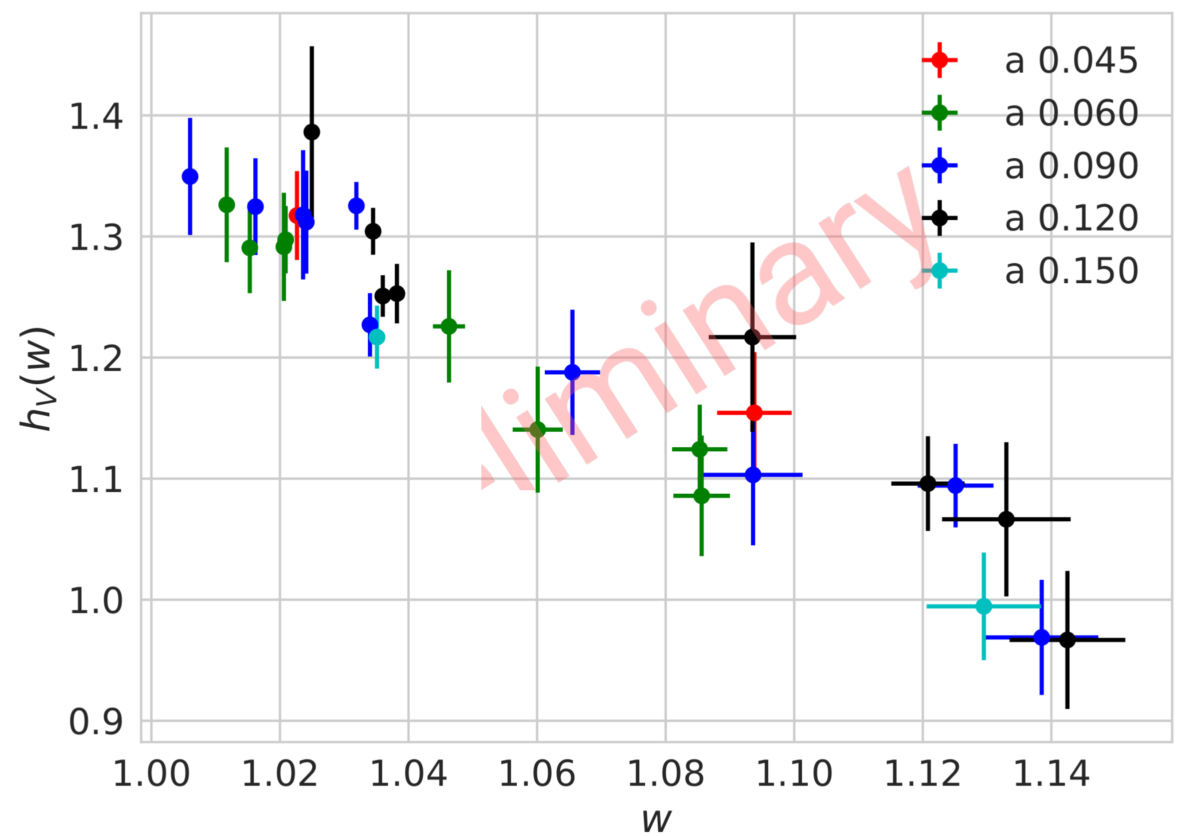
$$\frac{\langle D^*(p_{D^*},\varepsilon^\nu)|\,\mathscr{V}^\mu\,|\bar{B}(p_B)\rangle}{\sqrt{M_B M_{D^*}}} = \varepsilon^*_\nu \varepsilon^{\mu\nu}{}_{\rho\sigma} v^\rho_B v^\sigma_{D^*} h_V(w)$$

$$\frac{\langle D^*(p_{D^*},\varepsilon^\nu)|\,\mathscr{A}^\mu\,|\bar{B}(p_B)\rangle}{\sqrt{M_B M_{D^*}}} = i\varepsilon^*_\nu \left\{ g^{\mu\nu}\,(1+w)\,h_{A_1}(w) - v^\nu_B \left[v^\mu_B h_{A_2}(w) + v^\mu_{D^*} h_{A_3}(w) \right] \right\}$$

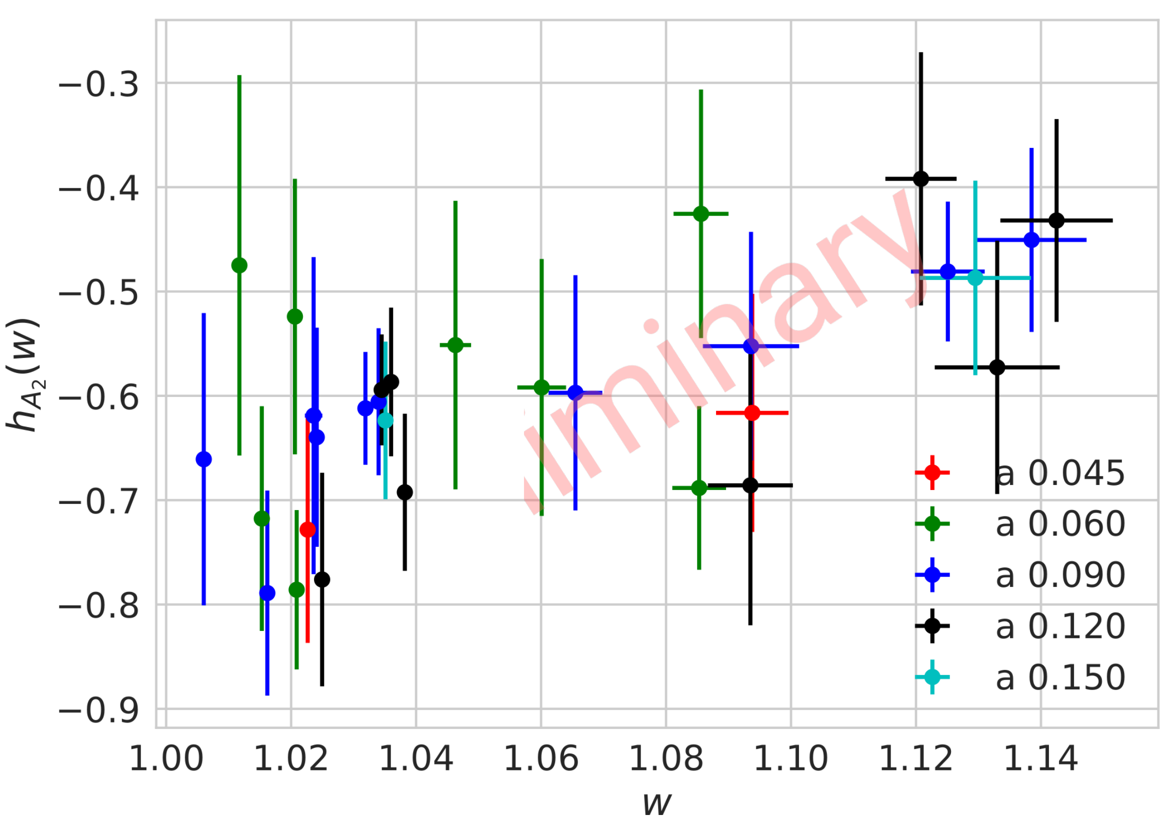
- Linear combination of h_{A_2} & h_{A_3} appears in $\mathrm{d}\Gamma/\mathrm{d}w$ suppressed by m_l^2;

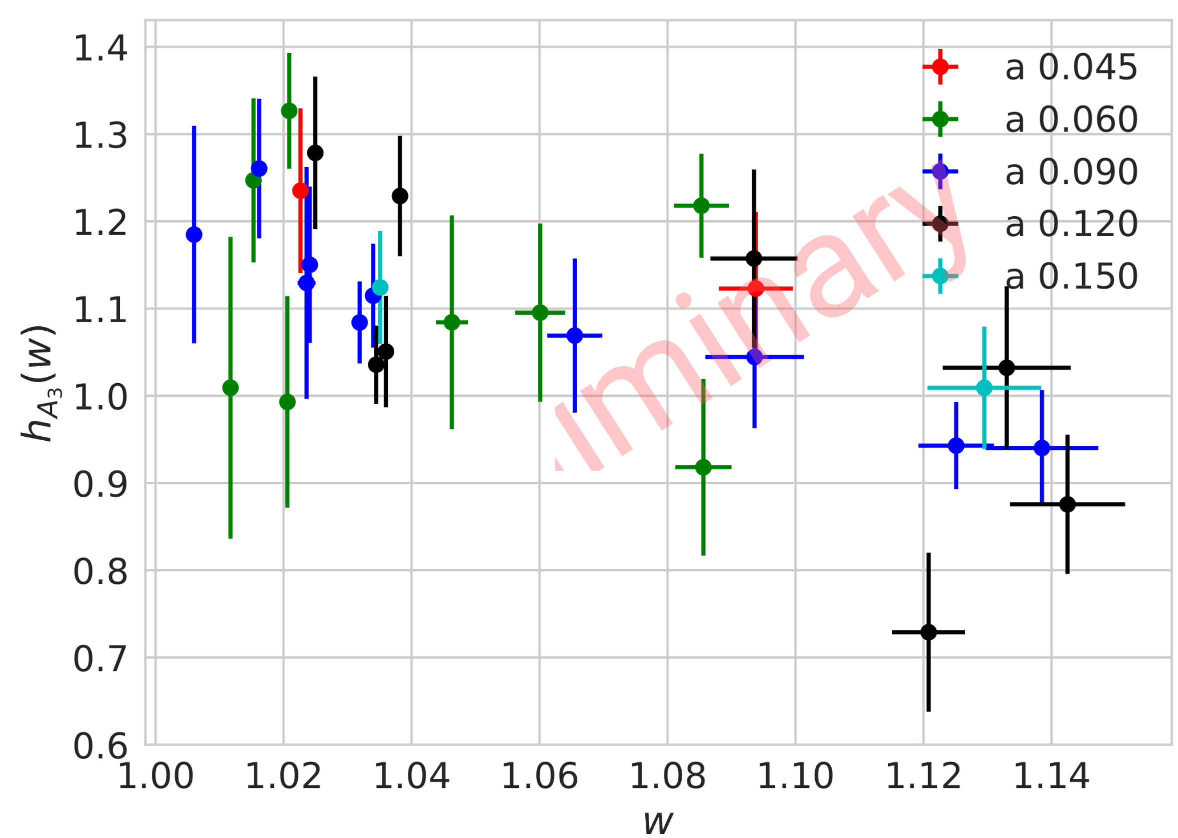
- charged Higgs $\mathscr{A} \propto m_l$, which could thus alter the w distribution for $\tau\nu$.





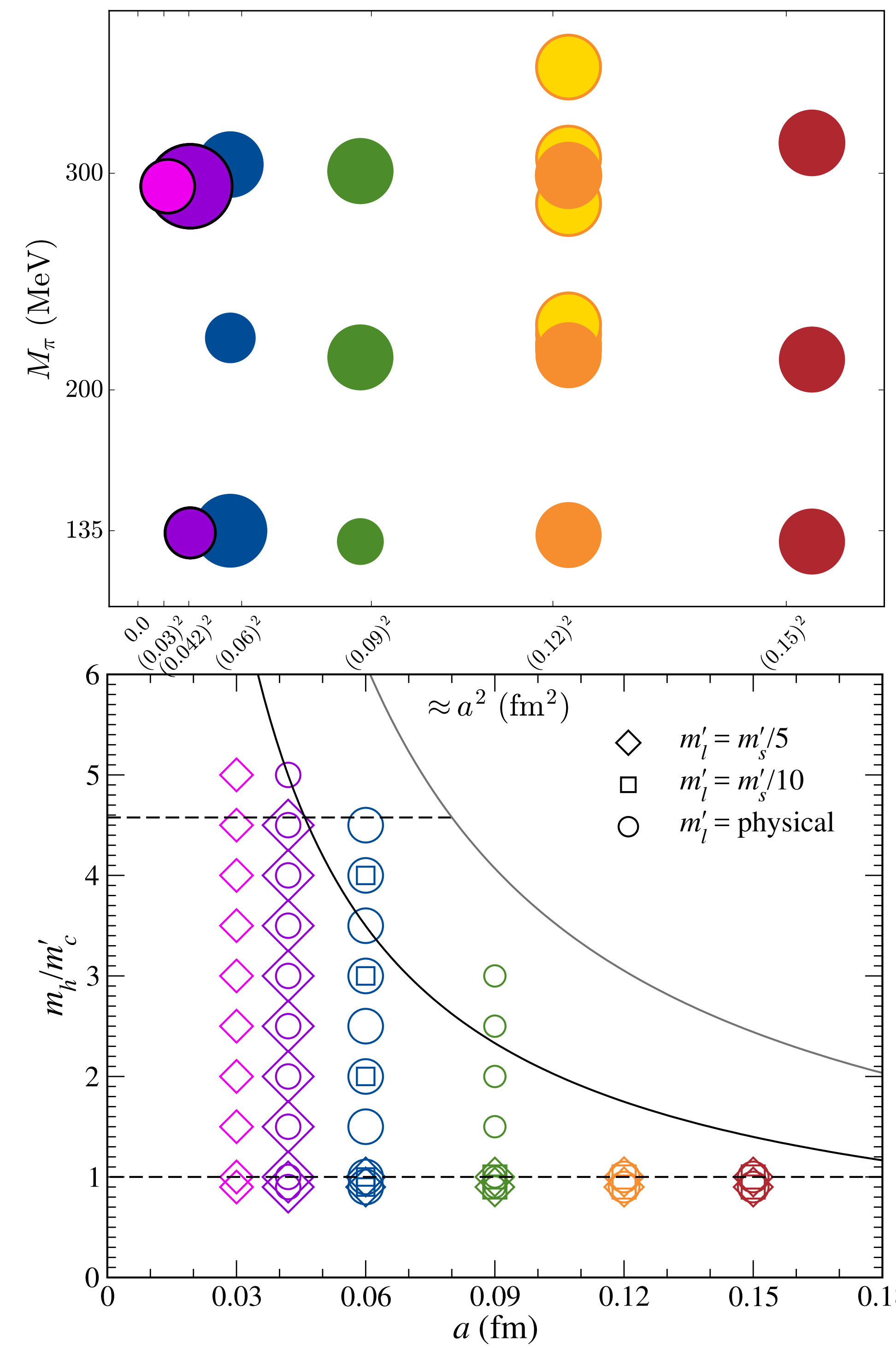
A. Vaquero Avilés-Casco *et alia* (Fermilab/MILC), **PRELIMINARY**



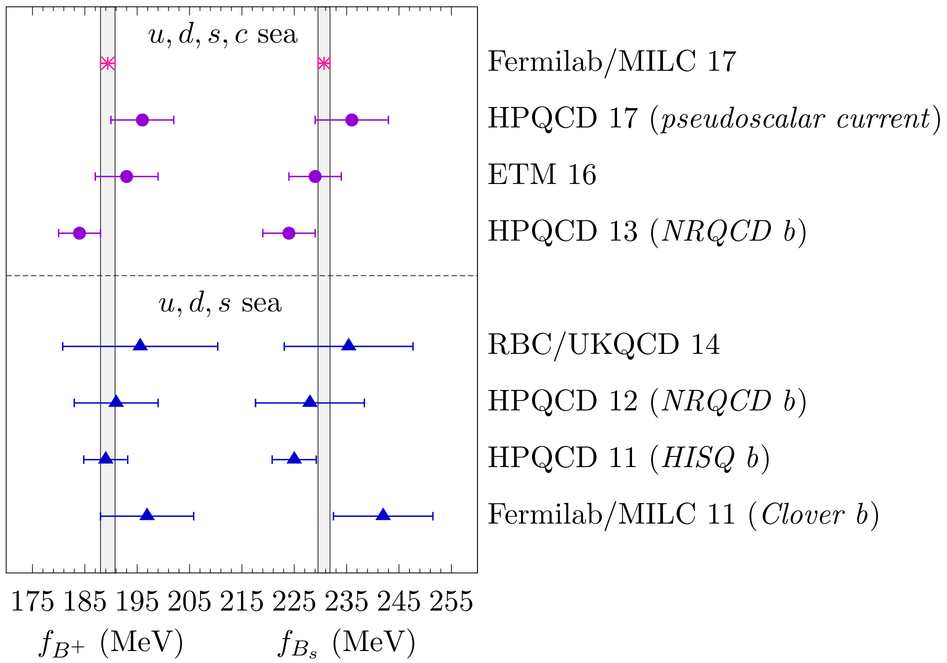


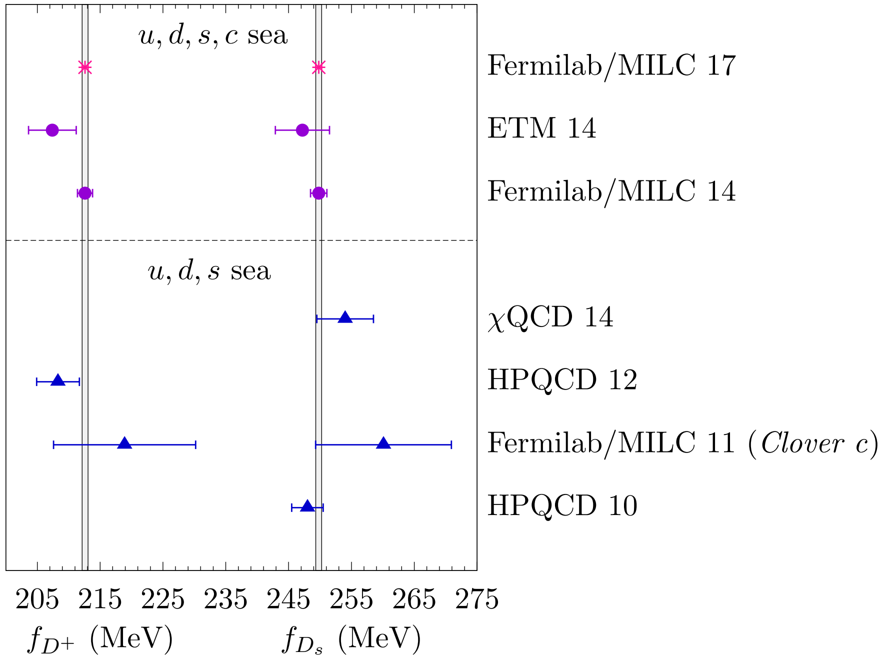
Leptonic Decays: $B \to \tau\nu$, $D_s \to l\nu$; $B_s \to \mu^+\mu^-$

- Simplest flavor-physics for lattice QCD.
- Amplitude($B \to \tau\nu$) $\propto |V_{ub}|f_B$ and, so far "yields" that is too high.
- Amplitude($B \to \mu^+\mu^-$) $\propto |V_{ts}||V_{tb}|f_B$×box, so could have BSM loop too.
- New calculation from Fermilab Lattice & MILC Collaborations [arXiv:1712.09262].
- 20 (2+1+1) HISQ ensembles.
- Huge slab in parameter space.



Results for Decay Constants



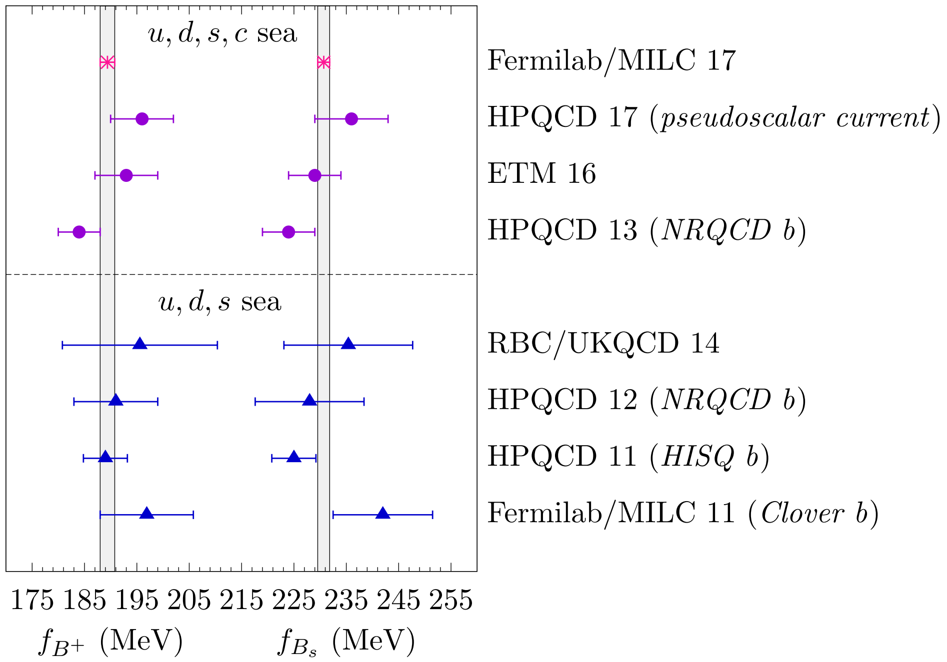


- Fermilab Lattice & MILC [arXiv:1712.09262]:

$$f_{D^0} = 211.5(0.3)_{\text{stat}}(0.3)_{\text{syst}}(0.2)_{f_{\pi,\text{PDG}}}\ \text{MeV}$$
$$f_{D^+} = 212.6(0.3)_{\text{stat}}(0.3)_{\text{syst}}(0.2)_{f_{\pi,\text{PDG}}}\ \text{MeV}$$
$$f_{D_s} = 249.8(0.3)_{\text{stat}}(0.3)_{\text{syst}}(0.2)_{f_{\pi,\text{PDG}}}\ \text{MeV}$$
$$f_{B^+} = 189.4(0.8)_{\text{stat}}(1.1)_{\text{syst}}(0.3)_{f_{\pi,\text{PDG}}}\ \text{MeV}$$
$$f_{B^0} = 190.5(0.8)_{\text{stat}}(1.0)_{\text{syst}}(0.3)_{f_{\pi,\text{PDG}}}\ \text{MeV}$$
$$f_{B_s} = 230.7(0.8)_{\text{stat}}(0.8)_{\text{syst}}(0.2)_{f_{\pi,\text{PDG}}}\ \text{MeV}$$

- Overall uncertainty: ~0.2% for D mesons,
  ~0.7% for B mesons.

Archaeology



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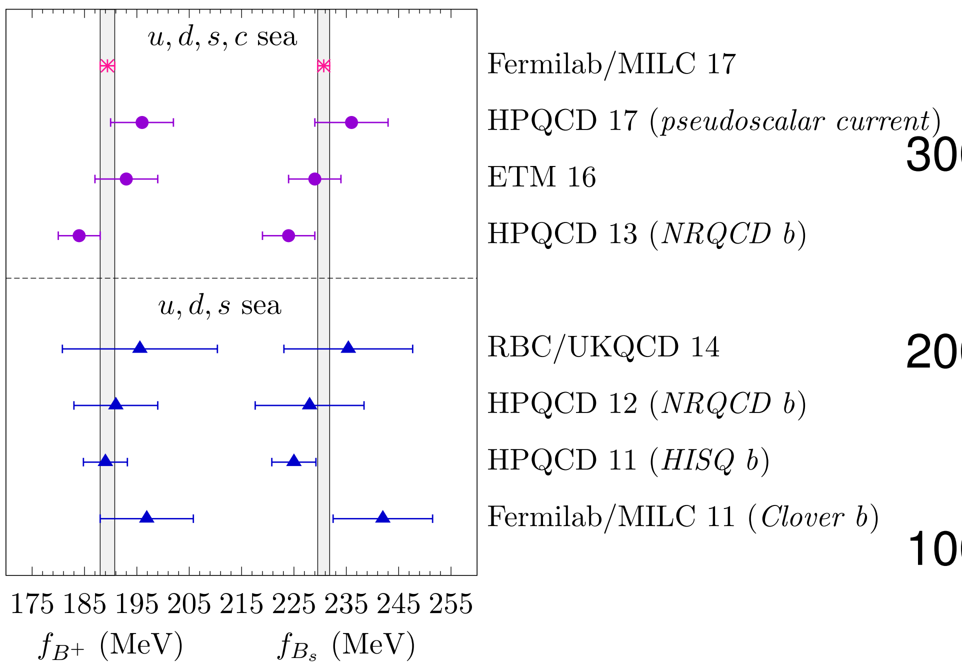

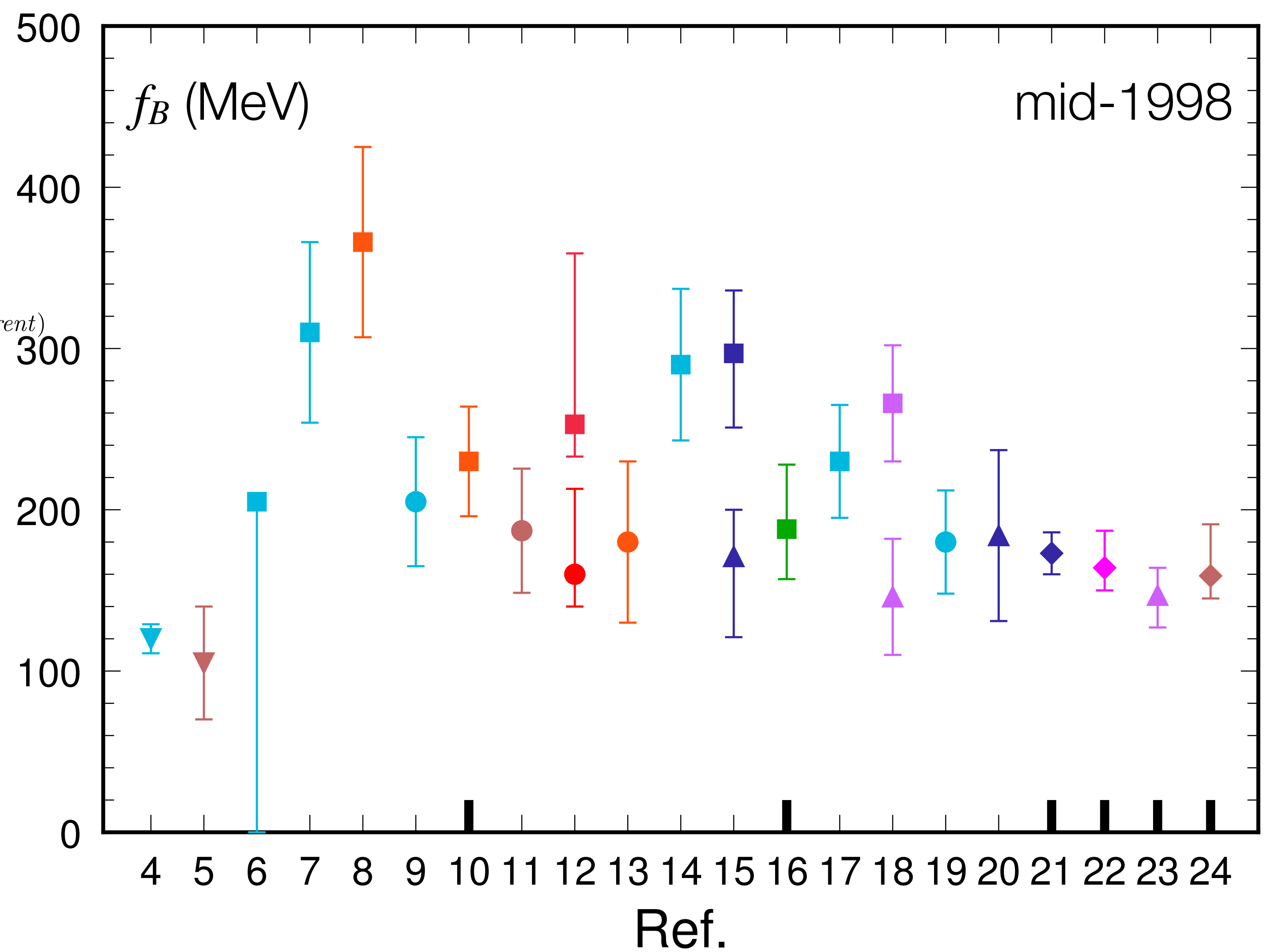

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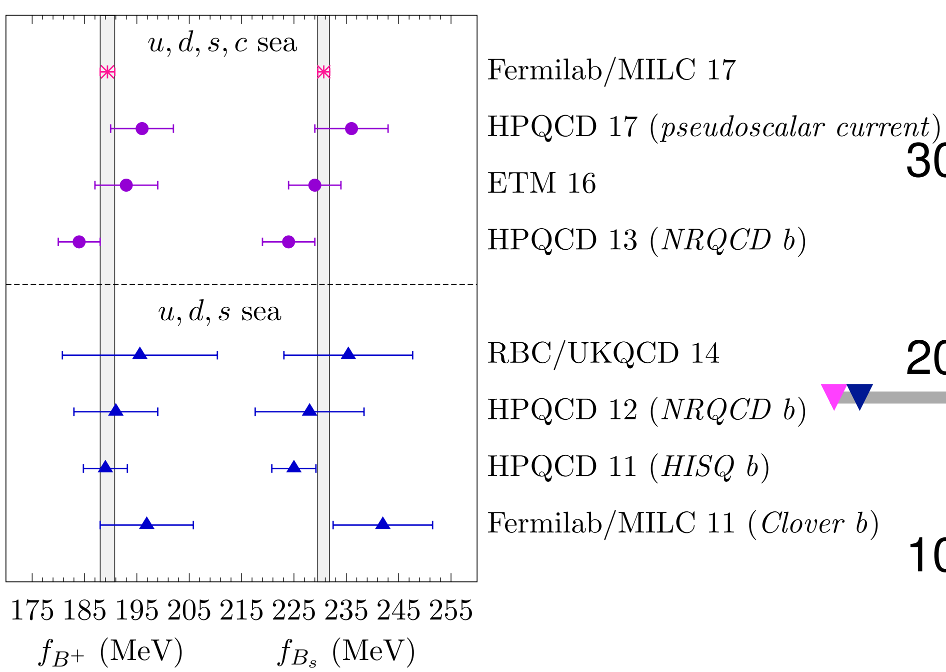

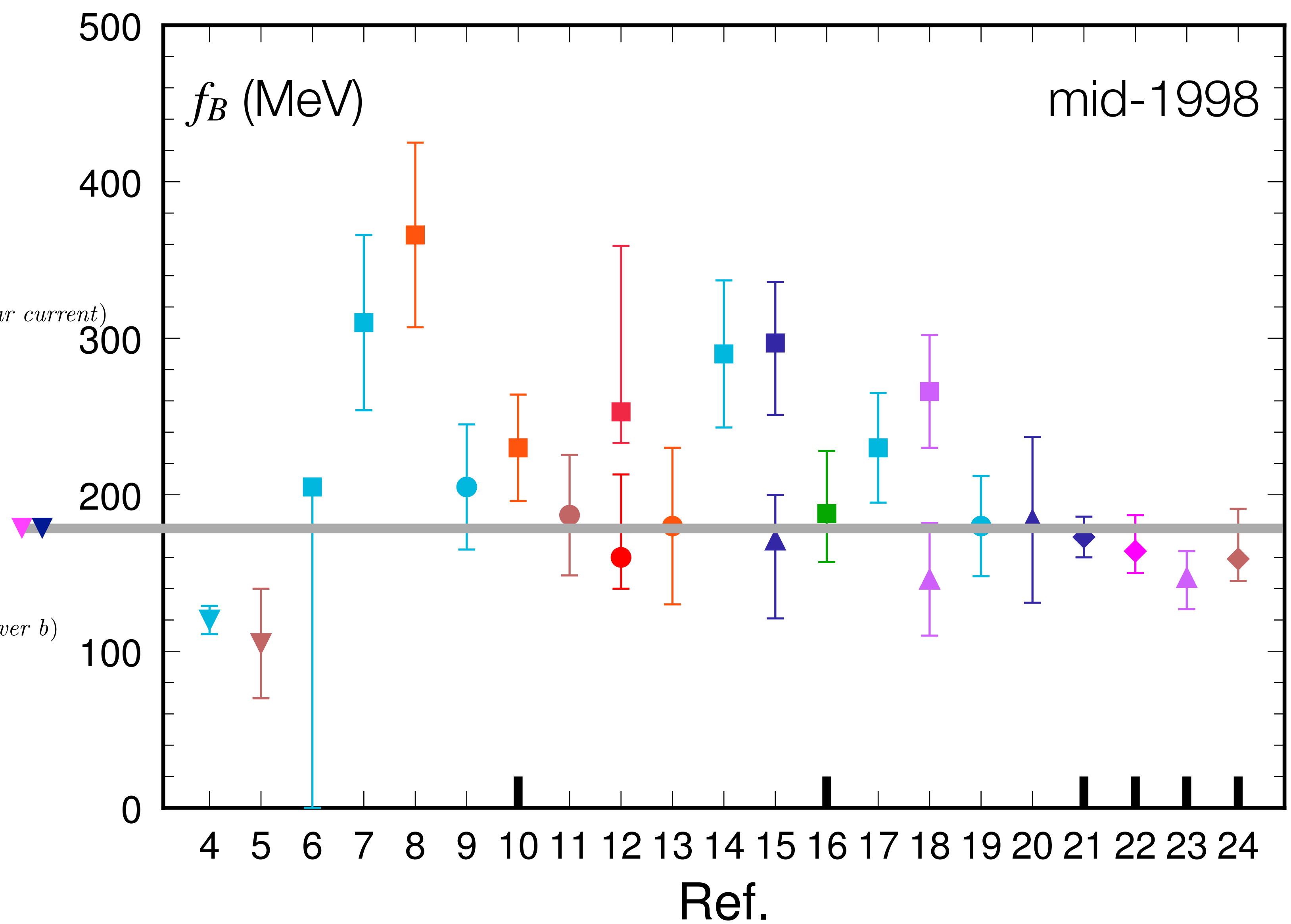

Quark Masses

Heavy-light Meson Masses in HQET

- From HQET (or other approaches to the $1/m_h$ expansion):

$$M_{H_J} = m_h + \bar{\Lambda} + \frac{\mu_\pi^2}{2m_h} - d_J \frac{\mu_G^2(m_h)}{2m_h}$$

- For ~20 years, I've wanted to vary m_h and use this formula to determine $\bar{\Lambda}$, μ_π^2, and $\mu_G^2(m_b)$ from lattice QCD [arXiv:hep-ph/0006345].

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mass of spin-J meson

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mass of spin-J meson

mass of heavy quark

energy of gluons and light quarks

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1 for B, $-\frac{1}{3}$ for B^*

spin-orbit interaction

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What’s a Quark Mass?

- You can’t put a quark on a scale and weigh it.
- Need definition, preferably regularization-independent, in QFT.
- Natural candidate is the “perturbative pole mass.” Alas, ambiguous:
  - physics—infrared gluons need to find a sink;
  - mathematics—obstruction to Borel summation of the perturbative series;
  - jargon—infrared renormalon;
  - numbers—$m_{\mathrm{b,pole}}/\bar{m}_b = (1, 1.093, 1.143, 1.183, 1.224)$.

Short-Distance Definitions

- Usual work-around is to use a "short-distance" mass.
- The $\overline{\text{MS}}$ mass in dimensional regularization, $m_{h,\overline{\text{MS}}}(\mu)$; $\bar{m}_h \equiv m_{h,\overline{\text{MS}}}(\bar{m}_h)$:
  - spoils HQET power counting: $m_{\text{pole}} - \bar{m}_h \propto \alpha_s(\bar{m}_h)\bar{m}_h$.
- Other definitions subtract out infrared part:
  - "kinetic mass" (Uraltsev) via a Wilsonian renormalization;
  - "renormalon subtracted mass" (Pineda) subtracts out renormalon;
  - "MSR mass" (Hoang, Jain, Scimemi, Stewart) similarly
  - all need another scale $1\ \text{GeV} < \nu_f < m_h$, or yet another $\alpha_s(\mu)$.

Renormalon-a-Ding-Dong

- The n^{th} coefficient in the relation between m_{pole} and $\overline{m}$ grows like $n!$:

$$r_n \sim R_n = R_0(2\beta_0)^n \frac{\Gamma(n+1+b)}{\Gamma(1+b)}, \quad n \geq 0, \quad b = \frac{\beta_1}{2\beta_0^2} = \frac{231}{645} \text{ for } (n_f = 4)$$

- Theory of asymptotic series leads to (α_{g} is α_{s} with simpler algebra):

$$\mu \sum_{n=0}^{\infty} R_n \alpha_{\text{g}}(\mu) = \frac{R_0}{2\beta_0} \mu \int_0^{\infty} dz \, \frac{e^{-z/(2\beta_0 \alpha_{\text{g}}(\mu))}}{(1-z)^{1+b}}$$

$$\equiv \mathscr{I}(\mu)$$

and note that the integration is ill-defined for $z > 1$.

- Break the integral into an unambiguous part $z \in [0,1]$ and a totally ambiguous part $z \in [1,\infty]$.

Minimal Renormalon Subtraction

arXiv:1712.04983

- Splitting the integral (Brambilla, **Komijani**, ASK, Vairo):

$$\mathscr{J}(\mu) = \mathscr{J}_{\text{MRS}}(\mu) + \delta m$$

$$\mathscr{J}_{\text{MRS}}(\mu) = \frac{R_0}{2\beta_0}\mu \int_0^1 dz\, \frac{e^{-z/[2\beta_0 \alpha_{\text{g}}(\mu)]}}{(1-z)^{1+b}}$$

$$\delta m = \frac{R_0}{2\beta_0}\mu \int_1^\infty dz\, \frac{e^{-z/[2\beta_0 \alpha_{\text{g}}(\mu)]}}{(1-z)^{1+b}} = -(-1)^b \frac{R_0}{2^{1+b}\beta_0}\Gamma(-b)\Lambda_{\overline{\text{MS}}}$$

- Minimal renormalon-subtracted (MRS) mass:

$$m_{\text{MRS}} \equiv m_{\text{pole}} - \delta m$$

$$= \bar{m}\left(1 + \sum_{n=0}^{\infty}\left[r_n - R_n\right]\alpha_{\text{g}}^{n+1}(\bar{m})\right) + \mathscr{J}_{\text{MRS}}(\bar{m})$$

Remarks

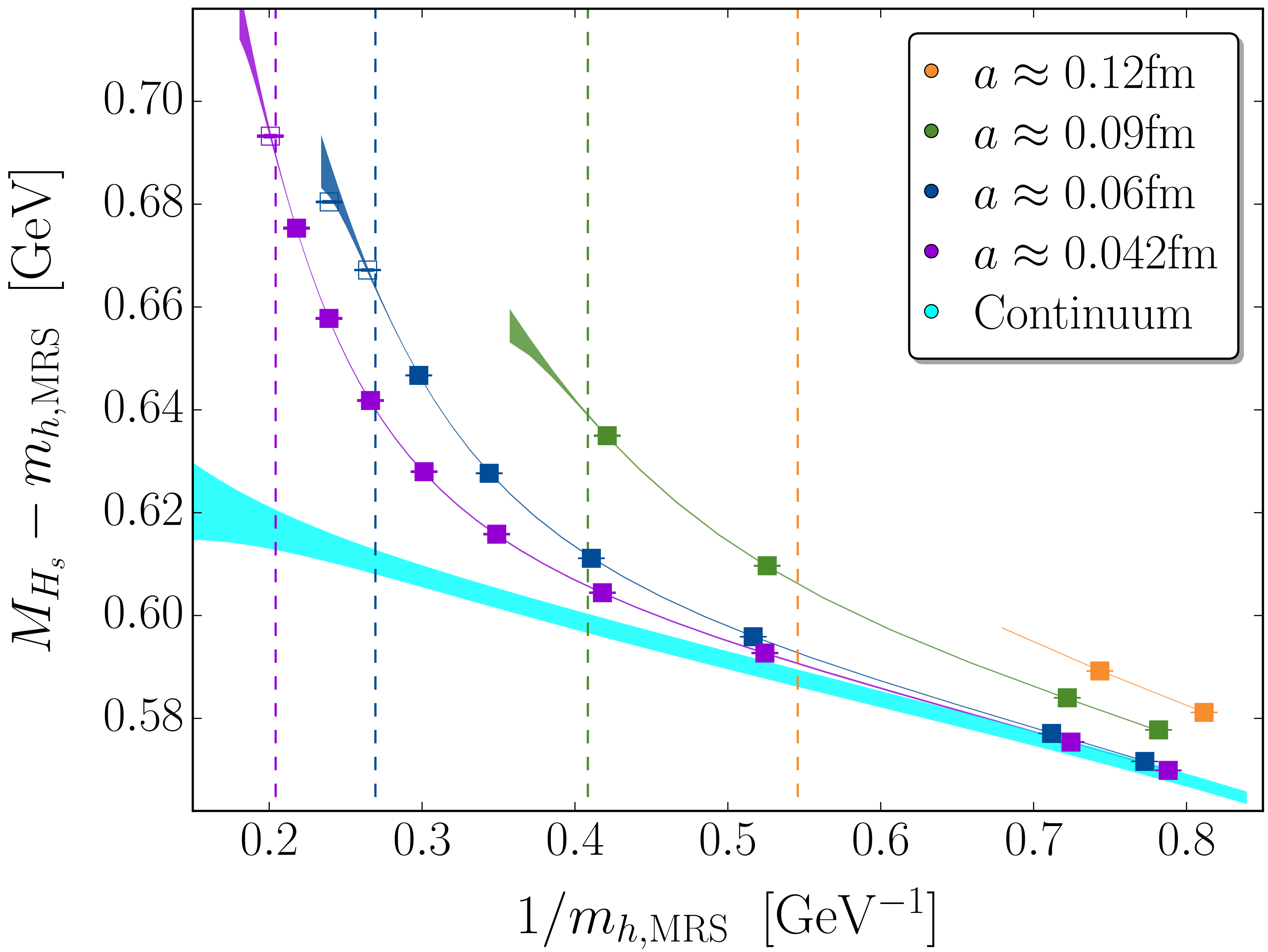
- MRS mass is a short-distance mass: subtract off long-range δm.
- No new scale: trim long-range field at $1/m_h$, not $1/\nu_f$.
- Numerically very stable: $m_{b,\text{MRS}}/\bar{m}_b = (1.157, 1.133, 1.131, 1.132, 1.132)$.
- Use new formula [Kojimani, arXiv:1701.00347] for R_0 to evaluate $\mathscr{J}_{\text{MRS}}$.
- Makes HQET formula unambiguous (to order $1/m_h$):

$$M_{H_J} = m_{h,\text{MRS}} + \bar{\Lambda}_{\text{MRS}} + \frac{\mu_\pi^2}{2m_h} - d_J \frac{\mu_G^2(m_h)}{2m_h}$$

- Next step: fit this formula to lattice-QCD data!

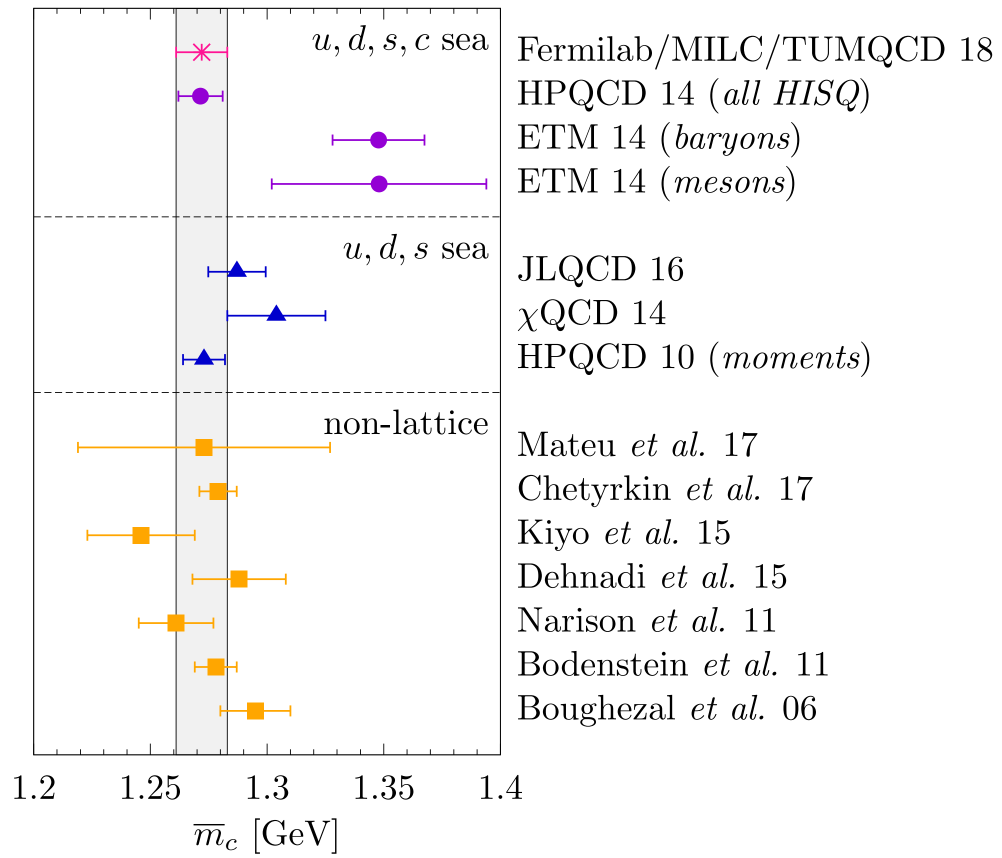
HQET Fit ⊕ Symanzik EFT ⊕ χPT

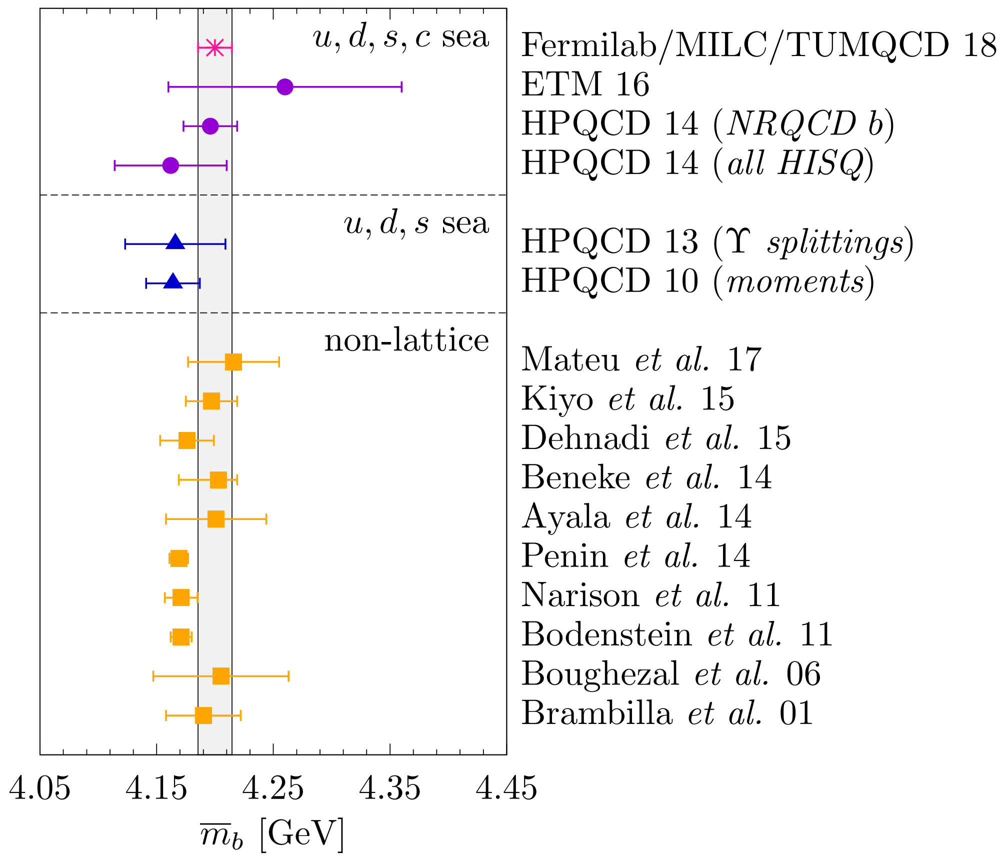
- Same correlators as decay constants.
- 20 ensembles
- 0.005–0.12% on meson *M*
- 5 (6) lattice spacings
- Snapshot at right ☛



Results & Comparisons

- Not quite finished, so these preliminary results are indicative:



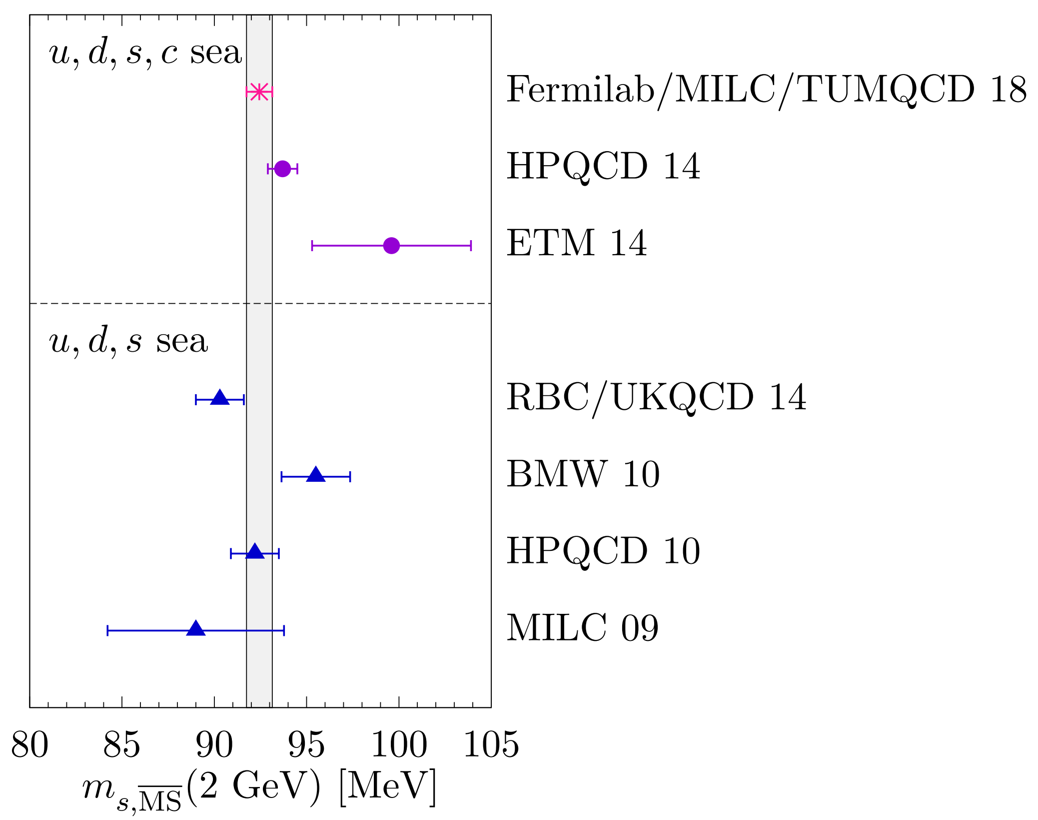


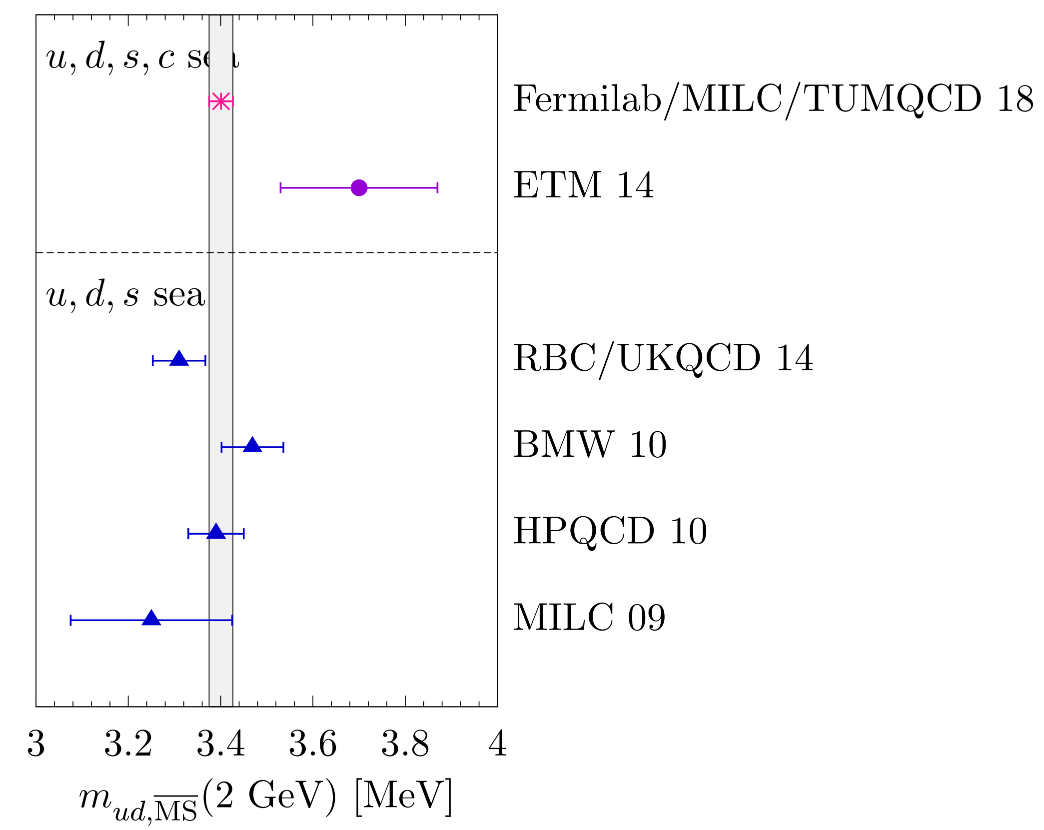
- To our knowledge, first results w/ order-α_s^5 running & order-α_s^4 matching.

- Precision: 0.3% for bottom to 0.5% for charm.

Results & Comparisons 2

- With mass ratios from light pseudoscalar meson:





- Most precise strange and "light" quark masses to date.

- Most precise quark masses for all quarks except top ($m_u > 50\sigma$).

Outlook

Outlook

- Pressing issue is $R(D^*)$. Perhaps in time for CKM 2018?
- Decay constants are done?
- The "all HISQ" technology can be extended to vector & axial-vector form factors, *i.e.*, absolutely normalized currents:
  - good news for $|V_{cb}|^4$ and $|V_{ub}|$.
- Neutral meson mixing still falls short of experimental precision: matrix elements have anomalous dimensions ⇒ perturbative QCD to get to $\overline{\text{MS}}$.
- Omitted here: theoretical and computational progress in $B \to K^*(K\pi)\mu\mu$.

Merci vielmals!