

# Non-local effects in exclusive $b \rightarrow sll$ decays

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# Introduction

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# Motivation

Experimental measurements on  $b \rightarrow s\ell\ell$

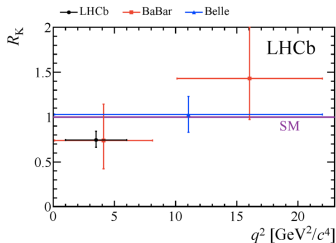
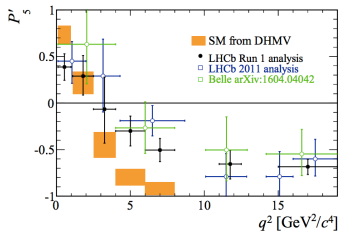
- ▶ LHCb measurements  $B \rightarrow K\mu\mu$ ,  $B \rightarrow K^*\mu\mu$ ,  $B_s \rightarrow \phi\mu\mu$
- ▶ Analogous measurements by Belle, ATLAS and CMS
- ▶ Lepton-Flavor Non-Universality

Raised a lot of interest, lot of work from theory + experiment

- ▶ Mostly: Interest in "Anomalies" and New Physics
- ▶ Here: strive to get a better handle on hadronic matrix elements of non-local operators

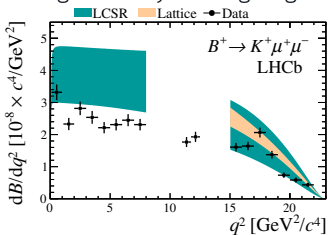
# Motivation

Intriguing "anomalies" in some observables

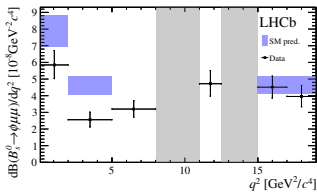


[Phys. Rev. Lett. 113, 151601 (2014)]

Less significant yet intriguing deviations in branching ratios



[LHCb JHEP06(2014)133]

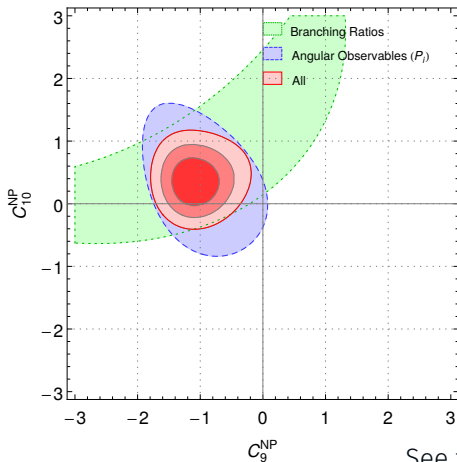


[LHCb JHEP09(2015)179]

# Motivation

## Significant SM pulls in global fits

[Descotes-Genon, Hofer, Matias, Virto 2015 + others]



Significance already at the level of  $\sim 5\sigma$  \*\*\*\*\*

# Effective Theory

For  $\Lambda_{EW}, \Lambda_{NP} \gg M_B$  : Flavour and CP mediated by  $D = 6$  ops :

$$\mathcal{L}_W = \mathcal{L}_{QCD} + \mathcal{L}_{QED} + \frac{4G_F}{\sqrt{2}} \left[ \lambda_t \sum_i \mathcal{C}_i \mathcal{O}_i + \lambda_c \sum_i \mathcal{C}_i^c \mathcal{O}_i^c + \lambda_u \sum_i \mathcal{C}_i^u \mathcal{O}_i^u \right]$$

$$\mathcal{O}_7 = \frac{e}{16\pi^2} m_b (\bar{s} \sigma^{\mu\nu} P_R b) F_{\mu\nu}$$

$$\mathcal{O}_8 = \frac{g_s}{16\pi^2} m_b (\bar{s} \sigma^{\mu\nu} P_R T^A b) G_{\mu\nu}^A$$

$$\mathcal{O}_{9\ell} = \frac{\alpha}{4\pi} (\bar{s} \gamma_\mu P_L b) (\bar{\ell} \gamma^\mu \ell)$$

$$\mathcal{O}_{10\ell} = \frac{\alpha}{4\pi} (\bar{s} \gamma_\mu P_L b) (\bar{\ell} \gamma^\mu \gamma_5 \ell)$$

$$\mathcal{O}_1^c = (\bar{c} \gamma_\mu P_L b) (\bar{s} \gamma^\mu P_L c)$$

$$\mathcal{O}_2^c = (\bar{c} \gamma_\mu P_L T^a b) (\bar{s} \gamma^\mu P_L T^a c)$$

$$\mathcal{O}_1^u = (\bar{u} \gamma_\mu P_L b) (\bar{s} \gamma^\mu P_L u)$$

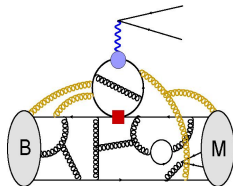
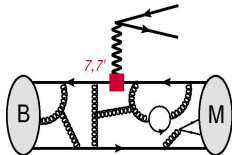
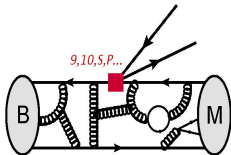
$$\mathcal{O}_2^u = (\bar{u} \gamma_\mu P_L T^a b) (\bar{s} \gamma^\mu P_L T^a u)$$

$$\mathcal{O}_i = (\bar{s} \gamma_\mu P_X b) \sum_q (\bar{q} \gamma^\mu q)$$

SM contributions to  $\mathcal{C}_i(\mu_b)$  known to NNLL [Bobeth, Misiak, Urban '99; Misiak, Steinhauser '04, Gorbahn,

Haisch '04; Gorbahn, Haisch, Misiak '05; Czakon, Haisch, Misiak '06]

# $B \rightarrow M\ell\ell$ Amplitudes



$$\mathcal{A}_\lambda^{L,R} = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2m_b M_B}{q^2} \left[ C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

► Local (Form Factors):  $\mathcal{F}_\lambda^{(T)}(q^2) = \langle \bar{M}_\lambda(k) | \bar{\Gamma}_\lambda^{(T)} b | \bar{B}(k+q) \rangle$

► Non-Local:  $\mathcal{H}_\lambda(q^2) = i P_\mu^\lambda \int d^4x e^{iq \cdot x} \langle \bar{M}_\lambda(k) | T \{ \mathcal{J}_{em}^\mu(x), C_i \mathcal{O}_i(0) \} | \bar{B}(q+k) \rangle$

► CKM structure:  $\mathcal{H}_\lambda = -\frac{\lambda_u}{\lambda_t} \mathcal{H}_\lambda^{(u)} - \frac{\lambda_c}{\lambda_t} \mathcal{H}_\lambda^{(c)}$

# Lorentz Decomposition

$$\begin{aligned}\mathcal{H}^\mu(q, k) &\equiv i \int d^4x e^{iq \cdot x} \langle \bar{K}^*(k, \eta) | T \{ \mathcal{J}_{\text{em}}^\mu(x), \mathcal{C}_i \mathcal{O}_i(0) \} | \bar{B}(k + q) \rangle \\ &\equiv M_B^2 \eta_\alpha^* \left[ S_\perp^{\alpha\mu} \mathcal{H}_\perp(q^2) - S_\parallel^{\alpha\mu} \mathcal{H}_\parallel(q^2) - S_0^{\alpha\mu} \mathcal{H}_0(q^2) \right]\end{aligned}$$

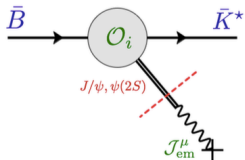
- $S_\lambda^{\alpha\mu}$  – basis of Lorentz structures (carefully chosen)
- $\mathcal{H}_\lambda$  – Lorentz invariant correlation functions
- $\lambda$  – polarization states ( $\perp, \parallel, 0$ ) [for vector meson]



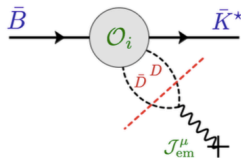
## A different approach

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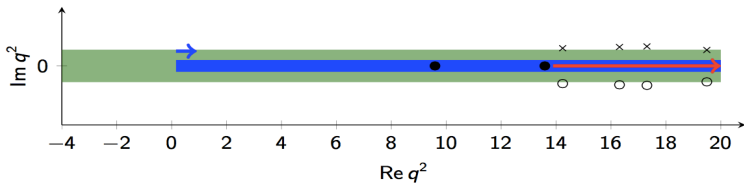
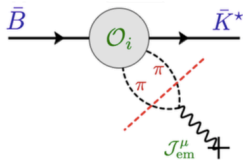
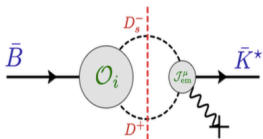
# Analytic structure



(a)

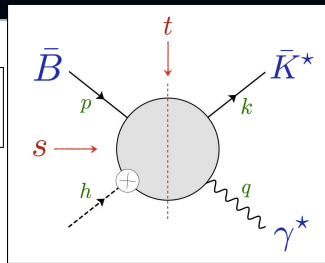


(b)



# Understanding the $p^2$ cut

**Trick:** Add spurious momentum  $h$  to  $\mathcal{O}_i$   
 Recover physical kinematics as  $h \rightarrow 0$



- ▶ consider Mandelstam variables

$$s \equiv (p + h)^2 \longrightarrow M_B^2$$

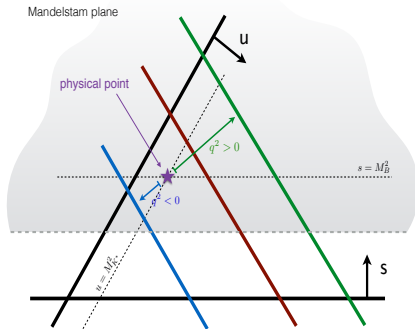
$$u \equiv (k - h)^2 \longrightarrow M_{K^*}^2$$

$$t \equiv (q - h)^2 \longrightarrow q^2$$

physical point

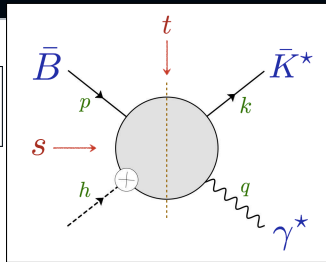
- ▶  $s$  independent of  $t$

- ▷ cut in  $s \sim p^2$  does not translate into cut in  $t \sim q^2$



# Understanding the $p^2$ cut

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$$s \equiv (p + h)^2 \longrightarrow M_B^2$$

$$u \equiv (k - h)^2 \longrightarrow M_{K^*}^2$$

$$t \equiv (q - h)^2 \xrightarrow{\text{physical point}} q^2$$

- ▶  $s$  independent of  $t$

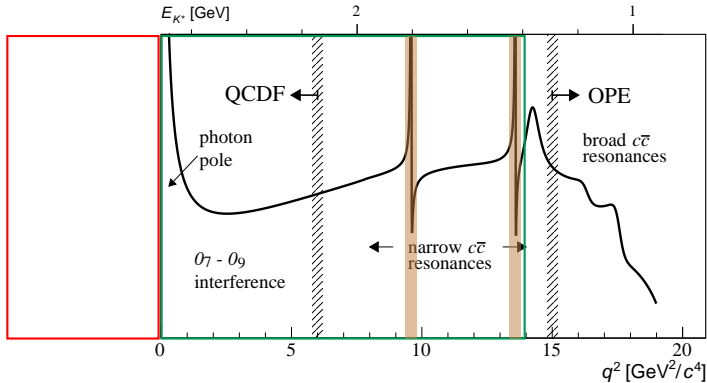
▷ cut in  $s \sim p^2$  does not translate into cut in  $t \sim q^2$

- two correlators:

$$\mathcal{H}_\lambda(q^2) \rightarrow \mathcal{H}_\lambda^{\text{real}}(q^2) + i\mathcal{H}_\lambda^{\text{imag}}(q^2)$$

- both are analytic at  $q^2 \leq 0$
- both have branch cuts at  $q^2 > 0$
- the same dispersion relation governs their  $q^2$ -dependence

# Strategy



[sketch from Blake, Gershon, Hiller 2015]

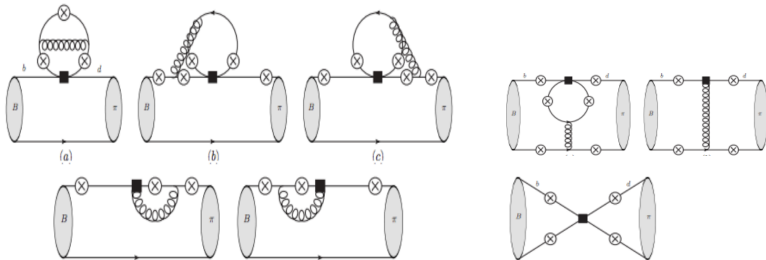
- ▶ **calculate** non-local matrix elements at  $q^2 < 0$
- ▶ **extrapolate** to  $q^2 > 0$  via some type of analytic continuation
- ▶ **constrain** two narrow resonances at  $q^2 > 0$  from data on  $B \rightarrow \psi_n K^*$

# Calculations at negative $q^2$

## ► QCD Factorization

[Beneke, Feldmann, Seidel 2001 & 2004]

$$\mathcal{H}_\lambda = C_\lambda \mathcal{F}_\lambda + \sum_{\pm} \int \frac{d\omega}{\omega} \Phi_{\pm}^B(\omega) \int_0^1 du T_\lambda^\pm(u, \omega) \phi_M^\pm(u) + \mathcal{O}(\Lambda/m_B, \Lambda/E)$$

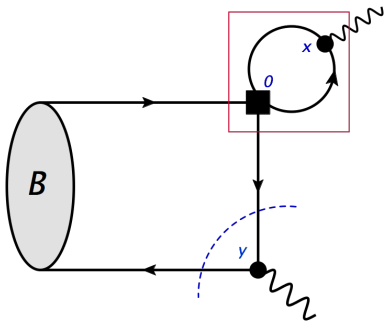


$$\mathcal{H}_\lambda(q^2) = \mathcal{H}_{\lambda;\text{fact,LO}}(q^2) + \mathcal{H}_{\lambda;\text{fact,NLO}}(q^2) + \mathcal{H}_{\lambda;\text{spect}}(q^2) + \mathcal{H}_{\lambda;\text{WA}}(q^2) + \dots$$

# Calculations at negative $q^2$

## ► LCSRs with $B$ -meson DAs

[Khodjamirian, Mannel, Pivovarov, Wang 2010]

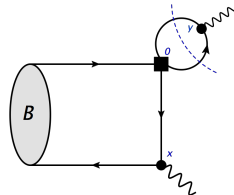
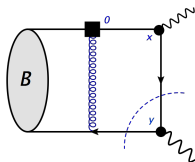
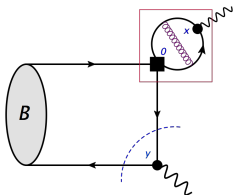


LC exp. of charm prop.

[Balitsky, Braun 1989]

$$\xrightarrow{q^2 \ll 4m_c^2} \underbrace{\left( \frac{C_1}{3} + C_2 \right) g(m_c^2, q^2)}_{\text{matching coeff}} [\bar{s} \Gamma b] + \dots$$

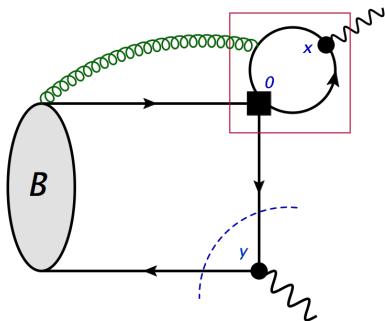
$$\Rightarrow \mathcal{H}_\lambda = (\text{matching coeff}) \times \mathcal{F}_\lambda^{\text{LC SR}}$$



# Calculations at negative $q^2$

## ► LCSRs with $B$ -meson DAs

[Khodjamirian, Mannel, Pivovarov, Wang 2010]



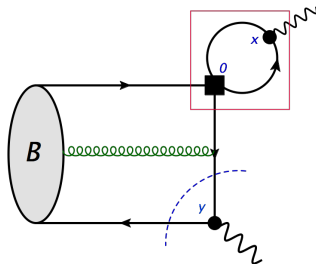
LC exp. of charm prop.

[Balitsky, Braun 1989]

$$q^2 \ll 4m_c^2 \rightarrow \underbrace{\left( \frac{C_1}{3} + C_2 \right) g(m_c^2, q^2)}_{\text{matching coeff}} [\bar{s} \Gamma b] +$$

$$+ (\text{coeff}) \times [\bar{s}_L \gamma^\alpha (i n_+ \cdot \mathcal{D})^n \tilde{G}_{\beta\gamma} b_L] + \dots$$

3-particle correction to  $\mathcal{F}_\lambda \rightarrow$





# Calculations at negative $q^2$

► At the end of the day

$$\mathcal{H}_\lambda(q^2) = \mathcal{H}_{\lambda;\text{fact,LO}}(q^2) + \mathcal{H}_{\lambda;\text{fact,NLO}}(q^2) + \mathcal{H}_{\lambda;\text{spect}}(q^2) + \mathcal{H}_{\lambda;\text{WA}}(q^2) + \\ + \mathcal{H}_{\lambda;\text{soft}}(q^2) + \mathcal{H}_{\lambda;\text{soft},O_8}(q^2) + \dots$$

- $\mathcal{H}_{\lambda;\text{soft}}$  and  $\mathcal{H}_{\lambda;\text{fact,LO}}$  similar in size with opposite signs:  
cancel to large extent
- $\mathcal{H}_{\lambda;\text{soft},O_8}$  contributions negligible

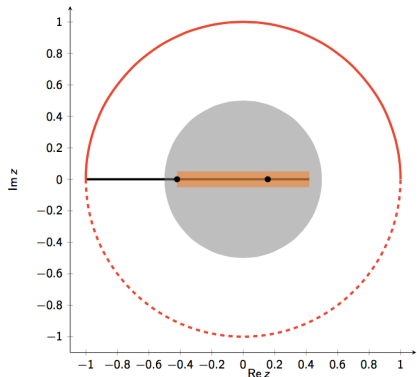
## Ansatz in $z$ valid below the $D\bar{D}$ threshold

[Bobeth, Chrzaszcz, van Dyk, Virto 2017]

Motivated by "z-parametrization" of form factors.

[Boyd et al '94, Bourelly et al '08]

1. Extract the poles :  $\hat{\mathcal{H}}_\lambda(q^2) = (q^2 - M_{J/\psi}^2)(q^2 - M_{\psi(2S)}^2) \mathcal{H}_\lambda(q^2)$



2.  $\hat{\mathcal{H}}_\lambda(q^2)$  is analytic except for  $D\bar{D}$  cut.
3. Perform conformal mapping  $q^2 \mapsto z(q^2)$ .
4.  $\hat{\mathcal{H}}_\lambda(z)$  analytic within unit circle.
5. Taylor expand  $\hat{\mathcal{H}}_\lambda(z)$  around  $z = 0$ .
6. Good convergence expected since  $|z| < 0.52$  for  $-7 \text{ GeV}^2 \leq q^2 \leq 14 \text{ GeV}^2$

## Some details for actual parametrisation

[Bobeth, Chrzaszcz, van Dyk, Virto 2017]

- ▶ Try to capture most features of the expansion (better convergence)
- ▶ Parametrize the ratios  $\mathcal{H}_\lambda(q^2)/\mathcal{F}_\lambda(q^2)$  instead
- ▶ The poles should not modify the asymptotic behaviour at  $|q^2| \rightarrow \infty$

$$\mathcal{H}_\lambda(z) = \frac{1 - z z_{J/\psi}^*}{z - z_{J/\psi}} \frac{1 - z z_{\psi(2S)}^*}{z - z_{\psi(2S)}} \hat{\mathcal{H}}_\lambda(z)$$

$$\hat{\mathcal{H}}_\lambda(z) = \left[ \sum_{k=0}^K \alpha_k^{(\lambda)} z^k \right] \mathcal{F}_\lambda(z)$$

where  $\alpha_k^{(\lambda)}$  are complex coefficients, and the expansion is truncated after the term  $z^K$ . We will take  $K = 2$  (16 real parameters).

- ▶ the modified EOS source code is available upon request (public [repo](#) and [web page](#) should be updated soon!)

## Experimental constraints

[Bobeth, Chrzaszcz, van Dyk, Virto 2017]

- ▶ The residues of the poles are given by  $B \rightarrow K^* \psi_n$  :

$$\mathcal{H}_\lambda(q^2 \rightarrow M_{\psi_n}^2) \sim \frac{M_{\psi_n} f_{\psi_n}^* \mathcal{A}_\lambda^{\psi_n}}{M_B^2 (q^2 - M_{\psi_n}^2)} + \dots$$

- ▶ Angular analyses determine

[Belle, Babar, LHCb]

$$|r_\perp^{\psi_n}|, |r_\parallel^{\psi_n}|, |r_0^{\psi_n}|, \arg\{r_\perp^{\psi_n} r_0^{\psi_n*}\}, \arg\{r_\parallel^{\psi_n} r_0^{\psi_n*}\},$$

where  $r_\lambda^{\psi_n} \equiv \text{Res}_{q^2 \rightarrow M_{\psi_n}^2} \frac{\mathcal{H}_\lambda(q^2)}{\mathcal{F}_\lambda(q^2)} \sim \frac{M_{\psi_n} f_{\psi_n}^* \mathcal{A}_\lambda^{\psi_n}}{M_B^2 \mathcal{F}_\lambda(M_{\psi_n}^2)}$

- ▶ We produce correlated pseudo-observables from a fit (5+5).

## (Prior) Fit to Experimental and theoretical pseudo-observables

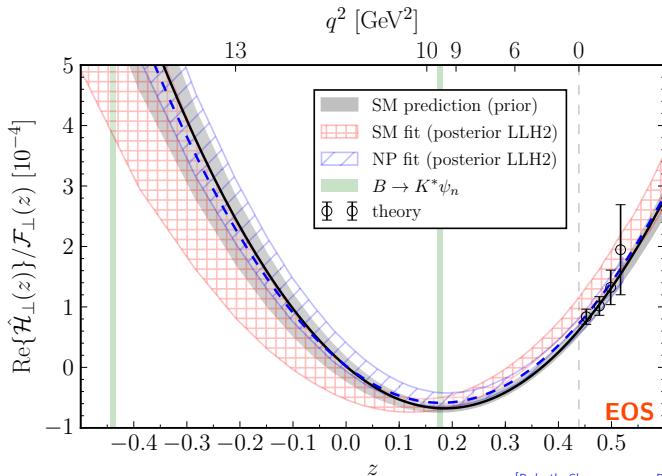
[\[Bobeth, Chrzaszcz, van Dyk, Virto 2017\]](#)

$k$	0	1	2
$\text{Re}[\alpha_k^{(\perp)}]$	$-0.06 \pm 0.21$	$-6.77 \pm 0.27$	$18.96 \pm 0.59$
$\text{Re}[\alpha_k^{(\parallel)}]$	$-0.35 \pm 0.62$	$-3.13 \pm 0.41$	$12.20 \pm 1.34$
$\text{Re}[\alpha_k^{(0)}]$	$0.05 \pm 1.52$	$17.26 \pm 1.64$	–
$\text{Im}[\alpha_k^{(\perp)}]$	$-0.21 \pm 2.25$	$1.17 \pm 3.58$	$-0.08 \pm 2.24$
$\text{Im}[\alpha_k^{(\parallel)}]$	$-0.04 \pm 3.67$	$-2.14 \pm 2.46$	$6.03 \pm 2.50$
$\text{Im}[\alpha_k^{(0)}]$	$-0.05 \pm 4.99$	$4.29 \pm 3.14$	–

**Table 1:** Mean values and standard deviations (in units of  $10^{-4}$ ) of the prior PDF for the parameters  $\alpha_k^{(\lambda)}$ .

# Confronting LHCb Data

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SM predictions and Fit including  $B \rightarrow K^* \mu^+ \mu^-$  data and  $C_9^{\text{NP}}$ 

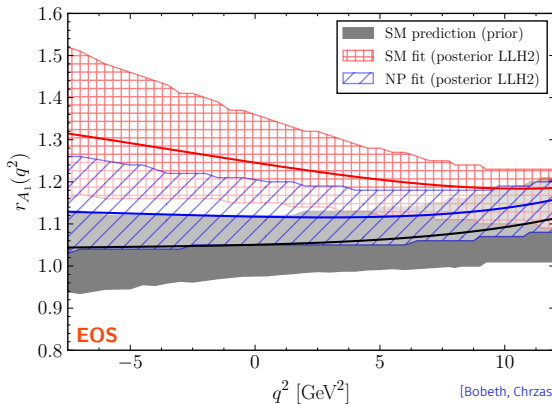
[Bobeth, Chrzaszcz, van Dyk, Virto 2017]

► parametrisation does not provide enough freedom in the SM fit in order to deviate substantially from the prior

SM predictions and Fit including  $B \rightarrow K^* \mu^+ \mu^-$  data and  $C_9^{\text{NP}}$

$$r_{A_1} \equiv \frac{A_1(q^2)}{V(q^2)} \times \text{kinematics}$$

► expected to be 1 in limit  $m_b, E_{K^*} \rightarrow \infty$



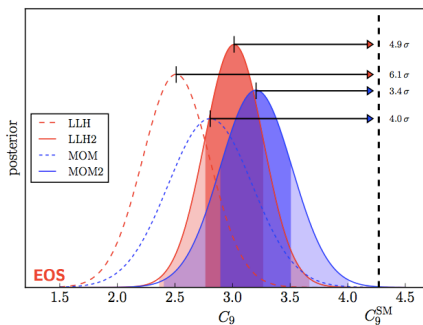
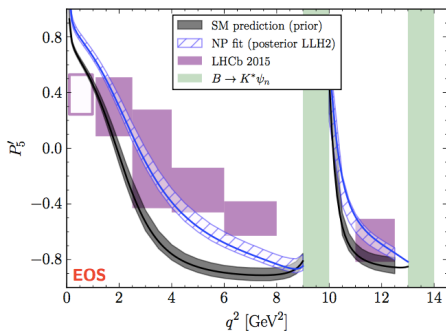
[Bobeth, Chrzaszcz, van Dyk, Virto 2017]

► in SM fit to all  $B \rightarrow K^* \mu^+ \mu^-$  data, fit prefers to change local form factor  $V$  over non-local correlators



## SM predictions and Fit including $B \rightarrow K^* \mu^+ \mu^-$ data and $C_9^{\text{NP}}$

[Bobeth, Chrzaszcz, van Dyk, Virto 2017]



The NP hypothesis with  $C_9^{\text{NP}} \sim -1$  is strongly favoured by the fit

- ▶ pulls  $> 3.4\sigma$  in 1D posterior of the parameter
- ▶ posterior odds (for some fits strongly) in favour of NP interpretation

# Prospects for the LHCb Upgrade and Belle II

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## Sensitivity to New Physics in $C_9$ in $B \rightarrow K^* \mu^+ \mu^-$ from an unbinned fit

[Chrzaszcz, Mauri, Serra, Silva Coutinho, van Dyk w.i.p]

- ▶ close collaboration with LHCb members
  - ▷ preparation for unbinned analysis within LHCb
  - ▷ sensitivity study ongoing for LHCb and Belle II prospects
  - ▷ extracting  $C_9$  in presence of  $z^3$  will require **simultaneous analysis** of theory constraints + data
- ▶ first contacts with Belle II members on this topic

## Sensitivity to Parameters in $B \rightarrow K^* \mu^+ \mu^-$ from an unbinned fit

[Chrzaszcz, Mauri, Serra, Silva Coutinho, van Dyk w.i.p]

### Preliminary

- ▶ sensitivity studies for the SM and with  $C_9^{\text{NP}} = -1$  as benchmark point
  - ▶ use central theory inputs exactly as in pheno analysis
- [Bobeth, Chrzaszcz, van Dyk, Virto 2017]
- ▶ (some) sensitivity to  $z$  coefficients **in absence of any theory priors!!**
  - ▶ demonstrate sensitivity to coefficients of  $z^3$ 
    - ▷ of course: in absence of theory inputs large increase in  $C_9^{\text{NP}}$  uncertainty
    - ▷ nevertheless: further inference of information on non-local corr. seems possible

# Summary

- ▶ Non-local effects are half of the amplitude. Must include them!
- ▶ Technically challenging, but good advances since 2001
- ▶ Theory calculations most reliable at spacelike  $q^2$ 
  - ▷ QCD Factorization [Beneke, Feldmann, Seidel 2001 & 2004]
  - ▷ LCSRs [Khodjamirian, Mannel, Pivovarov, Wang 2010]
- ▶ Access timelike  $q^2$  via analytic continuation (z-param. / dispersion rel.)
  - ▷ Global analyses to exploit parametrical correlations
  - ▷ Experimental colleagues have begun work to incorporate z-parametrization in their analyses
- ▶ Did not discuss large  $q^2$ !!

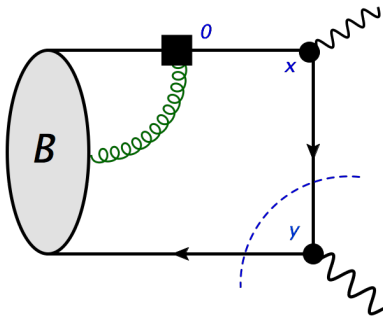


# Calculations at negative $q^2$

## ► LCSRs with $B$ -meson DAs

[Khodjamirian, Mannel, Wang 2012]

Soft gluon correction to  $O_8$  contribution



Simpler calculation than charm-loop

Numerically very small

# Accessing $q^2 > 0$ : dispersion relations

Dispersion relation relating  $\mathcal{H}(q_0^2 < 0)$  to  $\mathcal{H}(q^2 > 0)$

[Khodjamirian, Mannel, Pivovarov, Wang 2010] [Hambrock, Khodjamirian, Rusov 2015]

$$\mathcal{H}^{(p)}(q^2) - \mathcal{H}^{(p)}(q_0^2) = (q^2 - q_0^2) \left[ \sum_V \frac{f_V \mathcal{A}^P(B \rightarrow VM)}{(m_V^2 - q_0^2)(m_V^2 - q^2 - im_V \Gamma_V^{\text{tot}})} + \int_{s_h}^{\infty} ds \frac{\rho_h^{(p)}(s)}{(s^2 - q_0^2)(s - q^2 - i\epsilon)} \right]$$

$(p = u, c)$

$$V = \rho, \omega, \phi, J/\psi, \psi(2S)$$

► For  $b \rightarrow s \Rightarrow$  Neglect  $\lambda_u$  and OZI suppressed contributions

$\Rightarrow \mathcal{A}^c(B \rightarrow VM_s) \sim \mathcal{A}(B \rightarrow \psi_n M_s)$  can be determined from data.

► Light-hadron spectral density  $\Rightarrow$  QH-Duality

► Open-charm spectral density  $\simeq a_p + b_p \frac{q^2}{4m_D^2} + \dots$  (expansion for  $q^2 < 4m_D^2$ )

► Not well-suited for fits:

▷ Only one theory input:  $\mathcal{H}^{(p)}(q_0^2)$

▷ reminder:  $\frac{m_{J/\psi}^2}{4m_D^2} \sim 0.69$ , bad convergence expected even below the  $J/\psi$

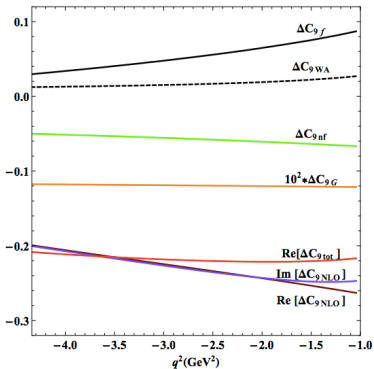


# Calculations at negative $q^2$

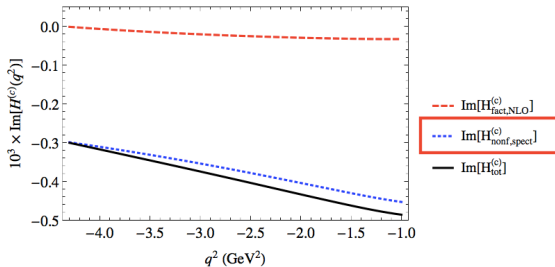
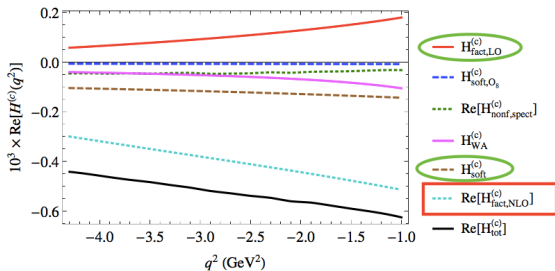
## ► Results for $\mathcal{H}^{(c)}$

$$B^- \rightarrow \pi^- \ell \ell \longrightarrow$$

$$B^- \rightarrow K^- \ell \ell$$



[Khodjamirian, Mannel, Wang 2012], [Hambrock, Khodjamirian, Rusov 2015]



# Calculations at negative $q^2$

## ► Results for $\mathcal{H}^{(u)}$

$$B^- \rightarrow \pi^- \ell \ell$$

[Hambrock, Khodjamirian, Rusov 2015]

$$B^0 \rightarrow \pi^0 \ell \ell$$

