Enhanced electromagnetic correction to $B_s \to \mu^+\mu^-$

M. Beneke (TU München)

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1708.09152, with C. Bobeth and R. Szafron

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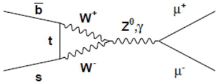
Status of $B_s \to \mu^+\mu^-$

"Instantaneous", "non-radiative" branching fraction



$$\mathrm{Br}(B_s \to \mu^+\mu^-) = \frac{G_F^2\alpha^2}{64\pi^3} f_{B_s}^2 \tau_{B_s} m_{B_s}^3 |V_{tb}V_{ts}^*|^2 \sqrt{1-\frac{4m_\mu^2}{m_{B_s}^2}}$$
$$\times \left\{ \left| \frac{2m_\mu}{m_{B_s}}(C_{10}-C_{10}') + (C_P - C_P') \right|^2 \left(1-\frac{4m_\mu^2}{m_{B_s}^2}\right) |C_S - C_S'|^2 \right\}$$

- SM only C_{10} ⇒ helicity suppression. Sensitive to scalar couplings.
- SM C_{10} calculations includes NNLO QCD, NLO EW matching corrections at EW scale, NNLL renormalization-group evolution to the b-quark mass scale including QED logarithms
- LHCb [1703.05747] $(3.0^{+0.7}_{-0.6}) \times 10^{-9}$ vs. Theory [Bobeth et al., 1311.0903] $(3.65 \pm 0.23) \times 10^{-9}$

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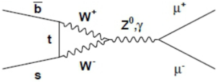

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Theory uncertainties [Bobeth et al., 1311.0903]

- Parametric: f_B (4.0%), CKM (4.3%), m_t (1.6%), $\tau_{B_s^H}$ (1.3%), α_s(0.1%)
- Non-parametric: Higher-order corrections at m_W (0.4%), QED scale variation (0.3%), m_t pole-$\overline{\mathrm{MS}}$ conversion (0.3%), other (0.5%) [e.g. dim-8 operators] – total of 1.5%

Some facts about $B_q \to \ell^+\ell^-$

- Long-distance QCD effects are very simple. Local annihilation. Only

$$\langle 0|\bar{q}\gamma^\mu\gamma_5 b|\bar{B}_q(p)\rangle = i f_{B_q} p^\mu$$

Task for lattice QCD (1.5% [Aoki et al. 1607.00299]).

- Only the operator Q_{10} from the weak effective Lagrangian enters.

- No scalar lepton current $\bar{\ell}\ell$, only $\bar{\ell}\gamma_5\ell \Longrightarrow$

$$\mathcal{A}^\lambda_{\Delta\Gamma} = 1 \qquad C_\lambda = S_\lambda = 0$$

$$\frac{\Gamma(B_s(t) \to \mu^+_\lambda \mu^-_\lambda) - \Gamma(\bar{B}_s(t) \to \mu^+_\lambda \mu^-_\lambda)}{\Gamma(B_s(t) \to \mu^+_\lambda \mu^-_\lambda) + \Gamma(\bar{B}_s(t) \to \mu^+_\lambda \mu^-_\lambda)} = \frac{C_\lambda \cos(\Delta M_{B_s} t) + S_\lambda \sin(\Delta M_{B_s} t)}{\cosh(y_s t/\tau_{B_s}) + \mathcal{A}^\lambda_{\Delta\Gamma} \sinh(y_s t/\tau_{B_s})}$$

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None of these are exactly true in the presence of electromagnetic corrections

Electromagnetic corrections: Motivations

- Electromagnetic corrections violate isospin symmetry.
  Can fake small electroweak penguin amplitudes in charmless B decays

- Large logarithmic enhancements $\ln m_b^2/\Lambda^2$, $\ln m_b^2/m_\ell^2$ possible
  Can fake lepton-flavour universality

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Factorization theorems for electromagnetic correction don't exist. Theory still needs to be developed.

- Start with the simplest decay $B_s \to \ell^+\ell^-$ (+soft γ).

- Theory framework: SCET with electromagnetism, fermion masses and power-suppressed interactions

Electromagnetic corrections to $B_s \to \mu^+\mu^-$, previous results

IR finite observable is

$$\mathcal{B}_{\rm phys} = \sum_{n=0}^{\infty} \mathcal{B}(B_s \to \mu^+\mu^- + n\gamma, \sum_n E_{\gamma,n} < E_{\gamma,\rm max}) \equiv \omega(E_{\gamma,\rm max})\mathcal{B}_{\rm non-rad}.$$

Signal window $|m_{B_s} - m_{\mu^+\mu^-}| < \Delta \quad \Longrightarrow \quad E_{\gamma,\rm max} = \Delta,\ \Delta \approx 60\,\rm MeV$

- Bloch-Nordsieck factor [Buras et al. 1208.0934] due to soft photon radiation

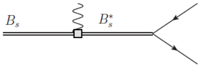
$$\omega(E_{\gamma,\rm max}) \approx \exp\left(-\frac{2\alpha}{\pi}\ln\frac{m_{B_s}}{E_{\gamma,\rm max}}\left[\ln\frac{m_{B_s}^2}{m_\mu^2} - 1\right]\right) \approx 0.89 \quad \left(\text{for } E_{\gamma,\rm max} = 60\,\rm MeV\right)$$

Treats real emission correctly, but assumes B-meson point-like for virtual photons up to energy m_{B_s}.

- "Structure-dependent real emission" [Aditya, Healey, Petrov, 1212.4166]

$$\mathcal{B}_{\rm SD} \approx \mathcal{B}(B_s \to B_s^*\gamma \to \mu^+\mu^- + \gamma, E_\gamma < E_{\gamma,\rm max}) \approx 1.6 \times 10^{-12}$$

No helicity suppression, but kinematic suppression, no large logs.

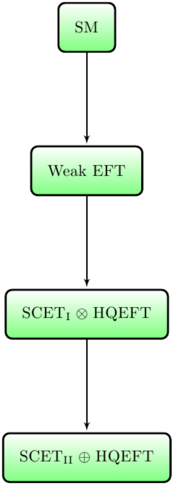


Scales and EFTs

Multiple scales:

- Electroweak scale, m_W
- Hard scale, m_b
- Hard-collinear scale, $\sqrt{m_b \Lambda}$
- Collinear scale, m_μ
- Soft scale, Λ
- "Ultrasoft" scale, $E_{\gamma,\max}$ (B-meson point-like)

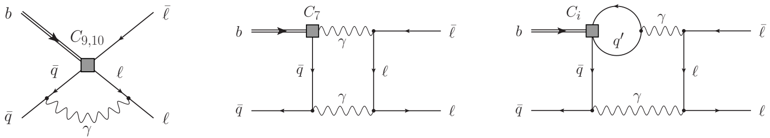
Take $m_\mu \sim \Lambda$ for counting purposes.



- Virtual QED corrections below the m_b scale not included, estimated to be 0.3%.

Enhanced electromagnetic correction

Surprise: m_B/Λ power-enhanced and logarithmically enhanced, purely virtual correction



$$i\mathcal{A} = m_\ell f_{B_q} \mathcal{N}\, C_{10}\, \bar{\ell}\gamma_5 \ell + \frac{\alpha_{\rm em}}{4\pi} Q_\ell Q_q\, m_\ell f_{B_q} \mathcal{N}\, \bar{\ell}(1+\gamma_5)\ell$$

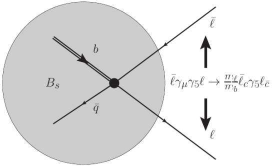
$$\times \left\{ \int_0^1 du\,(1-u)\, C_9^{\rm eff}(u m_b^2)\, m_B \int_0^\infty \frac{d\omega}{\omega}\, \phi_{B+}(\omega) \left[\ln \frac{m_b \omega}{m_\ell^2} + \ln \frac{u}{1-u} \right] \right.$$

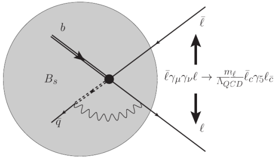
$$\left. - Q_\ell C_7^{\rm eff} m_B \int_0^\infty \frac{d\omega}{\omega}\, \phi_{B+}(\omega) \left[\ln^2 \frac{m_b\omega}{m_\ell^2} - 2 \ln \frac{m_b \omega}{m_\ell^2} + \frac{2\pi^2}{3} \right] \right\} + \ldots$$

The virtual photon probes the B meson structure. B-meson LCDA and $1/\lambda_B$ enters.

$$\frac{m_B}{\lambda_B} \equiv m_B \int_0^\infty \frac{d\omega}{\omega}\, \phi_{B+}(\omega) \sim 20 \qquad \ln \frac{m_b\omega}{m_\mu^2} \sim 6$$

Interpretation of the enhanced correction





$$\langle 0|\bar{q}\gamma^\mu\gamma_5 b|\bar{B}_q(p)\rangle$$

Local annihilation and helicity flip.

$$\langle 0|\int d^4x\, T\{j_{\rm QED}(x), \mathcal{L}_{\Delta B=1}(0)\}|\bar{B}_q\rangle$$

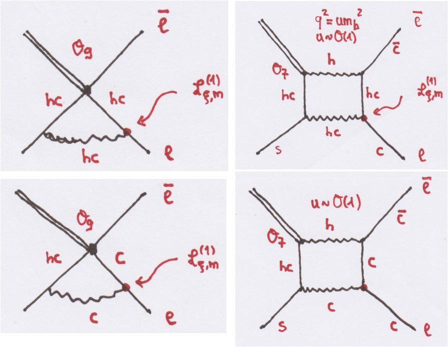
Helicity-flip and annihilation delocalized by a hard-collinear distance

The virtual photon probes the B meson structure. Annihilation/helicity-suppression is “smeared out” over light-like distance $1/\sqrt{m_B\Lambda}$. Still short-distance.

Logarithms are not the standard soft logarithms, but due to hard-collinear, collinear and soft regions.

SCET interpretation

- Typical SCET_{II} problem
  - hard-collinear $p_{hc}^2 \sim m_b\Lambda$
  - collinear $p_c^2 \sim \Lambda^2, m_\mu^2$
  - soft $p_s^2 \sim p_c^2$
- Matching to SCET_{II} non-zero only at sub-leading power (helicity-flip required)
- After tree-level matching to SCET_{I} need matrix element of



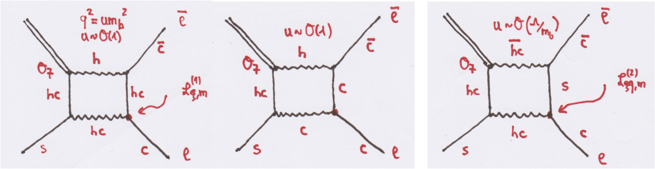
$$\overset{\text{SCET}_{\text{I}}}{\rightarrow} \int_0^1 du \left(C_9^{\text{eff}}(u) + \frac{C_7^{\text{eff}}}{u} \right) \bar{\chi}_{\text{hc}}(\bar{u}p_\ell)\Gamma h_v \, \bar{\ell}(up_\ell)\Gamma' \ell(p_{\bar{\ell}})$$

- Sum of hard-collinear and collinear loop in SCET_{II} gives a collinear logarithm $\ln(m_b\Lambda/m_\mu^2)$
- Endpoint divergence for $u \to 0$ in C_7^{eff} term

SCET interpretation (II)

Endpoint divergence is cancelled by the one-loop matrix element of the $\mathrm{SCET_I}$ operator

$$\bar{\chi}_{\rm hc}(p_\ell)\gamma_\perp^\mu h_v\, \mathcal{A}^\gamma_{{\rm hc},\perp\mu}(p_{\bar\ell}) \qquad \text{(third diagram below)}$$



- Involves power-suppressed SCET interactions and soft fermion (lepton) exchange
- Endpoint divergence results in another power of $\ln(m_b\Lambda/m_\mu^2)$. Fully calculable in perturbation theory, since the spectator quark is highly virtual (hard-collinear).

- Rather different from the standard double loagrithms.
  Expect modification of the simple radiation factor $\omega(E_{\gamma,\max})$, since virtual effects see the internal structure.

Numerical size of the correction

Include through the substitution

$$\overline{\mathcal{B}}(B_s \to \ell^+\ell^-) = \frac{\tau_{B_q} m_{B_q}^3 f_{B_q}^2}{8\pi}\, |\mathcal{N}|^2\, \frac{m_\ell^2}{m_{B_q}^2} \sqrt{1 - \frac{4m_\ell^2}{m_{B_q}^2}}\, |C_{10}|^2, \qquad C_{10} \to C_{10} + \frac{\alpha_{\rm em}}{4\pi} Q_\ell Q_q \Delta_{\rm QED}$$

where

$$\Delta_{\rm QED} = (33\ldots119) + i\,(9\ldots23)$$

- Reduction of the branching fraction by

$$0.3 - 1.1\%$$

  Uncertainty entirely due to B-meson LCDA.

- Cancellation of a factor of three between the $C_9^{\rm eff}(um_b^2)$ and double-log enhanced $C_7^{\rm eff}$ term:

$$-0.6\% = 1.1\%\,(C_9^{\rm eff}) - 1.7\%\,(C_7^{\rm eff})$$

- Significantly larger than previously estimated QED correction.
  QED uncertainty almost as large as other non-parametric uncertainties (1.2%)

Numerical size of the correction (II)

New SM value for the un-tagged, time-integrated branching fraction (including parameter update)

$$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)_{\rm SM} = (3.57 \pm 0.17) \cdot 10^{-9}$$

$(\alpha_s^{(5)}(m_Z) = 0.1181(11)$, $1/\Gamma_H^s = 1.609(10)$ ps $f_{B_s} = 228.4(3.7)$ MeV $[N_f = 2+1]$, $|V_{tb}^* V_{ts}/V_{cb}| = 0.982(1)$ $|V_{cb}| = 0.04200(64))$

Theory uncertainties

- Parametric: ± 0.167
- Non-parametric/non-QED: ± 0.043
- QED: ${}^{+0.022}_{-0.030}$ (from B-meson LCDA)

Observables related to the time-dependent helicity rate asymmetry

$$\mathcal{A}^\lambda_{\Delta\Gamma} = 1 - r^2 |\Delta_{\rm QED}|^2 \approx 1 - 1.0 \cdot 10^{-5}$$

$$C_\lambda = -\eta_\lambda\, 2r\, {\rm Re}(\Delta_{\rm QED}) \approx \eta_\lambda\, 0.6\%$$ $\left(r \equiv \frac{\alpha_{\rm em}}{4\pi} \frac{Q_\ell Q_q}{C_{10}} \text{ and } \eta_{L/R} = \pm 1\right)$

$$S_\lambda = 2r\, {\rm Im}(\Delta_{\rm QED}) \approx -0.1\%,$$

Summary & Outlook

I) Power-enhanced electromagnetic corrections, complex pattern of logarithms, because photon acts as a weak probe of the QCD structure of the B-meson.

- More long-distance QCD than f_B
- Effect of the same order as the non-parametric uncertainty, larger than previously estimated QED uncertainty
- $\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)_{\rm SM} = (3.57 \pm 0.17) \cdot 10^{-9}$

II) Interesting effect from the SCET point of view: power-enhancement from power-suppressed interactions)

III) Effect vanishes in $B^+ \to \ell^+\nu_\ell$ due to V-A structure of the charged current

IV) Outlook:

- Non-power enhanced QED corrections (including the standard soft logs and next-to-leading power virtual effects)
- Electron and τ lepton final state
- Electromagnetic effects in $B \to K^{(*)}\ell\ell$ (non-power enhanced)