QCD running in neutrinoless double beta decay

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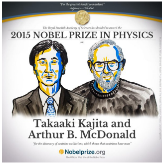
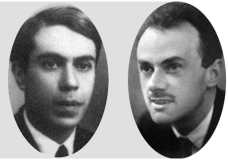
1 November 2017

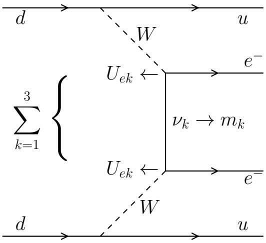
Introduction

- Neutrino oscillation $\Rightarrow$ Neutrinos have masses
- Open question: Dirac or Majorana?
- Is Lepton Number conserved in nature?





Introduction II



$$\langle m_\nu \rangle = \sum_j U_{ej}^2 m_j \equiv m_{ee}$$

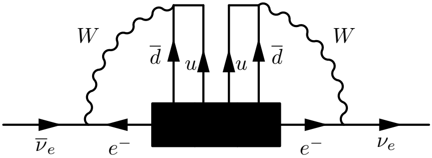
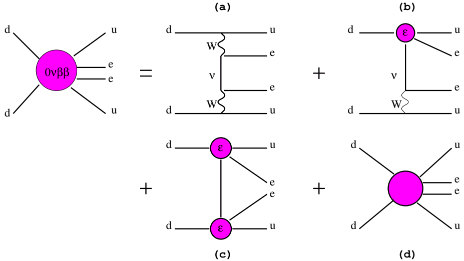


Figure : Black Box Theorem [Schechter & Valle, 1982]

Introduction III

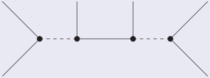
- Left-Right Symmetry
- R-Parity Violating Supersymmetry
- Leptoquarks
- Extradimensions
- etc.

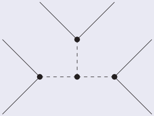


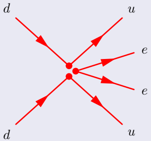
Picture taken from:
J. Phys. G **39**, 124007 (2012)

High Energy Examples

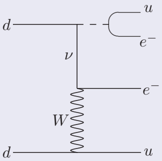
Short-Range

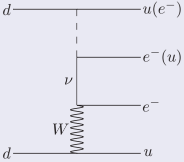


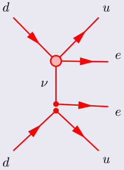




Long-range

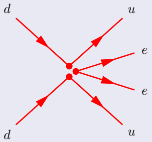






Short-Range Mechanisms (SRM)

Effective Lagrangian



$$\mathcal{L}_{\text{eff}}^{0\nu\beta\beta} = \frac{G_F^2}{2m_p} \sum_i C_i^{XY}(\mu) \cdot \mathcal{O}_i(\mu), \tag{1}$$

with the operator basis:

$$\mathcal{O}_1^{XY} = 4(\bar{u}P_X d)(\bar{u}P_Y d)\, j, \tag{2}$$

$$\mathcal{O}_2^{XX} = 4(\bar{u}\sigma^{\mu\nu}P_X d)(\bar{u}\sigma_{\mu\nu}P_X d)\, j, \tag{3}$$

$$\mathcal{O}_3^{XY} = 4(\bar{u}\gamma^{\mu}P_X d)(\bar{u}\gamma_{\mu}P_Y d)\, j, \tag{4}$$

$$\mathcal{O}_4^{XY} = 4(\bar{u}\gamma^{\mu}P_X d)(\bar{u}\sigma_{\mu\nu}P_Y d)\, j^{\nu}, \tag{5}$$

$$\mathcal{O}_5^{XY} = 4(\bar{u}\gamma^{\mu}P_X d)(\bar{u}P_Y d)\, j_{\mu} \tag{6}$$

$$j = \bar{e}(1 \pm \gamma_5)e^c\,, \quad j_\mu = \bar{e}\gamma_\mu\gamma_5 e^c\ .$$

Half-life in the SRM

Applying standard nuclear theory methods, one finds for the half-life:

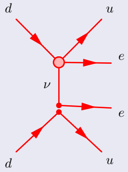
Half-life

$$\left[T_{1/2}^{0\nu\beta\beta}\right]^{-1} = G_1 \left|\sum_{i=1}^{3} C_i(\mu_0)\mathcal{M}_i\right|^2 + G_2 \left|\sum_{i=4}^{5} C_i(\mu_0)\mathcal{M}_i\right|^2 \qquad (7)$$

Here, $G_1 = G_{01}$ and $G_2 = (m_e R)^2 G_{09}/8$ are phase space factors in the convention of [Doi et al., 1985], and $\mathcal{M}_i = \langle A_f | \mathcal{O}_i^{\mathrm{h}} | A_i \rangle$ are the nuclear matrix elements defined in Ref. [Päs et al., 2001].

Long-Range Mechanisms (LRM)

Effective Lagrangian



$$\mathcal{L}_{\rm eff}^{d=6} = \frac{G_F}{\sqrt{2}} \left(j^\mu J^\dagger_\mu + \sum_i C_i^X(\mu) \mathcal{O}_i^{(6)X}(\mu) \right) \quad (8)$$

$$\mathcal{O}_1^{(6)X} = \bar{u} P_X d \cdot \bar{e} P_R \nu^C, \quad (9)$$

$$\mathcal{O}_2^{(6)X} = \bar{u} \sigma^{\mu\nu} P_X d \cdot \bar{e} \sigma^{\mu\nu} P_R \nu^C, \quad (10)$$

$$\mathcal{O}_3^{(6)X} = \bar{u} \gamma_\mu P_X d \cdot \bar{e} \gamma^\mu P_R \nu^C \quad (11)$$

$$j^\mu = \bar{e}\gamma^\mu(1-\gamma_5)\nu, \quad J_\mu = \bar{d}\gamma^\mu(1-\gamma_5)u$$

Half-life in the LRM

Applying standard nuclear theory methods, one finds for the half-life:

Half-life

$$[T_{1/2}^{0\nu\beta\beta}]^{-1} = |C_i \; ME|^2 \tag{12}$$

Currently the best bounds are

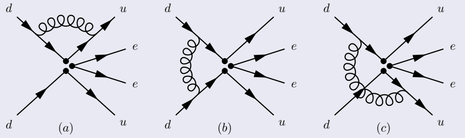
$$\begin{aligned} \text{KamLAND-Zen} \quad &: \quad T_{1/2}^{0\nu}(^{136}\mathrm{Xe}) = 1.07 \times 10^{26} \text{ ys } (90\%\mathrm{C.L.}), \\ \text{GERDA Phase-II} \quad &: \quad T_{1/2}^{0\nu}(^{76}\mathrm{Ge}) \; = 5.2 \;\; \times 10^{25} \text{ ys } (90\%\mathrm{C.L.}). \end{aligned}$$

SRM v/s LRM

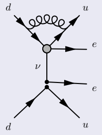
SRM
Elementary quark-level
$(uudddd)$
$dd \to uu + 2e^-$
Hadronic level
$nn \to pp + 2e^-$

LRM
Elementary quark-level
$d \to u + e^- \nu$
Hadronic level
$nn \to pp + 2e^-$

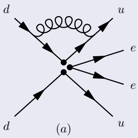
QCD corrections: Short-Range Mechanisms



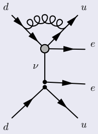
QCD corrections: Long-Range Mechanisms



QCD corrections: Short-Range Mechanisms



QCD corrections: Long-Range Mechanisms



$$T^a_{\alpha\beta} T^a_{\gamma\rho} = -\frac{1}{2N}\delta_{\alpha\beta}\delta_{\gamma\rho} + \frac{1}{2}\delta_{\alpha\rho}\delta_{\beta\gamma} \tag{13}$$

Color Mismatch effect.

Renormalization Group Equations

[Buras, 1998] :

RGE of WC

$$\frac{d\vec{C}(\mu)}{d\ln\mu} = \hat{\gamma}^T \vec{C}(\mu), \tag{14}$$

$$\hat{\gamma}(\alpha_s) = -2\alpha_s \frac{\partial \hat{Z}_1(\alpha_s)}{\partial \alpha_s}, \tag{15}$$

$$\vec{C}(\mu) = \hat{U}(\mu, M_W) \cdot \vec{C}(M_W). \tag{16}$$

Results: SRM

Without QCD

$$\left[T_{1/2}^{0\nu\beta\beta}\right]^{-1} = G_1 \left|\sum_{i=1}^{3} C_i(\mu_0)\mathcal{M}_i\right|^2 + G_2 \left|\sum_{i=4}^{5} C_i(\mu_0)\mathcal{M}_i\right|^2 \qquad (17)$$

With QCD

$$\begin{aligned}\left[T_{1/2}^{0\nu\beta\beta}\right]^{-1} &= G_1 \Big| \beta_1^{XX} C_1^{XX}(\Lambda) + \beta_1^{LR} C_1^{LR}(\Lambda) + \beta_2^{XX} C_2^{XX}(\Lambda) + \\ &\quad + \beta_3^{XX} C_3^{XX}(\Lambda) + \beta_3^{LR} C_3^{LR}(\Lambda) \Big|^2 \\ &+ G_2 \left| \beta_4^{XX} C_4^{RR}(\Lambda) + \beta_4^{LR} C_4^{LR}(\Lambda) + \beta_5^{XX} C_5^{RR}(\Lambda) + \beta_5^{LR} C_5^{LR}(\Lambda) \right|^2 ,\end{aligned}$$

where

$$\beta_1^{XX} = \mathcal{M}_1 \overbrace{U_{11}^{XX}}^{1} + \mathcal{M}_2 \overbrace{U_{21}^{XX}}^{0}$$

^{A}X	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3^{(+)}$	$\mathcal{M}_3^{(-)}$	$\lvert\mathcal{M}_4\rvert$	$\lvert\mathcal{M}_5\rvert$
^{76}Ge	9.0	-1.6×10^3	1.3×10^2	2.1×10^2	$\lvert 1.9 \times 10^2 \rvert$	$\lvert 1.9 \times 10^1 \rvert$
^{136}Xe	4.5	-8.5×10^2	6.9×10^1	1.1×10^2	$\lvert 9.6 \times 10^1 \rvert$	$\lvert 9.3 \rvert$

Table : The numerical values of the nuclear matrix elements $\mathcal{M}_i$ taken from Ref. [Deppisch et al., 2012].

	With QCD		Without QCD	With QCD		Without QCD
^{A}X	$\|C_1^{XX}(\Lambda_1)\|$	$\|C_1^{XX}(\Lambda_2)\|$	$\|C_1^{XX}\|$	$\|C_1^{LR,RL}(\Lambda_1)\|$	$\|C_1^{LR,RL}(\Lambda_2)\|$	$\|C_1^{LR,RL}\|$
^{76}Ge	5.0×10^{-10}	3.8×10^{-10}	$\mathbf{2.6 \times 10^{-7}}$	1.5×10^{-8}	9.1×10^{-9}	$\mathbf{2.6 \times 10^{-7}}$
^{136}Xe	3.4×10^{-10}	2.6×10^{-10}	$\mathbf{1.8 \times 10^{-7}}$	9.7×10^{-9}	6.1×10^{-9}	$\mathbf{1.8 \times 10^{-7}}$
^{A}X	$\|C_2^{XX}(\Lambda_1)\|$	$\|C_2^{XX}(\Lambda_2)\|$	$\|C_2^{XX}\|$	–	–	–
^{76}Ge	3.5×10^{-9}	5.2×10^{-9}	1.4×10^{-9}	–	–	–
^{136}Xe	2.4×10^{-9}	3.5×10^{-9}	9.4×10^{-10}	–	–	–
^{A}X	$\|C_3^{XX}(\Lambda_1)\|$	$\|C_3^{XX}(\Lambda_2)\|$	$\|C_3^{XX}\|$	$\|C_3^{LR,RL}(\Lambda_1)\|$	$\|C_3^{LR,RL}(\Lambda_2)\|$	$\|C_3^{LR,RL}\|$
^{76}Ge	1.5×10^{-8}	1.6×10^{-8}	1.1×10^{-8}	2.0×10^{-8}	2.1×10^{-8}	1.8×10^{-8}
^{136}Xe	9.7×10^{-9}	1.1×10^{-8}	7.4×10^{-9}	1.4×10^{-8}	1.4×10^{-8}	1.2×10^{-8}
^{A}X	$\|C_4^{XX}(\Lambda_1)\|$	$\|C_4^{XX}(\Lambda_2)\|$	$\|C_4^{XX(0)}\|$	$\|C_4^{LR,RL}(\Lambda_1)\|$	$\|C_4^{LR,RL}(\Lambda_2)\|$	$\|C_4^{LR,RL(0)}\|$
^{76}Ge	5.0×10^{-9}	3.9×10^{-9}	$\mathbf{1.2 \times 10^{-8}}$	1.7×10^{-8}	1.9×10^{-8}	1.2×10^{-8}
^{136}Xe	3.4×10^{-9}	2.7×10^{-9}	$\mathbf{7.9 \times 10^{-9}}$	1.2×10^{-8}	1.3×10^{-8}	7.9×10^{-9}
^{A}X	$\|C_5^{XX}(\Lambda_1)\|$	$\|C_5^{XX}(\Lambda_2)\|$	$\|C_5^{XX}\|$	$\|C_5^{LR,RL}(\Lambda_1)\|$	$\|C_5^{LR,RL}(\Lambda_2)\|$	$\|C_5^{LR,RL}\|$
^{76}Ge	2.3×10^{-8}	1.4×10^{-8}	$\mathbf{1.2 \times 10^{-7}}$	3.9×10^{-8}	2.8×10^{-8}	$\mathbf{1.2 \times 10^{-7}}$
^{136}Xe	1.6×10^{-8}	9.5×10^{-9}	$\mathbf{8.2 \times 10^{-8}}$	2.8×10^{-8}	2.0×10^{-8}	$\mathbf{8.2 \times 10^{-8}}$

Table : Individual upper limits on Wilson Coefficients

Results:LRM

without QCD

$$\left[T_{1/2}^{0\nu\beta\beta}\right]^{-1} = G_{0k}|C_i(\mu_0)\cdot ME|^2 \tag{18}$$

With QCD

$$\left[T_{1/2}^{0\nu\beta\beta}\right]^{-1} = G_{0k}|U_i(\mu_0,\Lambda)C_i(\Lambda)\cdot ME|^2 \tag{19}$$

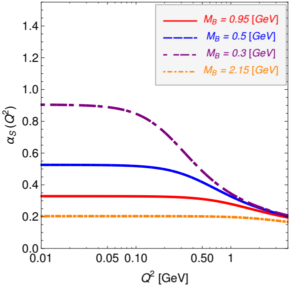
with

$$U_1 = 1.6, \quad U_2 = 0.6, \quad U_3 = 0.8$$

	Without QCD		With QCD	
	^{76}Ge	^{136}Xe	^{76}Ge	^{136}Xe
C_{V-A}^{V+A}	2.2×10^{-9}	1.5×10^{-9}	2.7×10^{-9}	1.9×10^{-9}
C_{V+A}^{V+A}	3.4×10^{-7}	2.4×10^{-7}	4.3×10^{-7}	3.0×10^{-7}
C_{S-P}^{S+P}	5.3×10^{-9}	3.7×10^{-9}	3.3×10^{-9}	2.3×10^{-9}
C_{S+P}^{S+P}	5.3×10^{-9}	3.7×10^{-9}	3.3×10^{-9}	2.3×10^{-9}
$C_{T_R}^{T_L}$	3.1×10^{-10}	2.2×10^{-10}	5.0×10^{-10}	3.5×10^{-10}
$C_{T_R}^{T_R}$	8.2×10^{-10}	5.7×10^{-10}	1.4×10^{-9}	9.2×10^{-10}

Table : Individual upper limits on Wilson coefficients, without QCD and with QCD running for comparison

Freezing *work in progress*



$$\mu^2 \to \mu^2 + M_B^2, \qquad \tilde{\alpha}_s(\mu^2) = \frac{\alpha_s(\lambda)}{1 + \beta_0 \frac{\alpha_s(\lambda)}{4\pi} \log \frac{\mu^2 + M_B^2}{\lambda^2}} \tag{20}$$

Preliminary results I *work in progress*

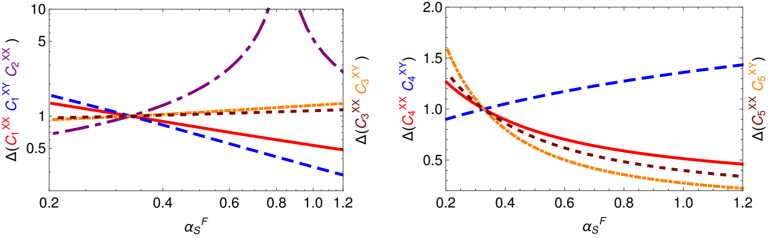


Figure : Relative change of the limit on the short-range coefficients with respect to the "frozen" value of α_S at low energies. Here α_S^F represents $\tilde{\alpha}_S(Q^2)$ for $Q^2 \leq 0.1$ GeV^2. $\Delta(C_i^{AB})$ is calculated normalizing with respect to the value of the coefficient derived without "freezing" and using an $\alpha_S(1\ \mathrm{GeV}^2) \simeq 0.32$.

$$\boxed{\Delta(C_i^{XY}) = C_i^{XY}(\alpha_S^F)/C_i^{XY}(\alpha_S(1\ \mathrm{GeV}))} \qquad (21)$$

Preliminary results II *work in progress*

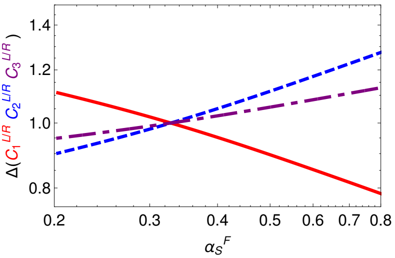


Figure : Relative change of the limit on the long-range coefficients with respect to the "frozen" value of α_S at low energies. Note the change of scale with respect to the short-range figure.

Summary and Conclusions

- We have calculated QCD running to the complete set of Lorentz-invariant operators for the short-range (SR) part and for the long-range part of the NDBD amplitude.
- We showed that the QCD corrections are indeed important in the SR principally because of the operator mixing
- We showed that in the long-range part the QCD correction is not as important as in the short-range part
- We analysed the IR behaviour of QCD running. With the exception of C_2^{XX} (in the SRM), the Wilson coefficients depend only moderately on the exact value of α_S^F and it seems we can extract reliable limits on these coefficients from $0\nu\beta\beta$

Thanks for your attention!

Back Up #1
A non-trivial example: $\not{R}_p$ SUSY

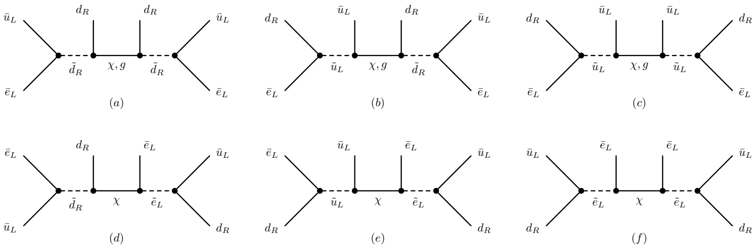


Figure : The six different Feynman diagrams in R-parity violating supersymmetry that contribute to $0\nu\beta\beta$ decay.

Back Up #2
A non-trivial example: $R\!\!\!/_p$ SUSY

$\tilde{g}$*-exchange contribution*:

$$\mathcal{L}_{eff}^{\tilde{g}} = \frac{G_F^2}{2m_p}\left(C_{\tilde{g}a}\,\mathcal{O}_a + C_{\tilde{g}b}\,\mathcal{O}_b + C_{\tilde{g}c}\,\mathcal{O}_c\right) = \tag{22}$$

$$= \frac{G_F^2}{2m_p}\frac{1}{48}\left[\left(2C_{\tilde{g}a} + 2C_{\tilde{g}c} - 7C_{\tilde{g}b}\right)\mathcal{O}_1^{RR} - \frac{1}{4}\left(2C_{\tilde{g}a} + 2C_{\tilde{g}c} + C_{\tilde{g}b}\right)\mathcal{O}_2^{RR}\right].$$

χ*-exchange contribution*:

$$\mathcal{L}_{eff}^{\tilde{g}} = \frac{G_F^2}{2m_p}\sum_{i=a\cdots f} C_{\# i}\,\mathcal{O}_{\# i} = \tag{23}$$

$$= \frac{G_F^2}{2m_p}\frac{1}{128}\left[4\left(C_b + C_c + C_a + 4C_f - 2C_d - 2C_e\right)\mathcal{O}_1^{RR} + \right.$$

$$\left. + \left(C_b - C_c - C_a\right)\mathcal{O}_2^{RR}\right].$$

Back Up #3
A non-trivial example: $\not{R}_p$ SUSY

$$C_{\tilde{g}c} = \frac{\kappa_3}{m_{\tilde{g}}} \frac{1}{m^4_{\tilde{u}_L}}, \quad C_{\tilde{g}a} = \frac{\kappa_3}{m_{\tilde{g}}} \frac{1}{m^4_{\tilde{d}_R}}, \quad C_{\tilde{g}b} = -\frac{\kappa_3}{m_{\tilde{g}}} \frac{1}{m^2_{\tilde{u}_L} m^2_{\tilde{d}_R}}, \tag{24}$$

$$C_b = \frac{\kappa_2}{m_\chi} \frac{\epsilon_L(u)\epsilon_R(d)}{m^2_{\tilde{u}_L} m^2_{\tilde{d}_R}}, \quad C_c = \frac{\kappa_2}{m_\chi} \frac{\epsilon^2_L(u)}{m^4_{\tilde{u}_L}}, \quad C_a = \frac{\kappa_2}{m_\chi} \frac{\epsilon^2_R(d)}{m^4_{\tilde{d}_R}}, \tag{25}$$

$$C_f = \frac{\kappa_2}{m_\chi} \frac{\epsilon^2_L(e)}{m^4_{\tilde{e}_L}}, \quad C_d = \frac{\kappa_2}{m_\chi} \frac{\epsilon_L(e)\epsilon_R(d)}{m^2_{\tilde{e}_L} m^2_{\tilde{d}_R}}, \quad C_e = \frac{\kappa_2}{m_\chi} \frac{\epsilon_L(e)\epsilon_L(u)}{m^2_{\tilde{e}_L} m^2_{\tilde{u}_L}}, \tag{26}$$

with

$$\kappa_2 = \lambda'^2_{111} 4\pi\alpha_2 \frac{m_p}{G_F^2}, \qquad \kappa_3 = \lambda'^2_{111} 16\pi\alpha_s \frac{m_p}{G_F^2}, \tag{27}$$

$$\epsilon_L(\psi) = \tan\theta_W [T_3(\psi) - Q(\psi)], \quad \epsilon_R(\psi) = \tan\theta_W Q(\psi), \tag{28}$$

Back Up #4
A non-trivial example: $R\!\!\!/_p$ SUSY

$$\tilde{g} - \text{exchange}: \qquad \lambda'_{111Ge} \leq 1.0 \times 10^{-2} \left(\frac{m_{\tilde{q}}}{1\text{TeV}}\right)^2 \left(\frac{m_{\tilde{g}}}{1\text{TeV}}\right)^{1/2}, \qquad (29)$$

$$\chi - \text{exchange}: \qquad \lambda'_{111Ge} \leq 7.3 \times 10^{-1} \left(\frac{m_{\tilde{e}}}{1\text{TeV}}\right)^2 \left(\frac{m_{\tilde{\chi}}}{1\text{TeV}}\right)^{1/2} \qquad (30)$$

$$\tilde{g} - \text{exchange}: \qquad \lambda'_{111Ge} \leq 9.3 \times 10^{-2} \left(\frac{m_{\tilde{q}}}{1\text{TeV}}\right)^2 \left(\frac{m_{\tilde{g}}}{1\text{TeV}}\right)^{1/2}, \qquad (31)$$

$$\chi - \text{exchange}: \qquad \lambda'_{111Ge} \leq 5.2 \left(\frac{m_{\tilde{e}}}{1\text{TeV}}\right)^2 \left(\frac{m_{\tilde{\chi}}}{1\text{TeV}}\right)^{1/2} \qquad (32)$$

This is about $\sim$10 ($\sim$7) weaker than the limits for gluino (neutralino) cases in Eqs. (29), (30) taking into account the QCD running.

Talk references

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