The FEL option for the pump radiation of the Gamma-factory

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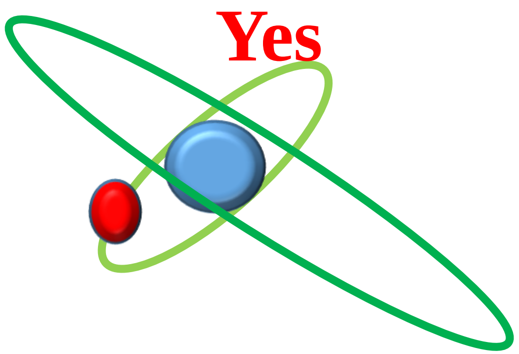
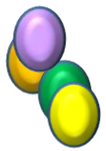
I thank Cesare, Luca, Andrea, Alberto, Camilla, Illya,
Mario, Marcello

To substitute the laser with a FEL as pump radiation for the gamma factory is a **very good idea!!**





ω_{trans} depends only on the transition chosen

This is the readiation frequency seen by the ion:

$$\omega^{PSI}_{FEL}=2\omega^{LAB}_{FEL}\gamma^{i}_{PSI}$$

They have to be equal.

With an FEL, the tuning between the transition and the pump radiation frequencies can be found very easily, without touching the ions.

How?

By simply changing the FEL's radiation frequency operating on the linac electron energy!

For saying something more about the option of using an FEL for pumping the gamma-factory, I started from the two examples of Bessonov' document (see also his PRL 1996 with Kwang-Je Kim).

Look the red and green quantities I took from it:

SPS Xenon

Laser wavelength	**532 nm**
laser photon energy	2.33 eV
Laser beam diameter at IP	3.4 mm
Rayleigh length	68.23 m
Laser beam length	100 cm
Length of laser resonator	272.92 m
Laser beam relative bandwidth	$6\ 10^{-4}$
Saturation intensity	$2.69\ 10^{6}$ W

LHC lead

Transition energy gap	68.57 keV
Laser wavelength	**108 nm**
laser photon energy	11.45 eV
Laser beam diameter at IP	$5\ 10^{-3}$ cm
Rayleigh length	7.5 cm
Laser beam length (power or field?)	15 cm
Laser beam relative bandwidth	$2\ 10^{-4}$
Saturation intensity	2.24 10^5 W
Laser energy	56 μJ
Repetition rate	**7.4 MHz**

I noticed that it is possible to conceive an unique FEL for both wavelengths. For the gamma-factory examples, from the basic FEL resonance relation, we can evaluate:

$$\lambda = \frac{\lambda_w}{2\gamma_e{}^2}(1+a_w^2) \qquad \gamma_e = \sqrt{\frac{\lambda_w}{2\lambda}(1+a_w^2)}$$

$\lambda_w = 1.5\,cm$; $a_w = 0.7$ ⇒

$\lambda = 108\,nm$ ⇒ $\gamma_e = 321 \quad E = 164\,MeV$

$\lambda = 532\,nm$ ⇒ $\gamma_e = 145 \quad E = 74\,MeV$

Many devices exist or existed that operate from the IR-optical to the near UV range, for instance…..

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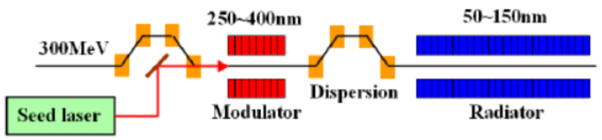
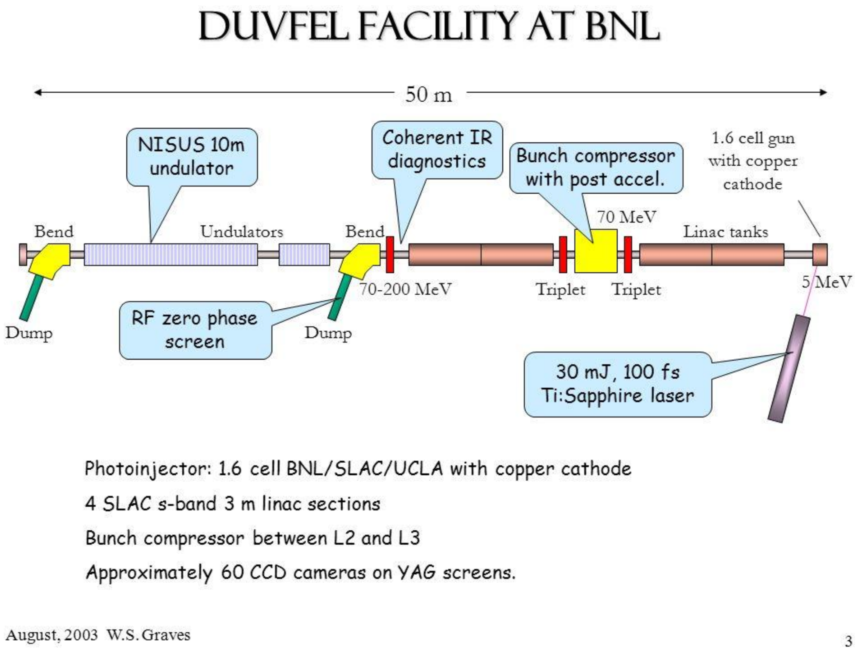
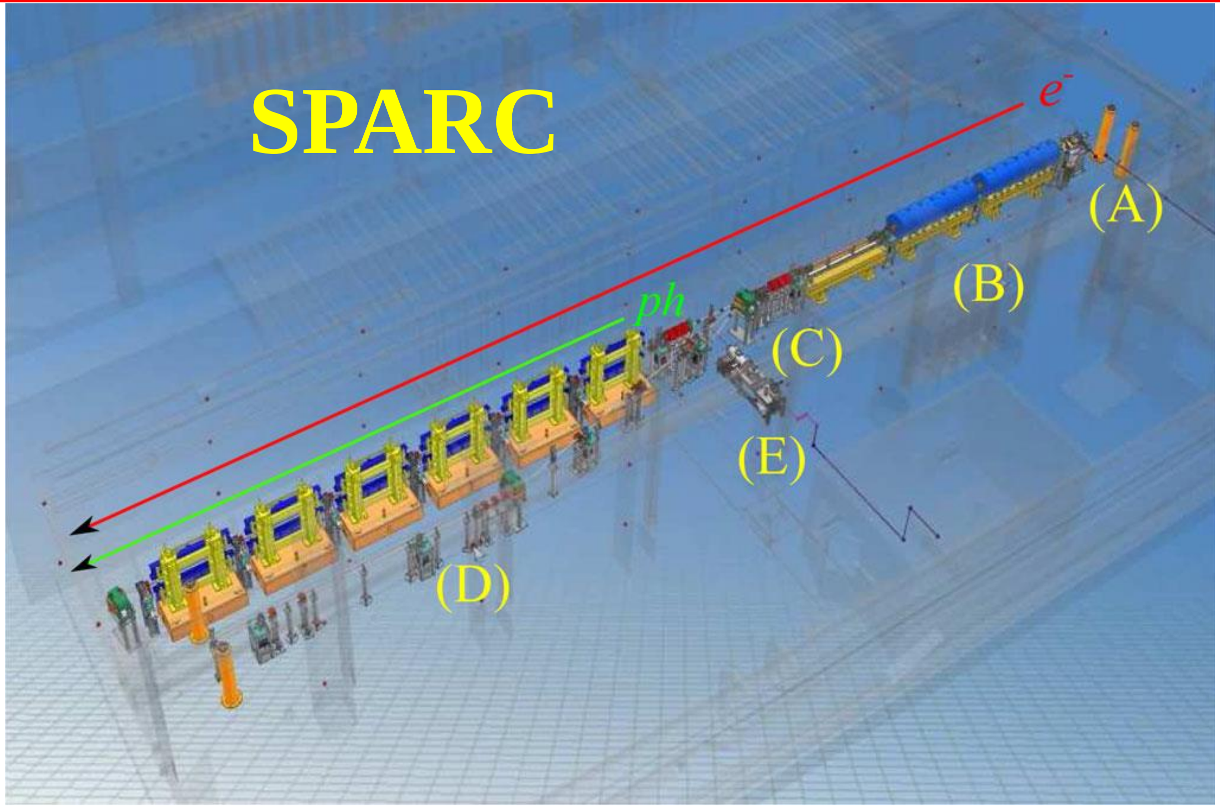






….but none of them, has the right repetition rate. Apart from the Duke FEL, that has not enought flux….

SPARC

Repetition rate 10 Hz

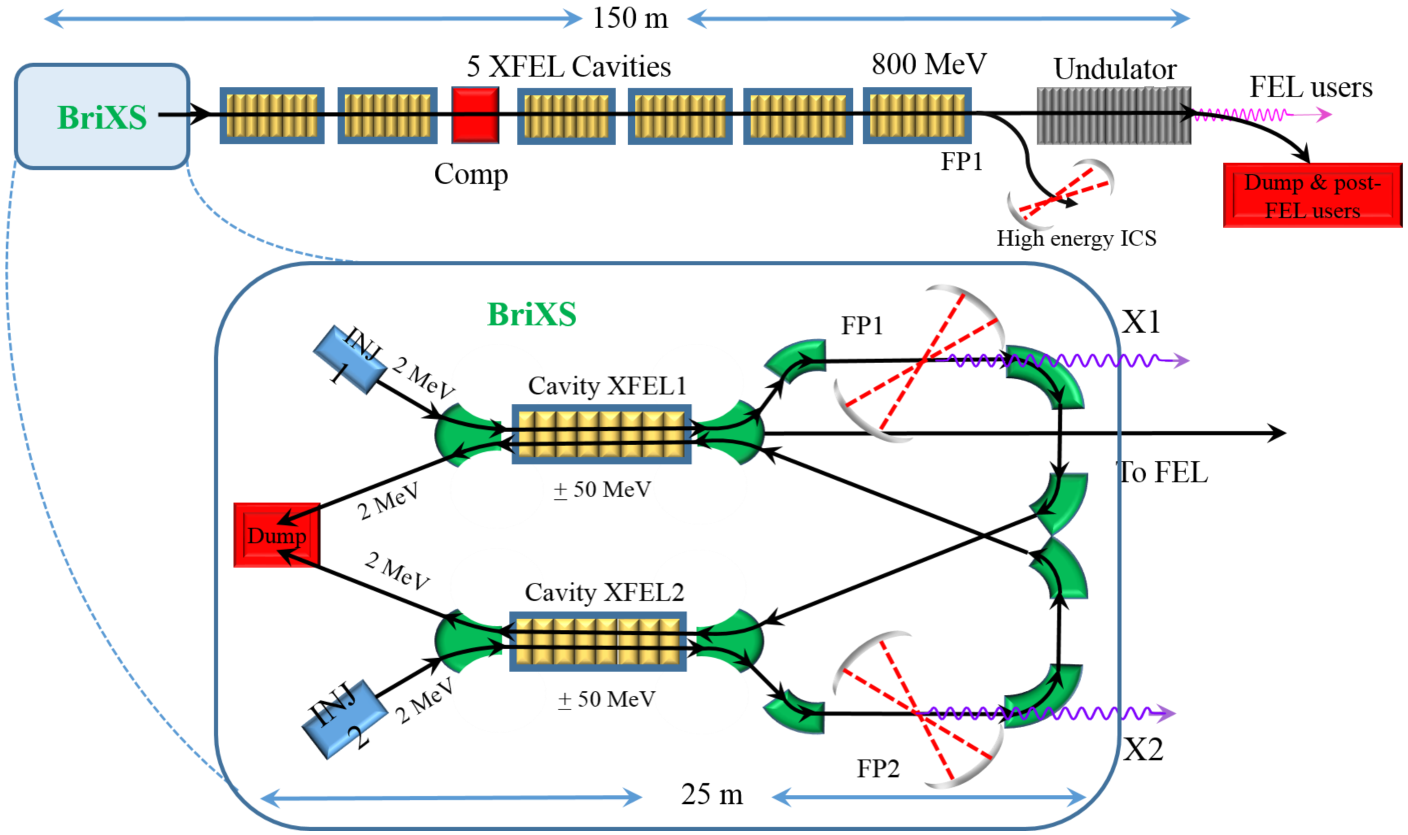
Sparc upgrade

Repetition rate 100 Hz

LHC lead

Repetition rate 7.4 MHz

…This instead is **MARIX**, the project we are developing in Milano for the ex-EXPO site, a combined Compton source-X ray FEL….

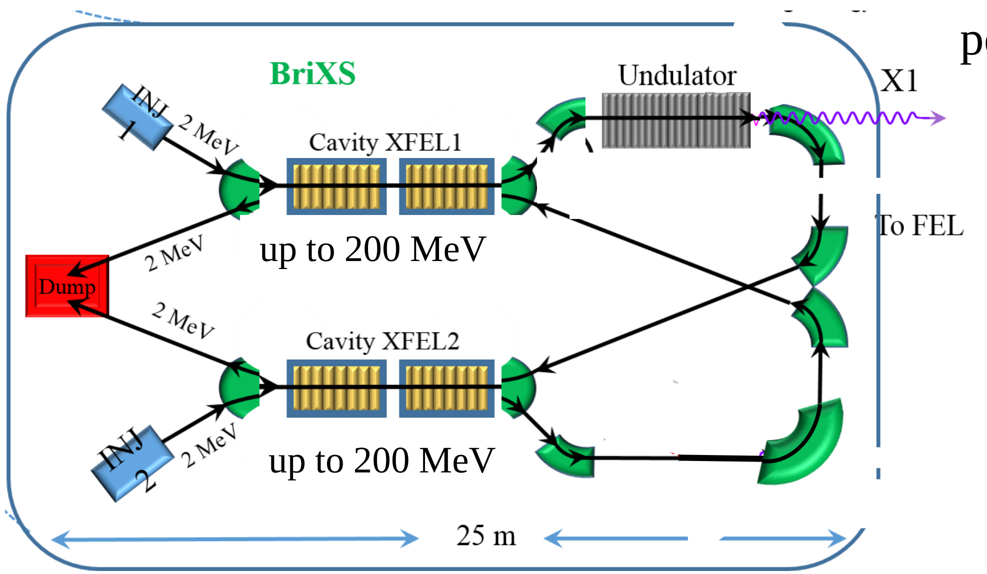


…..and it can be adapted for the gamma-factory !! Look this scheme:

Warm Photocathode Repetion rate up to 100 MHz. Final electron energy 2-6 MeV

Superconductive injector with energy recovery. Repetion rate up to 100 MHz. Final electron energy 200 MeV

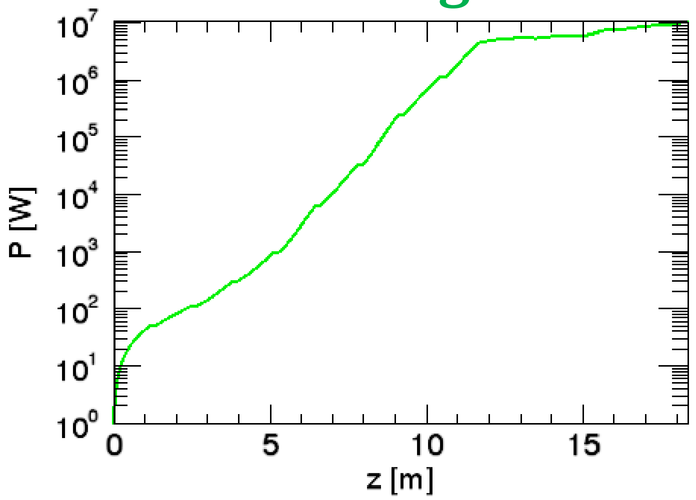
Permanent magnets 10 m undulator with period 1.5 cm

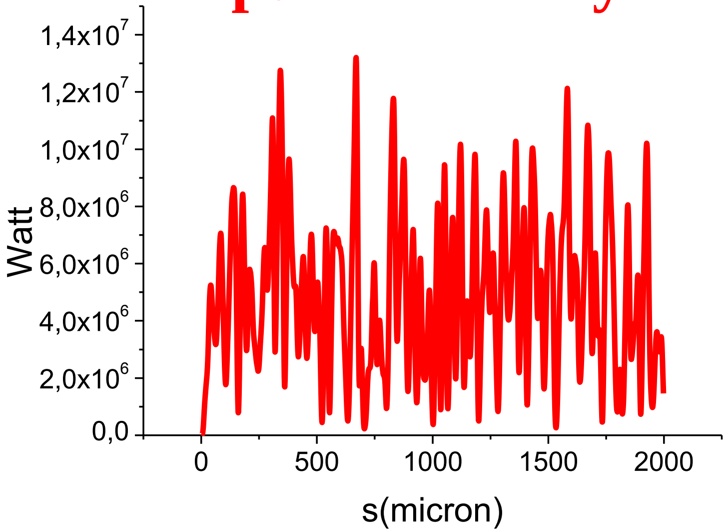


If You have time, give a look to our web site: http://eng.fisica.unimi.it/ecm/home/research/marix.

So, I have simulated a MARIX based FEL for the gamma-factory with GENESIS 1.3.

These are the **growth** and the **pulse intensity** obtained with:





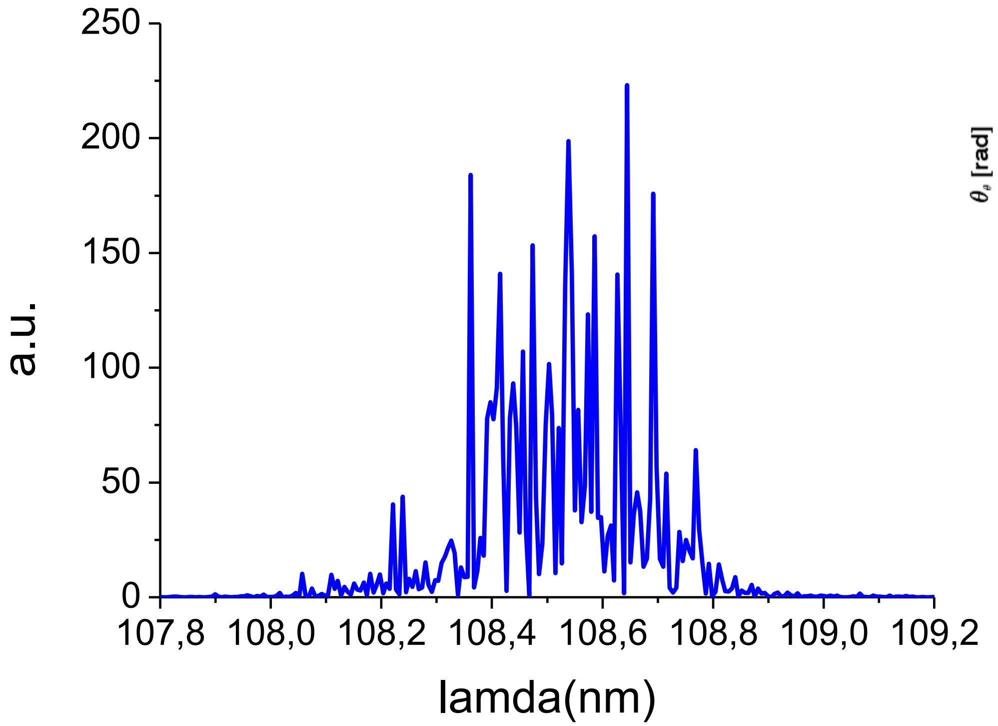
λ_w (cm)	1.5	a_w	0.7
g	321	Ee(MeV)	164
$Emit_{x,y}$ (μm)	0.8	ΔE/E	$5 10^{-4}$
I(A)	100	Q(pC)	600
Δt (ps)	6.6	L(mm)	2

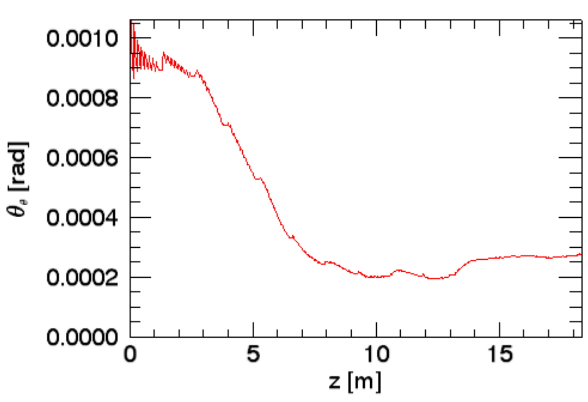
Saturation is achieved in L=12 m, a quite small dimension.

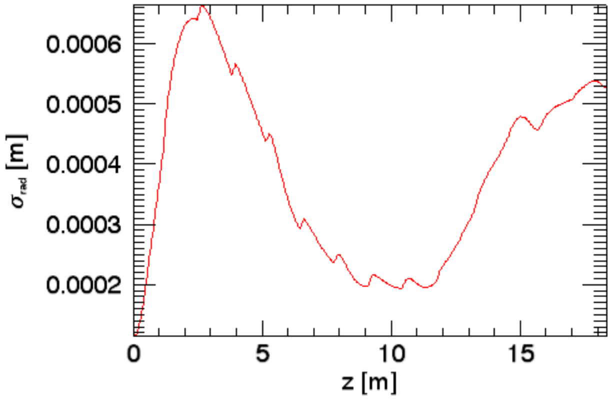
The saturation energy is E=40 μJ

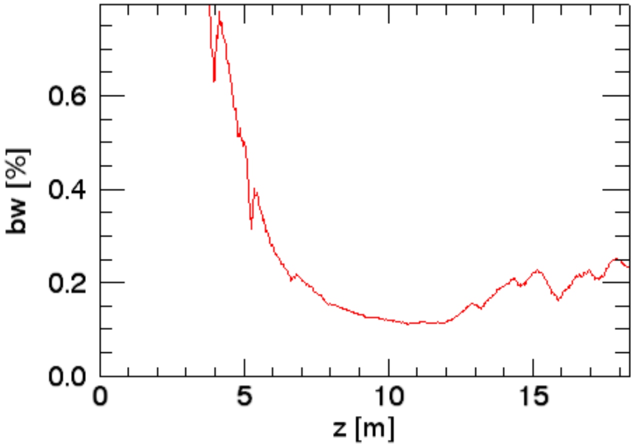
The total number of photon is $N_{FEL}=2.1\ 10^{13}$

There are other interesting quantities: this is the **spectrum**, with the typical SASE oscillations, the **bandwidth**, **rad size** and **divergence**:





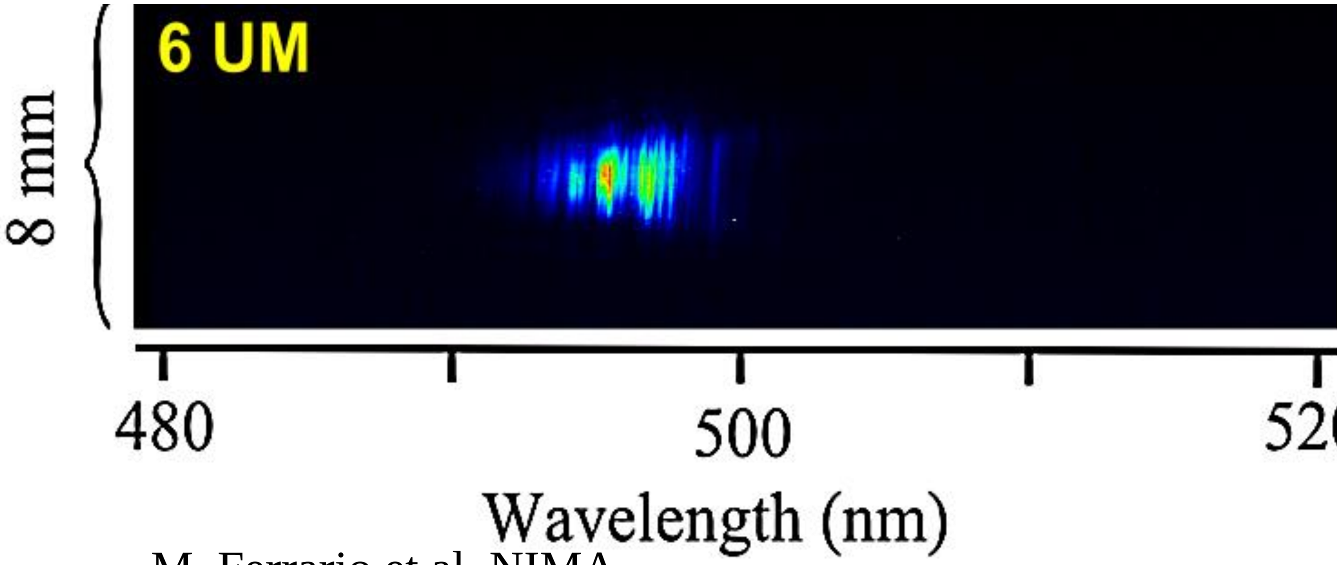




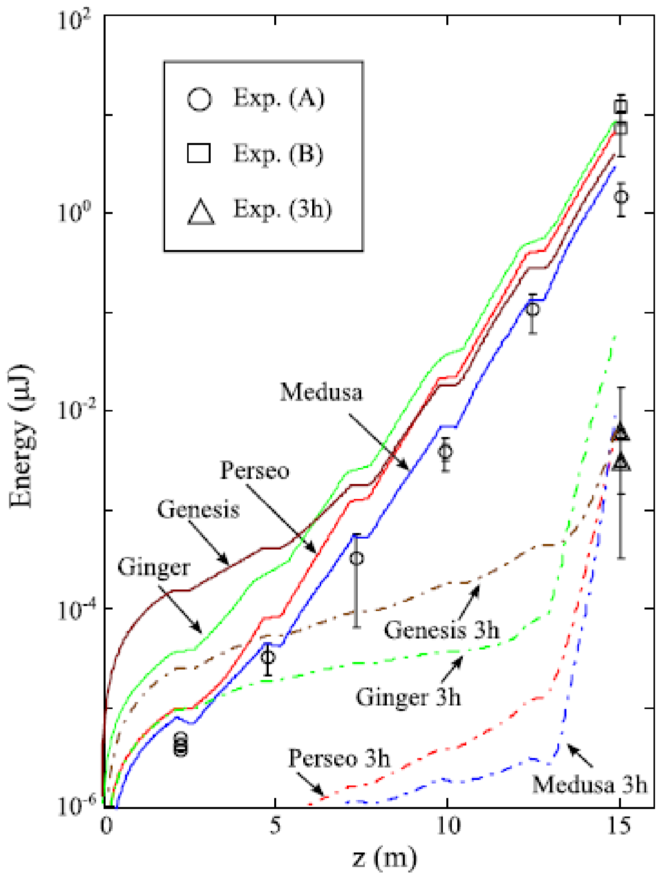
L_g1d= 0.59576D+00 L_g3d= 0.85794D+00 L_cop= 0.14921D-04

ro_1d= 0.11568D-02 ro_3d= 0.80330D-03 pow= 0.14640D+08

Maybe someone will think: 'Are these SASE fluctuations a mere theoretical invention?' No, they are really existing. In fact, this is the spectrum of the FEL SASE radiation we measured at SPARC_LAB with a spectrometer constructed indeed here in Padova*.



M. Ferrario et al. NIMA
L. Giannessi et al. PRL
V. Petrillo et al. PRL
*Poletto, Frassetto



At SPARC we did also a comparison between SASE and seeded spectra.
(A. Petralia et al. PRL 2015)

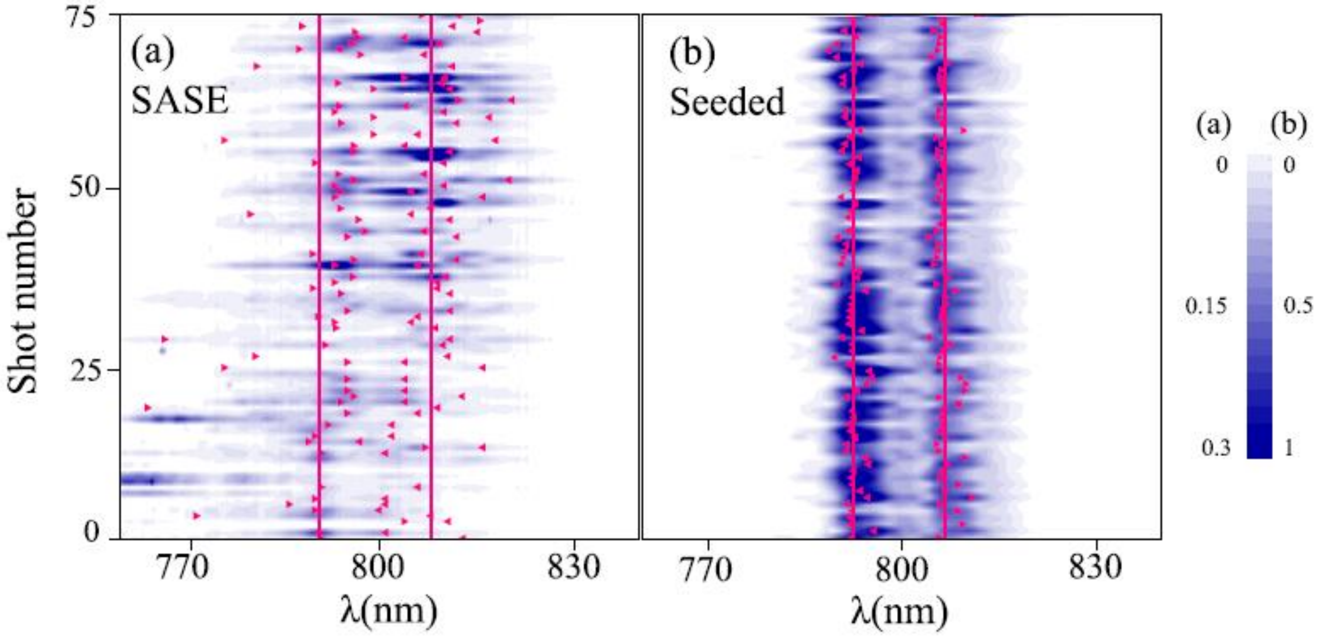


Figure 4: Spectral intensity vs wavelength, sequence of shots without (left) and with (right) seed. Pink lines: average wavelength $\langle\lambda_{1,2}\rangle$, pink triangles: peak values of each shot.

Let us summarize the comparison between laser and FEL.
In general:

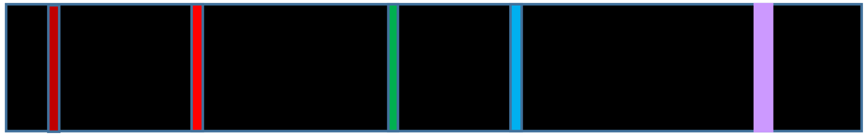
Laser radiation **versus** **FEL radiation**

Lasers exist only at 800 nm, 1030 nm… and harmonics

Low tunability

High spatial coherence

High longitudinal coherence



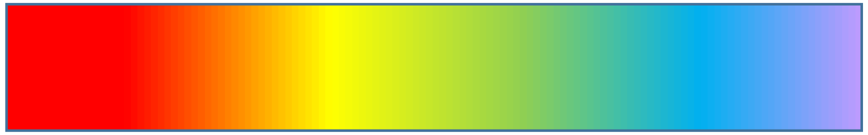
A unique FEL covers a wide range of frequencies

High tunability changing the e-beam energy

Spatial coherence given by propagation

Low longitudinal coherence

Spectral and temporal oscillations



And for the gamma factory:

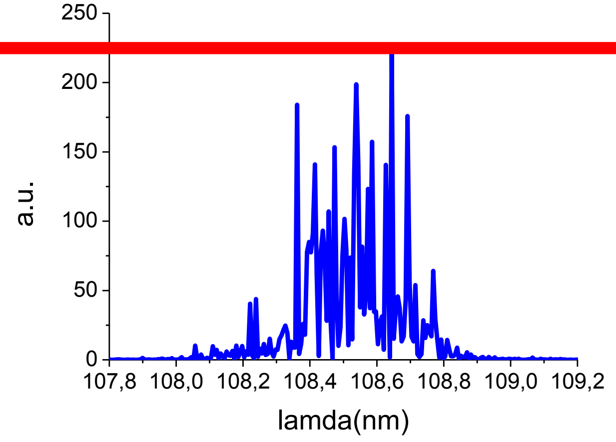
Laser radiation versus FEL radiation

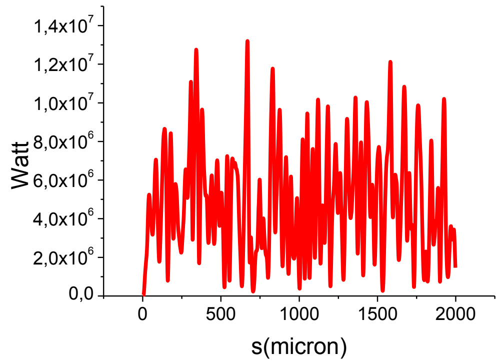
LHC lead		LHC lead	
Transition energy gap	68.57 keV	Transition energy gap	68.57 keV
Laser wavelength	**108 nm**	**FEL wavelength**	**108 nm**
Laser photon energy	11.45 eV	FEL photon energy	11.45 eV
Laser beam diameter at IP	$5\ 10^{-3}$ cm	Laser beam diameter at IP	$5\ 10^{-3}$ cm
Rayleigh length	7.5 cm	Rayleigh length	2.32 m
Laser beam length (power or field?)	15 cm	FEL beam length	3 mm
Laser beam relative bandwidth	$2\ 10^{-4}$	Laser beam relative bandwidth	10^{-3}
Laser energy	56 μJ	FEL energy	40 μJ
Repetition rate	**7.4 MHz**	**Repetition rate**	**7.4 MHz**

This is the FEL in the laboratory system….

But, what does the electron of the lead PSI see?

$$\tau_e\omega_e=t\omega$$

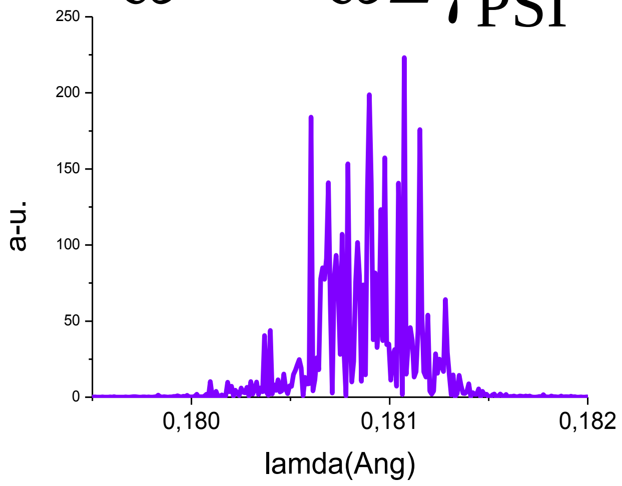




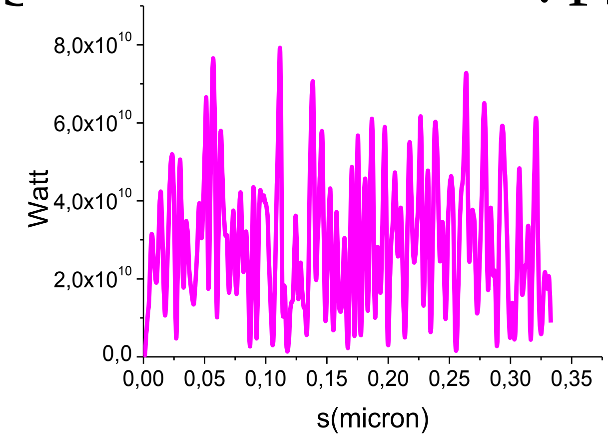
$$\omega^{PSI}=\omega2\gamma_{PSI}$$

$$\lambda^{PSI}=\lambda/2\gamma_{PS}$$

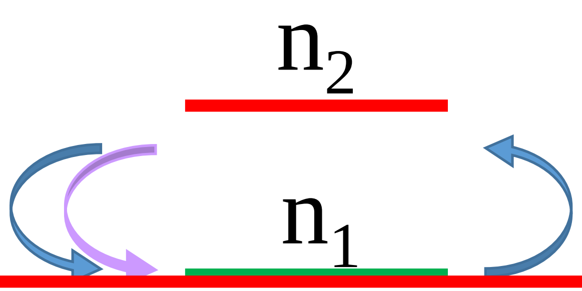
$$\tau^{PSI}=\tau/2\gamma_{PSI}$$



…look the abscisse!



So I wondered:' How the FEL characteristics can affect the rate of event?' How to study this problem?



These are the Einstein equation in the electron reference frame:

$$\frac{dn_2}{dt} = n_1 B_{12}\rho(\omega) - n_2 A_{21} - n_2 B_{21}\rho(\omega)$$

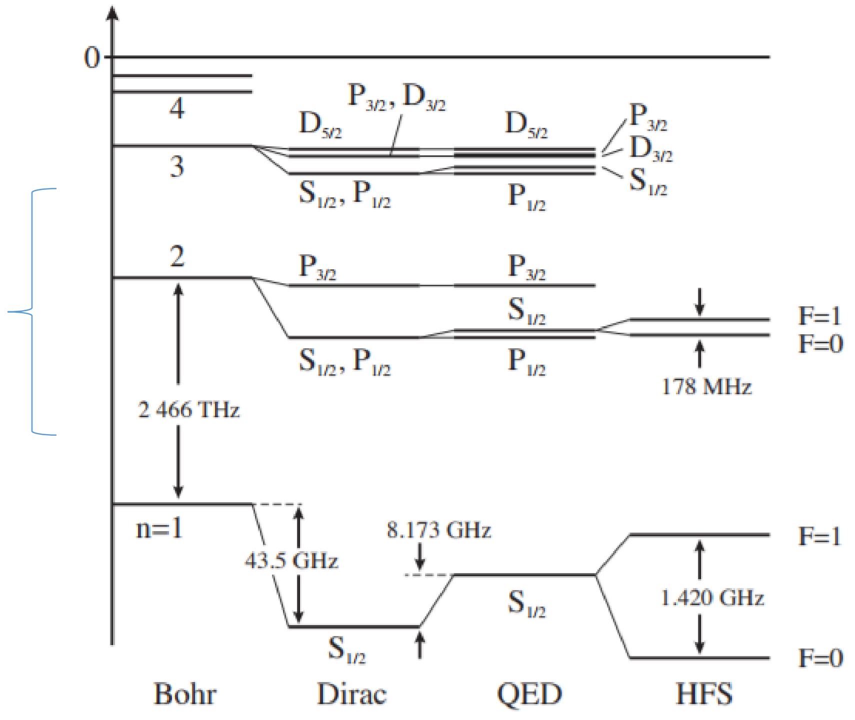
$$n_1 + n_2 = 1$$

$$\frac{dn_{ph}^{spont}}{dt} = A_{21} n_0 n_2$$

ρ(ω) is the energy density per frequency units

This is very important: Only the spontaneous radiation has to be considered because the stimulated emission has the same direction of the laser and when Lorentz tranformed does not experience the frequency upshift.

Since I needed to revise my atomic physics memories, I began from zero.



In the Bohr model:

For Hydrogen the Lyman-alpha is

$\Delta E = 10.2$ eV $\qquad \nu_{21} = 2.466\ 10^{15}\ \mathrm{sec}^{-1}$

For Lead ions:

$\Delta E = 68.5$ keV $\qquad \nu_{21} = Z^2\ 2.466\ 10^{15}\ \mathrm{sec}^{-1}$

$= 1.658\ 10^{19}\ \mathrm{sec}^{-1}$

$\omega_{21} = 1.041\ 10^{20}\ \mathrm{sec}^{-1}$

$\lambda_{21} = 1.809\ 10^{-11}$ m

Fabrizio remembered to us the existence of the spin-orbit effect and calculated the correction due also to relativistics effect and zitterbewegung to the energy gap:

For the n=1, l=0 (1s) state the energy shift is,

$$\Delta E_{1,1/2,m_j,0} = -\frac{1}{4} Z^4 \alpha^2 \,\mathrm{Ry}.$$ **2 states**

while for the n=2, l=1 (2p) states, we have:

$$\Delta E_{2,1/2,m_j,1} = -\frac{5}{64} Z^4 \alpha^2 \,\mathrm{Ry},$$ **3 states**

$$\Delta E_{2,3/2,m_j,1} = -\frac{1}{64} Z^4 \alpha^2 \,\mathrm{Ry}.$$ **3 states**

For Lead ions, for j=1/2:

ΔE=68.5 keV+5.63keV=74.13 keV

ν_{21} =1.79 10^{19} sec^{-1}

ω_{21}=1.127 10^{20} sec^{-1}

λ_{21}=1.675 10^{-11} m

Or for j=3/2

ΔE=68.5 keV+7.67keV=76.17 keV

ν_{21} =1.83 10^{19} sec^{-1}

ω_{21}=1.155 10^{20} sec^{-1}

λ_{21}=1.64 10^{-11} m

Furthermore we revised those cumbersome calculations for arriving to the event rate…

The transition probability can be obtained by means of the time-dependent perturbation theory of a two level system under an electromagnetic wave (see for instance Cohen-Tannoudji, pg 1304):

$$H_0 = \frac{p^2}{2m_e} + V_0(r), \qquad H = H_0 + H_1(t), \qquad A_0\,\epsilon\cos(\mathbf{k}\cdot\mathbf{r} - \omega t),$$

$$H_1 \simeq -\frac{e\,\mathbf{A}\cdot\mathbf{p}}{m_e} = -\frac{e\,A_0\,\epsilon\cdot\mathbf{p}}{2\,m_e}\left[\exp(\mathrm{i}\mathbf{k}\cdot\mathbf{r} - \mathrm{i}\omega t) + \exp(-\mathrm{i}\mathbf{k}\cdot\mathbf{r} + \mathrm{i}\omega t)\right].$$

$$P^{abs}_{i\to f}(t) = \frac{\pi\,e^2\,\rho(\omega_{fi})}{\epsilon_0\,\hbar^2\,m_e^2\,\omega_{fi}^2}\,\left|\langle f|\epsilon\cdot\mathbf{p}\exp(\mathrm{i}\mathbf{k}\cdot\mathbf{r})|i\rangle\right|^2 t.$$

Remember the time-dependent perturbation theory?

For the absorption the transition rate is (i:n=1,l=0,f: n=2,l=1, m=-1,0,1):

And this?

$$w^{abs}_{i\to f} \equiv \frac{dP^{abs}_{i\to f}}{dt} = \frac{\pi\, e^2\, \rho(\omega_{fi})}{\epsilon_0\, \hbar^2\, m_e^2\, \omega_{fi}^2}\, |\langle f | \boldsymbol{\epsilon}\cdot\mathbf{p}\, \exp(i\,\mathbf{k}\cdot\mathbf{r}) | i\rangle|^2$$

$\rho(\omega_{fi})$ is the energy density of the em field

ε is the polarization vector of the em field

First and second Hydrogen-like Lead orbital functions

… and the dipole approximation?

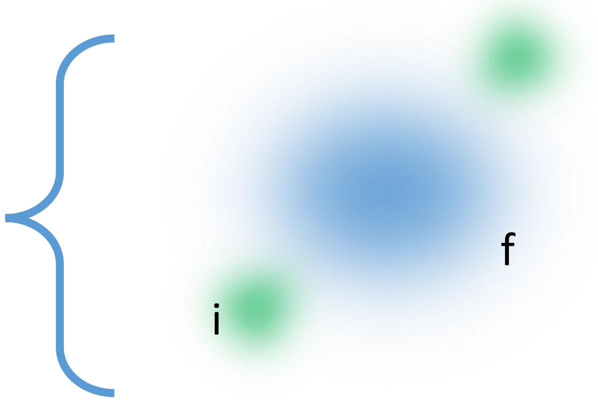
$$|\langle f|\boldsymbol{\epsilon}\cdot\mathbf{p}\exp(i\mathbf{k}\cdot\mathbf{r})|i\rangle|^2$$

$$\exp(i\mathbf{k}\cdot\mathbf{r}) = 1 + i\mathbf{k}\cdot\mathbf{r} + \cdots,$$

$$\langle f|\boldsymbol{\epsilon}\cdot\mathbf{p}\exp(i\mathbf{k}\cdot\mathbf{r})|i\rangle \simeq \boldsymbol{\epsilon}\cdot\langle f|\mathbf{p}|i\rangle.$$

Is it here still valid?

For hydrogen : r=a= 1A



λ=121 nm

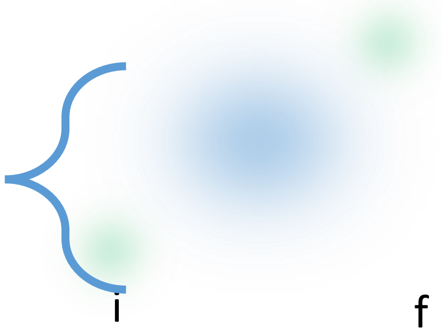
5.19 10^{7}m-1

kr= 5.19 10^{-3}

And for lead

r=a= 1/Z A

= 1.2 10^{-12} m



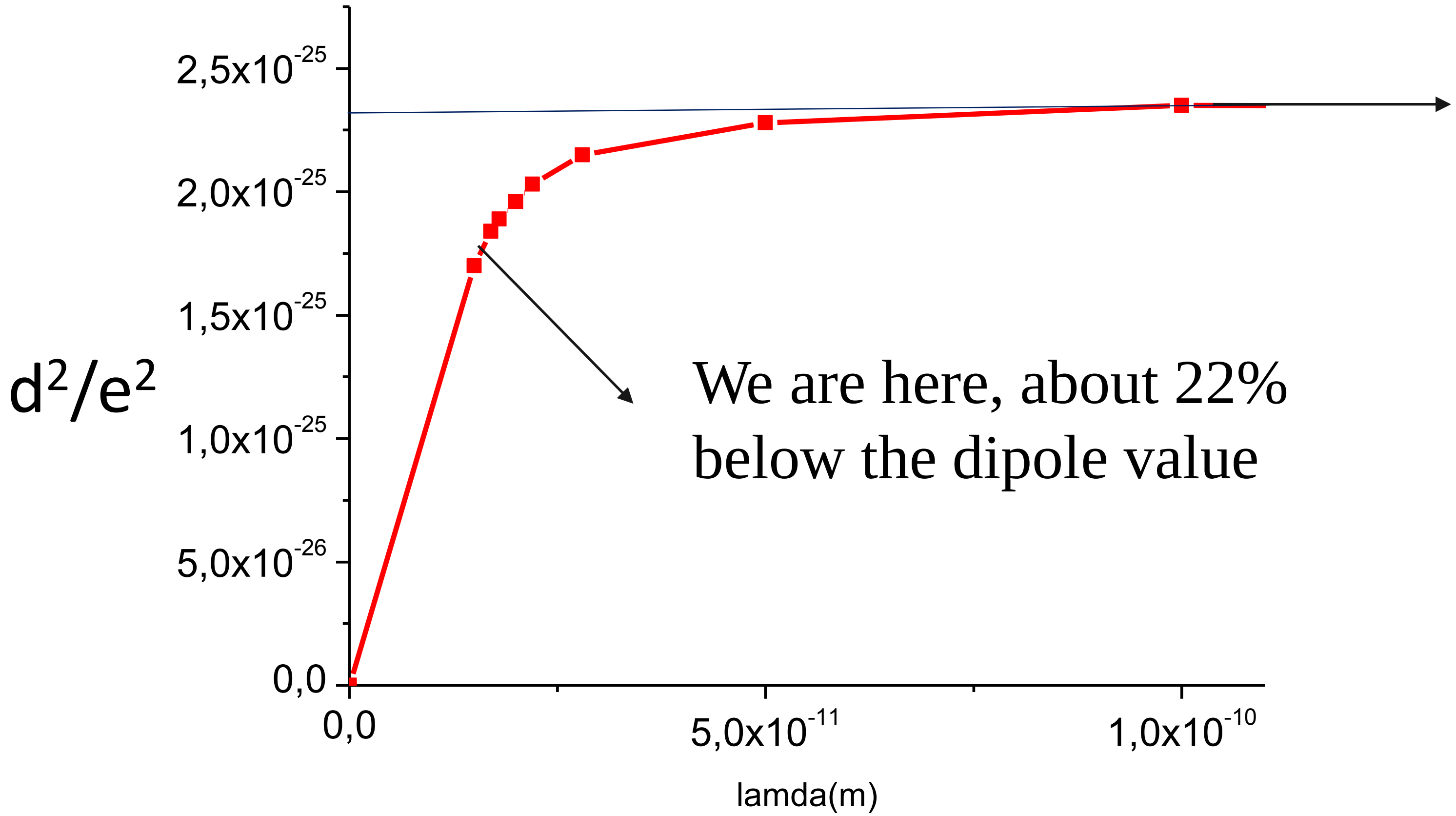
λ=108/6000 nm

k=3.5 10^{11}m-1

kr= 0.41

The dipole approximation is not fully valid, and the probability rate is lower, but luckily **only by 22%**!

The numerical calculation shows that the decrement is only 22%.



Dipole approx. for lead
2.309 10^-25

With corrections
1.78 10^-25

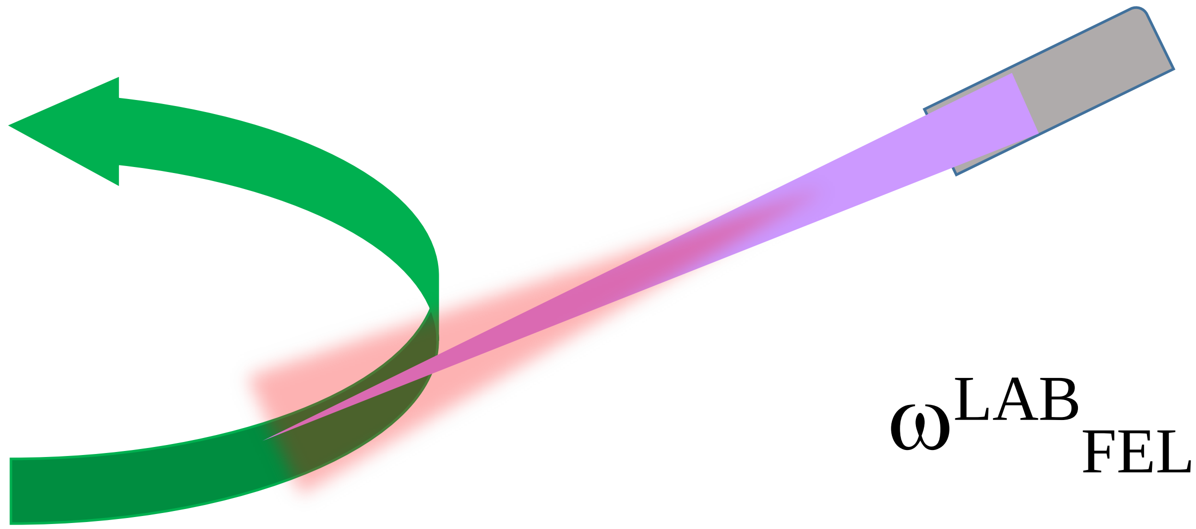
So, we have controlled some of the effects of the High Z, that are summarized here:

Corrections of 10% to the transition energy gap due to relativistic, spin-orbit and zitterbewegung effects (they scale as Z^4).

Corresponding change in the factors g1 and g2 due to the solution of the degeneration.

Corrections of 22% to the transition rate due to the break down of the dipole approximation

There are also other practical considerations, for instance the impact of the radiation on the undulator….



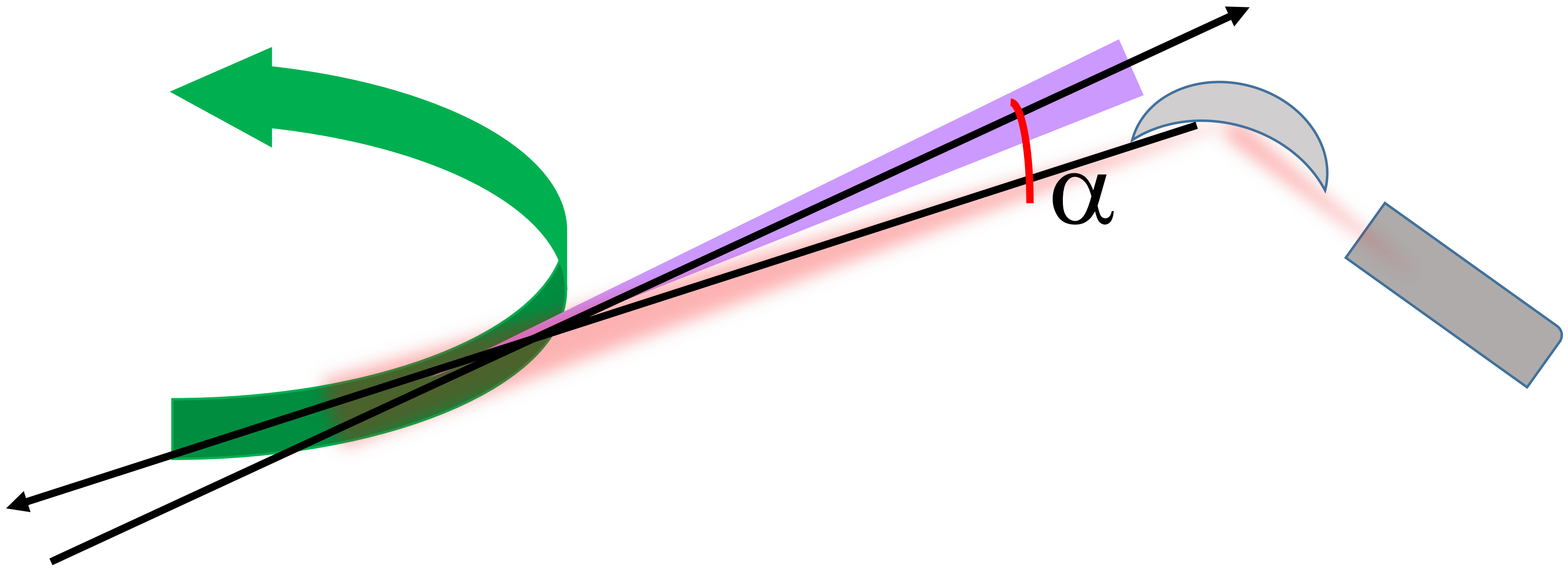
$$\omega_{12}=\omega_{FEL}\gamma^{i}_{PSI}(1+\cos\alpha)$$

$$\omega^{LAB}_{FEL}=2\omega_{FEL}(\gamma^{i}_{PSI})^{2}(1+\cos\alpha)$$

Lot of gamma rays on the magnets is not so safe…..

….maybe it is necessary an off-axis collision. Since the resonant frequency is fixed by the transition, is the pump that has to be tuned, remembering that, said α the collision angle, the frequency seen by the PSI's electron is:

$$\omega_{12}=\omega_{FEL}\gamma^{i}{}_{PSI}(1+\cos\alpha)$$



Stimulated absorption and emission are modeled by:

http://www.tcm.phy.cam.ac.uk/~bds10/aqp/handout_atomic.pdf

$$w^{abs}_{i\to f} \equiv \frac{dP^{abs}_{i\to f}}{dt} = \frac{\pi\, e^2\, \rho(\omega_{fi})}{\epsilon_0\, \hbar^2\, m_e^2\, \omega_{fi}^2}\; |\langle f|\boldsymbol{\epsilon}\cdot\mathbf{p}\, \exp(\mathrm{i}\,\mathbf{k}\cdot\mathbf{r})|i\rangle|^2$$

$$w^{abs}_{i\to f} = \frac{\pi}{\epsilon_0\, \hbar^2} |\boldsymbol{\epsilon}\cdot\mathbf{d}_{if}|^2\, \rho(\omega_{fi}),$$

$$= 1.45\ 10^{17}\ \rho(\omega)$$

$$w^{stm}_{i\to f} = \frac{\pi}{\epsilon_0\, \hbar^2} |\boldsymbol{\epsilon}\cdot\mathbf{d}_{if}|^2\, \rho(\omega_{if}),$$

$$\mathbf{d}_{if} = \langle f|e\,\mathbf{r}|i\rangle$$

These are just the B_{12} and B_{21} coefficients (a part from g1 and g2)!

And, for the spontaneous emission by lead , the Einstein model foresees:

$$w_{2P \to 1S} = \frac{\omega^3 d^2}{3\pi\,\epsilon_0\,\hbar\,c^3}, = 2.17\ 10^{16}\ \text{sec-1}$$

Very close to Bessonov's value.

This is just the Einstein coefficient A_{21}

Furthermore, I found this book (maybe quite old, but very interesting with the details of a huge number of transitions):

Atomic Transition Prob lities

Volume I Hydrogen Through Neon

A Critical Data Compilation

W. L. Wiese, M. W. Smith, and B. M. Glennon

Institute for Basic Standards
National Bureau of Standards
Washington, D.C.

H – Table A. $(n)_i - (n)_k$ Transitions (Average Values)

Transition	λ(Å)	E_i(cm^{-1})	E_k(cm^{-1})	g_i	g_k	A_{ki}(sec^{-1})	f_{ik}	S(a.u.)	log gf	Accuracy	Source
1–2 (L_α)	1215.67	0	82259	2	8	4.699×10^8	0.4162	3.330	−0.0797	AA	1
1–3 (L_β)	1025.72	0	97492	2	18	5.575×10^7	7.910×10^{-2}	0.5339	−0.8008	AA	1

Corresponds to the value found with the dipole correction

For Hydrogen the Lyman-alpha is

$A_{21}=4.699\ 10^{8}\ \text{sec}^{-1}$

For Lead ions:

$A_{21}=Z^{4}\ 4.699\ 10^{8}\ \text{sec}^{-1}$

$=2.124\ 10^{16}\ \text{sec}^{-1}$

Which means:

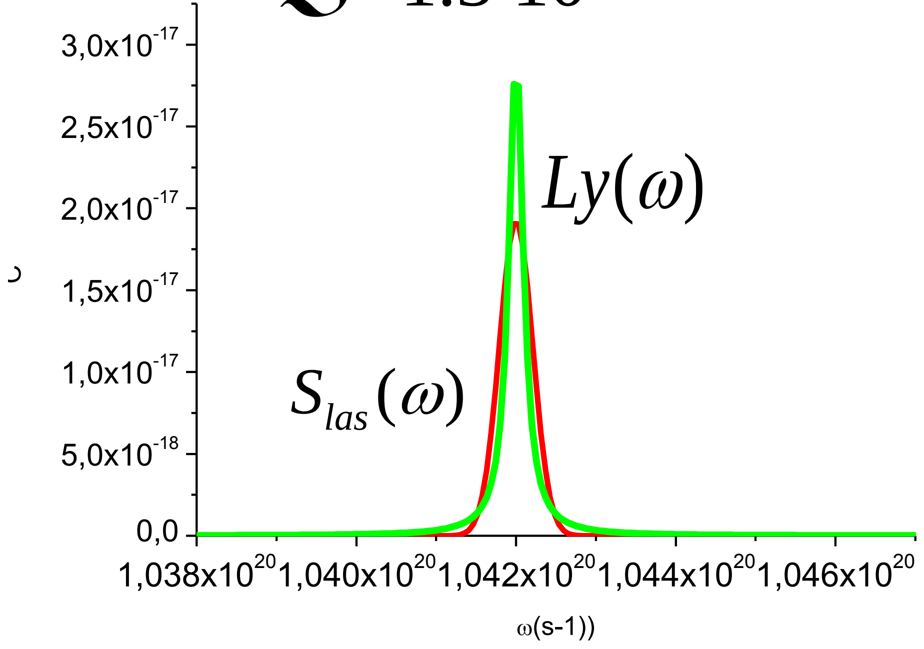
$\tau_{21}=1/A_{21}=4.708\ 10^{-17}\ \text{m}$

The FEL spectrum, with the SASE spikes, worried me a lot!

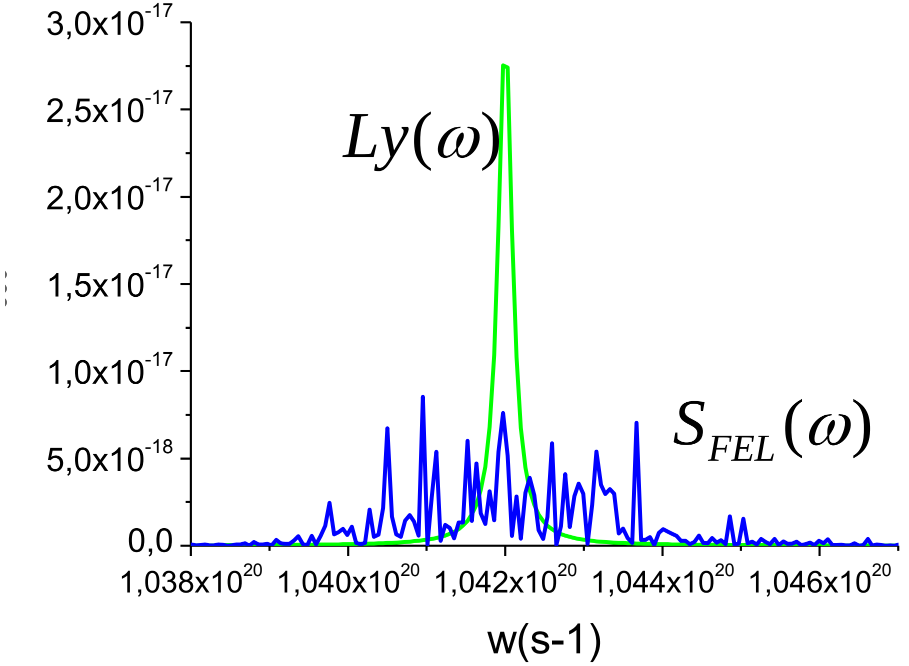
$$\rho(\omega) = \frac{h\nu_{12}N_{ph}}{\Delta V}\int S(\omega)Ly(\omega)d\omega \qquad \Im = \int S(\omega)Ly(\omega)d\omega \qquad Ly(\omega) = \frac{A_{21}/2\pi}{(\omega-\omega_{21})^2 + (A_{21}/2)^2}$$

For Laser with bandwidth $2\,10^{-4}$

$\Im = 1.3\ 10^{-17}$



For FEL: $\Im = 3.59\ 10^{-18}$



At the end, it leads to a factor 30 less then the Bessonov's case.

I thought that $\Im = \int S(\omega) Ly(\omega) d\omega$ **was larger!**
Why is so low?

Simply because,

if the bandwidth is << transition width $\Im \approx \frac{2}{\pi A_{21}} = 2.9\ 10^{-17}$

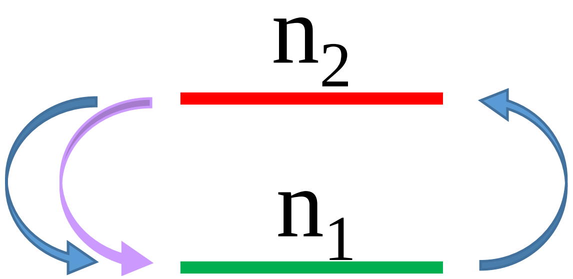
if the bandwidth is >> transition width $\Im \approx \frac{1}{\sqrt{2\pi bw \omega_{12}}} = 4 10^{-18}$

Furthermore, there is also the energy spread of the PSIs so $\Im$ **is more complicated, something like:**

$$\Im = \iint S(\omega) Ly(\omega) I\left(\sqrt{\frac{\varpi}{2\omega}}\right) d\omega d\varpi$$ **?????**

Let us come back to the Einstein equation in the PSI's electron reference frame:

$$\frac{dn_2}{dt} = n_1 B_{12}\rho(\omega) - n_2 A_{21} - n_2 B_{21}\rho(\omega)$$



$$n_1 + n_2 = 1$$

$$\frac{dn_{ph}^{spont}}{dt} = A_{21} n_0 n_2$$

$\rho(\omega)$ is the energy density per frequency units

$$\rho(\omega) = \frac{h\nu_{12} N_{ph}}{\Delta V}\mathfrak{I}$$

We have evaluated all the terms , in particular :

A_{21}=2.17 10^{16}, B_{12}=1.45$10^{17}$, B_{21} =g_1/g_2 1.45$10^{17}$

Now, let us do a simple analysis of these equations:

$$\frac{dn_2}{dt} = n_1 B_{12}\rho(\omega) - n_2 B_{21}\rho(\omega) \qquad n_1 + n_2 = 1$$

$$\frac{dn_{ph}^{spont}}{dt} = A_{21} n_0 n_2$$

First, let us suppose that ρ(ω) is very large

$$\frac{dn_2}{dt} = n_1 B_{12}\rho(\omega) - n_2 B_{21}\rho(\omega) \qquad n_1 + n_2 = 1$$

$$\frac{dn_{ph}^{spont}}{dt} = A_{21} n_0 n_2$$

and let us look for the stationary condition:

$$0 \approx n_1 B_{12}\rho(\omega) - n_2 B_{21}\rho(\omega) \qquad n_1 + n_2 = 1 \qquad \frac{dn_{ph}^{spont}}{dt} = A_{21} n_0 n_2$$

and:

$$n_2 \approx \frac{g_2}{g_1 + g_2}$$

$$\frac{dn_{ph}^{spont}}{dt} \approx A_{21} n_0 \frac{g_2}{g_1 + g_2}$$

so:

$$n_{ph}^{spont} \approx A_{21} n_0 \frac{g_2}{g_1 + g_2} \Delta t_{pump}^{PSI} \qquad n_{ph}^{spont} \approx A_{21} n_0 \frac{g_2}{g_1 + g_2} \frac{\Delta t_{pump}^{LAB}}{2\gamma_{PSI}}$$

And now, let us suppose now that ρ(ω) is very little and, again, let us look for the stationary solution:

$$0 \approx (1-n_2)B_{12}\rho(\omega) - n_2 A_{21} - n_2 B_{21}\rho(\omega) \qquad n_1 + n_2 = 1$$

$$\frac{dn_{ph}^{spont}}{dt} = A_{21} n_0 n_2$$

$$n_2 \approx \frac{B_{12}\rho(\omega)}{(A_{12} \qquad\qquad)} \qquad\qquad n_2 \approx \frac{B_{12}\rho(\omega)}{A_{12}}$$

Continuing with this simple analysis:

$$\frac{dn_{ph}^{spont}}{dt} \approx A_{21} n_0 \frac{B_{12}\rho(\omega)}{A_{21}} \qquad \frac{n_{ph}^{spont}}{n_0} = B_{12}\rho(\omega)\Delta t_{FEL}^{PSI} = \frac{B_{12}\rho(\omega)}{2\gamma_{PSI}}\Delta t_{FEL}^{Lab}$$

$$\rho(\omega) \approx \Im \frac{h\nu_{12} N_{ph,FEL}}{2\pi\sigma_{FEL}^2 c\Delta t_{FEL}^{PSI}} \qquad \frac{n_{ph}^{spont}}{n_0} = \Im B_{12} \frac{2\gamma_{PSI} h N_{ph,FEL}}{2\pi\sigma_{FEL}^2 \lambda_{FEL}^{LAB}}$$

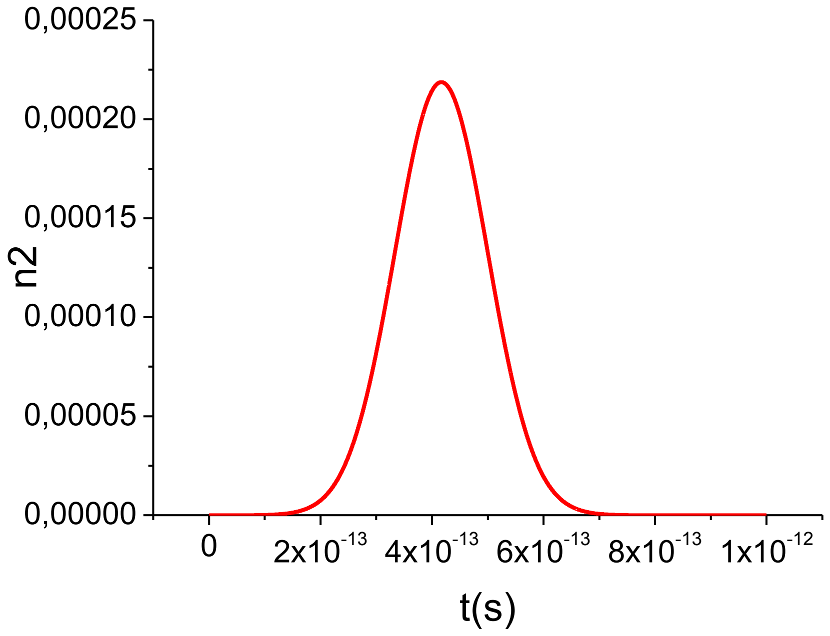
We arrive to two approximate expressions. The first, in the limit of low pump intensity, is:

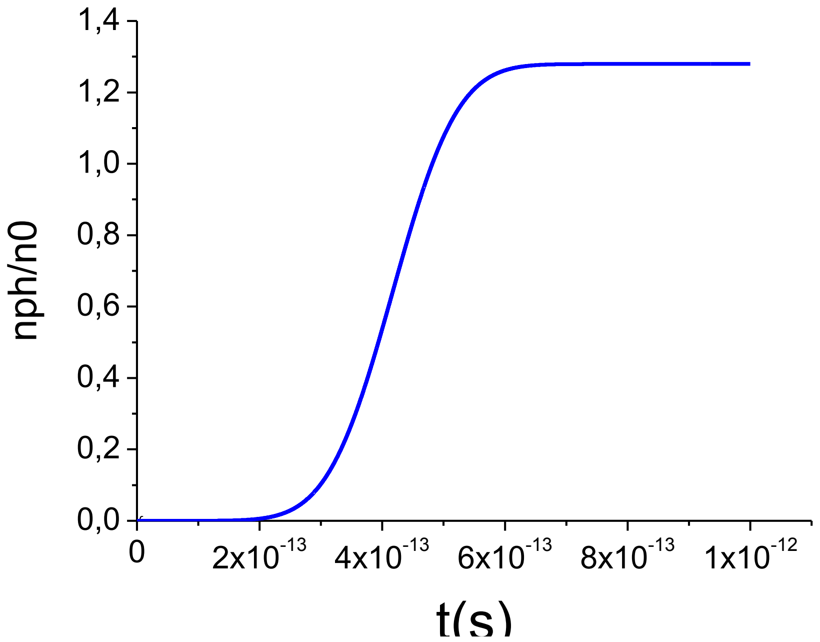
$$\frac{n_{ph}^{spont}}{n_0} = \Im B_{12} \frac{2\gamma_{PSI} h\nu_{FEL}^{LAB} N_{ph,FEL}}{2\pi\sigma_{FEL}^2 c}$$

While for an intense pump :

$$\frac{n_{ph}^{spont}}{n_0} \approx A_{21} \frac{g_2}{g_1 + g_2} \frac{\Delta t_{pump}^{LAB}}{2\gamma_{PSI}}$$

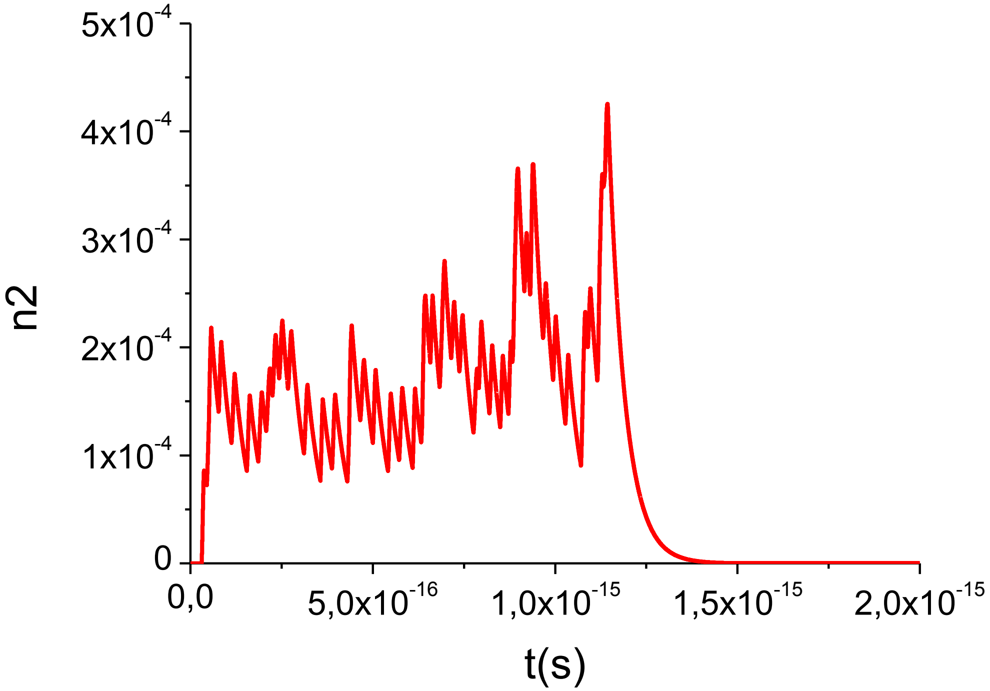
The Einstein equations for the lead Bessonov's case give:

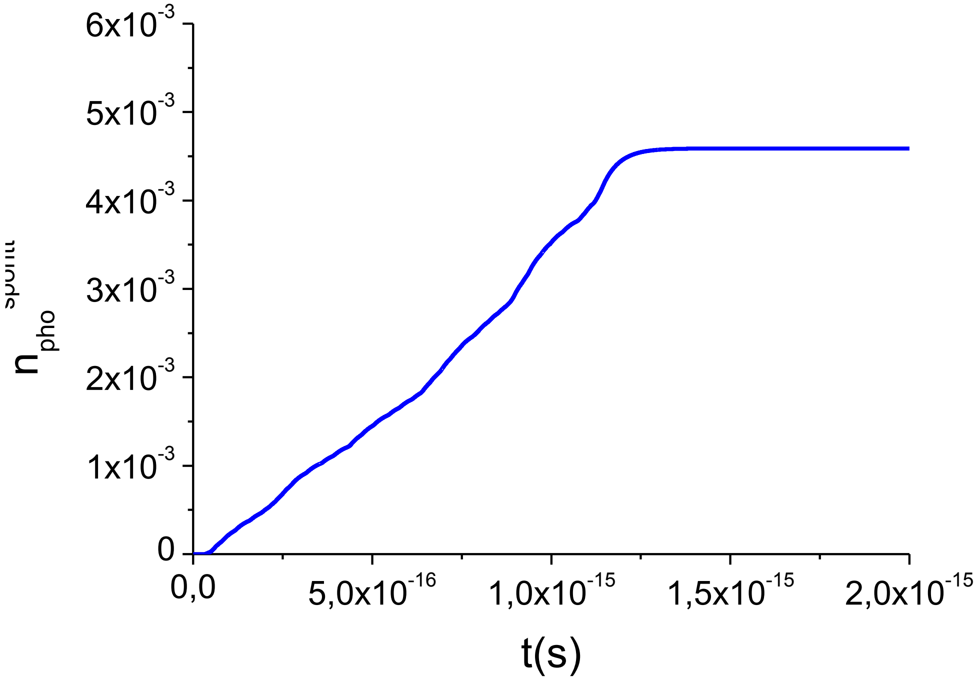




$N^{spont}_{ph}/n_0=1.28$ $N^{stim}_{ph}/n_0=6.59\ 10^{-5}$ $N_{ph}=8.90\ 10^{14}$

while for the FEL in the PSI's frame (for one ion) in the case of the FEL radiation shown above:

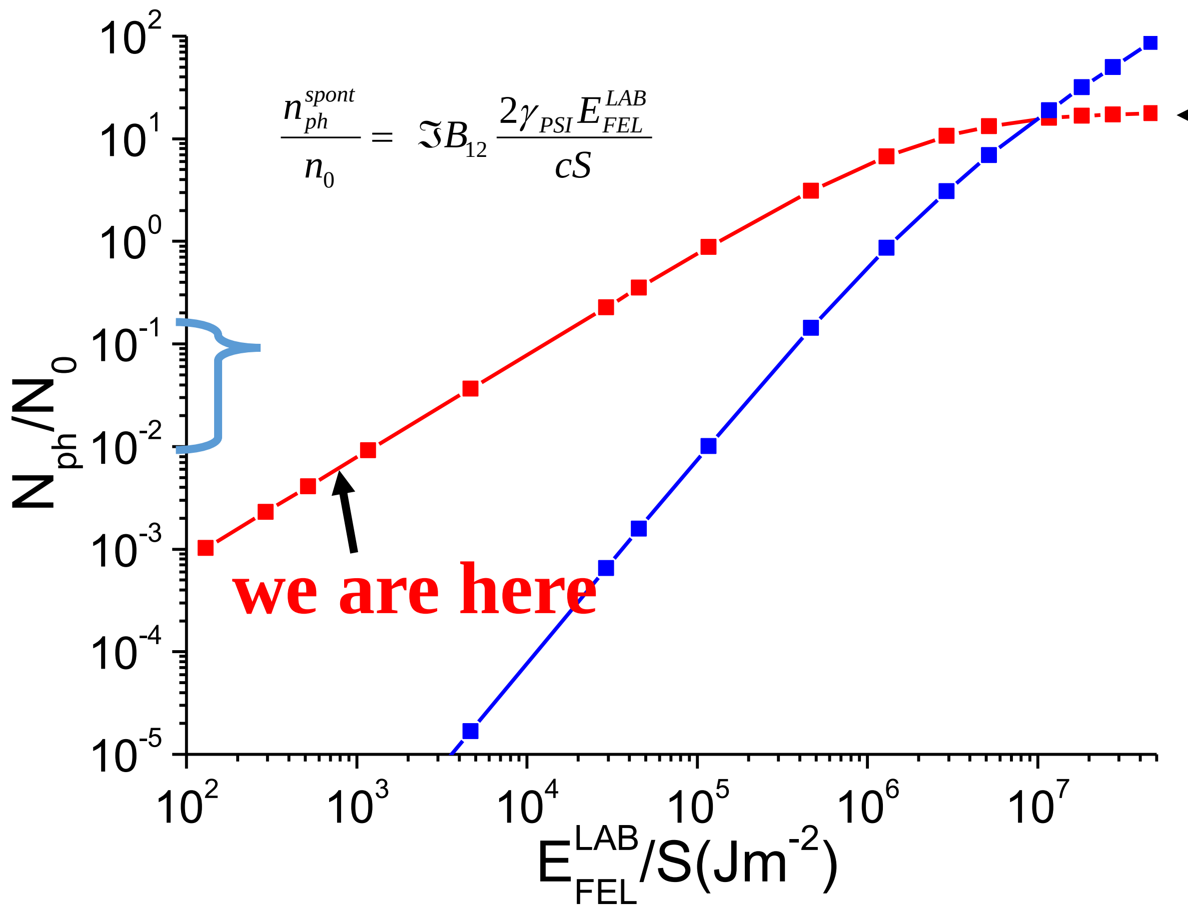




$N^{spont}_{ph}/n_0=0.0046$ $N^{stim}_{ph}/n_0=3.\ 10^{-7}$ $N_{ph}=3\ 10^{12}$

This is the dependence of the emission rate on the LAB FEL intensity at fixed bandwidth

In blu: the stimulated radiation. In red: the spontaneous emission.



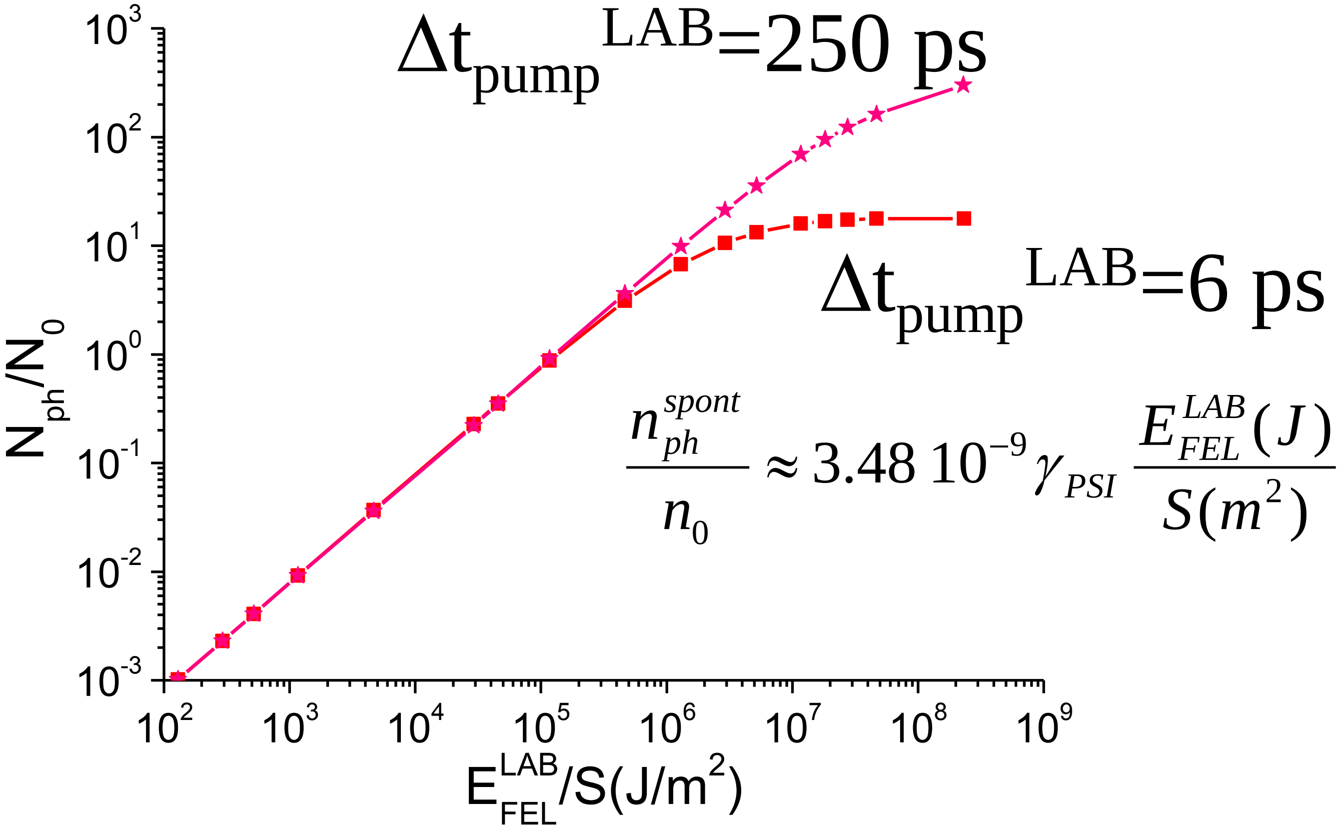
$$\frac{n_{ph}^{spont}}{n_0} \approx A_{21} \frac{g_2}{g_1+g_2} \frac{\Delta t_{pump}^{LAB}}{2\gamma_{PSI}}$$

$$= 2.17\ 10^{16} \frac{3}{4} \frac{6\ 10^{-12}}{6000} \approx 16.2$$

For the case shown before $E_{FEL}/S = 10^3\ m^{-3}$

The max focusing is 160 micron

And for different FEL time durations, same number of FEL photons:



$$\frac{n_{ph}^{spont}}{n_0} \approx 3.48\ 10^{-9} \gamma_{PSI} \frac{E_{FEL}^{LAB}(J)}{S(m^2)}$$

$$\frac{n_{ph}^{spont}}{n_0} \approx A_{21} \frac{g_2}{g_1 + g_2} \frac{\Delta t_{pump}^{LAB}}{2\gamma_{PSI}}$$

Using the FEL scaling laws for energy and spectrum:

$$\rho = \frac{1}{2\gamma}\sqrt[3]{\frac{I}{I_A}\left(\frac{JJ\lambda_w a_w}{2\pi\sigma_x}\right)^2}$$

$$E_{FEL}^{LAB}(J) \approx 7.210^{-7}\rho Q(pC)\gamma$$

$$bw \approx \rho$$

$$\Im \approx \frac{1}{\omega_{12}\rho\sqrt{2\pi}}$$

$$\frac{n_{ph}^{spont}}{n_0} \approx \Im B_{12}\frac{2\gamma_{PSI}E_{FEL}^{LAB}}{cS} \approx \frac{B_{12}}{\omega_{12}\sqrt{2\pi}}\frac{7.210^{-7}\gamma_{PSI}\;Q(pC)\gamma}{c2\pi\sigma_{FEL}^2(\mu m^{-2})}$$

Final considerations:

High FEL energy flux

$$\frac{n_{ph}^{spont}}{n_0} \approx A_{21} \frac{g_2}{g_1 + g_2} \frac{\Delta t_{pump}^{LAB}}{2\gamma_{PSI}}$$

Low FEL energy flux

$$\frac{n_{ph}^{spont}}{n_0} \approx \Im B_{12} \frac{2\gamma_{PSI} E_{FEL}^{LAB}}{cS} \approx \frac{B_{12}}{\omega_{12}\sqrt{2\pi}} \frac{7.210^{-7}\gamma_{PSI}\, Q(pC)\gamma}{c2\pi\sigma_{FEL}^2(\mu m^{-2})}$$

$$\approx \cos t \frac{Q(pC)}{\sigma_{FEL}^2(\mu m^{-2})}$$

$Q(pC)$ → **Limited by the injector**

$\sigma_{FEL}^2(\mu m^{-2})$ → **Limited by the divergence**

I have disregarded:

The hyperfine structure of the transition

All the transverse dynamics of ions and radiation

The ionization due to double absorption

Other million effects

A FEL is, in principle, able to solve some of the difficulties of the gamma-factory…..

…in particular those connected with the tuning of the pump frequency with the resonance.

Furthermore, one single device could serve both working points.

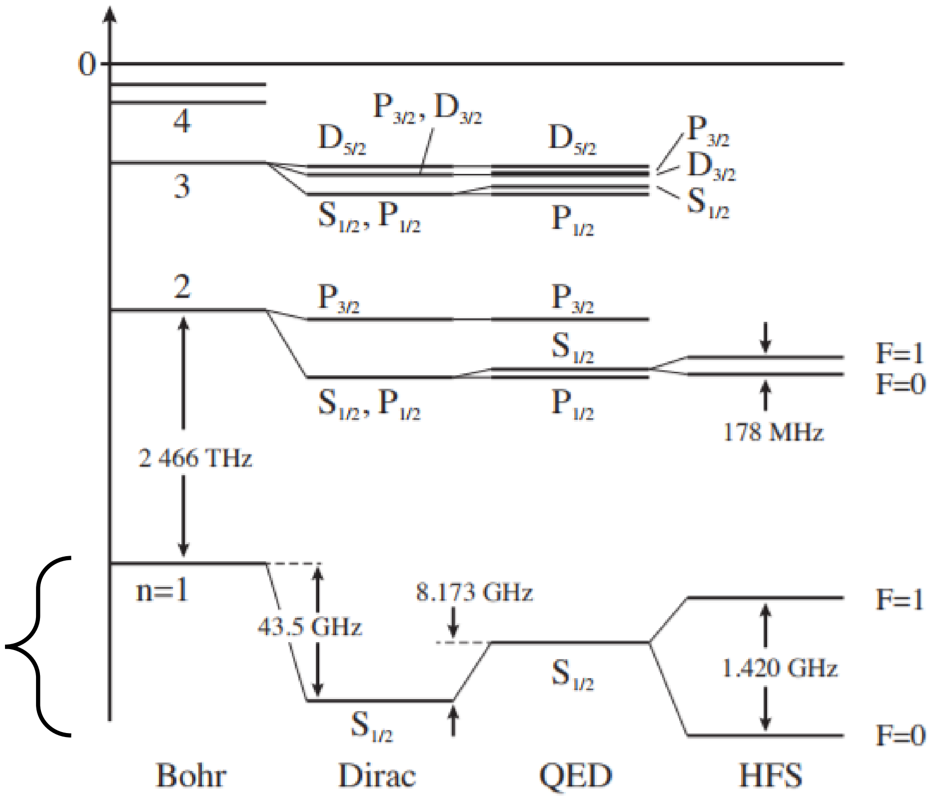
But, due to the limited length of the pulse, the efficiency does not seem particularly striking (two or three order of magnitude less than the laser case)!!

Remember these are only preliminary considerations!!!
Thank You and good by.

Relativistic corrections

for n=1, l=0 there is only this correction

$$\langle \hat{H}_1 \rangle_{n\ell m} = -\frac{mc^2}{2}\left(\frac{Z\alpha}{n}\right)^4\left(\frac{n}{\ell+1/2}-\frac{3}{4}\right).$$



For lead:

n=1,l=0 $\langle \hat{H}_1 \rangle_{n\ell m} = -\frac{mc^2}{2}\left(\frac{Z\alpha}{n}\right)^4\left(\frac{n}{\ell+1/2}-\frac{3}{4}\right)$ =-8.19 keV

Quite large!!!

n=2,l=1 $\langle \hat{H}_1 \rangle_{n\ell m} = -\frac{mc^2}{2}\left(\frac{Z\alpha}{n}\right)^4\left(\frac{n}{\ell+1/2}-\frac{3}{4}\right)$ =-1.19 keV

Spin-orbit correction (thanks to Fabrizio Castelli)

$$\langle \hat{H}_2 \rangle_{n,j=\ell\pm 1/2,m_j,\ell} = \frac{1}{4} mc^2 \left(\frac{Z\alpha}{n}\right)^4 \frac{n}{j+1/2} \begin{pmatrix} \frac{1}{j} \\ -\frac{1}{j+1} \end{pmatrix}$$

Only for l>0

For n=2, l=1, j=3/2,1/2

Zitterbewegung (Darwin) correction

$$\langle \hat{H}_3 \rangle_{njm_j\ell} = \frac{Ze^2}{4\pi\epsilon_0} \frac{\hbar^2}{8(mc)^2} 4\pi |\psi_{\ell n}(0)|^2 = \frac{1}{2} mc^2 \left(\frac{Z\alpha}{n}\right)^4 n\delta_{\ell,0} \, .$$