# Scientific Case for a Low Energy Gamma-Gamma Collider

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## Photon-photon scattering was one of the early predictions of QED and the first searches started around 1930

In that year, Hughes and Jauncey tried to detect photon-photon scattering, in an attempt that was totally disconnected from the positron theory of Dirac (1928)

*AUGUST 15, 1930* *PHYSICAL REVIEW* *VOLUME 36*

### AN ATTEMPT TO DETECT COLLISIONS OF PHOTONS

By A. L. Hughes and G. E. M. Jauncey
Department of Physics, Washington University

(Received June 16, 1930)

Abstract

Assuming that light is corpuscular and that collisions between light corpuscles (i.e. photons) can occur, it is shown that two photons of identical frequency $\nu$, moving along paths which make an angle $2\theta$ with each other, will on collision give rise to a photon of frequency $\nu(1+\cos\theta)$ travelling forward along the line bisecting the angle. To test this, two beams of sunlight (one suitably deflected by a mirror), filtered through red glass, were passed through lenses 24 cm in diameter and of 33 cm focal length, so that the beams, whose axes made an angle of 120° with each other, intersected at a common focus. The point of intersection of the beams was examined through a green filter with the dark-adapted eye. No light was detected. Calculations show that if the photon has a cross section, its area must be less than $3\times10^{-20}$ cm$^2$. From a result of Lord Rayleigh, some writers have suggested an area of the order of $\lambda^2$ for the photon. Our result shows the area to be of the order of $10^{-10}\lambda^2$.

In another early effort (Proc. Royal Soc. London. Series A, 125, (1929), 345-351.) William H. Watson proposed to measure the effect of a transverse magnetic field on the propagation of light, and therefore the scattering between real and virtual photons, on the basis that

*... The simplest particle properties which one can postulate are those of electric moment and magnetic moment; free electric charge is excluded by the fact that light is not deflected in a uniform electric or magnetic field ...*

and he set out

*... with the object of detecting, if possible, the existence of the magnetic moment of a photon ...*

*The Effect of a Transverse Magnetic Field on the Propagation of Light in vacuo.*

By WILLIAM H. WATSON, Carnegie Research Fellow.

(Communicated by Sir Ernest Rutherford, P.R.S.—Received June 21, 1929.)

Already in these very early experiments we notice a branching of two different experimental lines:

- optical tests at very low energy
- photon-photon scattering

This branching shows up in theory as well, with different formalisms at very low and at not-so-low energy

- effective Lagrangians
- calculation of scattering amplitudes and cross-sections

## optical methods (Lagrangian formulation)

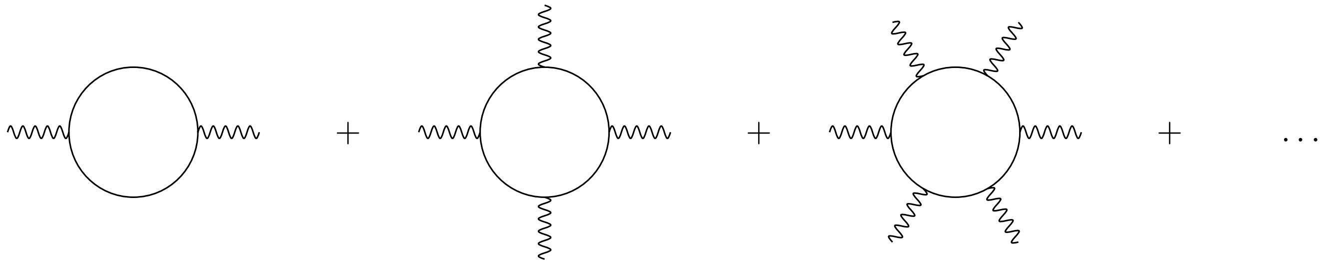
- **1936**: Heisenberg and Euler give the full closed-form expression of the nonlinear correction to the Maxwell EM field Lagrangian that takes into account the fluctuations of a spin ½ fermion field
- **1936**: Weisskopf presents a similar computation that takes into account the the fluctuations of a spin 0 scalar field. Both HE and W note that the fluctuations of the background field are expected to influence the propagation of light.
- **1942**: Schrödinger studies nonlinear optics and photon splitting associated with the Born-Infeld Lagrangian
- **1950-51**: Schwinger formalizes the HE Lagrangian in the language of QED;
- **1966**: Erber considers photon splitting with an approach that is partly based on the Lagrangian formulation
- **1970-1**: Bialinicka-Birula & Bialinicki-Birula, and Adler, study the interaction of photons with external magnetic fields and cast photon dispersion and photon splitting in the same framework;
- 1986: Maiani, Petronzio & Zavattini consider the effect of axion-like particles on the propagation of light in a magnetic field

## The calculations of Heisenberg and Euler, and of Weisskopf are slightly different in scope

- Heisenberg and Euler consider spin 1/2 particles
- Weisskopf mostly considers spin 0 particles

## However, they have many points in common

- Both HE and W calculate corrections to the classical Maxwell Lagrangian
- They compute **one-loop corrections to all orders**



- ***The resulting Lagrangians are "effective Lagrangians"; this means that in a theory of charged particles AND electromagnetic fields – where we assume that the energy of EM quanta is much less than the rest energy of the particles – the particle degrees of freedom are eliminated, and the originally linear theory acquires an "effective" nonlinear character***

## Nonlinear one-loop correction term obtained by Heisenberg and Euler for spin 1/2 particles

$$\mathcal{L}^{(1)}_{\rm sp} = -\frac{1}{hc}\int_0^\infty \frac{d\eta}{\eta^3}\exp(-\eta e\mathcal{E}_c)\left\{e^2\eta^2 a\cot(ea\eta)\, b\coth(eb\eta) - 1 - \frac{e^2\eta^2}{3}(b^2-a^2)\right\}$$

## Nonlinear one-loop correction term obtained by Weisskopf for spin 0 particles

$$\mathcal{L}^{(1)}_{\rm scalar} = \frac{1}{2hc}\int_0^\infty \frac{d\eta}{\eta^3}\exp(-\eta e\mathcal{E}_c)\left\{e^2\eta^2 a\,\mathrm{cosec}(ea\eta)\, b\,\mathrm{cosech}(eb\eta) - 1 + \frac{e^2\eta^2}{6}(b^2-a^2)\right\}$$

$$\text{where}\quad a^2-b^2 = -2\mathcal{F} = \mathbf{E}^2-\mathbf{B}^2;\quad ab = -\mathcal{G} = \mathbf{E}\cdot\mathbf{B};\quad \text{and}\quad \mathcal{E}_c = \frac{m^2c^3}{e\hbar}$$

scalar invariant

pseudoscalar invariant

critical electric field

(notation as in G. V. Dunne, “Heisenberg–Euler Effective Lagrangians: Basics and Extensions”, in “From Fields to Strings: Circumnavigating Theoretical Physics - Ian Kogan Memorial Collection” (World Scientific, 2005) 445-552.)

## Interestingly, the different terms can be related to specific features of the theory

the integration variable is the "proper time" variable (fully developed later by Stückelberg, Feynman and Schwinger)

subtraction of the infinite free-field effective action

$$\mathcal{L}_{\rm sp}^{(1)} = -\frac{1}{hc}\int_0^\infty \frac{d\eta}{\eta^3}\exp(-\eta e\mathcal{E}_c)\left\{e^2\eta^2 a\cot(ea\eta)\, b\coth(eb\eta) - 1 - \frac{e^2\eta^2}{3}(b^2-a^2)\right\}$$

this corresponds to a log term in the integrated Lagrangian: it is an embryonic form of charge renormalization

The Lagrangian correction terms can be expanded as follows, up to second order (natural units throughout)

$$\mathcal{L}_{\mathrm{sp}}^{(1)} \approx \frac{2\alpha^2}{45m^4}\left[(\mathbf{E}^2-\mathbf{B}^2)^2+7(\mathbf{E}\cdot\mathbf{B})^2\right]+\frac{16\pi\alpha^3}{315m^8}(\mathbf{E}^2-\mathbf{B}^2)\left[2(\mathbf{E}^2-\mathbf{B}^2)^2+13(\mathbf{E}\cdot\mathbf{B})^2\right]$$

$$\mathcal{L}_{\mathrm{scalar}}^{(1)} \approx \frac{\alpha^2}{360m^4}\left[7(\mathbf{E}^2-\mathbf{B}^2)^2+4(\mathbf{E}\cdot\mathbf{B})^2\right]+\frac{\pi\alpha^3}{630m^8}(\mathbf{E}^2-\mathbf{B}^2)\left[31(\mathbf{E}^2-\mathbf{B}^2)^2+77(\mathbf{E}\cdot\mathbf{B})^2\right]$$

Using the equations of motion of the fields one finds the **D** and **H** fields

$$\mathbf{D}=\frac{1}{\epsilon_0}\frac{\partial\mathcal{L}}{\partial\mathbf{E}}; \quad \mathbf{H}=-\mu_0\frac{\partial\mathcal{L}}{\partial\mathbf{B}}$$

and the corresponding effective values for the electric and magnetic polarizabilities (using the first term of the expansion only, for spin ½ particles)

$$\varepsilon_{ij}\approx\delta_{ij}+\frac{4\alpha^2}{45m^4}\left[2\left(\mathbf{E}^2-\mathbf{B}^2\right)\delta_{ij}+7B_iB_j\right]; \qquad \mu_{ij}\approx\delta_{ij}+\frac{4\alpha^2}{45m^4}\left[2\left(\mathbf{B}^2-\mathbf{E}^2\right)\delta_{ij}+7E_iE_j\right]$$

## As noted both by Heisenberg and Euler, and by Weisskopf, the modified polarizabilities describe a QED vacuum that behaves as a dielectric medium.

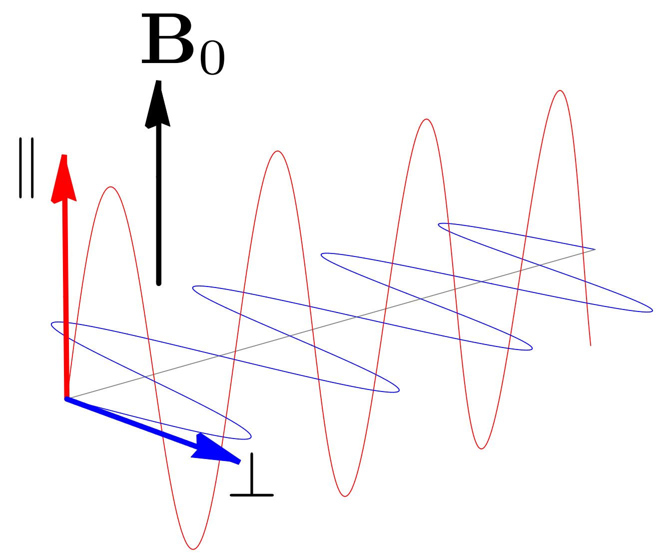
For instance, Weisskopf wrote:

"Under these circumstances, the electromagnetic properties of the vacuum can be represented by a field-dependent electric and magnetic polarizability of empty space which leads, for example, to the refraction of light in electric fields or to the scattering of light from light."

**These dielectric properties imply the birefringence of vacuum.**

Assuming that light propagates in a uniform, dipolar magnetic field $\mathbf{B}_0$, we find

$$n_{\parallel} = 1 + 7 A_e B_0^2$$
$$n_{\perp} = 1 + 4 A_e B_0^2$$




and eventually

$$\Delta n = n_{\parallel} - n_{\perp} = 3 A_e B_0^2$$

where

$$A_e = \frac{2\alpha^2 \lambda\!\!\!^{-2}}{45 \mu_0 m_e c^2} \approx 1.32 \times 10^{-24} \ \mathrm{T}^{-2}$$

***so that the magnetized vacuum of QED is birefringent.***

Notice that

$$A_e = \frac{\alpha}{90\pi}\left(\frac{1}{\mathcal{B}_c^2}\right)$$

i.e., the $A_e$ coefficient can be written as a function of the critical magnetic field

$$\mathcal{B}_c = \frac{\mathcal{E}_c}{c} = \frac{m_e^2 c^2}{e\hbar} \approx 4.4 \times 10^9 \mathrm{T}$$

**Resulting birefringence for near-visible (Nd-YAG laser, wavelength 1064 nm)**

$$\Delta n = 3A_e B_0^2 = \frac{\alpha}{30\pi}\left(\frac{B}{\mathcal{B}_c}\right)^2 \approx 7.7\ 10^{-5}\left(\frac{B}{\mathcal{B}_c}\right)^2$$

$$\Delta n|_{B=2.5\ \mathrm{T}} \approx 2.5\ 10^{-23}$$

It is interesting to note that the QED Lagrangian is only one of a class. Another one is the **Born-Infeld Lagrangian** (originally introduced to solve the divergence of electron EM self-energy)

$$
\begin{aligned}
\mathcal{L}_{BI} &= b^2 \left( \sqrt{-\det \eta_{\mu\nu}} - \sqrt{-\det \eta_{\mu\nu} + F_{\mu\nu}/b} \right) \\
&= b^2 \left( 1 - \sqrt{1 + 2\mathcal{F}/b^2 - \mathcal{G}^2/b^4} \right) \\
&\approx -\mathcal{F} + \frac{1}{2b^2}\mathcal{F}^2 + \frac{1}{2b^2}\mathcal{G}^2 \\
&= -\frac{1}{2}(\mathbf{E}^2 - \mathbf{B}^2) + \frac{1}{8b^2}(\mathbf{E}^2 - \mathbf{B}^2)^2 + \frac{1}{2b^2}(\mathbf{E} \cdot \mathbf{B}^2)^2
\end{aligned}
$$

$$
\text{then} \quad c_1^{(BI)} = c_2^{(BI)} = 1/2b^2
$$

Notably, the BI Lagrangian surfaces in low-energy extrapolations of string theories.

***An important and unique feature of the BI Lagrangian is that magnetized vacuum <u>DOES NOT</u> become birefringent.***

The generic Lagrangian

$$\mathcal{L} = -\mathcal{F} + c_1 \mathcal{F}^2 + c_2 \mathcal{G}^2$$

satisfies the requirements of Lorentz invariance and P-invariance.

The **Chern-Simons Lagrangian** does not belong to this class of Lagrangians

$$\mathcal{L}_{MCS} = -\mathcal{F} + k_\mu A_\nu \tilde{F}^{\mu\nu}$$

where the *k* four-vector is fixed, it corresponds to a preferred direction in space, and this Lagrangian is not Lorentz-invariant.

***It is interesting to notice that the CS Lagrangian leads to a birefringence of vacuum even in the absence of background fields.***

## *The PVLAS experiment:* started back at CERN in the '80's

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

Proposal D2
9 June 1980

EXPERIMENTAL DETERMINATION OF VACUUM POLARIZATION EFFECTS
ON A LASER LIGHT-BEAM PROPAGATING IN A STRONG MAGNETIC FIELD

E. Iacopini, P. Lazeyras, M. Morpurgo, E. Picasso,
B. Smith and E. Zavattini

CERN, Geneva, Switzerland

and

E. Polacco

Università di Pisa, Italy

**After the initial CERN proposal, the experiment has moved, to BNL first (BFRT collaboration), then to LNL (PVLAS collaboration), and now it is located in a clean room inside the Physics Dept. of the University of Ferrara**

The Physics Department in Ferrara, the present site of the PVLAS experiment.

**the PVLAS collaboration**

- **F. Della Valle**, University of Trieste and INFN-Trieste,
- **A. Ejlli**, University of Ferrara and INFN-Ferrara,
- **U. Gastaldi**, University of Ferrara and INFN-Ferrara,
- **E. Milotti**, University of Trieste and INFN-Trieste,
- **R. Pengo**, Laboratori Nazionali di Legnaro – INFN
- **G. Ruoso**, Laboratori Nazionali di Legnaro – INFN
- **G. Zavattini**, University of Ferrara and INFN-Ferrara

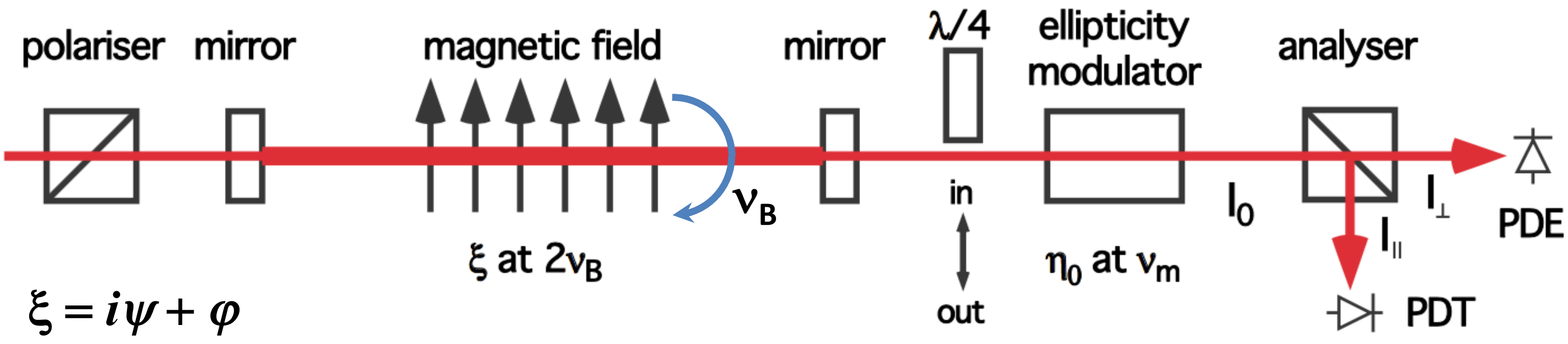
## The predicted QED effect is exceedingly small and in its effort to detect it PVLAS uses the following strategies

- increase the magnetic field as much as possible (the physical effect is proportional to $\mathbf{B}^2$)

- **increase the optical path length as much as possible (you fold the light path – and here you have the choice between a non resonant multipass cavity and a resonant cavity, a Fabry-Perot interferometer)**

- modulate the physical signal to linearize the system's response and beat noise

- Single pass ellipticity $\psi = \frac{\pi \Delta n_B L}{\lambda} \sin 2\theta(t) = \psi_0 \sin 2\theta(t)$
- Photons traverse a magnetic field region of length *L* and $\Delta n_B \propto B^2$

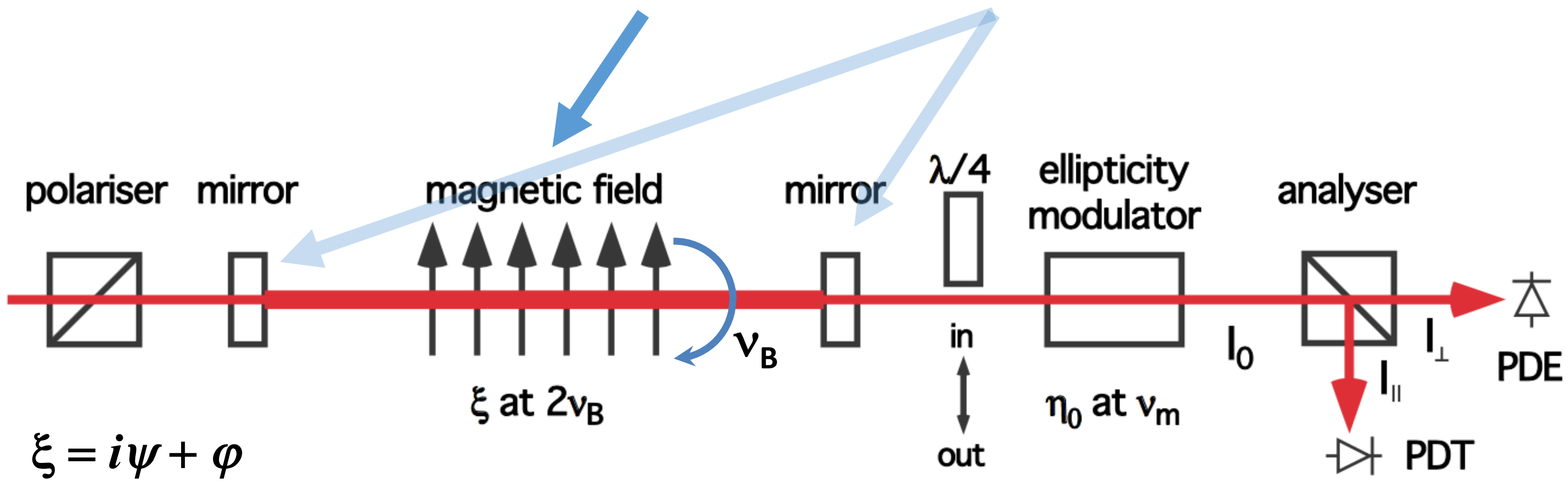



- We desire to determine the optical path difference $\mathrm{OPL} = \Delta n_B L$ between the two orthogonal polarisation states by measuring the induced ellipticity $\psi$
- The Fabry-Perot cavity amplifies $\psi$ by a factor $N = 2\mathcal{F}/\pi$
- Heterodyne detection linearizes the ellipticity to be measured and allows distinction between a rotation and an ellipticity (the apparatus is capable of detecting dichroism as well as ellipticity)
- The rotating magnetic field modulates the desired signal

adapted from the slides presented by G. Zavattini at Physics Beyond Colliders, CERN, Nov. 21-22 2017

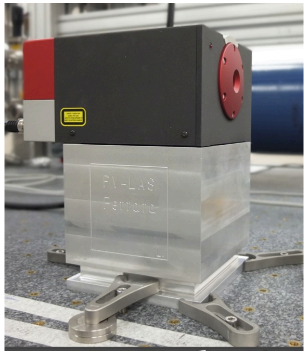
$$I_{\rm out} \approx I_0 \left| i\eta(t) + iN\psi_0 \sin 2\theta(t) + iN\alpha(t) + \varphi(t) + \ldots \right|^2$$

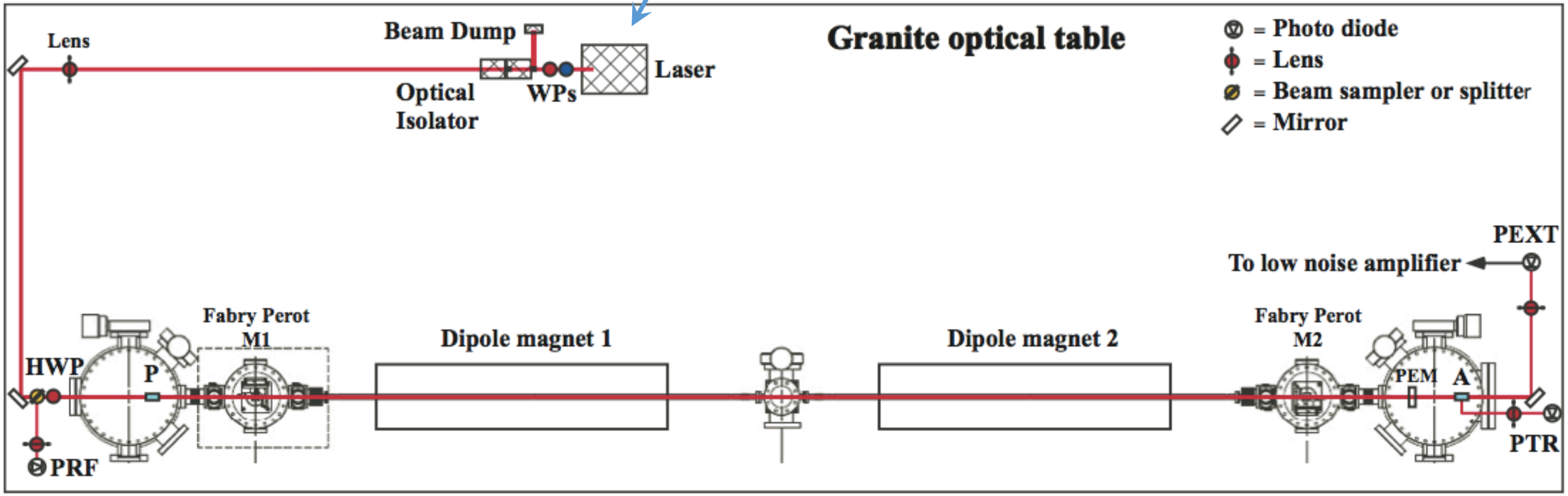
modulator signal noise rotation



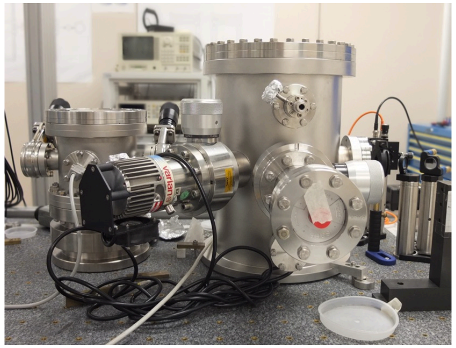

$$I_{\rm out} \approx I_0 \left\{ \eta^2(t) + 2\eta(t)N\psi_0 \sin 2\theta(t) + 2\eta(t)N\alpha(t) + \varphi^2(t) + \ldots \right\}$$

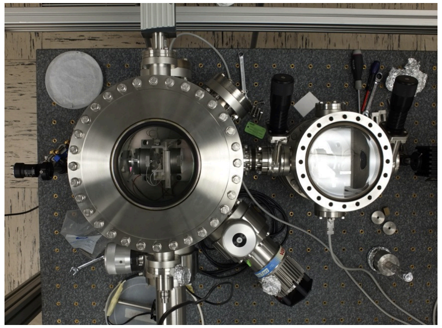
- Ellipticity signals beat with the modulator
- Rotation signals do not beat (they do when QWP is inserted)
- After demodulation at the demodulator frequency:
  - A pure sinusoidal signal appears at frequency $2\nu_B$
  - Need to understand noise contribution at $2\nu_B$
  - We are now convinced that mirrors are an important origin of noise




**3.3 m long Fabry Perot cavity**





# PVLAS @ Ferrara

$$\nu_{\mathrm{FSR}} = \frac{c}{2L} \approx 45.5 \text{ MHz}$$

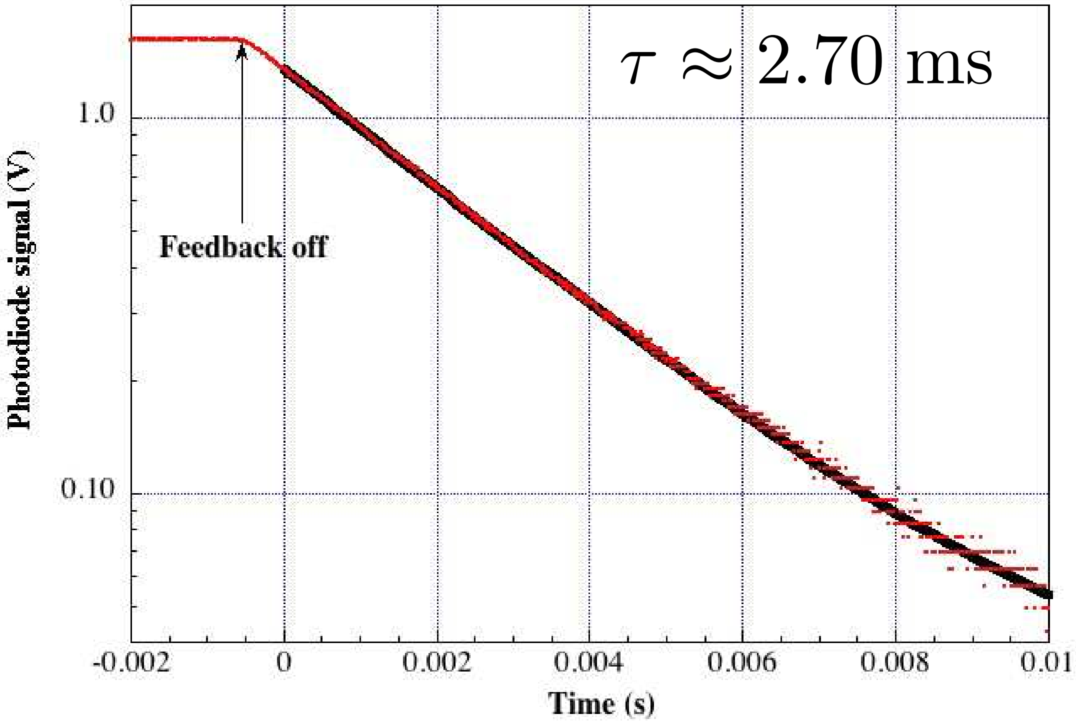


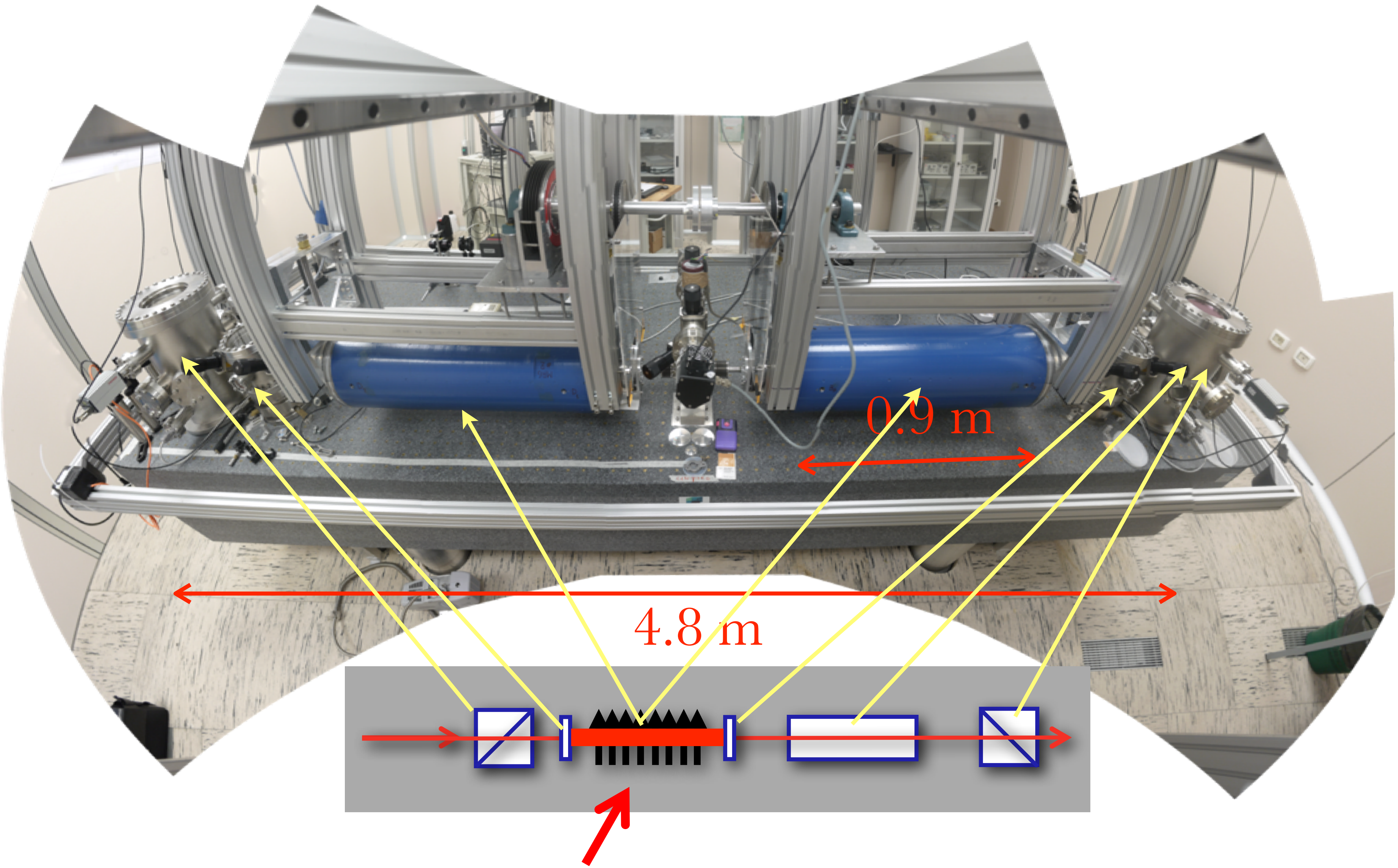

Fig. 3. Decay of the light transmitted from the cavity after switching off the laser frequency locking system. The decay is fitted with the exponential function $a + be^{-t/\tau_d}$, and gives for the decay time $\tau_d = 2.70 \pm 0.02$ ms.

$$\text{finesse} \approx 770000$$

$$c\tau \approx 810 \text{ km}$$

$$N \approx 490000$$

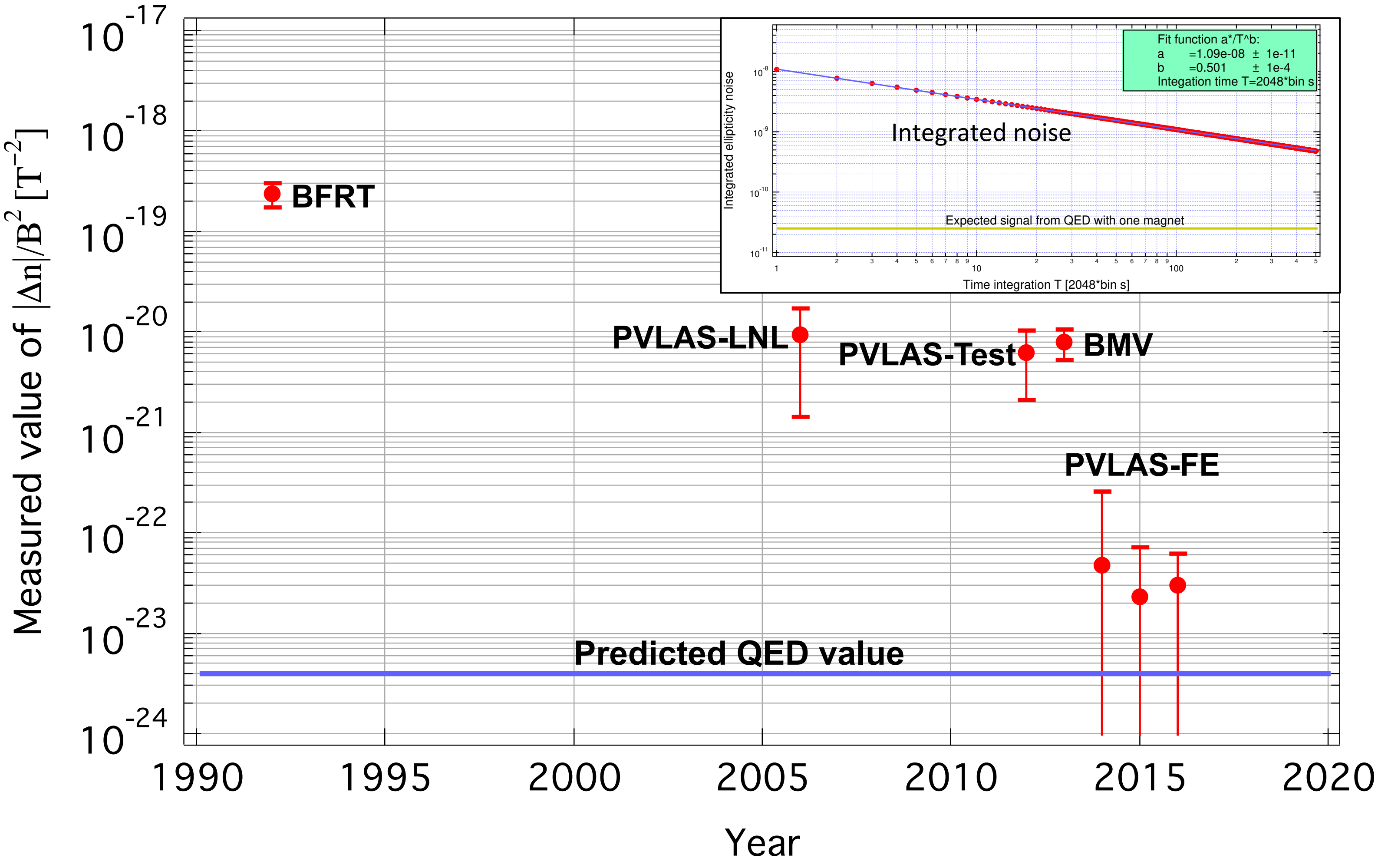
The heart of the PVLAS apparatus is the delicate Fabry-Perot interferometer.




The FP interferometer is here

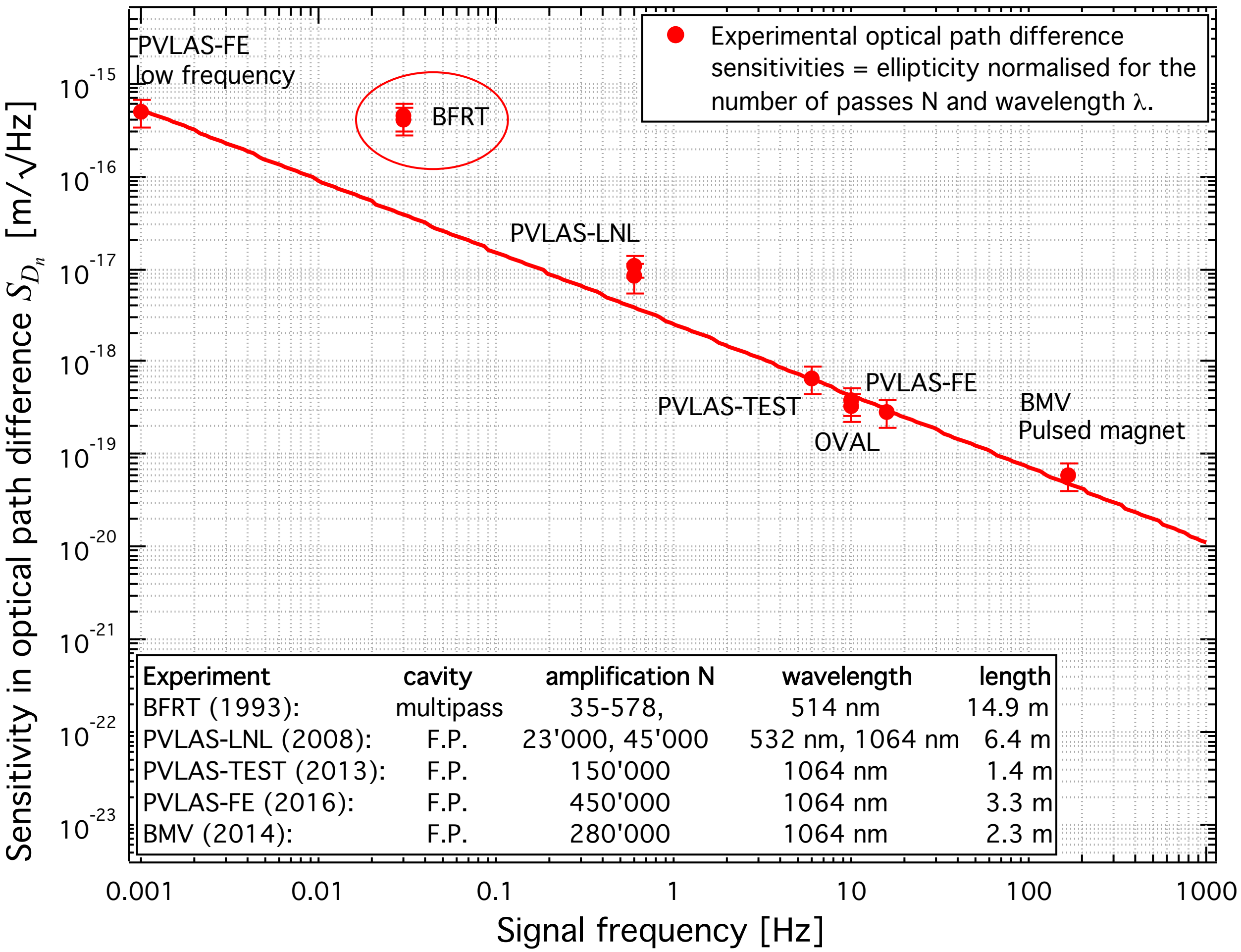
The PVLAS experiment has **record breaking FP resonator, and has achieved the best bounds on the magnetic birefringence of vacuum**

$$\Delta n = (-1.5 \pm 3.0) \times 10^{-22}$$




adapted from the slides presented by G. Zavattini at Physics Beyond Colliders, CERN, Nov. 21-22 2017

## Sensitivity in optical path difference between two perpendicular polarisations




**Extrapolating this to the field produced by LHC magnets and using a fast modulation method, one finds that vacuum polarization could be detected in about one day**

adapted from the slides presented by G. Zavattini at Physics Beyond Colliders, CERN, Nov. 21-22 2017

When the IRIDE project was first proposed, the first detection of the photon-photon elastic scattering process seemed to make a strong scientific case, but **if** very-low-energy, optical experiments successfully carry out the first detection of the birefringence of vacuum in a laboratory environment, and have the additional potential of discovering low-mass axion-like particles, **what is left for a gamma-gamma low-energy collider to do** (no first detection, not suited for very light particles)?

And do not forget the LHC/Atlas detection and the astronomical measurements of the Padova group ...

**It is important to note that the two approaches are not really equivalent, first, a gamma-gamma collider would provide a QED test with photons that are all real (two virtual photons in optical measurements), moreover, in the Lagrangian approach the degrees of freedom of the electronic field disappear and give rise to nonlinerity, so the Lagrangian approach is not suited to studies that involve the electronic field properties.**

nature photonics

LETTERS

PUBLISHED ONLINE: 18 MAY 2014 | DOI: 10.1038/NPHOTON.2014.95

# A photon–photon collider in a vacuum hohlraum

O. J. Pike[1]*, F. Mackenroth[1,2], E. G. Hill[1] and S. J. Rose[1]

**The ability to create matter from light is amongst the most striking predictions of quantum electrodynamics. Experimental signatures of this have been reported in the scattering of ultra-relativistic electron beams with laser beams[1,2], intense laser-plasma interactions[3] and laser-driven solid target scattering[4]. However, all such routes involve massive particles. The simplest mechanism by which pure light can be transformed into matter, Breit–Wheeler pair production ($\gamma\gamma' \rightarrow e^+e^-$)[5], has never been observed in the laboratory. Here, we present the design of a new class of photon–photon collider in which a gamma-ray beam is fired into the high-temperature radiation field of a laser-heated hohlraum. Matching experimental parameters to current-generation facilities, Monte Carlo simulations suggest that this scheme is capable of producing of the order of $10^5$ Breit–Wheeler pairs in a single shot. This would provide the first realization of a pure photon–photon collider, representing the advent of a new type of high-energy physics experiment.**

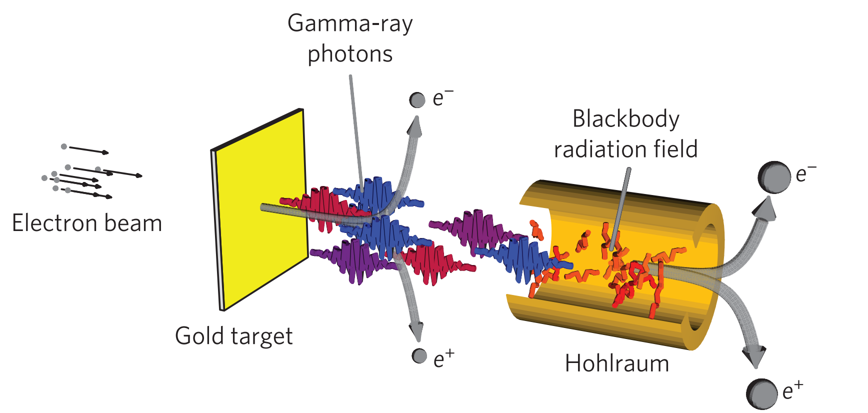
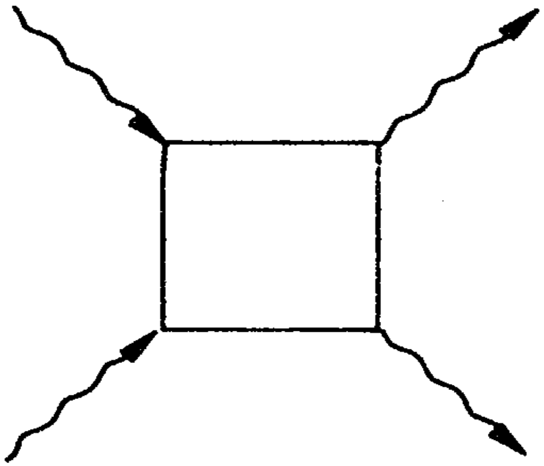
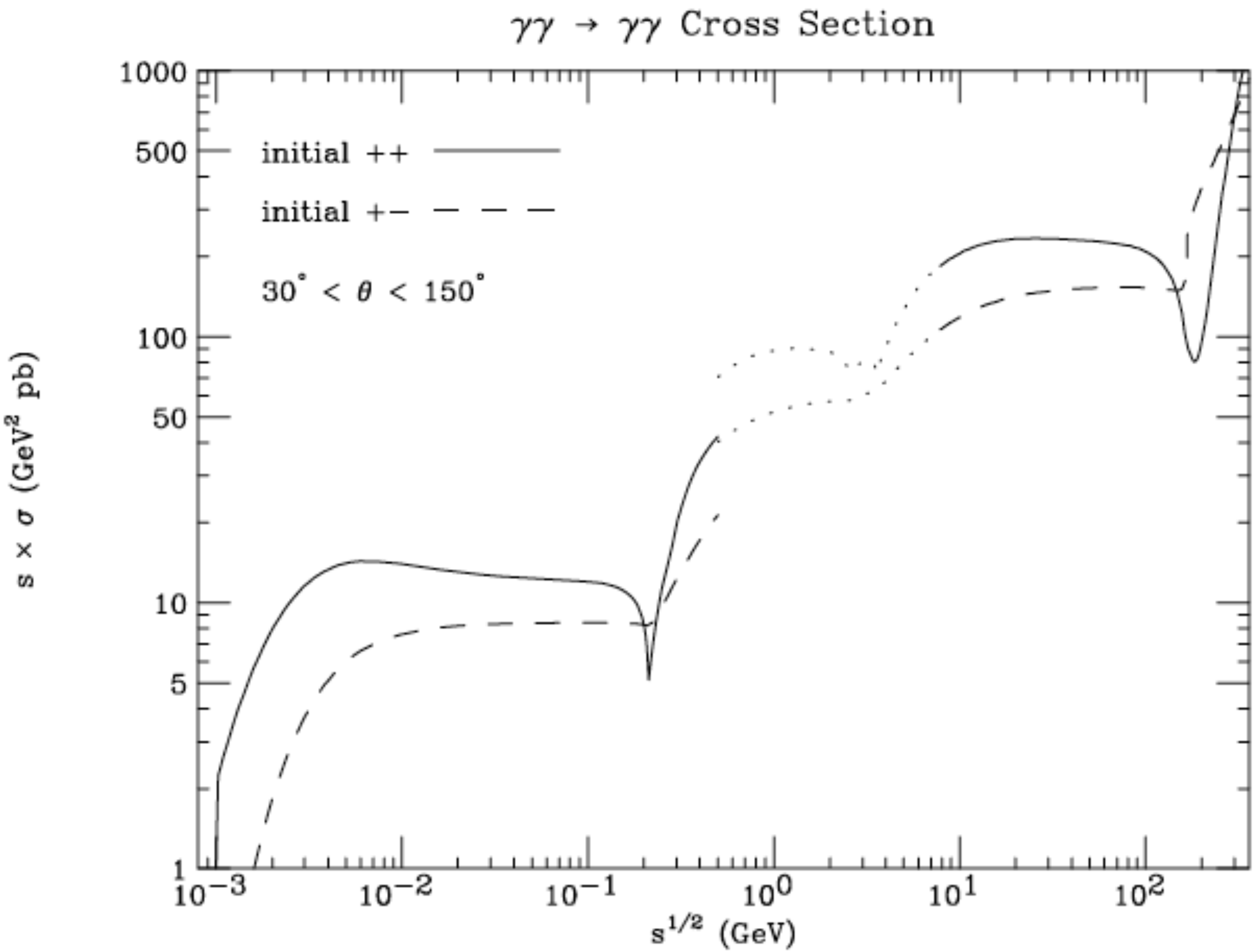



**Figure 1 | Schematic of the photon–photon collider.** Bremsstrahlung emission of ultra-relativistic electrons passing through a solid gold target is used to create a high-energy photon beam. This is fired into a vacuum hohlraum, where it interacts with a high-temperature thermal radiation field. Electrons and positrons emerging from the back surface of the gold target are deflected away from the hohlraum using a magnetic field. Breit–Wheeler pairs produced in the hohlraum are both narrowly collimated and highly energetic (Supplementary Fig. 2). On exiting the hohlraum, the positrons may be separated using a magnetic field and detected using, for example, Čerenkov radiation. The electron beam is generated and the hohlraum heated using high-intensity and high-energy lasers, respectively.



$\gamma\gamma \to \gamma\gamma$ **scattering**

- **1935**: Euler and Kochel provide a first general formula for the photon-photon scattering cross-section also for energies lower than the $2m_e$ threshold;
- **1936**: Euler provides the details of the cross-section formula (work done by Euler for his PhD thesis in Leipzig);
- **1936-37**: Akhiezer, Landau and Pomerancuk generalize the cross-section formula to high energies;
- **1950-51**: Karplus and Neuman carry out a thorough analysis using Feynman diagrams;
- **1964-65**: DeTollis utilizes dispersion relation techniques to give compact formulas for the scattering amplitudes;

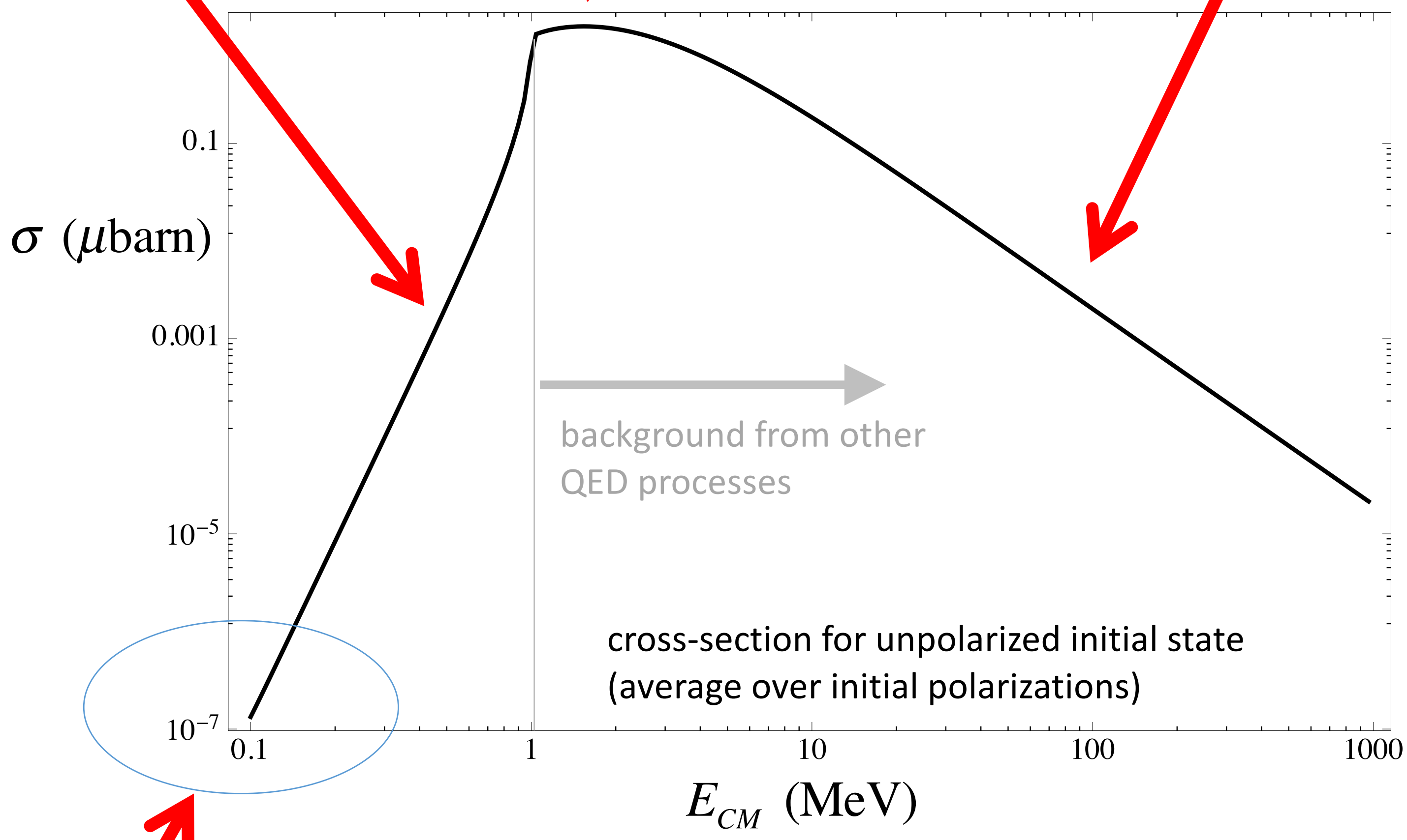



from Bern & al., JHEP 11 (2001) 031

$$\sigma \approx 0.13\left(\frac{\hbar\omega}{m_e c^2}\right)^6 \mu\text{barn}$$

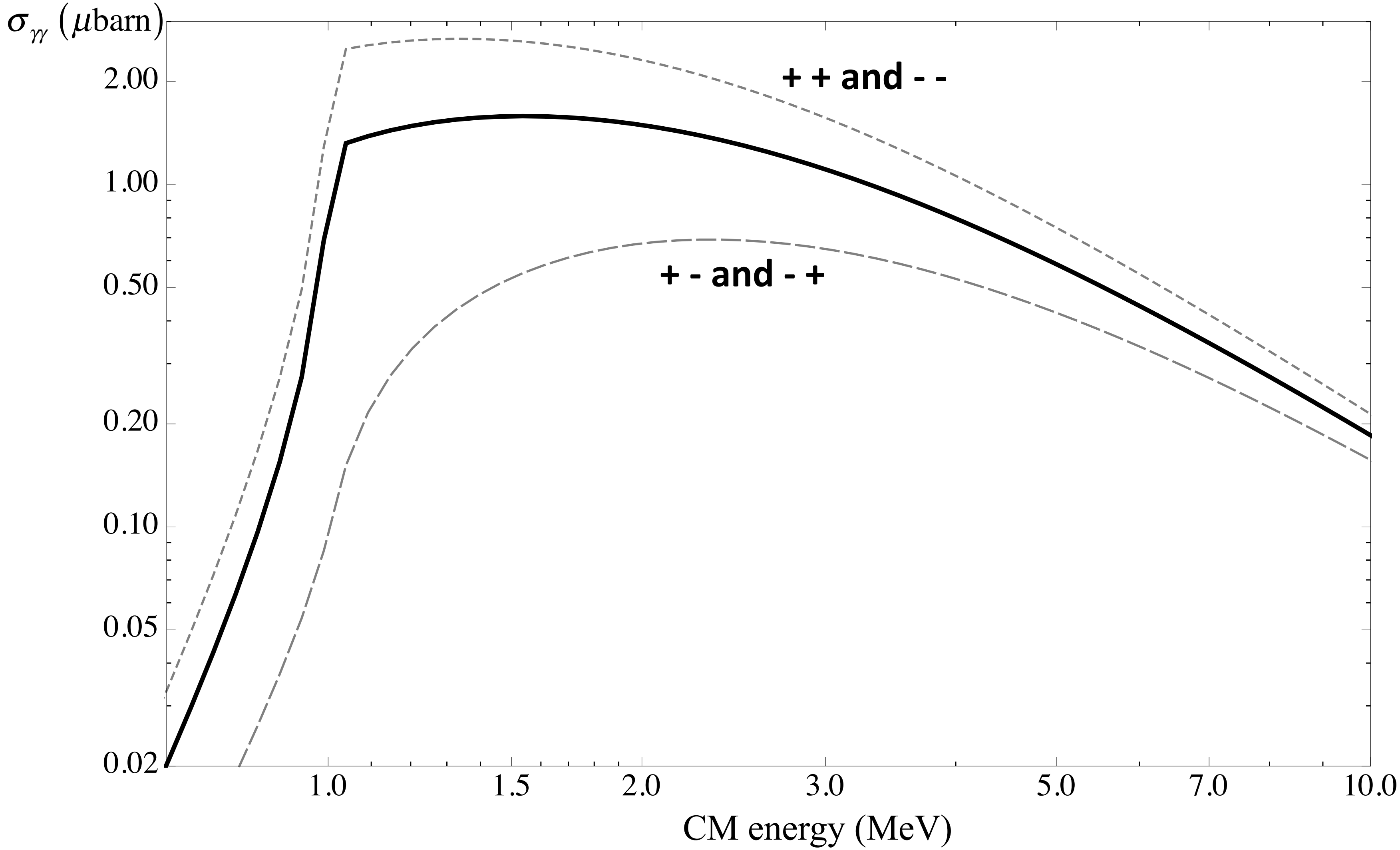
peak cross section, ≈1.6 µbarn at $\hbar\omega \approx 1.5 m_e c^2$

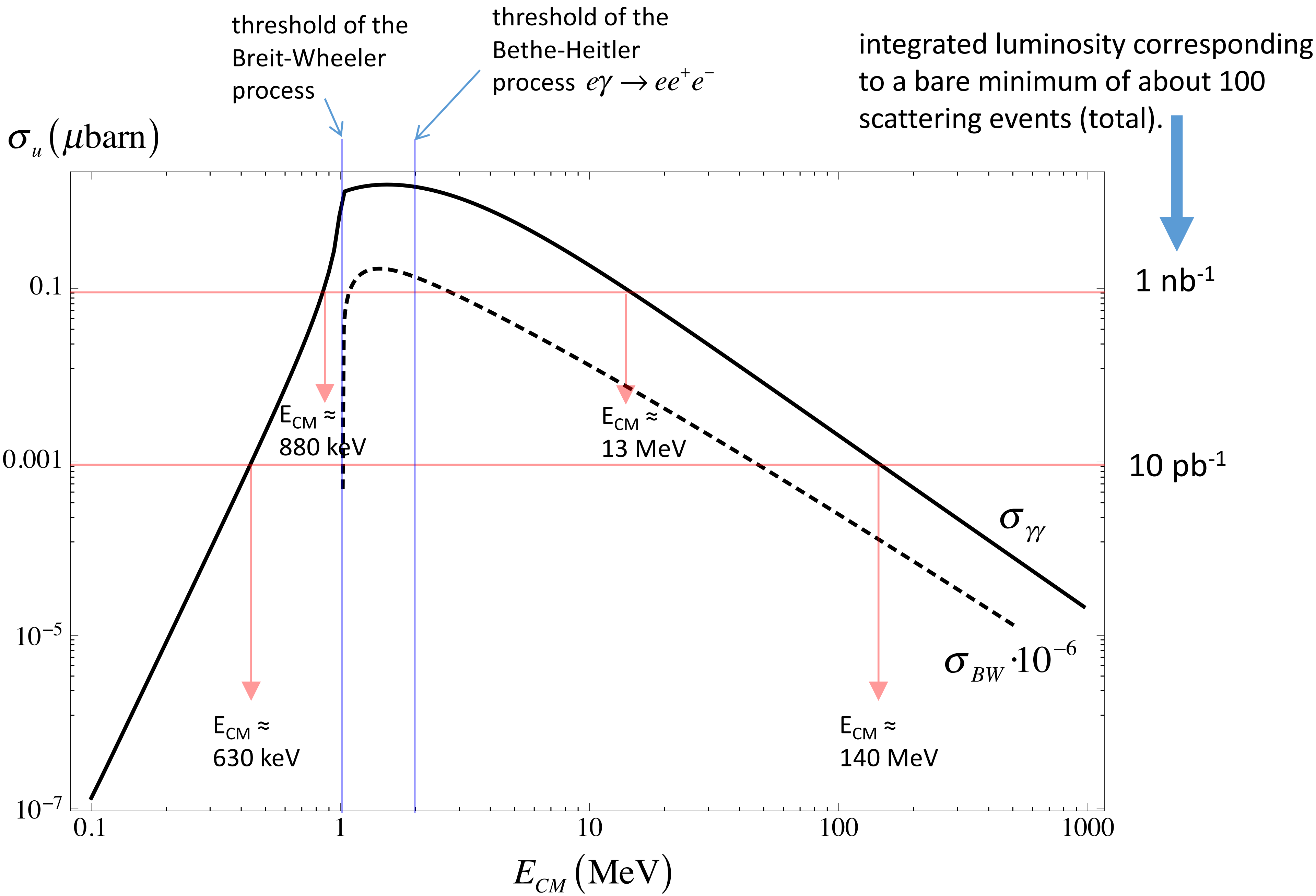
$$\sigma \approx 20\left(\frac{m_e c^2}{\hbar\omega}\right)^2 \mu\text{barn}$$




very low energy region: cross section is extremely small, but photon numbers can be very large, there is no background from other QED processes

## Unpolarized and (circularly) polarized initial photons




threshold of the
Breit-Wheeler
process
threshold of the
Bethe-Heitler
process $e\gamma \to ee^+e^-$
integrated luminosity corresponding
to a bare minimum of about 100
scattering events (total).
$\sigma_u\,(\mu\text{barn})$
0.1
0.001
$10^{-5}$
$10^{-7}$
1 $\text{nb}^{-1}$
10 $\text{pb}^{-1}$
$E_{CM} \approx$
880 keV
$E_{CM} \approx$
13 MeV
$E_{CM} \approx$
630 keV
$E_{CM} \approx$
140 MeV
$\sigma_{\gamma\gamma}$
$\sigma_{BW} \cdot 10^{-6}$
0.1
1
10
100
1000
$E_{CM}\,(\text{MeV})$

# Predicted positions of resonances close to the $e^+e^-$ production threshold

(Darewych & al., PRD 45 (1992) 675 )

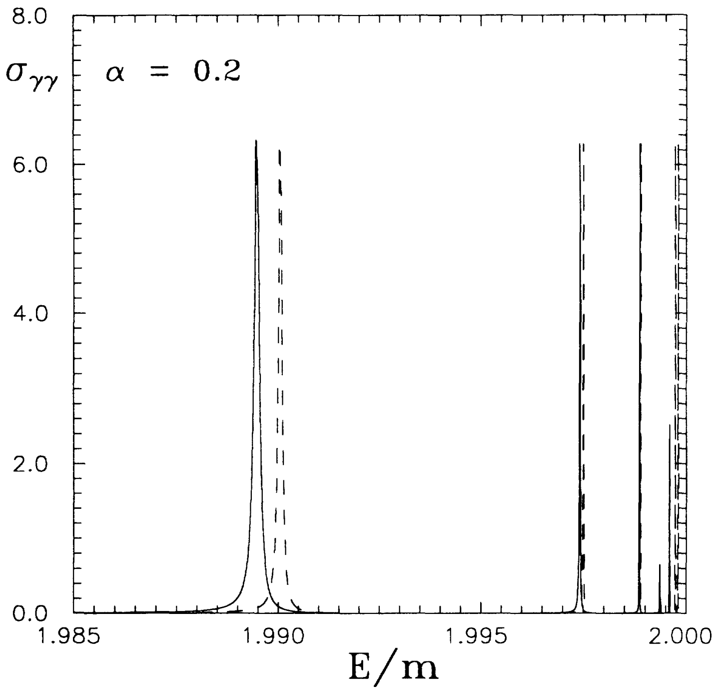



**FIG. 1.** Photon-photon scattering cross section in units of $1/m^2$ as a function of total energy for a QED coupling constant of $\alpha=0.2$. Solid line: calculated with kernel (30), dashed line: no transverse-photon exchange in Eq. (29) included; see text.

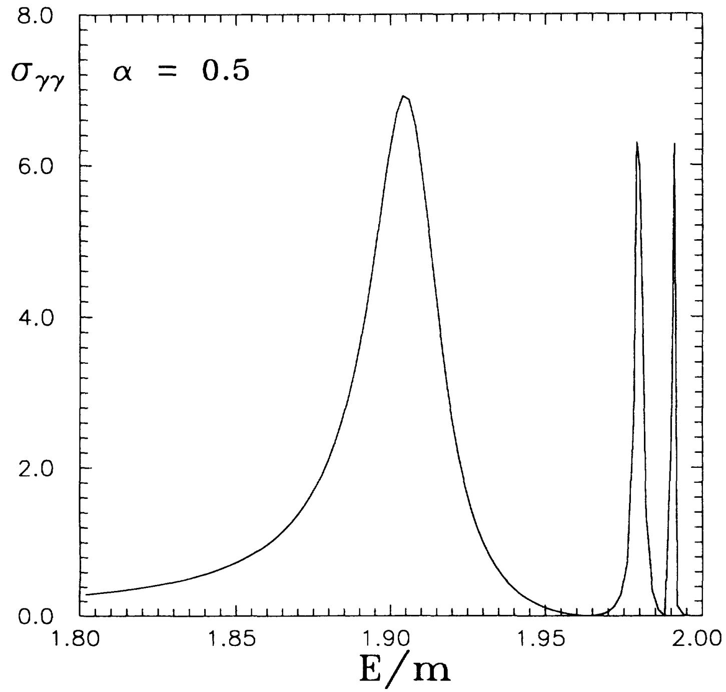



**FIG. 2.** Same as in Fig. 1 for $\alpha=0.5$. The strong deviation in the ground-state resonance position from the nonrelativistic value of $E/m=2-\alpha^2/4=1.9375$ is obvious.

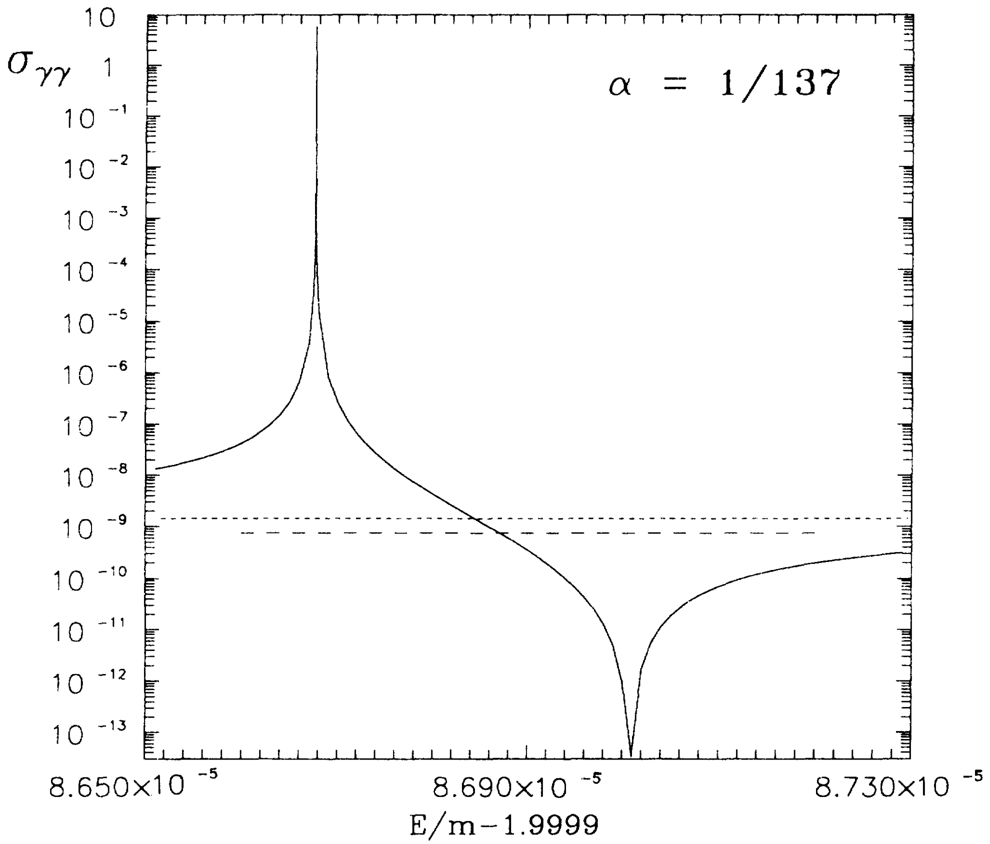



FIG. 4. Photon-photon scattering cross section for $\alpha = 1/137$ in the vicinity of the ground-state resonance. Solid line: present result with kernel (30), short dashed line: present result with kernel set equal to zero (no photon exchanges in virtual $e^+e^-$ channel), long dashed line: perturbative result [3]. On the energy scale 1.9999 has been subtracted to facilitate the display of labels.

Peak resonant cross-section

$$\sigma_{\gamma\gamma} \approx 10^4 \mathrm{barn}$$

Peak is extremely narrow, but the peak value is about **10 orders of magnitude larger than the peak nonresonant cross-section**

The positron resonances are extremely narrow, however:

- resonant amplification is extremely large

- Compton backscattering offers the possibility of using optical methods in a particle physics context. For example, by modulating the laser beam energy, then the backscattered photon is energy-modulated as well $\hbar\omega_{\mathrm{BS}} \approx 4\gamma^2\hbar\omega$

  Beam energy spread would act as an effective noise: maybe laser modulation could help detection.

- Access to positronium resonances could also mean access to the physics of the “mirror world” (Lee and Yang, 1956), i.e., to the world of “millicharged particles” and “dark photons”.

# My conclusions

a low-energy gamma-gamma collider in the MeV region may not be able to achieve the first laboratory detection of photon-photon elastic scattering, but

- it could actually produce e+e- pairs with the Breit–Wheeler process
- it could achieve the first ever lab detection of photon-photon elastic scattering with photons that are all real
- it would have the potential to study higher order in-loop corrections (inaccessible to effective Lagrangian methods and probably also to optical experiments)
- a gamma-gamma collider with polarized beams has the potential to study Born-Infeld corrections to photon-photon elastic scattering
- it could open up a new field of study of positron resonances, if
    - beam energy spread is sufficiently small
    - optical modulation helps in detection/measurements
- the Ps system has long been studied because of the – now vanished – orthopositronium decay time anomaly, however it could still be an interesting playground for searches related to the "mirror world"

On the whole, the long-term usefulness of such a machine (for fundamental physics) lies on precision ...