

CERN Summer School 2010, Raseborg, Finland

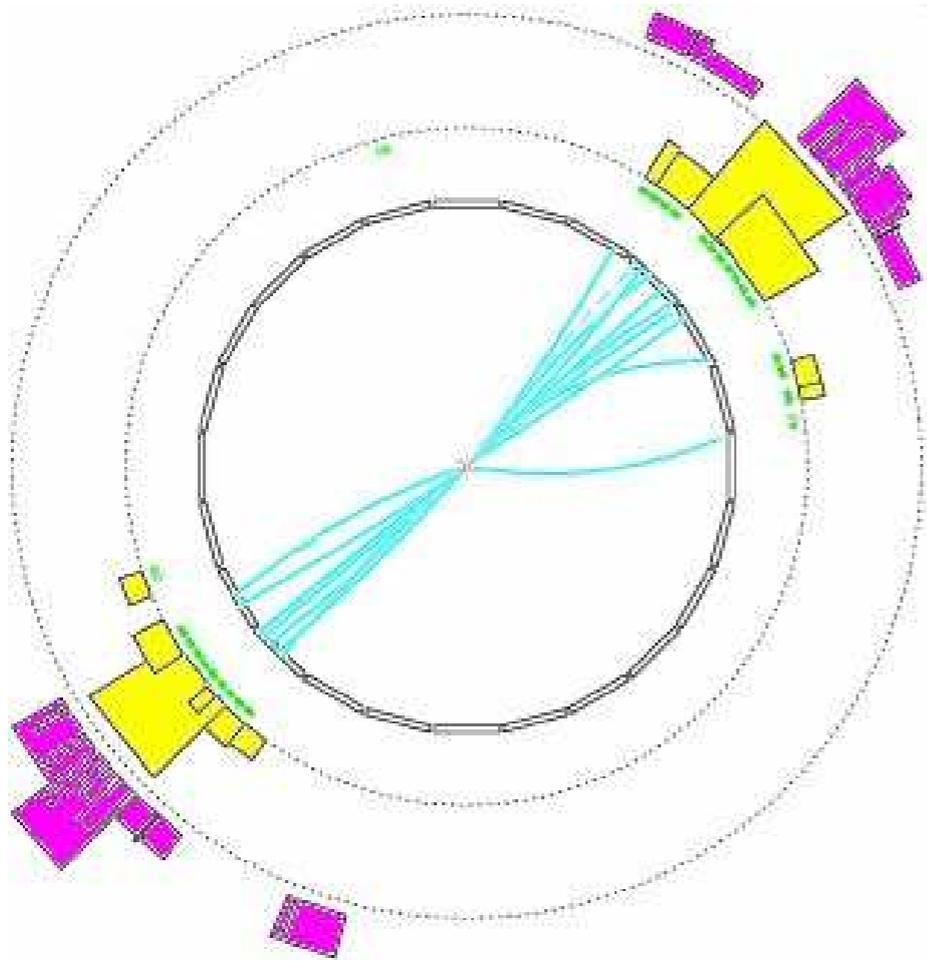
QCD

Lecture 5

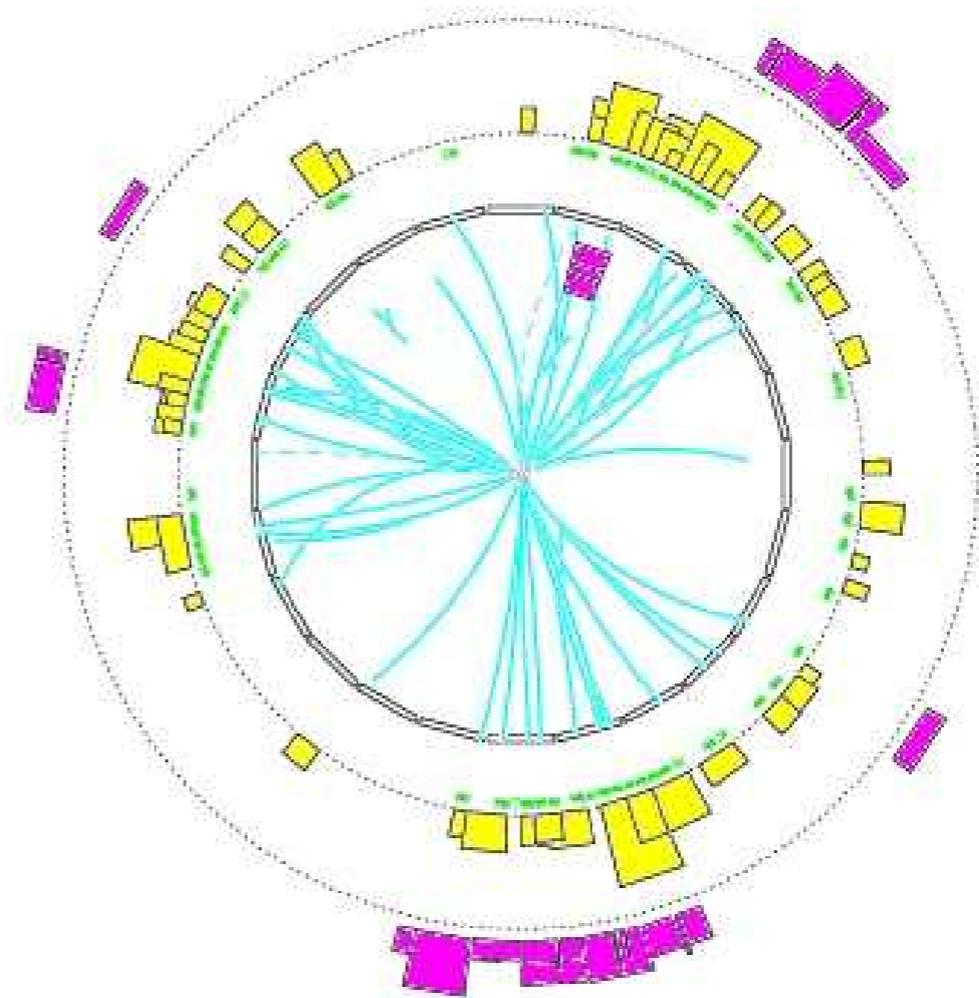
Jets and Matching

P. Skands

"Seeing" vs Defining Jets

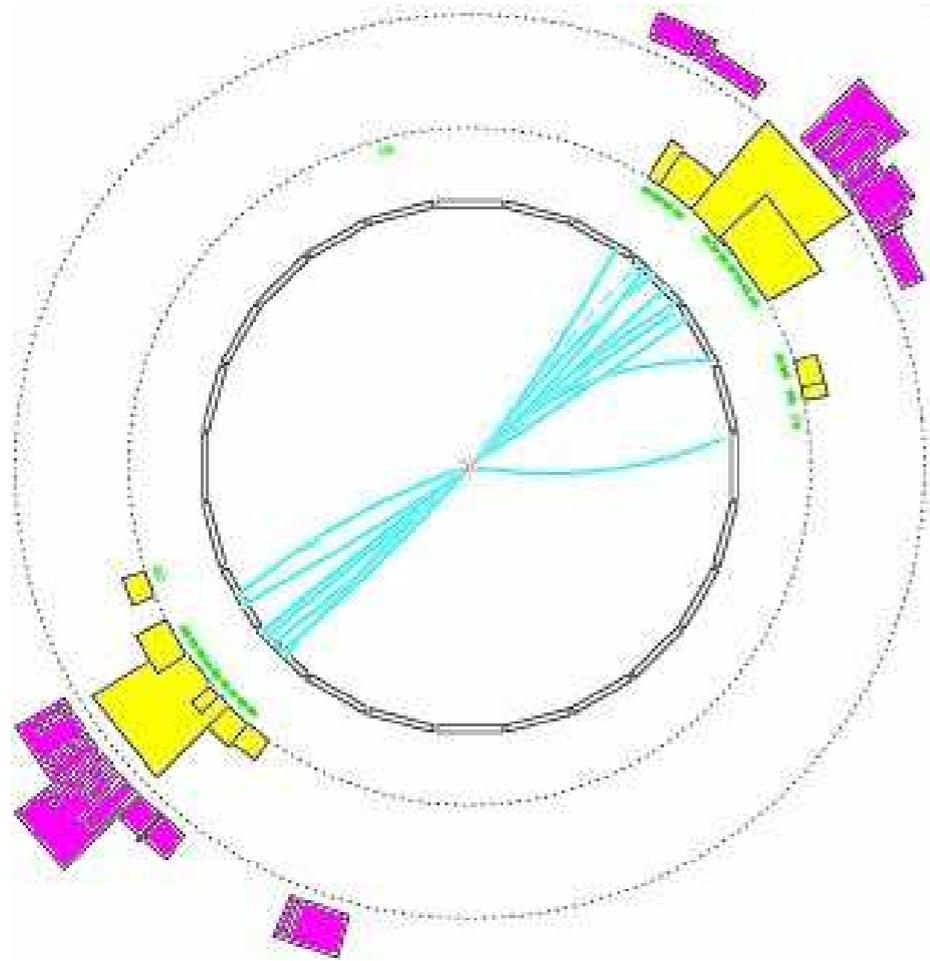


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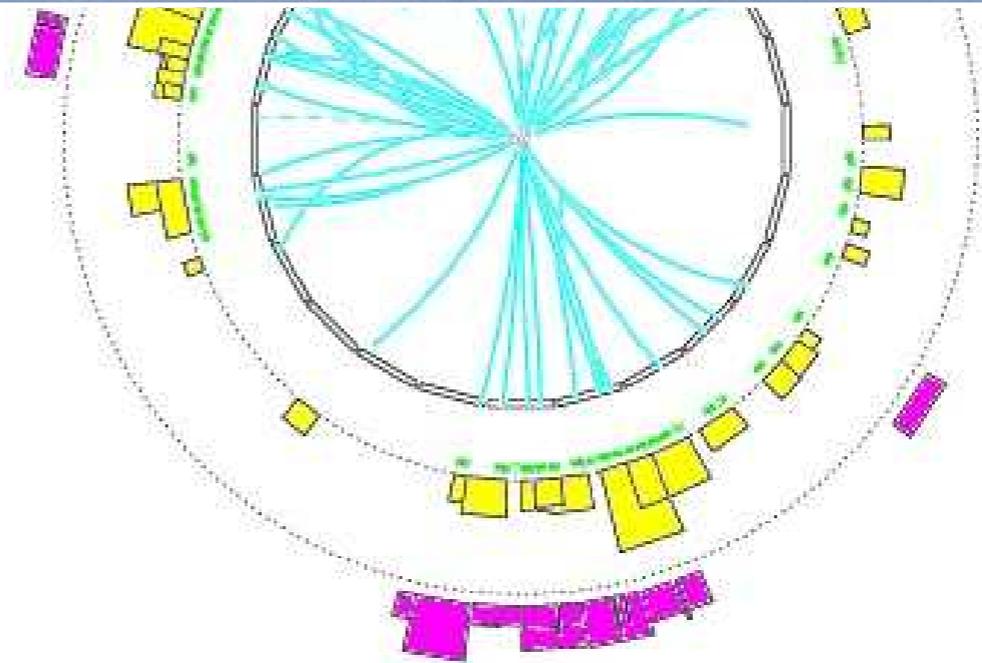
How many jets can you see?

"Seeing" vs Defining Jets



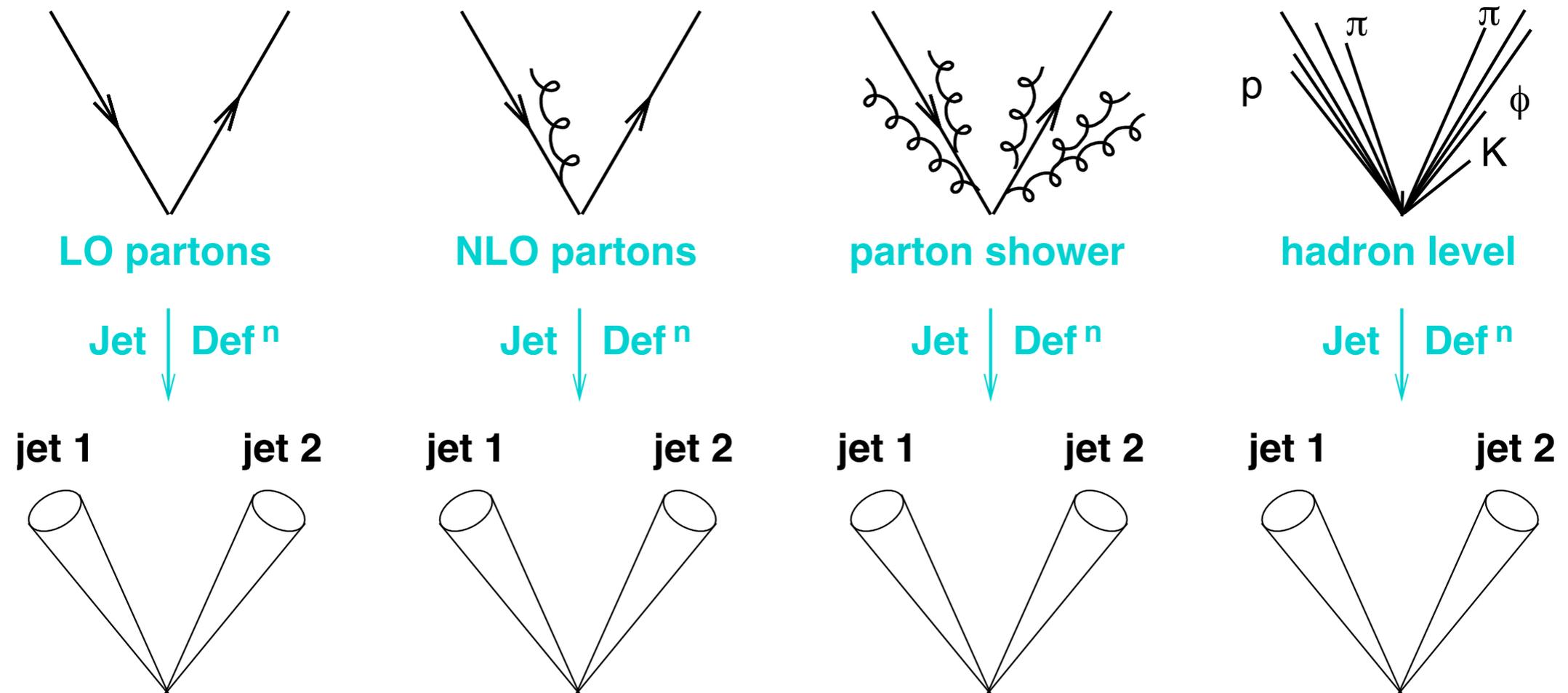
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Do you really want to ask yourself this for 10^9 events?



How many jets can you see?

Jets as Projections



Projections to jets provides a universal view of event

Illustrations by G. Salam

There is no unique or “best” jet definition

YOU decide how to look at event

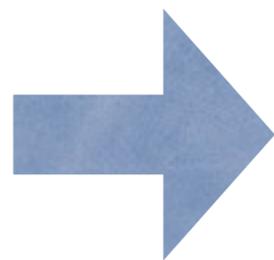
The construction of jets is inherently ambiguous

1. Which particles get grouped together?

JET ALGORITHM (+ parameters)

2. How will you combine their momenta?

RECOMBINATION SCHEME (e.g., 'E' scheme: add 4-momenta)



Jet Definition

Ambiguity complicates life, but gives flexibility
in one's view of events → Jets non-trivial!

Types of Algorithms

1. Sequential Recombination

Take your 4-vectors

Combine the vectors that have the lowest 'distance measure'

Different names for different distance measures

Durham k_T : $\min(k_{Ti}^2, k_{Tj}^2) \times \Delta R_{ij}^2$

[$k_{Ti}^2 = E_i^2(1 - \cos\theta_{ij})$] (+ beam treated as non-emitting)

Cambridge/Aachen : ΔR_{ij}^2

Anti- k_T : $\Delta R_{ij}^2 / \max(k_{Ti}^2, k_{Tj}^2)$

ArClus : $p_T^2 = s_{ij}s_{jk}/s$ [NB: ARCLUS is 3→2 instead of 2→1 ⇒ can keep all partons on shell, but more possibilities to try]

→ Now you have a new set of (n-1) 4-vectors

Iterate until A or B (you choose which):

A: all distance measures larger than something

B: you reach a specified number of jets

Look at event at:

specific resolution

specific n_{jets}

Why k_T (or p_T or ΔR)?

$$k_{Tij}^2 = E_i^2(1 - \cos\theta_{ij}) \quad p_{Tj}^2 = s_{ij}s_{jk}/s_{ijk} \quad (\text{note: there are also other } p_T \text{ defs})$$

Attempt to (approximately) capture universal jet-within-jet-within-jet... behaviour

Approximate full matrix element

$$\frac{|M_{X+1}^{(0)}(s_{i1}, s_{1k}, s)|^2}{|M_X^{(0)}(s)|^2} \sim 4\pi\alpha_s C_F \left(\frac{2s_{ik}}{s_{i1}s_{1k}} + \dots \right)$$

"Eikonal"
(universal, always there)

by Leading-Log limit of QCD → universal dominant terms

$$\frac{ds_{i1}ds_{1k}}{s_{i1}s_{1k}} \rightarrow \frac{dp_{\perp}^2}{p_{\perp}^2} \frac{dz}{z(1-z)} \rightarrow \frac{dE_1}{\min(E_i, E_1)} \frac{d\theta_{i1}}{\theta_{i1}} \quad (E_1 \ll E_i, \theta_{i1} \ll 1) \dots$$

Rewritings in soft/collinear limits

"smallest" k_T (or p_T or θ_{ij} , or ...) → largest Eikonal

Types of Algorithms

2. "Cone" type

Motivated by idea of partons \approx "invariant" directed energy-flow (most of which ends up within a "cone")

Take your 4-vectors

Select a procedure for which "test cones" to draw

Different names for different procedures

Seeded : start from hardest 4-vectors (and possibly combinations thereof, e.g., CDF midpoint algo) = "seeds"

Unseeded : smoothly scan over entire event, trying everything

Sum momenta inside test cone \rightarrow new test cone direction

Iterate until stable (test cone direction = momentum sum direction)

Warning: seeded algorithms are INFRARED UNSAFE

Infrared Safety

Definition

An observable is infrared safe if it is insensitive to

SOFT radiation:

Adding any number of infinitely soft particles (zero-energy) should not change the value of the observable

COLLINEAR radiation:

Splitting an existing particle up into two comoving particles (conserving the total momentum and energy) should not change the value of the observable

(Not accidentally, these are the two singular limits we encountered before)

IR Safety

Theorem:

For all “IR Safe Observables”, hadronization corrections (non-perturbative corrections) are POWER SUPPRESSED

$$\text{IR Safe Corrections} \propto \frac{Q_{\text{IR}}^2}{Q_{\text{UV}}^2}$$

All “non-IR Safe Observables” receive logarithmically divergent pQCD corrections in the IR, which must be canceled by large hadronization corrections → more sensitive to UV→IR transition

$$\text{IR Sensitive Corrections} \propto \alpha_s^n \log^m \left(\frac{Q_{\text{UV}}^2}{Q_{\text{IR}}^2} \right), \quad m \leq 2n$$

IR Safety

Compare an IR safe and unsafe Jet

May look pretty similar in experimental environment

(proof that nature has no trouble canceling all divergencies, no matter what the observable)

So what's the trouble?

It's not nice to your theory friends ...

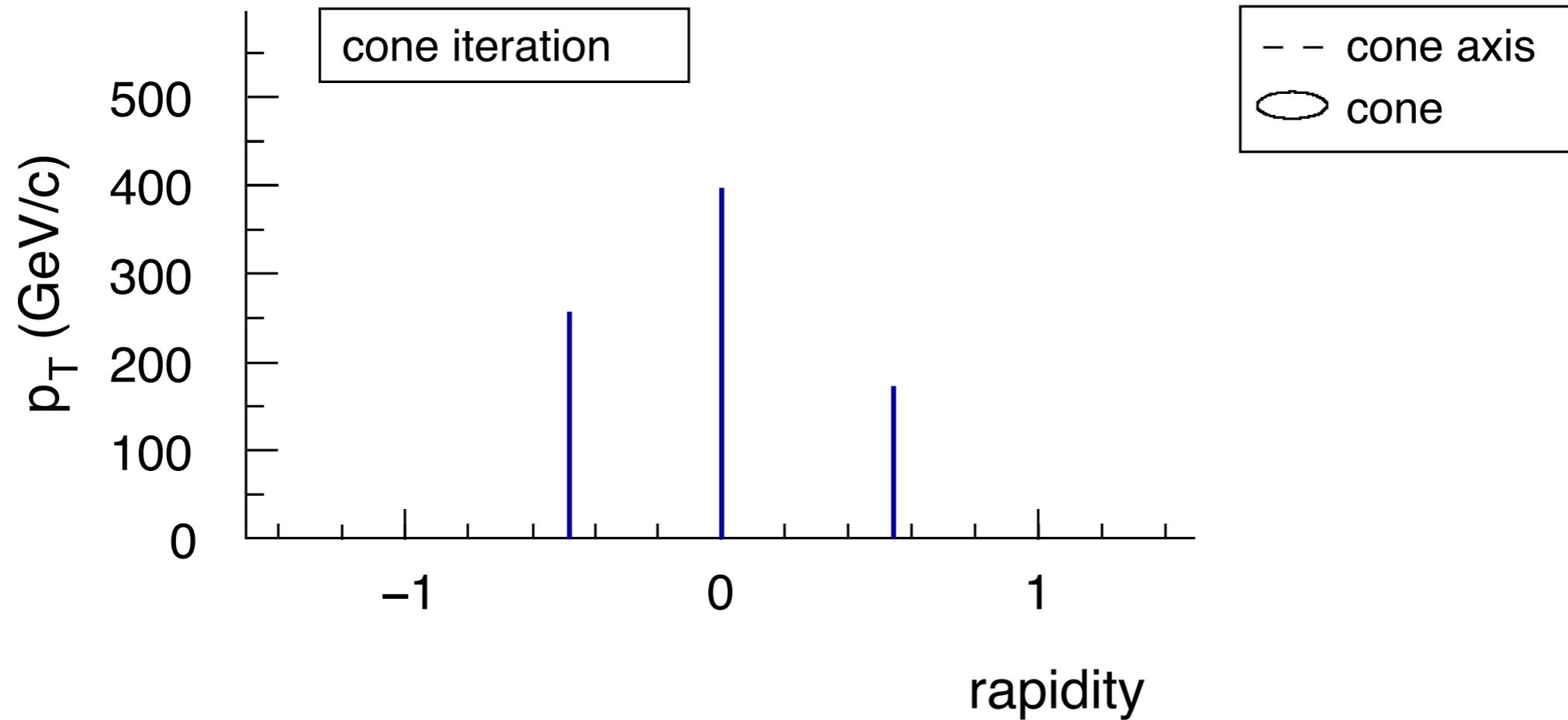
If they use a truncation of the theory (i.e., pQCD)

pQCD badly divergent if IR unsafe, but only power corrections if IR safe

Even if they have a hadronization model

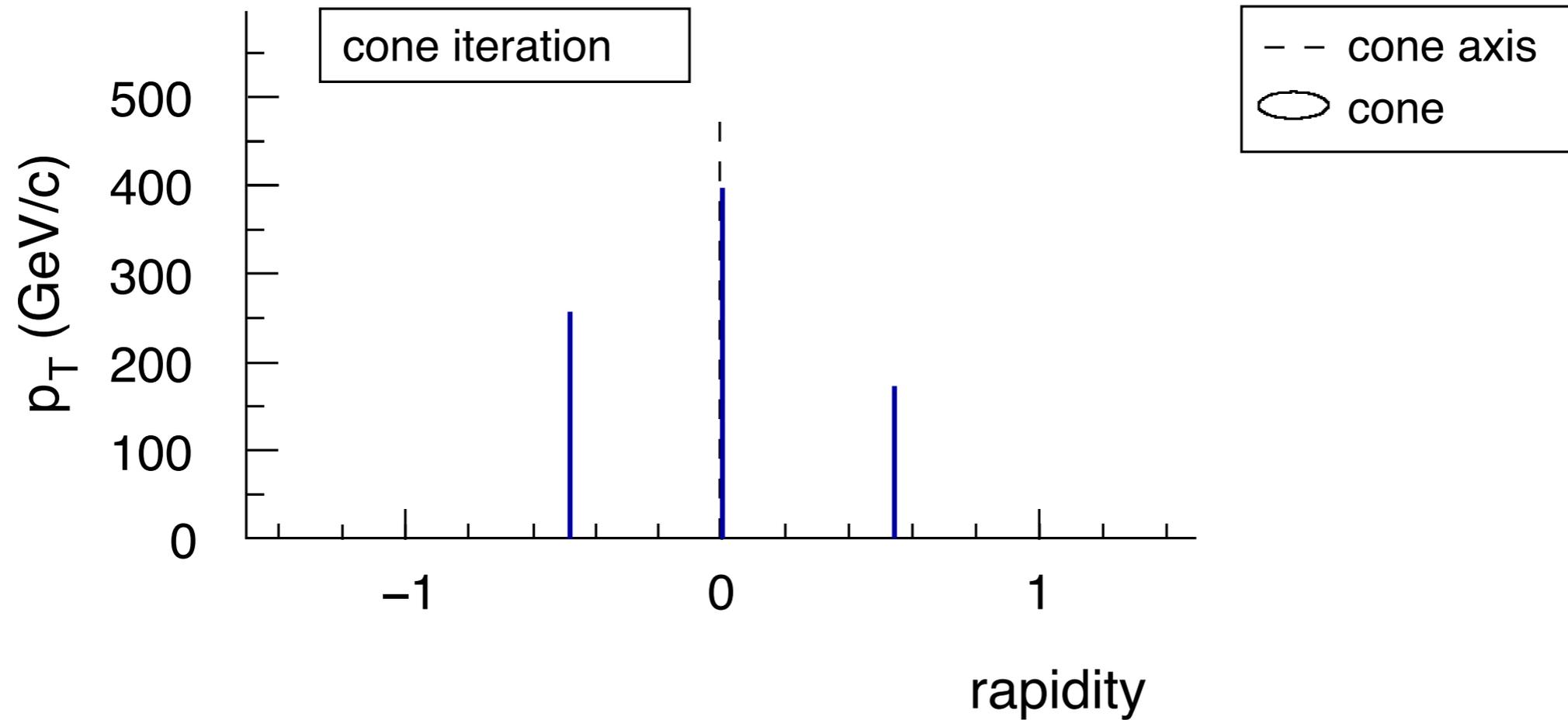
Dependence on hadronization model → larger uncertainty

ICPR iteration issue



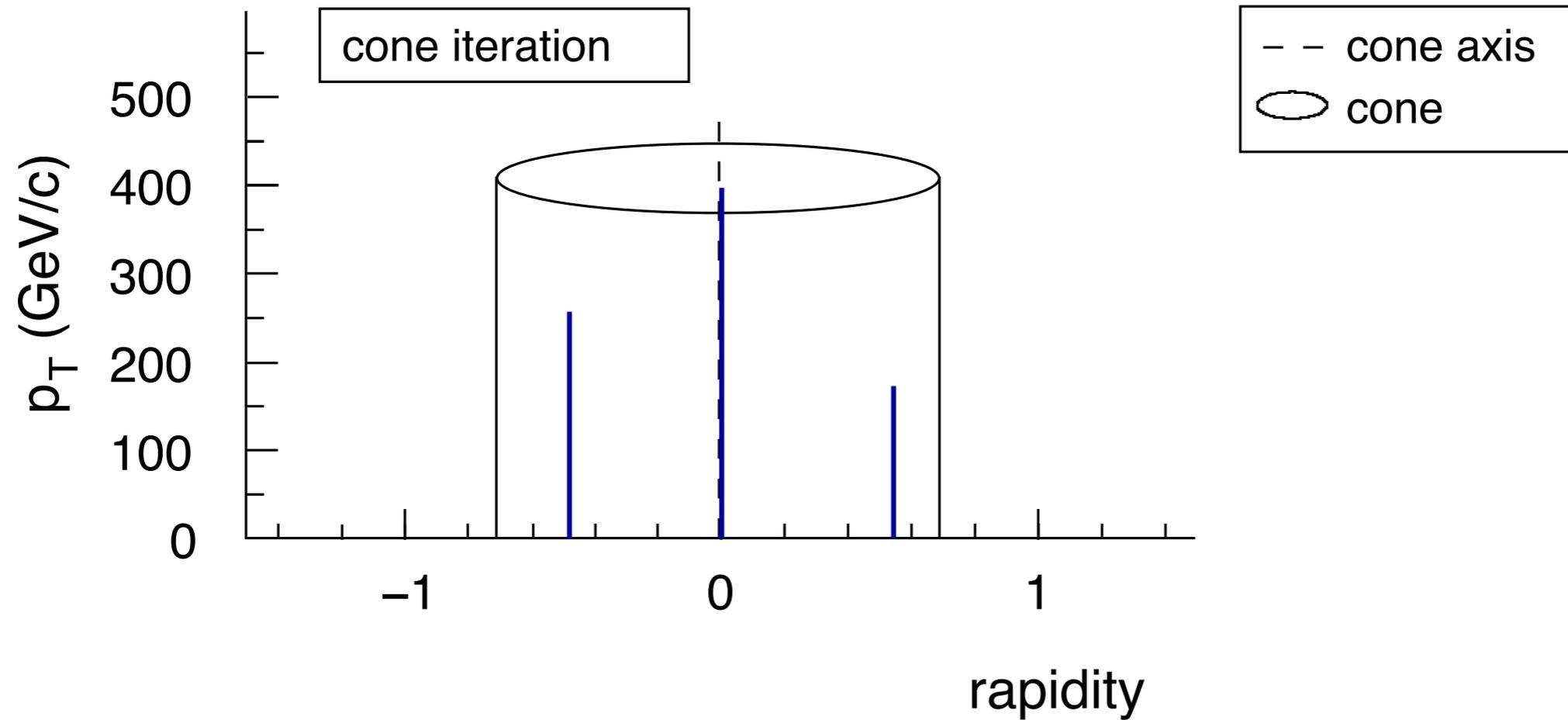
Collinear splitting can modify the hard jets: ICPR algorithms are collinear unsafe \implies perturbative calculations give ∞

ICPR iteration issue



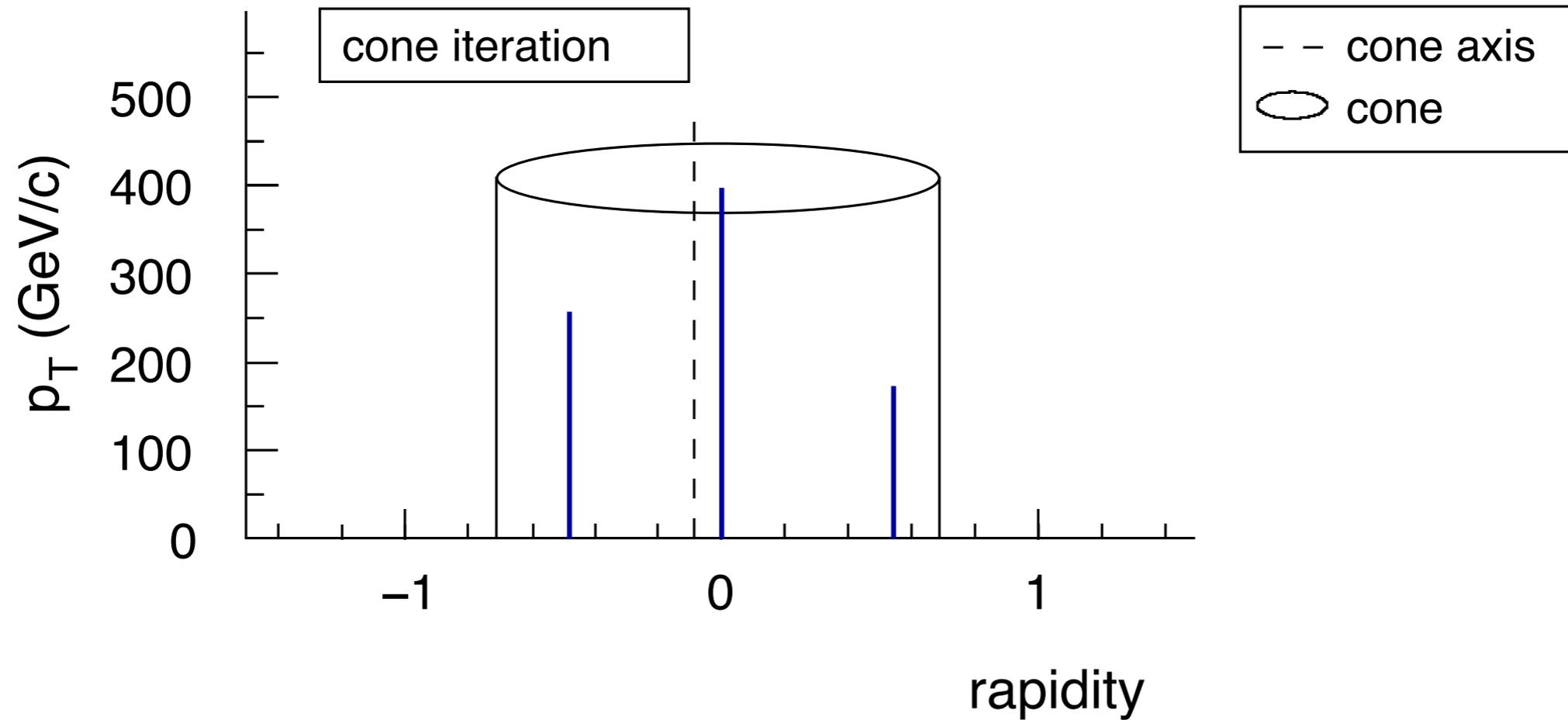
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ICPR iteration issue



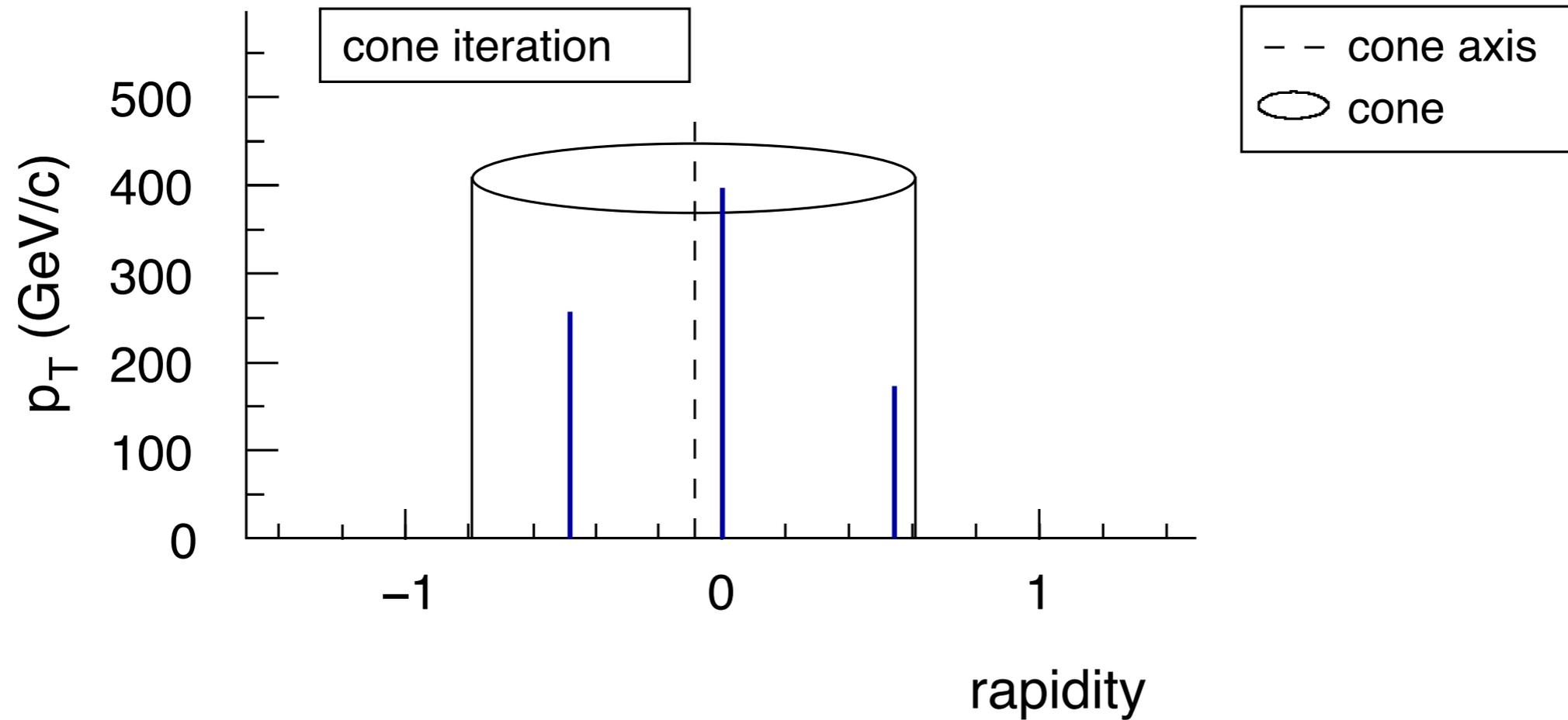
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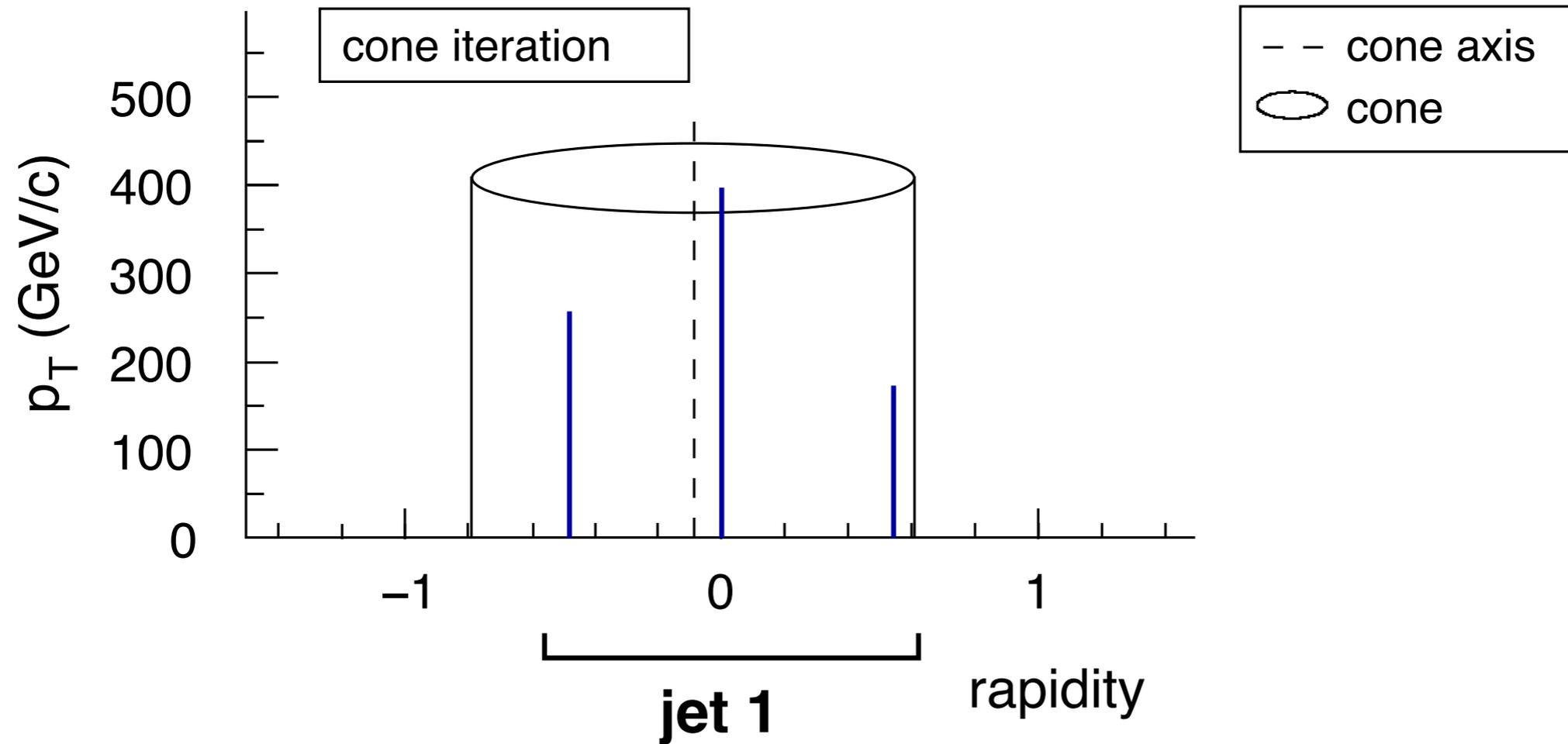
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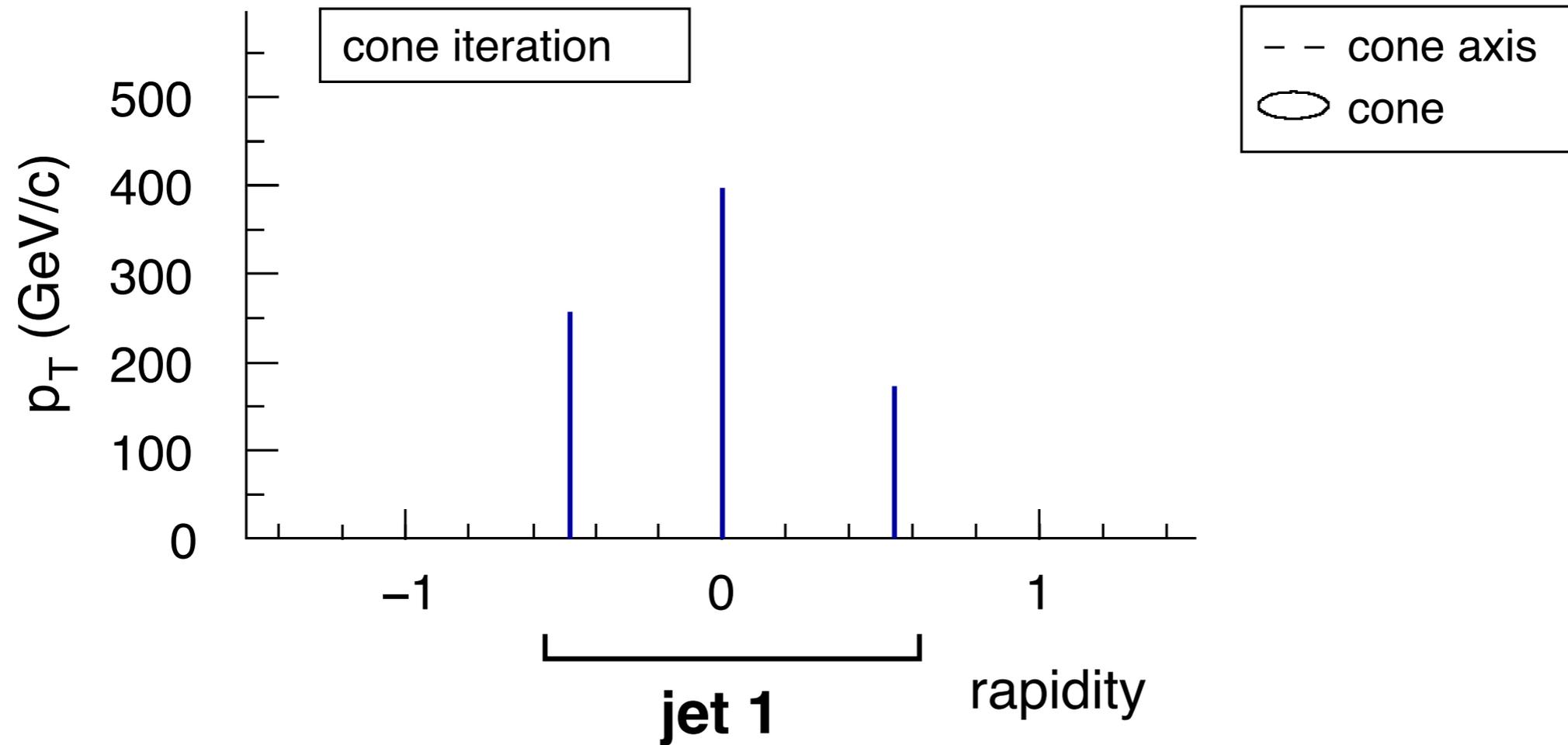
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ICPR iteration issue



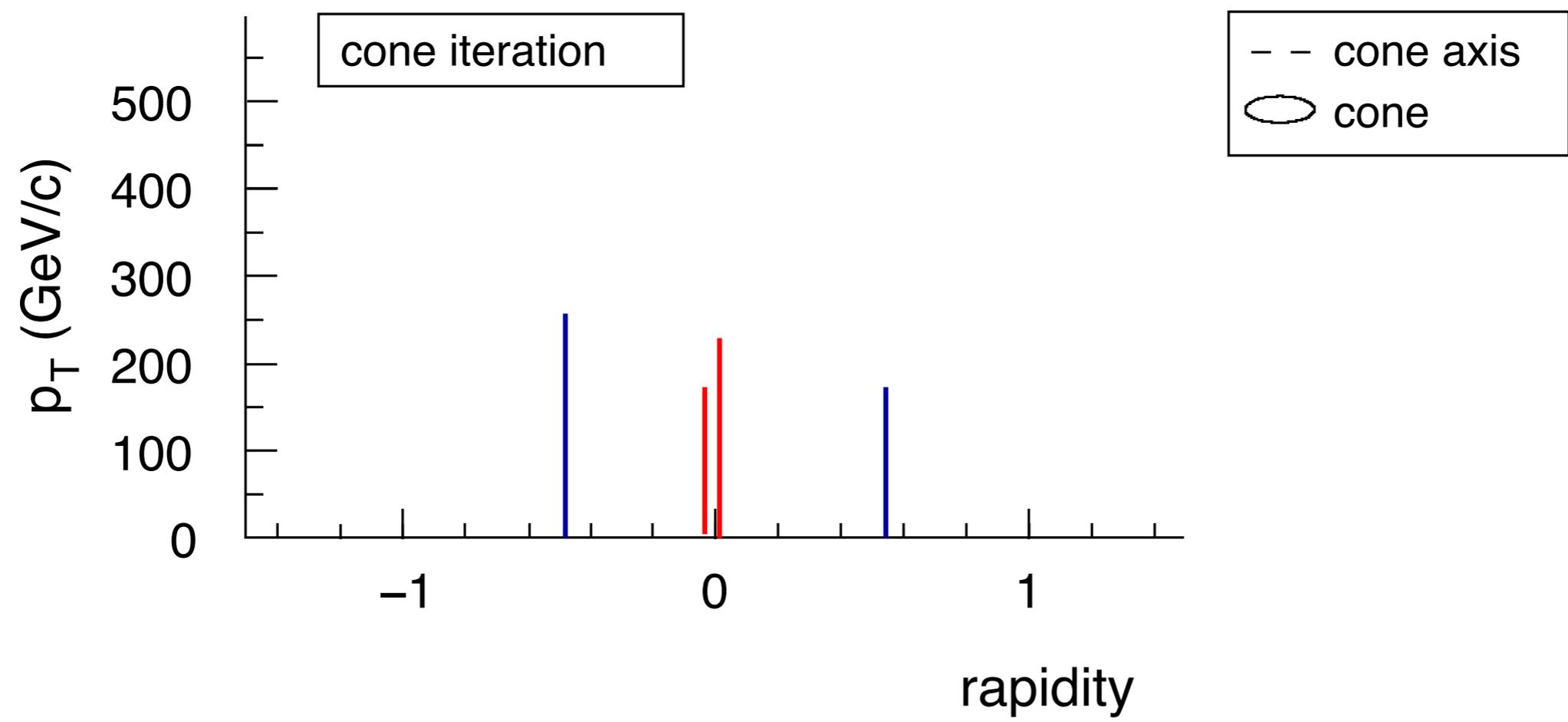
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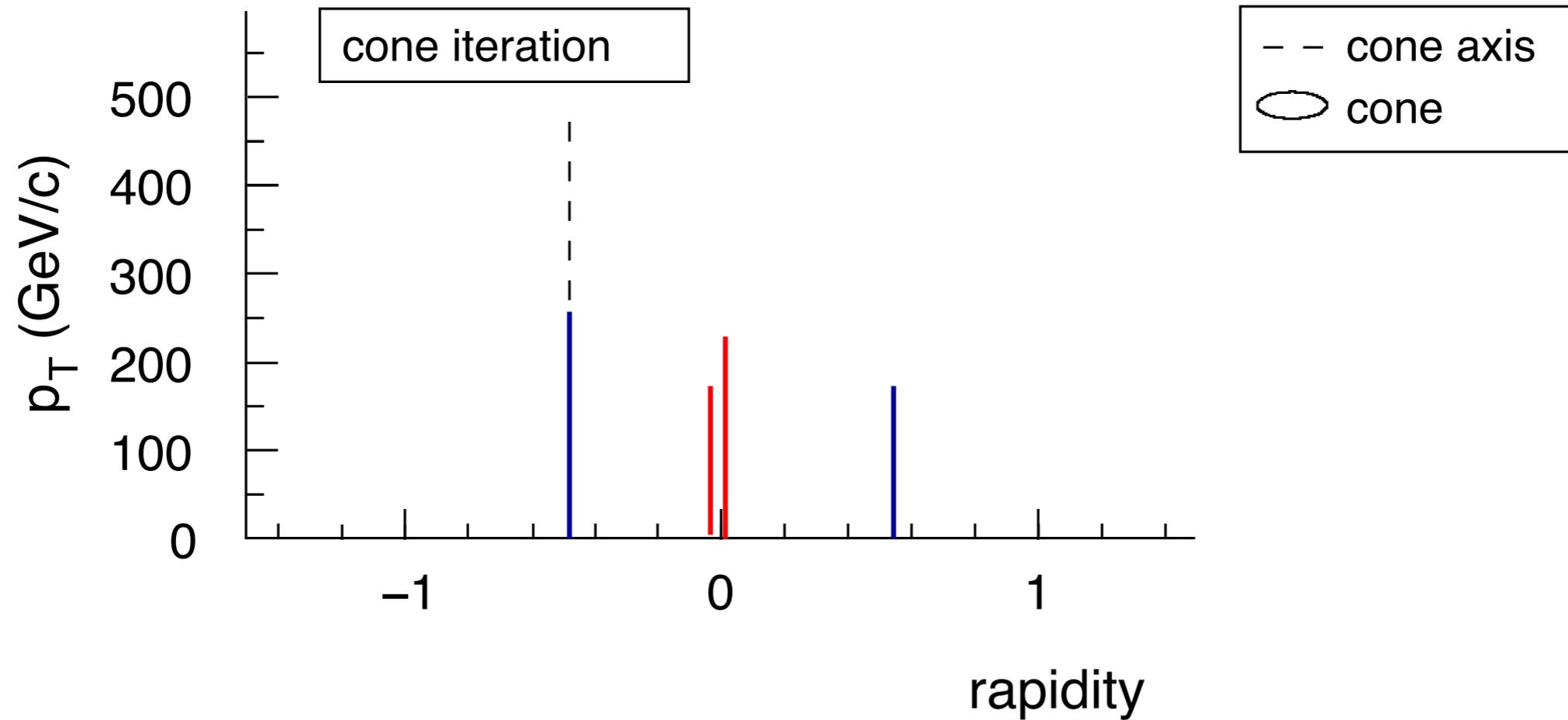
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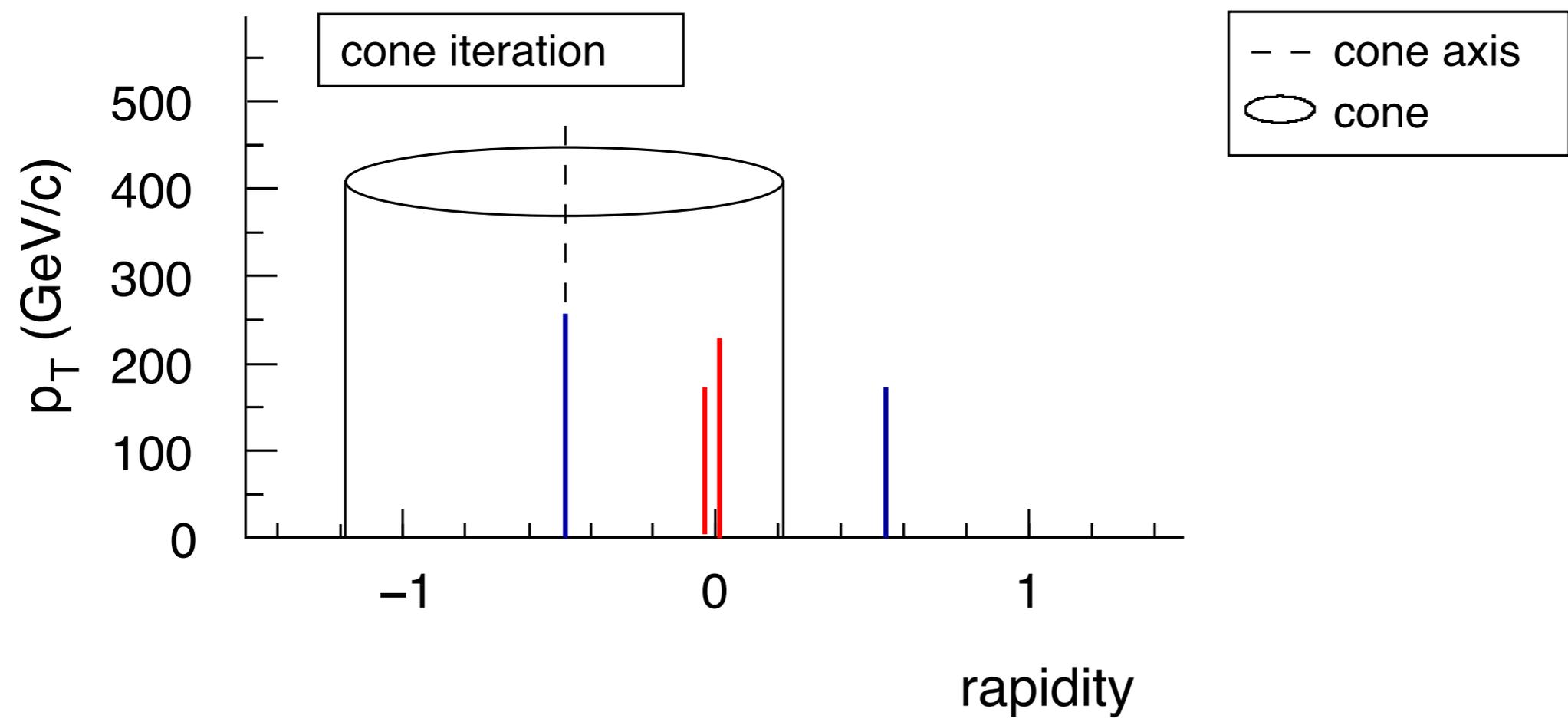
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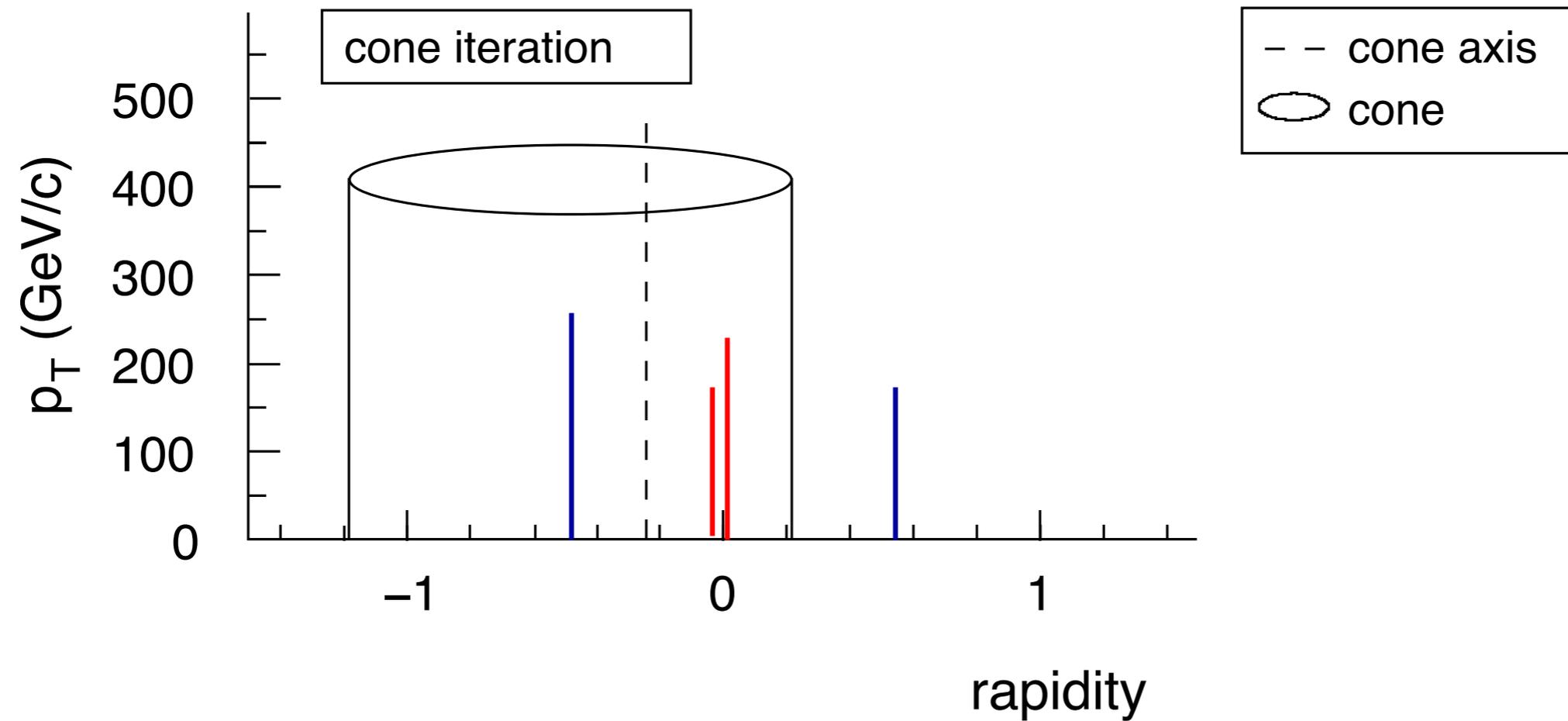
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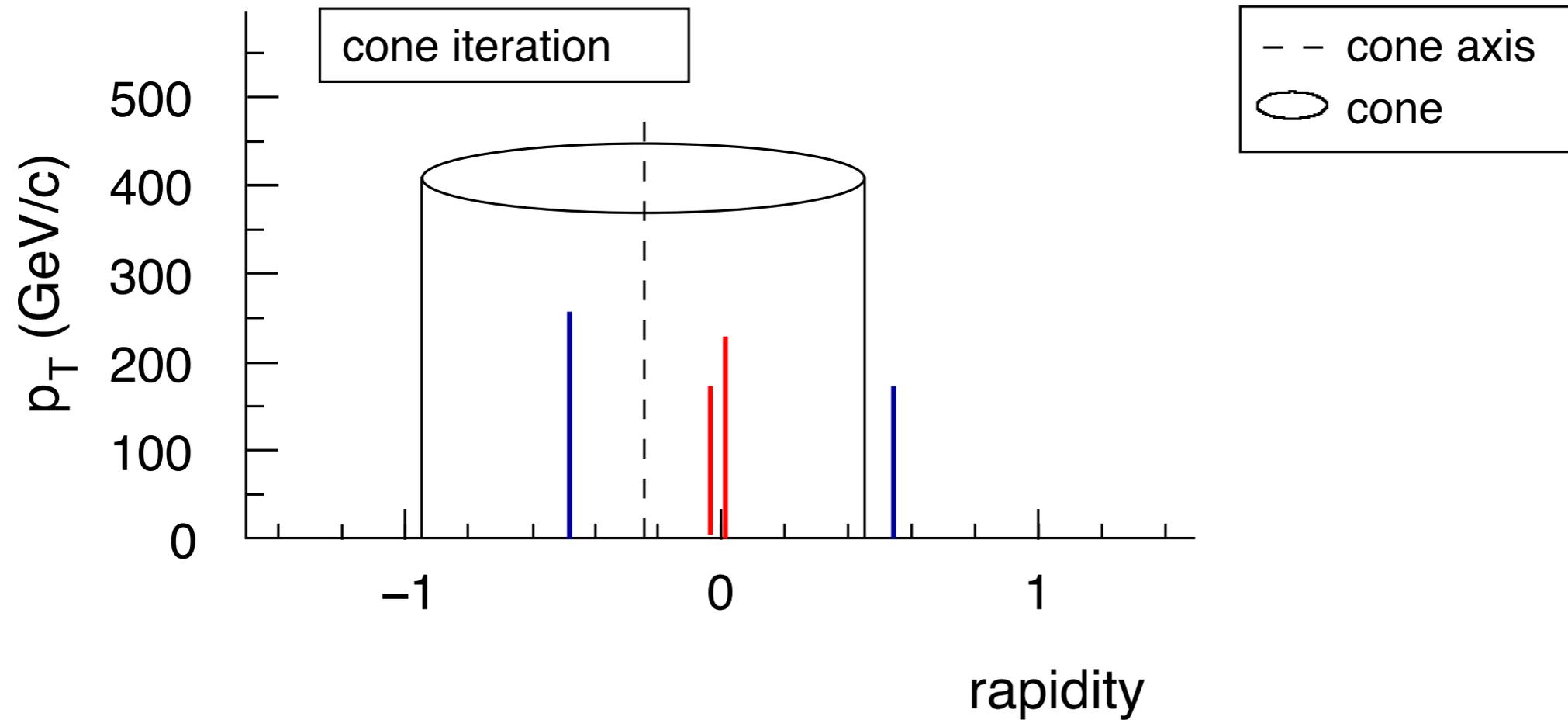
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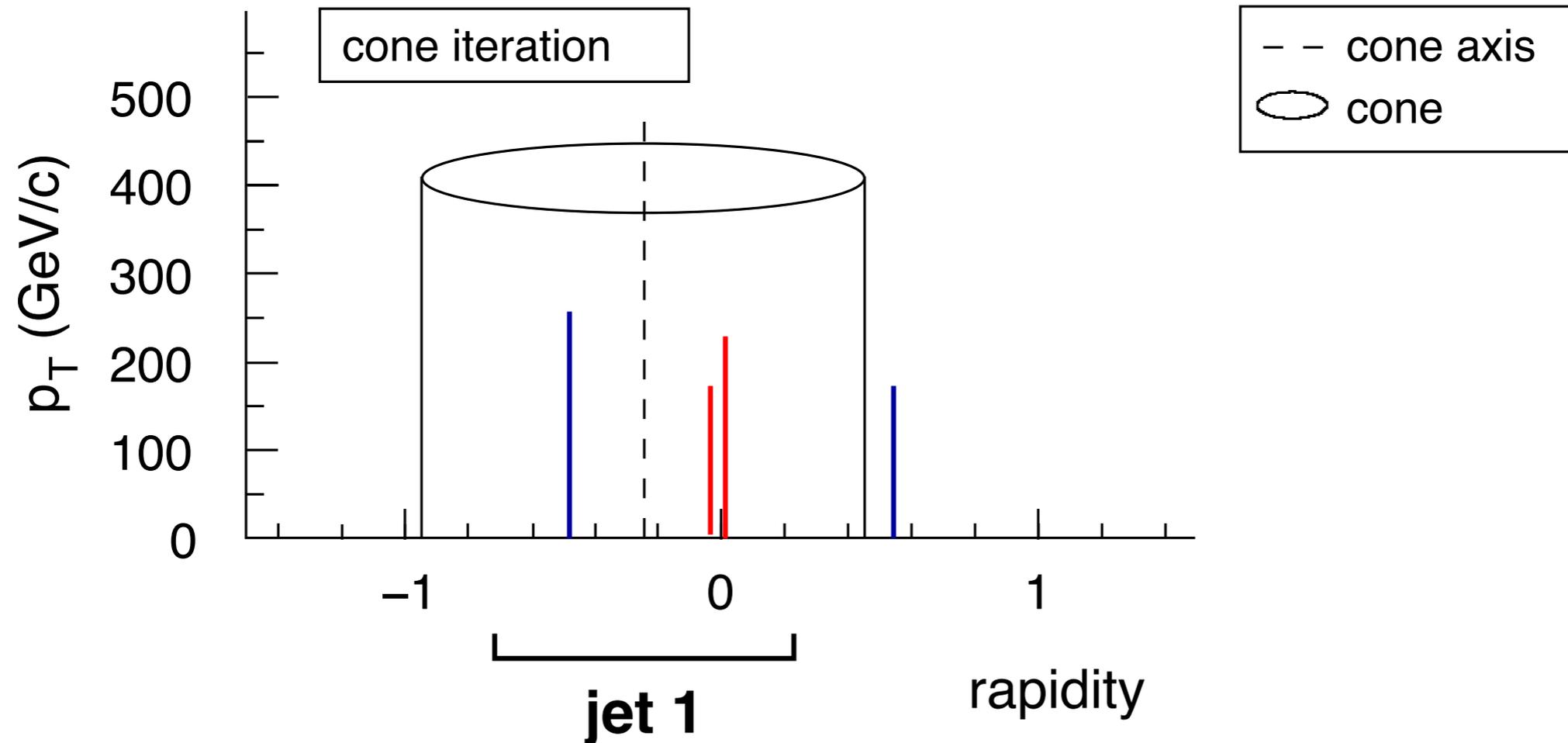
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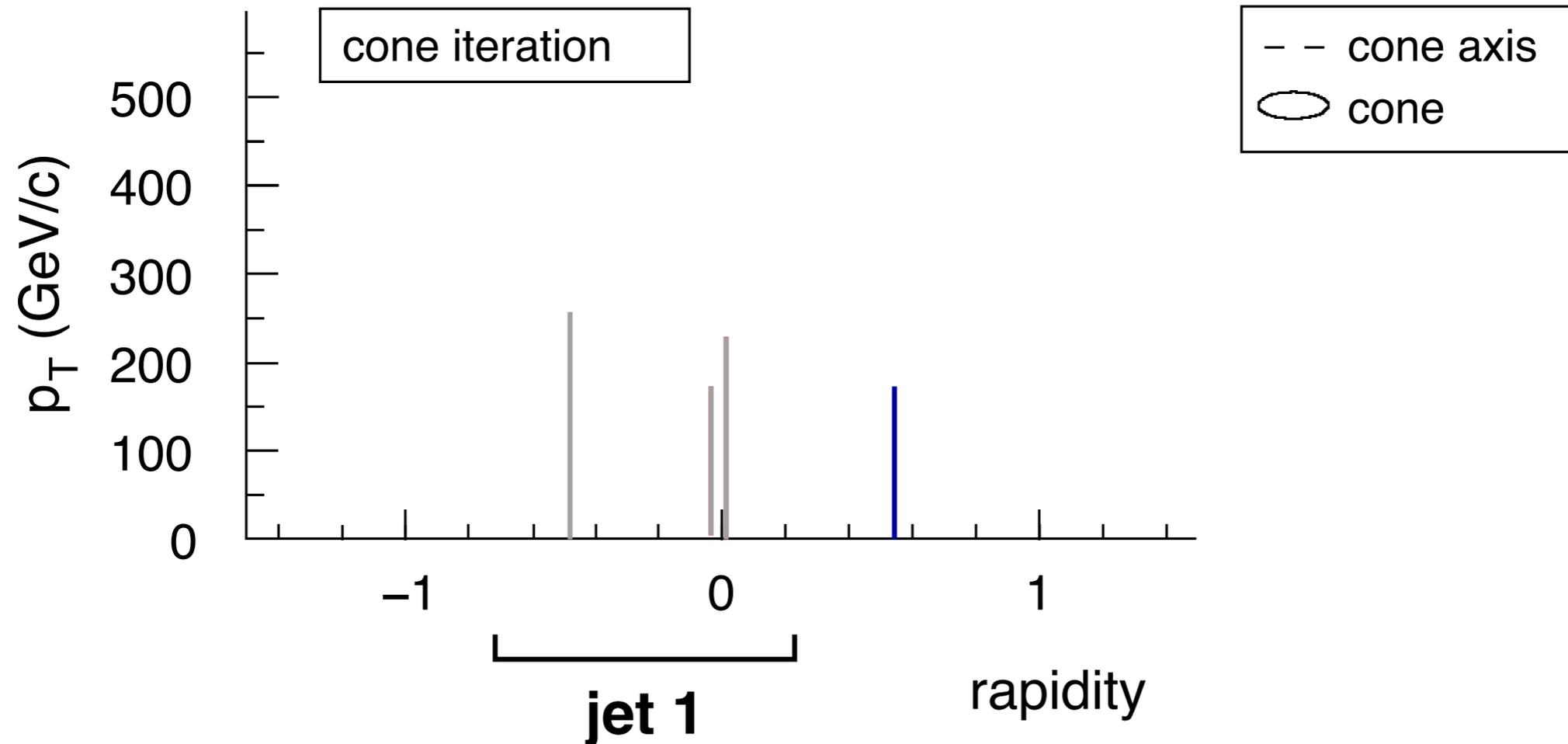
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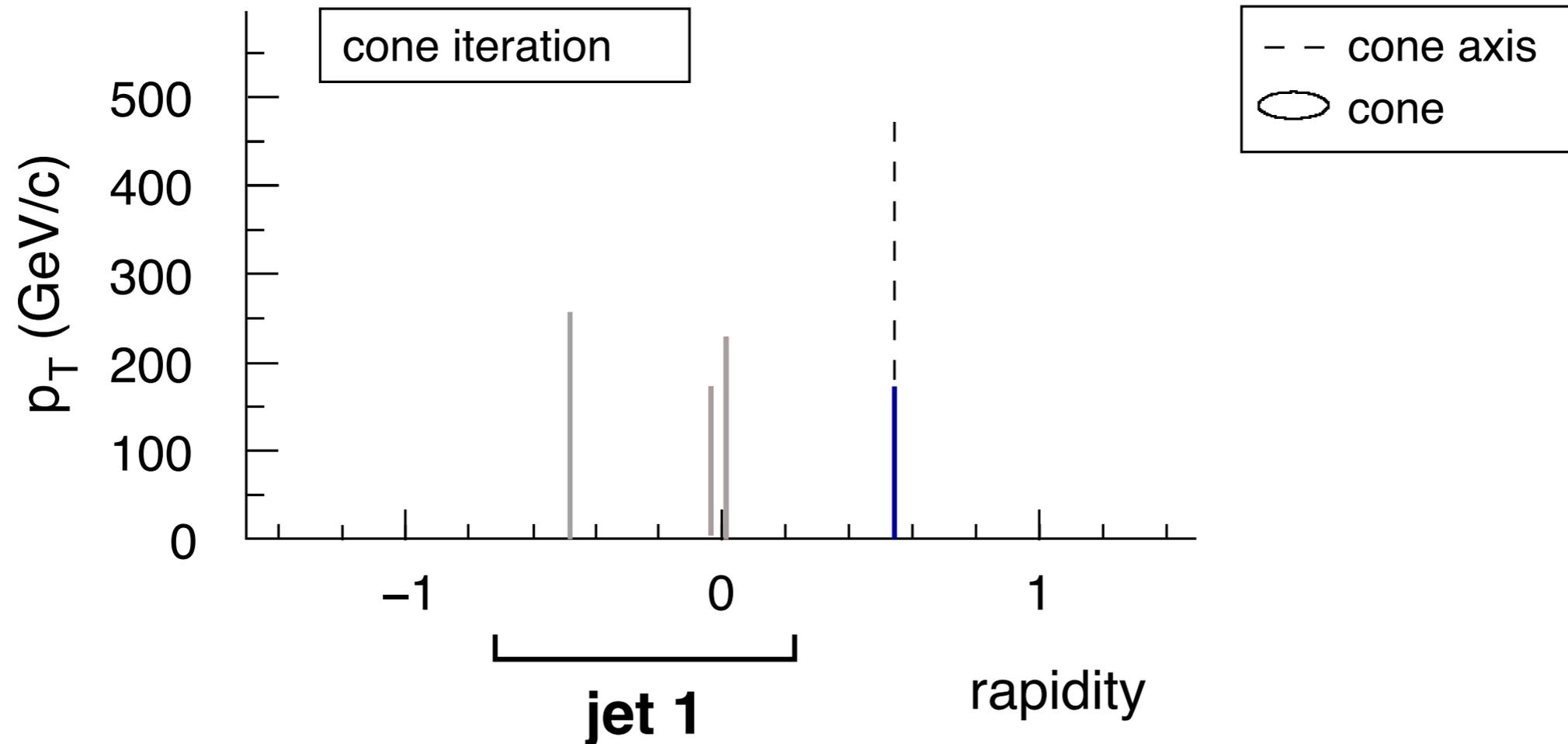
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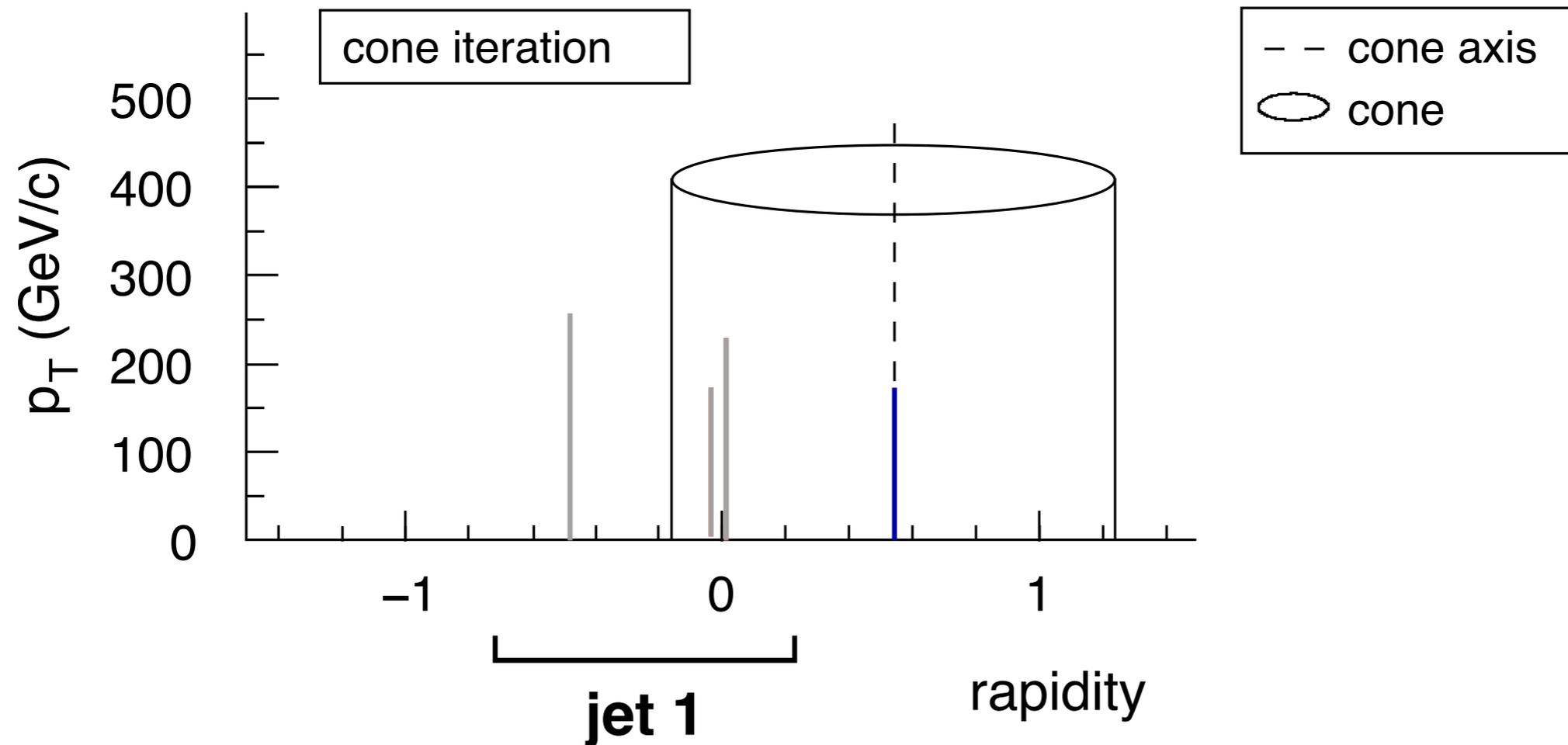
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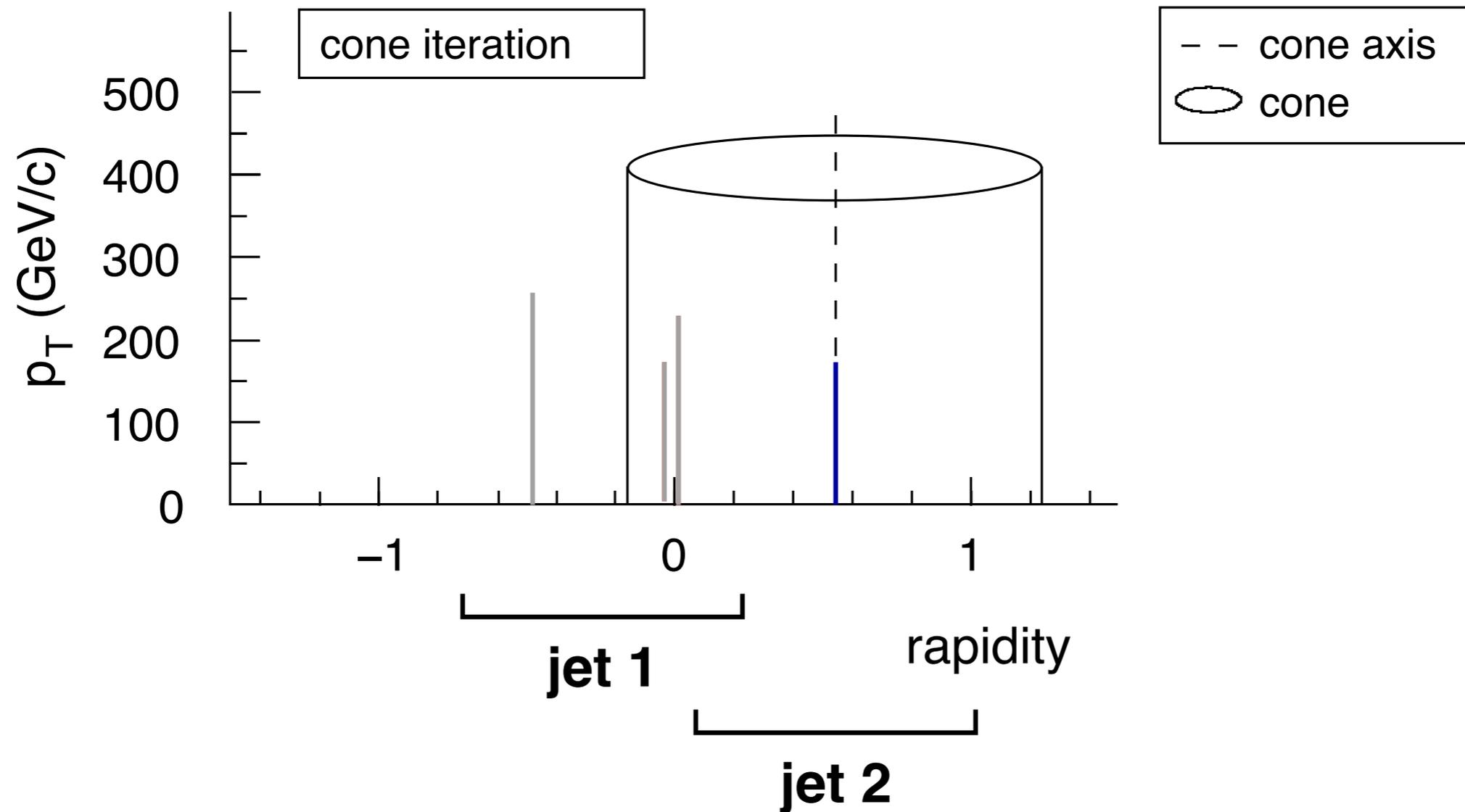
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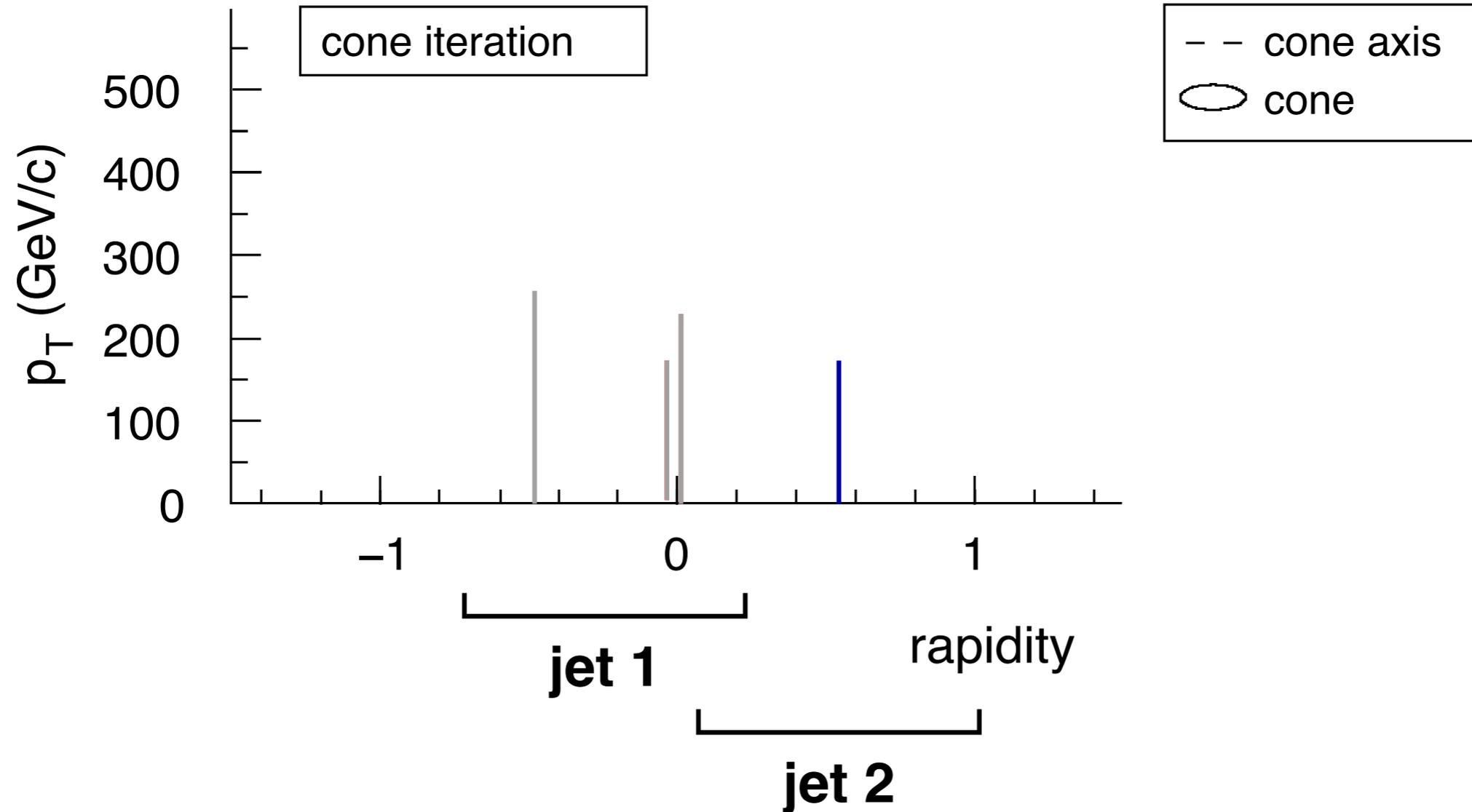
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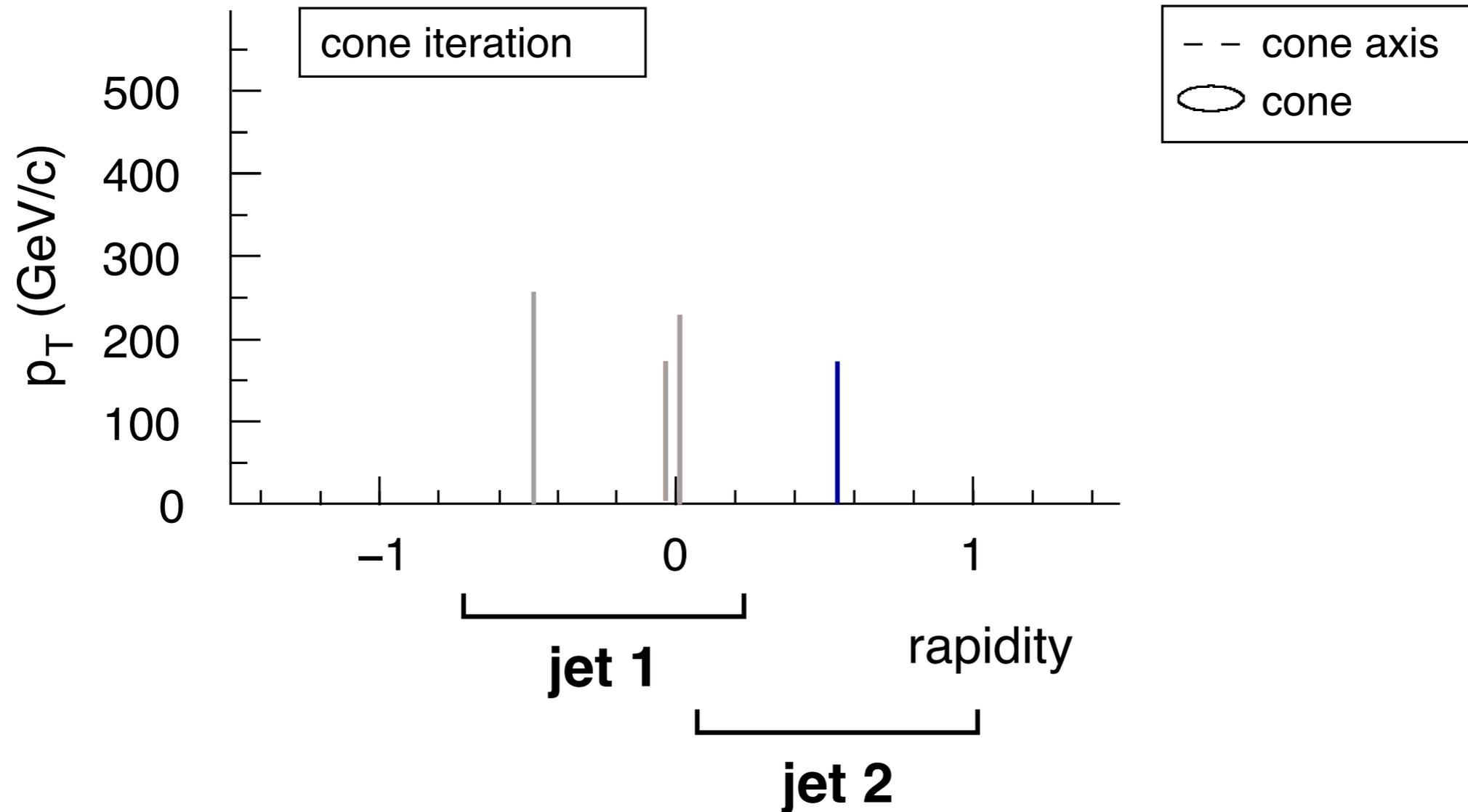


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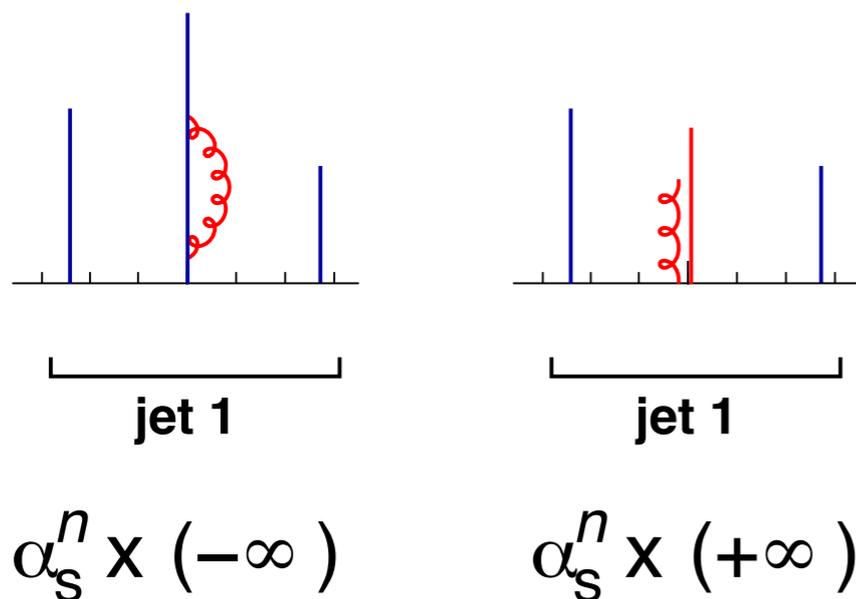
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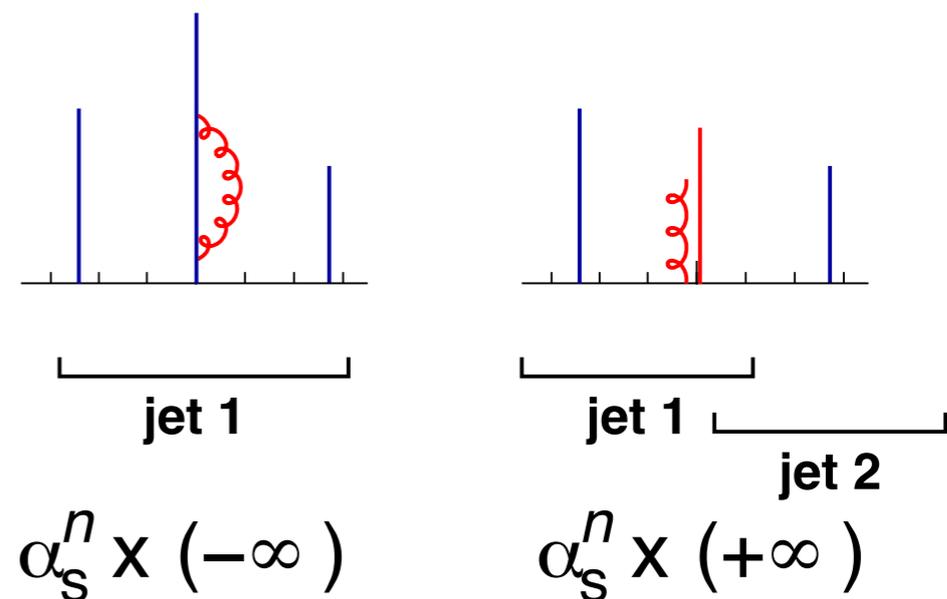
Consequences of Collinear Unsafety

Collinear Safe



Infinities cancel

Collinear Unsafe



Infinities do not cancel

Invalidates perturbation theory

IR Safety & Real Life

Real life does not have infinities, but pert. infinity leaves a real-life trace

$$\alpha_s^2 + \alpha_s^3 + \alpha_s^4 \times \infty \rightarrow \alpha_s^2 + \alpha_s^3 + \alpha_s^4 \times \ln p_t/\Lambda \rightarrow \alpha_s^2 + \underbrace{\alpha_s^3 + \alpha_s^3}_{\text{BOTH WASTED}}$$

Among consequences of IR unsafety:

	<i>Last meaningful order</i>			Known at
	JetClu, ATLAS cone [IC-SM]	MidPoint [IC _{mp} -SM]	CMS it. cone [IC-PR]	
Inclusive jets	LO	NLO	NLO	NLO (→ NNLO)
W/Z + 1 jet	LO	NLO	NLO	NLO
3 jets	none	LO	LO	NLO [nlojet++]
W/Z + 2 jets	none	LO	LO	NLO [MCFM]
m_{jet} in $2j + X$	none	none	none	LO

NB: 50,000,000\$/£/CHF/€ investment in NLO

Stereo Vision

Use IR safe algorithms

To study short-distance physics

These days, \approx as fast as IR unsafe algos and widely implemented (e.g., FASTJET), including

“Cone-like”: SiSCone, Anti-kT, ...

“Recombination-like”: kT, Cambridge/Aachen, ...

Then use IR sensitive observables

E.g., number of tracks, identified particles, ...

To explicitly check hadronization and other IR models

Jet Rates

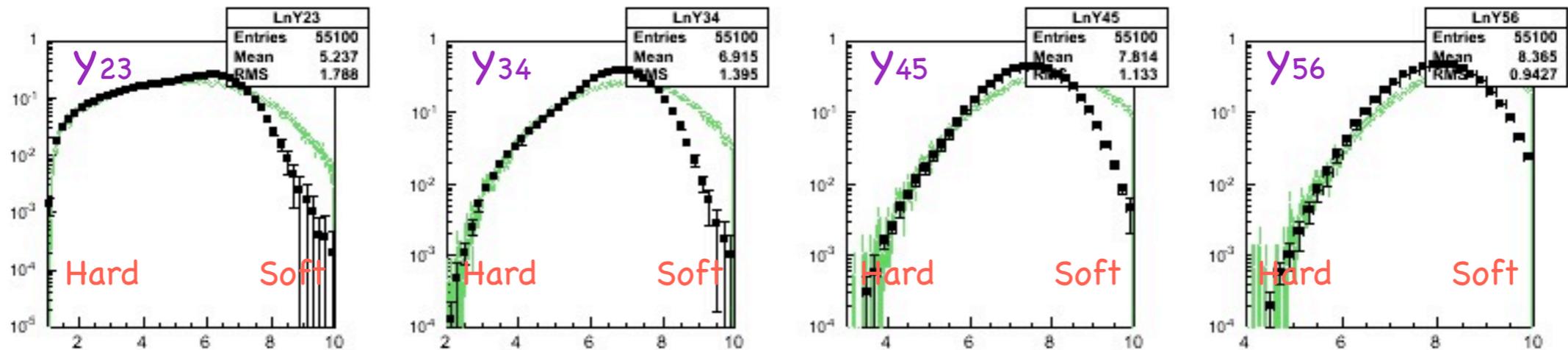
At $E_{vis} = 91$ GeV
 $y=2 \rightarrow k_T \approx 33$ GeV
 $y=4 \rightarrow k_T \approx 12$ GeV
 $y=6 \rightarrow k_T \approx 4.5$ GeV
 $y=8 \rightarrow k_T \approx 1.6$ GeV
 $y=10 \rightarrow k_T \approx 0.6$ GeV

Jet Resolution

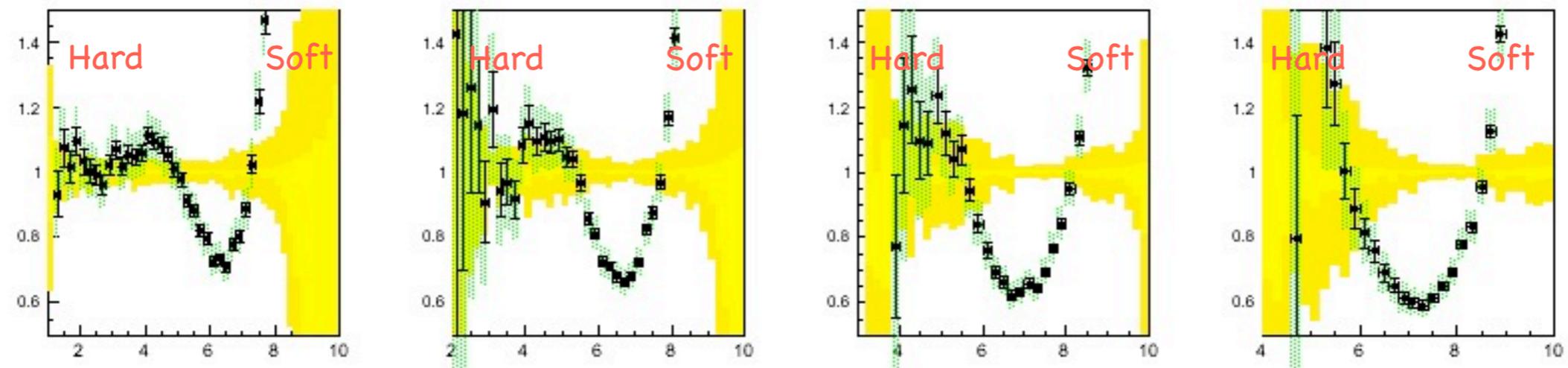
Parton Level

E.g., $y_{23} = k_T^2 / E_{vis}^2 =$ scale where event goes from having 2 to 3 jets

Theory vs LEP



Theory/LEP



(default PYTHIA 8.135)

Jet Rates

At $E_{vis} = 91 \text{ GeV}$
 $y=2 \rightarrow k_T \approx 33 \text{ GeV}$
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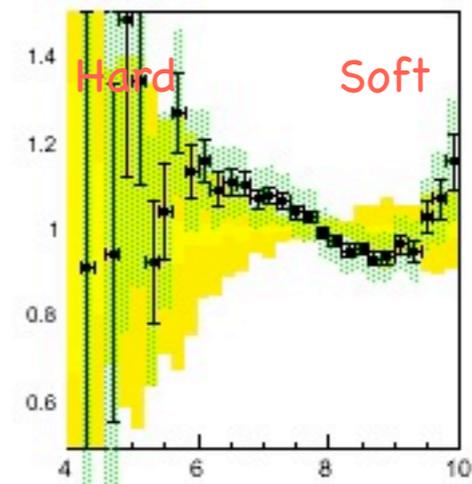
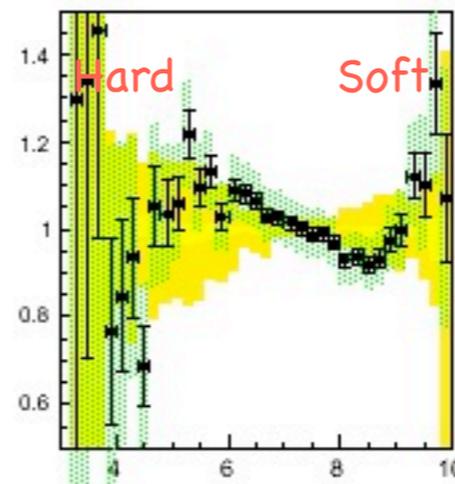
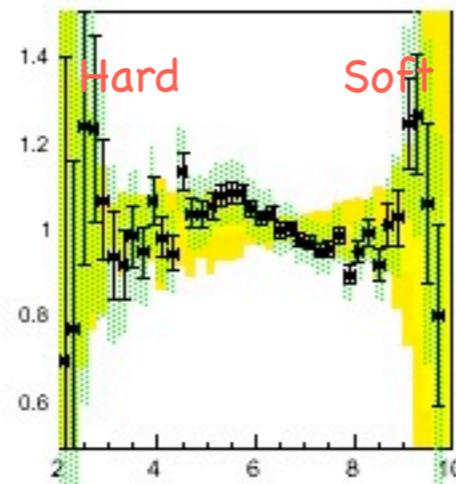
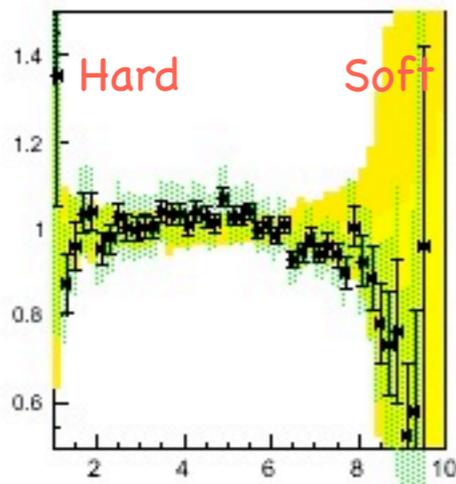
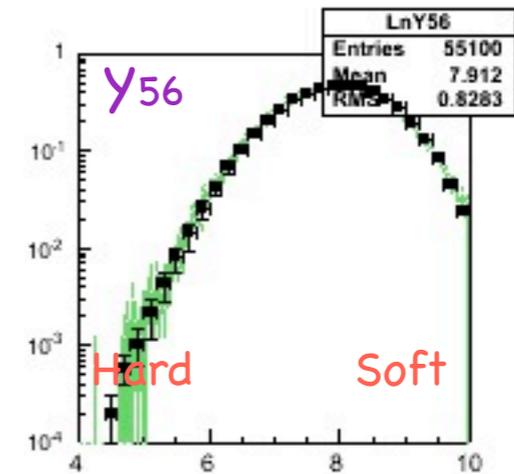
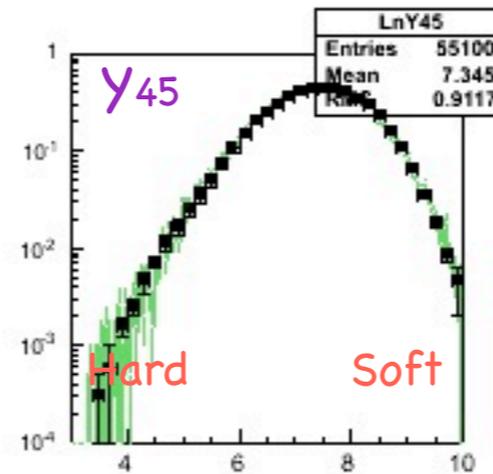
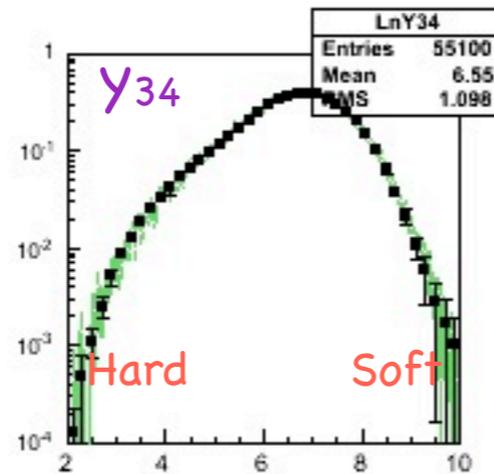
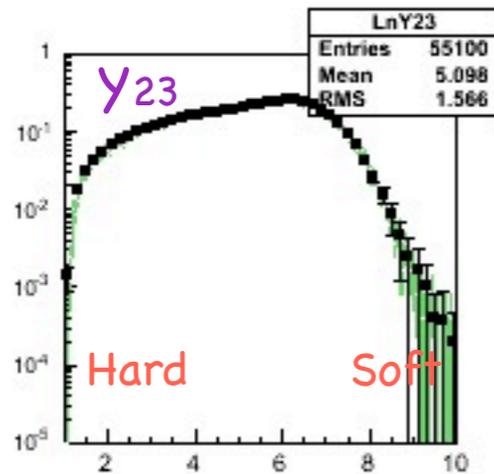
Jet Resolution

Hadron level

E.g., $y_{23} = k_T^2 / E_{vis}^2 = \text{scale where event goes from having 2 to 3 jets}$

Theory vs LEP

Theory/LEP

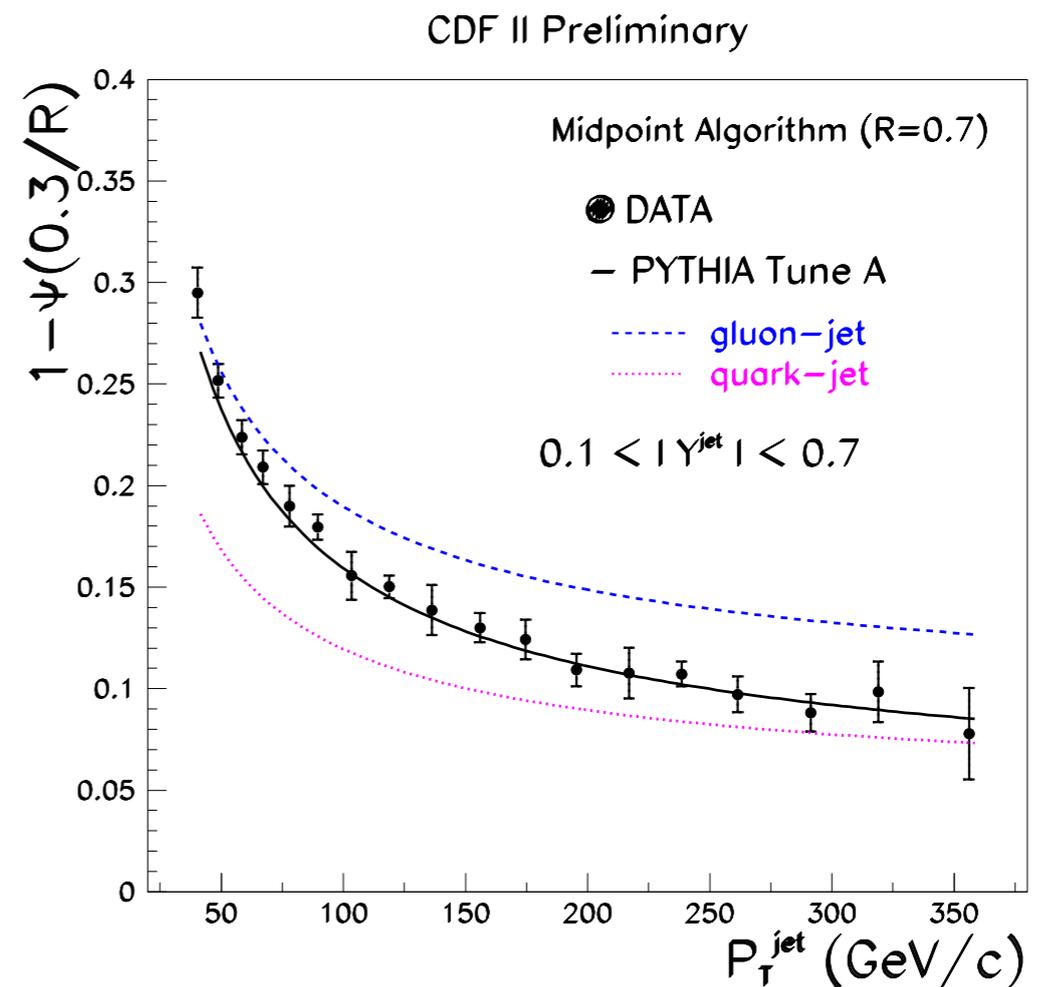
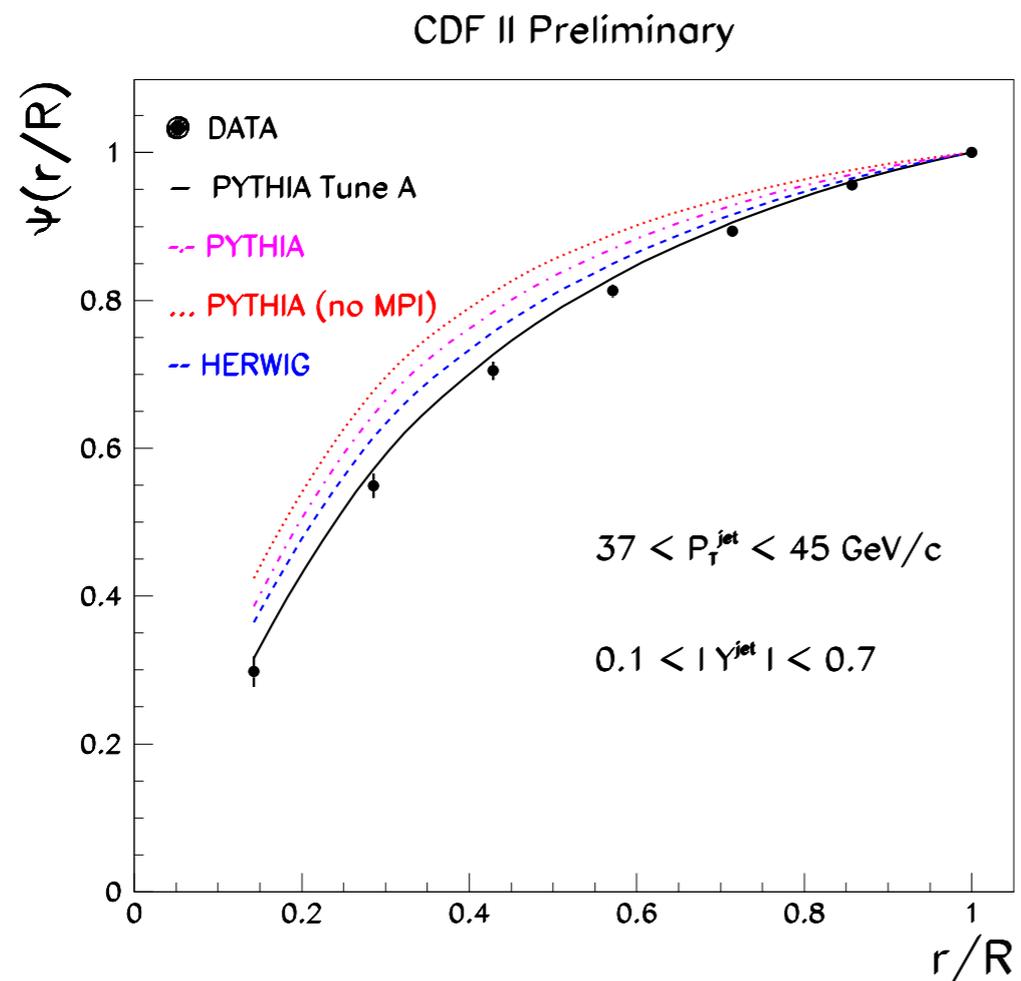


(default PYTHIA 8.135)

Jet Universality

At LEP: mostly quark jets with lots of c & b

At Tevatron/LHC: mostly gluon jets and light-quark jets



Merging Parton Showers and Matrix Elements

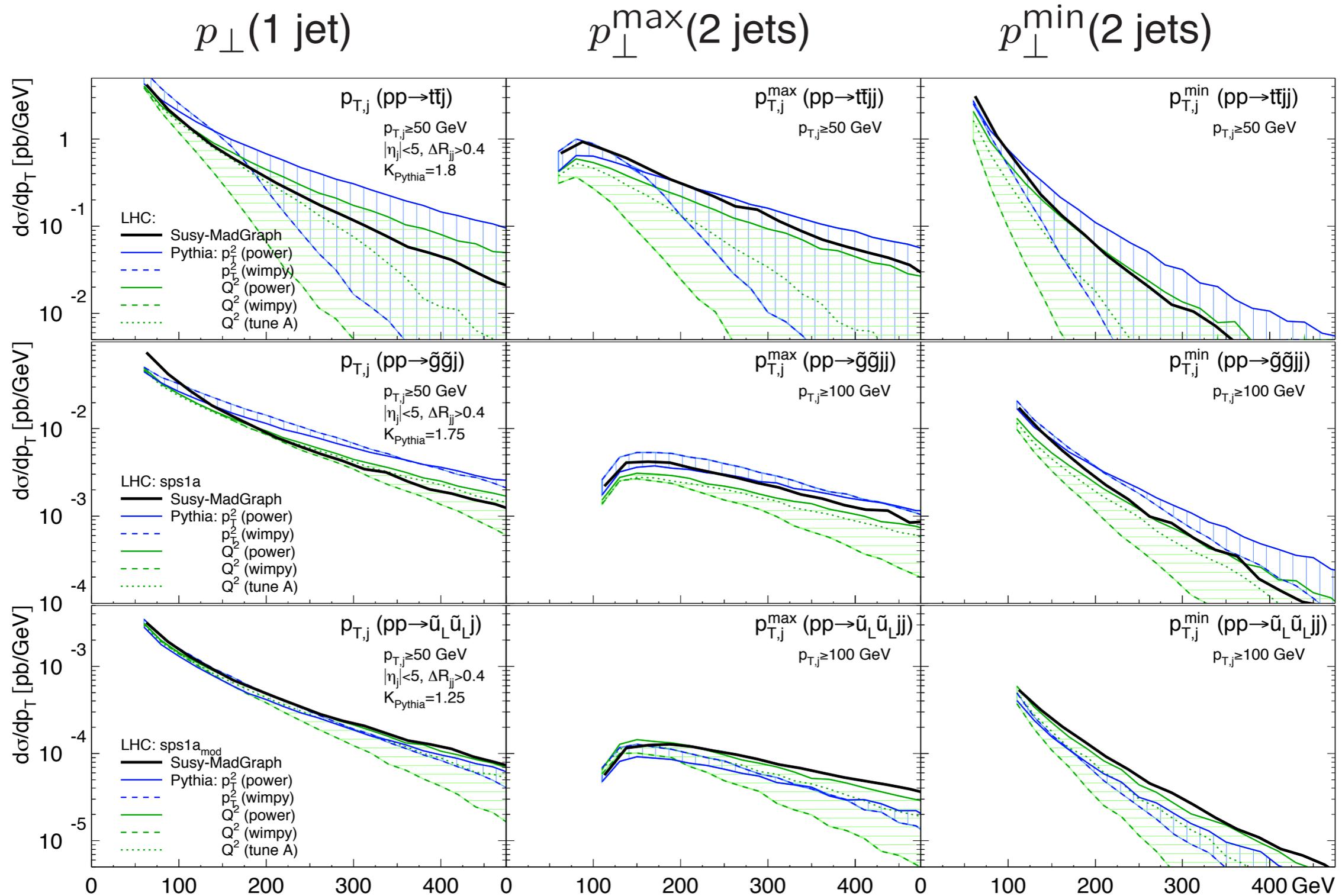
Matching

Note: tough subject

Not required to understand everything

Don't loose yourselves in the details,

Just try to understand the overall reasoning



power: $Q_{\max}^2 = s$; wimpy: $Q_{\max}^2 = m_{\perp}^2$; tune A: $Q_{\max}^2 = 4m_{\perp}^2$
 $m_t = 175$ GeV, $m_{\tilde{g}} = 608$ GeV, $m_{\tilde{u}_L} = 567$ GeV

(T. Plehn, D. Rainwater, P. Skands)

Matching

► A (Complete Idiot's) Solution – Combine

1. $[X]_{ME}$ + showering
2. $[X + 1 \text{ jet}]_{ME}$ + showering
3. ...

Run generator for X (+ shower)
Run generator for $X+1$ (+ shower)
Run generator for ... (+ shower)
Combine everything into one sample

Matching

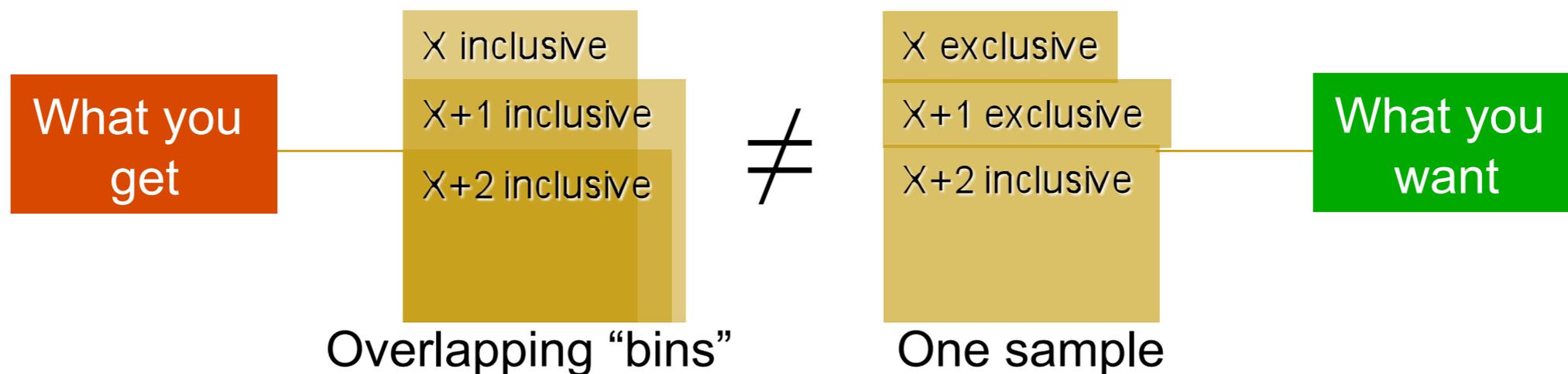
► A (Complete Idiot's) Solution – Combine

1. $[X]_{ME}$ + showering
2. $[X + 1 \text{ jet}]_{ME}$ + showering
3. ...

Run generator for X (+ shower)
Run generator for $X+1$ (+ shower)
Run generator for ... (+ shower)
Combine everything into one sample

► Doesn't work

- $[X]$ + shower is inclusive
- $[X+1]$ + shower is also inclusive

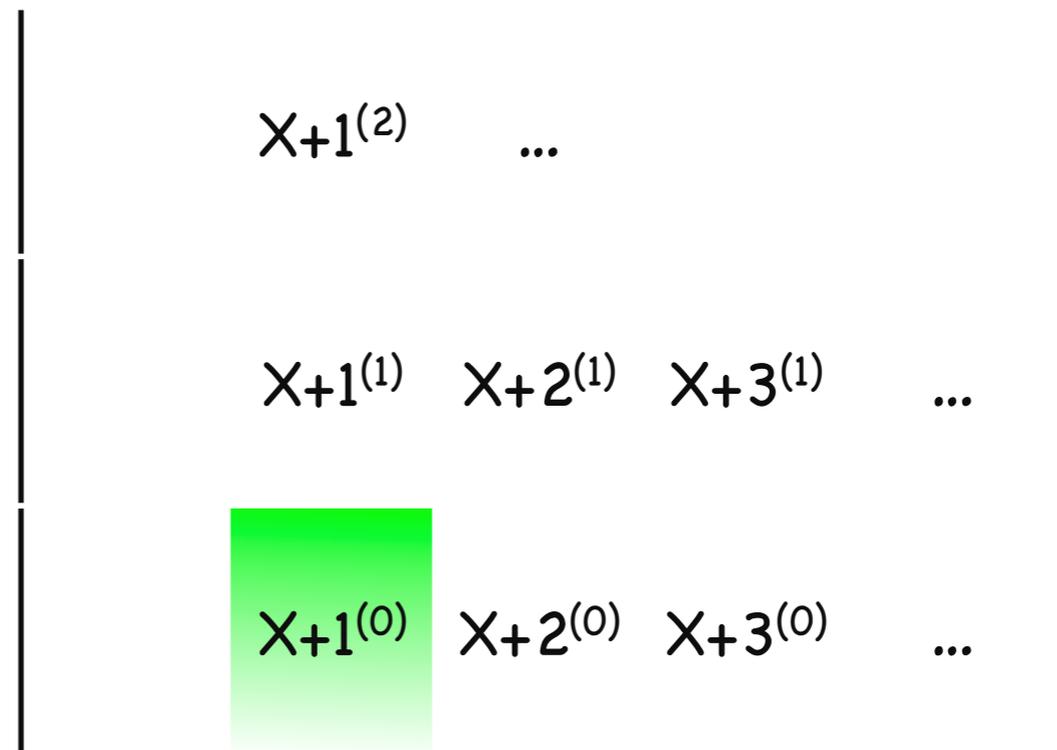


Loops and Legs

Born \times Shower



X_{+1} @ LO



... Fixed-Order Matrix Element

... Shower Approximation

... Fixed-Order ME above p_T cut & nothing below

... Fixed-Order ME above p_T cut & Shower Approximation below

Loops and Legs

Born \times Shower

$X^{(2)}$	$X_{+1}^{(2)}$...		
$X^{(1)}$	$X_{+1}^{(1)}$	$X_{+2}^{(1)}$	$X_{+3}^{(1)}$...
Born	$X_{+1}^{(0)}$	$X_{+2}^{(0)}$	$X_{+3}^{(0)}$...

...	Fixed-Order Matrix Element
...	Shower Approximation

X_{+1} @ LO \times Shower

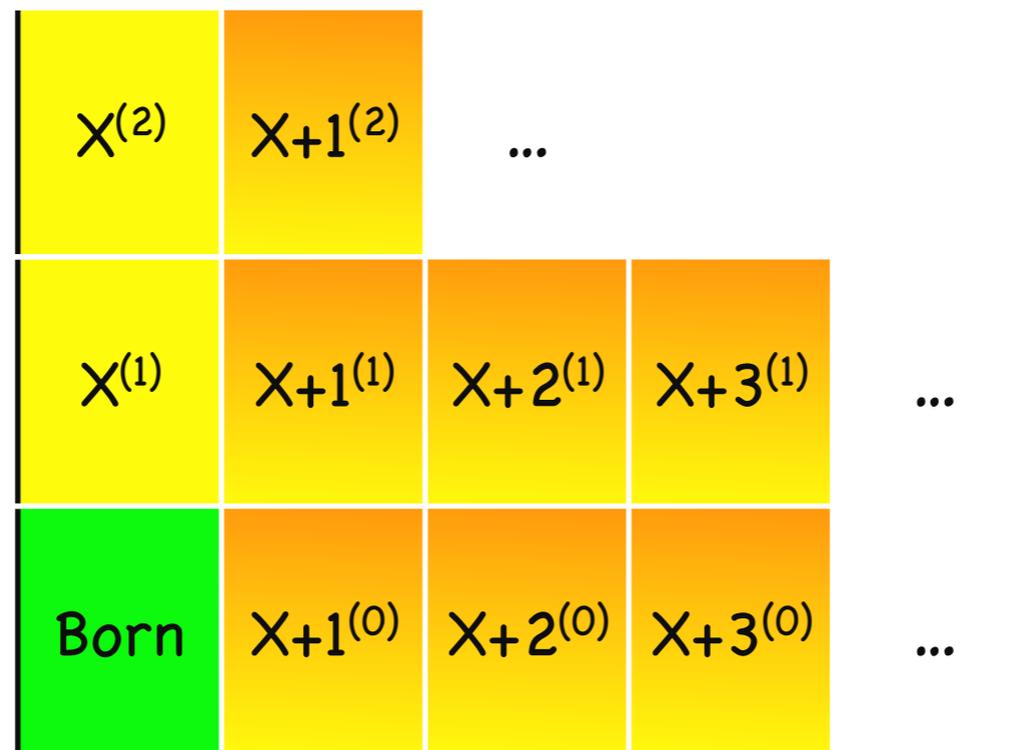
$X_{+1}^{(2)}$...		
$X_{+1}^{(1)}$	$X_{+2}^{(1)}$	$X_{+3}^{(1)}$...
$X_{+1}^{(0)}$	$X_{+2}^{(0)}$	$X_{+3}^{(0)}$...

...	Fixed-Order ME above p_T cut & nothing below
...	Fixed-Order ME above p_T cut & Shower Approximation below

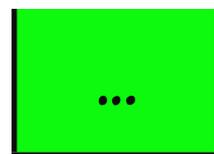
Loops and Legs

$$\text{Born} \times \text{Shower} + (X+1) \times \text{shower}$$

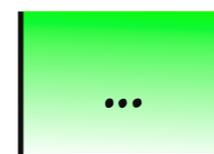
Double Counting of terms present in both expansions



Worse than useless



Fixed-Order Matrix Element



Fixed-Order ME above p_T cut & nothing below



Shower Approximation



Fixed-Order ME above p_T cut & Shower Approximation below

Phase Space Slicing

(with "matching scale")

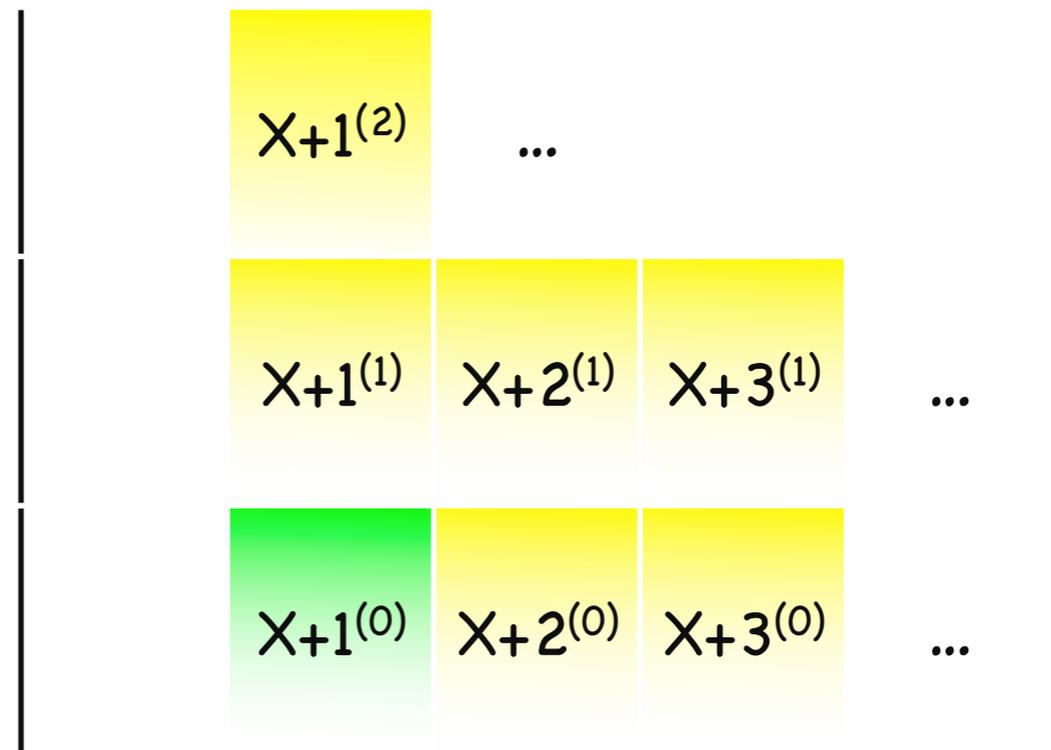
Born \times Shower

+ shower veto above p_T



X_{+1} @ LO \times Shower

with 1 jet above p_T



... Fixed-Order Matrix Element

... Shower Approximation

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... Fixed-Order ME above p_T cut & Shower Approximation below

Phase Space Slicing

(with "matching scale")

Born \times Shower

+

$X+1$ @ LO \times Shower

+ shower veto above p_T

with 1 jet above p_T

$X+1$ now correct in both soft and hard limits



Attention!
Must use the SAME p_T cut in both samples

But still ... :
 α_s and "splitting functions" usually discontinuous



Fixed-Order Matrix Element



Shower Approximation



Fixed-Order ME above p_T cut & nothing below



Fixed-Order ME above p_T cut & Shower Approximation below

Multi-Leg Slicing

(a.k.a. CKKW or MLM matching)

CKKW: Catani, Krauss, Kuhn, Webber, JHEP 0111:063,2001.

MLM: Michelangelo L Mangano

Keep going

Veto all shower emissions above “matching scale”
(except for the highest-multiplicity matrix element)

LO: when all jets hard
LL: for soft emissions

$X^{(2)}$	$X_{+1}^{(2)}$...		
$X^{(1)}$	$X_{+1}^{(1)}$	$X_{+2}^{(1)}$	$X_{+3}^{(1)}$...
Born	$X_{+1}^{(0)}$	$X_{+2}^{(0)}$	$X_{+3}^{(0)}$...

→ Multileg Tree-level matching

Vetoed Parton Showers

(used in Phase Space Slicing, a.k.a. CKKW or MLM matching)

Common (at ME level):

1. Generate one ME sample for each of $\sigma_n(p_{T\text{cut}})$ (using large, fixed α_{s0})
2. Use a jet algorithm (e.g., k_T) to determine an approximate shower history for each ME event
3. Construct the would-be shower α_s factor and reweight

$$w_n = \text{Prod}[\alpha_s(k_{Ti})] / \alpha_{s0}^n$$

→ “Renormalization-improved” ME weights

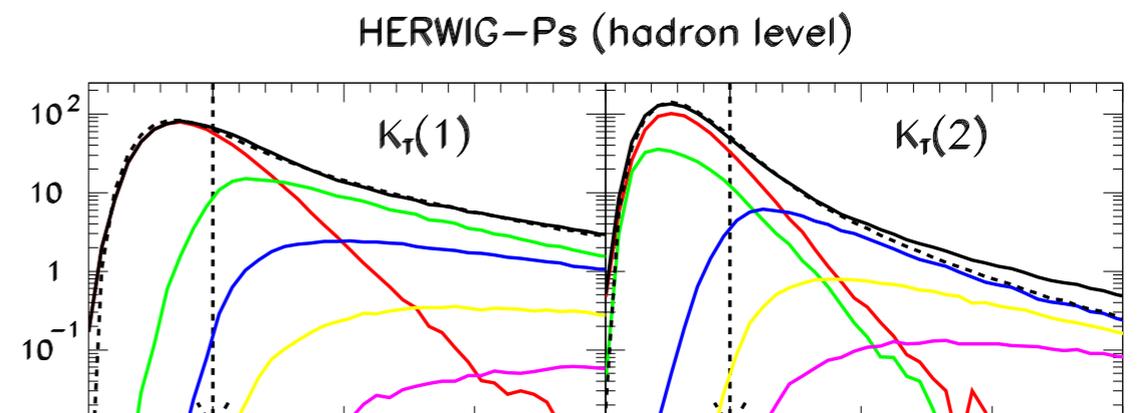
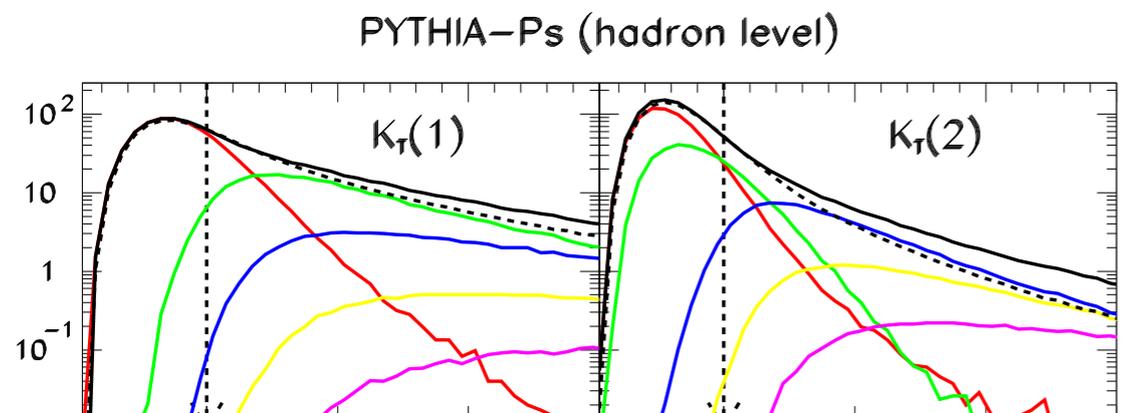
CKKW and CKKW-L

1. Apply Sudakov $\Delta(t_{\text{start}}, t_{\text{end}})$ for each reconstructed internal line (NLL for CCKW, trial-shower for CKKW-L)
2. Accept/Reject: $w_n \propto \text{Prod}[\Delta_i]$
3. Do parton shower, vetoing any emissions above cutoff

MLM

1. Do normal parton showers
2. Cluster showered event (cone)
3. Match ME partons to jets
4. If {all partons matched && $n_{\text{partons}} == n_{\text{jets}}$ } Accept : Reject;

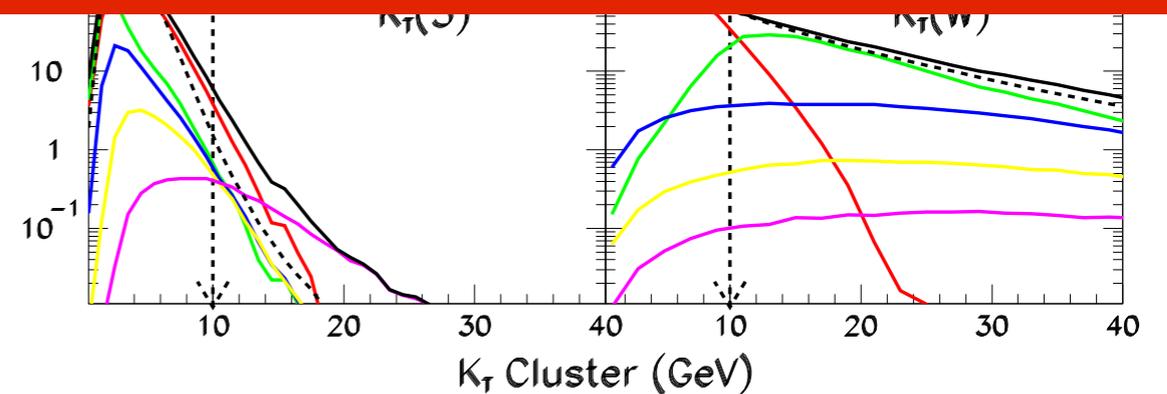
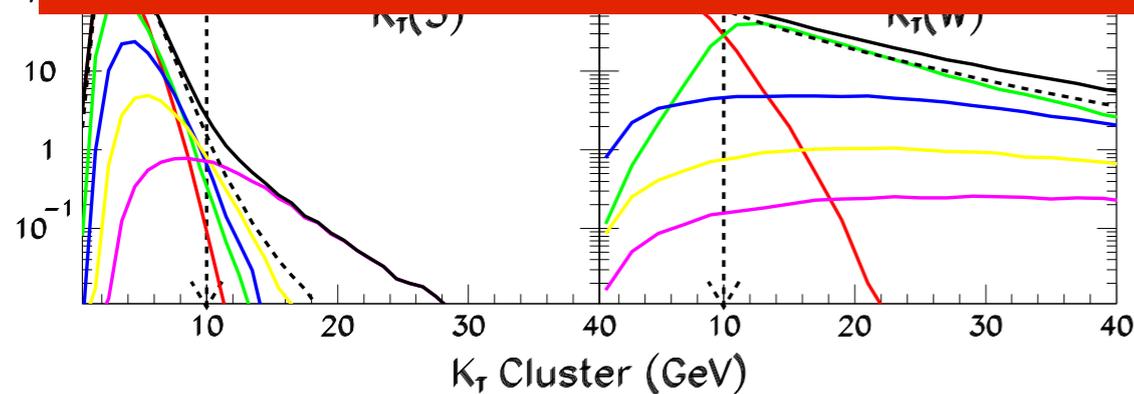
Multi-Jet Samples



Matching is mandatory when multiple hard jets

Note: precision still "only" LO

Note 2: resummation still "only" (N)LL



(S.Mrenna, P. Richardson)

"Additive" Matching

Born × Shower

$X^{(2)}$	$X_{+1}^{(2)}$...		
$X^{(1)}$	$X_{+1}^{(1)}$	$X_{+2}^{(1)}$	$X_{+3}^{(1)}$...
Born	$X_{+1}^{(0)}$	$X_{+2}^{(0)}$	$X_{+3}^{(0)}$...

NLO

$X^{(2)}$	$X_{+1}^{(2)}$...		
$X^{(1)}$	$X_{+1}^{(1)}$	$X_{+2}^{(1)}$	$X_{+3}^{(1)}$...
Born	$X_{+1}^{(0)}$	$X_{+2}^{(0)}$	$X_{+3}^{(0)}$...

... Fixed-Order Matrix Element

... Shower Approximation

"Additive" Matching

Born × Shower

$X^{(2)}$	$X_{+1}^{(2)}$...		
$X^{(1)}$	$X_{+1}^{(1)}$	$X_{+2}^{(1)}$	$X_{+3}^{(1)}$...
Born	$X_{+1}^{(0)}$	$X_{+2}^{(0)}$	$X_{+3}^{(0)}$...

...	Fixed-Order Matrix Element
...	Shower Approximation

NLO - Shower

$X^{(2)}$	$X_{+1}^{(2)}$...		
$X^{(1)}$	$X_{+1}^{(1)}$	$X_{+2}^{(1)}$	$X_{+3}^{(1)}$...
Born	$X_{+1}^{(0)}$	$X_{+2}^{(0)}$	$X_{+3}^{(0)}$...

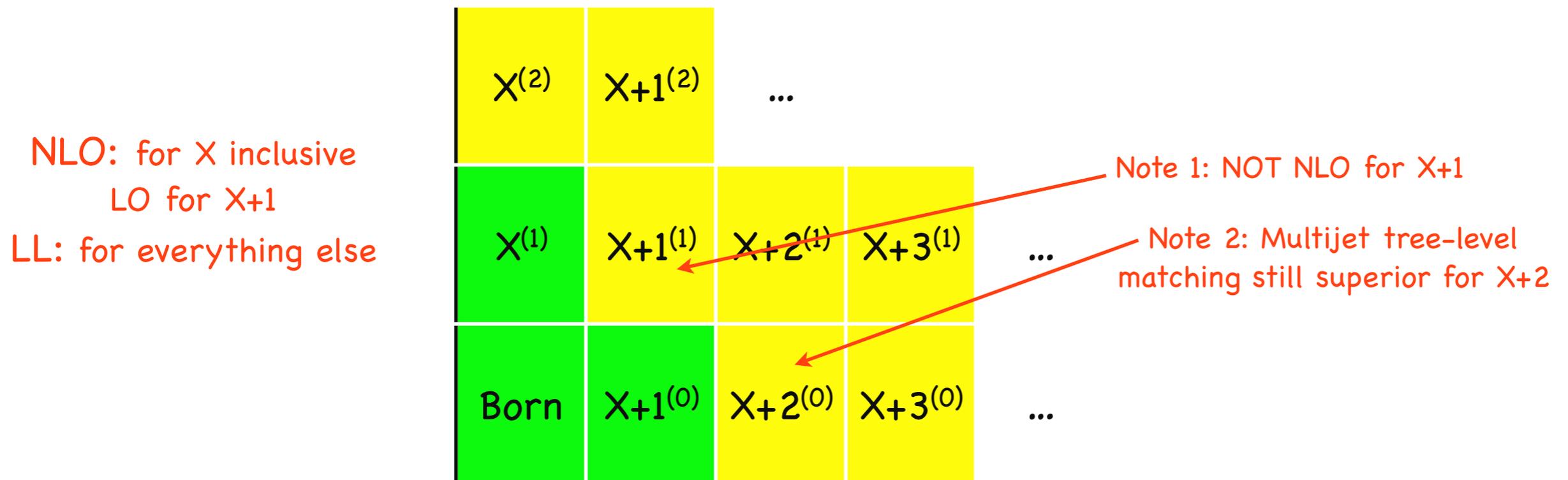
Expand shower approximation to NLO analytically, then subtract:

...	Fixed-Order ME minus Shower Approximation (NOTE: can be < 0!)
-----	---

“Additive” Matching

Add

Born + shower-subtracted $O(\alpha_s)$ matrix elements



→ NLO + parton shower
(however, the “correction events” can have $w < 0$)

"Additive" Matching

Born × Shower

$X^{(2)}$	$X_{+1}^{(2)}$...		
$X^{(1)}$	$X_{+1}^{(1)}$	$X_{+2}^{(1)}$	$X_{+3}^{(1)}$...
Born	$X_{+1}^{(0)}$	$X_{+2}^{(0)}$	$X_{+3}^{(0)}$...

...	Fixed-Order Matrix Element
...	Shower Approximation

NLO - Shower

$X^{(2)}$	$X_{+1}^{(2)}$...		
$X^{(1)}$	$X_{+1}^{(1)}$	$X_{+2}^{(1)}$	$X_{+3}^{(1)}$...
Born	$X_{+1}^{(0)}$	$X_{+2}^{(0)}$	$X_{+3}^{(0)}$...

Expand shower approximation to NLO analytically, then subtract:

...	Fixed-Order ME minus Shower Approximation (NOTE: can be < 0!)
-----	---

PYTHIA / POWHEG

"Merging"

Born \times First-Order Corrected Shower

$X^{(2)}$	$X_{+1}^{(2)}$...		
$X^{(1)}$	$X_{+1}^{(1)}$	$X_{+2}^{(1)}$	$X_{+3}^{(1)}$...
Born	$X_{+1}^{(0)}$	$X_{+2}^{(0)}$	$X_{+3}^{(0)}$...

$X^{(2)}$	$X_{+1}^{(2)}$...		
$X^{(1)}$	$X_{+1}^{(1)}$	$X_{+2}^{(1)}$	$X_{+3}^{(1)}$...
Born	$X_{+1}^{(0)}$	$X_{+2}^{(0)}$	$X_{+3}^{(0)}$...

 Fixed-Order Matrix Element
 Shower Approximation

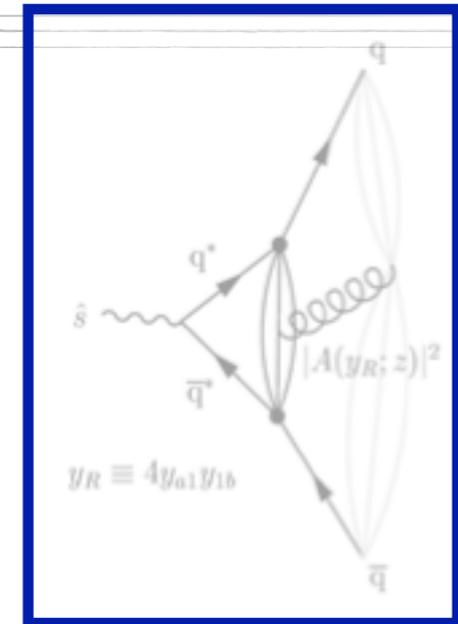
Use exact (process-dependent) splitting function for first splitting
 Fixed-Order ME minus Shower Approximation

NLO Matching in 1 Slide

► First Order Shower expansion

$$\text{PS} \quad \int d\Phi_2 \quad \boxed{\text{Born}} \int_{Q_{\text{had}}^2}^s \frac{d\Phi_3}{d\Phi_2} \quad \boxed{\text{LL}} \delta(\mathcal{O} - \mathcal{O}(\{p\}_3))$$

Unitarity of shower \rightarrow 3-parton real = 2-parton "virtual"



► 3-parton real correction ($A_3 = |M_3|^2/|M_2|^2 + \text{finite terms}; \alpha, \beta$)

$$\begin{aligned} \boxed{\chi_{+1^{(0)}}} &= \boxed{\chi_{+1^{(0)}}} - \left(\frac{\boxed{\chi_{+1^{(0)}}}}{\boxed{\text{Born}}} + \frac{4\pi\alpha_s \hat{C}_F}{s} \left(\alpha + \beta \frac{s_{ar} + s_{rb}}{s} \right) \right) \boxed{\text{Born}} \\ &= -\frac{4\pi\alpha_s \hat{C}_F}{s} \left(\alpha + \beta \frac{s_{ar} + s_{rb}}{s} \right) |M_2^{(0)}|^2 \quad \Rightarrow \end{aligned}$$

Finite terms cancel in 3-parton \mathcal{O}

► 2-parton virtual correction (same example)

$$\begin{aligned} \boxed{\chi^{(1)}} &= \boxed{\chi^{(1)}} + \boxed{\text{Born}} \int_0^s \frac{d\Phi_3}{d\Phi_2} \quad \boxed{\text{LL}} + \int_0^{Q_{\text{had}}^2} \frac{d\Phi_3}{d\Phi_2} \quad \boxed{\chi_{+1^{(0)}}} \\ &= \frac{\alpha_s \hat{C}_F}{2\pi} \left(2I_{q\bar{q}}^{(1)}(\epsilon, s) - 4 - 2I_{q\bar{q}}^{(1)}(\epsilon, s) + \frac{19 + \alpha + \frac{2}{3}\beta}{4} \right) \boxed{\text{Born}} \\ &= \frac{\alpha_s}{\pi} \left(1 + \frac{1}{3} \left(\alpha + \frac{2}{3}\beta \right) \right) \boxed{\text{Born}} \quad \Rightarrow \end{aligned}$$

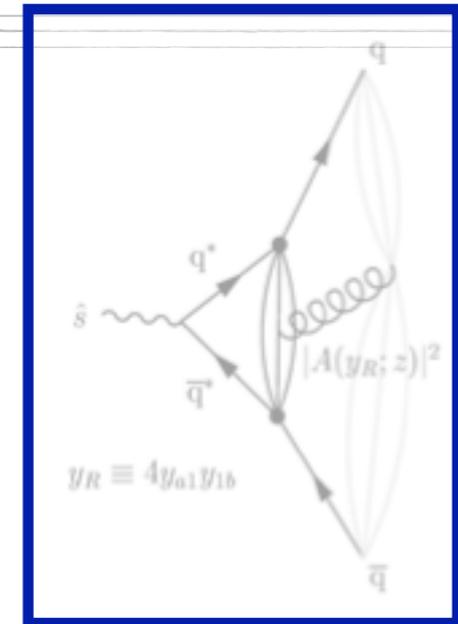
Finite terms cancel in 2-parton \mathcal{O} (normalization)

NLO Matching in 1 Slide

► First Order Shower expansion

PS $\int d\Phi_2 |M_2^{(0)}|^2 \int_{Q_{\text{had}}^2}^s \frac{d\Phi_3}{d\Phi_2} A_{q\bar{q}}(\dots) \delta(\mathcal{O} - \mathcal{O}(\{p\}_3))$

Unitarity of shower \rightarrow 3-parton real = 2-parton "virtual"



► 3-parton real correction ($A_3 = |M_3|^2/|M_2|^2 + \text{finite terms}; \alpha, \beta$)

$$w_3^{(R)} = |M_3^{(0)}|^2 - \left(A_3^0(\dots) + \frac{4\pi\alpha_s \hat{C}_F}{s} \left(\alpha + \beta \frac{s_{ar} + s_{rb}}{s} \right) \right) |M_2^{(0)}|^2$$

$$= -\frac{4\pi\alpha_s \hat{C}_F}{s} \left(\alpha + \beta \frac{s_{ar} + s_{rb}}{s} \right) |M_2^{(0)}|^2 \quad \Rightarrow$$

Finite terms cancel in 3-parton \mathcal{O}

► 2-parton virtual correction (same example)

$$w_2^{(V)} = 2\text{Re} [M_2^{(1)} M_2^{(0)*}] + |M_2^{(0)}|^2 \int_0^s \frac{d\Phi_3}{d\Phi_2} A_{q\bar{q}}(\dots) + \int_0^{Q_{\text{had}}^2} \frac{d\Phi_3}{d\Phi_2} w_3^{(R)}$$

$$= \frac{\alpha_s \hat{C}_F}{2\pi} \left(2I_{q\bar{q}}^{(1)}(\epsilon, s) - 4 - 2I_{q\bar{q}}^{(1)}(\epsilon, s) + \frac{19 + \alpha + \frac{2}{3}\beta}{4} \right) |M_2^{(0)}|^2$$

$$= \frac{\alpha_s}{\pi} \left(1 + \frac{1}{3} \left(\alpha + \frac{2}{3}\beta \right) \right) |M_2^{(0)}|^2 \quad \Rightarrow$$

Finite terms cancel in 2-parton \mathcal{O} (normalization)

Matching - Summary

LL Showers are correct

When all emissions are strongly ordered
(= dominant QCD structures)

But they are unproductive for hard jets

Often too soft (but not guaranteed! Can be too hard!)

Matrix elements are correct

When all jets are hard and no hierarchies
(single-scale problem)

(= small corner of phase space, but an important one!)

But they are unproductive for strongly ordered emissions

ME-PS matching → study both regions with ONE sample

Approaches on the Market

Hw/Py standalone

1st order matching for many processes, especially resonance decays

AlpGen + Hw/Py

MLM with HW or PY

NOTE: If you just write "AlpGen" on a plot, we assume AlpGen standalone! (no showering or matching!) - very different from Alp+Py/Hw

MadGraph + Hw/Py

MLM with HW or PY

Sherpa

CKKW + CS-dipole showers

Ariadne

CKKW-L + Lund-dipole showers

MC@NLO

NLO with subtraction, 10% $w < 0$
+ Herwig showers

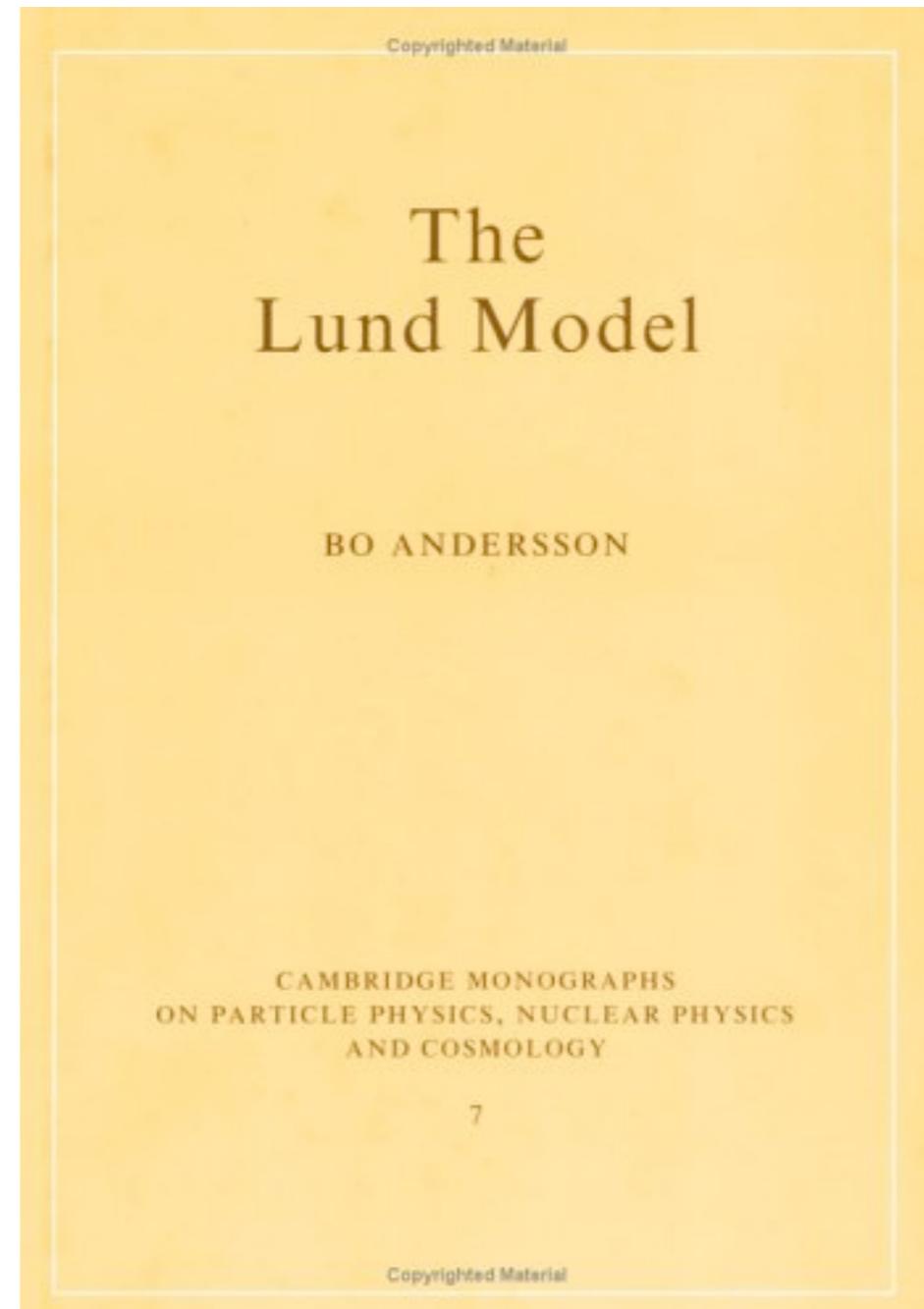
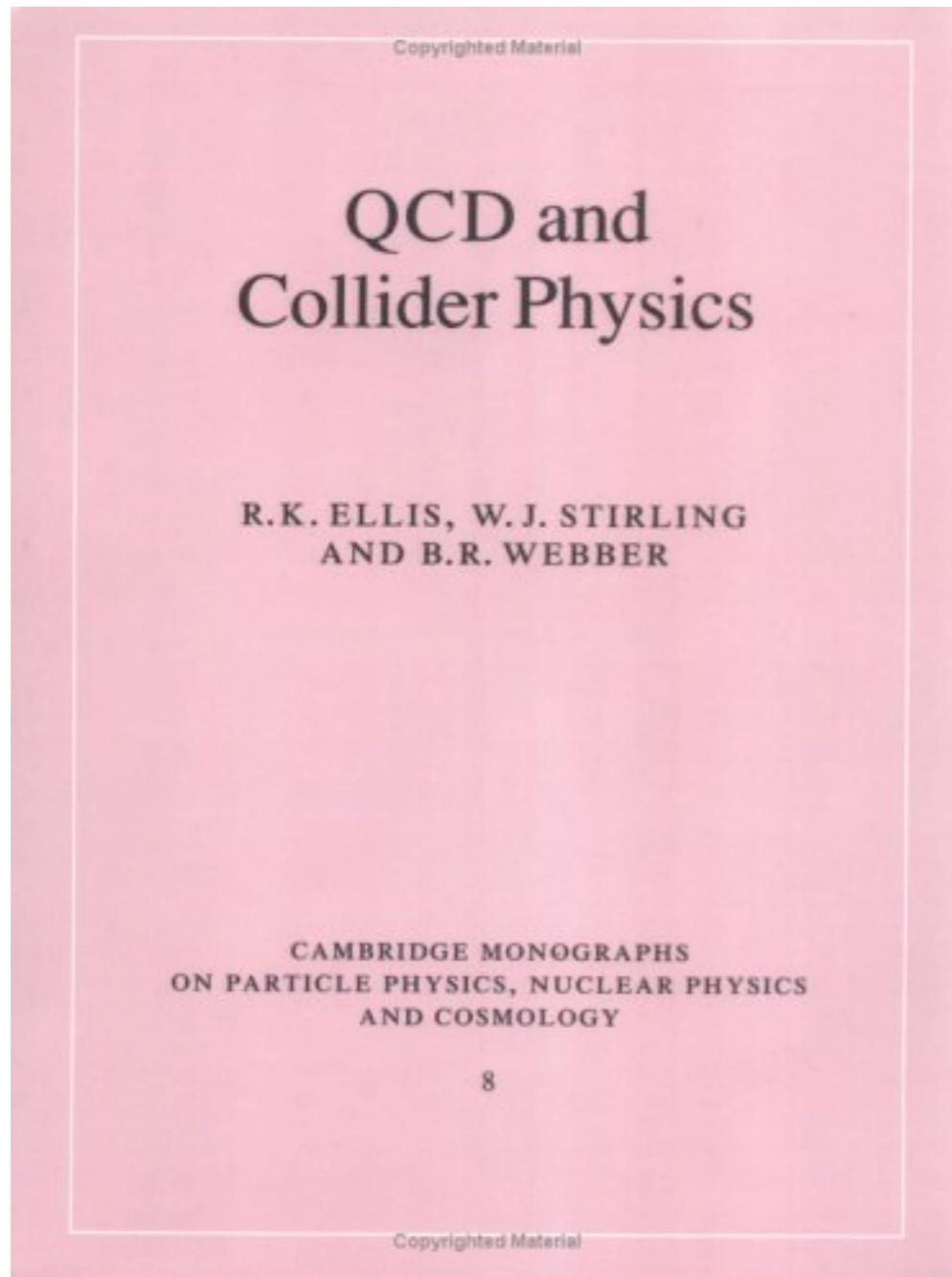
POWHEG

NLO with merging; 0% $w < 0$
+ "truncated" showers

(Vincia+Py8)

NLO + multileg a la GeeKS
+ dipole-antenna showers

Recommended Reading



Recommended Reading

RTFM : Pythia 6.4 physics and manual

Contains useful pheno-oriented introductions to many topics, \approx 600p
Sjöstrand, Mrenna, PS; [hep-ph/0603175](#)

Les Houches Guidebook to MC Generators

Sections on PDFs, matching, fixed order, etc (+ MCnet update \approx 2011)
M. Dobbs et al., [hep-ph/0403045](#)

Les Houches Accords for generators

Les Houches conventions + LHEF file structure
[hep-ph/0109068](#) (org LHA conventions) + [hep-ph/0609017](#) (LHEF) + [hep-ph/0712.3311](#) (BSM-LHEF)

Papers on Multiple Parton Interactions

Sjöstrand, van Zijl; [Phys.Rev.D36\(1987\)2019](#) (main ideas + org MPI model + pheno)
Sjöstrand, PS; [hep-ph/0408302](#) (interleaved model) & [hep-ph/0402078](#) (beam remnants)
Butterworth, Forshaw, Seymour [hep-ph/9601371](#) (JIMMY), + see hepforge

Additional Slides

PDF DGLAP : Details

We Wrote:
$$\frac{dq(x, \mu^2)}{d \ln \mu^2} = \frac{\alpha_s}{2\pi} \underbrace{\int_x^1 dz P_{qq}(z) \frac{q(x/z, \mu^2)}{z}}_{P_{qq} \otimes q},$$

More properly, it's a gain-loss equation (same equation, rewritten):

$$\frac{dq(x, \mu^2)}{d \ln \mu^2} = \frac{\alpha_s}{2\pi} \int_x^1 dz p_{qq}(z) \frac{q(x/z, \mu^2)}{z} - \frac{\alpha_s}{2\pi} \int_0^1 dz p_{qq}(z) q(x, \mu^2)$$

p_{qq} is real $q \leftarrow q$ splitting kernel: $p_{qq}(z) = C_F \frac{1+z^2}{1-z}$

First term: some partons flow from higher $y=x/z$ to x (POSITIVE)

Second term: some partons at x flow to lower $y=zx$ (NEGATIVE)

How can they be the same equation?

PDF DGLAP : Details

Awkward to write real and virtual parts separately. Use more compact notation:

$$\frac{dq(x, \mu^2)}{d \ln \mu^2} = \frac{\alpha_s}{2\pi} \underbrace{\int_x^1 dz P_{qq}(z) \frac{q(x/z, \mu^2)}{z}}_{P_{qq} \otimes q}, \quad P_{qq} = C_F \left(\frac{1+z^2}{1-z} \right)_+$$

This involves the *plus prescription*:

$$\int_0^1 dz [g(z)]_+ f(z) = \int_0^1 dz g(z) f(z) - \int_0^1 dz g(z) f(1)$$

$z = 1$ divergences of $g(z)$ cancelled if $f(z)$ sufficiently smooth at $z = 1$