Beyond the Standard Model

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Outline:

• The Standard Model: symmetries, consistency, and reasons for improvement
• Grand Unified Theories
• The strong CP-problem and axions
• The hierarchy problem
• Supersymmetry
• Composite/PGB Higgs and Higgsless models
• Extra dimensions
What you must know:

There is a relatively simple QFT that explains “almost” all data:

\[ \text{SU(3)xSU(2)xU(1)} \]

\[ Q_L : (3, 2, 1/3) \]
\[ u_R : (3, 1, 4/3) \]
\[ d_R : (3, 1, -2/3) \]
\[ l_L : (1, 2, -1) \]
\[ e_R : (1, 1, -2) \]
\[ H : (1, 2, 1) \]

Q = \( Y/2 + T_3 \)

+ **Gravity** (General Relativity)
Relatively simple lagrangian for the SM:

$$\mathcal{L}_{\text{SM}} = -\frac{1}{4g'^2} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4g^2} W_{\mu\nu} W^{\mu\nu} - \frac{1}{4g_s^2} G_{\mu\nu} G^{\mu\nu}$$

$$+ i\bar{Q}_L^i \hat{D} Q_L^i + i\bar{u}_R^i \hat{D} u_R^i + i\bar{d}_R^i \hat{D} d_R^i + i\bar{l}_L^i \hat{D} l_L^i + i\bar{e}_R^i \hat{D} e_R^i$$

$$+ |D_{\mu} H|^2$$

$$+ Y_u^{ij} Q_L^i \tilde{H} u_R^j + Y_d^{ij} Q_L^i H d_R^j + Y_e^{ij} l_L^i H e_R^j + \text{h.c.}$$

$$+ V(H)$$

+ we are, for the moment, neglecting neutrino masses!
Apart from kin. terms + masses, it gives interactions:

**Gauge:**

**Yukawa:**

**Self-Higgs:**

$g, Y_f, \lambda = \text{dimensionless couplings}$
Only one unknown parameter:

**The Higgs mass**

(or $\lambda$)

**Experimental bounds:**

**LEP searches** + **EW Precision Tests**

$e^+ \rightarrow Z^* h$

$e^- \rightarrow Z^* h$

$M_h = 120 \text{ GeV}$

$M_h = 1 \text{ TeV}$

$\Rightarrow 114 \text{ GeV} < M_h < 186 \text{ GeV}$ (95%CL)
SM Lagrangian dictated by symmetries: **Gauge** + (local) **Poincare** symmetries

when gravity is included

Can explain “almost” everything from the biggest to the smallest...

“Symmetries are the keystone of the universe”
The SM has also extra “accidental” symmetries:
We didn’t ask for them, but they are there!

Are Global Symmetries: \( \psi \rightarrow e^{iB\theta} \psi \)

1) Baryon number \( B \):

\( B=1/3 \) (quarks), \( B=0 \) (leptons, Higgs)

Proton \( B=1 \): Cannot decay to leptons

\( \uparrow \uparrow \uparrow \) \( \uparrow \uparrow \uparrow \)

\( \text{caveat: This symmetry is “anomalous” and proton could decay but with an extremely small rate} \)

2) Lepton number \( L_e, L_\mu, L_\tau \):

\( L_e=1 \) (for e), \( L_\mu=1 \) (for \( \mu \)), \( L_\tau=1 \) (for \( \tau \)) \( \) (zero for the rest)

\( \mu \) cannot decay to e+photon
Some **accidental** symmetries are approximate (broken by small couplings)

1) **Custodial symmetry:**

- In the limit $Y_f = 0$ and $g' = 0$

  **Extra global SU(2):** $H$ being a doublet

  when it gets a VEV: $\text{SU}(2)_L \times \text{SU}(2) \rightarrow \text{SU}(2)_c$

  $(W^+, W^-, Z)$ are a triplet of $\text{SU}(2)_c$ $\Rightarrow m_W = m_Z$

- For $Y_f \neq 0$ and $g' \neq 0$: $\frac{m_W^2}{m_Z^2 c_\theta^2 W} \equiv \rho \simeq 1.0$
2) Family symmetry:

In the limit all $Y_f = 0$:

$$U(3)_Q \times U(3)_u \times U(3)_d \times U(3)_L \times U(3)_e$$

In the limit $Y_f = 0$ for 1st + 2nd family:

$$U(2)_Q \times U(2)_u \times U(2)_d \times U(2)_L \times U(2)_e$$

$\rightarrow$ Small $K$-$\bar{K}$ mixing
...but these **accidental symmetries** of the SM are only symmetries of the dimension-4 operators:

Dimensional analysis \( (\hbar = c = 1) \) tell us that

\[
[S = \int \mathcal{L} \, d^4 x] = M^0
\]
\[
[\mathcal{L}] = M^4
\]
\[
[\partial_\mu] = M
\]
\[
[H] = [A_\mu] = M
\]
\[
[\psi] = M^{3/2}
\]

All SM terms in the Lagrangian have dimension 4
Why we don’t include terms like

e.g. \((W^{\mu\nu}W_{\mu\nu})^2\) ?

They are allowed by symmetries!

It has dim=8, so in the Lagrangian should be written as

\[
\frac{1}{\Lambda^4}(W^{\mu\nu}W_{\mu\nu})^2
\]

\(\Lambda = \) some scale suppressing the higher-dim terms

This new terms spoil the predictivity of the SM:

We have infinite of them!

It’s OK, for physics at scales smaller than \(\Lambda\):

\[
\frac{1}{\Lambda^4}(W^{\mu\nu}W_{\mu\nu})^2 \rightarrow \text{small effects}
\]
... but, even worse, higher-dim terms don’t respect the accidental symmetries of the SM:

**L violation:**

\[
\frac{1}{\Lambda} \bar{l}^c_L H_i H_j l^j_L
\]

**B violation:**

\[
\frac{1}{\Lambda^2} \epsilon^{\alpha\beta\gamma} [\bar{Q}^c_L \gamma^\mu u_R \beta] [\bar{d}^c_R \gamma^\mu l_L \alpha]
\]

**Proton decay:**

\[ p \rightarrow \pi^0 e^+ \]

Exp. \( \tau_p > 10^{34} \text{ years} \) \( \Rightarrow \Lambda > 10^{15} \text{ GeV} \)
Lessons so far:

• The SM Lagrangian (based on local symmetries) has extra global symmetries (B,L,...)

• Extra terms (suppressed by $\Lambda$) could be added (preserving local symmetries) but are dangerous since break the symmetries (B,L,...)

We have to require $\Lambda$ be very large

$\Rightarrow$ can we take it to be infinity?
Is there any need to go beyond the SM ($\Lambda \neq \infty$)?

Theoretical: Consistency of the theory?

Experimental: Data that cannot be explained?
Could it be the the SM the final theory?

We must use Einstein “Gedankenexperiment” (thought experiments):

“...at the age of sixteen: If I pursue a beam of light with the velocity c (velocity of light in a vacuum), I should observe such a beam of light as an electromagnetic field at rest though spatially oscillating. There seems to be no such thing...”
Scattering at high-energies $\gg M_w$

\[ \hbar \leftrightarrow \lambda \hbar + \text{loops} \sim \hbar \lambda(Q) \hbar \]

Dictated by RG evolution:

\[ \frac{d\lambda}{d \ln Q} = \frac{1}{16\pi^2} \left( 24\lambda^2 + 12\lambda Y_t^2 - 6Y_t^4 \right) + \cdots \]

where \( Q \sim E_{cm} \)

“velocity” of growth of \( \lambda(Q) \)

Espinosa
• If $\lambda(Q)$ grows, as we increase $Q$, it can become too large at some scale $\Lambda$:

$$\lambda(Q=\Lambda) \sim \pi$$

(perturbation theory not valid anymore)

• If $\lambda(Q)$ decreases, it can become negative at some scale $Q = \Lambda$:

$\Rightarrow$ Unstable Higgs potential

\[ \Lambda \equiv "\text{Cut-off scale}" \quad \Rightarrow \quad \text{I cannot trust my theory at } Q > \Lambda \]

Since

$$M_h^2 = 2\lambda(Q = M_H)v^2$$

for each Higgs mass there is a scale $\Lambda$
Figure 2: The scale $\Lambda$ at which the two-loop RGEs drive the quartic SM Higgs coupling non-perturbative, and the scale $\Lambda$ at which the RGEs create an instability in the electroweak vacuum ($\lambda < 0$). The width of the bands indicates the errors induced by the uncertainties in $m_t$ and $\alpha_S$ (added quadratically). The perturbativity upper bound (so sometimes referred to as 'triviality' bound) is given for $\lambda = \pi$ (lower bold line [blue]) and $\lambda = 2\pi$ (upper bold line [blue]). Their difference indicates the size of the theoretical uncertainty in this bound. The absolute vacuum stability bound is displayed by the light shaded [green] band, while the less restrictive finite-temperature and zero-temperature metastability bounds are medium [blue] and dark shaded [red], respectively. The theoretical uncertainties in these bounds have been ignored in the plot, but are shown in Fig. 3 (right panel). The grey hatched areas indicate the LEP [1] and Tevatron [2] exclusion domains.

We also consider the prospects for gathering more information about the fate of the SM in the near future. The Tevatron search for the SM Higgs boson will extend its sensitivity to both higher and lower $M_H$, and the LHC will extend the game. It is anticipated that the LHC has the sensitivity to extend the Tevatron exclusion down to 127 GeV or less with 1 fb$^{-1}$ of well-understood data at 14 TeV centre-of-mass energy [9]. This would decrease the relative likelihood of the 'survival' scenario, but not sufficiently to exclude it with any significance. On the other hand, discovery of a Higgs boson weighing 120 GeV or less would

$$\Lambda = 10^{19} \text{ GeV}$$
... but as $Q \sim 10^{19}$ GeV, gravitons are also important:

$$G_N = \frac{1}{M_P^2}$$

$G_N = \text{Newton's constant}$

$M_P = \text{Planck's mass} \sim 1.2 \times 10^{19}$ GeV

at $Q > M_P$ violation of unitarity
~ quantum loops of gravitons important

SM+GR not a consistent quantum theory at $Q > M_P$!

New physics expected (at least)
at energies $\sim 10^{19}$ GeV!
Very similar to Fermi’s theory:

\[ \sim G_F Q^2 \]

\[ G_F = \text{Fermi's constant} \]

We know what happened at \( Q \sim 1/\sqrt{G_F} \sim 300 \text{ GeV} \):

There was **New physics** (beyond Fermi’s theory):

We discovered the W/Z particles, the SM!
Could it be the SM the final theory? NO!
What could we find at $M_P \sim 10^{19}$ GeV?

A possibility (the only one?): **STRINGS**

Particles are the lowest-energy modes of a string
Two types of strings:

gravitons, gauge bosons and matter appear as massless excitations of the strings

theory of unification
Predictions:

1) The space must be 1+9 dimensional

2) There are string excitations of higher-energy:

\[ M_P \geq M_{\text{string}} \]

\[ 0 \]

“\text{The only prediction of string theory is that there are no predictions}”

Anonymous

... we will come back later to further explore these implications!
Data unexplained by the SM

1) Neutrino masses
2) Dark matter
3) Cosmological Inflationary epoch
4) Matter/Antimatter asymmetry in the universe

Nevertheless all these evidences could be explained by physics close to the Planck Scale. No deep reasons for a lower value of $\Lambda \sim M_P$. 
e.g. neutrino masses:

\[ \frac{1}{\Lambda} l_L H C l_L H \]

\[ m_\nu \sim \frac{v^2}{\Lambda} \sim 0.06 \text{ eV} \left( \frac{10^{15} \text{GeV}}{\Lambda} \right) \]
But there are other important reason to go beyond the SM

Search for a “natural” explanation of SM coupling-constants and masses
Search for a “natural” explanation of SM coupling-constants and masses:

1) **Cosmological constant:** $\int \Lambda_{\text{cosmo}} \sqrt{g} \, d^4x$

   $\Lambda_{\text{cosmo}} \sim 10^{-47} \, \text{GeV}^4 \ll \Lambda^4 \sim M_P^4 \sim 10^{76} \, \text{GeV}^4$

2) **Higgs mass term:** $V(H) = -\mu^2 |H|^2 + ...$

   $\mu^2 \sim v^2 \sim 10^4 \, \text{GeV}^2 \ll \Lambda^2 \sim M_P^2 \sim 10^{38} \, \text{GeV}^2$

3) **Charge quantization:**

   $Q_e + Q_p < 10^{-21}$

4) **Strong CP problem:** $\int \theta FF \, d^4x$

   $\theta < 10^{-13}$
5) Fermion masses and mixing angles:

\[ V_{\text{CKM}} = \begin{pmatrix}
0.97419 \pm 0.00022 & 0.2257 \pm 0.0010 & 0.00359 \pm 0.00016 \\
0.2256 \pm 0.0010 & 0.97334 \pm 0.00023 & 0.0415^{+0.0010}_{-0.0011} \\
0.00874^{+0.00026}_{-0.00037} & 0.0407 \pm 0.0010 & 0.999133^{+0.000044}_{-0.000043}
\end{pmatrix} \]

6) Gauge couplings:

\[ g' \sim 0.35 \quad g \sim 0.65 \quad g_s \sim 1.12 \quad \text{at } Q \sim M_Z \]

7) Number of families:

\[ N_f = 3 \]
## Search for a “natural” explanation

<table>
<thead>
<tr>
<th>New physics scale</th>
<th>Cosmological constant</th>
<th>Higgs potential</th>
<th>Charge quantization</th>
<th>Strong CP problem</th>
<th>Fermion masses/mixing angles</th>
<th>Gauge couplings</th>
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<tr>
<td></td>
<td>?</td>
<td>~ TeV</td>
<td>~ $10^{15}$ GeV</td>
<td>~ $10^{12}$ GeV</td>
<td>TeV - $M_p$</td>
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### Search for a "natural" explanation

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To be discussed here
Grand Unified Theories (GUT)
We want to explain:

\[ |q_p + q_e|/e \]

See DYLLA 73 for a summary of experiments on the neutrality of matter. See also “n CHARGE” in the neutron Listings.

<table>
<thead>
<tr>
<th>VALUE</th>
<th>DOCUMENT ID</th>
<th>COMMENT</th>
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<tr>
<td>(&lt;1.0 \times 10^{-21})</td>
<td>8</td>
<td>Neutrality of SF6</td>
</tr>
<tr>
<td>• • • We do not use the following data for averages, fits, limits, etc. • • •</td>
<td></td>
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<tr>
<td>(&lt;3.2 \times 10^{-20})</td>
<td>9</td>
<td>binary pulsar</td>
</tr>
<tr>
<td>(&lt;0.8 \times 10^{-21})</td>
<td>MARINELLI</td>
<td>Magnetic levitation</td>
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\(^8\) Assumes that \(q_n = q_p + q_e\).

\(^9\) SENGUPTA 00 uses the difference between the observed rate of of rotational energy loss by the binary pulsar PSR B1913+16 and the rate predicted by general relativity to set this limit. See the paper for assumptions.

\[ \Rightarrow \text{suggest that the charge is quantized: } Q_p = - Q_e \]

\[ Q = Y/2 + T_3 \]

\[ u_R, d_R, Q_L, e_L, e_R: \quad Y = (4/3, -2/3, 1/3, -1, -2) \]
The $U(1)$ hypercharges will be quantized if it is embedded in a non-abelian group $SU(2)$.

Quantized since it comes from a non-abelian group $SU(2)$.

The $U(1)$ hypercharges will be quantized if it is embedded in a **non-abelian group:**

Minimal case: $SU(4) \times SU(2) \times SU(2)$  

Pati-Salam 74

Simple group: $SU(5)$

Glashow, Georgi 74
SU(5) model

Embedding: \( SU(3) \times SU(2) \times U(1) \subset SU(5) \)

Extra gauge bosons \( X, Y \) associated to the new generators: \( 24-8-3-1=12 \) fields

Complex fields of SM charges = \( (3, 2, -5/3) \)

Not seen \( \rightarrow \) must be massive: mass = \( M_{GUT} \)
Matter embedding: \( 15 \text{ fields} \subset \mathbf{5} + \mathbf{10} \)

\[
\mathbf{5} = \begin{pmatrix} d^c_1 \\ d^c_2 \\ d^c_3 \\ e^- \\ -\nu_e \end{pmatrix} \quad \quad 10 = \begin{pmatrix} 0 & u^c_3 & -u^c_2 & -u_1 & -d_1 \\ 0 & u^c_1 & -u_2 & -d_2 \\ -u_3 & -d_3 & 0 & -e^c \\ 0 & 0 & 0 & 0 \end{pmatrix}
\]

Fit like a glove!

Not the same simplicity for the Higgs
(Doublet-triplet splitting problem)
The GUT-gauge symmetry must be broken (not seen in nature the $X,Y$ bosons):

\[ \text{SU}(5) \rightarrow \text{SU}(3) \times \text{SU}(2) \times \text{U}(1) \]

Extra “Higgs” in 24 getting VEV

Give mass only to $X,Y$ bosons: $M_{X,Y} = M_{\text{GUT}}$
SU(5) predictions:

1) Charge quantization

2) Gauge-coupling unification:

\[ g_5 = g_s = g = \sqrt{5/3} \, g' \quad \text{at} \quad Q \geq M_{\text{GUT}} \]

3) Proton decay:

\[ p \rightarrow \pi^0 e^+ \quad \text{proton} \]

\[ \Lambda \sim M_{\text{GUT}} \]

Exp. \( \tau_p > 10^{34} \, \text{years} \)

\[ M_{\text{GUT}} > 3 \times 10^{15} \, \text{GeV} \]

\[ d \rightarrow \pi^0 e^+ \quad \text{pion} \]
2) Gauge-coupling unification:

\[ g_5 = g_s = g = \sqrt{5/3} \, g' \quad \text{at} \quad Q \geq M_{\text{GUT}} \]

What are the values of the SM gauge-couplings at high-energies?

\[ A \xrightarrow{\text{+ loops}} \sim A \xrightarrow{g(Q)} f \]

\( g \) dependence with \( Q \) dictated by the SM spectrum can be calculated
RG equations: \[
\frac{dg_i^{-2}}{d \ln Q} = - \frac{b_i}{8\pi^2}
\]
\[
g_1 = \sqrt{\frac{5}{3}} g'
\]
\[
g_2 = g
\]
\[
g_3 = g_s
\]

b-coefficients depend on the particle spectrum

<table>
<thead>
<tr>
<th></th>
<th>SM</th>
<th>MSSM</th>
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</table>
| $b_i$    | \[
\begin{pmatrix}
\frac{41}{10} \\
-\frac{19}{6} \\
-7
\end{pmatrix}
\] | \[
\begin{pmatrix}
\frac{66}{10} \\
1 \\
-3
\end{pmatrix}
\] |
\[ g_1 = \sqrt{\frac{5}{3}} g' \]
\[ g_2 = g \]
\[ g_3 = g_s \]

\[ \alpha = \frac{g^2}{4\pi} \]

**Standard Model**

\[ \alpha_1^{-1} \]
\[ \alpha_2^{-1} \]
\[ \alpha_3^{-1} \]

**Logarithmic Scale**

\[ \log_{10}(Q/\text{GeV}) \]
SM+SUSY partners (to be discussed later):

\[ \alpha = \frac{g^2}{4\pi} \]

Supersymmetric Standard Model

\[ M_{\text{SUSY}} = M_Z \]

Too good to be true?  
Langacker, Polonsky 93
Search for proton decay
The Super-Kamiokande detector

• Stainless-steel tank
• 39m diameter and 42m tall
• Filled with 50,000 tons of ultra pure water.
• About 13,000 photo-multipliers on the tank wall
• At 1000 meter underground in the Kamioka-mine, Hida-city, Gifu, Japan.

Present experimental limit:

$$\tau_p > 10^{34} \text{ years}$$

$$\Rightarrow M_{\text{GUT}} > 3 \times 10^{15} \text{ GeV}$$
Other GUT’s beauties:

- Bottom-tau unification: $M_b = M_\tau$ at $Q \geq M_{GUT}$
  
  works reasonably well in the Supersymmetric SM
  
  ...but don’t work for other fermions

- $SO(10)$ model: Matter $16 = \bar{5} + 10 + 1$
  
  right-handed neutrino
  
  see-saw mechanism for neutrino masses
Implications: Majorana masses for neutrino

Neutrinoless Double Beta Decay:
The strong CP Problem
Dimension 4 operator allowed in QCD:

$$\theta \frac{g_s^2}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} G_{\mu\nu} \cdot G_{\rho\sigma}$$

Violates CP and induce a large EDM for the neutron.
Experimental limits give:

$$\theta \lesssim 10^{-10}$$

Why so small?
Peccei-Quinn axion

Promote $\theta$ to a scalar-field $a(x) \equiv \text{axion}$:

$$a(x) \frac{g_s^2}{32\pi^2} \frac{1}{f_a} \varepsilon^{\mu\nu\rho\sigma} G_{\mu\nu} \cdot G_{\rho\sigma} + \text{kinetic term}$$

No other couplings (possible by global symmetries: $a=\text{PGB}$)

At low-energies ($\sim \text{GeV}$) a potential will be generated:

$$V(a) \propto a(x)^2 + \cdots \quad \Rightarrow \quad a(x) = 0 \quad \Rightarrow \quad \theta = 0$$

The axion gets also a mass:

$$m_a = \frac{f_\pi}{f_a} \frac{\sqrt{m_u m_d}}{m_u + m_d} m_\pi$$

the larger $f_a$, the smaller its coupling to SM states, and the smaller its mass
Main searches through its coupling to 2 photons:

![Diagram of photon coupling](image)

Strong constraints from limits on energy losses in stars, SN,...

If \( a \) exists, the sun will lose energy by emitting it.
Excluded regions: 
(slightly model dependent)
CAST Experiment

Detecting axions coming from the sun
**ADMX Experiment**

If *axions* are DM:

- Halo axions enter cavity
- Axions scatter off B field
- Resonantly convert to microwave photons
- Excess photons observed above thermal noise