5. Flavour Dynamics

- **Fermion Masses**
- **Fermion Generations**
- **Quark Mixing**
- **Lepton Mixing**
- **Standard Model Parameters**
- CP **Violation**

Quarks Leptons Bosons

FERMION MASSESS

Scalar – Fermion Couplings allowed by Gauge Symmetry

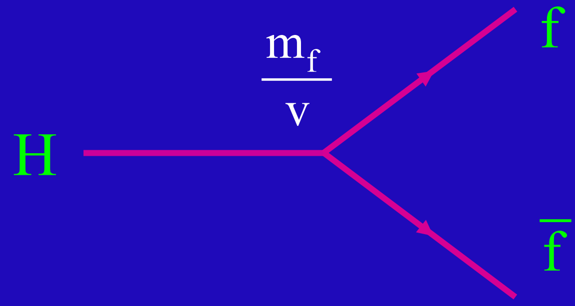
$$\mathcal{L}_Y = -(\bar{q}_u, \bar{q}_d)_L \left[c^{(d)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} (q_d)_R + c^{(u)} \begin{pmatrix} \phi^{(0)\dagger} \\ -\phi^{(+)\dagger} \end{pmatrix} (q_u)_R \right] - (\bar{\nu}_l, \bar{l})_L \, c^{(l)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} l_R + \text{h.c.}$$

SSB

$$\mathcal{L}_Y = -\left(1 + \frac{H}{v}\right) \left\{ m_{q_d} \, \bar{q}_d \, q_d + m_{q_u} \, \bar{q}_u \, q_u + m_l \, \bar{l} \, l \right\}$$

Fermion Masses are New Free Parameters

$$\left[m_{q_d}, m_{q_u}, m_l \right] = \left[c^{(d)}, c^{(u)}, c^{(l)} \right] \frac{v}{\sqrt{2}}$$



Couplings Fixed: $g_{Hf\bar{f}} = \frac{m_f}{v}$

FERMION GENERATIONS

$N_G = 3$ Identical Copies

Masses are the only difference

$$\begin{matrix} Q=0 \\ Q=-1 \end{matrix} \quad \begin{pmatrix} \nu'_j & u'_j \\ l'_j & d'_j \end{pmatrix} \quad \begin{matrix} Q=+2/3 \\ Q=-1/3 \end{matrix} \qquad (j = 1, \cdots, N_G)$$

WHY ?

$$\mathcal{L}_Y = -\sum_{jk} \left\{ \left(\bar{u}'_j, \bar{d}'_j\right)_L \left[c_{jk}^{(d)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} d'_{kR} + c_{jk}^{(u)} \begin{pmatrix} \phi^{(0)\dagger} \\ -\phi^{(+)\dagger} \end{pmatrix} u'_{kR} \right] - \left(\bar{\nu}'_j, \bar{l}'_j\right)_L c_{jk}^{(l)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} l'_{kR} \right\} + \text{h.c.}$$

SSB

$$\mathcal{L}_Y = -\left(1 + \frac{H}{\text{v}}\right) \left\{ \bar{d}'_L \cdot \mathbf{M}'_d \cdot d'_R + \bar{u}'_L \cdot \mathbf{M}'_u \cdot u'_R + \bar{l}'_L \cdot \mathbf{M}'_l \cdot l'_R + \text{h.c.} \right\}$$

Arbitrary Non-Diagonal Complex Mass Matrices

$$\left[\mathbf{M}'_d \,,\, \mathbf{M}'_u \,,\, \mathbf{M}'_l \right]_{jk} = \left[c_{jk}^{(d)} \,,\, c_{jk}^{(u)} \,,\, c_{jk}^{(l)} \right] \frac{\text{v}}{\sqrt{2}}$$

DIAGONALIZATION OF MASS MATRICES

$$\mathbf{M}'_d = \mathbf{H}_d \cdot \mathbf{U}_d = \mathbf{S}_d^\dagger \cdot \mathcal{M}_d \cdot \mathbf{S}_d \cdot \mathbf{U}_d$$
$$\mathbf{M}'_u = \mathbf{H}_u \cdot \mathbf{U}_u = \mathbf{S}_u^\dagger \cdot \mathcal{M}_u \cdot \mathrm{S}_u \cdot \mathbf{U}_u$$
$$\mathbf{M}'_l = \mathbf{H}_l \cdot \mathbf{U}_l = \mathbf{S}_l^\dagger \cdot \mathcal{M}_l \cdot \mathbf{S}_l \cdot \mathbf{U}_l$$

$$\mathbf{H}_\mathrm{f} = \mathbf{H}_\mathrm{f}^\dagger$$
$$\mathbf{U}_\mathrm{f} \cdot \mathbf{U}_\mathrm{f}^\dagger = \mathbf{U}_\mathrm{f}^\dagger \cdot \mathbf{U}_\mathrm{f} = 1$$
$$\mathbf{S}_\mathrm{f} \cdot \mathbf{S}_\mathrm{f}^\dagger = \mathbf{S}_\mathrm{f}^\dagger \cdot \mathbf{S}_\mathrm{f} = 1$$

$$\mathcal{L}_Y = -\left(1 + \frac{H}{\mathrm{v}}\right) \left\{ \bar{\mathrm{d}} \cdot \mathcal{M}_d \cdot \mathrm{d} + \bar{\mathrm{u}} \cdot \mathcal{M}_u \cdot \mathrm{u} + \bar{l} \cdot \mathcal{M}_l \cdot l \right\}$$

$$\mathcal{M}_u = \mathrm{diag}\,(m_u, m_c, m_t) \quad ; \quad \mathcal{M}_d = \mathrm{diag}\,(m_d, m_s, m_b) \quad ; \quad \mathcal{M}_l = \mathrm{diag}\,(m_e, m_\mu, m_\tau)$$

$$\mathrm{d}_L \equiv \mathbf{S}_d \cdot \mathrm{d}'_L \quad ; \quad \mathrm{u}_L \equiv \mathbf{S}_u \cdot \mathrm{u}'_L \quad ; \quad l_L \equiv \mathbf{S}_l \cdot l'_L$$
$$\mathrm{d}_R \equiv \mathbf{S}_d \cdot \mathbf{U}_d \cdot \mathrm{d}'_R \quad ; \quad \mathrm{u}_R \equiv \mathbf{S}_u \cdot \mathbf{U}_u \cdot \mathrm{u}'_R \quad ; \quad l_R \equiv \mathbf{S}_l \cdot \mathbf{U}_l \cdot l'_R$$

Mass Eigenstates
$\neq$
Weak Eigenstates

$$\bar{\mathrm{f}}'_L \, \mathrm{f}'_L = \bar{\mathrm{f}}_L \, \mathrm{f}_L \quad ; \quad \bar{\mathrm{f}}'_R \, \mathrm{f}'_R = \bar{\mathrm{f}}_R \, \mathrm{f}_R \quad \Longrightarrow \quad \mathcal{L}'_{\mathrm{NC}} = \mathcal{L}_{\mathrm{NC}}$$

$$\bar{\mathrm{u}}'_L \, \mathrm{d}'_L = \bar{\mathrm{u}}_L \cdot \mathbf{V} \cdot \mathrm{d}_L \quad ; \quad \mathbf{V} \equiv \mathbf{S}_u \cdot \mathbf{S}_d^\dagger \quad \Longrightarrow \quad \mathcal{L}'_{\mathrm{CC}} \neq \mathcal{L}_{\mathrm{CC}}$$

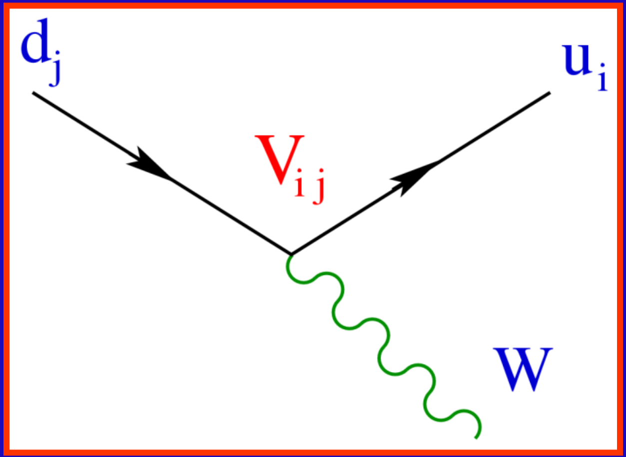
QUARK MIXING

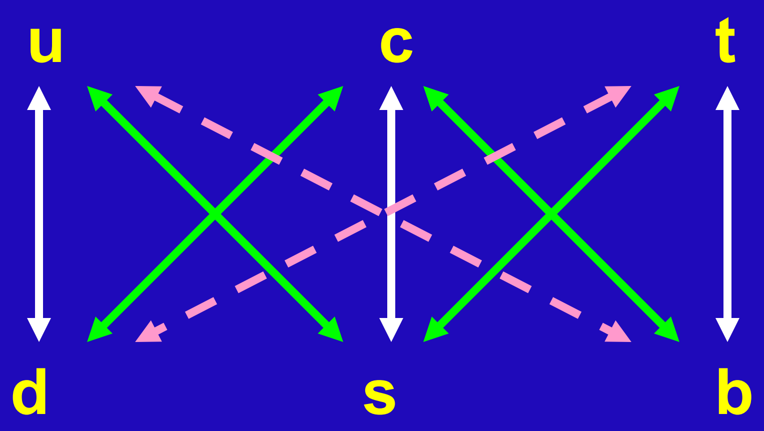
$$\mathcal{L}_{NC}^{Z} = -\frac{e}{2\sin\theta_W \cos\theta_W} Z_\mu \sum_f \bar{f}\, \gamma^\mu \left[v_f - a_f\, \gamma_5 \right] f$$

Flavour Conserving Neutral Currents

$$\mathcal{L}_{CC} = -\frac{g}{2\sqrt{2}} W_\mu^\dagger \left[\sum_{ij} \bar{u}_i\, \gamma^\mu (1-\gamma_5)\, \mathbf{V}_{ij}\, d_j + \sum_l \bar{\nu}_l\, \gamma^\mu (1-\gamma_5)\, l \right] + \text{h.c.}$$

Flavour Changing Charged Currents





LEPTON MIXING

$$\mathcal{L}_{CC}^{(l)} = -\frac{g}{2\sqrt{2}}\, W_\mu^\dagger \sum_{ij} \bar{\nu}_i\, \gamma^\mu (1-\gamma_5)\, \mathbf{V}_{ij}^{(l)}\, l_j + \text{h.c.}$$

- **IF** $m_{\nu_i} = 0$ ➡ $\mathcal{L}_{CC}^{(l)} = -\frac{g}{2\sqrt{2}}\, W_\mu^\dagger \sum_l \bar{\nu}_l\, \gamma^\mu (1-\gamma_5)\, l + \text{h.c.}$

$\bar{\nu}_{l_j} \equiv \bar{\nu}_i \; \mathbf{V}_{ij}^{(l)}$

Separate Lepton Number Conservation (Minimal SM without ν_R)

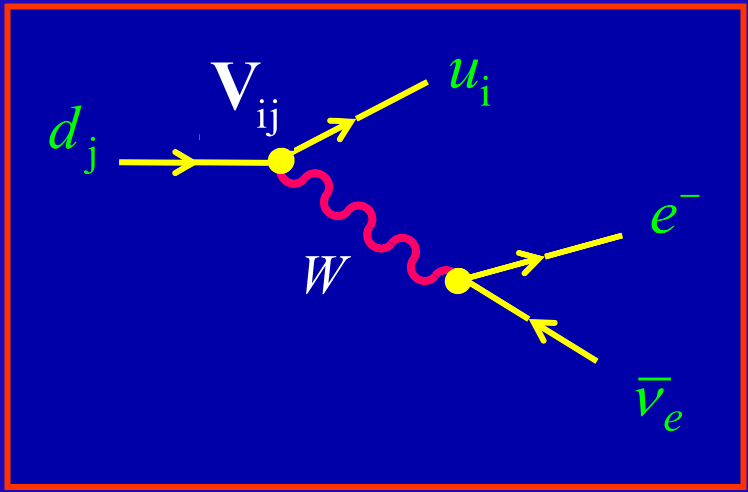
- **IF** ν_R^i exist and $m_{\nu_i} \neq 0$

$\not{L}_e\,,\ \not{L}_\mu\,,\ \not{L}_\tau$ $\quad (L_e + L_\mu + L_\tau$ Conserved$)$

BUT $\quad \text{Br}(\mu \to e\gamma) < 1.2\times 10^{-11} \quad ; \quad \text{Br}(\tau \to \mu\gamma) < 4.4\times 10^{-8}$

(90 % CL)

Measurements of V_{ij}



$$\Gamma(d_j \to u_i\, e^-\, \bar{\nu}_e) \propto |V_{ij}|^2$$

We measure decays of hadrons (no free quarks)

Important QCD Uncertainties

V_{ij} DETERMINATIONS

CKM entry	Value	Source
$\|V_{ud}\|$	0.97425 ± 0.00022 0.9746 ± 0.0019 0.9741 ± 0.0026	Nuclear β decay $n \to p e^- \bar{\nu}_e$ $\pi^+ \to \pi^0 e^+ \nu_e$
$\|V_{us}\|$	0.2246 ± 0.0012 0.2165 ± 0.0031 0.2259 ± 0.0015 0.2244 ± 0.0012	$K \to \pi e^- \bar{\nu}_e$ τ decays $K/\pi \to \mu\nu$, Lattice
$\|V_{cd}\|$	0.230 ± 0.011 0.229 ± 0.026	$\nu d \to c X$ $D \to \pi l \nu$, Lattice
$\|V_{cs}\|$	0.985 ± 0.104	$D \to K l \nu$, Lattice
$\|V_{cb}\|$	0.0386 ± 0.0011 0.0415 ± 0.0007 0.0407 ± 0.0007	$B \to D^* / D l \bar{\nu}_l$ $b \to c l \bar{\nu}_l$
$\|V_{ub}\|$	0.0034 ± 0.0004 0.0041 ± 0.0003 0.0038 ± 0.0003	$B \to \pi l \bar{\nu}_l$ $b \to u l \bar{\nu}_l$
$\|V_{tb}\| / \sqrt{\sum_q \|V_{tq}\|^2}$ $\|V_{tb}\|$	> 0.89 > 0.74 ; < 1	$t \to bW / qW$ $p\bar{p} \to tb + X$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9995 \pm 0.0010$$

$$\sum_j \left(|V_{uj}|^2 + |V_{cj}|^2 \right) = 2.002 \pm 0.027 \quad \text{(LEP)}$$

QUARK MIXING MATRIX

- **Unitary** $N_G \times N_G$ **Matrix:** N_G^2 **parameters**

$$\mathbf{V} \cdot \mathbf{V}^\dagger = \mathbf{V}^\dagger \cdot \mathbf{V} = \mathbf{1}$$

- $2\,N_G - 1$ **arbitrary phases:**

$$u_i \to e^{i\phi_i}\, u_i \quad ; \quad d_j \to e^{i\theta_j}\, d_j \qquad \Rightarrow \qquad \mathbf{V}_{ij} \to e^{i(\theta_j - \phi_i)}\, \mathbf{V}_{ij}$$

$\Downarrow$

$\mathbf{V}_{ij}$ **Physical Parameters:**

$\frac{1}{2} N_G (N_G - 1)$ **Moduli** ; $\frac{1}{2} (N_G - 1)(N_G - 2)$ **phases**

- **$N_f = 2$:** 1 angle, 0 phases (Cabibbo)

$$V = \begin{bmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{bmatrix} \;\Rightarrow\; \text{No } \not{CP}$$

- **$N_f = 3$:** 3 angles, 1 phase (CKM)

$c_{ij} \equiv \cos\theta_{ij} \;;\; s_{ij} \equiv \sin\theta_{ij}$

$$V = \begin{bmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, e^{-i\delta_{13}} \\ -s_{12}\, c_{23} - c_{12}\, s_{23}\, s_{13}\, e^{i\delta_{13}} & c_{12}\, c_{23} - s_{12}\, s_{23}\, s_{13}\, e^{i\delta_{13}} & s_{23}\, c_{13} \\ s_{12}\, s_{23} - c_{12}\, c_{23}\, s_{13}\, e^{i\delta_{13}} & -c_{12}\, s_{23} - s_{12}\, c_{23}\, s_{13}\, e^{i\delta_{13}} & c_{23}\, c_{13} \end{bmatrix}$$

$$\approx \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3\,(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\,\lambda^2 \\ A\lambda^3\,(1-\rho-i\eta) & -A\,\lambda^2 & 1 \end{bmatrix} + \mathcal{O}\left(\lambda^4\right)$$

$\lambda \approx \sin\theta_C \approx 0.225 \;;\; A \approx 0.81 \;;\; \sqrt{\rho^2+\eta^2} \approx 0.37$

$\delta_{13} \neq 0 \quad (\eta \neq 0) \;\Rightarrow\; \not{CP}$

Standard Model Parameters

QCD: $\alpha_s(M_Z)$ **1**

EW Gauge / Scalar Sector: **4**

$$g\,,g'\,,\mu^2\,,h \;\Leftrightarrow\; \alpha\,,\theta_W\,,M_W\,,M_H \;\Leftrightarrow\; \alpha\,,G_F\,,M_Z\,,M_H$$

Yukawa Sector: **13**

$$m_e\;,\;m_\mu\;,\;m_\tau$$

$$m_d\;,\;m_s\;,\;m_b$$

$$m_u\;,\;m_c\;,\;m_t$$

$$\theta_1\,,\theta_2\,,\theta_3\,,\delta$$

➡ **18 Free Parameters** (+ Neutrino Masses / Mixings ?)

TOO MANY !

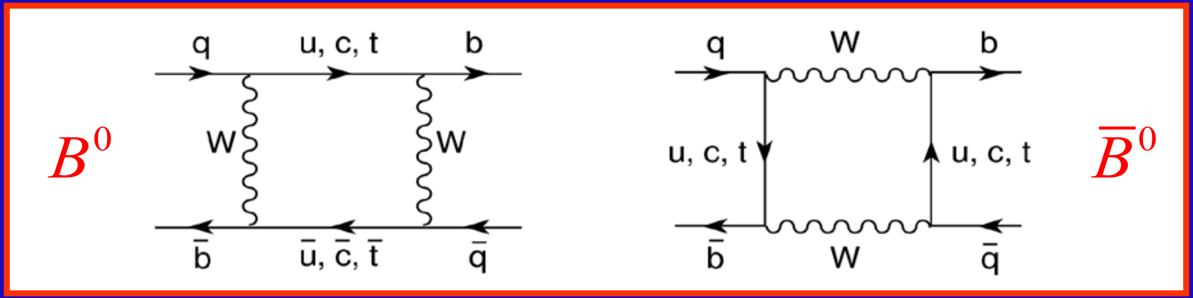
- $\mathcal{C}$, $\mathcal{P}$: Violated maximally in weak interactions
- $\mathcal{CP}$: Symmetry of nearly all observed phenomena
- Slight (~ 0.2 %) $\not{\mathcal{CP}}$ in K^0 decays (1964)
- Sizeable $\not{\mathcal{CP}}$ in B^0 decays (2001)
- Huge Matter — Antimatter Asymmetry in our Universe ➡ Baryogenesis

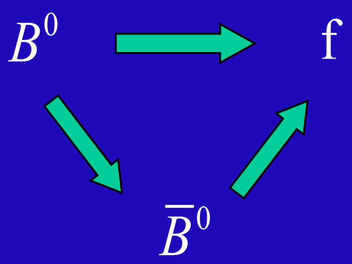
$\mathcal{CPT}$ Theorem: $\not{\mathcal{CP}}$ ⬌ $\not{\mathcal{T}}$

Thus, $\not{\mathcal{CP}}$ requires:

- Complex Phases
- Interferences

Meson – Antimeson Mixing

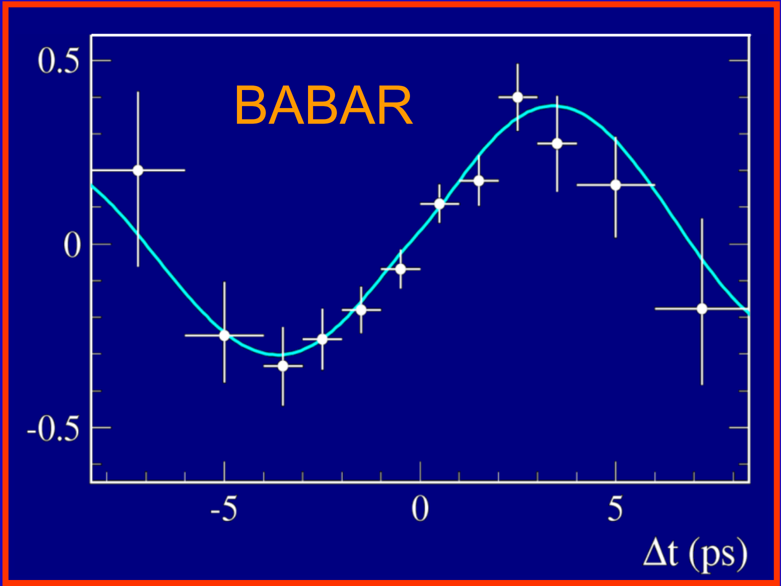




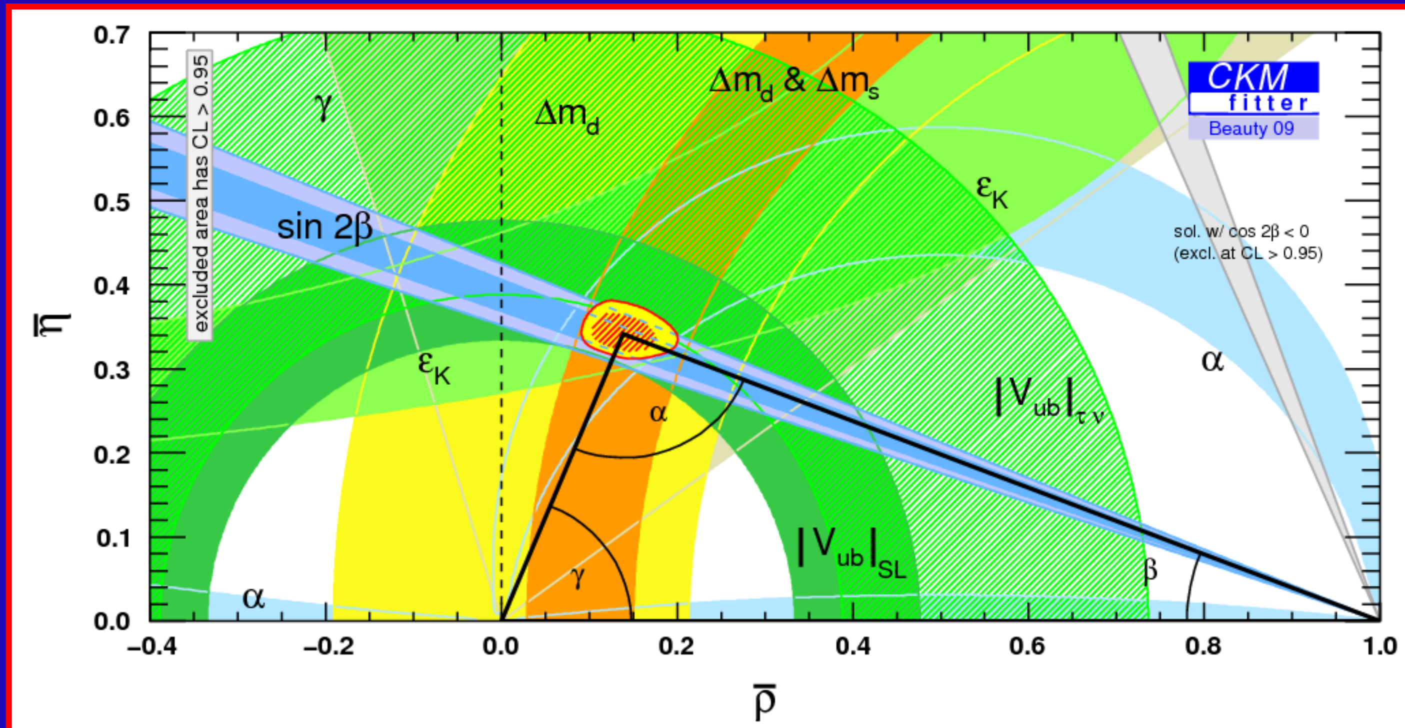
2 Interfering Amplitudes

$\not{CP}$ Signal

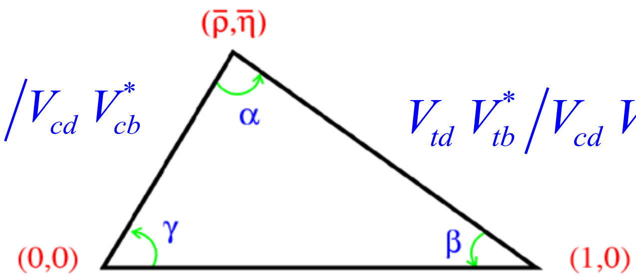
$$\frac{\Gamma(B^0 \to J/\psi\, K_S) - \Gamma(\overline{B}^0 \to J/\psi\, K_S)}{\Gamma(B^0 \to J/\psi\, K_S) + \Gamma(\overline{B}^0 \to J/\psi\, K_S)} \neq 0$$



$$V_{ud}\,V_{ub}^{*} + V_{cd}\,V_{cb}^{*} + V_{td}\,V_{tb}^{*} = 0$$



$V_{ud}\,V_{ub}^{*}\,/\,V_{cd}\,V_{cb}^{*}$



$V_{td}\,V_{tb}^{*}\,/\,V_{cd}\,V_{cb}^{*}$

UT*fit*

$$\bar{\eta} \equiv \eta \left(1-\frac{1}{2}\lambda^2\right) = 0.342 \pm 0.014$$

$$\bar{\rho} \equiv \rho \left(1-\frac{1}{2}\lambda^2\right) = 0.154 \pm 0.022$$

$$\alpha = 92.0 \pm 3.4^\circ \;;\; \beta = 22.0 \pm 0.8^\circ \;;\; \gamma = 65.6 \pm 3.3^\circ$$

Standard Model Mechanism of $\not{CP}$

Complex phases in Yukawa couplings only:

$$\mathcal{L}_Y = -\sum_{jk} (\bar{u}'_j, \bar{d}'_j)_L \left[c^{(d)}_{jk} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} d'_{kR} + c^{(u)}_{jk} \begin{pmatrix} \phi^{(0)\dagger} \\ -\phi^{(+)\dagger} \end{pmatrix} u'_{kR} \right] + \text{h.c.}$$

SSB $\left[\langle \phi^{(0)} \rangle = v/\sqrt{2} \right]$

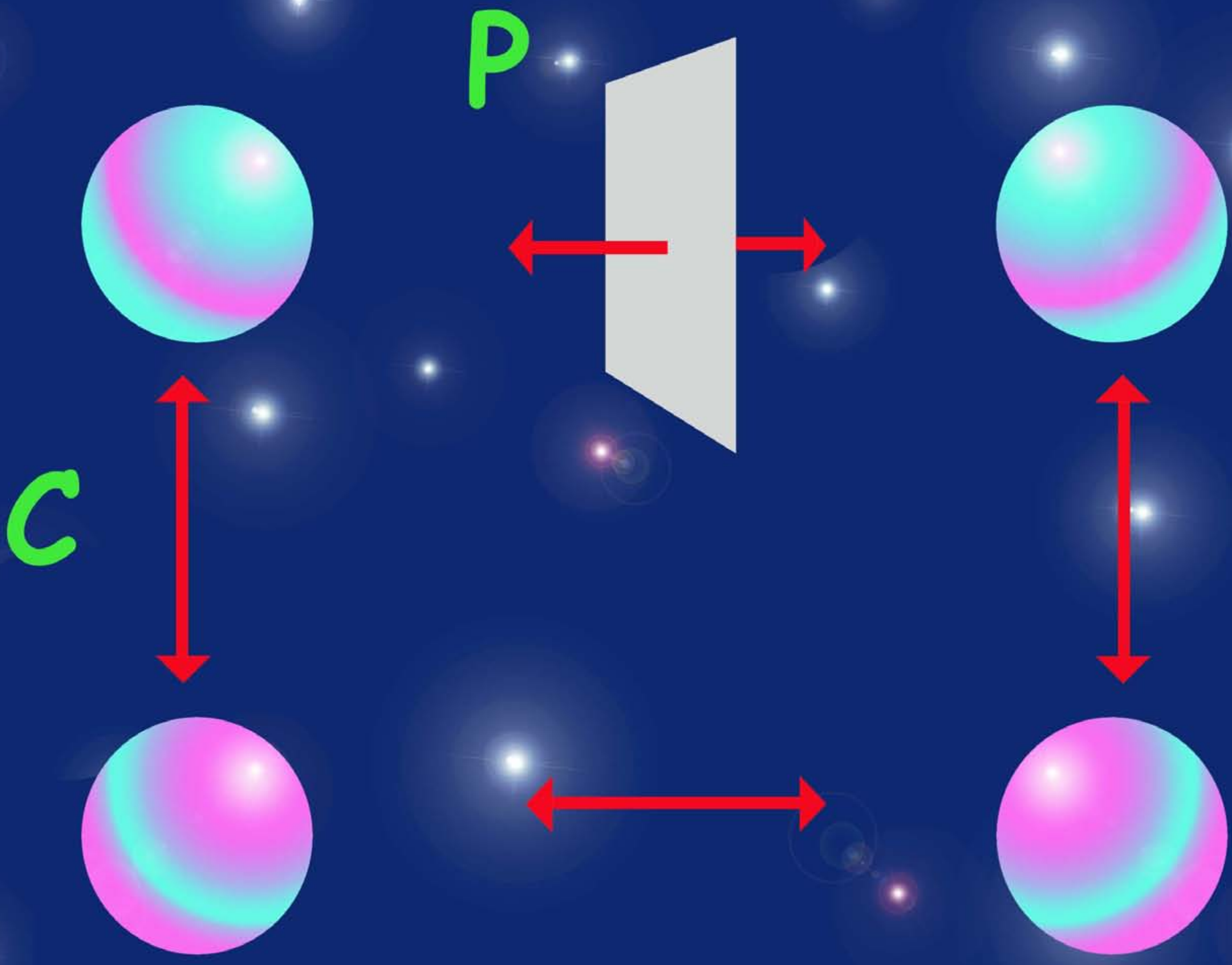
$$\mathcal{L}_Y = -\left(1+\frac{H}{v}\right) \frac{v}{\sqrt{2}} \left\{ \bar{d}'_{jL}\, c^{(d)}_{jk}\, d'_{kR} + \bar{u}'_{jL}\, c^{(u)}_{jk}\, u'_{kR} + \text{h.c.} \right\}$$

$c^{(q)}_{jk}$ diagonalization

$$\mathcal{L}_Y = -\left(1+\frac{H}{v}\right) \left\{ \bar{d}_{jL}\, m_{d_j}\, d_{jR} + \bar{u}_{jL}\, m_{u_j}\, u_{jR} + \text{h.c.} \right\}$$

$$\mathcal{L}_{CC} = -\frac{g}{2\sqrt{2}} W^\dagger_\mu \sum_{ij} \bar{u}_i \gamma^\mu (1-\gamma_5)\, \mathbf{V}_{ij}\, d_j + \text{h.c.}$$

The CKM matrix $\mathbf{V}_{ij}$ is the only source of $\not{CP}$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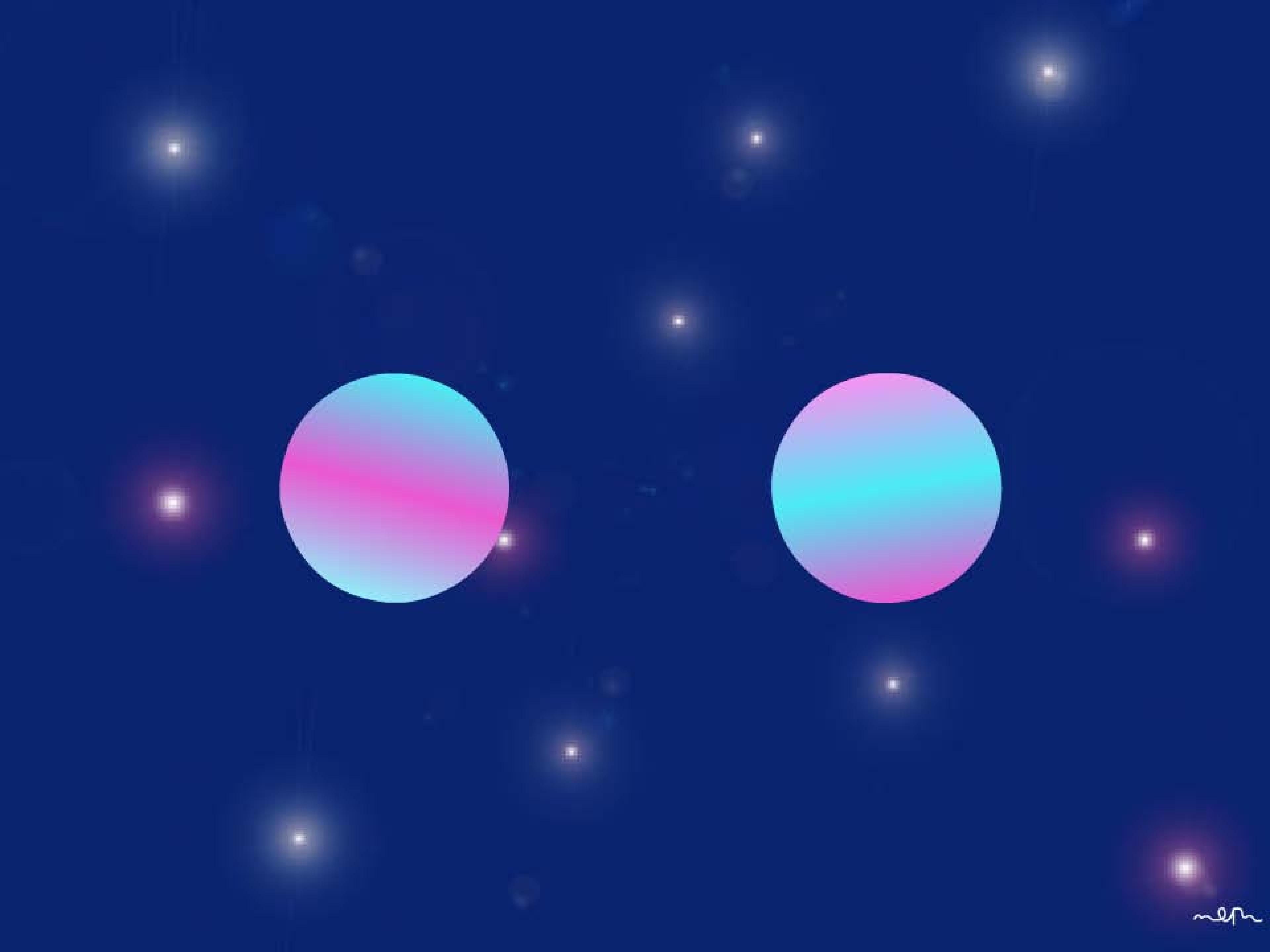
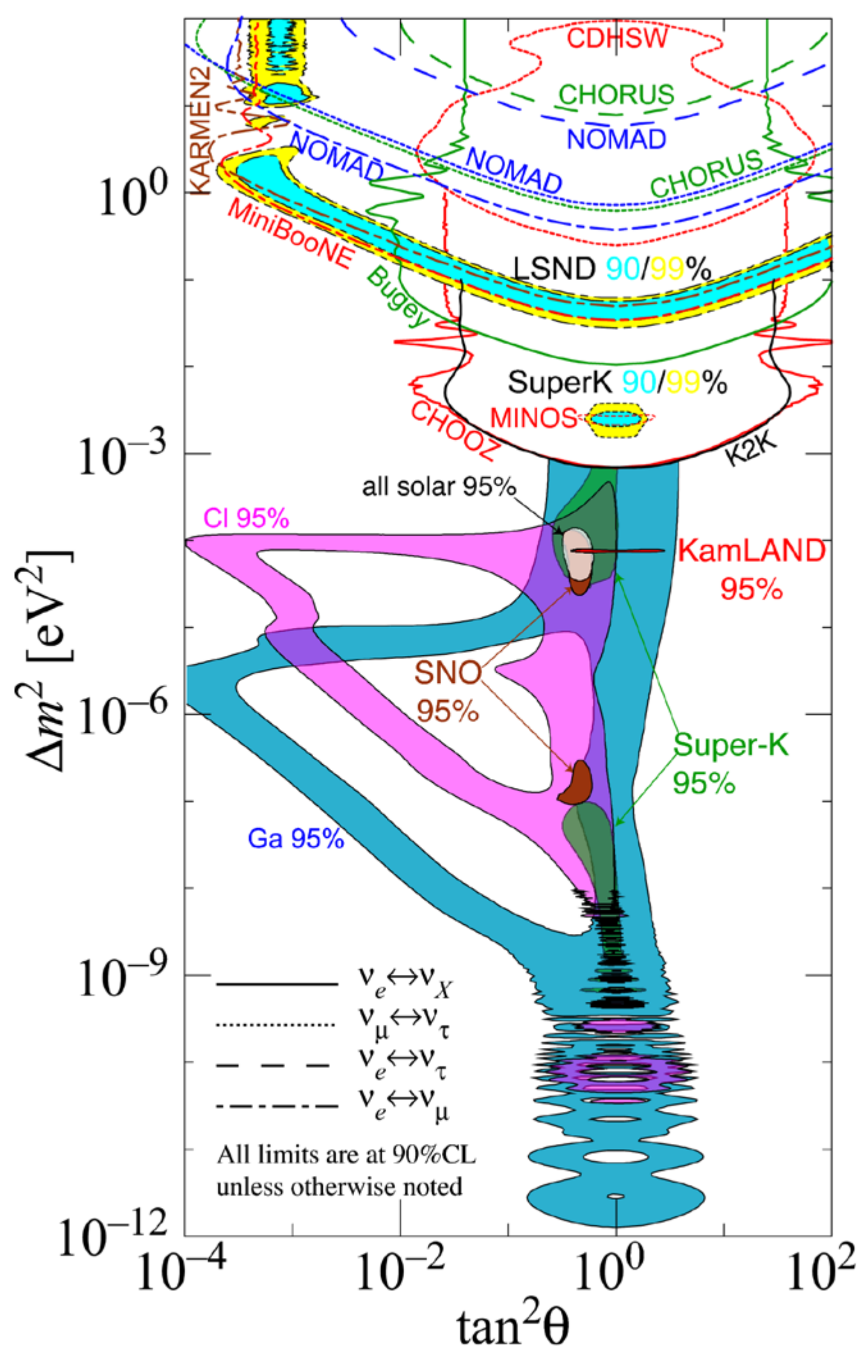
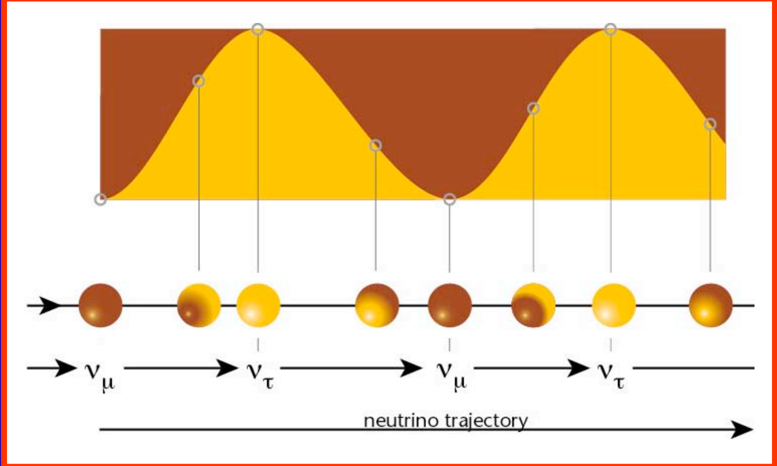


Neutrino Oscillations



http://hitoshi.berkeley.edu/neutrino



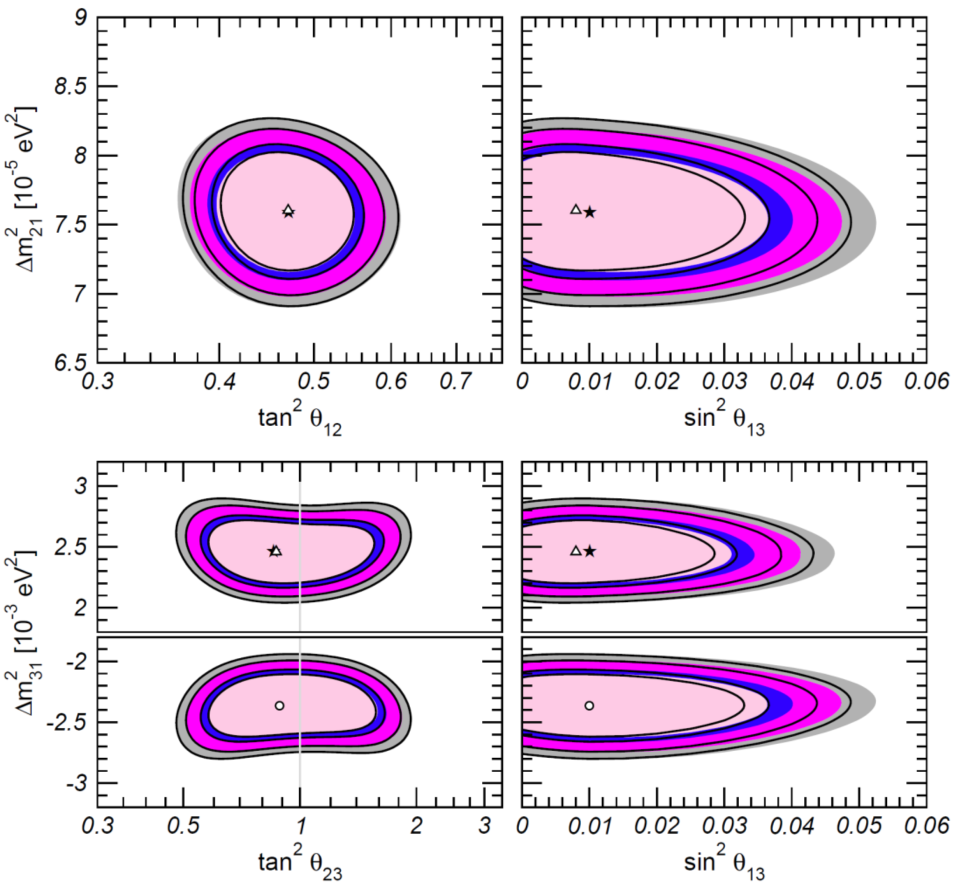
Lepton Mixing

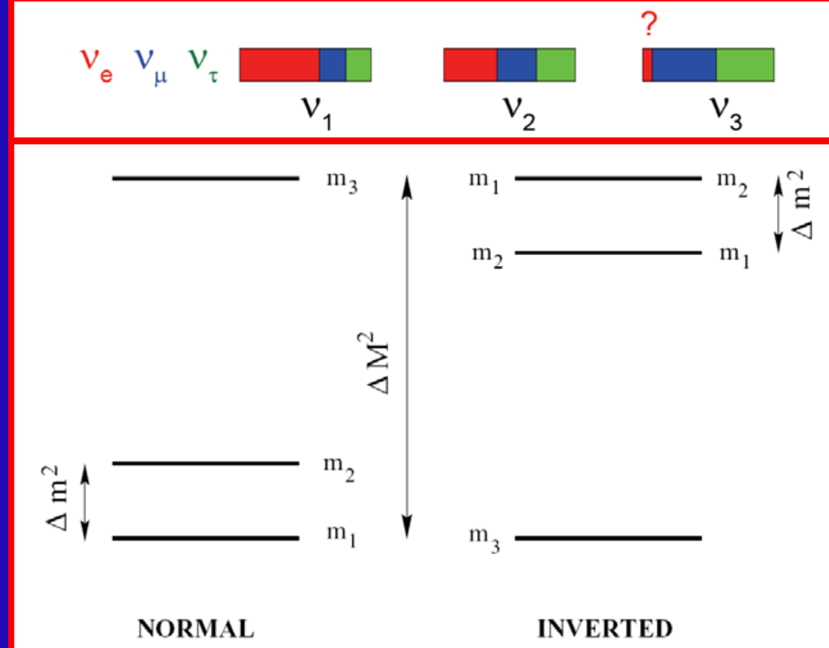
ν_R , $\not{CP}$?

NEW PHYSICS

Neutrino Oscillations

González-García, Maltoni, Salvado, 2010





$$\Delta m^2_{21} = (7.59 \pm 0.21) \cdot 10^{-5}\ \mathrm{eV}^2$$

$$\left|\Delta m^2_{32}\right| = (2.43 \pm 0.13) \cdot 10^{-3}\ \mathrm{eV}^2$$

$$\sin^2(2\theta_{12}) = 0.87 \pm 0.04$$

$$\sin^2(2\theta_{23}) > 0.92$$

$$\sin^2(2\theta_{13}) < 0.19$$

THE STANDARD THEORY OF FUNDAMENTAL INTERACTIONS

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$

Electroweak + Strong Forces

- Gauge Symmetry → Dynamics
- 3 Gauge Parameters: $\alpha_s(M_Z^2)$, α , θ_W
- All Known Experimental Facts Explained
- Problem with Mass Scales / Mixings:

- 15 Additional Parameters
- Why 3 Families ?
- Why Left $\neq$ Right ?
- Why $m_t > M_Z$?
- Does the Higgs Exist ?
- Flavour Mixing
- $\mathcal{CP}$ Violation
- Neutrino Masses / Oscillations


WANTED
Higgs
GREAT REWARD
STOCKHOLM