The hierarchy problem

(On the origin of the Higgs potential)
Electroweak symmetry breaking (EWSB) in the SM is triggered by the Higgs VEV:

\[ V(h) = -\frac{1}{2} \mu^2 h^2 + \frac{1}{4} \lambda h^4 \]

\[ \mu^2 = \lambda v^2 = \frac{\lambda}{g^2} 4M_W^2 \sim 10^4 \text{ GeV}^2 \ll M_P^2 \sim 10^{38} \text{ GeV}^2 \]

**Why so different?**
Even worse, at the quantum level, scalar masses are extremely sensitive to heavy states

\[ \sim \frac{1}{16\pi^2} M^2 \]

mass of the particle in the loop
Not the same situation for fermions or gauge bosons

\[ \text{gauge symmetries can protect them} \]

\textbf{No symmetry in the SM protects the Higgs mass}

In general:

\[ m\bar{\Psi}_L \Psi_R \quad \text{vs} \quad \mu^2 |H|^2 \]

Not a singlet if \( \Psi_R \) transform:

\[ \Psi_R \rightarrow e^{i\theta} \Psi_R \]  
(chiral symmetry)

Always a singlet under phase transformations

\textbf{Expected:} \( \mu^2 \sim \) heavier scale\(^2 \sim M_{\text{GUT}}^2, M_P^2, M_{\text{string}}^2 \)

\[ \Rightarrow \text{This is the hierarchy problem} \]
Let me emphasize that is not a problem of consistency but of naturalness.

Example:

Fine-tune system
**Analogy with Superconductivity**

EWSB \leftrightarrow \text{ Breaking of } U(1)_{\text{EM}}

Higgs Model \leftrightarrow \text{ GL Model} \quad \langle h \rangle = \langle e^- e^- \rangle

Give the GL Model a good description of superconductors?

![Phase Diagram](image)
Analogy with Superconductivity

\[ \text{EWSB} \Leftrightarrow \text{Breaking of } U(1)_{\text{EM}} \]
\[ \text{Higgs Model} \Leftrightarrow \text{GL Model} \quad \langle h \rangle = \langle e^- e^- \rangle \]

Give the GL Model a good description of superconductors?

\[ \text{NO, it only works close to the critical line} \]

only there \( \langle h \rangle \) is small and it makes sense to Taylor-expand the potential:

\[ V(h) = m^2 |h|^2 + \lambda |h|^4 + \cdots \]
Possibilities that theorists envisage to tackle the Hierarchy Problem:

1) **Supersymmetry:** Protecting the Higgs mass by a symmetry

2) **Composite Higgs:** The Higgs is not elementary:
   
   As in superconductivity: $h \sim ee$
   
   or QCD: pions $\sim q\bar{q}$

3) **Large extra dimensions:**
   
   Gravity strong at the EW-scale: $\Lambda \sim M_{\text{string}} \sim \text{TeV}$

=> In all cases **New Physics at $\sim\text{TeV}$**

Strong motivation for the LHC!
Supersymmetry

We want a symmetry to protect the Higgs mass:

**Idea:** Scalar $\rightarrow$ Fermion

since fermion masses protected by chiral symmetry

It exists, it is a **Super**symmetry:

Simplest case:

$$\mathcal{L} = |\partial_\mu \Phi|^2 + i \frac{1}{2} \bar{\Psi} \delta \Phi \Psi$$

Invariant under:

$$\Phi \rightarrow \Phi + \delta \Phi$$

$$\Psi \rightarrow \Psi + \delta \Psi$$

$$\delta \Phi \rightarrow \bar{\xi} (1 - \gamma_5) \Psi$$

$$\delta \Psi \rightarrow i (1 - \gamma_5) \gamma^\mu \bar{\xi} \partial_\mu \Phi$$

The scalar must be massless!!
Supersymmetry Algebra
(Maximal extension of Poincare in a QFT)

Minimal SUSY (N=1): One extra generator Q

\[ Q|\text{Boson}\rangle = |\text{Fermion}\rangle, \quad Q|\text{Fermion}\rangle = |\text{Boson}\rangle \]

Schematic form:

\[
\begin{align*}
[Q, M_{\mu\nu}] &= Q \\
\{Q, Q^\dagger\} &= P^\mu, \\
\{Q, Q\} &= \{Q^\dagger, Q^\dagger\} = 0, \\
[P^\mu, Q] &= [P^\mu, Q^\dagger] = 0,
\end{align*}
\]

Q commutes with \( P^2 \) and any generator of the gauge symmetries:

The Fermion and Boson have equal masses and charges
Minimal Supersymmetric SM (MSSM)

Imposing supersymmetry to the SM $\Rightarrow$ MSSM

The spectrum is doubled:

- SM fermion $\Rightarrow$ New scalar (s-”...”)
- SM boson $\Rightarrow$ New majorana fermion (“ ...“-ino)

![Diagram showing Standard and SUSY particles]
... but not yet realistic:
The model has a quantum anomaly (due to the Higgsino) and the down-quarks and leptons are massless

Extra Higgs needed
➡ Two Higgs doublets:

\[ H_u : (1, 2, 1) \]
\[ H_d : (1, 2, -1) \]

➡ give mass to the up quarks
➡ give mass to the down quarks and leptons

+ two Higgsino doublets:

\[ \tilde{H}_u : (1, 2, 1) \]
\[ \tilde{H}_d : (1, 2, -1) \]
The generic nomenclature for a spin-1/2 superpartner is to add an asterisk to the superpartner of the Standard Model particle to which it is assigned. The chiral and gauge supermultiplets in Tables 1.1 and 1.2 make up the MSSM. The lightest superpartner of the lighter Standard Model particle is called the Lsp, or lightest superpartner. The Lsp is the particle that is expected to be the lightest superparticle. The right-handed quarks and leptons (and their superpartners) have indices $\mu, \nu, e$; left-handed quarks, leptons, and their superpartners have indices $\ell, a, i$.

### MSSM Spectrum

<table>
<thead>
<tr>
<th>Squarks</th>
<th>Sleptons</th>
<th>Higgsinos</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\tilde{u}_L, \tilde{d}_L)$</td>
<td>$(\tilde{\nu}, \tilde{\ell}_L)$</td>
<td>$(H_u^+, H_u^0)$</td>
</tr>
<tr>
<td>$\tilde{u}_R^*$</td>
<td>$\tilde{\ell}_R^*$</td>
<td>$(H_d^0, H_d^-)$</td>
</tr>
<tr>
<td>$\tilde{d}_R^*$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(\tilde{H}_u^+, \tilde{H}_u^0)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(\tilde{H}_d^0, \tilde{H}_d^-)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gauginos</td>
<td>Gauginos</td>
<td></td>
</tr>
<tr>
<td>$\tilde{g}$</td>
<td>$g$</td>
<td></td>
</tr>
<tr>
<td>$\tilde{W}^\pm, \tilde{W}^0$</td>
<td>$W^\pm, W^0$</td>
<td></td>
</tr>
<tr>
<td>$\tilde{B}^0$</td>
<td>$B^0$</td>
<td></td>
</tr>
</tbody>
</table>

- **Particles**: $R$-parity = 1
- **Superpartners**: $R$-parity = -1

1) Superpartners interact in pairs
2) Lightest superpartner is stable
Type of interactions

Getting them from “supersymmetrization”:

- For a scalar propagator in a theory with exact supersymmetry as indicated by the second and third terms in eq. (3.50).

- The part of the Lagrangian is used to tie together in a Feynman diagram is traditionally drawn as a solid fermion.

- Figure 3.3c shows the coupling of a gaugino to a chiral fermion.

- The analogous term as indicated by the second and third terms in eq. (3.50).

- The analogous term as indicated by the second and third terms in eq. (5.3). For variety, the fermion mass terms occur in any gauge theory because of the argument based on anomaly cancellation mentioned in the Introduction.
Up to scalar trilinear and quartics:
How supersymmetry works?

Fermion loop
\[ \mu^2 = +A \]

Boson loop
\[ \mu^2 = -A \]

\[ \mu^2_{\text{total}} = 0 \]
It's not the first time that symmetries force doubling the known spectrum:

**Relativistic quantum field theories:**

Particle → Antiparticles

Made the electron-mass corrections not *linearly divergent*:

\[ \Delta m_e \propto m_e \]
But if supersymmetry is exact:
\[ \mathbf{M}_F = \mathbf{M}_B \implies \text{e.g. } M_e = M_{\tilde{e}} \]

It must be broken to give masses to the superpartners.

Supersymmetry breaking must afford “soft terms”:
(terms that do not spoil the good UV properties of the Susy)

\[
- \frac{1}{2} \left( \mathbf{M}_3 \tilde{g}\tilde{g} + \mathbf{M}_2 \tilde{W}\tilde{W} + \mathbf{M}_1 \tilde{B}\tilde{B} + \text{c.c.} \right) \\
- \left( \tilde{u} a_u \tilde{Q} H_u - \tilde{d} a_d \tilde{Q} H_d - \tilde{e} a_e \tilde{L} H_d + \text{c.c.} \right) \\
- \tilde{Q}^\dagger m_Q^2 \tilde{Q} - \tilde{L}^\dagger m_L^2 \tilde{L} - \tilde{u} m_u^2 \tilde{u}^\dagger - \tilde{d} m_d^2 \tilde{d}^\dagger - \tilde{e} m_e^2 \tilde{e}^\dagger \\
- m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d - (b H_u H_d + \text{c.c.}) \right) + \mu \tilde{H}_u \tilde{H}_d
\]

for 3 families, more than 100 terms are possible!!
How supersymmetry works?
(including soft-masses)

Fermion loop
\[ \mu^2 = +A \]

Boson loop
\[ \mu^2 = -A + m_{\text{stop}}^2 \]

\[ \mu^2_{\text{total}} \sim m_{\text{stop}}^2 \]

Superpartners expected around \( v \sim 100 \text{ GeV} \)
Constraints on superpartner masses from flavor physics:

Breaking of lepton symmetry:

Exp: \( BR(\mu \to e\gamma) < 10^{-11} \) \( \Rightarrow m_{\tilde{e}} \simeq m_{\tilde{\mu}} \)

up to 1% - 0.1%

Large contributions to K-\( \bar{K} \) mixing:

\( m_{\tilde{s}} \simeq m_{\tilde{d}} \) up to 0.1% - 0.001%
Soft terms must be generated in a clever way

Most interesting possibilities:

1) Gauge mediation

2) Gravity/Moduli/Extra-dim mediation
The famous scenario "minimal sugra" not a model, just an Ansatz:

At $Q = M_{\text{GUT}}$

- All gaugino masses equal $= M_{1/2}$
- All scalar masses equal $= M_0$
- All trilinear equal $= A_0$

At $Q = M_Z$

I don’t know, but experimentalists like it a lot!
I) Gauge mediation

New sector
Susy breaking sector

MSSM

Gauge interactions are “flavor blind”:
Universal masses for squarks/sleptons with equal charges
Very predictive (in the minimal case).
Just calculate loops:

Depends on 3 parameters:
1) $\mu$-term: Higgsino mass
2) Susy-breaking scale: $F$
3) Scale where the soft-terms are induced: $M$

Giudice, Rattazzi 97
Predicts a very light gravitino = Mass suppressed by $M_P$:  

$\Rightarrow$ partner of the graviton

$$m_{3/2} = \frac{F}{k\sqrt{3}M_P} = \frac{1}{k} \left( \frac{\sqrt{F}}{100 \text{ TeV}} \right)^2 2.4 \text{ eV}$$

$k = \text{model-dependent coefficient}$

$\Rightarrow$ Lightest Superparticle
2) Gravity/Moduli/Extra-dim mediation:

...to be discussed later

Spectrum at high-energies \((Q \sim 1/R)\) model dependent

An example: Scalar masses = 0 (at tree-level)
Gaugino masses = \(M_{1/2} \neq 0\)

Lightest superpartner: **Neutralino** (mixture of gaugino and Higgsino)
Higgs sector

Only 3 parameters:

\[
V = (|\mu|^2 + m_{H_u}^2)(|H_u^0|^2 + |H_u^+|^2) + (|\mu|^2 + m_{H_d}^2)(|H_d^0|^2 + |H_d^-|^2) \\
+ [b(H_u^+ H_d^- - H_u^0 H_d^0) + \text{c.c.}] \\
+ \frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2)^2 + \frac{1}{2}g^2|H_u^+ H_d^0|^2 + |H_u^0 H_d^-|^2.
\]

quartic coupling related to gauge-couplings

Spectrum:

2 x 4 = 8 scalars = 3 Goldstones (eaten by W, Z) 
+3 neutral Higgs = h, H, A 
+ Charged Higgs = H^+, H^-
2 unknown parameters (since \( v^2 = \langle H_u \rangle^2 + \langle H_d \rangle^2 \)):

1) \( \tan \beta = \frac{\langle H_u \rangle}{\langle H_d \rangle} \)

2) \( m_A \)

At tree-level:

\[
\begin{align*}
\begin{cases}
  m_{H^+}^2 = m_A^2 + m_W^2 \\
  m_{h,H}^2 = & \frac{1}{2} \left( m_A^2 + m_Z^2 \pm \sqrt{(m_A^2 - m_Z^2)^2 + 4 \sin^2 2\beta m_A^2 m_Z^2} \right)
\end{cases}
\end{align*}
\]

\[ m_h \leq m_Z \]

Was a great prediction for Higgs hunters at LEP!
... but quantum effects (mostly loops of top/stop) modify the bound:

Upper bound \( \sim 130 \text{ GeV} \)

\[
M_Φ \ [\text{GeV}] \quad X_t = \sqrt{6} M_S
\]

\[\tan β = 3 \quad \tan β = 30\]

\( H \)

\( H^\pm \)

\( h \)
LEP searches:

with decays to taus and bottoms

Theoretically

Excluded by LEP

Theoretically Inaccessible

m_{h^0}-max

m_{A^0}-max

Excluded by LEP
After LEP, a heavy stop is essential to keep the MSSM alive

Higgs bound:

\[ m_h^2 < m_Z^2 + \frac{3m_t^4}{2\pi^2v^2} \ln\left(\frac{m_{\text{stop}}}{m_t}\right) + \cdots \]

Needed to be large to be above the experimental bound

Higgs searches rules out a big chunk of the parameter space of the MSSM!
In minimal Sugra:

\[ \frac{M_0^2}{\mu^2} \]

Only the thin “withe spike” is left!
MSSM Higgs hunting at the LHC

**Bad news:** $h$ too light to decay to $WW/ZZ$

- $A, H^+$ have very small couplings to $WW/ZZ$
- $H$ small regions with sizable couplings to $WW/ZZ$

**Good news:** Regions where the decays of $H, A, H^+$ to leptons are enhanced (Large $\tan\beta$ region)

Due to: $m_\tau = Y_\tau \langle H_d \rangle$

- can be larger than in the SM, if $\langle H_d \rangle$ is smaller than $v$
Near future:

More interestingly: MSSM could be ruled out if a Higgs $\rightarrow$ WW/ZZ with mass $\sim 160$ GeV is discovered in the first LHC run.
In the long run....
Superpartners at Hadron Colliders
Superpartners at Hadron Colliders

Neutral gaugino + Higgsino mix:
Mass-eigenstate = $\chi^0$ neutralino

Charged gaugino + Higgsino mix:
Mass-eigenstate = $\chi^+$ chargino
Main consideration:
Due to $R$-parity superpartners are produced in pairs, and decay, in cascade, down to the lightest one (neutral) that, being stable, goes away from the detector.

A classic:

3 isolated leptons
+ 2 b-jets
+ 4 jets
+ $E_T^{\text{miss}}$
Strategy: Detect leptons or jets + Missing $E_T$

3 isolated leptons
+ 2 b-jets
+ 4 jets
+ $E_T^{\text{miss}}$

Final states with same-charge dilepton due to the Majorana nature of the gluino
Tevatron

From P. Wittich at “Physics at the LHC 2010” conference
• Large production cross section, bkgnds from multi-jet, $Z \rightarrow \nu\nu$, top
• Optimize searches as a function of $(\text{Missing } E_T, n_{\text{jet}})$
• No excess seen so far
• Limits for 2 (2.1)/fb of data for CDF (D0)
• interpret results in mSUGRA-like SUSY scenario
Trileptons: Chargino-Neutralino Search, 3.2/fb

- Very clean signature:
  - Missing $E_T$ due to undetected $\nu$, $\chi^{0}_1$
  - 3 isolated leptons, *lower momentum*

- Rejection using kinematic selections on: $m_{l^+l^-}$, $n_{jets}$, Missing $E_T$, $\Delta\phi$ between leptons...

Good agreement between data and SM prediction $\rightarrow$ *set limit*

<table>
<thead>
<tr>
<th>Channel</th>
<th>SM expected</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trilepton</td>
<td>$1.5 \pm 0.2$</td>
<td>1</td>
</tr>
<tr>
<td>Lepton+trk</td>
<td>$9.4 \pm 1.2$</td>
<td>6</td>
</tr>
</tbody>
</table>
Chargino-neutralino results

- Interpret null result in mSugra SUSY scenario as a convenient/conventional benchmark

\[
m_0 = 60 \text{ GeV}, \tan \beta = 3, A_0 = 0, \mu > 0
\]

CDF Run II Preliminary, 3.2 fb\(^{-1}\)

- Excludes \(m_{\chi^\pm_1} < 164\) (154 Exp.) GeV/c\(^2\)

Limit on relative \(\chi_0^0-\tilde{\ell}\) masses
- \(m_{\chi^2} > m_{\tilde{\ell}}\), increases BR to e/\(\mu\)
- \(m_{\chi^2} \approx m_{\tilde{\ell}}\), reduces acceptance

2 b jets + $E_T^{\text{Miss}} - \sim q$ and LQ

- Final state familiar from Higgs searches
  - missing $E_T$ and b quarks
- Also good signal for leptoquarks and SUSY
- event selection:
  - b tagging (D0: neural-net algo)
  - two b-tagged jets, $E_T^{\text{miss}}$, Sign., $\Sigma E_T$
  - optimize $p_T$, $E_T^{\text{miss}}$, $H_T$, $X_{jj}$ for SUSY/LQ3 signals

$Z H \rightarrow \nu \bar{\nu} b \bar{b}$

$\bar{b}_1 \bar{b}_1 \rightarrow b \tilde{\chi}_1^0 b \tilde{\chi}_1^0$

$LQ_3 \rightarrow \nu_\tau b$

$$X_{jj} = \frac{p_{jet1}^T + p_{jet2}^T}{H_T}$$

Data
- $b\bar{b}$, $LQ_3$ 240 GeV
- Z+jets
- W+jets
- Top
- Diboson
- Multijet

Events / 10 GeV

$E_T$ (GeV)

Higgs boson mass for SUSY models.
Supersymmetric top in the e+μ+bb+MET, 3.1/fb

- 3rd generation again - special role in SUSY
- Look for decay mode in e μ final state with $E_T^{\text{Miss}} > 18$ GeV
  - Low SM backgrounds ($Z \to \tau \tau, t\bar{t}$)
  - Reject with $\delta \Phi($lepton, $E_T^{\text{Miss}})$ cuts
- No explicit b tag required

- Consider small and large $\delta m($stop, sneutrino$)$
  - Drives kinematics of accepted events

- Bin events in two kinematic variables
  - HT: scaler sum of jet $p_T$
  - ST: scalar sum of lepton $p_T$, $E_T^{\text{Miss}}$

- Null result: set limits in sneutrino/stop mass plane

\[ p\bar{p} \rightarrow t_1 \bar{t}_1 \]

\[ B(t_1 \rightarrow \tilde{\nu} b l) = 100\% \]

R parity conserving

D0 Preliminary Result

LEP II excluded

LEP I excluded

Blue: this result

D0 Conference Note 5937-CONF
LHC

From P. Jenni at “Physics at the LHC 2010” conference
The initial LHC running will already match (maybe exceed) end 2010 the Tevatron reach

A typical example; note that the missing transverse energy performance enters directly the ‘Effective Mass’, detectors must be well understood for these measurements
Ultimate discovery reach for SUSY particles at the LHC (indicative plots, model-dependent…)

The mass scale probed for squarks and gluinos will be typically 2.5 TeV by 2017
Other MSSM goodies:

- Gauge coupling unification
- The lightest supersymmetric particle (LSP) can be Dark matter
- Local supersymmetry must incorporate gravity:

\[ \{Q, Q^\dagger\} = P^\mu \]

- Fits well EWPT from LEP/Tevatron

⇒ It has allowed us to write more than 20,000 papers