Extra dimensions
Kaluza (1921) and Klein (1926) proposed an **extra spacial dimension** trying to unify EM with Gravity:

In 5 dimensions the gravitational field has more components:

\[
h_{MN} = (h_{\mu\nu}, h_{5\mu}, h_{55})
\]

- \(M = \mu, 5 = 1, 2, 3, 4, 5\)
- symmetric under \(M \leftrightarrow N\)

The extra spacial-dimension had to be compactified

\[R \sim 1/M_p\]

Einstein liked the idea:

"Long live the fifth dimension"  
Einstein's letter to Ehrenfest 21/1/1928

...but didn’t work: Not possible to incorporate matter
The formalism developed however was useful:
For periodic extra dim., one can perform a Fourier expansion of 5D fields:

\[ \Phi(x, x_5) = \sum_{n=-\infty}^{\infty} e^{inx_5/R} \Phi(n)(x) \]

QM \[ e^{ip_5x_5}, p_5 = n/R \]
States with 5D momentum \(\sim\) mass in 4D: \( p_\mu^2 = p_5^2 \)

5D field = 4D massless state (n=0) + infinity tower of 4D massive states

Kaluza-Klein states

...equivalent of harmonics
• The next incarnation of extra dimensions came around 1980, when **string theory** was developed as a theory of quantum gravity.

![String Theory Diagram](image)

• It was shown that string theory could only be made consistent if the **spacetime dimensions were larger than four**. The **extra dimensions** were supposed to be compactified close to the Planck scale:

![Compactification Diagram](image)

radius \sim \frac{1}{M_P}

Therefore not testable in near-future experiments
In 1998 Arkani-Hamed, Dimopoulos and Dvali (ADD) realize that extra dimensions could explain the weakness of gravity: \( G_N << G_F \)

**How?** Gauss’s law:

\[
\int_S d\Phi \sim Q_{int}
\]

\[
S \sim r^{2+d}
\]

\[
d = \text{number of extra dimensions}
\]

At large distances, the strength of a force becomes smaller in higher dimensions.
BUT:

1) Only gravity could propagate in these extra dimensions (otherwise all forces will be weak)

Possible in “Brane Worlds” (String constructions):
2) We see at large distances only 3 spacial dimensions, so these extra dimensions have to be compactified.

**How large can they be?**

Surprisingly, we have not measured very well gravity at distances smaller $\sim 0.1$ mm

**Constrains on extra forces:**

$$F_{KK}(r) = -\alpha G_N \frac{m_1 m_2}{r} e^{-r/\lambda},$$

$\alpha$ = measures the strength of the interaction ($\alpha = 1 \rightarrow$ Gravit. strength)

$\lambda$ = range of the interaction

Gravity could be different at sub-mm scales: Extra dim of radius $R \sim 0.04$ mm possible!
ADD proposed the following scenario:

1) Cutoff of the SM $\sim \Lambda \sim M_{\text{string}}$: Scale at which gravity is strong

\begin{align*}
\text{EW scale} & \quad \sim 100 \text{ GeV} \\
\text{Gravity strength} & \quad \sim 100 \text{ GeV} \sim \text{TeV}
\end{align*}

Since $M_W \sim \Lambda \sim M_{\text{string}}$, no big hierarchy problem!

2) Gravity propagating in $d$ extra dimensions of size $R$:

Gravity strength:

\[ G_N = \frac{1}{M_P^2} \sim \frac{1}{M_{\text{string}}^2} \left( \frac{1}{M_{\text{string}}^2 2\pi R^d} \right) \]

dilution factor due to the spreading of the gravitational field lines in $d$ extra dimensions
\[ G_N = \frac{1}{M_P^2} \sim \frac{1}{M_{\text{string}}^2} \frac{1}{(M_{\text{string}} 2\pi R)^d} \]

\[ \downarrow \]

\[ M_{\text{string}} \sim \text{TeV} \]

\[ d=1 \rightarrow R \sim 10^8 \text{ Km} \quad \text{Not possible} \]

\[ d=2 \rightarrow R \sim 0.1 \text{ mm} \quad \sim \text{at the verge of the exp. bounds} \]

\[ \ldots \]

\[ d=6 \rightarrow R \sim 1/ \text{ MeV} \quad \text{OK} \]
Predictions:

1) For $d=2$, we expect deviations from Newtonian gravity at distances smaller than $\sim 0.1\text{mm}$

   The 1st KK-graviton of mass $\sim 1/(0.1\text{mm})$ give a “new” interaction (a “new” force)

2) String theory at the reach of the LHC

   But we already said...

   “The only prediction of string theory is that there are no predictions”

   Anonymous

The only generic ones:

1) The space must be $1+9$ dimensional
2) There are string excitations of higher-energy

No clear predictions on what to expect!
Model-independent signals from gravity at \( \sim \)TeV:

Gravity becomes strong at \( \sim \) TeV energies:

\[
\sum_{n} G_{N} Q^{2} \sim \frac{1}{M_{\text{string}}^{2}} Q^{2}
\]

To be seen at LHC as deviations in Drell-Yan cross-sections for SM processes. Example: deviations in the \( \gamma\gamma \) invariant mass distribution of \( pp \rightarrow \gamma\gamma \):

arXiv:hep-ex/0310020v1
KK-Graviton production:

Search for:
Mono-jet + Missing energy

$\sqrt{s} = 14$ TeV

Events / 20 GeV

- $jW(\text{ev})$, $jW(\mu\nu)$
- $jW(\tau\nu)$
- $jZ(\nu\nu)$
- total background

- signal $\delta=2$, $M_D = 4$ TeV
- signal $\delta=2$, $M_D = 8$ TeV
- signal $\delta=3$, $M_D = 5$ TeV
- signal $\delta=4$, $M_D = 5$ TeV

arXiv:hep-ex/0310020v1
In 1999 Randall and Sundrum had a different idea:

Use gravitational redshift factors to explain the difference between $M_P$ and the EW-scale.

Assume that the extra-dimensional geometry is non-flat.

Scales shrink as we move in the extra dimension.

Here the observer sees that the atomic transition is less energetic.
Qualitatively can be understood as the photon loosing kinetic energy as it climbs up the gravitational potential well:

\[ E_2 < E_1 \]

Similarly...
Randall-Sundrum Idea

Placing the Higgs in the interior of the extra dimension could explain why its mass-term is smaller than $M_P$:

In this boundary the scales can be very large $\sim M_P$

**Anti-de-Sitter (AdS):**

$$a = \frac{L}{z}$$

$$ds^2 = a(z)^2 [dx^2 + dz^2]$$

In this boundary the scales are smaller $\sim M_W$

Two scale model
Alternative understanding by looking at the wave-function of a graviton in a AdS-space.

As in QM: Small overlapping of wave-functions = small couplings

⇒ gravity is weak for the Higgs!!
In 1999 when Sundrum was explaining this idea in a conference in Santa Barbara, E. Witten stood up and more or less said: “This is as having a composite Higgs made of strongly-coupled fields of a conformal field theory (CFT)”

**What did he have in mind?**

**The AdS/CFT correspondence:**

- Strongly coupled 4D theories in certain limits
- Weakly coupled gravity theories in higher-dimensions

Composite states have similar dynamics as particles in a curved extra dimension

⇒ **Holography:** ”4D composite states encode 5D information”

Maldacena 97
One can check certain things for a Higgs in an AdS-extra dimension:

Example:

The form factor of a 5D Higgs follows the expectation for a composite state.
Five Dimensional composite Higgs model

$\text{AdS}_5$

Gravity

$+ \text{SM gauge bosons}$

$\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$

$+ \text{SM fermions}$

UV-bound.

$M_P$-scales

IR-bound.

HIGGS

TeV-scales
Small masses for fermions (e.g. electron) easy to generate by having the wave-functions picked towards the opposite boundary to the Higgs.

\[ M_{5D} > k/2 \]

Nice “geometrical” explanation of the smallness of some of the SM fermion masses
the higher the spin, the higher the mass
Five Dimensional composite (PGB) Higgs model

The bosonic sector:

UV-bound. \[ \text{SU}(2)_L \otimes \text{U}(1)_Y \]

AdS$_5$

SO(5) \otimes \text{U}(1)

IR-bound.

SO(4) \otimes \text{U}(1)

extra dim
Why this symmetry breaking pattern?

We are in 5D: \( A_M = (A_\mu, A_5) \)

Massless boson spectrum:

- \( A_\mu \) of \( SU(2)_L \otimes U(1)_Y = \) SM Gauge bosons
- \( A_5 \) of \( SO(5)/SO(4) = 2 \) of \( SU(2)_L = \) SM Higgs

\[ \rightarrow \quad \text{Higgs-gauge unification} \]

Hosotani mechanism

Higgs mass protected by 5D gauge invariance!

\[ A_5 \rightarrow A_5 + \partial_5 \theta \]

shifts as a PGB
Predictions

Light Higgs + KK resonances for each SM field in complete reps of the bulk group $SO(5)$

top: $5 = 2_{7/6} + 2_{1/6} + 1_{2/3}$

exotic states of $Q=5/3$
the higher the spin, the higher the mass
How to see the KK at Hadron Colliders?
(and similarities/distinction with other models)
Higgsless

TC  5D models

Composite/PGB Higgs

Little Higgs  5D Higgs

$W', Z'$

Decay:
$W', Z' \rightarrow \text{leptons}$

$W', Z' \rightarrow \text{tops, } W_{long}, Z_{long}, h$

Possible to see up to 2 TeV
Decay: \( g' \rightarrow t\bar{t} \)

Agashe et al
Higgsless

TC

5D models

Composite/PGB Higgs

Little Higgs

5D Higgs

Decay: \( t'_R \rightarrow W_{long} b \)

feasible to see up to 1-2 TeV
Higgsless

TC  5D models

Composite/PGB Higgs

Little Higgs  5D Higgs

\[ T_{5/3} \]

Decay: \[ T_{5/3} \rightarrow W_{long} t \]

feasible to see up to 1-2 TeV
If this fermion is light, it can be double produced:

two like-sign leptons

masses up to 1 TeV reached with an integrated luminosity of 20/fb

Contino, Servant, see also Saavedra, Wulzer, Disertori