On accuracy of central mass energy determination for FCCee_z_202_nosol_13.seq

A. Bogomyagkov

Budker Institute of Nuclear Physics
Novosibirsk

FCC-ee polarization workshop
October 2017

Introduction: different energies

Circumference:	Π	
Design energy:	E_0	magnets fields
Average energy:	$\langle E \rangle = \oint E(s) \frac{ds}{\Pi}$	
Measured energy:	$E_{meas} = f(W)$	function of spin tune
Invariant mass: (central mass energy)	M	

Introduction: spin precession frequency

Ω_0 is revolution frequency.
W is spin precession frequency.
Gyromagnetic ratio: $q = q_0 + q' = \frac{e}{mc} + q'$.

$$W = \frac{1}{2\pi} \oint \left(\frac{q_0}{\gamma} + q' \right) B_\perp(\theta) d\theta = \Omega_0 \cdot \left(1 + \frac{q'}{q_0} \frac{\langle B_\perp \rangle}{\langle B_\perp / \gamma \rangle} \right)$$
$$\approx \Omega_0 \cdot \left(1 + \langle \gamma \rangle \frac{q'}{q_0} \right) ,$$

$$\frac{q'}{q_0} = \frac{g-2}{2} = 1.1596521859 \cdot 10^{-3} \pm 3.8 \cdot 10^{-12} .$$

$$E[MeV] = 440.64843(3) \left(\frac{W}{\Omega_0} - 1 \right) .$$

Spin distribution width: synchrotron oscillations

Synchrotron oscillations: $\delta = \Delta E/E_0 = a \cdot \cos(\omega_{syn} t)$.

$$W = \Omega_0 \left(1 + \nu_0 - \alpha_0 \nu_0 \frac{a^2}{2}\right) + \Omega_0 \left(\nu_0 (1 - \alpha_0) - \alpha_0\right) \sin(\omega_{syn} t) + \alpha_0 \Omega_0 \nu_0 \frac{a^2}{2} \cos(2\omega_{syn} t)$$

FCCee_z_202_nosol_13

Spin precession frequency distribution shifts and becomes wider by

$$\left\langle \frac{W - \Omega_0(1 + \nu_0)}{\Omega_0(1 + \nu_0)} \right\rangle = \left\langle -\frac{\alpha_0 \nu_0 \frac{a^2}{2}}{1 + \nu_0} \right\rangle = -\frac{\alpha_0 \nu_0 \sigma_\delta^2}{1 + \nu_0} = -2 \cdot 10^{-12}$$

$$\frac{\Delta E}{E_0} = -2 \cdot 10^{-14}$$

Energy dependent momentum compaction

Momentum compaction: $\alpha = \alpha_0 + \alpha_1 \delta$

Synchrotron oscillations: $\ddot{\delta} = -\omega_{syn}^2 \delta - \omega_{syn}^2 \dfrac{\alpha_1}{\alpha_0} \delta^2$

Average and RMS: $\langle \delta \rangle = -\frac{\alpha_1}{\alpha_0} \sigma^2 \, , \, \langle \delta^2 \rangle = \sigma^2$

Average W: $\langle W \rangle_\delta = \gamma_0 \Omega_0 \frac{q'}{q_0} \left(1 - \alpha_0 \sigma^2 - \frac{\alpha_1}{\alpha_0} \sigma^2 \right)$

Average energy: $\langle E \rangle = E_0 \left(1 - \frac{\alpha_1}{\alpha_0} \sigma^2 \right)$

Measured energy: $E_{meas} = E_0 \left(1 - \frac{\alpha_1}{\alpha_0} \sigma^2 - \alpha_0 \sigma^2 \right)$

Energy dependent momentum compaction

FCCee_z_202_nosol_13

$E_0 = 45.6$ GeV, $\alpha_0 = 1.5 \cdot 10^{-5}$, $\alpha_1 = -9.8 \cdot 10^{-6}$, $\sigma = 3.8 \cdot 10^{-4}$

$$\frac{\langle E \rangle - E_{meas}}{E_0} = \alpha_0 \sigma^2 = 2 \cdot 10^{-12}$$

$$\frac{\langle E \rangle - E_0}{E_0} = -\frac{\alpha_1}{\alpha_0}\sigma^2 = 1 \cdot 10^{-7}$$

Longitudinal field compensation

Detector field is $B_0 = 2$ T.
Deviation of compensating field is $\Delta B_c = 0.1$ T.
Length of compensating solenoid is $L_c = 0.75$ m.
$B\rho = 152.105\,\mathrm{T\cdot m}$, $E_0 = 45.6$ GeV, $\nu = 103.484$.

FCCee_z_202_nosol_13

$$\Delta\nu = \frac{\varphi^2}{8\pi}\cot(\pi\nu) \approx \frac{1}{8\pi}\cot(\pi\nu)\left(\frac{\Delta B_c}{B_0}\frac{2B_0L_c}{B\rho}\right)^2 \approx 2\times10^{-9}\,.$$

$$\frac{\Delta E}{E_0} = \frac{\Delta\nu\cdot 440.65}{E_0} \approx 2\times10^{-11}\,.$$

Spin distribution width: horizontal betatron oscillations

Ya.S. Derbenev, et al., "Accurate calibration of the beam energy in a storage ring based on measurement of spin precession frequency of polarized particles", Part. Accel. 10 (1980) 177-180

FCCee_z_202_nosol_13

Sextupole fields introduce additional $B_\perp \propto x^2$, $K2 = \dfrac{1}{B\rho}\dfrac{\partial^2 B_y}{\partial x^2}$.

Spin precession frequency distribution shifts and becomes wider by

$$\frac{\Delta\nu}{\nu} = -\frac{1}{2\pi}\oint \left(\varepsilon_x\beta_x(s) + \eta_x(s)^2\sigma_\delta^2\right) K2(s)ds\,.$$

$$\frac{\Delta\nu}{\nu} = \frac{\Delta E}{E_0} = -2.5\cdot 10^{-7}\,.$$

Vertical magnetic fields: horizontal correctors

One corrector with deflection χ: $\dfrac{\Delta E}{E_0} = -\dfrac{\chi \eta_x}{\alpha \Pi}\,, \quad \chi = \oint \dfrac{\Delta B_y}{B\rho} ds\,.$

RMS of energy shift: $\sigma\left(\dfrac{\Delta E}{E_0}\right) = \dfrac{2\sqrt{2}\sin(\pi\nu_x)}{\alpha \Pi} \dfrac{\langle \eta_x \rangle}{\langle \beta_x \rangle} \sigma_x$

σ_x is RMS of horizontal orbit variation.

FCCee_z_202_nosol_13

$$\sigma\left(\frac{\Delta E}{E_0}\right) = -1.2 \cdot 10^{-3}[m^{-1}] \cdot \sigma_x[m]\,,$$

$\sigma\left(\frac{\Delta E}{E_0}\right) = 10^{-6}$ demands stability of the horizontal orbit between calibrations

$$\sigma_x = 0.8 \text{ mm.}$$

Vertical magnetic fields: quadrupoles

Shifted quadrupole: $\frac{\Delta E}{E_0} = -\frac{\chi \eta_x}{\alpha \Pi}, \quad \chi = K1L \cdot \Delta x, K1 = \frac{1}{B\rho}\frac{\partial B_y}{\partial x}.$

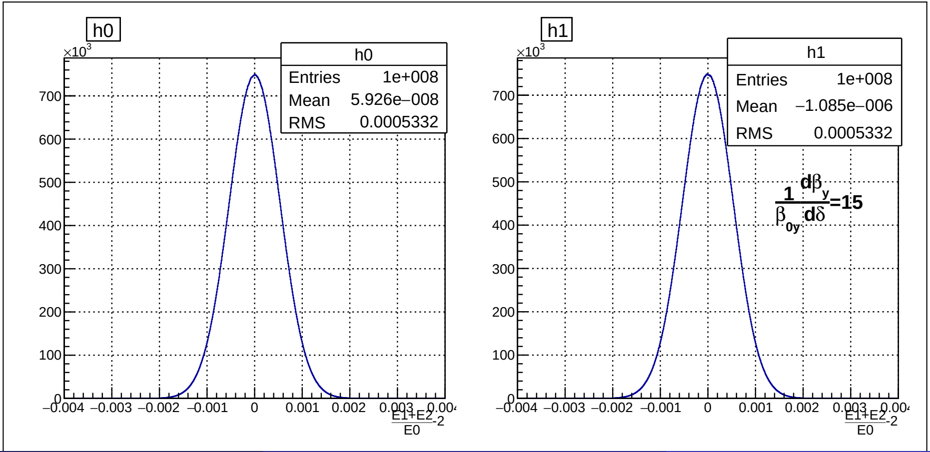
FCCee_z_202_nosol_13

$\frac{\Delta E}{E_0} = 10^{-6}$ demands stability of quadrupoles position between calibrations (10 min)

Quadrupole	Δx, m
QC7.1:	$2 \cdot 10^{-4}$
QY2.1:	$7.6 \cdot 10^{-5}$
QFG2.4:	$1.6 \cdot 10^{-4}$
QF4.1:	$1.4 \cdot 10^{-4}$
QG6.1:	$3.5 \cdot 10^{-5}$
QF4:	$\Delta x / \sqrt{720} = 5 \cdot 10^{-6}$

Central mass energy: β chromaticity

Invariant mass: $M^2 = (E_1 + E_2)^2 \cos^2(\theta) + O(m_e^2) + O(\sigma_\alpha^2) + O(\sigma_E^2)$.
Beta function chromaticity at IP: $\beta_{x,y} = \beta_{0\,x,y} + \beta_{1\,x,y}\delta$, $\sigma^2_{x,y} = \varepsilon_{x,y}\beta_{x,y}$.
Particles with energy deviation have higher collision rate.



Central mass energy: β chromaticity

FCCee_z_202_nosol_13

$\frac{1}{\beta_x}\frac{d\beta_x}{d\delta}$	$\frac{1}{\beta_y}\frac{d\beta_y}{d\delta}$	ΔM, keV	$\frac{\Delta M}{E_0}$
0	15	-49 ± 2.4	$-1.1 \cdot 10^{-6} \pm 5 \cdot 10^{-8}$
200	0	-26 ± 2.4	$-5.7 \cdot 10^{-7} \pm 5 \cdot 10^{-8}$
200	15	-75 ± 2.4	$-1.6 \cdot 10^{-6} \pm 5 \cdot 10^{-8}$

Need to measure and adjust $\frac{1}{\beta_{0y}}\frac{d\beta_y}{d\delta}$.

Energy dependence on azimuth: full tapering

Two diametrically opposite RF cavities, U_0 — energy loss per revolution, $E(0)$ — after RF cavity. Full tapering — magnets fields are adjusted to keep design curvature, quadrupole strength etc.

$$\frac{dE}{ds} \propto E^4\,, \quad E(s) = \frac{E(0)}{(1+k\cdot s)^{\frac{1}{3}}}\,, \qquad k \approx \frac{3}{\Pi}\frac{U_0}{E(0)} + \frac{3}{\Pi}\frac{U_0^2}{E(0)^2} + O(U_0^3)$$

Average energy: $\langle E\rangle \approx E(0) - \dfrac{U_0}{4} - \dfrac{U_0^2}{12E(0)}\,.$

Energy at the IP: $E(IP) = E(0) - \dfrac{U_0}{4}$.

The difference: $\dfrac{\langle E\rangle - E(IP)}{E(0)} \approx -\dfrac{1}{12}\dfrac{U_0^2}{E(0)^2} = 5\cdot 10^{-8}$, for $E_0 = 45.6$ GeV (Z).

The difference: $\dfrac{\langle E\rangle - E(IP)}{E(0)} \approx -\dfrac{1}{12}\dfrac{U_0^2}{E(0)^2} = 2\cdot 10^{-7}$, for $E_0 = 80.5$ GeV (WW).

Energy dependence on azimuth: partial tapering

Partial tapering (ΔK_0) — fields of magnets groups are adjusted to keep approximately design curvature (K_0).

Equations of motion (canonical variables)

$$\begin{cases} \sigma' = -K_0 x\,, \\ p_t{}' = \left(-\dfrac{eV_0}{p_0 c}\right)\sin\left(\phi_s + \dfrac{2\pi}{\lambda_{RF}\sigma}\right)\delta(s - s_0) - \dfrac{2}{3}\dfrac{e^2\gamma^4}{p_0 c}K_0^2\sigma\,. \end{cases}$$

Solution: $p_t(s) = p_{0t} - f(s)$.

$$\sigma = 0 = -\int_0^{\Pi} K_0(s)x(s)ds = -p_{0t}\alpha\Pi + \Pi\left\langle (K_0 f + \Delta K_0)\eta \right\rangle_s\,.$$

$$p_{0t} = \frac{1}{\alpha}\left\langle (K_0 f + \Delta K_0)\eta \right\rangle_s\,.$$

Energy dependence on azimuth: partial tapering

For simple (symmetrical) cases we do need to know function $f(s)$, just at certain points.

Two RF cavities and symmetrical arcs

$$\begin{cases} \langle p_t \rangle = p_{0t} - \langle f \rangle = p_{0t} - \dfrac{U_0}{4E_0} = \dfrac{\langle E \rangle - E_0}{E_0}, \\ p_t(IP) = p_{0t} - f(IP) = p_{0t} - \dfrac{U_0}{4E_0} = \dfrac{E_{IP} - E_0}{E_0}, \end{cases}$$

$$\begin{cases} \langle E \rangle = E_0 + E_0 p_{0t} - \dfrac{U_0}{4}, \\ E_{IP} = E_0 + E_0 p_{0t} - \dfrac{U_0}{4}. \end{cases}$$

There is no difference between $\langle E \rangle$ and E_{IP} in the first order. Numerical calculations are needed for not symmetrical arcs, magnet misalignments.

Collective field of the own bunch

Electron in the field of own bunch will have potential energy

$$U[eV] = \frac{N_p e^2[Gs]}{\sqrt{2\pi}\sigma_z[cm]}\left(\gamma_e + \ln(2) - 2\ln\left(\frac{\sigma_x+\sigma_y}{r}\right)\right)\frac{10^{-7}}{e[C]}\,,$$

$\gamma_e = 0.577$ Euler constant, $N_p = 4\cdot 10^{10}$ — bunch population, $r_{ip} = 15$ mm and $r_{arc} = 20$ mm — vacuum chamber radius at IP and in the arcs, $\sigma_{x,IP} = 6.2\cdot 10^{-6}$ m, $\sigma_{y,IP} = 3.1\cdot 10^{-8}$ m, , $\sigma_{x,arc} = 1.9\cdot 10^{-4}$ m, $\sigma_{y,arc} = 1.2\cdot 10^{-5}$ m.

$$\frac{U_{ip}}{E_0} = \frac{192keV}{45.6GeV} = 4.2\cdot 10^{-6}\,,$$

$$\frac{U_{arc}}{E_0} = \frac{120keV}{45.6GeV} = 2.6\cdot 10^{-6}\,.$$

Collective field of the opposite bunch

Potential energy at the center of the bunch $\{x, y, s, z = s - ct\} = \{0, 0, 0, 0\}$

$$U(x, y, s, ct) = -\frac{\gamma N_p r_e mc^2}{\sqrt{\pi}} \int_0^\infty dq \frac{\exp\left[-\frac{(x+s\cdot 2\theta)^2}{2\sigma_x^2+q} - \frac{y^2}{2\sigma_y^2+q} - \frac{\gamma^2(s+ct)^2}{2\gamma^2\sigma_s^2+q}\right]}{\sqrt{2\sigma_x^2+q}\sqrt{2\sigma_y^2+q}\sqrt{2\gamma^2\sigma_s^2+q}},$$

$$\frac{U(0, 0, 0, 0)}{E_0} = -\frac{0.4MeV}{45.6GeV} = -9.3 \cdot 10^{-6}.$$

Invariant mass in the external field

The four-momentum: $P^\mu = (E - e\varphi, \vec{p}) = (E - e\varphi, \vec{\mathcal{P}} - \frac{e}{c}\vec{A})$,

Energy-momentum relation: $(E - e\varphi)^2 = m^2 c^4 + c^2 (\vec{p})^2$.

Invariant mass:

$$M^2 = (P_1^\mu + P_2^\mu)^2 = 2E_1 e_1 \varphi + 2E_2 e_2 \varphi + 2E_1 E_2 - (e_1\varphi)^2 - (e_2\varphi)^2 - 2p^{\vec{(1)}} p^{\vec{(2)}} .$$

Longitudinal momentum ($\delta = (E_i - E_0)/E_0$, $u = e_i\varphi/E_0$):

$$\begin{aligned} p_{i,s} &= \sqrt{(E_i - e_i\varphi)^2 - p_{i,x}^2 - p_{i,y}^2} \\ &= E_0 \sqrt{(1 + \delta_i - u)^2 - \left(\frac{p_{i,x}}{E_0}\right)^2 - \left(\frac{p_{i,y}}{E_0}\right)^2} . \end{aligned}$$

Invariant mass

Average values

$$\left\langle M^2 \right\rangle = 4E_0^2 \cos^2(\theta)(1 - u^2) - 2E_0^2\sigma_{px}^2 \cos(2\theta) - 2E_0^2\sigma_{py}^2 \cos(2\theta)$$

$$\langle M \rangle = 2E_0 \cos(\theta) \left(1 - \frac{u^2}{2}\right) - \frac{E_0}{2} \left(\sigma_\delta^2 \cos(\theta) + \sigma_{px}^2 \cos(\theta) + \sigma_{py}^2 \frac{\cos(2\theta)}{\cos(\theta)}\right)$$

$$\left\langle M^2 \right\rangle - \langle M \rangle^2 = 2E_0^2 \cos^2(\theta) \left(\sigma_\delta^2 + \sigma_{px}^2 \tan^2(\theta)\right)$$

Invariant mass shift due to beam potentials

$$\frac{\langle M \rangle - 2E_0 \cos(\theta)}{2E_0 \cos(\theta)} = \left(1 - \frac{(e\varphi)^2}{2E_0^2}\right) \approx 4 \times 10^{-10}$$

Summary

What is not estimated?

1. Vertical orbit distortions. They will change the spin tune.
2. Electron positron energy difference due to synchrotron radiation in not identical arcs, energy loss due to not identical impedance of the vacuum chamber in the arcs. Requires impedance estimations.
3. Influence of wrong RF cavities (LEP) (wrong phase, misalignment etc.)
4. Local separation of the beams (change of the orbit length and spin tune, nonlinear elements in the bump).
5. Beam separation in presence of the opposite sign dispersion.

Summary

Largest corrections and errors

1. Beta function chromaticity (correction, tunable) $\sim 2 \times 10^{-6}$.
2. Horizontal correctors and shift of quadrupoles (error) $\sim 10^{-6}$ with position stability of arc quadrupoles $\Delta x < 5 \times 10^{-6}$ between calibrations (every 10 minutes).
3. Horizontal betatron oscillations and sextupole fields $2.5 \sim 10^{-7}$.

References

1. V.V. Danilov et al., “Longitudinal Beam-Beam Effects for an Ultra-High Luminosity Regime”, proceedings of PAC 1991, p. 526.
2. F. Zimmerman and Tor O. Raubenheimer, “Longitudinal space charge in final focus systems for linear colliders”, SLAC-PUB-7304 (1997).
3. V.E. Blinov et al., “Analysis of errors and estimation of accuracy in the experiment on precise mass measurement of J/psi, psi’ mesons and tau lepton on the VEPP-4M collider”, NIM A 494 (2002) 68-74.
4. V.E. Blinov et al., “Absolute calibration of particle energy at VEPP-4M ”, NIM A 494 (2002) 81-85.