Using an RF dipole to measure the beam energy — what is measured and then what?

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The following material describes a framework, based on spin-orbit tracking simulations for the effect of a radio frequency dipole, aimed at measuring electron energies at the interaction points (IP). No numerical estimates are offered, just the concept.

1) Before using a radio-frequency (rf) magnetic field to measure a spin tune in order to work back to the beam energy at an IP, we need an understanding of the meaning of spin tune.

2) For a particle on the 6-D closed orbit (CO) of a ring (which will always differ from the design orbit), the CO spin tune can be trivially extracted from the complex eigenvalues of the 1-turn 3x3 spin-rotation matrix on the CO. In standard modern notation, the spin tune on the CO is denoted by ν_0.

   Note that the CO spin tune takes account of the "energy saw tooth" and resulting transverse CO shape arising from the loss of energy by radiation and its replenishment by the rf cavities.

3) But particles execute syncho-betatron (s-b) motion away from the CO and then the instantaneous rate of spin precession and the axis of precession varies around the ring. An instantaneous rate of precession is not a spin tune. An "eigentune" extracted from a 1-turn 3x3 matrix on a s-b trajectory depends on the orbital phases and is therefore also not a spin tune! Instead one needs the concept of the amplitude dependent spin tune (ADST).

   For a good survey on this see the book by Georg Hoffstaetter [1], Mathias Vogt's thesis [2] or Barber, Ellison and Heinemann [3].

   The ADST is defined for integrable motion and is a function of only the 3 orbital amplitudes (J_1, J_2, J_3)— NOT the phases!!! In standard modern notation the ADST is denoted by $\nu(\vec{J})$. [1] The ADST is the average number of precessions around the *invariant spin field* per turn.

   See page 163 of the GH book or page 107 of the MV thesis for an example for protons at very high energy for single amplitude vertical motion.

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[1]LET US AGREE TO STICK TO STANDARD MODERN NOTATION IN ORDER TO FACILITATE COMMUNICATION.

4) The ADST in that example was calculated using methods detailed in those sources.

   However, it can also be discovered with a fast Fourier transform (FFT) of the spin motion.

   The ADST is, in fact, an equivalence class consisting of a single tune the "preferred spin tune" (PST)) which reduces to ν_0 as the amplitudes go to zero, together with a countable infinity of tunes obtained by adding linear combinations of integer multiples of orbital tunes and integers to the PST.

   In general an FFT will expose a few spectral lines from the ADST, including the PST.

   Spin-orbit resonance is properly defined in terms of $\nu(\vec{J})$, not ν_0!!!!

$$\nu(\vec{J}) = k_0 + k_1 Q_1 + k_2 Q_2 + k_3 Q_3$$

   In fact those pages 163 and 107 illustrate a second order resonance and show the spin-tune gap. If the system is close to spin-orbit resonance with one member of the ADST, it is close to resonance with all other members.

5) Energy oscillations due to synchrotron motion contribute oscillations in the instantaneous precession rate – but the members of the ADST DO NOT oscillate — they depend only on the amplitude of the synchrotron motion and the other two amplitudes.

   Instead, an FFT exposes lines close to the PST separated from the PST by multiples of the synchrotron tune (which for protons is usually very small). These nearby lines give rise to the so-called synchrotron sideband resonances.

6) Spin flipping with a weak rf magnetic field takes place when the rf tune matches a member of the ADST! – not when it matches the CO spin tune. See Barber's talk at Spin2010 [4] where it is shown how the so-called single resonance model still applies.

   The physics is clear: the perturbing rf field should be coherent with the long-term spin motion encoded in the ADST.

   Moreover with synchrotron motion, to discover the PST, one should check that one is not sitting at a sideband.

7) The above items should be well known to people working seriously with spin-orbit dynamics.

8) Of course, in a beam with a smooth distribution of amplitudes, there is no single PST. Instead the FFT will display a spread of lines centered on a line associated with an average of the orbital amplitudes.

   In general the distribution will not be centered on the CO spin tune. So a spin-flipping exercise will not expose the CO spin tune.

9) The shift of the PSTs away from the CO spin tune will have contributions from sources as listed, for example, by Anton Bogomyagkov and in the papers about measurements at VEPP-4M [5, 6]

   For example at higher order the instantaneous rate of spin precession can depend quadratically on the fractional energy deviation due to synchrotron motion and it can then be that the ADST is not so simple to define as with the purely linear dependence of the rate of precession on the fractional energy deviation.

For electrons there is orbital damping and noise due to radiation. Nevertheless, an FFT should expose a tune spectrum for the long-term spin motion.

10) So what is measured when an rf magnetic field resonantly kills the polarization in an electron ring?

11) This can be discovered by Monte-Carlo simulations which include the rf magnetic field.

    For that one needs a very high quality spin-orbit tracking code. Several codes are, of course, available but we suggest that the most suitable is Bmad by David Sagan. As well as providing spin-orbit tracking, it is a tool kit whose tools can be combined for a wide range of studies. With this one can obtain the distribution of the PST, or its generalization, with an FFT. Bmad has been central to the continual improvement of the performance of CESR at cornell.

    Then, one can simulate the effect of the rf magnetic field and discover how the rf tune which causes depolarization is related to the centre of the distribution of generalised PSTs.

    The simulations with Bmad should include all known effects on the orbital and spin motion. This inclusion would probably entail a some careful extension of the basic spin-orbit tracking code and it would require the attention of an expert. In any case a proper study would involve a deep understanding of particle dynamics in storage rings and would provide material for a substantial PhD thesis.

    Any interference between effects would be automatically taken into account and it would be straightforward to check the sensitivity of the depolarizing rf tune to parameters and to compare with rough hand-made estimates

    With this we would know what the rf magnetic field exposes/measures and how far the rf-depolarizing tune is from the CO spin tune and the design-orbit spin tune.

12) As David Sagan will confirm, Bmad is ideally suited. So far:

    – It can use high order symplectic tracking (before damping).
    – It can simulate radiation.
    – It uses full 3-D spin motion.
    – It handles misalignments.
    – It handles crab crossing.
    – It handles space charge.
    – It handles wake fields.
    – It handles wigglers.
    – Etc.
    – It is written in modern Fortran.
    – and it will be the essential tool for a substantial contract recently awarded by the DOE for joint work with the University of New Mexico and JLab.

11) Finally, with the inclusion of the effects of the collective fields of an electron's own bunch and of the oncoming bunch, this approach, using Monte-Carlo simulation, should lead to an understanding of how the distribution of particle energies at the interaction points is related to the rf-depolarizing tune.

This kind of approach to understanding and studying spin-motion in storage rings has been standard at DESY since 1982 although the early software was not too sophisticated.

References

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