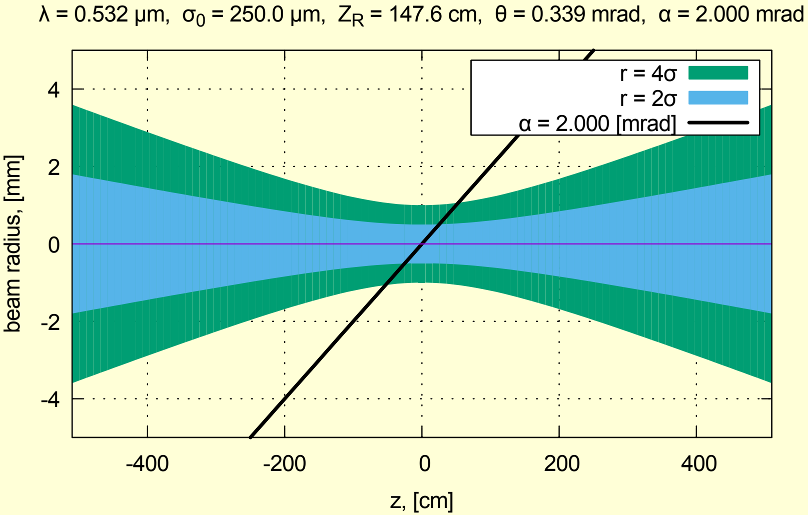
Compton Polarimeter: intensity of Inverse Compton scattering at FCC-ee

An example



λ = 0.532 μm, σ0 = 250.0 μm, ZR = 147.6 cm, θ = 0.339 mrad, α = 2.000 mrad
r = 4σ
r = 2σ
α = 2.000 [mrad]
beam radius, [mm]
4
2
0
-2
-4
-400
-200
0
200
400
z, [cm]

Gaussian beam

The optical intensity [W/cm^2] in a Gaussian beam is:

$$I(r,z) = \frac{2P}{\pi w(z)^2} \exp\left(-\frac{2r^2}{w(z)^2}\right),$$

The beam radius $w(z)$ is the distance from the beam axis where the intensity drops to $1/e^2$ ($\simeq$13.5%) of the maximum value.

$$w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_R}\right)^2},$$

$z_R = \dfrac{\pi w_0^2}{\lambda}$ is the Rayleigh length: $I(0, z_R) = I(0,0)/2$.

Gaussian beam

The optical intensity [W/cm^2] in a Gaussian beam is:

$$I(r,z) = \frac{P}{2\pi\sigma(z)^2}\exp\left(-\frac{r^2}{2\sigma(z)^2}\right).$$

The beam size $\sigma(z) = w(z)/2$ is the distance from the axis where the intensity drops to $1/e$ ($\simeq$36.8%) of the maximum value.

$$\sigma(z) = \sigma_0\sqrt{1+\left(\frac{z}{z_R}\right)^2},\ \text{where } z_R = \frac{4\pi\sigma_0^2}{\lambda}.$$

Far field divergence: $\theta = \dfrac{\sigma_0}{z_R} = \dfrac{\lambda}{4\pi\sigma_0}$

An electron in the field of laser beam head sea

Radiation power is a number of laser photons emitted per second:

$$P = dE/dt = h\nu \cdot dN/dt \ \ [\mathsf{J\ s}^{-1}]$$

Thus the longitudinal density of laser photons along z is:

$$\rho_{\|} = \frac{dN}{dz} = \frac{P\lambda}{hc^2} \ \ [\mathsf{cm}^{-1}]$$

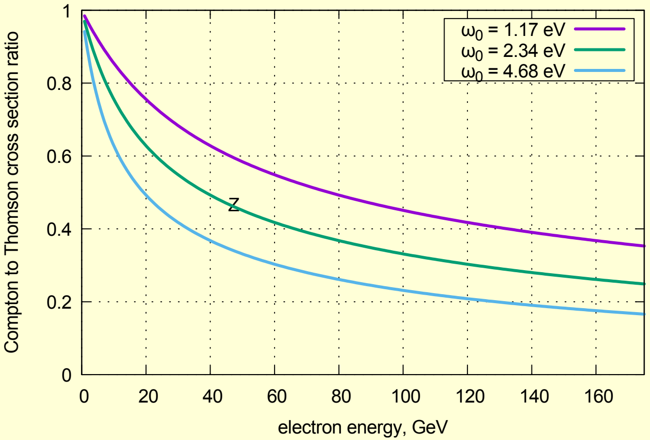
Consider an electron $(v/c \simeq 1)$ propagating towards the laser head sea with small incident angle α.

The photon target density seen by this electron will be defined as:

$$\rho_{\perp} = \rho_{\|} \frac{(1+\cos\alpha)}{2\pi\sigma_0^2} \int\limits_{-\infty}^{\infty} \frac{\exp\left(-\frac{z^2\tan^2\alpha}{2\sigma(z)^2}\right)}{1+(z/z_R)^2} dz \ \ [\mathsf{cm}^{-2}]$$

Compton scattering cross section

The probability W of the Compton scattering is determined by the product of $\rho_\perp$ and the scattering cross section.



Inverse Compton scattering probability

The maximum scattering probability W_{max} is reached in head-on case ($\alpha = 0$) and at low energy with $\sigma_T = 0.665$ barn.

$$W_{max} = \frac{\sigma_T}{\pi\sigma_0^2}\frac{P\lambda}{hc^2}\int\limits_{-\infty}^{\infty}\frac{dz}{1+(z/z_R)^2} =$$

$$= \frac{4\sigma_T P}{hc^2}\int\limits_{-\infty}^{\infty}\frac{dx}{1+x^2} = \frac{4\pi\sigma_T P}{hc^2} = \frac{P}{P_c},$$

where $\boxed{P_c = \hbar c^2/2\sigma_T \simeq 0.7124 \cdot 10^{11}\ \text{[W]}}$ is the power of laser radiation required for 100% scattering probability.

We see that W_{max} depends on laser power only!

Scattering probability with CW laser

The loss in scattering probability with $\alpha \neq 0$ is defined as:

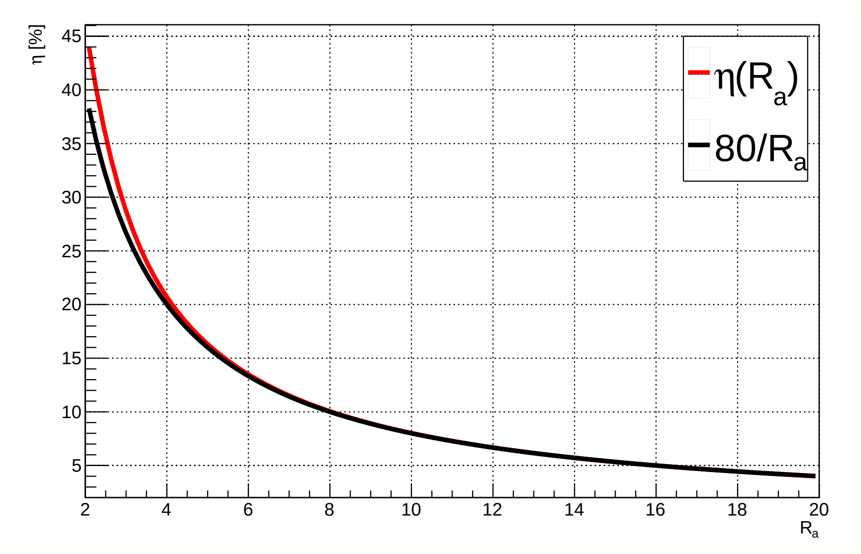
$$\eta(\alpha) \equiv \frac{W(\alpha)}{W_{max}} = \frac{1+\cos\alpha}{2\pi} \int\limits_{-\infty}^{\infty} \exp\left(-\frac{x^2 \tan^2\alpha}{2\theta_0^2(1+x^2)}\right) \frac{dx}{1+x^2}.$$

$\alpha \ll 1$ is preferable and $\alpha > \theta_0$:

$$\eta(R_a) = \frac{1}{\pi} \int\limits_{-\infty}^{\infty} \exp\left(-\frac{x^2 R_a^2}{2(1+x^2)}\right) \frac{dx}{1+x^2},$$

where $\boxed{R_a = \alpha/\theta_0}$ is the "Ratio of angles"

Scattering efficiency vs scattering angle



Scattering probability: pulsed laser

Laser pulse propagation:

$$\rho_{\parallel}(s,t) = \frac{N_\gamma}{\sqrt{2\pi} c\tau_L} \exp\left\{-\frac{1}{2}\left(\frac{s-ct}{c\tau_L}\right)^2\right\}, \quad N_\gamma = E_L\lambda/hc.$$

Scattering probability for $\alpha = 0$:

$$W = \frac{(E_L/\sqrt{2\pi}\tau_L)}{P_c} \times \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\exp\{-2(x\, z_R/c\tau_L)^2\}}{1+x^2}\, dx$$

$$= \frac{P_L}{P_c} \times \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\exp\{-2(xR_l)^2\}}{1+x^2} dx, \text{ where } P_L = E_L/\sqrt{2\pi}\tau_L$$

and $\boxed{R_l = z_R/c\tau_L}$ is the “Ratio of lengths”

Scattering probability: pulsed laser

Scattering probability for arbitrary α is defined as:

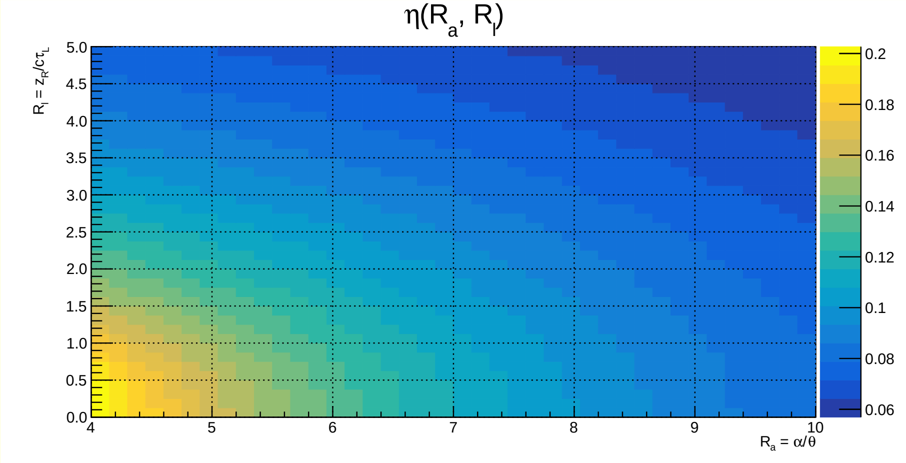
$$W = \frac{P_L}{P_c} \times \frac{1}{\pi} \int\limits_{-\infty}^{\infty} \frac{\exp\left\{-x^2 \left(2R_l^2 + \frac{R_a^2}{2(1+x^2)}\right)\right\}}{1+x^2} dx,$$

where

$$P_L = E_L/\sqrt{2\pi}\tau_L, \quad P_c \simeq 0.7124 \cdot 10^{11} \text{ [W]}$$

$$R_l = z_R/c\tau_L, \quad R_a = \alpha/\theta_0 = \frac{4\pi\sigma_0}{\alpha}.$$

Scattering probability: pulsed laser



FCC-ee beam parameters

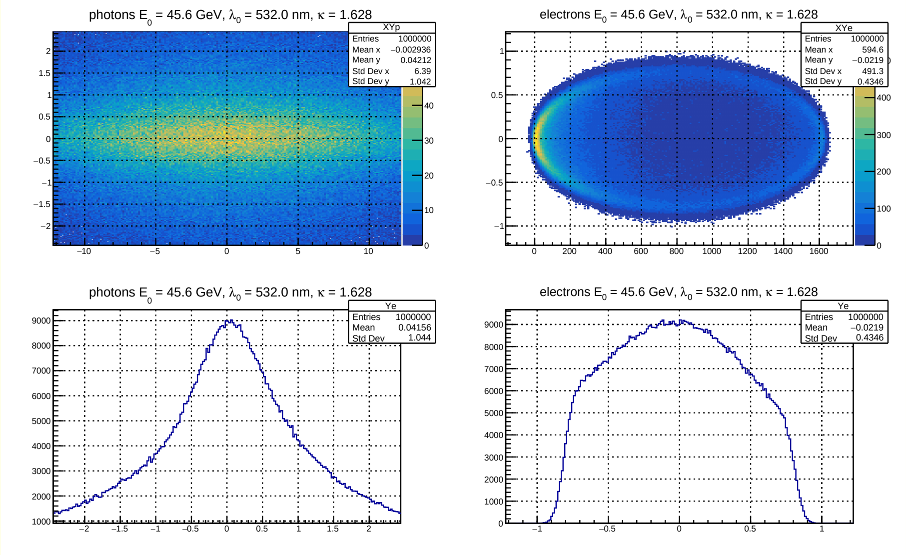
$$\beta_x = 100\ \text{m};\ \epsilon_x = 0.27\ \text{nm}: \qquad \sigma_{x,\beta} = \sqrt{\epsilon_x \beta_x} = 0.165\ \text{mm}$$
$$\eta_x = 100\ \text{mm};\ \sigma_E/E = 4 \cdot 10^{-4}: \quad \sigma_{x,\eta} = \eta_x \sigma_e/E = 0.04\ \text{mm}$$

Example

- Laser wavelength $\lambda = 532$ nm.
- Waist size $\sigma_0 = 0.250$ mm. Rayleigh length $z_R = 148$ cm.
- Far field divergence $\theta = 0.169$ mrad
- Interaction angle $\alpha = 1.000$ mrad
- Compton cross section correction 0.5
- Pulse energy: $E_L = 1$ [mJ]; τ_L=5 [ns] (sigma)
- Pulse power: $P_L = 80$ [kW]
- Ratio of angles $R_a = 5.905249$
- Ratio of lengths $R_l = 0.984208$
- $P_L/P_c = 1.1 \cdot 10^{-6}$
- "efficiency" $= 0.13$
- Scattering probability $W \simeq 7 \cdot 10^{-8}$

- With 10^{10} electrons and 3 kHz rep. rate: $\dot{N}_\gamma \simeq 2 \cdot 10^6$

Scattered photons and electrons (MC)



Example: 10^6 scattered particles

$$\frac{dP_\perp}{P_\perp} = \frac{\sigma_y}{\bar{y}\sqrt{N}} \text{ (stat. only)}$$

- $P_\perp = 10\% : dP_\perp/P_\perp = 10\%$(fit of e gives 7%)
- $P_\perp = 20\% : dP_\perp/P_\perp = 6\%$(fit of e gives 3.7%)
- $P_\perp = 30\% : dP_\perp/P_\perp = 4\%$
- $P_\perp = 40\% : dP_\perp/P_\perp = 2.8\%$

Scattered electrons MC and fit result