The relationships $\nu_{spin} = a\gamma$ holds for a purely planar ring.

The effect of closed orbit distortion has been evaluated for LEP by using a simplified model by R. Assmann who found that for half-integer $\nu_s^0$ it is $\Delta\nu_s = 0$ in first and second order in the extra-spin rotations. For $\nu_s^0 \neq 0.5$ it is

$$< \Delta\nu_s > = \frac{\cot \pi \nu_s^0 (a\gamma)^2}{8\pi} \left[ < \Sigma_q (K \ell)_q^2 y_q^2 > + < \Sigma_k \theta_k^2 > \right]$$

$y_q =$ effective beam position at the quadrupole

Evaluating this expression over 10 seeds
K. Yokoya (1988) spin tune shift

\[ \Delta \nu_s^{(1)} = \frac{1}{2\pi} R (a\gamma + 1) \int_0^{2\pi} d\theta (\hat{n}_0 \cdot \hat{y}) x''_{co} \]

that is \( \Delta \nu_s^{(1)} = 0 \) always for a planar designed ring. The second order term is

\[ \Delta \nu_s^{(2)} = \frac{1}{4\pi} R^2 (a\gamma + 1)^2 \mathcal{S} \left[ \frac{1}{e^{-i2\pi\nu_s^0} - 1} \int_0^{2\pi} d\theta h^*(\theta) y''_{co} \int_\theta^{\theta+2\pi} d\theta' h(\theta') y''_{co} \right] \]

with

\[ h(\theta) = (\hat{m}_0 + i\hat{l}_0) \cdot \hat{x} \]

\[ y'' = -K (y - \delta_y^Q) + \left( \frac{\Delta B}{B\rho} \right)_{cor} \]

Equivalent expressions were also found by Barber et al. in 1994. Yokoya expression may be used for comparison with Assmann one. Spin tune is computed by SITF from the transport matrix: it may be used in presence of distortions, when more statistics is available.
Effect of RF electric field (term $\vec{\beta} \times \vec{E}_{RF}$ in BMT-equation)$^a$

<table>
<thead>
<tr>
<th>$\Delta E$ (KeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>45 GeV 2 $\times y'_{rms}$</td>
</tr>
<tr>
<td>80 GeV 16 $\times y'_{rms}$</td>
</tr>
</tbody>
</table>

$y'_{rms} =$ rms slope in mrad. With

$$< y'_{rms} > \simeq \sqrt{\frac{< \gamma_y >}{< \beta_y >}} < y_{rms} > \simeq 0.1 < y_{rms} >$$

the contribution from the RF electric field should be small.

$^a$From Yu. I. Eidelman et al. formulas