The Z boson line shape at the FCCee

From 1.5 loops at LEP to 2.5 loops in future

– ZFITTER/TOPAZ0/KKMC $\longrightarrow$ SM^2 = Standard Model S-Matrix approach –

How-to and Prospects

Tord Riemann[a], University of Silesia, Katowice, Poland

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I. Dubovyk[a] + A. Freitas + J. Gluza[a] + M. Grünewald + S. Riemann + J. Usovitsch

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Preface

S-matrix approach to the 2-loop SM contributions, 1991-1992, 2006-2018

- Stuart 1991 [10]: Guiding principle for 2-loop calculations in presence of a resonance.
- Helen Veltman 1992 [11]: Scheme worked out.
- Awramik, Czakon, Freitas 2006+/- [12]: Scheme applied to effective weak mixing angles.

S-matrix approach to the Z line shape, 1991-1992, 2016+

- Developed for a model-independent analysis tool of $e^+e^- \to (\gamma, Z) \to f^+f^-$ **around the Z boson resonance**
- Aim: determinations of M_Z and Γ_Z in correlation with the γZ-interference
- Refs.: Leike/Riemann/Rose 1991 [1], Riemann 1992 [2], Kirsch/Riemann 1994 [3], First application: L3 1993 [4], also: TOPAZ, VENUS, OPAL, . . .
- Fortran software: stand-alone ZPOLE (Leike/Riemann 1991, unpublished) and SMATASY/ZFITTER (Grünewald/Kirsch/Riemann 1994→2005) [3, 5, 6, 7, 8, 9]
- This talk Dubovyk, Freitas, Gluza Riemann, Usovitsch 2016-2018: Join the two approaches for a scheme ready for FCC-ee accuracy; see PoS LL2016 [13].

Present interest in precision approaches to the Z boson I

Belle-II

$\sqrt{s} \sim 10$ GeV
$\rightarrow$ **Belle-II** will measure 10^9 $\mu^+\mu^-$ events
See e.g. T.Ferber [14]

Fcc-ee

$\sqrt{s} \sim M_Z$
$\rightarrow$ **Fcc-ee** expects 10^{13} events at the Z resonance
See e.g. A. Freitas [15].
Much work on weak two-loop contributions to the Z resonance has been done by Hollik et al., Czakon et al., Freitas et al.
see [16] and many refs. therein.

Model-independent alternative: How to do?
$\rightarrow$ Request by the Fcc-ee physics study group
$\rightarrow$ **S-matrix approach** a la SMATASY/ZFITTER
See T. Riemann [17] – Talk triggered by request from Fulvio, made me remind the stuff in 2015 2015.

Introduction

The analysis tool for the Z resonance: ZFITTER

- Complete electroweak radiative corrections
- QED corrections by convolution with some $\sigma_0(s')$ or for initial-final state interferences with some $\sigma_0(s,s')$
- **semi-analytical QED** integrations, using

$$\frac{1}{|s-M_Z^2+iM_Z\Gamma_Z|^2} \sim \frac{i}{\Gamma_Z}\left[\frac{1}{s-M_Z^2+iM_Z\Gamma_Z}-\frac{1}{s-M_Z^2-iM_Z\Gamma_Z}\right] \tag{1}$$

- free choice of $\sigma_0(s')$ by **user interfaces**
- Standard Model interfaces: four weak form factors $\rho, \kappa_e, \kappa_e, \kappa_{ef}$

ZFITTER is **well-tested**, **flexible**, **accurate** and **fast** at the same time.
$\rightarrow$ Interest e.g. at Belle-II for 10^9 events at $\sqrt{s} \sim 10$ GeV.
Compare: Fcc-ee expects 10^{13} Z events

Introduction

Stuart 1991 [10], S-Matrix ansatz for $e^+e^- \longrightarrow Z \longrightarrow f^+f^-$

$$M = \frac{R}{s - s_0} + F(s), \qquad s_0 = M_Z^2 - iM_Z\Gamma_Z \tag{2}$$

Allows to study, but mostly not tackled by the 2-loop people:

- Mass M_Z and width Γ_Z

  $\rightarrow$ Leike/Riemann/Rose 1991 [1]
- What are the resulting cross sections and how many independent degrees of freedom of the line shape are to be introduced?

  $\rightarrow$ Leike/Riemann/Rose 1991 [1]
- Who is correlated with whom?

  $\rightarrow$ Leike/Riemann/Rose 1991 [1] and Kirsch/S.Riemann, L3 [18, 4]
- What about asymmetries and QED corrections?

  $\rightarrow$ Riemann [2]
- But also: How to define gauge-invariant mass and width of Z at higher orders of perturbation theory?

  $\rightarrow$ Denner 2014 [19], Freitas 2014 [16] and Fcc-ee [15], Degrassi FCC-ee [20] and refs. therein

Introduction

The reaction

$$e^+e^- \to (\gamma, Z) \to f^+f^- \;+\; (n\,\gamma) \tag{3}$$

allows to study the Z boson, its mass M_Z, its width Γ_Z, its couplings, and potentially deviations from the Standard Model.

Need correct "model"

See experiences with *constant* and *s-dependent* Z width:

$$\frac{1}{[s - M_Z^2 + iM_Z\,\Gamma_Z(s)]} \quad \textbf{versus} \quad \frac{1}{[s - M_Z^2 + iM_Z\,\Gamma_Z]} \tag{4}$$

To a very good accuracy, it holds: $\Gamma_Z(s) \approx s/M_Z^2 \times \Gamma_Z$

see next slide, $\to$ Bardin/Leike/Riemann/Sachwitz 1988 [21]; also: Berends/Burgers/Hollik/v.Neerven 1988 [22]

Need correct unfolding ..

.. of *Realistic Observables* in order to get *Pseudo Observables*. $\to$ e.g.: Borrelli/Consoli/Maiani/Sisto 1990 [23], Later: Bardin/Passarino 1999 [24], Bardin/Grünewald/Passarino 1999 [25], Passarino 2003 [26], Passarino 2013 [27] and refs. therein.

Lesson: The model influences numerical results

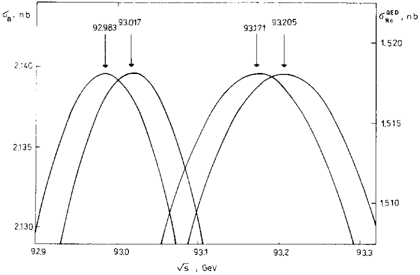


Fig. 1. Total cross sections σ_B, σ_{RC}^{QED} for the reactions $e^+e^- \to \mu^+\mu^-(\gamma)$ in Born approximation (left scale) and including $O(\alpha)$ QED corrections right scale). Peaks of σ with energy-dependent width $\Gamma_Z(s)$ are shifted by 34 MeV to the left.

Total cross section for $e^+e^- \to \mu^+\mu^-$ production at LEP . . .

. . . without (left) and with (right) QED corrections. Both sample data produced with an energy-dependent Z width.
The assumptions on the Z-propagator in the fit formulas influence the location of the peak, but not the “experimental errors”.
Fig.: from [21], license Number: 3557090997554.

	Born		Born + QED	
from fit: $\rightarrow$	M_Z	Γ_Z	M_Z	Γ_Z
$\Gamma_Z(s)$	93.000 ± 0.013	2.498 ± 0.009	93.000 ± 0.016	2.498 ± 0.011
Γ_Z	92.966 ± 0.013	2.498 ± 0.009	92.966 ± 0.016	2.498 ± 0.011

- Beware: **Gfitter/GSM (2007-2011) is an illegal clone of ZFITTER**, available at `http://zfitter-gfitter.desy.de/` and `http://fh.desy.de/projekte/gfitter01/Gfitter01.htm`.
  See also: `http://zfitter.education`, `http://zfitter.com`.

Total cross sections

There are immediate questions, from an experimental point:

- What about the photon exchange?
- What about QED corrections, e.g. the $2 \to 3$ part of the cross sections?
- What about asymmetries, besides σ_{tot}?

We have to describe

$$e^+e^- \longrightarrow (\gamma, Z) \longrightarrow f^+f^-(\gamma) \tag{5}$$

Ansatz in the complex energy plane, for **four helicity matrix elements**:

$$\mathcal{M}^i(s) = \frac{R^i_\gamma}{s} + \frac{R^i_Z}{s - s_Z} + F^i(s), \quad i = 1, \ldots 4. \tag{6}$$

Beware: Eqn. (19) is mathematically not consistent → Böhm/Sato 2004 [29]

The poles of $\mathcal{M}$ have complex residua R_Z and R_γ, the latter corresponding to the photon, and the background $F(s)$ is an analytic function without poles:

$$F^i(s) = \sum_{n=0}^{\infty} F^i_n (s - s_0)^n \tag{7}$$

Introductory Remarks – the SM^2 Approach I

Our aim is to combine several theoretical results into a frame for the analysis of Z line shape data obtained at the FCCee:

For analyzing the precision data from running the FCCee at the Z peak,

$$\sqrt{s} \approx M_Z^2.$$

Aim at a theoretical accuracy of about 10^{-4}, i.e. much better than the LEP 1 accuracy of about 10^{-3}.

This means in practice going from considering 1.5 loop orders to 2.5 loop orders.

2.5 loop orders means here that we aim at accurate calculations up to 2 loops, and add also the numerically most important 3 loop (or higher order) terms.

We will demonstrate that one has to combine several elements:

From the theory side I

Determination of the $Z \to f\bar{f}$ vertex – i.e. v_f, a_f – at 2 to 3 loops

Disentangling so-called pseudo-observables:

$\to$ E.g. write asymmetries like

$$A_{FB}^0 \equiv A_{FB}(e^+e^- \to f\bar{f}) = \frac{\sigma_{FB}^0}{\sigma_{tot}^0} \approx \frac{3}{4} A_e A_f$$

in terms of factorizing building blocks

$$A_f \equiv \frac{2\Re e \frac{v_f}{a_f}}{1+\left(\Re e \frac{v_f}{a_f}\right)^2} \approx \frac{2v_f a_f}{v_f^2+a_f^2} = \frac{1-4|Q_f|\sin^2\theta_{\rm eff}^{\rm f}}{1-4|Q_f|\sin^2\theta_{\rm eff}^{\rm f}+8Q_f^2\left(\sin^2\theta_{\rm eff}^{\rm f}\right)^2}$$

These A_f are expressible by effective weak mixing angles due to the definition

$$\Re e \frac{v_f}{a_f} \equiv 1 - 4|Q_f|\sin^2\theta_{eff}^f$$

$\to$ E.g. determine partial Z widths and the total Z width as functions of building blocks

$$\Gamma_f \sim v_f^2 + a_f^2 = 2a_f^2\left[1 - 4|Q_f|\sin^2\theta_{\rm eff}^{\rm f} + 8Q_f^2\left(\sin^2\theta_{\rm eff}^{\rm f}\right)^2\right]$$

From the theory side II

The specific problem from the theory side is:

Organize a consistent perturbation expansion at several loop orders in presence of an unstable intermediate state: the Z boson

Consistent means:

Well-defined, unitary, gauge-invariant, analytic

Maybe not necessarily unique in details

Since the LEP studies around 1990 we know that such a scheme may be based on an S-matrix element:

$$M_Z \sim \frac{R}{s - s_0^2} + \sum_{n=0}^{\infty} (s - s_0^2)^n M_n,$$

$$s_0^2 \equiv M_Z^2 + i M_Z \Gamma_Z$$

This is a Laurent expansion.

From the theory side III

Laurent expansions have one singular point, here at $s = s_0$, with a pole of first order.
The R and M_n are complex constants.
Remark: There is no room for a photon exchange term, $M_\gamma = Const/s$.

From the experimental side I

Use cross sections

$$\sigma^{realistic}(e^+e^- \to f\bar{f} + n\,\gamma + m\ gluons + light\ pairs\ etc.)$$

for an extraction of the above-mentioned pseudo-observables, i.e.

$$\sigma^0_{tot}(e^+e^- \to f\bar{f}), \quad \sigma^0_{FB}(e^+e^- \to f\bar{f}), \quad \cdots$$

and the quantities like

$$\Gamma_Z^{tot}, \sin^2\theta_{f,eff}, R_b, \cdots$$

The specific problem from the experimental side is:

Determine the Born and the real emission contributions such that the S-matrix language may be consistently applied. Example for initial state corrections:

$$\sigma_{tot,FB} = \int \frac{ds'}{s} \rho^{ini}_{tot,FB}(s'/s)\sigma^0_{tot,FB}(s') \text{ + corrections}$$

The $\rho^{ini}_{tot}(s'/s)$ and $\rho^{ini}_{FB}(s'/s)$ and $\rho^{fin}_{tot,FB,LR,pol}(s'/s)$ and $\rho^{ini-fin}_{tot,FB}(s'/s)$ are certain universal radiator functions, characteristic of distribution type and cut dependent.

From the experimental side II

Another approach are Monte Carlo programs, $\rightarrow$ see talk by Stashek.
The $\sigma^0_{tot}(s')$ and $\sigma^0_{FB}(s')$ etc. are $2 \rightarrow 2$ pseudo cross sections derived, here, from an S-matrix ansatz.
For initial-final state interferences, the $\sigma^0_{tot,FB}(s,s')$ has two arguments s, s'.
Etc.
$\rightarrow$ This means that a so-called "model-independent" data analysis does, e.g., not explicitly rely on simple effective Born Standard Model variables.
Because such an ansatz would bias the analysis to some extent, which bias should be compared to the anticipated accuracy of about 10^{-4}.

Let us call the approach the **Standard Model S-Matrix approach:** **the SM^2 approach**.

The claim is that the SM^2 approach is the unique way how to do an analysis of data with a precision of 2 electroweak loops or more loops.
So we consider some discussion of the SM^2 approach in the CDR to be important.

Although all building blocks have been studied and applied since long, their systematic combination is invented here for the first time.

From hep-ph/0608099:

In higher-order calculations, occurrences of unstable intermediate particles need to be treated carefully in order to preserve gauge-invariance and unitarity. Currently, the only scheme proven to fulfill both requirements to all orders in perturbation theory is the *pole scheme* [30, 31, 10, 11, 32, 33, 34]

It involves a systematic Laurent expansion around the complex pole $\mathcal{M}^2 = M^2 - iM\Gamma$ associated with the propagator of the unstable particle with mass M and width Γ. In the case of the process $e^+e^- \to f\bar{f}$, $e \neq f$, near the Z pole, the amplitude is written as

$$\mathcal{A}[e^+e^- \to f\bar{f}] = \frac{R}{s - \mathcal{M}_Z^2} + S + (s - \mathcal{M}_Z^2)S' + \ldots \tag{8}$$

with

$$\mathcal{M}_Z^2 = \overline{M}_Z^2 - i\overline{M}_Z\overline{\Gamma}_Z. \tag{9}$$

Owing to the analyticity of the S-matrix, all coefficients of Laurent expansion, $R, S, S', \ldots$ and the pole location $\mathcal{M}_Z^2$ are individually gauge-invariant, UV- and IR-finite, when soft and collinear real photon emission is added.

The first term in (8) corresponds to a Breit-Wigner parametrization of the Z line shape with a constant decay width.

Experimentally, however, the gauge-boson mass is determined based on a Breit-Wigner function with a running (energy-dependent) width,

$$\mathcal{A} \propto \frac{1}{s - M_Z^2 + is\Gamma_Z/M_Z}. \tag{10}$$

As a consequence of these different parameterizations, there is a shift between the experimental mass parameter, M_Z, and the mass parameter of the pole scheme, $\overline{M}_Z$, [21]

$$\overline{M}_Z^2 = M_Z^2/(1 + \Gamma_Z^2/M_Z^2), \tag{11}$$

amounting to $\overline{M}_Z \approx M_Z - 34.1$ MeV. In the following, barred quantities always refer to pole scheme parameters.
The evaluation of higher order contributions in the pole scheme involves a simultaneous expansion around the pole location and in the perturbation order α. Since near the Z pole α, Γ_Z and $(s - \mathcal{M}_Z^2)$ are all of the same order, for a next-to-next-leading order calculation R needs to be determined to $\mathcal{O}(\alpha^2)$, S only to $\mathcal{O}(\alpha)$, while a tree-level result is sufficient for S'.
The effective weak mixing angle is contained in the pole term residue R in (8).
...

In experimental studies, the program ZFITTER [7] is widely used for prediction of the contributions from QED and QCD corrections, s-channel photon exchange and γ-Z interference, off-shellness of the Z-boson and box contributions. In ZFITTER, the radiative corrections to the process $e^+e^- \to f\bar{f}$ are parametrized by four form factors $\rho_{\rm ef}$, $\kappa_{\rm e}$, $\kappa_{\rm f}$, $\kappa_{\rm ef}$,

$$\begin{aligned}
\mathcal{A}[e^+e^- \to f\bar{f}] = 4\pi i\, \alpha\, \frac{Q_{\rm e}Q_{\rm f}}{s}\, \gamma_\mu \otimes \gamma^\mu \\
+ i \frac{\sqrt{2} G_\mu M_{\rm Z}^2}{1 + i\Gamma_{\rm Z}/M_{\rm Z}}\, I_{\rm e}^{(3)}\, I_{\rm f}^{(3)}\, \frac{1}{s - \overline{M}_{\rm Z}^2 + i\overline{M}_{\rm Z}\overline{\Gamma}_{\rm Z}} \\
\times \rho_{\rm ef} \Big[\gamma_\mu(1+\gamma_5) \otimes \gamma^\mu(1+\gamma_5) \\
- 4|Q_{\rm e}| s_{\rm W}^2\, \kappa_{\rm e}\, \gamma_\mu \otimes \gamma^\mu(1+\gamma_5) \\
- 4|Q_{\rm f}| s_{\rm W}^2\, \kappa_{\rm f}\, \gamma_\mu(1+\gamma_5) \otimes \gamma^\mu \\
+ 16|Q_{\rm e}Q_{\rm f}| s_{\rm W}^4\, \kappa_{\rm ef}\, \gamma_\mu \otimes \gamma^\mu \Big]
\end{aligned} \tag{12}$$

Note that apart from the Z propagator, the gauge boson masses are defined according to the running width prescription (un-barred symbols) instead of the pole scheme definition (barred symbols). As a result the form factors κ_{e}, κ_{f}, κ_{ef} can differ from the corresponding form factors $\overline{\kappa_{\mathrm{e}}}$, $\overline{\kappa_{\mathrm{f}}}$, $\overline{\kappa_{\mathrm{ef}}}$ in the pole scheme. In the following, the relation between the two sets of quantities will be worked out.

End of quote from hep-ph/0608099

The amplitude may be further rewritten, in order to introduce the familiar couplings v_f, a_f, which now will cover the loop corrections:

$$\mathcal{A}_Z^{eff}(s,t) \sim i\, e^2 \frac{\chi_Z(s)}{s} a_e a_f \rho_{ef}(s,t) \Big(\gamma_\mu\gamma_5 \otimes \gamma_\mu\gamma_5 \tag{13}$$

$$+ \frac{v_f}{a_f}\gamma_\mu \otimes \gamma_\mu\gamma_5 + \frac{v_e}{a_e}\gamma_\mu\gamma_5 \otimes \gamma_\mu + \frac{v_{ef}}{a_e a_f}\gamma_\mu \otimes \gamma_\mu \Big). \tag{14}$$

Here, we made the choice that the axial couplings remain Born like,

$$a_e = -\frac{1}{2}, \tag{15}$$

$$a_f = \pm\frac{1}{2}. \tag{16}$$

This choice means that the axial couplings remain to be real constants here, and that the (axial $\times$ axial) radiative corrections coming from a product of two vertices like (28) will be collected in the definition of ρ_{ef},

$$\delta\rho_{ef}^{Z\bar{e}eZ\bar{b}b} = \frac{g_{\mathrm{A}}^{e}(k^2)_{k^2=M_Z^2}}{a_e} \frac{g_{\mathrm{A}}^{f}(k^2)_{k^2=M_Z^2}}{a_f}, \tag{17}$$

two issues related to the photon I

There are **two issues related to the photon** to be solved accurately:

Ph-1 Photon exchange amplitude in the s-channel with running α_{QED}
Discussed here

Ph-2 Real emissions
Discussed later

Concerning photon issue PH-1:

Already in Born approximation, the photon exchange contributions as a part of the Z amplitude lead to non-factorization. The effect is numerically not small and has to be taken into account.

There are 3 options to solve photon issue PH-1:

Ph-1 A – Take two interfering amplitudes, one is just the photon.
Is not conform with the S-matrix ansatz which knows only ONE pole. See **Warning** at next slide.
Is experimentally preferred.

two issues related to the photon II

Conventionally, one works with **two interfering amplitudes** as it is described in detail in the publications on the ZFITTER project [38, 28, 37, 7, 9].

Ph-1 B – Put the complete photon amplitude into the Z amplitude $\gamma \otimes \gamma$ structure.
Conforms with the S-matrix ansatz which knows only ONE pole.
But is experimentally disfavoured.
This was exemplified in [39].

Ph-1 C – Split the non-factorizing v_{ef}-term into 2 terms,

$$[v_{ef}]\gamma \otimes \gamma = [v_e\ v_f]\gamma \otimes \gamma + [v_{ef} - v_e\ v_f]\gamma \otimes \gamma,$$

Put $(v_e v_f)\gamma \otimes \gamma$ into the Z amplitude, which is now factorizing, and the $(v_{ef} - v_e v_f)\gamma \otimes \gamma$ may be added (with correct pre-factor) to the photon amplitude.
Again we have two interfering amplitudes and are not conform with the S-matrix idea.
See **Warning** at next slide.

This modification of the photon amplitude is **appropriate when calculating MC programs** where the couplings Q_f, v_v, a_v are assumed to maybe running, but also assumed to factorize.

two issues related to the photon III

Warning

In the S-matrix approach there is only ONE amplitude with only ONE pole at $s_0 = M_Z^2 - iM_Z\gamma_Z$, and no pole at $s_\gamma = 1/s$!

Solution, 3 Feb. 2015

Look at the 2 amplitudes, where only the second is a Laurent series:

$$\mathcal{M}^i(s) = \frac{R_\gamma^i}{s} + \frac{R_Z^i}{s - s_Z} + F^i(s), \quad i = 1, \dots 4. \tag{18}$$

(Beware: Eqn. (18) is mathematically not consistent $\rightarrow$ Robin Stuart 1991 and also Böhm/Sato 2004 [29])

and

$$\mathcal{M}^i(s) = \frac{R_Z^i}{s - s_Z} + F^i(s), \quad i = 1, \dots 4. \tag{19}$$

two issues related to the photon IV

$$
\begin{aligned}
\frac{R^i_\gamma(s)}{s} &= \frac{\sum_{n=0}^{\infty} R^i_n (s-s_0)^n}{s} \\
&= \frac{\sum_{n=0}^{\infty} R^i_n (s-s_0)^n}{s_0-(s_0-s)} \\
&= \sum_{n=0}^{\infty} R^i_n (s-s_0)^n \, \frac{1}{s_0} \, \frac{1}{1-\frac{s_0-s}{s_0}} \\
&= \sum_{n=0}^{\infty} R^i_n (s-s_0)^n \, \frac{1}{s_0} \left[1 + \frac{s_0-s}{s_0} + \left(\frac{s_0-s}{s_0}\right)^2 \cdots \right]
\end{aligned}
\tag{20}
$$

The term $R^i_\gamma(s)/s$ is part of the the background term $B(s)$.

- It is useful to sum up a selected part of self-energy insertions in the propagators in order to derive the Breit-Wigner resonance form,

two issues related to the photon V

- It is useful to sum up a selected part of the photonic background of the Z resonance in order to take explicit notice of physically known pieces of the input expressions.

End of the comment on photon exchange.

Copy from PoS LL2016, partly taken from 0608099. Modified here.

In Born approximation everything looks relatively easy. Neglecting here photon exchange, and using a Breit-Wigner resonance Z propagator with a priori mass M_Z and width Γ_Z of the Z boson, one derives a qualitatively good description of the Z line shape close to the peak:

$$\frac{d\sigma^B}{d\cos\theta} \sim G_F^2 \left| \frac{s}{s - M_Z^2 + iM_Z\Gamma_Z} \right|^2 \times \left[(a_e^{B\,2} + v_e^{B\,2})(a_b^{B\,2} + v_b^{B\,2})(1 + \cos^2\theta) + (2a_e^B v_e^B)(2a_b^B v_b^B)(2\cos\theta) \right]. \tag{21}$$

Symmetric or anti-symmetric integration over $\cos\theta$ allows to determine the two independen contributions. One of them is the total cross section,

$$\sigma_{tot}^B \equiv \int_{-1}^{1} d\cos\theta \frac{d\sigma^B}{d\cos\theta} \sim \left| \frac{s}{s - M_Z^2 + iM_Z\Gamma_Z} \right|^2 G_F^2 \left(a_e^{B\,2} + v_e^{B\,2} \right) \left(a_b^{B\,2} + v_b^{B\,2} \right) \sim \Gamma_e^B \Gamma_b^B,$$

and the other one the forward-backward asymmetry,

$$\sigma^B_{FB} \equiv \left[\int_0^1 - \int_{-1}^0\right] d\cos\theta \frac{d\sigma^B}{d\cos\theta} \sim \left|\frac{s}{s - M_Z^2 + iM_Z\Gamma_Z}\right|^2 G_F^2 (2a_e^B v_e^B)\ (2a_b^B v_b^B), \quad (22)$$

$$A^B_{FB} \equiv \frac{\sigma^B_{FB}}{\sigma^B_T} = \frac{3}{4} \frac{2a_e^B v_e^B}{a_e^{B\,2} + v_e^{B\,2}} \frac{2a_b^B v_b^B}{a_b^{B\,2} + v_b^{B\,2}} \equiv \frac{3}{4} A_e^B A_b^B. \quad (23)$$

We observe the factorization of σ_T^B into the product of two partial widths,

$$\Gamma_f^B = \frac{G_F M_Z^3}{\sqrt{2}\, 6\pi} c_f \left(a_f^{B\,2} + v_f^{B\,2}\right), \quad (24)$$

and of A^B_{FB} into the product of two asymmetry functions,

$$A_f^B = \frac{2a_f^B v_f^B}{a_f^{B\,2} + v_f^{B\,2}}. \quad (25)$$

In Born approximation it is $a_f^B = \pm\frac{1}{2}$, $Q_e = -1$, c_f the color factor, and

$$\frac{v_f^B}{a_f^B} = 1 - 4|Q_f| \sin^2\theta_W. \quad (26)$$

The vector and axial vector couplings will get loop corrections, which may be calculated from the vertex diagrams $\mathbf{V}_\mu^{Zf\bar{f}}(k^2)$; for the $Z\bar{b}b$-vertex:

$$g_{\rm V}^b(k^2)|_{k^2=M_Z^2} = \frac{1}{2(2-D)k^2}\,{\rm Tr}[\gamma^\mu\, \not{p}_1\, \mathbf{V}_\mu^{Zb\bar{b}}\, \not{p}_2]|_{k^2=M_Z^2}, \tag{27}$$

$$g_{\rm A}^b(k^2)_{k^2=M_Z^2} = \frac{1}{2(2-D)k^2}\,{\rm Tr}[\gamma_5\,\gamma^\mu\, \not{p}_1\, \mathbf{V}_\mu^{Zb\bar{b}}\, \not{p}_2]_{k^2=M_Z^2}. \tag{28}$$

Here, we relate vertex corrections to effective couplings. In reality, realistic cross sections are measured, and one has to relate their couplings to $g_{\rm V}^b(k^2)$ and $g_{\rm A}^b(k^2)$.

Fitting programs like Gfitter are relating "experimental" values of e.g. Γ_b, A_b with their theoretical predictions, e.g. in the standard model [35]. But does this fit the original pseudo-observables? To some approximation, it does, as can be seen in the Born formulae. But one has to control the quantum corrections safely. We know since long how to relate pseudo-observables in a strict way to the loop corrections [36, 28, 37, 7, 9]. The amplitude for e^+e^- annihilation into two massless fermions, and we assume here the final state to be massless, may be described to all orders of perturbation theory by four complex-valued form factors, which depend on the masses and the invariants s and t, and which are chosen here to be $\rho_{ef}, \kappa_e, \kappa_f, \kappa_{ef}$; we quote from [7], eq. (3.3.1):[1]

$$\mathcal{A}_Z^{eff}(s,t) \sim i\, e^2\, 4 I_e^{(3)} I_f^{(3)} \frac{\chi_Z(s)}{s} \rho_{ef}(s,t) \Big\{ \gamma_\mu (1+\gamma_5) \otimes \gamma_\mu (1+\gamma_5) \\ -4|Q_e| s_W^2 \kappa_e(s,t) \gamma_\mu \otimes \gamma_\mu (1+\gamma_5) - 4|Q_f| s_W^2 \kappa_f(s,t) \gamma_\mu (1+\gamma_5) \otimes \gamma_\mu \\ +16|Q_e Q_f| s_W^4 \kappa_{ef}(s,t) \gamma_\mu \otimes \gamma_\mu \Big\}. \tag{29}$$

We use the definitions

$$\chi_Z(s) = \frac{G_F M_Z^2}{\sqrt{2}\, 2\pi\alpha} \rho_Z(s), \tag{30}$$

$$\rho_Z(s) = \frac{s}{s - M_Z^2 + i M_Z \Gamma_Z(s)}. \tag{31}$$

For the complete amplitude one sums over all relevant diagrams so that the form factors are perturbative series:

$$\rho_{ef} = 1 + \delta\rho_{ef} = 1 + \frac{\alpha}{\pi}\delta\rho_{ef}^{(1)} + \cdots , \tag{32}$$

$$\kappa_a = 1 + \delta\kappa_a = 1 + \frac{\alpha}{\pi}\delta\kappa_a^{(1)} + \cdots , \quad a = e, f, ef. \tag{33}$$

Compared to a "naive" notation, we split from the rest of the amplitude the form factor ρ_{ef} multiplicatively. If a diagram is represented by an original set $\{\rho_{ef}, \bar{\kappa}_e, \bar{\kappa}_f, \bar{\kappa}_{ef}\}$, this yields re-definitions for all the κ_a:

$$\kappa_a = \frac{\bar{\kappa}_a}{\rho_{ef}}, \quad a = e, f, ef. \tag{34}$$

The form factors, if introduced as it is done here, may be used for definitions of an effective Fermi constant and three effective weak mixing angles:

$$G_F^{\rm eff} = \rho_{ef}(s,t)\; G_F, \tag{35}$$

$$\sin^2\theta_{W,e}^{\rm eff} = \kappa_e(s,t)\; \sin^2\theta_W, \tag{36}$$

$$\sin^2\theta_{W,f}^{\rm eff} = \kappa_f(s,t)\; \sin^2\theta_W, \tag{37}$$

$$\sin^2\theta_{W,ef}^{\rm eff} = \sqrt{\kappa_{ef}(s,t)}\; \sin^2\theta_W, \tag{38}$$

where

$$\sin^2\theta_W \equiv 1 - \frac{M_W^2}{M_Z^2}. \quad (39)$$

The unique definition of an effective weak mixing angle is lost.
The Breit-Wigner propagator $\rho_Z(s)$ contains a width function which is predicted by perturbation theory. Its calculation deserves special attention, and for the moment the notation $\Gamma_Z(s)$ emphasizes that it originates from summing over self-energies like $\Sigma_Z(s)$.
The amplitude may be further rewritten, in order to introduce the familiar couplings v_f, a_f, which now will cover the loop corrections:

$$\mathcal{A}_Z^{eff}(s,t) \sim i\,e^2\frac{\chi_Z(s)}{s}a_e a_f \rho_{ef}(s,t)\Big(\gamma_\mu\gamma_5 \otimes \gamma_\mu\gamma_5 \quad (40)$$

$$+\frac{v_f}{a_f}\gamma_\mu \otimes \gamma_\mu\gamma_5 + \frac{v_e}{a_e}\gamma_\mu\gamma_5 \otimes \gamma_\mu + \frac{v_{ef}}{a_e a_f}\gamma_\mu \otimes \gamma_\mu\Big). \quad (41)$$

Here, we made the choice that the axial couplings remain Born like,

$$a_e = -\frac{1}{2}, \tag{42}$$

$$a_f = \pm\frac{1}{2}. \tag{43}$$

This choice means that the axial couplings remain to be real constants here, and that the (axial $\times$ axial) radiative corrections coming from a product of two vertices like (28) will be collected in the definition of ρ_{ef},

$$\delta\rho_{ef}^{Z\bar{e}eZ\bar{b}b} = \frac{g_{\rm A}^e(k^2)_{k^2=M_Z^2}}{a_e} \; \frac{g_{\rm A}^f(k^2)_{k^2=M_Z^2}}{a_f}, \tag{44}$$

while the vector couplings are understood to contain radiative corrections. From a final state $Z\bar{b}b$ vertex loop correction, one then gets e.g.:

$$v_e = v_e^B, \tag{45}$$

$$v_b = v_b^B + \delta v_f^{Z\bar{b}b} = v_b^B + g_{\rm V}^b(k^2)_{k^2=M_Z^2}, \tag{46}$$

$$v_{eb} = v_e^B\left(v_b^B + \delta v_{ef}^{Z\bar{b}b}\right) = v_e^B\left(v_b^B + \delta v_f^{Z\bar{b}b}\right). \tag{47}$$

The above situation retains factorization.
From the above definitions we get three relations between the vector couplings and the form factors κ:

$$\frac{v_e}{a_e} = 1 - 4|Q_e|s_W^2\kappa_e, \quad (48)$$

$$\frac{v_f}{a_f} = 1 - 4|Q_f|s_W^2\kappa_f, \quad (49)$$

$$\frac{v_{ef}}{a_e a_f} = \frac{v_e v_f}{a_e a_f} + \Delta_{ef}, \quad (50)$$

with

$$\Delta_{ef} = 16|Q_e Q_f|s_W^4(\kappa_{ef} - \kappa_e\kappa_f). \quad (51)$$

If $\kappa_{ef} - \kappa_e\kappa_f = 0$, there is factorization. Factorization is broken by photonic corrections and by box diagrams, while it is respected by weak vertex corrections and self-energies. Having defined the amplitude, one may calculate, with standard text book methods, a $2 \to 2$ cross section. For unpolarized scattering one gets [28]:

$$\frac{d\sigma^{eff}}{d\cos\theta} = \frac{\pi\alpha^2}{2s}|\chi_Z(s)|^2\left[(1+\cos^2\theta)k_T + 2\cos\theta k_{FB}\right]. \quad (52)$$

The symmetric part depends on

$$k_T = |\rho_{ef}|^2 \left[|a_e a_f|^2 + |v_e a_f|^2 + |a_e v_f|^2 + |v_{ef}|^2\right] \tag{53}$$
$$= |\rho_{ef}|^2 |a_e|^2 |a_f|^2 \left[(1+|\frac{v_e}{a_e}|^2)(1+|\frac{v_f}{a_f}|^2) + \Delta_T\right],$$

with

$$\Delta_T = |\Delta_{ef}|^2 + 2\Re e\left(\frac{v_e}{a_e}\frac{v_f}{a_f}\Delta_{ef}^*\right). \tag{54}$$

Assuming factorization, this becomes

$$k_T = |\rho_{ef}|^2 \left[(|a_e|^2 + |v_e|^2)(|a_f|^2 + |v_f|^2)\right], \tag{55}$$

and finally, neglecting additionally the imaginary parts of v_e and v_f (and of Δ_T):

$$k_T = |\rho_{ef}|^2 \left[(a_e^2 + v_e^2)(a_f^2 + v_f^2)\right]. \tag{56}$$

This is the formula usually applied to analyses.

Similarly, for the anti-symmetric cross section part:

$$k_{FB} = |\rho_{ef}|^2 a_e a_f \Re e(v_e v_f^* + v_{ef}) \tag{57}$$
$$= a_e a_f \left(2\Re e \left[\frac{v_e}{a_e} \right] 2\Re e \left[\frac{v_f}{a_f} \right] + \Delta_{FB} \right),$$

with

$$\Delta_{FB} = 2\Re e \Delta_{ef}, \tag{58}$$

and after again neglecting non-factorizing terms and imaginary parts:

$$k_{FB} = 2|\rho_{ef}|^2 (2a_e v_e)\ (2a_f v_f). \tag{59}$$

The cross section formula (52) is the exact result from the amplitude square and averaging over the initial final helicity states.
So far, everything is completely independent of how we derived the form factors:

- As a Born (or effective Born) amplitude
- A la ZFITTER ($\rightarrow$ 1.5 loops) – numerical discussion see Bardin 1999 [25]
- A la SM^2 (Standard Model S-Matrix, $\rightarrow$ 2.5 loops) – discussion to be done

Under the assumption that the form factors are independent of the scattering angle (*remember: box diagrams do depend on the scattering angle*), we get for the total cross section and the forward-backward asymmetry:

$$\sigma^0_{tot} = \frac{4\pi\alpha^2}{3s}|\chi_Z|^2\, k_T, \tag{60}$$

$$\sigma^0_{FB} = \frac{\pi\alpha^2}{s}|\chi_Z|^2\, k_{FB}, \tag{61}$$

and the forward-backward asymmetry becomes

$$A^0_{FB} = \frac{3}{4}\,\frac{k_{FB}}{k_T}. \tag{62}$$

If the form factors depend on the scattering angles, as it is the case for corrections from box diagrams, one has to study the numerical effect of that.
Further observables may be introduced for polarized scattering, where the amplitude (30) is taken between helicity projected states. This may be easily investigated following [28].

The loop-corrected asymmetry parameter A_b as defined in (25) will be set in relation to loop-corrected pseudo-observables at $s = M_Z^2$, in terms of the angular integrals $\sigma^0_{FB}, \sigma^0_{tot}$ as defined in (53) and (57):

$$
\begin{aligned}
A_{FB}^{0,\bar{b}b} &= \frac{\sigma^0_{FB}}{\sigma^0_{tot}} \\
&= \frac{3}{4}\,\frac{\Re e[2a_e v_e\ 2a_b v_b + 4\sin^2\theta_W |Q_e Q_b|^2(\kappa_{eb} - \kappa_e\kappa_b)]}{|a_e a_b|^2 + |v_e a_b|^2 + |a_e v_b|^2 + |v_{eb}|^2} \quad + \quad \text{corrections} \\
&= \frac{3}{4}\, A_\mathrm{e}\, A_\mathrm{b} \quad + \quad \text{corrections}.
\end{aligned}
\tag{63}
$$

The first “corrections” are due to neglected angular dependences of the form factors, and the second “corrections” are due to neglected non-factorizations and imaginary parts.

As discussed in detail in [12], as well as in earlier work [10, 11], the weak mixing angle $\sin^2\theta^\mathrm{b}_\mathrm{eff}$ and A_b are determined from the residue R of the leading part of the resonance matrix element $\bar{\mathcal{M}}$ (65). This residue may be determined in a very good and controlled approximation from the renormalized vector and axial vector couplings of the vertices $V_\mu^{Ze\bar{e}}$ and $V_\mu^{Zb\bar{b}}$. To do so, we have to understand how the form factors are composed. Besides the terms from s channel Z boson exchange, they contain terms from s channel photon exchange, from box diagrams (with weak bosons, but also with photon exchanges), vertices, self-energies.

footnote: We remark here that not only self-energies and vertices, but also arbitrary box diagrams may be inserted exactly into the form factors [36, 28, 7].

Some of these terms are enhanced by the resonance form of the transition, others are not. One has to understand some summation of terms which otherwise would explode at $s = M_Z^2$.

We will now discuss shortly the consequences of the fact that we are studying a resonance. Here arises the question, what is the correct and model-independent formulation of the amplitude from the point of view of general quantum field theory? In order to respect general principles – unitarity, analyticity, gauge invariance – one may use the pole scheme [10]. In the pole scheme, one makes the following ansatz for the amplitude $\bar{\mathcal{M}}$ as a function of the scattering energy, or as a function of the corresponding relativistic invariant s. In a sufficiently small neighborhood around the pole position, it is a Laurent expansion with position of the pole s_0 defined by the mass m_Z and its width g_Z,

$$s_0 \quad = \quad m_Z^2 - i m_Z g_Z, \tag{64}$$

and the residue R, plus a background term B. The latter is a Taylor expansion:

$$\bar{\mathcal{M}} \sim \frac{R}{s - m_Z^2 + i m_Z g_Z} + \sum_{n=0}^{\infty} \frac{b_n}{s_0} \left(1 - \frac{s}{s_0}\right)^n . \tag{65}$$

A kind of master formula for the residue term R of the Laurent series for the Z-amplitude is equation (12), together with (13), of [12, 11]:

We mark in red what is the 2-loop term from the final state Zff-vertex

$$\begin{aligned} R = {} & z_e^{(0)} R_{ZZ} z_f^{(0)} + \left[\hat{z}_e^{(1)}(M_Z^2)\, z_f^{(0)} + z_e^{(0)}\, \hat{z}_f^{(1)}(M_Z^2)\right]\left[1 + \Sigma_{ZZ}^{(1)\prime}(M_Z^2)\right] \\ & + \hat{z}_e^{(2)}(M_Z^2)\, z_f^{(0)} + z_e^{(0)}\, \hat{z}_f^{(2)}(M_Z^2) + \hat{z}_e^{(1)}(M_Z^2)\, \hat{z}_f^{(1)}(M_Z^2) \\ & - i M_Z \Gamma_Z \left[\hat{z}_e^{(1)\prime}(M_Z^2)\, z_f^{(0)} + z_e^{(0)}\, \hat{z}_f^{(1)\prime}(M_Z^2)\right], \end{aligned} \tag{66}$$

$$\begin{aligned} R_{ZZ} = {} & 1 - \Sigma_{ZZ}^{(1)\prime}(M_Z^2) \\ & - \Sigma_{ZZ}^{(2)\prime}(M_Z^2) + \left(\Sigma_{ZZ}^{(1)\prime}(M_Z^2)\right)^2 + i M_Z \Gamma_Z\, \Sigma_{ZZ}^{(1)\prime\prime}(M_Z^2) \\ & - \frac{1}{M_Z^4}\left(\Sigma_{\gamma Z}^{(1)}(M_Z^2)\right)^2 + \frac{2}{M_Z^2}\, \Sigma_{\gamma Z}^{(1)}(M_Z^2)\, \Sigma_{\gamma Z}^{(1)\prime}(M_Z^2). \end{aligned} \tag{67}$$

The $\Sigma_{V_1V_2}, V_i = Z, \gamma$ stand for self-energies, and the vertices are defined there as

$$\Gamma[Z_\mu f\bar{f}] \equiv z_{\mathrm{f},\mu} = i\gamma_\mu(v_\mathrm{f} + a_\mathrm{f}\gamma_5), \quad (68)$$

$$\Gamma[\gamma_\mu f\bar{f}] \equiv g_{\mathrm{f},\mu} = i\gamma_\mu(q_\mathrm{f} + p_\mathrm{f}\gamma_5), \quad (69)$$

$$\begin{aligned} \hat{z}_{\mathrm{f},\mu}(k^2) &= i\gamma_\mu \left[\hat{v}_\mathrm{f}(k^2) + \hat{a}_\mathrm{f}(k^2)\gamma_5\right] \\ &\equiv i\gamma_\mu \left[v_\mathrm{f}(k^2) + a_\mathrm{f}(k^2)\gamma_5\right] - i\gamma_\mu \left[q_\mathrm{f}(k^2) + p_\mathrm{f}(k^2)\gamma_5\right] \frac{\Sigma_{\gamma Z}(k^2)}{k^2 + \Sigma_{\gamma\gamma}(k^2)}. \quad (70) \end{aligned}$$

For details of notations, we refer to [12].

Here, $\bar{\mathcal{M}}$ stands for the functional form of the amplitude introduced in (27). Within our formalism, one may write in full generality:

$$\rho_{ef} = \frac{R_r}{s - s_0} + \sum_{n=0}^{\infty} \frac{b_{r,n}}{s_0} \left(1 - \frac{s}{s_0}\right)^n, \tag{71}$$

$$\kappa_e = \sum_{n=0}^{\infty} \frac{b_{e,n}}{s_0} \left(1 - \frac{s}{s_0}\right)^n, \tag{72}$$

$$\kappa_f = \sum_{n=0}^{\infty} \frac{b_{f,n}}{s_0} \left(1 - \frac{s}{s_0}\right)^n, \tag{73}$$

$$\kappa_{ef} = \sum_{n=0}^{\infty} \frac{b_{ef,n}}{s_0} \left(1 - \frac{s}{s_0}\right)^n. \tag{74}$$

Because ρ is chosen to be an overall factor, it is appropriate to include the resonating part of the amplitude here.

footnote: One might, instead, hold a resonating overall factor of the amplitude $\chi_Z(s)$ outside the form factors. This is done in ZFITTER. Then ρ has to be understood as a Taylor series, and if a specific contribution is non-resonating, e.g. because it is due to photon exchange, the first coefficient of ρ would vanish, $b_{r,0} = 0$ (see [39] for more details). From the generic formula (65), one gets all the expressions

for σ_T, A_{FB}, A_{LR} etc. as explained above [1, 2]. As a result, all these quantities σ_A have the same form, but depend on different terms $R_{A,r}$ and $b_{A,f,n}$, which are bi-linear compositions of the coefficients R_r and $b_{f,n}$.
The Breit-Wigner function used here deviates from the Breit-Wigner function as it was used by the LEP collaborations, where $\Gamma_Z(s) = s/M_Z^2\, \Gamma_Z$ was used instead of $\Gamma_Z(s) = g_Z$. The difference is not negligible and amounts to [21]:

$$m_Z = \frac{M_Z}{\sqrt{1+\Gamma_Z^2/M_Z^2}} \approx M_Z - \frac{1}{2}\Gamma_Z^2/M_Z \approx M_Z - 34\text{ MeV}, \tag{75}$$

$$g_Z = \frac{\Gamma_Z}{\sqrt{1+\Gamma_Z^2/M_Z^2}} \approx \Gamma_Z - \frac{1}{2}\Gamma_Z^3/M_Z^2 \approx \Gamma_Z - 1\text{ MeV}. \tag{76}$$

We have expansions both around the pole position s_0 and in the coupling constants α and α_s, and have to assume that α, α_s and g_Z/m_Z and also $1 - s/m_Z^2$ are of the same numerical order. As a consequence, in an electroweak calculation, R is needed to order $\mathcal{O}(\alpha^2)$, the coefficients b_0 to order $\mathcal{O}(\alpha)$, and the b_1 etc. to leading order only. One has to observe that also the quantity g_Z itself is a prediction of the theory, beginning at order $\mathcal{O}(\alpha^2)$.
A further complication comes from the fact that there are not so small higher order photonic corrections. There are two approaches to that. Either one assumes the

photon exchange amplitude as a separate quantity, which interferes with the Z boson amplitude, and takes this correctly into account. This was done in the ZFITTER approach [28, 40, 9]. To the perturbative orders covered by ZFITTER, this was a controlled approach. In general, it might be more consistent to work with only one amplitude and to understand photonic corrections a a part of background. Then, nevertheless it makes sense to calculate those parts of the photonic backgound with a precision needed by experiment, and to separate this from the unknown parts of the background, as was discussed in [41]. The above considerations help to understand the hierarchy of corrections. Weak vertex corrections as well as weak self-energies contribute to R, while all the photonic corrections and also the box diagrams go into the background B. This means that a two-loop calculation for the Z resonance has to include only vertices and self-energies at two loops – these are the factorizing corrections. An immediate consequence is that for the calculation of an asymmetry like A_{FB} close to the Z peak, one needs only the v_e/a_e and v_f/a_f, derived from (self-energy- and) vertex corrections, at two loops, and the other terms with less accuracy.

footnote: Strictly speaking, the A_f is dependent on the scattering channel for which it is measured or calculated. Using e.g. A_e as it is measured from muon pair production for the determination of A_b from A_{FB} as it is measured from $\bar{b}b$ production, one has check that this is consistent to the accuracy aimed at. The photonic corrections, as well as the box terms are not resonating and thus suppressed compared to the resonance residue. As a consequence, all the complicated two-loop boxes are negligible here, while the photonic corrections and the one-loop box terms are well-known and may be considered as a correction. In ZFITTER, this is organized in the various interfaces [37, 40]. The numerical details have been carefully studied in [25], and never again since then.

End of quote from PoS LL2016

[1] The left-projector is in that notations $L = (1 + \gamma_5)/2$, while it is usually $L = (1 - \gamma_5)/2$. We further stress that the notation covers all kinds of contributions, including also box diagrams.

Ansatz for realistic applications I

The analysis of the Z line shape will be based here on the cross section

$$\sigma(s) = \sum_{i=1}^{4} \sigma^i(s) = \frac{1}{4} \sum_{i=1}^{4} s |\mathcal{M}^i(s)|^2, \tag{77}$$

where the sum must be performed over four helicity amplitudes with different residua R_Z^i and functions $F^i(s)$. The result is, including the convolution with *initial state radiation* here:

Ansatz for realistic applications II

$$\sigma_{tot}^{real}(s) = \frac{4}{3}\pi\alpha^2 \int \frac{ds'}{s} \left[\frac{r^\gamma}{s'} + \frac{s' r_{tot} + (s'-M_Z^2) j_{tot}}{(s'-M_Z^2)^2 + M_Z^2\Gamma_Z^2} + background\right] \rho_{tot}^{ini}\left(\frac{s'}{s}\right) + corrections \quad (78)$$

$$\sigma_{FB}^{real}(s) = \pi\alpha^2 \int \frac{ds'}{s} \left[\frac{r^\gamma}{s'} + \frac{s' r_{FB} + (s'-M_Z^2) j_{FB}}{(s'-M_Z^2)^2 + M_Z^2\Gamma_Z^2} + background\right] \rho_{FB}^{ini}\left(\frac{s'}{s}\right) + corrections \quad (79)$$

The radiation connected with initial-final state interferences can be taken into account by an analogue formula to (78) with a slightly more complicated structure [6, 5]:

$$\sigma_{\rm int}(s) = \int ds' \sigma(s, s') \rho_{\rm int}(s'/s). \quad (80)$$

Ansatz for realistic applications III

As instructive examples, we reproduce here the $\mathcal{O}(\alpha)$ approximated radiator functions for initial state radiation for σ_{tot}^{real} and σ_{FB}^{real} [5]:

$$\rho_{tot}^{ini}\left(\frac{s'}{s}\right) \quad \sim \quad H_e(v) = Q_e^2 \frac{\alpha}{\pi}\left(L_e - 1\right) \frac{1+(1-v)^2}{v}, \tag{81}$$

compared to

$$\rho_{FB}^{ini}\left(\frac{s'}{s}\right) \quad \sim \quad h_e(v) = Q_e^2 \frac{\alpha}{\pi}\left(L_e - 1 - \ln\frac{1-v}{\left(1-\frac{v}{2}\right)^2}\right) \frac{1+(1-v)^2}{v} \frac{1-v}{\left(1-\frac{v}{2}\right)^2}. \tag{82}$$

Here it is $L_e = \ln(s/m_e^2)$ and $v = 1 - s'/s$.
The v vanishes in the soft photon limit, and then ρ_{FB} approaches ρ_{tot}.
The unfolding of realistic observables according to (81) and (82) can be performed with the analysis tools TOPAZ0 [42, 43, 44, 45] and ZFITTER. The latter one relies on the work quoted above for ρ_{tot} and ρ_{FB}. Evidently, the result of unfolding depends on the model chosen for the hard process $\sigma_{tot}^0(s')$ or $\sigma_{FB}^0(s')$. This fact is reflected by the various model-dependent so-called interfaces of ZFITTER.

Ansatz for realistic applications IV

Watch: the S-matrix model approach SM^2 was not included into ZFITTER.
The model-independent S-matrix approach without the SM^2 content is realized in the ZFITTER interface SMATASY.
Several collaborations at LEP and at TRISTAN used this interface package successfully.

An interlinking of SMATASY/ZFITTER with an SM^2 2.5 loop-library to be created may be envisaged.

I am in contact with the JINR ZFITTER/SANC group and try to understand if thy would have some interest to start such a project.
The point for them would be to create a frame with ZFITTER's QED part, SMATASY's effective Born part, and a strict complete one-loop SM library (may be created with SANC).

Some details

The correct ansatz for the S-matrix based cross section is:

$$\sigma(s,s') = \frac{1}{8} s' \sum_i \left[\mathcal{M}_i(s)\mathcal{M}_i^*(s') + \mathcal{M}_i^*(s)\mathcal{M}_i(s') \right] . \tag{83}$$

From Leike/Riemann/Rose:

$$\sigma(s) = \sum_A \sigma_A(s), \qquad A = Z, \gamma, F, \gamma Z, ZF, F\gamma, \tag{84}$$

with the contributions:

$$\begin{aligned}
\sigma_Z(s) &= \frac{s r_Z}{|s-s_Z|^2}, & r_Z &= \tfrac{1}{4}\sum |R_Z^i|^2, \\
\sigma_\gamma(s) &= \frac{r_\gamma}{s}, & r_\gamma &= |R_\gamma|^2, \\
\sigma_F(s) &= s r_F(s), & r_F(s) &= \tfrac{1}{4}\sum |F^i(s)|^2, \\
\sigma_{\gamma Z}(s) &= 2\mathrm{Re}\frac{C_\gamma^* C_Z}{s-s_Z}, & C_\gamma &= R_\gamma, \qquad C_Z = \tfrac{1}{4}\sum R_Z^i, \\
\sigma_{ZF}(s) &= 2\mathrm{Re}\frac{s C_{ZF}(s)}{s-s_Z}, & C_{ZF}(s) &= \tfrac{1}{4}\sum R_Z^i F^{i*}(s), \\
\sigma_{F\gamma}(s) &= 2\mathrm{Re}\left[C_\gamma^* C_F(s)\right], & C_F(s) &= \tfrac{1}{4}\sum F^i(s).
\end{aligned}$$

Some details

After making denominators real one remains with the following formula for the line shape:

$$\sigma(s) = \frac{R + (s - M_Z^2) I}{|s - s_Z|^2} + \frac{r_\gamma}{s} + r_0 + (s - M_Z^2) r_1 + \ldots \quad (85)$$

Besides M_Z, Γ_Z, the real constants R, I, r_0 and r_1 are introduced:

$$\begin{aligned}
R &= M_Z^2 \left[r_Z + 2(\Gamma_Z/M_Z) \left(\Im m C_R + M_Z \Gamma_Z \Re e\, e(C_R') \right) \right], \\
I &= r_Z + 2 \Re e\, e C_R, \\
C_R(s) &= C_\gamma^* C_Z + s_Z C_{ZF}(s), \\
r_0 &= M_Z^2 \left[r_F - M_Z \Gamma_Z \Im m(r_F') \right] + \Re e\, e C_r - M_Z \Gamma_Z \Im m C_r', \\
r_1 &= r_F + M_Z^2 \left[\Re e\, e(r_F') - (\Gamma_Z/M_Z) \Im m(r_F') \right] + \Re e\, e C_r', \\
C_r(s) &= C_\gamma^* C_F(s) + C_{ZF}(s).
\end{aligned} \quad (86)$$

The energy-dependent functions C_{ZF}, C_F, r_F, and their (primed) derivatives with respect to s have to be taken at $s = s_Z$. As may be seen, the cross section may be described by only six real parameters as long as one takes into account only the first two terms in the expansion of the functions $F^i(s)$ around $s = s_Z$ and at most terms of the order $(s - M_Z^2)^n, n = 0, 1$ in the cross section parametrization.

Asymmetries

On a Sunday in Summer 1992, I had a discussion with Luciano Maiani in the CERN library.
He had doubt that an analogue to the model-independent ansatz for σ_{tot} might be usefully formulated, especially in view of the QED corrections.

I believed one can do that, and I followed the rule “The proof of the pudding is in the eating” [2]. The result:

$$A(s) = A_0 + A_1 \left(\frac{s}{M_Z^2} - 1\right) + A_2 \left(\frac{s}{M_Z^2} - 1\right)^2 + \ldots \tag{87}$$

$$A_{FB} = \frac{\sigma_{FB}}{\sigma_T}, \qquad A_{pol} = \frac{\sigma_{pol}}{\sigma_T}. \tag{88}$$

The coefficients are in QED-Born approximation:

$$A_0 = \frac{R_A}{R_T}, \tag{89}$$

and

$$A_1 = \left[\frac{J_A}{R_A} - \frac{J_T}{R_T}\right] A_0. \tag{90}$$

Some details

$$\begin{aligned}
R_T &= \kappa^2(a_e^2+v_e^2)(a_f^2+v_f^2) + 2\kappa|Q_eQ_f|v_ev_f\frac{\Gamma_Z}{M_Z}\Im m\frac{\alpha(s)}{\alpha}, \\
R_{FB} &= 3\kappa^2 a_ev_ea_fv_f + \frac{3}{2}\kappa|Q_eQ_f|a_ea_f\frac{\Gamma_Z}{M_Z}\Im m\frac{\alpha(s)}{\alpha}, \\
R_{pol} &= -2\kappa^2(a_e^2+v_e^2)a_\tau v_\tau - 2\kappa|Q_eQ_f|v_ea_\tau\frac{\Gamma_Z}{M_Z}\Im m\frac{\alpha(s)}{\alpha}, \qquad (91) \\
J_A &= 2|Q_eQ_f|\Re e\, e\frac{\alpha(s)}{\alpha}\kappa\kappa_A, \\
\kappa_T &= v_ev_f, \qquad (92) \\
\kappa_{FB} &= \frac{3}{4}a_ea_f, \qquad (93) \\
\kappa_{pol} &= -v_ea_\tau, \qquad (94)
\end{aligned}$$

$$\kappa = \frac{G_\mu}{\sqrt{2}}\frac{M_Z^2}{8\pi\alpha} = 0.3724\left(\frac{M_Z}{91}\right)^2. \qquad (95)$$

QED corrections for asymmetries

In the vicinity of the Z boson peak, asymmetries behave relatively smoothly and may be described by a simple, universal formula [2, 3]

$$A(s) = A_0 + C(s)\, A_1 \left(\frac{s}{M_Z^2} - 1 \right) + \cdots . \tag{96}$$

The QED corrections are contained in the model-independent factor $C(s)$.

QED corrections for asymmetries

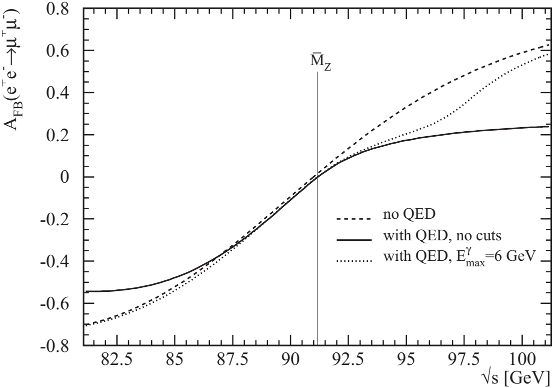


Figure 1: The forward-backward asymmetry for the process $e^+e^- \to \mu^+\mu^-$ near the Z boson peak. From Kirsch/Riemann 1994 [3], license Number: 3557090997554.

Summary

- The S-matrix approach may be strictly applied to the Standard Model.
  $\rightarrow$ Standard Model S-Matrix approach – SM^2 approach
- The degrees of freedom of a cross section are, at minimum:
  M_Z
  Γ_Z
  R – the residue of the Z resonance, *per scattering channel*
  J – the value of the γZ interference, *per scattering channel*
- So we have at least four degrees of freedom of the line shape.
  This deserves at least five data points as a function of s.
- Asymmetries may be described as well as σ_{tot}.
- For an exact numerical analysis of data, an accurate description of QED corrections is mandatory.
  This has been realized by combining SMATASY with ZFITTER, but must be improved by treating the 2.5-loop terms consistently.
- With so much more statistics at the Fcc-ee compared to LEP-1 and LEP-2:

  The SM^2 approach gets a mandatory approach at the Fcc-ee for the Z peak.

Applications

In Leike/S.Riemann/Riemann 1992 [46] correlations are discussed.

For the Z peak position s_{peak}, one may derive the relation:

$$\Delta\sqrt{s_{peak}} = \Delta M_Z + \frac{1}{4}\frac{\Gamma_Z^2}{M_Z}\,\Delta\left(\frac{J_T}{R_T}\right) + \ldots \tag{97}$$

between an uncertainty in M_Z and an uncertainty in the γZ interference.
The latter influences A_1.

Similarly, for a **hypothetical heavy gauge boson** Z', the effects from its virtual exchange transform after a partial fraction decomposition into simple shifts of the γZ interferences [46]:

$$\Delta\left(\frac{J_T}{R_T}\right) = -2\frac{g'^2}{g^2}\frac{M_{Z'}^2}{M_{Z'}^2 - M_Z^2}\frac{(a_e a'_e + v_e v'_e)(a_f a'_f + v_f v'_f)}{(a_e^2 + v_e^2)(a_f^2 + v_f^2)}, \tag{98}$$

Correlations

From Phys. Rept. 2006, section 2 [47]:
The extra free parameter j_{had}^{tot} is strongly anti-correlated with m_Z, resulting in errors on m_Z enlarged by a factor of almost three, as is observed in the existing S-matrix analyses of LEP-I data [77]. The dependence of m_Z on j_{had}^{tot} is given by:

$$j_{had}^{tot} = \frac{Gm_Z^2}{\sqrt{2}\pi\alpha(m_Z^2)} Q_e g_{V_e} \cdot 3 \sum_{q \neq t} Q_q g_{V_q} \tag{99}$$

$$\frac{\partial m_Z}{\partial j_{had}^{tot}} = -1.6\mathrm{MeV}/0.1 \tag{100}$$

...
Improved experimental constraints on the hadronic interference term are obtained by including measurements of the hadronic total cross-section at centre-of-mass energies further away from the Z pole than just the off-peak energies at LEP-I. Including the measurements of the **TRISTAN** *collaborations at KEK,* **TOPAZ** *[78] and* **VENUS** *[79], at $q(s) = 58$ GeV,* **the error on j_{had}^{tot} is about** *± 0.1, while its central value is in good agreement with the SM expectation.*

Correlations

From Phys. Rept. 2006, section 2 [47], continued:

Measurements at centre-of-mass energies above the Z resonance at **LEP-II** *[80-83] also provide constraints on j_{had}^{tot}, and in addition test modifications to the interference terms arising from the possible existence of a heavy Z' boson."*

[77] = L3, OPAL 1993 . . . 2003 [48, 49, 50] (see also K. Sachs, L3 [51, 52])
[78] = TOPAZ 1994 [53]
[79] = VENUS 1999 [54]
[80] => correct ref: ALEPH 1996 [55]
[81] = DELPHI 1999 [56]
[82] = L3 1993 . . . 2000 [4, 57, 52]
[83] = OPAL 1997 [58]

Correlations

From Phys. Rept. 2013, App. A [59]:

“In the LEP-I combination the measured values of **the Z boson mass**

$m_Z = 91.1929 \pm 0.0059$ **GeV**

agrees well with the results of the **standard nine parameter fit,**

$[m_Z =]\ 91.1876 \pm 0.0021$ **GeV**,

albeit with a significantly larger error, resulting from the correlation with the large uncertainty on j_{had}^{tot}.
This uncertainty is the dominant source of uncertainty on mZ in the S-Matrix fits.
The measured value of

$j_{had}^{tot} = -0.10 \pm 0.33$

also agrees with the prediction of the SM,

$[j_{had}^{tot} =]\ 0.2201^{+0.0032}_{-0.0137}$.”

Correlations

We miss an analysis of the LEP-2 data in terms of j_{had}^{tot}, but see Holt 2001 [60] and Sachs 2003 [51].

Including more measurements from LEP*II* solves this problem, reducing the correlation. The final result of $M_Z = 91\ 186.9 \pm 2.3$ MeV [8] is in very good agreement with the result of the standard lineshape fit $M_Z = 91\ 187.6 \pm 2.1$ MeV [9] with only slightly increased error.

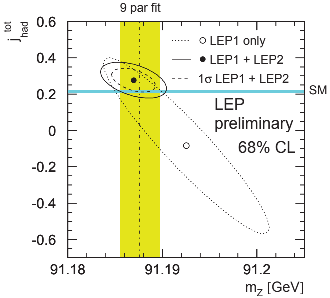


Figure 2: Correlation between the mass of the Z and j_{had}^{tot}. Results are shown for LEP*I* data only and for a combined fit to LEP*I* and LEP*II* data. The yellow band indicates the 1 σ error from the 9 parameter fit.

Figure 2: K. Sachs, "Standard model at LEP II", talk held at Moriond 2001, fig. 2 [51]

Correlations

LEP experiments use cross-section and forward-backward asymmetry results from $\sqrt{s} \sim \mathrm{M_Z}$ and LEP II. OPAL and L3 have reported preliminary results which are given in Table 1, and are compared to the value obtained by VENUS [9] using data at $\sqrt{s} \sim 60$ GeV and preliminary LEP I S-Matrix results. The results are consistent with each other, and with the $\mathcal{SM}$ prediction $\mathrm{j}_{\mathrm{had}}^{\mathrm{tot}} = 0.22$.

Expt	Data	$\mathrm{j}_{\mathrm{had}}^{\mathrm{tot}}$
L3:	LEP I + LEP II	0.30 ± 0.10
OPAL:	LEP I + LEP II	0.21 ± 0.12
VENUS:	VENUS + LEP I	0.20 ± 0.08

Table 1: Measurements of $\mathrm{j}_{\mathrm{had}}^{\mathrm{tot}}$

Figure 3: P. Holt, "Fermion pair production above the Z^0 resonance", talk held at HEP 2001, table 1 [60]

Fortran programs: ZPOLE and SMATASY/ZFITTER

An older version of the Fortran test package **ZPOLE** (Leike/Riemann, v.0.5, July 1991) is available on request.
It was used for the numerics of [1]

Older ZFITTER versions had an **S-Matrix interface ZUSMAT**.
ZUSMAT was used for analysing the total cross sections, but could not treat asymmetries.

Both codes got later replaced by **SMATASY**, in order to have the full functionality of the **ZFITTER** [7, 8, 9] radiative corrections.

The actual Fortran program for the S-matrix Z line shape approach

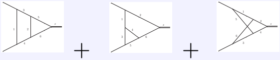
M. Grünewald, S. Kirsch, T. Riemann 1994 [3]
SMATASY v.6.42.01 = SMATA642 (2 June 2005)
available at `https://gruenew.web.cern.ch/gruenew/smatasy.html`

some backup

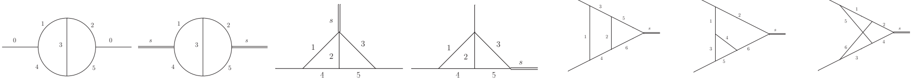
The Z-boson width

$$\Gamma(z \to \bar{b}b) \sim |\mathcal{M}_{1-loop} + \mathcal{M}_{2-loop} + \cdots|^2 + \cdots$$

$$\mathcal{M}_{2-loop} \sim \cdots + \bar{u}\, \mathbf{V}^{Zb\bar{b}}{}_{\mu}\, u\, \epsilon^{\mu} + \cdots$$

$\mathbf{V}_{\mu}^{Zb\bar{b}} \sim \cdots +$  $+ \cdots$

We calculate the bosonic integrals for the 2-loop diagrams for the vertex $\mathbf{V}_{\mu}^{Zb\bar{b}}$



The vector and axial vector components are, with $k^2 = M_Z^2$:

$$g_{\mathrm{V}}^{b}(k^2) = \frac{1}{2(2-D)k^2} \mathrm{Tr}[\gamma^{\mu}\, \not{p}_1\, \mathbf{V}_{\mu}^{Zb\bar{b}}\, \not{p}_2],$$

$$g_{\mathrm{A}}^{b}(k^2) = \frac{1}{2(2-D)k^2} \mathrm{Tr}[\gamma_5\, \gamma^{\mu}\, \not{p}_1\, \mathbf{V}_{\mu}^{Zb\bar{b}}\, \not{p}_2],$$

$\sin^2\theta_{\rm eff}^{\rm b}$

$$\Re e\,\frac{g_{\rm V}^b}{g_{\rm A}^b} \equiv 1 - 4|Q_b|\sin^2\theta_{\rm eff}^{\rm b}$$

$\sin^2\theta_{\rm eff}^{\rm b}$ and $\Delta\kappa_b$

$$\begin{aligned}
\sin^2\theta_{\rm eff}^{\rm b} &= \frac{1}{4|Q_b|}\left(1 - \Re e\,\frac{g_{\rm V}^b}{g_{\rm A}^b}\right) \\
&= \left(1 - \frac{M_W^2}{M_Z^2}\right)(1 + \Delta\kappa_b) \\
&= 0.281 \pm 0.016 \quad \text{LEP1 2005 [61]}
\end{aligned}$$

$\Delta\kappa_b$ and A_b

$$\begin{aligned}
A_{\rm b} &= \frac{2\Re e\,\frac{g_{\rm V}^b}{g_{\rm A}^b}}{1 + (\Re e\,\frac{g_{\rm V}^b}{g_{\rm A}^b})^2} \\
&= \frac{1 - 4|Q_b|\sin^2\theta_{\rm eff}^{\rm b}}{1 - 4|Q_b|\sin^2\theta_{\rm eff}^{\rm b} + 8Q_b^2\left(\sin^2\theta_{\rm eff}^{\rm b}\right)^2} \\
&= 0.899 \pm 0.013 \quad \text{LEP1 2005 [61]}
\end{aligned}$$

LEP1 $\to$ LEPEWWG-extended [61], using ZFITTER [9, 40, 8]

Higher-order references for the Zbb vertex

1-loop contributions – Akhundov:1985,Bernabeu:1987,Beenakker:1988,Jegerlehner:1988,Diakonos:1988 [62, 63, 64, 65, 66]

Fermionic electroweak two-loop corrections – Awramik:2008 [67]

$\mathcal{O}(\alpha\alpha_s)$ **QCD corrections** – Djouadi:1987,Djouadi:1988,Kniehl:1990,Kniehl:1991,Djouadi:1993, Czarnecki:1996,Harlander:1998 [68, 69, 70, 71, 72, 73, 74]

partial h.o. corr's. of order $\mathcal{O}(\alpha_t\alpha_s^2)$ – Avdeev:1994,Chetyrkin:1995 [75, 76]

partial h.o. corr's. of order $\mathcal{O}(\alpha_t\alpha_s^3)$ – Schroeder:2005,Chetyrkin:2006,Boughezal:2006 [77, 78, 79]

partial h.o. corr's. of order $\mathcal{O}(\alpha^2\alpha_t)$ **and** $\mathcal{O}(\alpha_t^3)$ – vanderBij:2000,Faisst:2003 [80, 81]

Standard Model prediction of M_W **from the Fermi constant** G_μ – Awramik:2003 [82].

Further references

Cite for the Z boson total width also Andrei Kataev (1992): [83].
Awramik:2006 [12], Freitas:2013 [84], Freitas:2014h [16], Awramik:2002 [85], Onishchenko:2002[86], Freitas:2002 [87], Bardin:1988 [21].

Present official numerical values: See Particle Data Group 2014 [88].

Lacking: Bosonic 2-loop vertex contributions to the various partial Z **vertices – have more scales than the fermionic 2-loop contributions**
Added 01/2018:
See Dubovyk, Freitas, Gluza, Riemann, Usovitsch 2016 [89]
Dubovyk, Freitas, Gluza, Riemann, Usovitsch 2018: Preliminary results presented by J. Gluza at this workshop [90], to be published.

Some backup slides I

boson
The reaction

$$e^+e^- \to (\gamma, Z) \to f^+f^- \ + \ (n\,\gamma) \tag{101}$$

allows to study the Z boson, its mass M_Z, its width Γ_Z, its couplings, and potentially deviations from the Standard Model.

The numerical outcome depends on the model applied.
See experiences with constant and s-dependent Z width

$\to$ Bardin/Leike/Riemann/Sachwitz 1988 [21]

Also: Berends/Burgers/Hollik/v.Neerven 1988 [22]

Unfolding of *Realistic Observables* is needed in order to get *Pseudo Observables*.

$\to$ e.g.:
Borrelli/Consoli/Maiani/Sisto 1990 [23]

Later: Bardin/Passarino 1999 [24], Bardin/Grünewald/Passarino 1999 [25], Passarino 2003 [26], Passarino 2013 [27] and refs. therein.
???????

Some backup slides II

Close to the Z-boson peak:

$$\frac{d\sigma}{d\cos\theta} \sim G_F^2 \left| \frac{s}{s - M_Z^2 + iM_Z\Gamma_Z} \right|^2 \left[(a_e^2 + v_e^2)(a_b^2 + v_b^2)(1 + \cos^2\theta) + (2a_e v_e)(2a_b v_b)(2\cos\theta) \right]$$

Symmetric integration over $\cos\theta$

$$\sigma_T = \int_{-1}^{1} d\cos\theta \frac{d\sigma}{d\cos\theta}$$
$$\sim \left| \frac{s}{s - M_Z^2 + iM_Z\Gamma_Z} \right|^2 G_F(a_e^2 + v_e^2) \ \mathbf{G_F(a_b^2 + v_b^2)} \qquad \sim \Gamma_e \ \Gamma_b$$

Some backup slides III

Anti-symmetric integration over $\cos\theta$

$$
\begin{aligned}
A_{F-B} &= \frac{\left[\int_0^1 d\cos\theta - \int_{-1}^0 d\cos\theta\right] \frac{d\sigma}{d\cos\theta}}{\sigma_T} \\
&\sim \frac{2a_e v_e}{a_e^2 + v_b^2} \; \frac{\mathbf{2a_b v_b}}{\mathbf{a_b^2 + v_b^2}} \qquad \sim \; A_e \; \mathbf{A_b}
\end{aligned}
$$

Correct renormalization uses S-matrix notations: $\mathcal{M} \sim R/(s - M_Z^2 + iM_Z\Gamma_Z)$ + Taylor expansion.
It is not my aim here to explain the renormalization of the Z-boson.
Let me just mention:
One has to calculate the $Z \to \bar{f}f$ transition vertex at $s = M_Z^2$ with all the vertex corrections.
For a decay into fermions with small masses - leptons, neutrinos, u, d, c, s, b - the amplitude is:

$$
\begin{aligned}
\mathcal{M} &= \epsilon_\mu \bar{u}\gamma^\mu \left(F_1\gamma_5 + F_2\right) u \\
&= \epsilon_\mu F_1 \bar{u}\gamma^\mu \left(\gamma_5 + F_2\right) u
\end{aligned} \qquad (102)
$$