pySecDec for phenomenological predictions

Sophia Borowka

CERN

European Research Council

Established by the European Commission

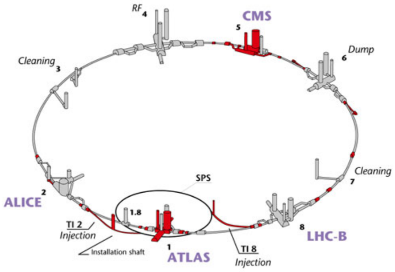
Based on:

Comput.Phys.Commun. 222 (2018) 313-326 **SB, G. Heinrich, S. Jahn, S.P. Jones, M. Kerner, J. Schlenk, T. Zirke**

1709.01266 C. Bogner, SB, T. Hahn, G. Heinrich, S. Jahn, S.P. Jones, M. Kerner, A. von Manteuffel, M. Michel, E. Panzer, V. Papara

CERN mini-workshop, CERN, Jan 11^{th} 2018

Higher order predictions



- we need accurate predictions at NLO and beyond
- we want differential predictions
- we want to take exact heavy quark mass dependences (and H, W, Z) into account
- phenomenological predictions involve complicated multi-loop multi-scale integrals

Techniques for multi-loop multi-scale integrals

- Diverse approaches on the market to compute these
  - Feynman parametrization, Mellin-Barnes representation, differential equations, difference equations, dispersion integrals, integrals in coordinate space, gluing, experimental mathematics, combination of several methods

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- contribution lookup table: **loop** encyclop**edia** C. Bogner, SB, T. Hahn, G. Heinrich, S.P. Jones, M. Kerner, A. von Manteuffel, M. Michel, E. Panzer, V. Papara '17

Loopedia

Loopedia start page

Loopedia

Ex.: Edge list [(1,2),(2,3),(2,3),(3,4)] or 1 2 2 3 2 3 3 4 — Nickel index e11|e|

Enter your graph by its edge list (adjacency list) or Nickel index

or browse:

Loops = any   Legs = any   Scales = any

Fulltext must contain: ______ must not contain: ______

Search   Reset

If you wish to add a new integral to the database, start by searching for its graph first.

The Loopedia Team is C. Bogner, S. Borowka, T. Hahn, G. Heinrich, S. Jones, M. Kerner, A. von Manteuffel, M. Michel, E. Panzer, V. Papara.
Software version of 06 Nov 2017 16:01 UTC. In case of technical difficulties with this site please contact Thomas Hahn or Viktor Papara.
This Web site uses the GraphState library [arXiv:1409.8227] for all graph-theoretical operations
and the neato component of Graphviz for drawing graphs.

Loopedia is free and open to everyone. To acknowledge and support the work put into keeping Loopedia up to date, please cite arXiv:1709.01266.

Loopedia Graph Browser

Results for loops = 2, legs = 4, all scales — Row 6»

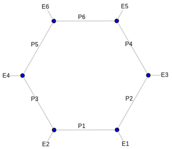
Prev Next Show 4 rows per page Home

Loopedia Record Display

Graph e12|e3|e4|e5|e5|e| — **Masses** 000|00|00|00|00|0|

Edit • : • | Edit • | Browse • : * | Home

Edge list:	(e,0|0) (0,1|0) (0,2|0) (e,1|0) (1,3|0) (e,2|0) (2,4|0) (e,3|0) (3,5|0) (e,4|0) (4,5|0) (e,5|0)
Nickel index:	e12|e3|e4|e5|e5|e|:000|00|00|00|00|0|
Database path:	1/6/6/e12|e3|e4|e5|e5|e|/8/000|00|00|00|00|0|



Propagator P1 m = 0

Propagator P2 m = 0

Propagator P3 m = 0

Propagator P4 m = 0

Propagator P5 m = 0

Propagator P6 m = 0

External Leg E1 $q^2 = 0$

External Leg E2 $q^2 = 0$

External Leg E3 $q^2 = 0$

External Leg E4 $q^2 = 0$

External Leg E5 $q^2 = 0$

External Leg E6 $q^2 = 0$

Choose Configuration

View public records for this configuration ↓BELOW↓ or choose different configuration ↑ABOVE↑

Reference: arXiv:1104.2787
Authors: Lance J. Dixon, James M. Drummond, Johannes M. Henn
Description: The massless one-loop on-shell hexagon integral is computed by use of a differential equation. The integral is IR- and UV-finite. The result involves classical polylogarithms.
Submitter: bogner@math.hu-berlin.de

Record 1502126180.wmqg
added 07 Aug 2017 17:16 UTC
last modified 31 Aug 2017 22:39 UTC

Reference: arXiv:1104.2781
Authors: Vittorio Del Duca, Claude Duhr, Vladimir Smirnov
Description: The massless one-loop on-shell hexagon integral is computed in $D = 6$ by use of Mellin-Barnes and direct integration techniques. The integral is IR- and UV-finite. The result involves

Record 1505802263.T4Pa
added 19 Sep 2017 06:24 UTC

Analytic vs. numerical approach

Analytical:

+ fast evaluation of result

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- intuitive understanding of result harder

PySecDec

Numerical evaluation using sector decomposition

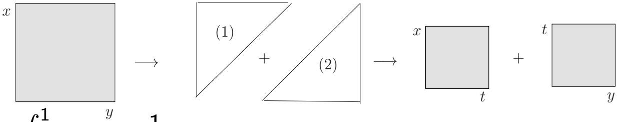
Public codes:

- sector_decomposition (uses GiNaC) Bogner & Weinzierl '07
  supplemented with `CSectors` Gluza, Kajda, Riemann, Yundin '10
  for construction of integrand in terms of Feynman parameters
- FIESTA* (uses Mathematica, C) A.V. Smirnov, V.A. Smirnov, Tentyukov '08 '09, A.V. Smirnov '13, A.V. Smirnov '15
- (PY)SECDEC* (uses Python, C++, FORM)
  Carter & Heinrich '10; SB, Carter, Heinrich '12; SB & Heinrich '13; SB, Heinrich, Jones, Kerner, Schlenk, Zirke '15; SB, Heinrich, Jahn, Jones, Kerner, Schlenk, Zirke '17

*Multi-scale integrals not limited to the Euclidean region
SB, J. Carter & G. Heinrich '12; A.V. Smirnov '13

The method of sector decomposition

- Idea and method of sector decomposition pioneered by
  Hepp '66, Denner & Roth '96, Binoth & Heinrich '00



$$\int_0^1 dx_1 \int_0^1 dx_2 \; \frac{1}{(x_1+x_2)^{2+\epsilon}}$$

$$= \int_0^1 dx_1 \int_0^1 dx_2 \; \frac{1}{(x_1+x_2)^{2+\epsilon}} (\theta(x_1-x_2)+\theta(x_2-x_1))$$

$$= \int_0^1 dx_1 \int_0^{x_1} dx_2 \; \frac{1}{(x_1+x_2)^{2+\epsilon}} + \int_0^1 dx_2 \int_0^{x_2} dx_1 \; \frac{1}{(x_1+x_2)^{2+\epsilon}}$$

$$= \int_0^1 dx_1 \int_0^1 dt \; \frac{x_1}{(x_1+x_1 t)^{2+\epsilon}} + \int_0^1 dx_2 \int_0^1 d\tilde{t} \; \frac{1}{x_2^{1+\epsilon}(\tilde{t}+1)^{2+\epsilon}}$$

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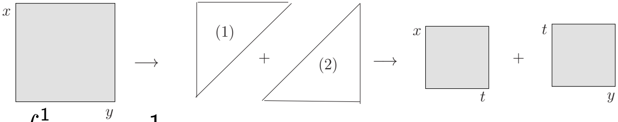

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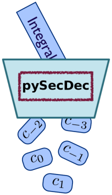
- iterative sector decomposition is highly automatable

pySecDec use cases

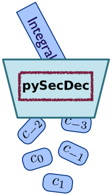


Feynman integral

parametric integral

- compute single
  - **multi-loop** multi-scale integrals
  - general **parametric** functions

pySecDec use cases



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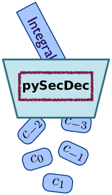
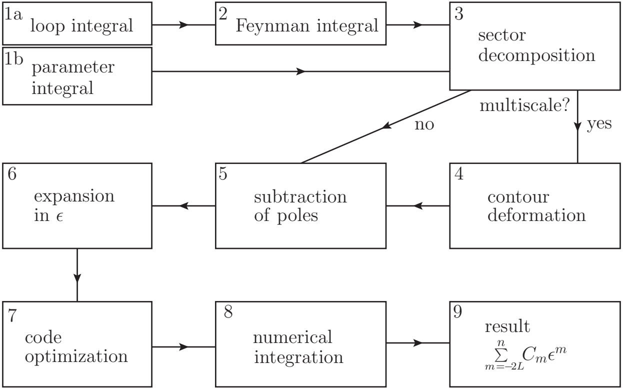


Feynman integral

parametric integral

- compute single
  - **multi-loop** multi-scale integrals
  - general **parametric** functions
- generate library of integrals
- use algebra package for symbolic manipulations on integrals

Operational sequence of the program pySecDec



numerical integration: CUBA library Hahn '04, CQUAD GSL library

pySecDec toolbox

Primary objective:

- facilitate generation of integral libraries

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Improved structure:

- dependences on open-source libraries only
  NUMPY www.numpy.org, SYMPY www.sympy.org,
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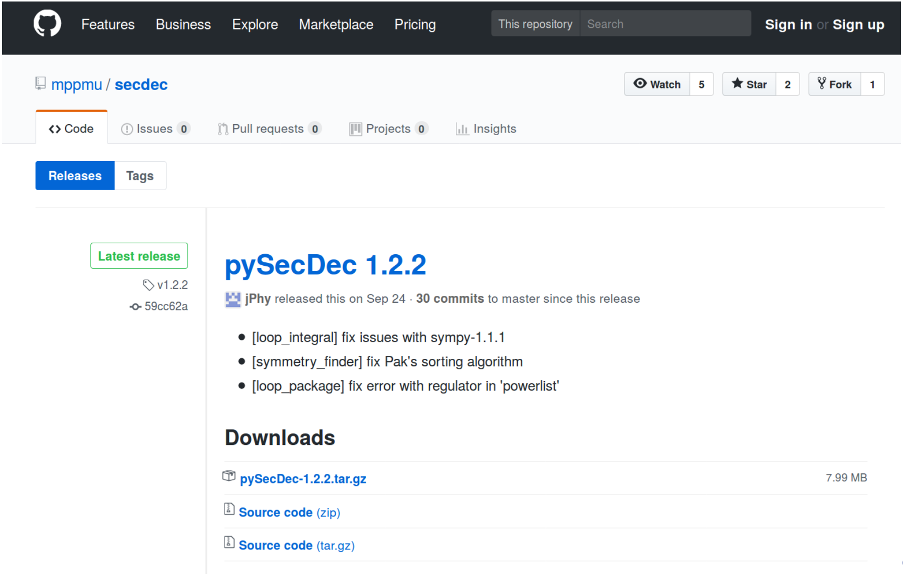
More power:

- multiple regulators allowed
- iterated remapping of singularities on integration boundary
- decreased number of iterated sectors by finding graph isomorphisms
- no distinction between loop & parametric functions
- interface for evaluation of whole amplitude

Download and Usage

Download pySecDec: https://github.com/mppmu/secdec



Install pySecDec

- **Install**:
  tar xzvf pySecDec-1.2.2.tar.gz
  cd pySecDec-1.2.2
  make

- **Prerequisites**:
  - python 2.7 or 3
  - python libraries numpy & sympy
  - C++ compiler

- **Optional prerequisites**:
  - geometric decomposition strategies:
    NORMALIZ Bruns, Ichim, Roemer, Soeger '12
  - NEATO http://www.graphviz.org

- **All other dependences are shipped for easier installation**

Input for pySecDec

- no input files needed, direct python interface
- many usage examples provided, e.g.

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```
#! /usr/bin/env python
from pySecDec.loop_integral import loop_package
import pySecDec as psd

li = psd.loop_integral.LoopIntegralFromGraph(
internal_lines = [['m',[3,4]],['m',[4,5]],['m',[3,5]],[0,[1,2]],[0,[4,1]],[0,[2,5]]],
external_lines = [['p1',1],['p2',2],['p3',3]],

replacement_rules = [
                        ('p1*p1', 0),
                        ('p2*p2', 0),
                        ('p3*p3', 's'),
                        ('p4*p4', 0),
                        ('p1*p2', 's/2'),
                        ('p2*p3', '-s/2'),
                        ('p1*p3', '-s/2'),
                        ('m**2', 'msq')
                    ]
)


Mandelstam_symbols = ['s']
mass_symbols = ['msq']
```

pySecDec as a library: example snippet

```
#include "yyyy_bubble/yyyy_bubble.hpp"
#include "yyyy_box6Dim/yyyy_box6Dim.hpp"


/*
 * pySecDec Master Integrals
 */

// one loop bubble
yyyy_bubble::nested_series_t<secdecutil::UncorrelatedDeviation<yyyy_bubble::integrand_return_t>> bubble(yyyy_bubble::real_t uORt)
{
    using namespace yyyy_bubble;

    const std::vector<real_t> real_parameters{uORt};
    const std::vector<complex_t> complex_parameters{};

    // optimize contour
    const std::vector<nested_series_t<yyyy_bubble::integrand_t>> integrands = yyyy_bubble::make_integrands(real_parameters, complex_parameters
        // The number of samples for the contour optimization, the minimal and maximal deformation parameters, and the decrease factor can be
        // optionally set here as additional arguments.
        );

    // add integrands of sectors (together flag)
    const yyyy_bubble::nested_series_t<yyyy_bubble::integrand_t> summed_integrands = std::accumulate(++integrands.begin(), integrands.end(), *int

    // define the integrator
    auto integrator = secdecutil::cuba::Vegas<std::complex<double>>();
    integrator.flags = 2; // verbose output
    integrator.epsrel = 1e-5;
    integrator.epsabs = 1e-7;
    integrator.maxeval = 1e7;

    // integrate
    return secdecutil::deep_apply(summed_integrands, integrator.integrate) * yyyy_bubble::prefactor(real_parameters, complex_parameters);
}
   ...
/*
 * numerical amplitude using pySecDec Master Integrals
 */
secdecutil::Series<secdecutil::UncorrelatedDeviation<std::complex<double>>> yyyy_numerical(double s, double
{
    return -8.*( 1. + (t*t + u*u)/s * box6Dim(t,u) +  (t-u)/s*( bubble(u)-bubble(t) ) );
}
```

Timings

Comparison of pySecDec to SecDec 3 & Fiesta

	pySecDec time (s) (algebraic, num.)	SecDec 3 time (s) (algebraic, num.)	Fiesta time (s) (algeb., num.)
triangle 2L	(40.5, 9.6)	(56.9, 28.5)	(211.4, 10.8)
triangle 3L	(110.1, 0.5)	(131.6, 1.5)	(48.9, 2.5)
box2L6 (Eucl.)	(8.2, 0.2)	(4.2, 0.1)	(4.9, 0.04)
box2L6 (Phys.)	(21.5, 1.8)	(26.9, 4.5)	(115.3, 4.4)
box2L7 (invprop)	(345.7, 2.8)	(150.4, 6.3)	(21.5, 8.8)

4-core Intel(R) Core(TM) i7-4770 CPU @ 3.40GHz

- relative accuracy goal set to 10^{-2}
- algebraic part usually takes longer, but is only done once in (py)SecDec

Applications of SecDec

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$\longrightarrow$ pySecDec is ready for many more applications! $\longleftarrow$

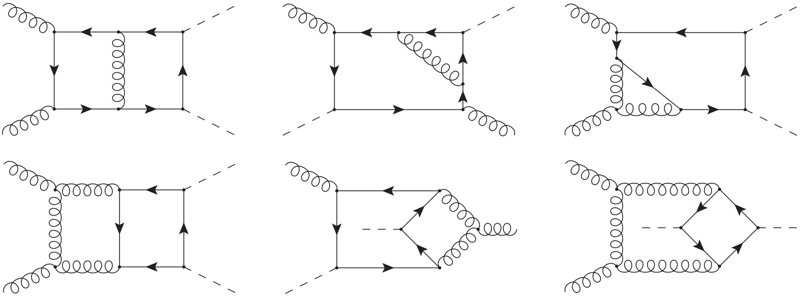
A bit of extra motivation

$gg \to hh$ @ NLO with full top mass dependence

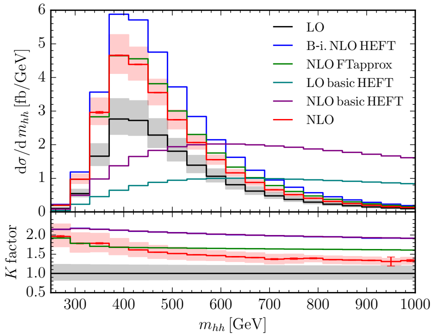
SB, N. Greiner, G. Heinrich, S.P. Jones, M. Kerner, J. Schlenk, U. Schubert, T. Zirke '16

Some of most complicated diagrams entering calculation:

- two-loop integrals, with numerators (4 independent mass scales: $\hat{s}$, $\hat{t}$, m_t^2, m_h^2 or 3 ratios), e.g.



Exact result vs approximations at 100 TeV



- NLO HEFT good approximation for $m_{hh} < 2m_t$
- scale uncertainties of HEFT and FT_{approx} do not enclose central value of full result in m_{hh} tail $\rightarrow$ HEFT breaks down
- fast evaluation using interpolated grid Heinrich, Jones, Kerner, Luisoni, Vryonidou '17; Jones, Kuttimalai '17

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Thank you!