merging & matching
Outline

- Showers
- Showers vs Matrix Elements
- ME corrections
- MEPS merging
- NLOPS matching
- Summary
Shower :: hard scattering / underlying Born xsec^n
Shower :: hard scattering / underlying Born xsec\(^n\)

- Simulation begins with generation of the hard scattering, aka underlying Born, according to LO pQCD:

- Partonic xsec\(^n\):

\[
\hat{d}\sigma_{lo} = \frac{1}{2s} \times M \times d\text{LiPS}
\]

- Hadronic xsec\(^n\):

\[
d\sigma_{lo} = \int dx_+ dx_- \ f_+(x_+, \mu_f) f_-(x_-, \mu_f) \ d\hat{\sigma}_B
\]
Shower :: hard scattering / underlying Born xsec

- Simulation begins with generation of the hard scattering, aka underlying Born, according to LO pQCD:

\[ \text{Process} \rightarrow H \]

- Phase space of the Born process are fully parametrised by some set of so-called `Born variables`.

- The set of Born variables is typically labelled $\Phi_B$

- In $gg \rightarrow H$ Born phase space is 1D: $\Phi_B = y_H$

- Given $y_H$ value you can fully reconstruct $gg \rightarrow H \text{ momenta}$
Simulation begins with generation of the hard scattering, aka underlying Born, according to LO pQCD:

\[ \frac{d\sigma_{\text{LO}}}{d\Phi_B} = B(\Phi_B) \]

In other words the simulation begins by generating a set of \( \Phi_B \) values from distribution:

\[ \frac{d\sigma_{\text{LO}}}{d\Phi_B} = \text{constants} \times f\left(\frac{m_H}{J_S} e^{+y_H}\right) \times f\left(\frac{m_H}{J_S} e^{-y_H}\right) \]

In our \( gg \rightarrow H \) example this means generating \( y_H \) [ = \( \Phi_B \)]
Simulation begins with generation of the hard scattering, aka underlying Born, according to LO pQCD:

LO $gg \rightarrow H$ generated this way look like

\[
\begin{array}{ccccccc}
1 & 21 & \cdots & 0 & 0 & 0 & p_{z,1} & E_1 \\
2 & 21 & \cdots & 0 & 0 & 0 & p_{z,2} & E_2 \\
3 & 25 & \cdots & 0 & 0 & 0 & p_{z,1} + p_{z,2} & E_1 + E_2 \\
\end{array}
\]

They’re 1D
Shower :: hard scattering / underlying Born xsec^n

- Simulation begins with generation of the hard scattering, aka underlying Born, according to LO pQCD:
  - Higgs has zero transverse momentum at LO \([O(\alpha_s^2)]\)
  - Short on usefulness
Shower :: hardest emission $\text{xsec}^n$

attach

parton shower
Shower :: hardest emission $\sigma^{n}$

- Diff $\sigma^{n}$ is total $\sigma^{n}$ times prob of hard scattering ...

$$\text{d}\sigma = B(y_H) \, dy_H$$

$\sigma_{\text{total}}$ times prob to produce Higgs in $[y_H, y_H + dy_H]$

$B(y_H) \equiv \frac{d\sigma}{dy_H}$
probability nothing happens ...

\[
\frac{d\sigma}{d^2y} = B(y_H) \, dy_H
\]

\[
x \exp \left[ - \sum_{l=1,2} \int_{\frac{Q^2}{p_T^2}}^\infty \frac{dz}{2\pi} \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} P_l(z, \phi) \frac{f_l \left( \frac{x_l}{2}, p_T \right)}{z f_l \left( x_l, p_T \right)} \right]
\]

Radiation ph.space parametrized by $p_T$, energy frac $n$, $z$, & $\phi$
Shower :: hardest emission $\times \sec^n$

- prob incoming gluon at $x_i$ due to emission with $p_T$ at $x_i/z$

\[ d\sigma_{ps} = B(y_H) dy_H \]

\[ \times \exp \left[ - \sum_{l=1,2} \int \frac{dP_T^2}{2\pi} \int dz \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} \frac{P_T(z,\phi)}{P_T^2} \frac{f_l(x, P_T)}{zf_l(x, P_T)} \right] \]

\[ \times \sum_{l=1,2} \frac{dP_T^2}{2\pi} \int dz \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} \frac{P_T(z,\phi)}{P_T^2} \frac{f_l(x, P_T)}{zf_l(x, P_T)} \]

prob incoming parton density, $f_l(x, P_T)$ $gg \rightarrow H$
due to `earlier` parton branching at $x_i/z$ in $d\Phi_{rad}$
Shower :: hardest emission xsec$^n$

- after first, hardest, emission fully differential MC xsec$^n$ is:

\[ d\sigma_{PS} = B(y_H) \, dy_H \]

Probability to produce Higgs in \([y_H, y_H + dy_H]\)

\[
\times \exp \left[ - \sum_{l=1,2} \int \frac{dP_T^2}{2\pi} \cdot dz \cdot \frac{d\phi}{2\pi} \cdot \frac{\alpha_s}{2\pi} \cdot \frac{P_l(z,\phi)}{P_T^2} \cdot \frac{f_l(x_H, P_T)}{z \cdot f_l(x_l, P_T)} \right]
\]

Corrects PDF evaluation s.t. PDFs evaluate at $x_i/z$, $p_T$ instead of $x_i$, $m_H$ as in $B(y_H)$

\[
\times \sum_{l=1,2} \int \frac{dP_T^2}{2\pi} \cdot dz \cdot \frac{d\phi}{2\pi} \cdot \frac{\alpha_s}{2\pi} \cdot \frac{P_l(z,\phi)}{P_T^2} \cdot \frac{f_l(x_H, P_T)}{z \cdot f_l(x_l, P_T)} \cdot d\Phi_{rad}
\]

Prob $gg \rightarrow H$ doesn't emit evolving bwds in interval $[m_H^2, p_T^2]$

Splitting $f^i_{g}$ for emission from Born leg $l$

Prob incoming parton density, $f_l(x_i, p_T)$ $gg \rightarrow H$ due to `earlier` parton branching at $x_i/z$ in $d\Phi_{rad}$
Shower :: hardest emission $\text{xsec}^n$

- much of the time this is written in the more digestible form:

$$d\sigma_{ps} = B(\Phi_B) \, d\Phi_B$$

Probability to produce Higgs in $[y_H, y_H+dy_H]$

$$\times \, \exp \left[ - \int_{p_T^1}^{Q^2} d\Phi_{rad} \, \frac{R^{ps}(\Phi_B, \Phi_{rad})}{B(\Phi_B)} \right]$$

Prob $gg \rightarrow H$ doesn’t emit evolving bwds in interval $[m_H^2, p_T^2]$

$$\times \, d\Phi_{rad} \, \frac{R^{ps}(\Phi_B, \Phi_{rad})}{B(\Phi_B)}$$

Prob incoming parton density, $f_i(x, p_T)$ $gg \rightarrow H$ due to `earlier’ parton branching at $x_i/z$ in $d\Phi_{rad}$
Shower :: hardest emission xsec^n :: unitarity
If parton shower momentum map preserves Higgs rapidity in each hard scattering, plotting a histo of $y_H$ of PS events will give back what you started from:

$$\frac{d\sigma_{PS}}{d\Phi_B} = \frac{d\sigma_{LO}}{dy_H}$$

Instructive for later on to see how this comes out in the maths.
Shower :: hardest emission xsec$^n$ :: unitarity

- first, hardest, emission xsec$^n$ again:

$$d\sigma_{PS} = B(\Phi_B) \, d\Phi_B$$

Probability to produce Higgs in $[y_H, y_H+dy_H]$

$$\times \exp \left[ - \int_{p_T^2}^{Q^2} d\Phi_{\text{rad}} \frac{R_{PS}(\Phi_B, \Phi_{\text{rad}})}{B(\Phi_B)} \right]$$

Prob $gg \rightarrow H$ doesn’t emit evolving bwds in interval $[m_H^2, p_T^2]$

$$\times d\Phi_{\text{rad}} \frac{R_{PS}(\Phi_B, \Phi_{\text{rad}})}{B(\Phi_B)}$$

Prob incoming parton density, $f_i(x_i, p_T)$ $gg \rightarrow H$ due to ‘earlier’ parton branching at $x_i/z$ in $d\Phi_{\text{rad}}$
Now integrate out $\Phi_{\text{rad}}$ in 2\textsuperscript{nd} & 3\textsuperscript{rd} lines to get $d\sigma_{PS} / dy_H$:

\[
\int d\Phi_{\text{rad}} \left( \frac{R_{i}^{PS}}{B} \times \exp \left[ -\int_{p_T^2}^{Q^2} d\Phi_{\text{rad}} \frac{R_{i}^{PS}}{B} \right] \right)
\]

\[
= \int_0^{Q^2} dp_T^2 \frac{d}{dp_T^2} \exp \left[ -\int_{p_T^2}^{Q^2} d\Phi_{\text{rad}} \frac{R_{i}^{PS}}{B} \right]
\]

\[
= \exp \left[ -\int_{0}^{Q^2} dp_T^2 \right] - \exp \left[ -\int_{0}^{Q^2} dp_T^2 \right]
\]

\[
= e^{-0} - e^{-\infty} = 1
\]
Shower :: hardest emission $\times \sec^n$ :: unitarity

- Drop that back in $d\sigma_{PS}$ from the previous page:

$$\int d\Phi_{rad} \, d\sigma_{PS} = B(y_H) \, dy_H \times \int d\Phi_{rad} \, \exp \left[ - \int_{P_T^2}^{Q^2} d\Phi_{rad} \, \frac{R^{PS}}{B} \right] \times \frac{R^{PS}}{B} = 1$$

$$\Rightarrow \quad \frac{d\sigma_{PS}}{d\Phi_B} = B(y_H) = \frac{d\sigma_{LO}}{dy_H}$$

shower is unitary $\rightarrow$ conserves $xsec^n$

differential in born vars $\Phi_B$
Shower :: hardest emission xsec^n

run shower
Event myEvent = myShower.hardScattering();

- Simulation begins with generation of the hard scattering, aka underlying Born, according to LO pQCD:

- Higgs has zero transverse momentum at LO \([O(\alpha_s^2)]\)

- Short on usefulness
myShower.emit(myEvent);

- first, hardest, emission xsec\textsuperscript{n} again:

\[
\frac{d\sigma}{d\Phi} = B \ d\Phi \\
\times \exp\left[-\int_{p_T^2}^{Q^2} d\Phi_{\text{rad}} \frac{R^{ps}}{B}\right] \sim 1+O(\alpha_s) \text{ at high } p_T
\]
myShower.emit(myEvent);

- Good: LO for fully inclusive obs and LL for exclusive obs

\( \frac{d\sigma_{PS}}{dy_H} \)

Still correct at LO \( [\alpha_s^2] \) by construction

\( \frac{d\sigma_{PS}}{d\rho_T^H} \)

Soft/Coll approx of ME, at least LL accurate
myShower.output(myEvent);
myShower.output(myEvent);

- From PoV of distns, shower takes events in elements $d\Phi_B$ [bins] and spreads them out across the $\Phi_{rad}$ in the same $d\Phi_B$ element.

\[ \frac{d\sigma_{PS}}{dy_H} \]

LO $[\alpha_s^2]$ accurate by construct,

unitarity + mapping

\[ \frac{d\sigma_{PS}}{dy_H} \]

Still LO $[\alpha_s^2]$ correct by construct

By $2\rightarrow 1$ mom. cons. all evts live in zero $p_T$ bin

\[ \frac{d\sigma_{PS}}{dp_T^H} \]

Soft/Coll approx of ME, at least LL accurate
myShower.output(myEvent);

- Good: LO for fully inclusive obs and LL$_\sigma$ for exclusive obs
myShower.output(myEvent);

- first, hardest, emission $\times \sec^n$ again:

$$d\sigma_{PS} = B(\Phi_B) \, d\Phi_B$$

$$\times \exp \left[ - \int_{p_T^2}^{Q^2} d\Phi_{rad} \, \frac{R_{PS}(\Phi_B, \Phi_{rad})}{B(\Phi_B)} \right]$$

$$\times \, d\Phi_{rad} \, \frac{R_{PS}(\Phi_B, \Phi_{rad})}{B(\Phi_B)}$$
myShower.output(myEvent);

- first, hardest, emission xsec\(^n\) again:

\[
d\sigma_{PS} = B(\Phi_B) \, d\Phi_B \\
\times \exp \left[ - \int_{P_T^2}^{Q^2} d\Phi_{rad} \, \frac{R^{PS}(\Phi_B, \Phi_{rad})}{B(\Phi_B)} \right] \\
\times d\Phi_{rad} \, \frac{R^{PS}(\Phi_B, \Phi_{rad})}{B(\Phi_B)}
\]
myShower.output(myEvent);

- first, hardest, emission $\alpha_s^n$ again:

\[
\frac{d\sigma_{ps}}{d\Phi_B} = B(\Phi_B) \left[ \exp \left( - \int_{p_T^2}^{Q^2} d\Phi_{rad} \frac{R_{ps}(\Phi_B, \Phi_{rad})}{B(\Phi_B)} \right) \right] \times d\Phi_{rad} \frac{R_{ps}(\Phi_B, \Phi_{rad})}{B(\Phi_B)}
\]

\[
\sim 1 + O(\alpha_s)
\]
myShower.output(myEvent);

- By unitarity we had that PS pred\textsuperscript{ns} for Born vars is exactly
  \[ d\sigma_{PS} = B(\Phi_B) \, d\Phi_B \]

- Expanding to \( O(\alpha_s) \) — valid at high \( p_T \) — PS real xsec\textsuperscript{n} is
  \[ d\sigma_{PS} \approx R_{PS}(\Phi_R) \, d\Phi_B \, d\Phi_{\text{rad}} \]
  ... with \( R_{PS} \) the real soft/collinear approx to the HJ xsec\textsuperscript{n}

- PS is LO+ LL for general inclusive observables

- Basic [crude?] desc\textsuperscript{n} of inclusive quantities & high \( p_T \) rad\textsuperscript{n} evts
matrix elements vs showers
ME corrections: shower

Good. Physical behaviour down to $p_T=0$

Good. Soft/Coll ME approx valid & combined with LL resummation

OK. Integrates to LO $\sigma_{total}$ [unitarity]

Bad. Soft/Coll approx of ME

$M_H = 120$ GeV

$\frac{d\sigma}{dp_T}$ vs. $p_T^H$ (GeV)
ME corrections : why not use HJ ME or a ggH NLO calc?
ME corrections: NLO fixed order

\[ \frac{d\sigma}{dp_T^H} \]

- **Good. Fully LO accurate**
- **O(\(\alpha_s^3\)): no soft/coll approx**

- **Bad. Counting powers of \(\alpha_s\) is nonsense below here:** need to sum large log terms
  \[ \sim \frac{1}{p_T^2} \alpha_s^n \ln^{2n-1} \frac{Q^2}{p_T^2} \]
  to all orders in \(\alpha_s\)

**M_H = 120 GeV**
ME corrections: matrix elements vs parton showers

\[ \frac{d\sigma}{dp_T^H} \]

NLO ggH shower

\( M_H = 120 \text{ GeV} \)
matrix elements

parton showers

矩陣 元件

夸克 膠子 陣雨
matrix element correction technique
ME corrections: improve hardest emission beyond soft/coll approx

NLO ggH shower

$M_H = 120$ GeV

Bad. Soft/Coll approx of ME

Good. Fully LO accurate $O(\alpha_s^3)$: no soft/coll approx
ME corrections: improve hardest emission beyond soft/coll approx

- first, hardest, emission xsec$^n$ from the shower:

\[
\frac{d\sigma}{d^3p} = B(\Phi_B) d\Phi_B \\
\times \exp \left[-\int_{P_T^2}^{Q^2} d\Phi_{rad} \frac{R^{PS}(\Phi_B, \Phi_{rad})}{B(\Phi_B)} \right] \\
\times d\Phi_{rad} \frac{R^{PS}(\Phi_B, \Phi_{rad})}{B(\Phi_B)}
\]
ME corrections: improve hardest emission beyond soft/coll approx

- first, hardest, emission xsec\textsuperscript{n} from the shower:

\[
\frac{d\sigma}{dy} = B(\Phi_B) \frac{d\Phi_B}{R_{PS}(\Phi_B, \Phi_{rad})} \times e^{-\int^Q_{p_T} d\Phi_{rad} \frac{R_{PS}(\Phi_B, \Phi_{rad})}{B(\Phi_B)}} \times d\Phi_{rad} \frac{R_{PS}(\Phi_B, \Phi_{rad})}{B(\Phi_B)}
\]
ME corrections: improve hardest emission beyond soft/coll approx

- first, hardest, emission xsec from the shower:

\[ d\sigma_{ps} = B(\Phi_B) \, d\Phi_B \]

\[ \times \exp \left[ - \int_{P_T^2}^{Q^2} \frac{R_{ps}(\Phi_B, \Phi_{rad})}{B(\Phi_B)} \, d\Phi_{rad} \right] \]

\[ \times d\Phi_{rad} \frac{R_{ps}(\Phi_B, \Phi_{rad})}{B(\Phi_B)} \]

\[ \sim 1 + O(\alpha_s) \]
ME corrections: improve hardest emission beyond soft/coll approx

- By unitarity PS pred\(^n\)s for Born vars is exactly
  \[ \sigma_{PS} = B(\Phi_B) \, d\Phi_B \]

- From last page at high \(p_T\) PS real xsec\(^n\) is
  \[ \sigma_{PS} \sim R^{PS}(\Phi_R) \, d\Phi_B \, d\Phi_{rad} \]
  ... with \(R^{PS}\) the real **soft/collinear** approx to the HJ xsec\(^n\)

- PS is LO+ LL for general inclusive observables

  - Basic [crude?] desc\(^n\) of inclusive quantities & high \(p_T\) rad\(^n\) evts
matrix element correction in a nutshell:

replace

\[ R^{\text{PS}} \sim R \]

for hardest emission when doing shower

- By defn $R = R^{\text{PS}}$ if rad$^n$ is soft/coll $\rightarrow$ no diff at low $p_T$
- By defn $R \neq R^{\text{PS}}$ if rad$^n$ isn't soft/coll $\rightarrow$ large diff at high $p_T$
ME corrections: improve hardest emission beyond soft/coll approx

- first, hardest, emission xsec
  from shower with ME corr:

\[
\frac{d\sigma}{dp_s} = B \, d\Phi_B \\
\times \exp \left[ - \int_{p_T}^{Q^2} d\Phi_{\text{rad}} \frac{R}{B} \right] \\
\times d\Phi_{\text{rad}} \frac{R}{B}
\]

- nothing but the same formulas as 5 slides back but \( R \rightarrow R^{PS} \)
ME corrections: improve hardest emission beyond soft/coll approx

- first, hardest, emission $\text{xsec}^n$ from shower with ME corr$^n$:

$$d\sigma_{ps} = B d\Phi_B$$

$$\times \exp \left[ -\int_{p_T^2}^{Q^2} d\Phi_{rad} \frac{R}{B} \right]$$

$$\times d\Phi_{rad} \frac{R}{B}$$

cancels

- nothing but the same formulas as 1-2 slides ago but $R \rightarrow R^{ps}$
ME corrections : improve hardest emission beyond soft/coll approx

- first, hardest, emission xsec\(^n\) from shower with ME corr\(^n\):
  
  \[
  d\sigma_{ps} = \cancel{B} \ d\Phi_B
  \]
  
  \[
  \times \exp \left[ - \int_{p_T^2}^{Q^2} d\Phi_{rad} \frac{R}{B} \right]
  \]
  
  \[
  \times d\Phi_{rad} \frac{R}{B}
  \]
  
  \[
  \sim 1 + O(\alpha_s)
  \]
  
  cancels

- nothing but the same formulas as 1–2 slides ago but R \(\rightarrow R_{PS}^{ps}\)
ME corrections: improve hardest emission beyond soft/coll approx

- Proof of unitarity **exactly** as before, predns for Born vars still:
  \[ \frac{d\sigma}{d\Phi_0} = B(\Phi_0) \, d\Phi_0 \]
  \[ \text{LO } gg \rightarrow H \frac{d\sigma}{dy_H} \]

- From last page at high p_T PS with MEC real xsec^n changes to
  \[ \frac{d\sigma}{d\Phi_0} = R(\Phi_R) \, d\Phi_0 \, d\Phi_{rad} \]
  ... with R the full expression for H+jet xsec^n [previously was R^{PS}]

- PS still LO + LL \([O(\alpha_s^2)]\) for general inclusive observables
  \[ \text{BUT } \text{promoted to } O(\alpha_s^3) \text{ accuracy in the rad}^n \text{ spectrum} \]
  \[ \text{Same desc}^n \text{ of incl obs but substantially better high p_T desc}^n \]
ME corrections: improve hardest emission beyond soft/coll approx

Good. Physical behaviour down to $p_T=0$

Good. Soft/Coll ME approx valid & combined with LL resummation

$M_H = 120$ GeV

NLO ggH shower

OK. Still integrates to LO $xsec^n$ [unitarity]

Bad. Soft/Coll approx of ME
ME corrections: improve hardest emission beyond soft/coll approx

$$d\sigma/dp_T^H$$

$$p_T^H$$ (GeV)

$M_H = 120$ GeV

- **NLO ggH shower + MEC**
- Good. Fully LO accurate $O(\alpha_s^3)$: no soft/coll approx
- OK. Integrates to LO $x\sec^n$ [unitarity]
- Good. Soft/Coll ME approx valid & combined with LL resummat^n
- Good. Physical behaviour down to $p_T=0$
Codes including selection of ME corrections

- W/Z/H prod^n and decay, & top-dec:
  1. Pythia & Pythia8
  2. Herwig & Herwig7

- As above but also ME corr^n for higher jet multiplicities, with possibility to interlink them, i.e. follow WJ ME corr^n for first emission with WJJ ME corr^n for the second emission
  3. Vincia
ME corrections at work: fHerwig Higgs production in gluon fusion

- **LEFT**: shower has soft tail, shower+MEC has soft tail & retains Sudakov shoulder
- **RIGHT**: shower+MEC & fixed order LO HJ ME calc agree very well at high $q_T$

Corcella & Moretti 2004
ME corrections at work: Pythia $W$ hadroproduction

- LEFT: shower FAR too soft, shower+MEC has soft tail & retains Sudakov shoulder
- LEFT: shower+MEC & fixed order LO $WJ$ ME calc agree very well at high $q_T$
- RIGHT: shower+MEC describing data well; shower would be hopeless at high $q_T$

Miu & Sjöstrand 1998
MEPS merging: shower

\[ \frac{d\sigma}{dp_T^J} \]

Good. Physical behaviour down to \( p_T = 0 \)

Good. Soft/Coll ME approx valid & combined with LL resummat\(^n\)

OK. Integrates to LO xsec\(^n\) [unitarity]

Bad. Soft/Coll approx of ME

\( M_H = 120 \text{ GeV} \)

\( p_T^J \text{ (GeV)} \)
Bad. Counting powers of $\alpha_S$ is nonsense below here: need to sum large log terms
$\sim \frac{1}{p_T^2} \alpha_S^n \ln^{2n-1} \frac{Q^2}{p_T^2}$ to all orders in $\alpha_S$
Something rather obvious suggests itself
1. select good *shower* events: $p_T < Q_{\text{cut}}$  \([40 \text{ GeV} \text{ in figure}]\)
MEPS merging first go

1. select good **shower** events: $p_T < Q_{\text{cut}}$  [40 GeV in figure]

2. shower the **HJ ME** events and select good ones: $p_T > Q_{\text{cut}}$
MEPS merging first go

1. select good **shower** events: $p_T < Q_{cut}$  [40 GeV in figure]
2. shower the **HJ ME** events and select good ones: $p_T > Q_{cut}$
3. join samples from 1 & 2 : put events in a file and mix them up
MEPS merging first go : merging scale dependence

- looks OK
- but there is a discontinuity around $Q_{\text{cut}}$
- and I just made up $Q_{\text{cut}} \rightarrow$ unphysical $\rightarrow$ pred$^n$s shouldn’t depend on it
- but they will; even the stupid total inclusive xsec$^n$ will change with $Q_{\text{cut}}$
MEPS merging first go: merging scale dependence

\[ \frac{d\sigma}{dp_T} \]

- integral of this ...

\[ Q_{cut} = 40 \text{ GeV} \]
MEPS merging first go : merging scale dependence

\[ \text{HJ ME shower} \]

\[ Q_{\text{cut}} = 30 \text{ GeV} \]

- is less than the integral of this ...
MEPS merging first go: merging scale dependence

which is less than the integral of this ...
MEPS merging first go: merging scale dependence

- and this ...