New research leading to possible elimination of transverse beam impedance by making beam equipment in the shape of higher order multipoles

Multipolar decomposition

Any function of two variables (x,y) can be approximated by a sum of multipolar terms. This is called “Multipolar decomposition” or “Holomorphic decomposition”.

The more multipolar terms (i.e. the higher the order) the better the approximation.

For each order there are two terms: a normal term and a skew term.

$$f(x,y) = \underbrace{a_0}_{\text{Zero order}} + \underbrace{a_{1,normal} \cdot x + a_{1,skew} \cdot y}_{\text{first order (dipole)}} + \underbrace{a_{2,normal} \cdot \left(\frac{x^2}{2} - \frac{y^2}{2}\right) + a_{2,skew} \cdot (x \cdot y)}_{\text{second order (quadrupole)}} \ldots$$

Here, the terms: a_0 , $a_{1,normal}$, $a_{1,skew}$, $a_{2,normal}$, $a_{2,skew}$ ... are all constants.

Multipolar decomposition

```
normalPotential[n_Integer]:=ComplexExpand[Re[(x+ⅈ y)^n]/n]
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skewPotential[n_Integer]:=ComplexExpand[Im[(x+ⅈ y)^n/n]]
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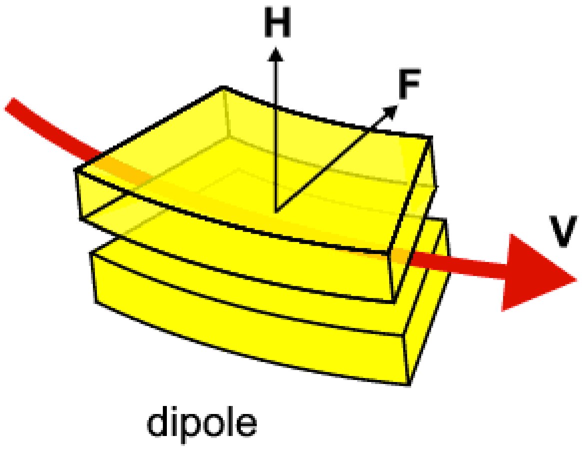
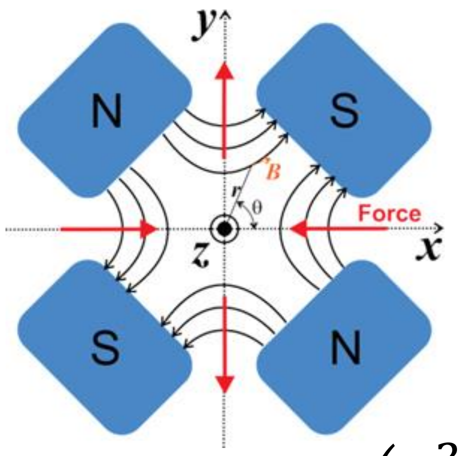
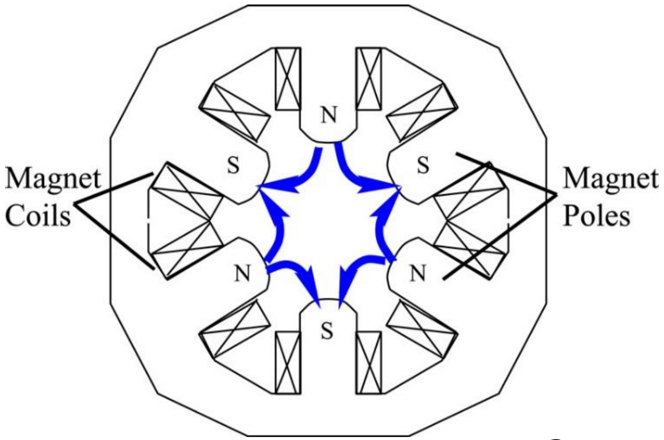
$$f(x,y) = a_0 + a_{1,normal} \cdot x + a_{1,skew} \cdot y + a_{2,normal} \cdot \left(\frac{x^2}{2} - \frac{y^2}{2}\right) + a_{2,skew} \cdot (x \cdot y) \ldots$$

We know multipolar fields from magnet design:







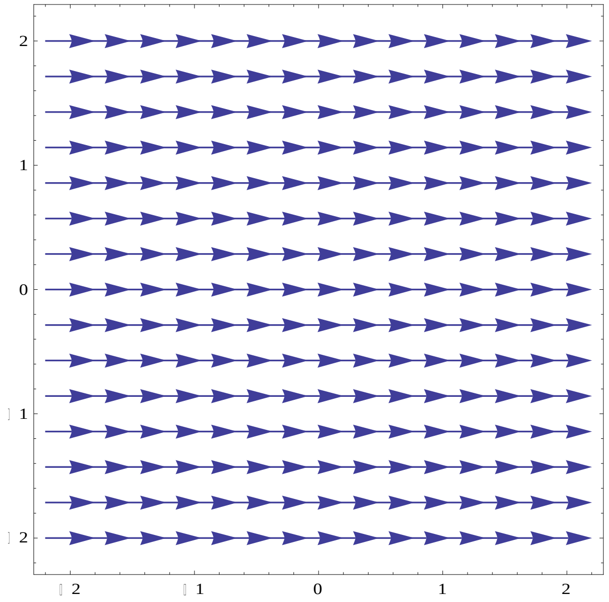
$$f(x,y) = a_{1,normal} \cdot x$$

$$f(x,y) = a_{2,normal} \cdot \left(\frac{x^2}{2} - \frac{y^2}{2}\right)$$

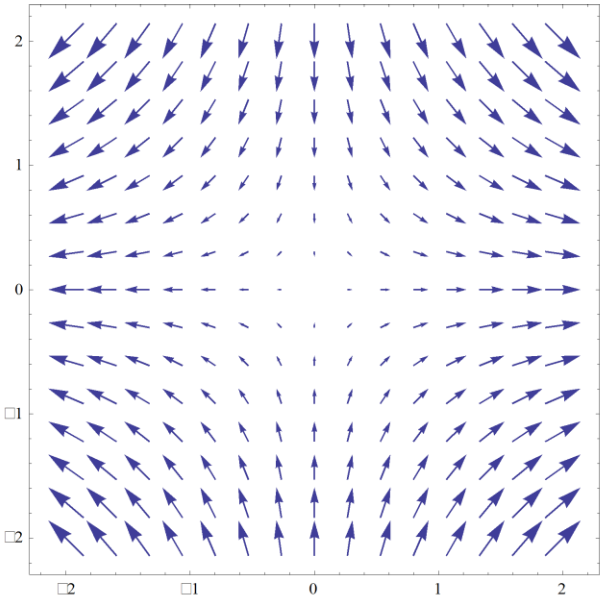
$$f(x,y) = a_{3,normal} \cdot \left(\frac{x^3}{3} - xy^2\right)$$

Multipolar decomposition

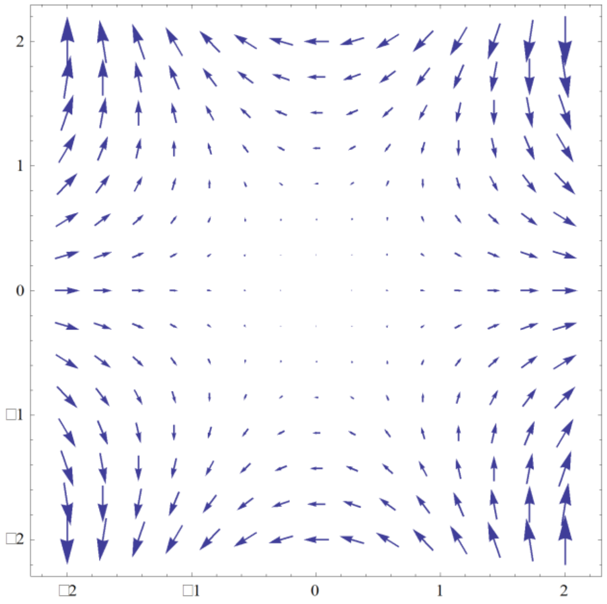
The higher the order of the multipole,
the more the field potentials goes to zero at the position of the beam



Dipole

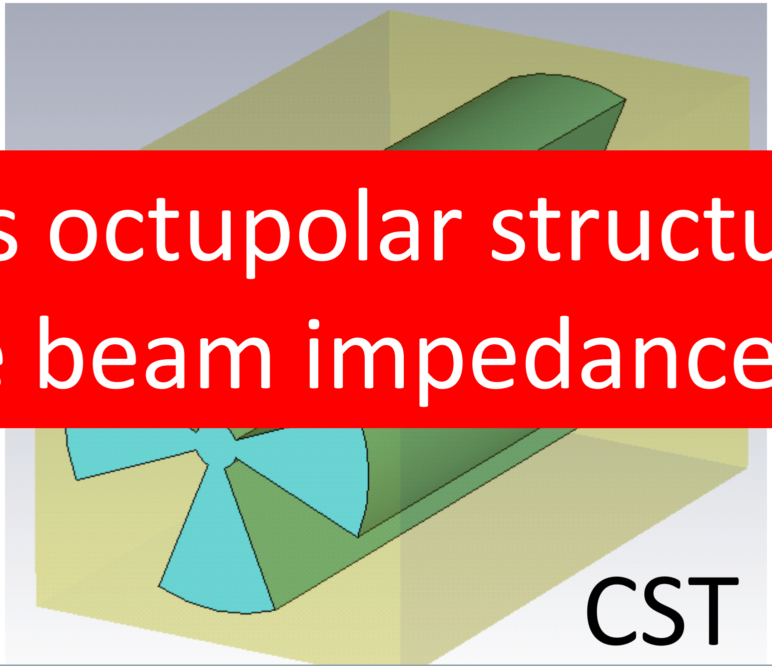


Quadrupole

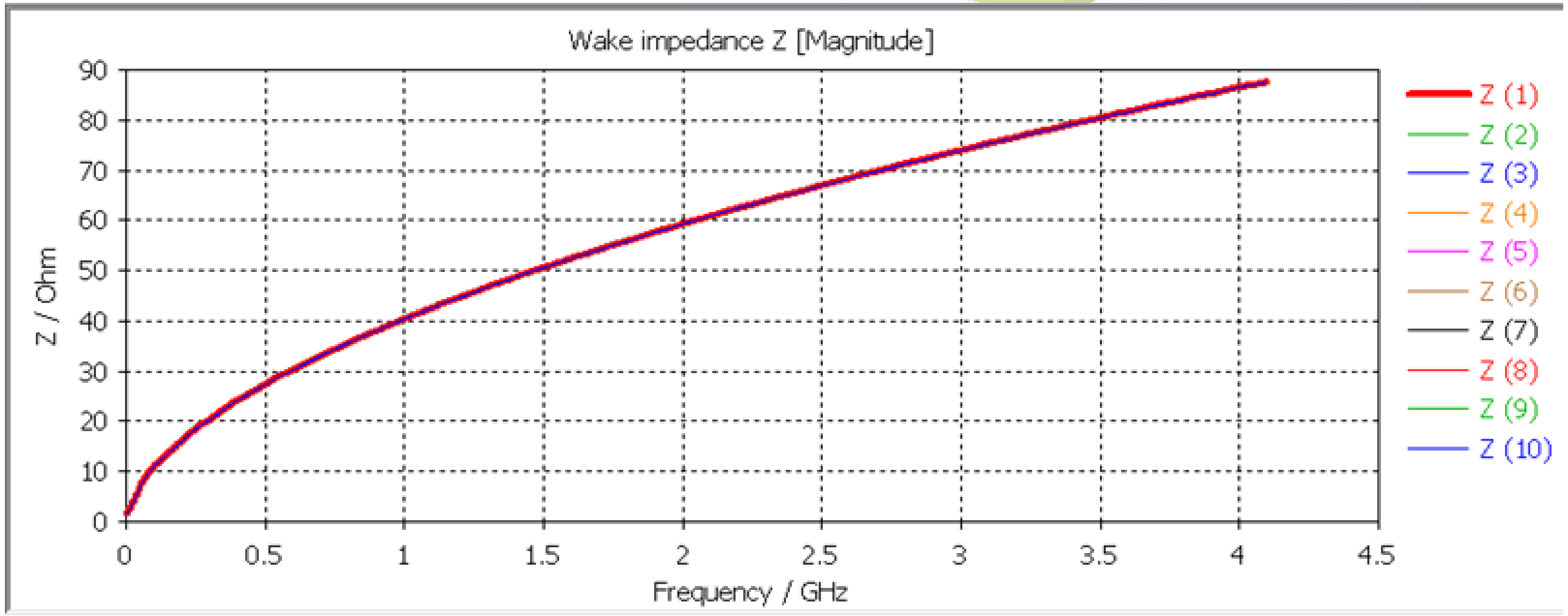


Sextupole

What happens to beam impedance?



With this octupolar structure of a collimator
The transverse beam impedance is zero up to first order



Result Navigator

3D Run ID	xbeam	xtest
1	0	0.0005
2	0	0.001
3	0	0.0015
4	0	0.002
5	0	0.0025
6	0.001	0.0005
7	0.001	0.001
8	0.001	0.0015
9	0.001	0.002
10	0.001	0.0025

References

[1] Measurement of transverse kick in CLIC accelerating structure in FACET Hao Zha, Andrea Latina, Alexej Grudiev

[2] Holomorphic decomposition. John Jowett
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[3] On single wire technique for transverse coupling impedance measurement. H. Tsutsui
http://cds.cern.ch/record/702715/files/sl-note-2002-034.pdf

[4] Longitudinal instability of a coasting beam above transition, due to the action of lumped discontinuities V. Vaccaro
https://cds.cern.ch/record/1216806/files/isr-66-35.pdf